Adaptation of my equations to a subdivided population. Notation, for a quantity Y that depends on two sites ($Y = e, d, Q$):

$$Y_{\text{self}} = Y_{i,i} \tag{1a}$$

$$Y_{\text{in}} = Y_{i,j}, \quad i \text{ and } j \neq i \text{ in the same deme;} \tag{1b}$$

$$Y_{\text{out}} = Y_{i,j}, \quad i \text{ and } j \text{ in different demes.} \tag{1c}$$

For a site i, G_i denotes the deme it is in, and notation $j \in G_i$ means that sites i and j are in the same deme.

The expected frequency of altruists in the population is given by

$$\mathbb{E}[\overline{X}] = p + \delta \frac{p(1-p)}{\mu} \left[\mathsf{b}(\beta^D - \beta^I) - \mathsf{c}(\gamma^D - \gamma^I) \right]. \tag{2}$$

Moran, Birth-Death

$$
\begin{aligned}
\beta_{\text{BD}}^D &= \sum_{k,\ell=1}^{N} \frac{1-\mu}{N} e_{k l} Q_{l k} \\
&= \sum_{k=1}^{N} \frac{1-\mu}{N} \left(e_{\text{self}} + (n-1) e_{\text{in}} Q_{\text{in}} + (N-n) e_{\text{out}} Q_{\text{out}} \right) \\
&= (1-\mu) \left(e_{\text{self}} + (n-1) e_{\text{in}} Q_{\text{in}} + (N-n) e_{\text{out}} Q_{\text{out}} \right).
\end{aligned} \tag{3a}
$$

$$\beta_{\text{BD}}^{I} = \sum_{j,k,l=1}^{N} \left(\frac{d_{lj}}{N} - \frac{\mu}{N^2} \right) e_{kl} Q_{jk}$$

$$= \frac{1}{N} \sum_{j=1}^{N} \left[\left(\sum_{l=1}^{N} d_{lj} e_{jl} - \frac{\mu}{N} \right) + \sum_{\substack{k \in G_j \\ k \neq j}} \left(\sum_{l=1}^{N} d_{lj} e_{kl} Q_{\text{in}} - \frac{\mu}{N} Q_{\text{in}} \right) + \sum_{k \notin G_j} \sum_{l=1}^{N} d_{lj} \left(e_{kl} Q_{\text{out}} - \frac{\mu}{N} Q_{\text{out}} \right) \right]$$

$$= \frac{1}{N} \sum_{j=1}^{N} \left[d_{\text{self}} e_{\text{self}} + (n-1) d_{\text{in}} e_{\text{in}} + (N-n) d_{\text{out}} e_{\text{out}} \right.$$

$$+ \sum_{\substack{k \in G_j \\ k \neq j}} \left(d_{\text{in}} e_{\text{self}} + d_{\text{self}} e_{\text{in}} + (n-2) d_{\text{in}} e_{\text{in}} + (N-n) d_{\text{out}} e_{\text{out}} \right) Q_{\text{in}}$$

$$\left. + \sum_{k \notin G_j} \left(d_{\text{self}} e_{\text{out}} + (n-1) d_{\text{in}} e_{\text{out}} + d_{\text{out}} e_{\text{self}} + (n-1) d_{\text{out}} e_{\text{in}} + (N-2n) d_{\text{out}} e_{\text{out}} \right) Q_{\text{out}} \right]$$

$$- \frac{\mu}{N} \left(1 + (n-1) Q_{\text{in}} + (N-n) Q_{\text{out}} \right)$$

$$= d_{\text{self}} e_{\text{self}} + (n-1) d_{\text{in}} e_{\text{in}} + (N-n) d_{\text{out}} e_{\text{out}}$$

$$+ (n-1) \left(d_{\text{in}} e_{\text{self}} + d_{\text{self}} e_{\text{in}} + (n-2) d_{\text{in}} e_{\text{in}} + (N-n) d_{\text{out}} e_{\text{out}} \right) Q_{\text{in}}$$

