Symposium 9: Fitness and evolution in a social environment: from theory to reality

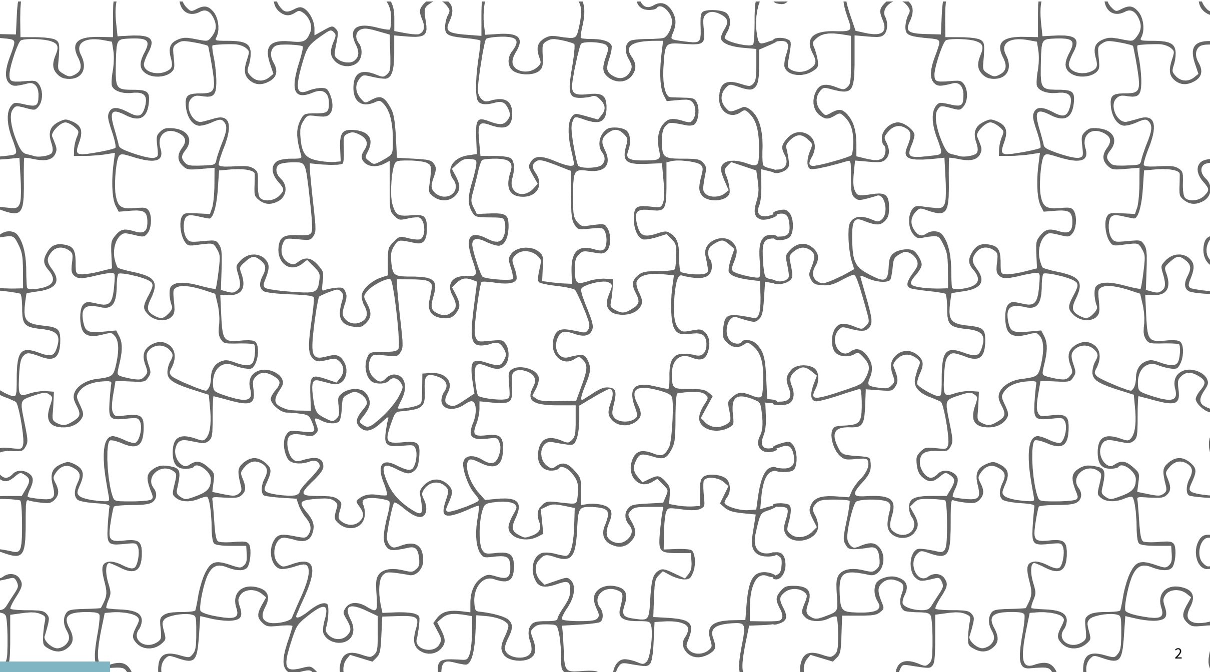
Fidelity of parent-offspring transmission and the evolution of social behavior in subdivided populations.

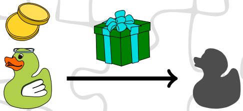
F. Débarre

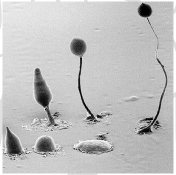
@flodebarre

CNRS

Centre de Recherches Interdisciplinaires en Biologie, Paris







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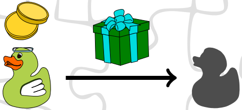

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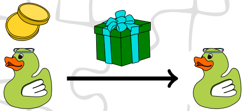
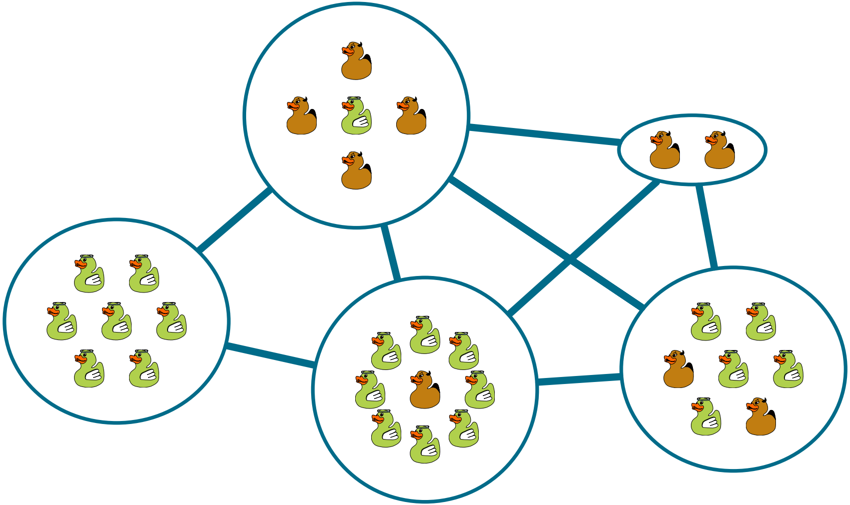


Spatial structure and altruism



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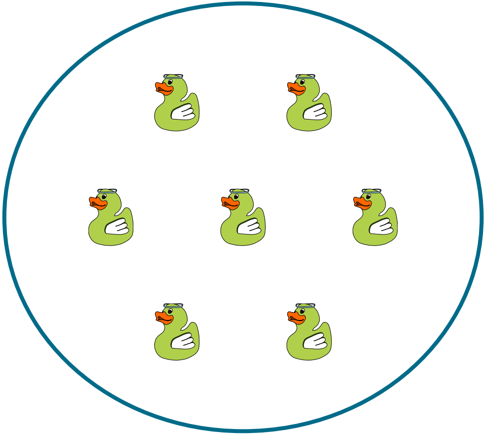

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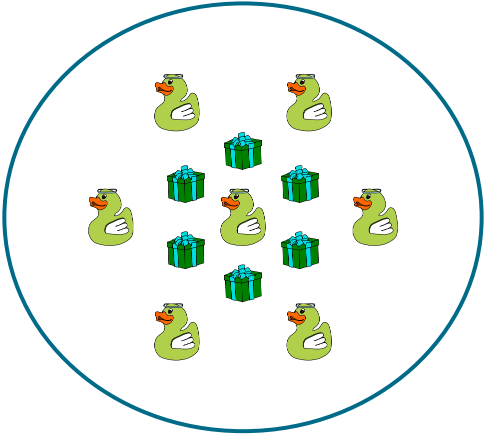

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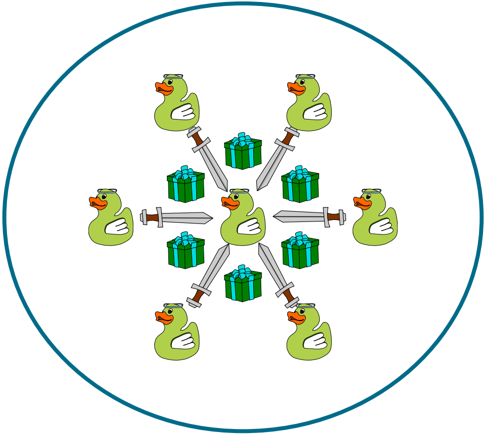

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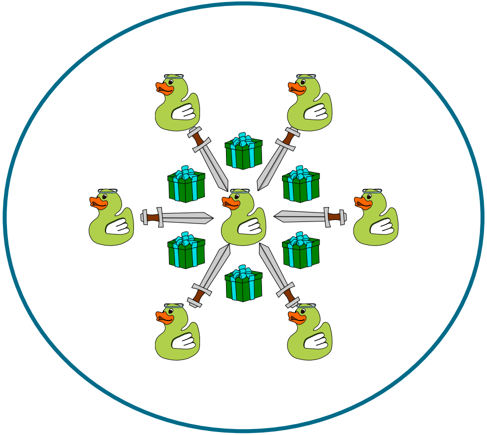
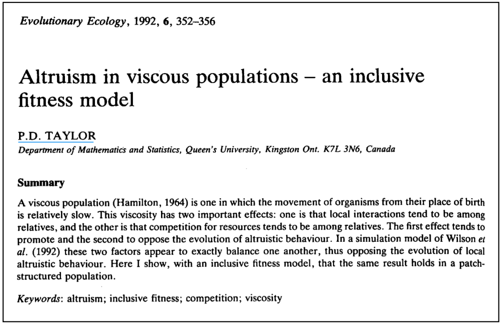




