Adaptation of my equations to a subdivided population. Notation, for a quantity Y that depends on two sites ($Y = e, d, Q$):

$$
\begin{aligned}
Y_{\mathrm{self}} &= Y_{i,i} && (1a)\\
Y_{\mathrm{in}} &= Y_{i,j}, \quad i \text{ and } j \neq i \text{ in the same deme;} && (1b)\\
Y_{\mathrm{out}} &= Y_{i,j}, \quad i \text{ and } j \text{ in different demes.} && (1c)
\end{aligned}
$$

For a site i, G_i denotes the deme it is in, and notation $j \in G_i$ means that sites i and j are in the same deme.

The expected frequency of altruists in the population is given by

$$
\mathbb{E}\left[\overline{X}\right] = p + \delta \frac{p(1-p)}{\mu}\left[\mathsf{b}\,(\beta^D - \beta^I) - \mathsf{c}\,(\gamma^D - \gamma^I)\right]. \tag{2}
$$

Moran, Birth-Death

$$
\begin{aligned}
\beta^D_{\mathrm{BD}} &= \sum_{k,\ell=1}^{N} \frac{1-\mu}{N} e_{kl} Q_{lk}\\
&= \sum_{k=1}^{N} \frac{1-\mu}{N}\Big(e_{\mathrm{self}} + (n-1) e_{\mathrm{in}} Q_{\mathrm{in}} + (N-n) e_{\mathrm{out}} Q_{\mathrm{out}}\Big)\\
&= (1-\mu)\Big(e_{\mathrm{self}} + (n-1) e_{\mathrm{in}} Q_{\mathrm{in}} + (N-n) e_{\mathrm{out}} Q_{\mathrm{out}}\Big). && (3a)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{I}_{\mathrm{BD}} =& \sum_{j,k,l=1}^{N} \left(\frac{d_{lj}}{N} - \frac{\mu}{N^2} \right) e_{kl} Q_{jk} \\
=& \frac{1}{N} \sum_{j=1}^{N} \Bigg[\left(\sum_{l=1}^{N} d_{lj} e_{jl} \right) + \sum_{\substack{k \in G_j \\ k \neq j}} \left(\sum_{l=1}^{N} d_{lj} e_{kl} Q_{\mathrm{in}} Q_{\mathrm{in}} \right) + \sum_{k \notin G_j} \sum_{l=1}^{N} d_{lj} \left(e_{kl} Q_{\mathrm{out}} Q_{\mathrm{out}} \right) \Bigg] \\
&+ \frac{\mu}{N^2} \sum_{j=1}^{N} \left(\sum_{l=1}^{N} e_{kl} \right) \left(\sum_{k=1}^{N} Q_{jk} \right) \\
=& \frac{1}{N} \sum_{j=1}^{N} \Bigg[d_{\mathrm{self}} e_{\mathrm{self}} + (n-1) d_{\mathrm{in}} e_{\mathrm{in}} + (N-n) d_{\mathrm{out}} e_{\mathrm{out}} \\
&\quad + \sum_{\substack{k \in G_j \\ k \neq j}} \left(d_{\mathrm{in}} e_{\mathrm{self}} + d_{\mathrm{self}} e_{\mathrm{in}} + (n-2) d_{\mathrm{in}} e_{\mathrm{in}} + (N-n) d_{\mathrm{out}} e_{\mathrm{out}} \right) Q_{\mathrm{in}} \\
&\quad + \sum_{k \notin G_j} \left(d_{\mathrm{self}} e_{\mathrm{out}} + (n-1) d_{\mathrm{in}} e_{\mathrm{out}} + d_{\mathrm{out}} e_{\mathrm{self}} + (n-1) d_{\mathrm{out}} e_{\mathrm{in}} + (N-2n) d_{\mathrm{out}} e_{\mathrm{out}} \right) Q_{\mathrm{out}} \Bigg] \\
&- \frac{\mu}{N} \left(1 + (n-1) Q_{\mathrm{in}} + (N-n) Q_{\mathrm{out}} \right) \left(e_{\mathrm{self}} + (n-1) e_{\mathrm{in}} + (N-n) e_{\mathrm{out}} \right) \\
=& d_{\mathrm{self}} e_{\mathrm{self}} + (n-1) d_{\mathrm{in}} e_{\mathrm{in}} + (N-n) d_{\mathrm{out}} e_{\mathrm{out}} \\
&+ (n-1) \left(d_{\mathrm{in}} e_{\mathrm{self}} + d_{\mathrm{self}} e_{\mathrm{in}} + (n-2) d_{\mathrm{in}} e_{\mathrm{in}} + (N-n) d_{\mathrm{out}} e_{\mathrm{out}} \right) Q_{\mathrm{in}} \\
&+ (N-n) \left(d_{\mathrm{self}} e_{\mathrm{out}} + (n-1) d_{\mathrm{in}} e_{\mathrm{out}} + d_{\mathrm{out}} e_{\mathrm{self}} + (n-1) d_{\mathrm{out}} e_{\mathrm{in}} + (N-2n) d_{\mathrm{out}} e_{\mathrm{out}} \right) Q_{\mathrm{out}} \\
&- \frac{\mu}{N} \left(1 + (n-1) Q_{\mathrm{in}} + (N-n) Q_{\mathrm{out}} \right) \left(e_{\mathrm{self}} + (n-1) e_{\mathrm{in}} + (N-n) e_{\mathrm{out}} \right). \qquad \text{(3b)}
\end{aligned}
$$