$$+ (N-n) \left(d_{\text{self}} e_{\text{out}} + (n-1) d_{\text{in}} e_{\text{out}} + d_{\text{out}} e_{\text{self}} + (n-1) d_{\text{out}} e_{\text{in}} + (N-2n) d_{\text{out}} e_{\text{out}} \right) Q_{\text{out}}$$

$$- \frac{\mu}{N} \left(1 + (n-1) Q_{\text{in}} + (N-n) Q_{\text{out}} \right). \tag{3b}$$

$$\gamma_{\text{BD}}^{D} = 1 - \mu. \tag{3c}$$

$$\gamma_{\text{BD}}^{I} = \frac{1}{N} \sum_{j,k=1}^{N} \left(d_{kj} - \frac{\mu}{N} \right) Q_{jk}$$

$$= \frac{1}{N} \sum_{j=1}^{N} \left[d_{\text{self}} - \frac{\mu}{N} + (n-1) \left(d_{\text{in}} - \frac{\mu}{N} \right) Q_{\text{in}} + (N-n) \left(d_{\text{out}} - \frac{\mu}{N} \right) Q_{\text{out}} \right]$$

$$= d_{\text{self}} + (n-1) d_{\text{in}} Q_{\text{in}} + (N-n) d_{\text{out}} Q_{\text{out}}$$

$$- \frac{\mu}{N} \left(1 + (n-1) Q_{\text{in}} + (N-n) Q_{\text{out}} \right) \tag{3d}$$

Moran, Death-Birth

$$\beta_{\text{DB}}^D = \frac{1-\mu}{N} \sum_{j,k=1}^{N} Q_{jk} e_{jk} = \beta_{\text{BD}}^D$$

$$= (1-\mu)\Big(e_{\text{self}} + (n-1)e_{\text{in}}Q_{\text{in}} + (N-n)e_{\text{out}}Q_{\text{out}}\Big). \tag{4a}$$

$$\beta_{\text{DB}}^I = \frac{1-\mu}{N} \sum_{i,j,k,l=1}^{N} d_{ji} d_{li} e_{kl} Q_{jk} \tag{4b}$$

Presented in the table in the appendix.

$$\gamma_{\text{DB}}^D = 1-\mu = \gamma_{\text{BD}}^D. \tag{4c}$$

$$\gamma_{\text{DB}}^I = (1-\mu) \sum_{i,j,k=1}^{N} \frac{d_{ji} d_{ki}}{N} Q_{jk}$$

$$= \frac{1-\mu}{N} \sum_{j=1}^{N} \sum_{i=1}^{N} \Big(d_{ji}d_{ji} + \sum_{\substack{k\neq j \\ k\in G_j}} d_{ji}d_{ki}Q_{\text{in}} + \sum_{k\notin G_j} d_{ji}d_{ki}Q_{\text{out}}\Big)$$

$$= \frac{1-\mu}{N} \sum_{j=1}^{N} \Bigg[d_{\text{self}}d_{\text{self}} + (n-1)d_{\text{in}}d_{\text{in}} + (N-n)d_{\text{out}}d_{\text{out}}$$

$$+ \Big(d_{\text{self}}d_{\text{in}} + d_{\text{in}}d_{\text{self}} + (n-2)d_{\text{in}}d_{\text{in}} + (N-n)d_{\text{out}}d_{\text{out}}\Big)Q_{\text{in}}$$

$$+ \Big(d_{\text{self}}d_{\text{out}} + (n-1)d_{\text{in}}d_{\text{out}} + d_{\text{out}}d_{\text{self}} + (n-1)d_{\text{out}}d_{\text{in}} + (N-2n)d_{\text{out}}d_{\text{out}}\Big)Q_{\text{out}} \Bigg]$$

$$\tag{4d}$$

Probabilities of identity by descent

WF est faux. Il faut utiliser les formules Fourier...!