Evolutionary Ecology, 1992, **6**, 352–356

Altruism in viscous populations – an inclusive fitness model

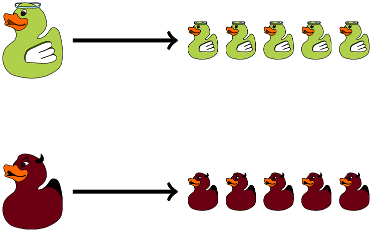
P.D. TAYLOR
Department of Mathematics and Statistics, Queen's University, Kingston Ont. K7L 3N6, Canada

Summary

A viscous population (Hamilton, 1964) is one in which the movement of organisms from their place of birth is relatively slow. This viscosity has two important effects: one is that local interactions tend to be among relatives, and the other is that competition for resources tends to be among relatives. The first effect tends to promote and the second to oppose the evolution of altruistic behaviour. In a simulation model of Wilson *et al.* (1992) these two factors appear to exactly balance one another, thus opposing the evolution of local altruistic behaviour. Here I show, with an inclusive fitness model, that the same result holds in a patch-structured population.

Keywords: altruism; inclusive fitness; competition; viscosity

A common feature of models



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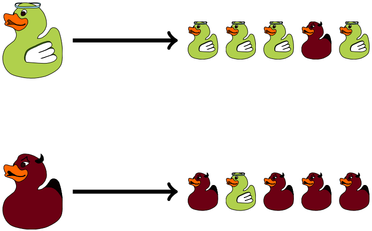
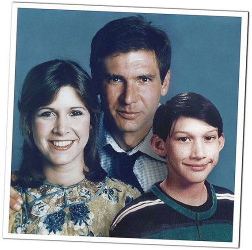


What is the effect of population viscosity on the evolution of altruism when parent-offspring strategy transmission is **imperfect**?

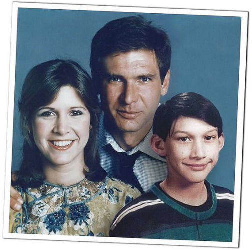
Fidelity of parent-offspring transmission



Causes of imperfect strategy transmission

- Mutation

Fidelity of parent-offspring transmission



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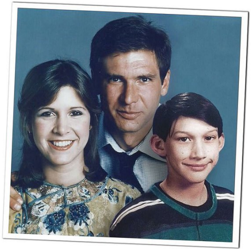

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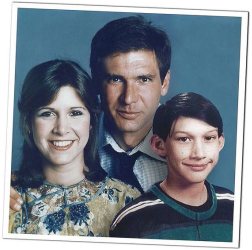

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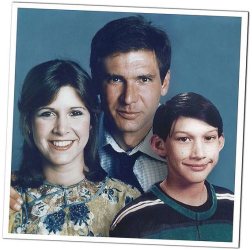
In the model

Parent

Fidelity of parent-offspring transmission



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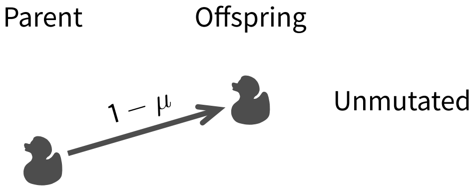

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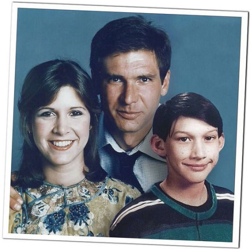

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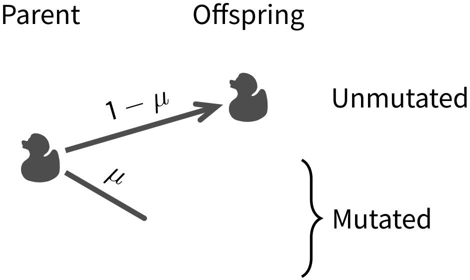

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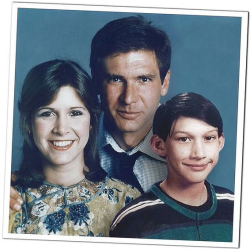

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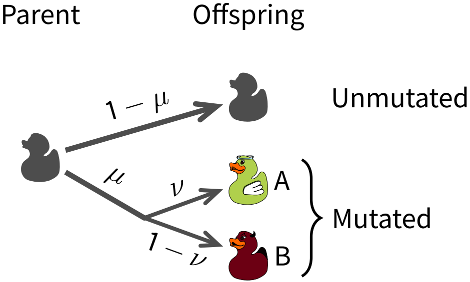
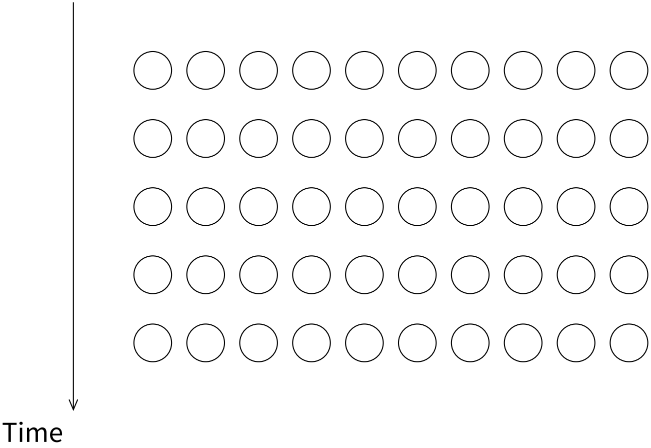


Genealogy, Identity by descent and Identity in state



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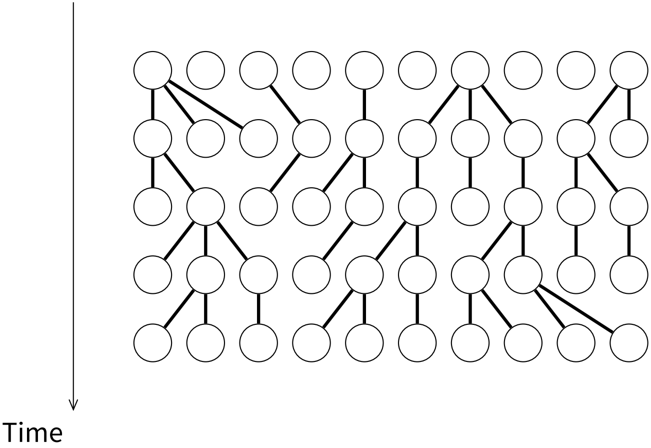

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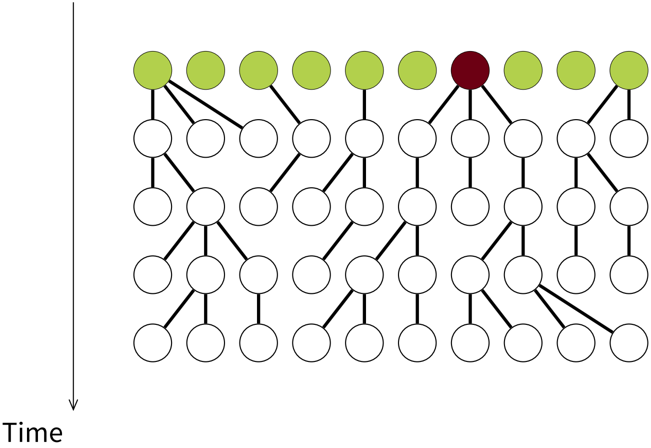