$$
\gamma^{D}_{\mathrm{BD}} = 1 - \mu. \qquad \text{(3c)}
$$

$$
\begin{aligned}
\gamma^{I}_{\mathrm{BD}} &= \frac{1}{N} \sum_{j,k=1}^{N} \left(d_{kj} - \frac{\mu}{N} \right) Q_{jk} \\
&= \frac{1}{N} \sum_{j=1}^{N} \left[d_{\mathrm{self}} - \frac{\mu}{N} + (n-1) \left(d_{\mathrm{in}} - \frac{\mu}{N} \right) Q_{\mathrm{in}} + (N-n) \left(d_{\mathrm{out}} - \frac{\mu}{N} \right) Q_{\mathrm{out}} \right] \\
&= d_{\mathrm{self}} + (n-1) d_{\mathrm{in}} Q_{\mathrm{in}} + (N-n) d_{\mathrm{out}} Q_{\mathrm{out}} \\
&\quad - \frac{\mu}{N} \left(1 + (n-1) Q_{\mathrm{in}} + (N-n) Q_{\mathrm{out}} \right) \qquad \text{(3d)}
\end{aligned}
$$

Moran, Death-Birth

$$\beta^D_{\mathrm{DB}} = \frac{1-\mu}{N}\sum_{j,k=1}^{N} Q_{jk}e_{jk} = \beta^D_{\mathrm{BD}}$$
$$= (1-\mu)\Big(e_{\mathrm{self}} + (n-1)e_{\mathrm{in}}Q_{\mathrm{in}} + (N-n)e_{\mathrm{out}}Q_{\mathrm{out}}\Big). \tag{4a}$$

$$\beta^I_{\mathrm{DB}} = \frac{1-\mu}{N}\sum_{i,j,k,l=1}^{N} d_{ji}d_{li}e_{kl}Q_{jk} \tag{4b}$$

Presented in the table in the appendix.

$$\gamma^D_{\mathrm{DB}} = 1-\mu = \gamma^D_{\mathrm{BD}}. \tag{4c}$$

$$\begin{aligned}
\gamma^I_{\mathrm{DB}} &= (1-\mu)\sum_{i,j,k=1}^{N}\frac{d_{ji}d_{ki}}{N}Q_{jk} \\
&= \frac{1-\mu}{N}\sum_{j=1}^{N}\sum_{i=1}^{N}\Big(d_{ji}d_{ji} + \sum_{\substack{k\neq j\\ k\in G_j}} d_{ji}d_{ki}Q_{\mathrm{in}} + \sum_{k\notin G_j} d_{ji}d_{ki}Q_{\mathrm{out}}\Big) \\
&= \frac{1-\mu}{N}\sum_{j=1}^{N}\Big[d_{\mathrm{self}}d_{\mathrm{self}} + (n-1)d_{\mathrm{in}}d_{\mathrm{in}} + (N-n)d_{\mathrm{out}}d_{\mathrm{out}} \\
&\quad + \Big(d_{\mathrm{self}}d_{\mathrm{in}} + d_{\mathrm{in}}d_{\mathrm{self}} + (n-2)d_{\mathrm{in}}d_{\mathrm{in}} + (N-n)d_{\mathrm{out}}d_{\mathrm{out}}\Big)Q_{\mathrm{in}} \\
&\quad + \Big(d_{\mathrm{self}}d_{\mathrm{out}} + (n-1)d_{\mathrm{in}}d_{\mathrm{out}} + d_{\mathrm{out}}d_{\mathrm{self}} + (n-1)d_{\mathrm{out}}d_{\mathrm{in}} + (N-2n)d_{\mathrm{out}}d_{\mathrm{out}}\Big)Q_{\mathrm{out}}\Big]
\end{aligned} \tag{4d}$$

Probabilities of identity by descent

WF est faux. Il faut utiliser les formules Fourier...!