Moran   For $i = \neq j$,

$$Q_{ij} = \frac{1-\mu}{2} \sum_{k=1}^{N} \big(d_{kj}Q_{ki} + d_{ki}Q_{kj}\big). \tag{5a}$$

For $j \neq i$, $j \in G_i$,

$$Q_{\text{in}} = \frac{1-\mu}{2}\Big((d_{\text{in}} + d_{\text{self}}Q_{\text{in}}) + (d_{\text{self}}Q_{\text{in}} + d_{\text{in}})$$
$$+ (n-2)(d_{\text{in}}Q_{\text{in}} + d_{\text{in}}Q_{\text{in}}) + (N-n)(d_{\text{out}}Q_{\text{out}} + d_{\text{out}}Q_{\text{out}}) \Big)$$
$$= (1-\mu)\Big(d_{\text{in}} + d_{\text{self}}Q_{\text{in}} + (n-2)d_{\text{in}}Q_{\text{in}} + (N-n)d_{\text{out}}Q_{\text{out}} \Big). \qquad (5b)$$

And for $j \notin G_i$,

$$Q_{\text{out}} = \frac{1-\mu}{2}\Big((d_{\text{out}} + d_{\text{self}}Q_{\text{out}}) + (n-1)(d_{\text{out}}Q_{\text{in}} + d_{\text{in}}Q_{\text{out}})$$
$$+ (d_{\text{self}}Q_{\text{out}} + d_{\text{out}}) + (n-1)(d_{\text{in}}Q_{\text{out}} + d_{\text{out}}Q_{\text{in}})$$
$$+ (N-2n)(d_{\text{out}}Q_{\text{out}} + d_{\text{out}}Q_{\text{out}}) \Big)$$
$$= (1-\mu)\Big(d_{\text{out}} + d_{\text{self}}Q_{\text{out}} + (n-1)(d_{\text{out}}Q_{\text{in}} + d_{\text{in}}Q_{\text{out}}) + (N-2n)d_{\text{out}}Q_{\text{out}} \Big)$$
$$(5c)$$

Wright-Fisher   For $j \neq i$,

$$Q_{ij} = (1-\mu)^2 \sum_{k,l=1}^{N} d_{ki}d_{lj}Q_{kl}. \qquad (6a)$$

When $j \neq i$, $j \in G_i$,

$$Q_{\text{in}} = (1-\mu)^2 \Bigg[\Big(d_{\text{self}} d_{\text{in}} + d_{\text{in}} d_{\text{self}} + (n-2) d_{\text{in}} d_{\text{in}} + (N-n) d_{\text{out}} d_{\text{out}} \Big)$$

$$+ \Big(d_{\text{self}} d_{\text{self}} + (n-2) d_{\text{self}} d_{\text{in}}$$

$$+ (n-1) d_{\text{in}} d_{\text{in}} + (n-2) d_{\text{in}} d_{\text{self}}$$

$$+ (n-2)(n-2) d_{\text{in}} d_{\text{in}} + (N-n)(n-1) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{in}}$$

$$+ \Big((N-n) d_{\text{self}} d_{\text{out}} + (N-n)(n-1) d_{\text{in}} d_{\text{out}}$$

$$+ (N-n) d_{\text{out}} d_{\text{self}} + (N-n)(n-1) d_{\text{out}} d_{\text{in}}$$

$$+ (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg]$$

$$= (1-\mu)^2 \Bigg[\Big(2 d_{\text{in}} d_{\text{self}} + (n-2) d_{\text{in}}^2 + (N-n) d_{\text{out}}^2 \Big)$$

$$+ \Big(d_{\text{self}}^2 + 2(n-2) d_{\text{self}} d_{\text{in}} + (n^2 - 3n + 3) d_{\text{in}}^2 + +(N-n)(n-1) d_{\text{out}}^2 \Big) Q_{\text{in}}$$