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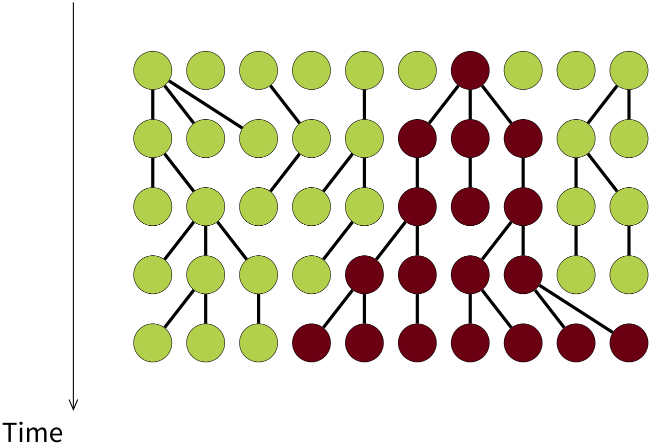

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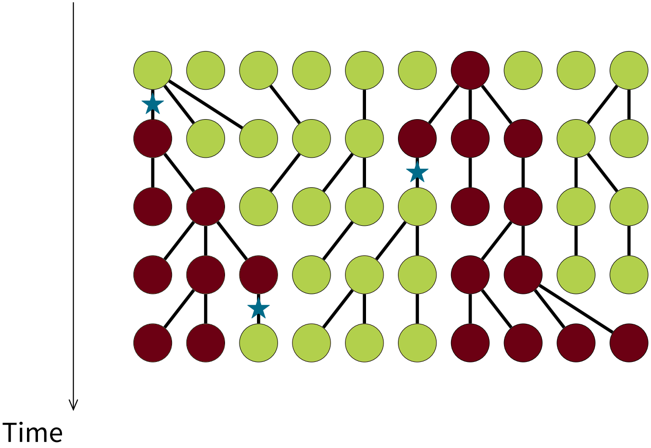

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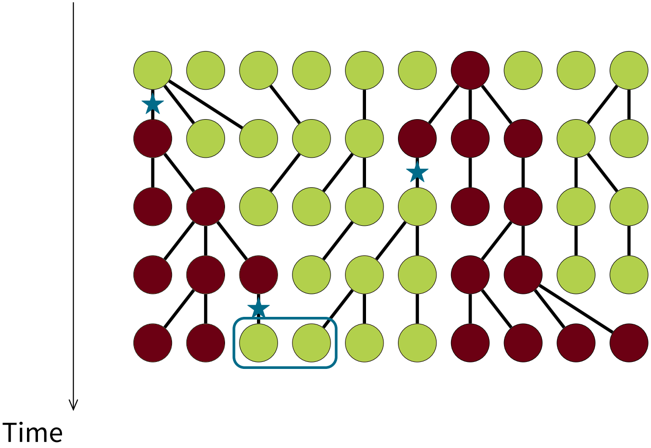

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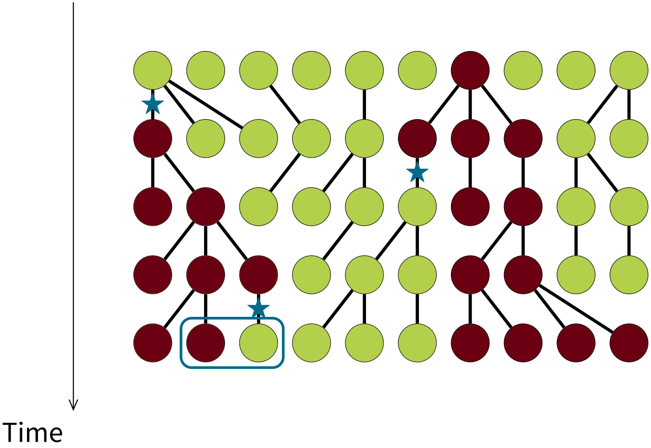

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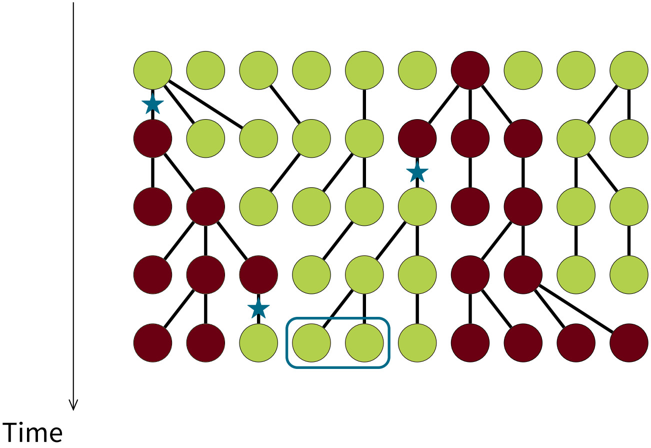

Expected state of pairs of sites and identity by descent

At neutrality (i.e., in the absence of selection, $\delta = 0$),

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P_{ij}

Expected state
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= Probability that the two
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At neutrality (i.e., in the absence of selection, $\delta = 0$),

$$P_{ij} = Q_{ij}\,\nu + (1 - Q_{ij})\nu^2$$

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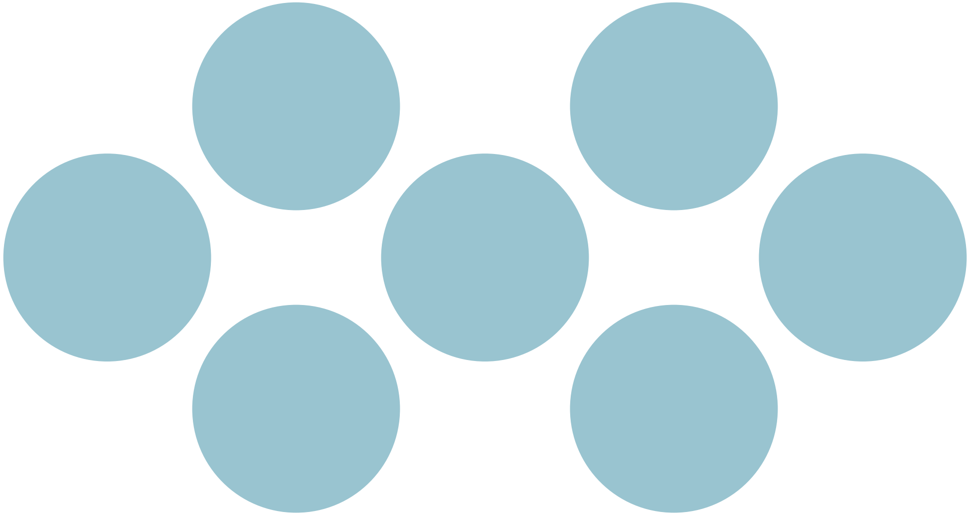
Q_{in}, Q_{out}