Moran

For $i =\neq j$,

$$Q_{ij} = \frac{1-\mu}{2}\sum_{k=1}^{N}\left(d_{kj}Q_{ki} + d_{ki}Q_{kj}\right). \tag{5a}$$

For $j \neq i$, $j \in G_i$,

$$\begin{aligned}
Q_{\text{in}} = \frac{1-\mu}{2}\Big(& (d_{\text{in}} + d_{\text{self}} Q_{\text{in}}) + (d_{\text{self}} Q_{\text{in}} + d_{\text{in}}) \\
& + (n-2)\,(d_{\text{in}} Q_{\text{in}} + d_{\text{in}} Q_{\text{in}}) + (N-n)\,(d_{\text{out}} Q_{\text{out}} + d_{\text{out}} Q_{\text{out}})\Big) \\
= (1-\mu)\Big(& d_{\text{in}} + d_{\text{self}} Q_{\text{in}} + (n-2) d_{\text{in}} Q_{\text{in}} + (N-n) d_{\text{out}} Q_{\text{out}}\Big).
\end{aligned} \tag{5b}$$

And for $j \notin G_i$,

$$\begin{aligned}
Q_{\text{out}} = \frac{1-\mu}{2}\Big(& (d_{\text{out}} + d_{\text{self}} Q_{\text{out}}) + (n-1)\,(d_{\text{out}} Q_{\text{in}} + d_{\text{in}} Q_{\text{out}}) \\
& + (d_{\text{self}} Q_{\text{out}} + d_{\text{out}}) + (n-1)\,(d_{\text{in}} Q_{\text{out}} + d_{\text{out}} Q_{\text{in}}) \\
& + (N-2n)\,(d_{\text{out}} Q_{\text{out}} + d_{\text{out}} Q_{\text{out}})\Big) \\
= (1-\mu)\Big(& d_{\text{out}} + d_{\text{self}} Q_{\text{out}} + (n-1)\,(d_{\text{out}} Q_{\text{in}} + d_{\text{in}} Q_{\text{out}}) + (N-2n) d_{\text{out}} Q_{\text{out}}\Big)
\end{aligned} \tag{5c}$$

Wright-Fisher For $j \neq i$,

$$Q_{ij} = (1-\mu)^2 \sum_{k,l=1}^{N} d_{ki} d_{lj} Q_{kl}. \tag{6a}$$

When $j \neq i,\ j \in G_i$,

$$
\begin{aligned}
Q_{\text{in}} = (1-\mu)^2 \Bigg[& \Big(d_{\text{self}} d_{\text{in}} + d_{\text{in}} d_{\text{self}} + (n-2) d_{\text{in}} d_{\text{in}} + (N-n) d_{\text{out}} d_{\text{out}} \Big) \\
& + \Big(d_{\text{self}} d_{\text{self}} + (n-2) d_{\text{self}} d_{\text{in}} \\
& \qquad + (n-1) d_{\text{in}} d_{\text{in}} + (n-2) d_{\text{in}} d_{\text{self}} \\
& \qquad + (n-2)(n-2) d_{\text{in}} d_{\text{in}} + (N-n)(n-1) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{in}} \\
& + \Big((N-n) d_{\text{self}} d_{\text{out}} + (N-n)(n-1) d_{\text{in}} d_{\text{out}} \\
& \qquad + (N-n) d_{\text{out}} d_{\text{self}} + (N-n)(n-1) d_{\text{out}} d_{\text{in}} \\
& \qquad + (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg] \\
= (1-\mu)^2 \Bigg[& \Big(2 d_{\text{in}} d_{\text{self}} + (n-2) d_{\text{in}}^2 + (N-n) d_{\text{out}}^2 \Big) \\
& + \Big(d_{\text{self}}^2 + 2(n-2) d_{\text{self}} d_{\text{in}} + (n^2-3n+3) d_{\text{in}}^2 + + (N-n)(n-1) d_{\text{out}}^2 \Big) Q_{\text{in}} \\
& + \Big(2(N-n) d_{\text{self}} d_{\text{out}} + 2(N-n)(n-1) d_{\text{in}} d_{\text{out}} \\
& \qquad + (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg]
\end{aligned}
\tag{6b}
$$