$$+ \Big(2(N-n) d_{\text{self}} d_{\text{out}} + 2(N-n)(n-1) d_{\text{in}} d_{\text{out}}$$

$$+ (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg] \tag{6b}$$

And when $j \notin G_i$, we have

$$Q_{\text{out}} = (1-\mu)^2 \Bigg[\Big(2 d_{\text{self}} d_{\text{out}} + 2(n-1) d_{\text{in}} d_{\text{out}} + (N-2n) d_{\text{out}}^2 \Big)$$

$$+ \Big(2(n-1) d_{\text{self}} d_{\text{out}} + 2(n-1)^2 d_{\text{in}} d_{\text{out}} + (N-2n)(n-1) d_{\text{out}}^2 \Big) Q_{\text{in}}$$

$$+ \Big(d_{\text{self}} d_{\text{self}} + (n-1) d_{\text{self}} d_{\text{in}} + (N-2n) d_{\text{self}} d_{\text{out}}$$

$$+ (n-1) d_{\text{in}} d_{\text{self}} + (n-1)^2 d_{\text{in}}^2 + (n-1)(N-2n) d_{\text{in}} d_{\text{out}}$$

$$+ (N-n) d_{\text{out}} d_{\text{self}} + (N-n)(n-1) d_{\text{out}} d_{\text{in}} + (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg] . \tag{6c}$$

<mark>PAS FINI</mark>

Appendix

All combinations for i, j, k, l. Notation: (i, j) means that i and j are in the same deme, but are different; G_i refers to the deme containing site i.

j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
$j=i$	$k=i$	$l=i$	$(i=j=k=l)$	1	d_{self}	d_{self}	e_{self}	1
$j=i$	$k=i$	$l\neq i; l\in G_i$	$(i=j=k,l)$	$n-1$	d_{self}	d_{in}	e_{in}	1
$j=i$	$k=i$	$l\notin G_i$	$(i=j=k),(l)$	$N-n$	d_{self}	d_{out}	e_{out}	1
$j=i$	$k\neq i; k\in G_i$	$l=i$	$(i=j=l,k)$	$n-1$	d_{self}	d_{self}	e_{in}	Q_{in}
$j=i$	$k\neq i; k\in G_i$	$l=k$	$(i=j,k=l)$	$n-1$	d_{self}	d_{in}	e_{self}	Q_{in}
$j=i$	$k\neq i; k\in G_i$	$l\neq i,k; l\in G_i$	$(i=j,k,l)$	$(n-1)(n-2)$	d_{self}	d_{in}	e_{in}	Q_{in}
$j=i$	$k\neq i; k\in G_i$	$l\notin G_i$	$(i=j,k),(l)$	$(n-1)(N-n)$	d_{self}	d_{out}	e_{out}	Q_{in}
$j=i$	$k\notin G_i$	$l=i=j$	$(i=j=l),(k)$	$(N-n)$	d_{self}	d_{self}	e_{out}	Q_{out}
$j=i$	$k\notin G_i$	$l\neq i, l\in G_i$	$(i=j,l),(k)$	$(N-n)(n-1)$	d_{self}	d_{in}	e_{out}	Q_{out}
$j=i$	$k\notin G_i$	$l=k$	$(i=j),(k=l)$	$(N-n)$	d_{self}	d_{out}	e_{self}	Q_{out}
$j=i$	$k\notin G_i$	$l\neq k; l\in G_k$	$(i=j),(k,l)$	$(N-n)(n-1)$	d_{self}	d_{out}	e_{in}	Q_{out}
$j=i$	$k\notin G_i$	$l\notin G_i, G_k$	$(i=j),(k),(l)$	$(N-n)(N-2n)$	d_{self}	d_{out}	e_{out}	Q_{out}
$j\neq i, j\in G_i$	$k=i$	$l=i$	$(i=k=l,j)$	$(n-1)$	d_{in}	d_{self}	e_{self}	Q_{in}
$j\neq i, j\in G_i$	$k=i$	$l=j$	$(i=k,j=l)$	$(n-1)$	d_{in}	d_{in}	e_{in}	Q_{in}
$j\neq i, j\in G_i$	$k=i$	$l\neq i,j; l\in G_i$	$(i=k,j,l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	Q_{in}
$j\neq i, j\in G_i$	$k=i$	$l\notin G_i$	$(i=k,j),(l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{out}	Q_{in}
$j\neq i, j\in G_i$	$k=j$	$l=i$	$(i=l,j=k)$	$(n-1)$	d_{in}	d_{self}	e_{in}	1
$j\neq i, j\in G_i$	$k=j$	$l=j$	$(i,j=k=l)$	$(n-1)$	d_{in}	d_{in}	e_{self}	1
$j\neq i, j\in G_i$	$k=j$	$l\neq i,j; l\in G_i$	$(i,j=k,l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	1
$j\neq i, j\in G_i$	$k=j$	$l\notin G_i$	$(i,j=k),(l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{out}	1
$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=i$	$(i=l,j,k)$	$(n-1)(n-2)$	d_{in}	d_{self}	e_{in}	Q_{in}
$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=j$	$(i,j=l,k)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	Q_{in}
$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=k$	$(i,j,k=l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{self}	Q_{in}
$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l\neq i,j,k; l\in G_i$	(i,j,k,l)	$(n-1)(n-2)(n-3)$	d_{in}	d_{in}	e_{in}	Q_{in}
$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l\notin G_i$	$(i,j,k),(l)$	$(n-1)(n-2)(N-n)$	d_{in}	d_{out}	e_{out}	Q_{in}