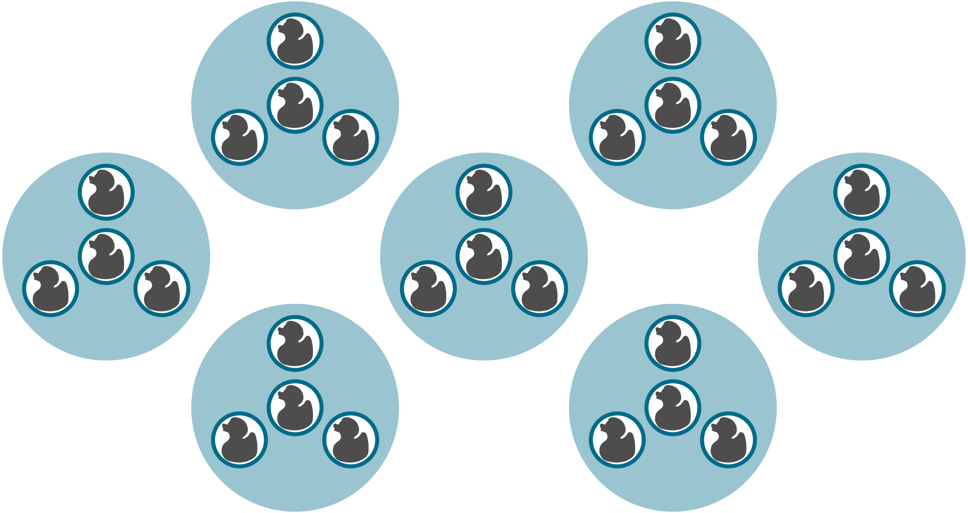
Subdivided population – Island model

N_d demes



Subdivided population – Island model

N_d demes of n individuals each (total population size $N = n\,N_d$)



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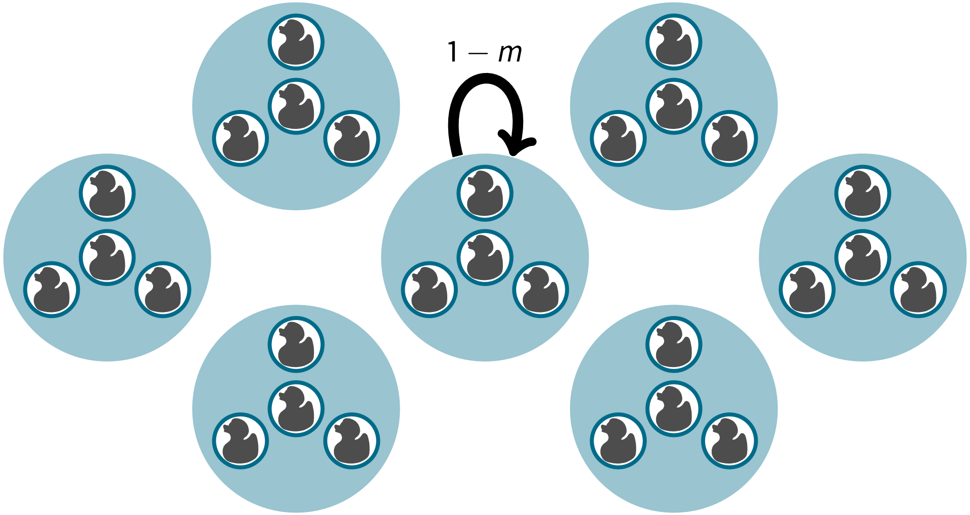

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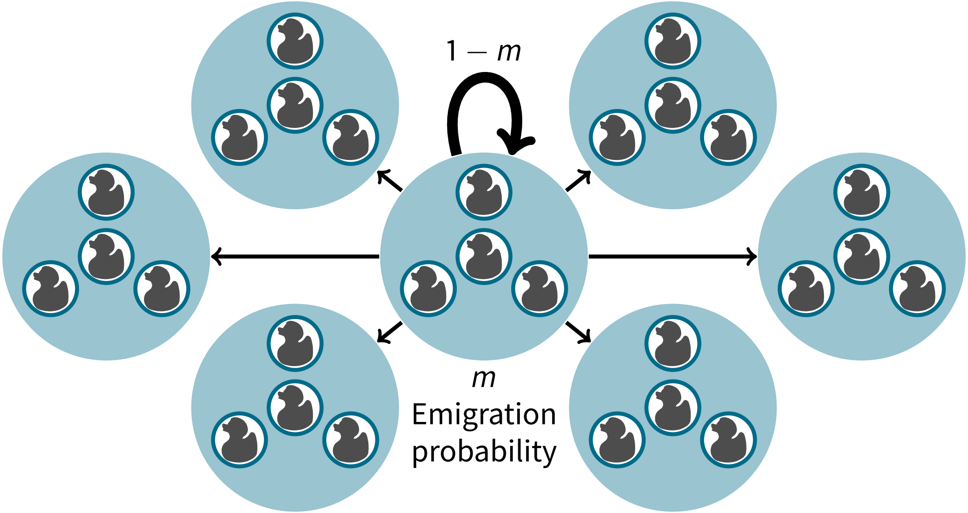

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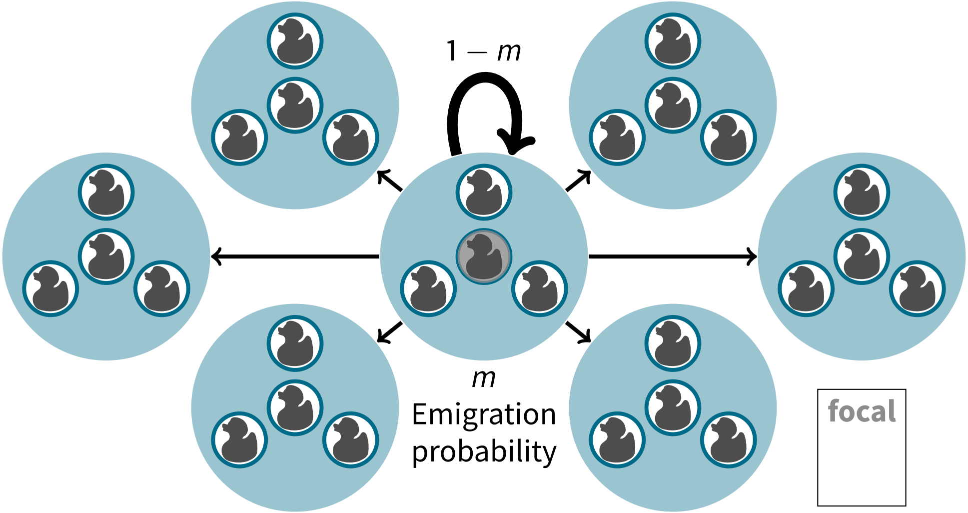

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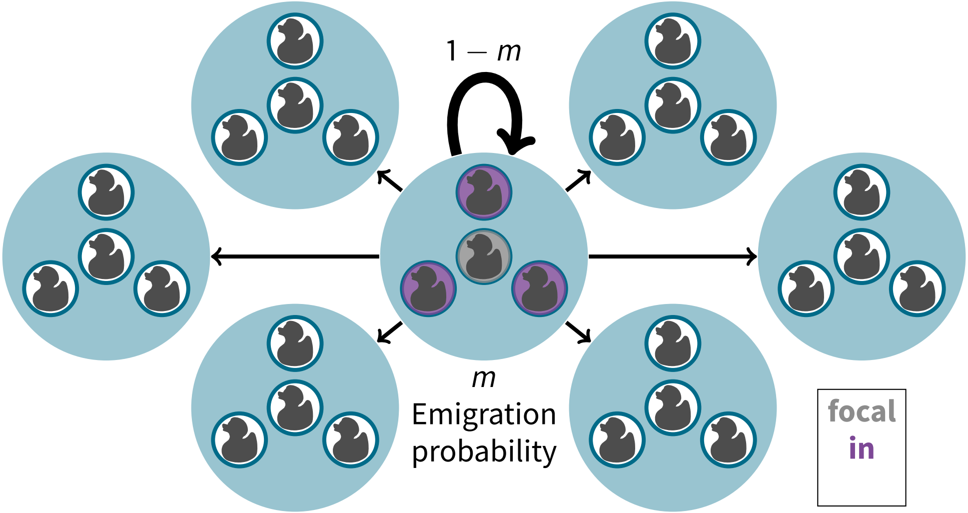