And when $j \notin G_i$, we have

$$
\begin{aligned}
Q_{\text{out}} = (1-\mu)^2 \Bigg[& \Big(2 d_{\text{self}} d_{\text{out}} + 2(n-1) d_{\text{in}} d_{\text{out}} + (N-2n) d_{\text{out}}^2 \Big) \\
& + \Big(2(n-1) d_{\text{self}} d_{\text{out}} + 2(n-1)^2 d_{\text{in}} d_{\text{out}} + (N-2n)(n-1) d_{\text{out}}^2 \Big) Q_{\text{in}} \\
& + \Big(d_{\text{self}} d_{\text{self}} + (n-1) d_{\text{self}} d_{\text{in}} + (N-2n) d_{\text{self}} d_{\text{out}} \\
& \qquad + (n-1) d_{\text{in}} d_{\text{self}} + (n-1)^2 d_{\text{in}}^2 + (n-1)(N-2n) d_{\text{in}} d_{\text{out}} \\
& \qquad + (N-n) d_{\text{out}} d_{\text{self}} + (N-n)(n-1) d_{\text{out}} d_{\text{in}} + (N-n)(N-2n) d_{\text{out}} d_{\text{out}} \Big) Q_{\text{out}} \Bigg].
\end{aligned}
\tag{6c}
$$

PAS FINI

Appendix

All combinations for i, j, k, l. Notation: (i, j) means that i and j are in the same deme, but are different; G_i refers to the deme containing site i.

	j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
1	$j=i$	$k=i$	$l=i$	$(i=j=k=l)$	1	d_{self}	d_{self}	e_{self}	1
2	$j=i$	$k=i$	$l\neq i; l\in G_i$	$(i=j=k,l)$	$n-1$	d_{self}	d_{in}	e_{in}	1
3	$j=i$	$k=i$	$l\notin G_i$	$(i=j=k),(l)$	$N-n$	d_{self}	d_{out}	e_{out}	1
4	$j=i$	$k\neq i; k\in G_i$	$l=i$	$(i=j=l,k)$	$n-1$	d_{self}	d_{self}	e_{in}	Q_{in}
5	$j=i$	$k\neq i; k\in G_i$	$l=k$	$(i=j,k=l)$	$n-1$	d_{self}	d_{in}	e_{self}	Q_{in}
6	$j=i$	$k\neq i; k\in G_i$	$l\neq i,k; l\in G_i$	$(i=j,k,l)$	$(n-1)(n-2)$	d_{self}	d_{in}	e_{in}	Q_{in}
7	$j=i$	$k\neq i; k\in G_i$	$l\notin G_i$	$(i=j,k),(l)$	$(n-1)(N-n)$	d_{self}	d_{out}	e_{out}	Q_{in}
8	$j=i$	$k\notin G_i$	$l=i=j$	$(i=j=l),(k)$	$(N-n)$	d_{self}	d_{self}	e_{out}	Q_{out}
9	$j=i$	$k\notin G_i$	$l\neq i, l\in G_i$	$(i=j,l),(k)$	$(N-n)(n-1)$	d_{self}	d_{in}	e_{out}	Q_{out}
10	$j=i$	$k\notin G_i$	$l=k$	$(i=j),(k=l)$	$(N-n)$	d_{self}	d_{out}	e_{self}	Q_{out}
11	$j=i$	$k\notin G_i$	$l\neq k; l\in G_k$	$(i=j),(k,l)$	$(N-n)(n-1)$	d_{self}	d_{out}	e_{in}	Q_{out}
12	$j=i$	$k\notin G_i$	$l\notin G_i, G_k$	$(i=j),(k),(l)$	$(N-n)(N-2n)$	d_{self}	d_{out}	e_{out}	Q_{out}
13	$j\neq i, j\in G_i$	$k=i$	$l=i$	$(i=k=l,j)$	$(n-1)$	d_{in}	d_{self}	e_{self}	Q_{in}
14	$j\neq i, j\in G_i$	$k=i$	$l=j$	$(i=k,j=l)$	$(n-1)$	d_{in}	d_{in}	e_{in}	Q_{in}
15	$j\neq i, j\in G_i$	$k=i$	$l\neq i,j; l\in G_i$	$(i=k,j,l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	Q_{in}
16	$j\neq i, j\in G_i$	$k=i$	$l\notin G_i$	$(i=k,j),(l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{out}	Q_{in}
17	$j\neq i, j\in G_i$	$k=j$	$l=i$	$(i=l,j=k)$	$(n-1)$	d_{in}	d_{self}	e_{in}	1
18	$j\neq i, j\in G_i$	$k=j$	$l=j$	$(i,j=k=l)$	$(n-1)$	d_{in}	d_{in}	e_{self}	1
19	$j\neq i, j\in G_i$	$k=j$	$l\neq i,j; l\in G_i$	$(i,j=k,l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	1
20	$j\neq i, j\in G_i$	$k=j$	$l\notin G_i$	$(i,j=k),(l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{out}	1
21	$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=i$	$(i=l,j,k)$	$(n-1)(n-2)$	d_{in}	d_{self}	e_{in}	Q_{in}
22	$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=j$	$(i,j=l,k)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{in}	Q_{in}
23	$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l=k$	$(i,j,k=l)$	$(n-1)(n-2)$	d_{in}	d_{in}	e_{self}	Q_{in}
24	$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l\neq i,j,k; l\in G_i$	(i,j,k,l)	$(n-1)(n-2)(n-3)$	d_{in}	d_{in}	e_{in}	Q_{in}
25	$j\neq i, j\in G_i$	$k\neq i,j; k\in G_i$	$l\notin G_i$	$(i,j,k),(l)$	$(n-1)(n-2)(N-n)$	d_{in}	d_{out}	e_{out}	Q_{in}