j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
$j \neq i; j \in G_i$	$k \notin G_i$	$l = i$	$(i = l, j), (k)$	$(n-1)(N-n)$	d_{in}	d_{self}	e_{out}	Q_{out}
$j \neq i; j \in G_i$	$k \notin G_i$	$l = j$	$(i, j = l), (k)$	$(n-1)(N-n)$	d_{in}	d_{in}	e_{out}	Q_{out}
$j \neq i; j \in G_i$	$k \notin G_i$	$l \neq i, j; l \in G_i$	$(i, j, l), (k)$	$(n-1)(N-n)(n-2)$	d_{in}	d_{in}	e_{out}	Q_{out}
$j \neq i; j \in G_i$	$k \notin G_i$	$l = k$	$(i, j), (k = l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{self}	Q_{out}
$j \neq i; j \in G_i$	$k \notin G_i$	$l \neq k; l \in G_k$	$(i, j), (k, l)$	$(n-1)(N-n)(n-1)$	d_{in}	d_{out}	e_{in}	Q_{out}
$j \neq i; j \in G_i$	$k \notin G_i$	$l \notin G_i, G_k$	$(i, j), (k), (l)$	$(n-1)(N-n)(N-2n)$	d_{in}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k = i$	$l = i$	$(i = k = l), (j)$	$(N-n)$	d_{out}	d_{self}	e_{self}	Q_{out}
$j \notin G_i$	$k = i$	$l \neq i; l \in G_i$	$(i = k, l), (j)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{in}	Q_{out}
$j \notin G_i$	$k = i$	$l = j$	$(i = k), (j = l)$	$(N-n)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k = i$	$l \neq j; l \in G_j$	$(i = k), (j, l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k = i$	$l \notin G_i, G_j$	$(i = k), (j), (l)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l = i$	$(i = l, k), (j)$	$(N-n)(n-1)$	d_{out}	d_{self}	e_{in}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l = k$	$(i, k = l), (j)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{self}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l \neq i, k; l \in G_i$	$(i, k, l), (j)$	$(N-n)(n-1)(n-2)$	d_{out}	d_{in}	e_{in}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l = j$	$(i, k), (j = l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l \neq j; l \in G_j$	$(i, k), (j, l)$	$(N-n)(n-1)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k \neq i; k \in G_i$	$l \notin G_i, G_j$	$(i, k), (j), (l)$	$(N-n)(n-1)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k = j$	$l = i$	$(i = l), (j = k)$	$(N-n)$	d_{out}	d_{self}	e_{out}	1
$j \notin G_i$	$k = j$	$l \neq i; l \in G_i$	$(i, l), (j = k)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{out}	1
$j \notin G_i$	$k = j$	$l = j$	$(i), (j = k = l)$	$(N-n)$	d_{out}	d_{out}	e_{self}	1
$j \notin G_i$	$k = j$	$l \neq j; l \in G_j$	$(i), (j = k, l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{in}	1
$j \notin G_i$	$k = j$	$l \notin G_i, G_j$	$(i), (j = k), (l)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{out}	1