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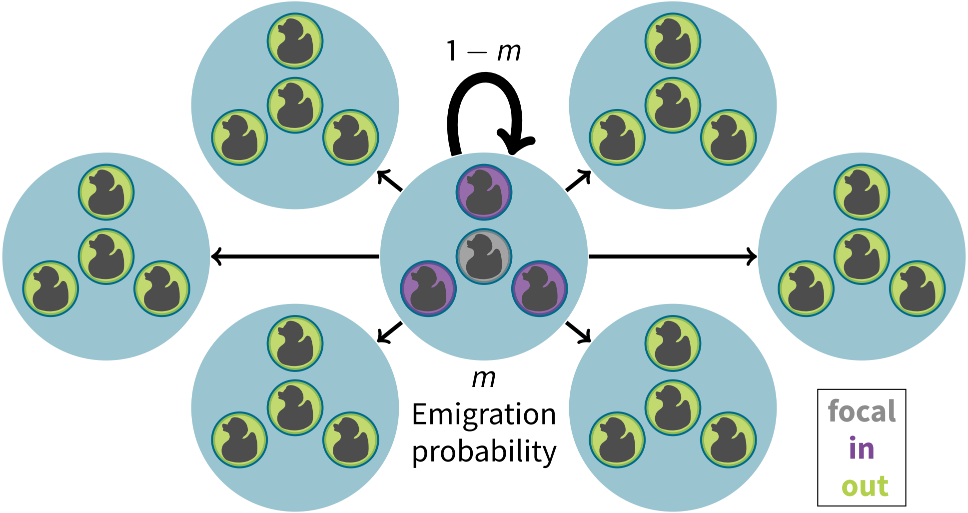

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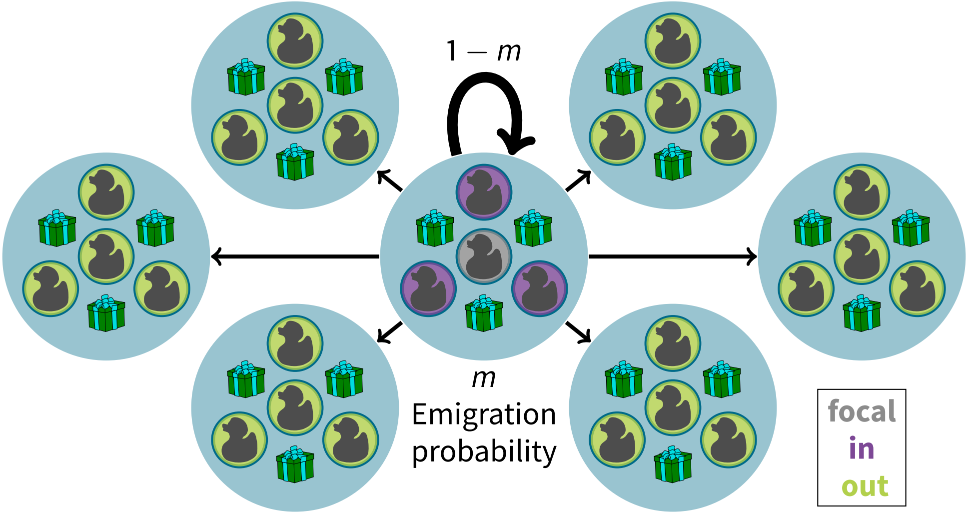

Updating the population

Constant population size (N), so between two time steps,

$\#$ 

$=$ $\#$ 

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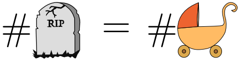

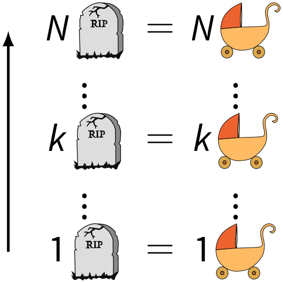

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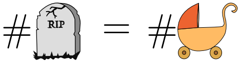
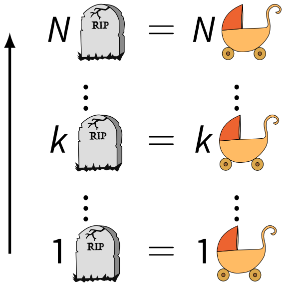


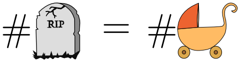


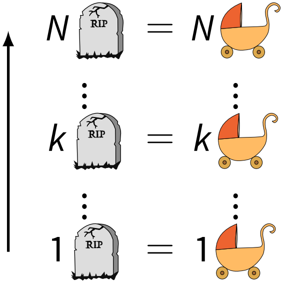
Wright-Fisher

Moran process

Updating the population

Constant population size (N), so between two time steps,





Wright-Fisher

Moran process

Life-cycle

Offspring production

Updating the population

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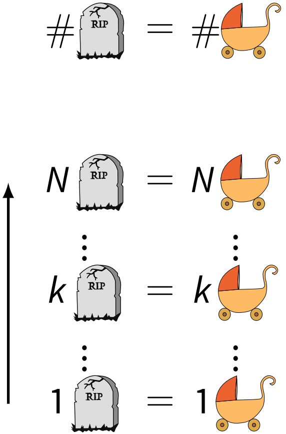

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Moran process

Life-cycle

Offspring production

Offspring dispersal

Updating the population

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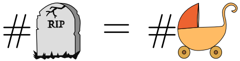

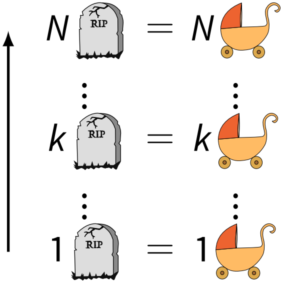

Wright-Fisher

Moran process

Life-cycle

Offspring production

Offspring dispersal

k parents die

Updating the population

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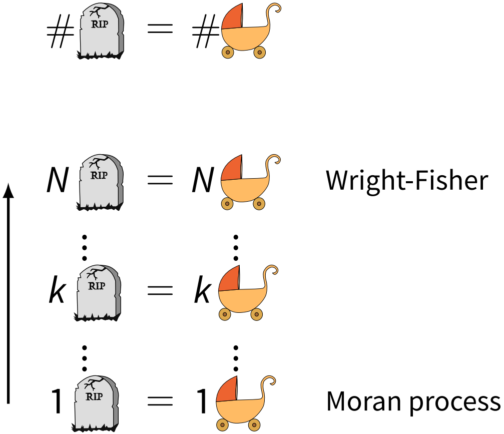
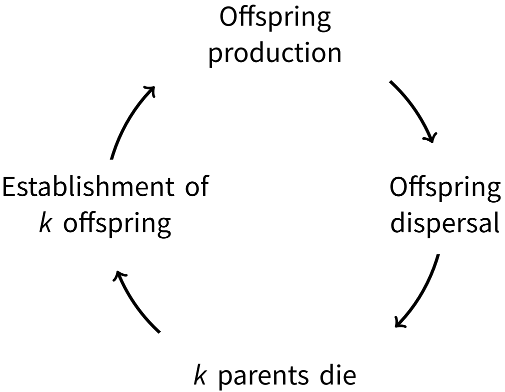


Life-cycle



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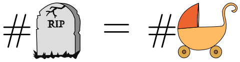