	j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
26	$j \neq i; j \in G_i$	$k \notin G_i$	$l = i$	$(i = l, j), (k)$	$(n-1)(N-n)$	d_{in}	d_{self}	e_{out}	Q_{out}
27	$j \neq i; j \in G_i$	$k \notin G_i$	$l = j$	$(i, j = l), (k)$	$(n-1)(N-n)$	d_{in}	d_{in}	e_{out}	Q_{out}
28	$j \neq i; j \in G_i$	$k \notin G_i$	$l \neq i, j; l \in G_i$	$(i, j, l), (k)$	$(n-1)(N-n)(n-2)$	d_{in}	d_{in}	e_{out}	Q_{out}
29	$j \neq i; j \in G_i$	$k \notin G_i$	$l = k$	$(i, j), (k = l)$	$(n-1)(N-n)$	d_{in}	d_{out}	e_{self}	Q_{out}
30	$j \neq i; j \in G_i$	$k \notin G_i$	$l \neq k; l \in G_k$	$(i, j), (k, l)$	$(n-1)(N-n)(n-1)$	d_{in}	d_{out}	e_{in}	Q_{out}
31	$j \neq i; j \in G_i$	$k \notin G_i$	$l \notin G_i, G_k$	$(i, j), (k), (l)$	$(n-1)(N-n)(N-2n)$	d_{in}	d_{out}	e_{out}	Q_{out}
32	$j \notin G_i$	$k = i$	$l = i$	$(i = k = l), (j)$	$(N-n)$	d_{out}	d_{self}	e_{self}	Q_{out}
33	$j \notin G_i$	$k = i$	$l \neq i; l \in G_i$	$(i = k, l), (j)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{in}	Q_{out}
34	$j \notin G_i$	$k = i$	$l = j$	$(i = k), (j = l)$	$(N-n)$	d_{out}	d_{out}	e_{out}	Q_{out}
35	$j \notin G_i$	$k = i$	$l \neq j; l \in G_j$	$(i = k), (j, l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
36	$j \notin G_i$	$k = i$	$l \notin G_i, G_j$	$(i = k), (j), (l)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
37	$j \notin G_i$	$k \neq i; k \in G_i$	$l = i$	$(i = l, k), (j)$	$(N-n)(n-1)$	d_{out}	d_{self}	e_{in}	Q_{out}
38	$j \notin G_i$	$k \neq i; k \in G_i$	$l = k$	$(i, k = l), (j)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{self}	Q_{out}
39	$j \notin G_i$	$k \neq i; k \in G_i$	$l \neq i, k; l \in G_i$	$(i, k, l), (j)$	$(N-n)(n-1)(n-2)$	d_{out}	d_{in}	e_{in}	Q_{out}
40	$j \notin G_i$	$k \neq i; k \in G_i$	$l = j$	$(i, k), (j = l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
41	$j \notin G_i$	$k \neq i; k \in G_i$	$l \neq j; l \in G_j$	$(i, k), (j, l)$	$(N-n)(n-1)(n-1)$	d_{out}	d_{out}	e_{out}	Q_{out}
42	$j \notin G_i$	$k \neq i; k \in G_i$	$l \notin G_i, G_j$	$(i, k), (j), (l)$	$(N-n)(n-1)(N-2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
43	$j \notin G_i$	$k = j$	$l = i$	$(i = l), (j = k)$	$(N-n)$	d_{out}	d_{self}	e_{out}	1
44	$j \notin G_i$	$k = j$	$l \neq i; l \in G_i$	$(i, l), (j = k)$	$(N-n)(n-1)$	d_{out}	d_{in}	e_{out}	1
45	$j \notin G_i$	$k = j$	$l = j$	$(i), (j = k = l)$	$(N-n)$	d_{out}	d_{out}	e_{self}	1
46	$j \notin G_i$	$k = j$	$l \neq j; l \in G_j$	$(i), (j = k, l)$	$(N-n)(n-1)$	d_{out}	d_{out}	e_{in}	1
47	$j \notin G_i$	$k = j$	$l \notin G_i, G_j$	$(i), (j = k), (l)$	$(N-n)(N-2n)$	d_{out}	d_{out}	e_{out}	1