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j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
$j \notin G_i$	$k \neq j; k \in G_j$	$l = i$	$(i=l),(j,k)$	$(N-n)(n-1)$	d_{out}	d_{self}	e_{out}	Q_{in}
$j \notin G_i$	$k \neq j; k \in G_j$	$l \neq i; l \in G_i$	$(i,l),(j,k)$	$(N-n)(n-1)(n-1)$	d_{out}	d_{in}	e_{out}	Q_{in}
$j \notin G_i$	$k \neq j; k \in G_j$	$l = j$	$(i),(j=l,k)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{in}	Q_{in}
$j \notin G_i$	$k \neq j; k \in G_j$	$l = k$	$(i),(j,k=l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{self}	Q_{in}
$j \notin G_i$	$k \neq j; k \in G_j$	$l \neq j,k; l \in G_j$	$(i),(j,k,l)$	$(N-n)(n-1)(n-2)$	d_{out}	d_{out}	e_{in}	Q_{in}
$j \notin G_i$	$k \neq j; k \in G_j$	$l \notin G_i, G_j$	$(i),(j,k),(l)$	$(N-n)(n-1)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{in}
$j \notin G_i$	$k \notin G_i, G_j$	$l = i$	$(i=l),(j),(k)$	$(N-n)(N-2n)$	d_{out}	d_{self}	e_{out}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l \neq i; l \in G_i$	$(i,l),(j),(k)$	$(N-n)(N-2n)(n-1)$	d_{out}	d_{in}	e_{out}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l = j$	$(i),(j=l),(k)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l \neq j; l \in G_j$	$(i),(j,l),(k)$	$(N-n)(N-2n)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l = k$	$(i),(j),(k=l)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{self}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l \neq k; l \in G_k$	$(i),(j),(k,l)$	$(N-n)(N-2n)(n-1)$	d_{out}	d_{out}	e_{in}	Q_{out}
$j \notin G_i$	$k \notin G_i, G_j$	$l \notin G_i, G_j, G_k$	$(i),(j),(k),(l)$	$(N-n)(N-2n)(N-3n)$	d_{out}	d_{out}	e_{out}	Q_{out}

A Island model

With self replacement

$$d_{\text{self}} = d_{\text{in}} = \frac{1-m}{n}, \tag{7a}$$

$$d_{\text{out}} = \frac{m}{N-n}. \tag{7b}$$

Without self-replacement

$$d_{\text{self}} = 0, \tag{8a}$$

$$d_{\text{in}} = \frac{1-m}{n-1}, \tag{8b}$$

$$d_{\text{out}} = \frac{m}{N-n}. \tag{8c}$$

B IDB

B.1 Moran

Using the formulas for a 2D graph in REF Debarre 2017,

$$\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}} = \sum_{l_1=0}^{N_1-1} \sum_{l_2=0}^{N_2-1} \tilde{d}_{\substack{l_1 \\ l_2}} \exp\left(-\iota \frac{2\pi q_1 l_1}{N_1}\right) \exp\left(-\iota \frac{2\pi q_2 l_2}{N_2}\right) \tag{9a}$$

$$\tilde{\mathcal{Q}}_{\substack{r_1 \\ r_2}} = \frac{1}{N} \sum_{q_1=0}^{N_1-1} \sum_{q_2=0}^{N_2-1} \frac{\mu \lambda_M'}{1 - (1-\mu)\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}}} \exp\left(\iota \frac{2\pi q_1 r_1}{N_1}\right) \exp\left(\iota \frac{2\pi q_2 r_2}{N_2}\right) \tag{9b}$$