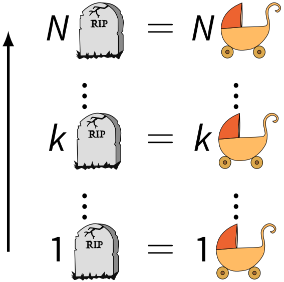

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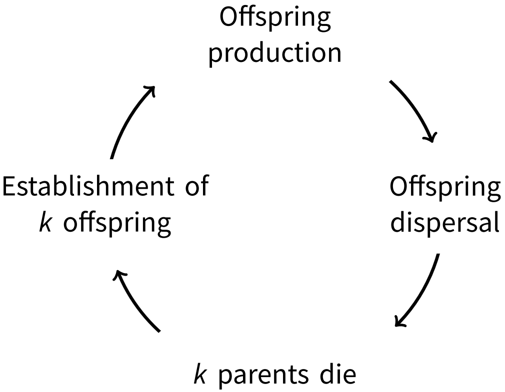
“Death-Birth” updating



Population

$X_i(t) = \begin{cases} 1 & \text{if site } i \text{ occupied by (see below)} \\ 0 & \text{if site } i \text{ occupied by (see below)} \end{cases}$

1 if site i occupied by  at time t $(1 \leq i \leq N)$

0 if site i occupied by  at time t $(1 \leq i \leq N)$

Population

$X_i(t) =$

- 1 if site i occupied by  at time t $(1 \leq i \leq N)$
- 0 if site i occupied by  at time t $(1 \leq i \leq N)$

We are interested in $\mathbb{E}[\overline{X}]$,
the expected ($\mathbb{E}$) proportion ($\overline{X}$) of altruists in the population.

Social interactions affect fecundity


In a deme with k :

$$f = 1 + \omega\left(b\,\frac{k-1}{n-1} - c\right),$$

$$f = 1 + \omega\left(b\,\frac{k}{n-1}\right).$$

Social interactions

Social interactions affect fecundity

In a deme with k :

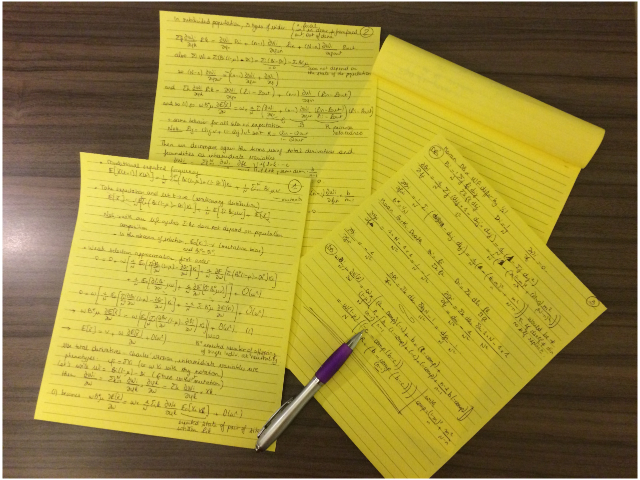
f  $= 1 + \omega\left(b\,\frac{k-1}{n-1} - c\right),$

f  $= 1 + \omega\left(b\,\frac{k}{n-1}\right).$

Selection is weak

$$\omega \ll 1.$$

“Field site”



Expected frequency of altruists in the population

$$\mathbb{E}[\overline{X}] = \nu + \delta\ \nu(1-\nu)\ \frac{1-\mu}{\mu}\ (1-Q_{\text{out}})\ \times$$
$$\Bigg(-c\ - (b-c)\left(\frac{(1-m)^2}{n} + \frac{m^2}{n\,(N_d-1)}\right)$$
$$+\ \frac{Q_{\text{in}} - Q_{\text{out}}}{1-Q_{\text{out}}}\left[\ b\ - (b-c)\,(n-1)\ \left(\frac{(1-m)^2}{n} + \frac{m^2}{n\,(N_d-1)}\right)\ \right]\Bigg)$$

Expected frequency of altruists in the population

Mutation-drift equilibrium

$$\mathbb{E}[\overline{X}] = \nu + \delta\ \nu(1-\nu)\ \frac{1-\mu}{\mu}\ (1-Q_{\text{out}})\ \times$$
$$\left(-c - (b-c)\left(\frac{(1-m)^2}{n} + \frac{m^2}{n\,(N_d-1)}\right) + \frac{Q_{\text{in}} - Q_{\text{out}}}{1-Q_{\text{out}}}\left[b - (b-c)\,(n-1)\left(\frac{(1-m)^2}{n} + \frac{m^2}{n\,(N_d-1)}\right) \right] \right)$$

Expected frequency of altruists in the population

Mutation-drift equilibrium

Selection strength

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Expected frequency of altruists in the population

Mutation-drift equilibrium

Selection strength

~~Population variance~~

Variance in the state of one site

$$\mathbb{E}[\bar{X}] = \nu + \delta\, \nu(1-\nu)\, \frac{1-\mu}{\mu}\, (1 - Q_{\text{out}}) \times$$

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$$

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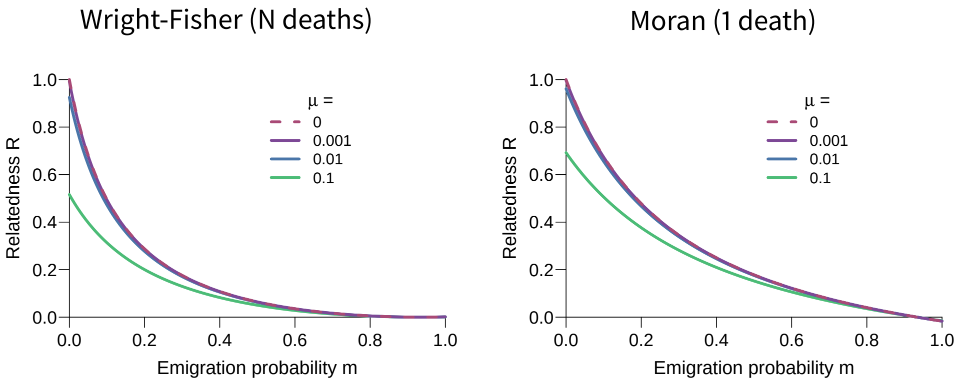
R $\qquad$ $\mathcal{B}$

How does relatedness R change with the emigration probability m?

How does relatedness *R* change with the emigration probability *m*?



$$(n = 4, N_d = 15)$$

Expected frequency of altruists in the population

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$R \searrow$ $\qquad \mathcal{B}$

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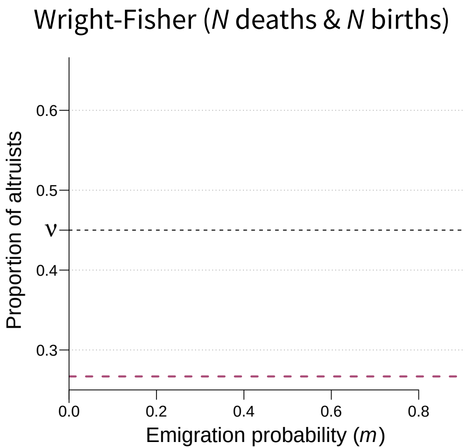
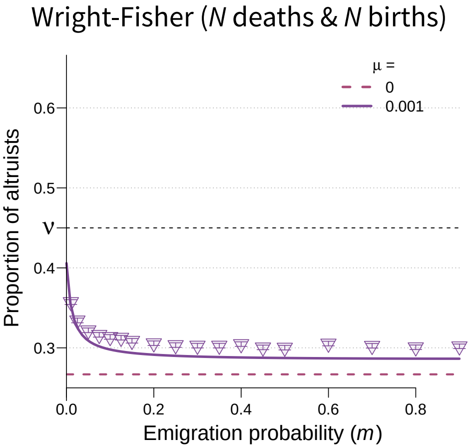
$R \searrow$ $\quad\quad$ $\mathcal{B} \nearrow$