	j	k	l	Notation	Count	d_{ji}	d_{li}	e_{kl}	Q_{jk}
48	$j \notin G_i$	$k \neq j; k \in G_j$	$l = i$	$(i = l), (j, k)$	$(N - n)(n - 1)$	d_{out}	d_{self}	e_{out}	Q_{in}
49	$j \notin G_i$	$k \neq j; k \in G_j$	$l \neq i; l \in G_i$	$(i, l), (j, k)$	$(N - n)(n - 1)(n - 1)$	d_{out}	d_{in}	e_{out}	Q_{in}
50	$j \notin G_i$	$k \neq j; k \in G_j$	$l = j$	$(i), (j = l, k)$	$(N - n)(n - 1)$	d_{out}	d_{out}	e_{in}	Q_{in}
51	$j \notin G_i$	$k \neq j; k \in G_j$	$l = k$	$(i), (j, k = l)$	$(N - n)(n - 1)$	d_{out}	d_{out}	e_{self}	Q_{in}
52	$j \notin G_i$	$k \neq j; k \in G_j$	$l \neq j, k; l \in G_j$	$(i), (j, k, l)$	$(N - n)(n - 1)(n - 2)$	d_{out}	d_{out}	e_{in}	Q_{in}
53	$j \notin G_i$	$k \neq j; k \in G_j$	$l \notin G_i, G_j$	$(i), (j, k), (l)$	$(N - n)(n - 1)(N - 2n)$	d_{out}	d_{out}	e_{out}	Q_{in}
54	$j \notin G_i$	$k \notin G_i, G_j$	$l = i$	$(i = l), (j), (k)$	$(N - n)(N - 2n)$	d_{out}	d_{self}	e_{out}	Q_{out}
55	$j \notin G_i$	$k \notin G_i, G_j$	$l \neq i; l \in G_i$	$(i, l), (j), (k)$	$(N - n)(N - 2n)(n - 1)$	d_{out}	d_{in}	e_{out}	Q_{out}
56	$j \notin G_i$	$k \notin G_i, G_j$	$l = j$	$(i), (j = l), (k)$	$(N - n)(N - 2n)$	d_{out}	d_{out}	e_{out}	Q_{out}
57	$j \notin G_i$	$k \notin G_i, G_j$	$l \neq j; l \in G_j$	$(i), (j, l), (k)$	$(N - n)(N - 2n)(n - 1)$	d_{out}	d_{out}	e_{out}	Q_{out}
58	$j \notin G_i$	$k \notin G_i, G_j$	$l = k$	$(i), (j), (k = l)$	$(N - n)(N - 2n)$	d_{out}	d_{out}	e_{self}	Q_{out}
59	$j \notin G_i$	$k \notin G_i, G_j$	$l \neq k; l \in G_k$	$(i), (j), (k, l)$	$(N - n)(N - 2n)(n - 1)$	d_{out}	d_{out}	e_{in}	Q_{out}
60	$j \notin G_i$	$k \notin G_i, G_j$	$l \notin G_i, G_j, G_k$	$(i), (j), (k), (l)$	$(N - n)(N - 2n)(N - 3n)$	d_{out}	d_{out}	e_{out}	Q_{out}

A Island model

With self replacement

$$d_{\text{self}} = d_{\text{in}} = \frac{1-m}{n}, \tag{7a}$$
$$d_{\text{out}} = \frac{m}{N-n}. \tag{7b}$$

Without self-replacement

$$d_{\text{self}} = 0, \tag{8a}$$
$$d_{\text{in}} = \frac{1-m}{n-1}, \tag{8b}$$
$$d_{\text{out}} = \frac{m}{N-n}. \tag{8c}$$