We have

$$\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}} = d_{\text{self}} + \sum_{l_2=1}^{N_2-1} d_{\text{in}} \exp\left(-\iota \frac{2\pi q_2 l_2}{N_2}\right) + \sum_{l_1=1}^{N_1-1} \sum_{l_2=0}^{N_2-1} d_{\text{out}} \exp\left(-\iota \frac{2\pi q_1 l_1}{N_1}\right) \exp\left(-\iota \frac{2\pi q_2 l_2}{N_2}\right)$$

$$= d_{\text{self}} + \left(\delta_{q_2}(N_2-1) + (1-\delta_{q_2})(-1)\right) d_{\text{in}} + \left(\delta_{q_1}(N_1-1) + (1-\delta_{q_1})(-1)\right)\left(\delta_{q_2} N_2\right) d_{\text{out}}$$

$$= d_{\text{self}} + \left(\delta_{q_2} N_2 - 1\right) d_{\text{in}} + \left(\delta_{q_1} N_1 - 1\right) \delta_{q_2} N_2 d_{\text{out}}. \tag{10a}$$

Whether there is self-replacement or not, we have $N_1 = D$ and $N_2 = n$, and

$$\tilde{\mathcal{D}}_{\substack{0 \\ 0}} = 1, \tag{11a}$$

$$\tilde{\mathcal{D}}_{\substack{q_1 \\ 0}} = 1 - m - \frac{m}{d-1} \quad (q_1 \not\equiv 0 \pmod{N_1}), \tag{11b}$$

$$\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}} = d_{\text{self}} - d_{\text{in}} \quad (q_2 \not\equiv 0 \pmod{N_2}). \tag{11c}$$

So for $\tilde{Q}$,

$$\tilde{Q}_{r_2}^{r_1} = \frac{\mu \lambda'_M}{N} \left[\frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_0^0} + \sum_{q_2=1}^{N_2-1} \frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{q_2}^0} \exp\left(-\iota\frac{2\pi q_2 r_2}{N_2}\right) + \sum_{q_1=1}^{N_1-1} \frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_0^{q_1}} \exp\left(-\iota\frac{2\pi q_1 r_1}{N_1}\right) \right.$$

$$\left. + \sum_{q_1=1}^{N_1-1} \sum_{q_2=1}^{N_2-1} \frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{q_2}^{q_1}} \exp\left(-\iota\frac{2\pi q_1 r_1}{N_1}\right) \exp\left(-\iota\frac{2\pi q_2 r_2}{N_2}\right) \right]$$

$$= \frac{\mu \lambda'_M}{N} \left[\frac{1}{1-(1-\mu)} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(\delta_{r_2} N_2 - 1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(\delta_{r_1} N_1 - 1) \right.$$

$$\left. + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(\delta_{r_1} N_1 - 1)(\delta_{r_2} N_2 - 1) \right]. \tag{12a}$$

In particular,

$$\tilde{Q}_0^0 = \frac{\mu \lambda'_M}{N} \left[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(n-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(D-1) \right.$$

$$\left. + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(D-1)(n-1) \right]$$

$$= 1. \tag{12b}$$

We find λ'_M using the above equation. When $r_1 = 0$, the two individuals are in the same deme. They are different when $r_2 \neq 0$:

$$Q_{\text{in}} = \frac{\mu \lambda'_M}{N} \left[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(D-1) \right.$$

$$\left. + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(D-1)(-1) \right]. \tag{12c}$$

And when $r_1 \neq 0$, the two individuals are in different demes:

$$Q_{\text{out}} = \frac{\mu \lambda'_M}{N} \left[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(-1) \right.$$

$$\left. + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})} \right]. \tag{12d}$$