Effect of the emigration probability m on the expected proportion of altruists



$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$

Effect of the emigration probability *m* on the expected proportion of altruists



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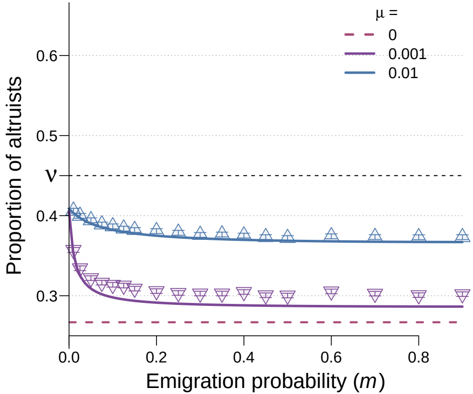
Wright-Fisher (*N* deaths & *N* births)



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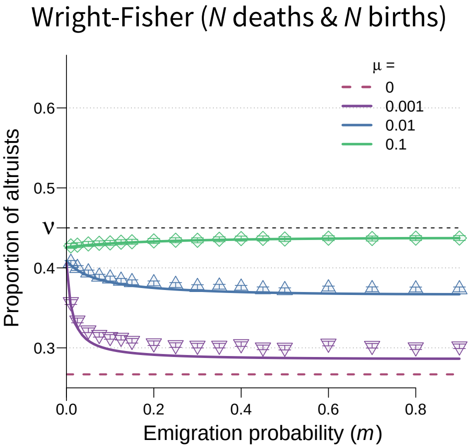

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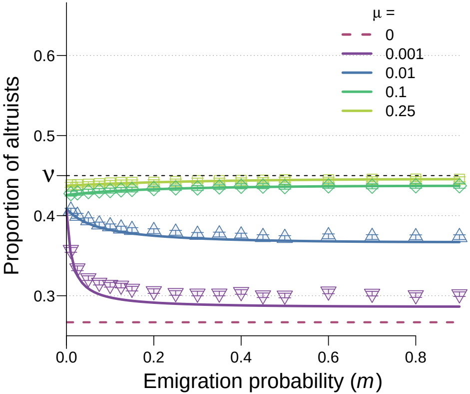

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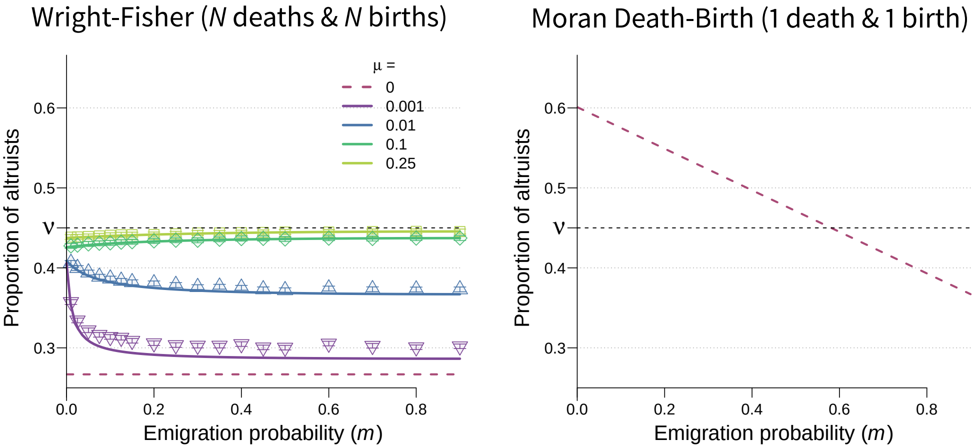

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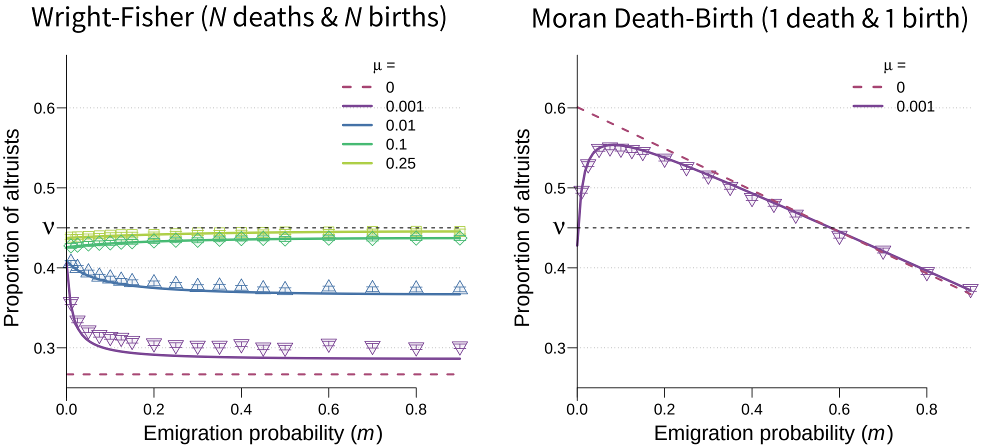

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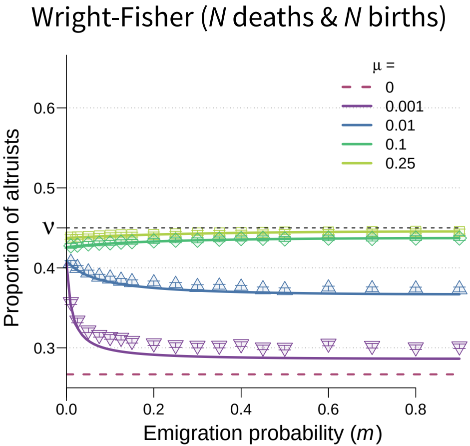

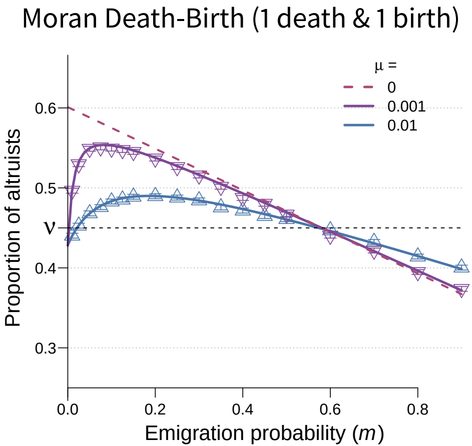

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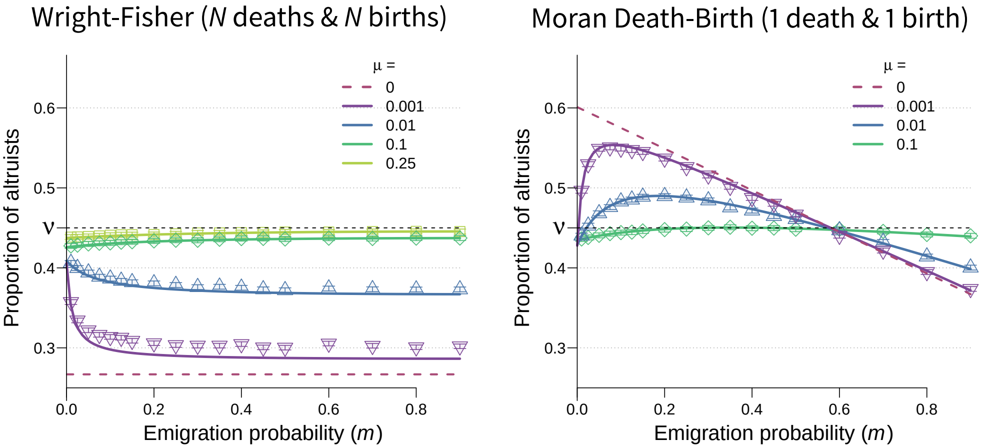