B IDB

B.1 Moran

Using the formulas for a 2D graph in REF Debarre 2017,

$$\tilde{\mathcal{D}}_{\substack{q_1\\q_2}} = \sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1} \tilde{d}_{\substack{l_1\\l_2}} \exp\left(-\imath\frac{2\pi q_1 l_1}{N_1}\right)\exp\left(-\imath\frac{2\pi q_2 l_2}{N_2}\right) \tag{9a}$$
$$\tilde{\mathcal{Q}}_{\substack{r_1\\r_2}} = \frac{1}{N}\sum_{q_1=0}^{N_1-1}\sum_{q_2=0}^{N_2-1} \frac{\mu\lambda'_M}{1-(1-\mu)\tilde{\mathcal{D}}_{\substack{q_1\\q_2}}} \exp\left(\imath\frac{2\pi q_1 r_1}{N_1}\right)\exp\left(\imath\frac{2\pi q_2 r_2}{N_2}\right) \tag{9b}$$

We have

$$\begin{aligned}\tilde{\mathcal{D}}_{\substack{q_1\\q_2}} &= d_{\text{self}} + \sum_{l_2=1}^{N_2-1} d_{\text{in}} \exp\left(-\imath\frac{2\pi q_2 l_2}{N_2}\right) + \sum_{l_1=1}^{N_1-1}\sum_{l_2=0}^{N_2-1} d_{\text{out}} \exp\left(-\imath\frac{2\pi q_1 l_1}{N_1}\right)\exp\left(-\imath\frac{2\pi q_2 l_2}{N_2}\right)\\
&= d_{\text{self}} + \left(\delta_{q_2}(N_2-1) + (1-\delta_{q_2})(-1)\right) d_{\text{in}} + \left(\delta_{q_1}(N_1-1)+(1-\delta_{q_1})(-1)\right)\left(\delta_{q_2}N_2\right) d_{\text{out}}\\
&= d_{\text{self}} + \left(\delta_{q_2}N_2-1\right) d_{\text{in}} + \left(\delta_{q_1}N_1-1\right)\delta_{q_2}N_2 d_{\text{out}}.\end{aligned} \tag{10a}$$

Whether there is self-replacement or not, we have $N_1 = D$ and $N_2 = n$, and

$$\tilde{\mathcal{D}}_{\substack{0\\0}} = 1, \tag{11a}$$
$$\tilde{\mathcal{D}}_{\substack{q_1\\0}} = 1 - m - \frac{m}{d-1} \quad (q_1 \not\equiv 0 \pmod{N_1}), \tag{11b}$$
$$\tilde{\mathcal{D}}_{\substack{q_1\\q_2}} = d_{\text{self}} - d_{\text{in}} \quad (q_2 \not\equiv 0 \pmod{N_2}). \tag{11c}$$

So for $\tilde{\mathcal{Q}}$,

$$\begin{aligned}
\tilde{\mathcal{Q}}_{\substack{r_1\\r_2}} &= \frac{\mu\lambda'_M}{N}\Bigg[\frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{\substack{0\\0}}} + \sum_{q_2=1}^{N_2-1}\frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{\substack{0\\q_2}}}\exp\left(-\imath\frac{2\pi q_2 r_2}{N_2}\right) + \sum_{q_1=1}^{N_1-1}\frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{\substack{q_1\\0}}}\exp\left(-\imath\frac{2\pi q_1 r_1}{N_1}\right)\\
&\quad + \sum_{q_1=1}^{N_1-1}\sum_{q_2=1}^{N_2-1}\frac{1}{1-(1-\mu)\tilde{\mathcal{D}}_{\substack{q_1\\q_2}}}\exp\left(-\imath\frac{2\pi q_1 r_1}{N_1}\right)\exp\left(-\imath\frac{2\pi q_2 r_2}{N_2}\right)\Bigg]\\
&= \frac{\mu\lambda'_M}{N}\Bigg[\frac{1}{1-(1-\mu)} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(\delta_{r_2}N_2-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(\delta_{r_1}N_1-1)\\
&\quad + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(\delta_{r_1}N_1-1)(\delta_{r_2}N_2-1)\Bigg]. \qquad (12a)
\end{aligned}$$

In particular,

$$\begin{aligned}
\tilde{\mathcal{Q}}_{\substack{0\\0}} &= \frac{\mu\lambda'_M}{N}\Bigg[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(n-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(D-1)\\
&\quad + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(D-1)(n-1)\Bigg]\\
&= 1. \qquad (12b)
\end{aligned}$$

We find λ'_M using the above equation. When $r_1 = 0$, the two individuals are in the same deme. They are different when $r_2 \not\equiv 0$:

$$\begin{aligned}
Q_{\text{in}} &= \frac{\mu\lambda'_M}{N}\Bigg[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(D-1)\\
&\quad + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(D-1)(-1)\Bigg]. \qquad (12c)
\end{aligned}$$