B.2 Wright-Fisher

$$\tilde{\mathcal{Q}}_{\substack{r_1 \\ r_2}} = \frac{1}{N} \sum_{q_1=0}^{N_1-1} \sum_{q_2=0}^{N_2-1} \frac{\mu \lambda'_{WF}}{1 - (1-\mu)^2 (\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}})^2} \exp\left(-\iota \frac{2\pi q_1 r_1}{N_1}\right) \exp\left(-\iota \frac{2\pi q_2 r_2}{N_2}\right)$$

$$= \frac{1}{N}\Bigg[\frac{\mu \lambda'_{WF}}{1 - (1-\mu)^2 (\tilde{\mathcal{D}}_{\substack{0 \\ 0}})^2} + \sum_{q_2=1}^{N_2-1} \frac{\mu \lambda'_{WF}}{1 - (1-\mu)^2 (\tilde{\mathcal{D}}_{\substack{0 \\ q_2}})^2} \exp\left(-\iota \frac{2\pi q_2 r_2}{N_2}\right)$$

$$+ \sum_{q_1=1}^{N_1-1} \frac{\mu \lambda'_{WF}}{1 - (1-\mu)^2 (\tilde{\mathcal{D}}_{\substack{q_1 \\ 0}})^2} \exp\left(-\iota \frac{2\pi q_1 r_1}{N_1}\right)$$

$$+ \sum_{q_1=1}^{N_1-1} \sum_{q_2=1}^{N_2-1} \frac{\mu \lambda'_{WF}}{1 - (1-\mu)^2 (\tilde{\mathcal{D}}_{\substack{q_1 \\ q_2}})^2} \exp\left(-\iota \frac{2\pi q_1 r_1}{N_1}\right) \exp\left(-\iota \frac{2\pi q_2 r_2}{N_2}\right) \Bigg] \quad (13)$$

$$= \frac{\mu \lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2} + \frac{1}{1-(1-\mu)^2 (d_{\text{self}} - d_{\text{in}})^2}(\delta_{q_2} N_2 - 1)$$

$$+ \frac{1}{1-(1-\mu)^2 (1-m-\frac{m}{d-1})^2}(\delta_{q_1} N_1 - 1)$$

$$+ \frac{1}{1-(1-\mu)^2 (d_{\text{self}} - d_{\text{in}})^2}(\delta_{q_1} N_1 - 1)(\delta_{q_2} N_2 - 1) \Bigg]$$

$$= \frac{\mu \lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2} + \frac{1}{1-(1-\mu)^2 (d_{\text{self}} - d_{\text{in}})^2}(\delta_{q_2} N_2 - 1)\delta_{q_1} N_1$$

$$+ \frac{1}{1-(1-\mu)^2 (1-m-\frac{m}{d-1})^2}(\delta_{q_1} N_1 - 1) \Bigg]. \quad (14)$$

To find λ'_{WF}, we solve

$$1 = \frac{\mu \lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2} + \frac{1}{1-(1-\mu)^2 (d_{\text{self}} - d_{\text{in}})^2}(N_2-1)N_1 + \frac{1}{1-(1-\mu)^2 (1-m-\frac{m}{d-1})^2}(N_1-1) \Bigg].$$
$$(15\text{a})$$

Then,

$$Q_{\text{in}} = \frac{\mu \lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2} - \frac{1}{1-(1-\mu)^2 (d_{\text{self}} - d_{\text{in}})^2}N_1 + \frac{1}{1-(1-\mu)^2 (1-m-\frac{m}{d-1})^2}(N_1-1) \Bigg].$$
$$(15\text{b})$$

and

$$Q_{\text{out}} = \frac{\mu \lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2} - \frac{1}{1-(1-\mu)^2 (1-m-\frac{m}{d-1})^2} \Bigg]. \quad (15\text{c})$$