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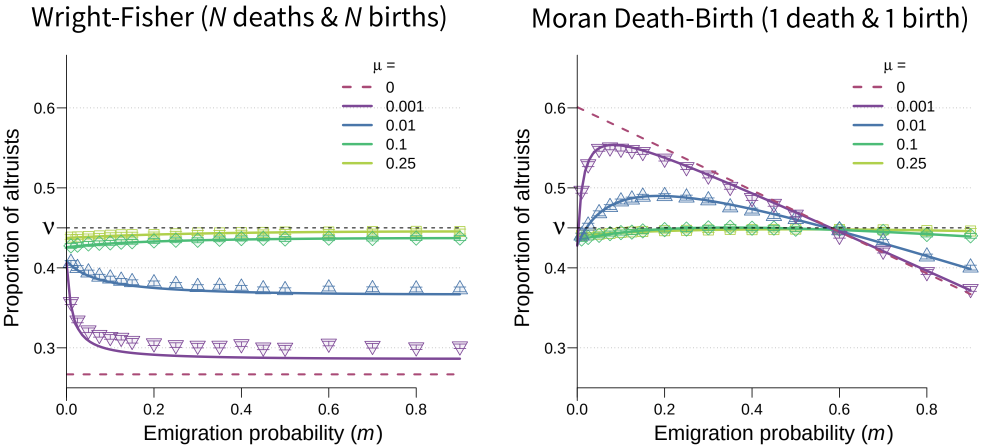

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Is the result robust?

Another life-cycle

Moran Birth-Death (1 birth & 1 death)

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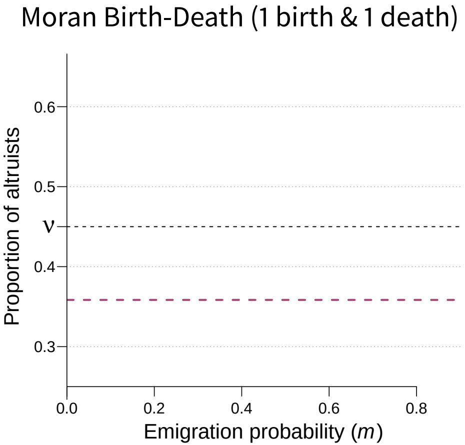
Another life-cycle



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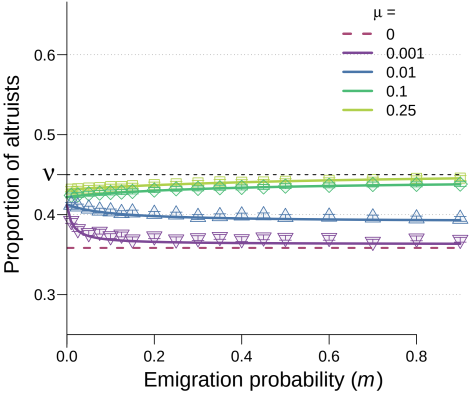

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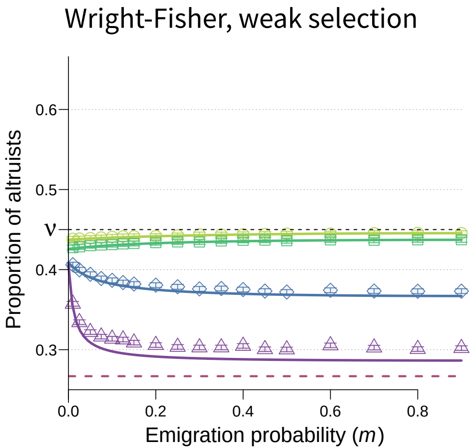
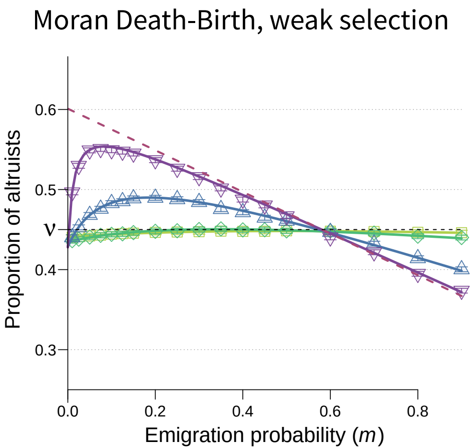
Strong selection





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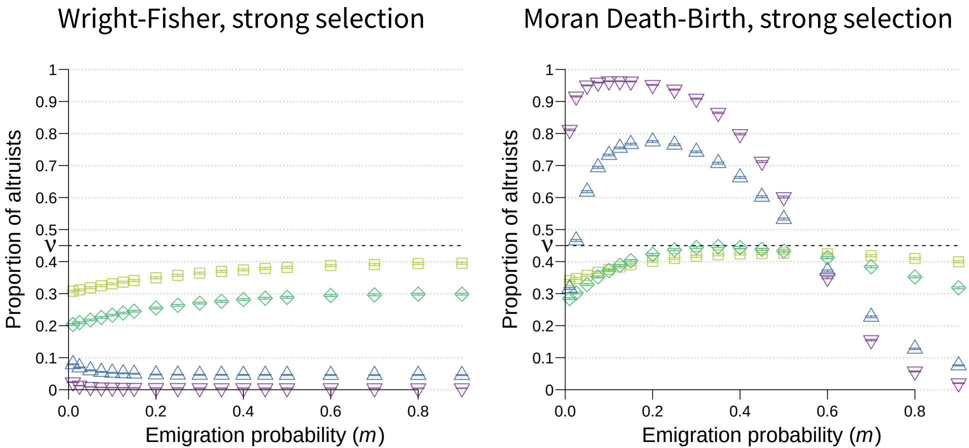
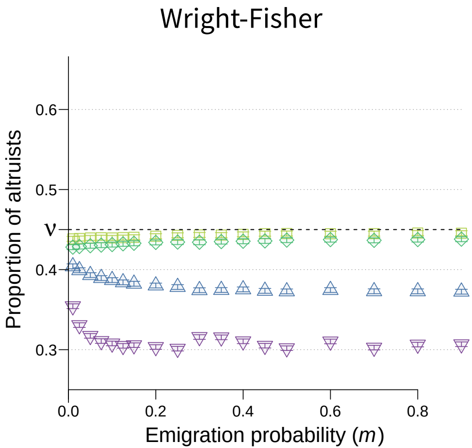
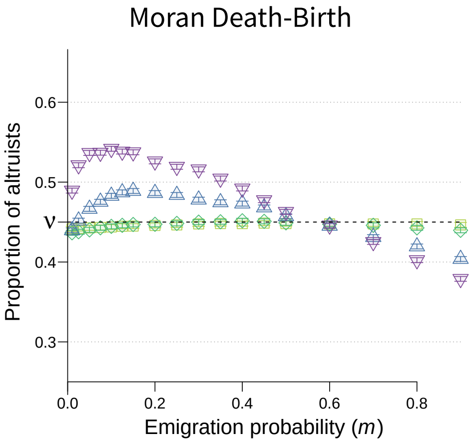


$$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.1)$$

Heterogeneous deme sizes ($\overline{n} = 4$ as before, but $2 \leq n \leq 5$)





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From theory to reality…

Take-Home Messages

- Under weak selection, it is possible to compute the expected frequency of social individuals, for any life-cycle, any regular population structure, any mutation probability. (Débarre, 2017, JTB)

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Funding & Thanks

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L. Kruuk & J. Reid
+ Ch. Mullon
for comments

and thank you for your attention!