And when $r_1 \not\equiv 0$, the two individuals are in different demes:

$$\begin{aligned}
Q_{\text{out}} &= \frac{\mu\lambda'_M}{N}\Bigg[\frac{1}{\mu} + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}(-1) + \frac{1}{1-(1-\mu)(1-m-\frac{m}{d-1})}(-1)\\
&\quad + \frac{1}{1-(1-\mu)(d_{\text{self}}-d_{\text{in}})}\Bigg]. \qquad (12d)
\end{aligned}$$

B.2 Wright-Fisher

$$
\begin{aligned}
\tilde{\mathcal{Q}}_{\substack{r_1\\r_2}} &= \frac{1}{N}\sum_{q_1=0}^{N_1-1}\sum_{q_2=0}^{N_2-1}\frac{\mu\lambda'_{WF}}{1-(1-\mu)^2(\tilde{\mathcal{D}}_{\substack{q_1\\q_2}})^2}\exp\left(-\imath\frac{2\pi q_1 r_1}{N_1}\right)\exp\left(-\imath\frac{2\pi q_2 r_2}{N_2}\right)\\
&= \frac{1}{N}\Bigg[\frac{\mu\lambda'_{WF}}{1-(1-\mu)^2(\tilde{\mathcal{D}}_{\substack{0\\0}})^2}+\sum_{q_2=1}^{N_2-1}\frac{\mu\lambda'_{WF}}{1-(1-\mu)^2(\tilde{\mathcal{D}}_{\substack{0\\q_2}})^2}\exp\left(-\imath\frac{2\pi q_2 r_2}{N_2}\right)\\
&\quad+\sum_{q_1=1}^{N_1-1}\frac{\mu\lambda'_{WF}}{1-(1-\mu)^2(\tilde{\mathcal{D}}_{\substack{q_1\\0}})^2}\exp\left(-\imath\frac{2\pi q_1 r_1}{N_1}\right)\\
&\quad+\sum_{q_1=1}^{N_1-1}\sum_{q_2=1}^{N_2-1}\frac{\mu\lambda'_{WF}}{1-(1-\mu)^2(\tilde{\mathcal{D}}_{\substack{q_1\\q_2}})^2}\exp\left(-\imath\frac{2\pi q_1 r_1}{N_1}\right)\exp\left(-\imath\frac{2\pi q_2 r_2}{N_2}\right)\Bigg] \qquad (13)\\
&= \frac{\mu\lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2}+\frac{1}{1-(1-\mu)^2(d_{\text{self}}-d_{\text{in}})^2}(\delta_{q_2}N_2-1)\\
&\quad+\frac{1}{1-(1-\mu)^2(1-m-\frac{m}{d-1})^2}(\delta_{q_1}N_1-1)\\
&\quad+\frac{1}{1-(1-\mu)^2(d_{\text{self}}-d_{\text{in}})^2}(\delta_{q_1}N_1-1)(\delta_{q_2}N_2-1)\Bigg]\\
&= \frac{\mu\lambda'_{WF}}{N}\Bigg[\frac{1}{1-(1-\mu)^2}+\frac{1}{1-(1-\mu)^2(d_{\text{self}}-d_{\text{in}})^2}(\delta_{q_2}N_2-1)\delta_{q_1}N_1\\
&\quad+\frac{1}{1-(1-\mu)^2(1-m-\frac{m}{d-1})^2}(\delta_{q_1}N_1-1)\Bigg]. \qquad (14)
\end{aligned}
$$

To find λ'_{WF}, we solve

$$
1=\frac{\mu\lambda'_{WF}}{N}\left[\frac{1}{1-(1-\mu)^2}+\frac{1}{1-(1-\mu)^2(d_{\text{self}}-d_{\text{in}})^2}(N_2-1)N_1+\frac{1}{1-(1-\mu)^2(1-m-\frac{m}{d-1})^2}(N_1-1)\right]. \qquad (15a)
$$

Then,

$$
Q_{\text{in}}=\frac{\mu\lambda'_{WF}}{N}\left[\frac{1}{1-(1-\mu)^2}-\frac{1}{1-(1-\mu)^2(d_{\text{self}}-d_{\text{in}})^2}N_1+\frac{1}{1-(1-\mu)^2(1-m-\frac{m}{d-1})^2}(N_1-1)\right]. \qquad (15b)
$$

and

$$
Q_{\text{out}}=\frac{\mu\lambda'_{WF}}{N}\left[\frac{1}{1-(1-\mu)^2}-\frac{1}{1-(1-\mu)^2(1-m-\frac{m}{d-1})^2}\right]. \qquad (15c)
$$