

Fidelity of parent-offspring transmission and the evolution of social behavior in subdivided populations.

F. Débarre



@flodebarre

CNRS

Centre de Recherches Interdisciplinaires en Biologie, Paris



Expected frequency of altruists in the population

$$\mathbb{E}[\bar{X}] = \nu + \delta \nu(1 - \nu) \frac{1 - \mu}{\mu} (1 - Q_{\text{out}}) \times$$

$$\left(-c - (b - c) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right.$$

$$\left. + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[b - (b - c)(n - 1) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right] \right)$$

Expected frequency of altruists in the population

Mutation-drift
equilibrium

$$\mathbb{E}[\bar{X}] = \nu + \delta \nu(1 - \nu) \frac{1 - \mu}{\mu} (1 - Q_{\text{out}}) \times$$
$$\left(-c - (b - c) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right.$$
$$\left. + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[b - (b - c)(n - 1) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right] \right)$$

Expected frequency of altruists in the population

Mutation-drift
equilibrium Selection
strength

$$\mathbb{E}[\bar{X}] = \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \nu(1 - \nu) \frac{1 - \mu}{\mu} (1 - Q_{\text{out}}) \times$$

$$\left(-c - (b - c) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right.$$

$$\left. + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[b - (b - c)(n - 1) \left(\frac{(1 - m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right) \right] \right)$$

Expected frequency of altruists in the population

Mutation-drift equilibrium
 Selection strength
 Population variance
 Variance in the state of one site

$$\mathbb{E}[\bar{X}] = \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times$$

$$\left(-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right] \right)$$

Expected frequency of altruists in the population

Mutation-drift equilibrium
 Selection strength
 Population variance
 Variance in the state of one site

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{\text{Variance in the state of one site}} - c \right. \\
 & \left. + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right] \right)
 \end{aligned}$$

Expected frequency of altruists in the population

Mutation-drift equilibrium
 Selection strength
 Population variance
 Variance in the state of one site

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{-c} \right. \\
 & \left. + \frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}} \left[\underbrace{b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{\mathcal{B}} \right] \right)
 \end{aligned}$$

Expected frequency of altruists in the population

Mutation-drift equilibrium
 Selection strength
 Population variance
 Variance in the state of one site

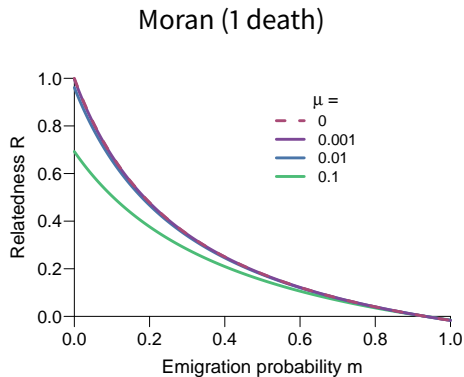
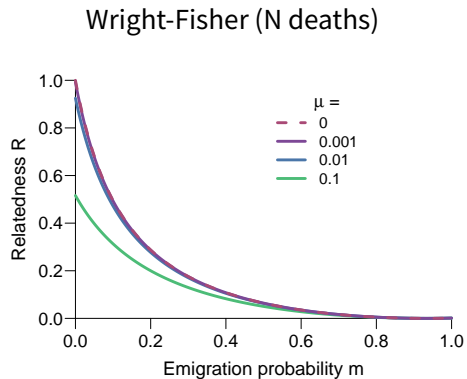
$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{\text{Variance in the state of one site}} - c \right. \\
 & \left. + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_R \left[\underbrace{b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_B \right] \right)
 \end{aligned}$$

Expected frequency of altruists in the population

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance: Variance in the state of one site}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c}_{\text{purple}} - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right) \underbrace{-c}_{\text{purple}} \\
 & + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_R \left[\underbrace{b}_{\text{green}} - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right] \underbrace{\quad}_{\mathcal{B}} \Big)
 \end{aligned}$$

How does relatedness R change with the emigration probability m ?

How does relatedness R change with the emigration probability m ?



$$(n = 4, N_d = 15)$$

Expected frequency of altruists in the population

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance: Variance in the state of one site}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c}_{\text{Cost}} - \underbrace{(b-c)}_{\text{Benefit}} \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right) - c \\
 & + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_R \left[\underbrace{b}_{\mathcal{B}} - \underbrace{(b-c)(n-1)}_{\mathcal{B}} \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right) \right] \Bigg)
 \end{aligned}$$

Expected frequency of altruists in the population

Mutation-drift equilibrium Selection strength Population variance
 Variance in the state of one site

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{\text{Population variance}} - c \right) \\
 & + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_R \left[\underbrace{b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{\mathcal{B}} \right]
 \end{aligned}$$

Expected frequency of altruists in the population

Mutation-drift equilibrium
 Selection strength
 Population variance
 Variance in the state of one site

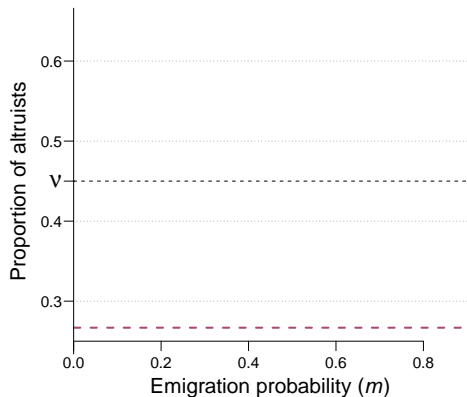
$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \frac{1-\mu}{\mu} (1-Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b-c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{-C \nearrow} \right. \\
 & \left. + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_{R \searrow} \left[\underbrace{b - (b-c)(n-1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d-1)} \right)}_{B \nearrow} \right] \right)
 \end{aligned}$$

Expected frequency of altruists in the population

$$\begin{aligned}
 \mathbb{E}[\bar{X}] = & \underbrace{\nu}_{\text{Mutation-drift equilibrium}} + \underbrace{\delta}_{\text{Selection strength}} \underbrace{\nu(1-\nu)}_{\text{Population variance}} \underbrace{\frac{1-\mu}{\mu}}_{\text{Variance in the state of one site}} (1 - Q_{\text{out}}) \times \\
 & \left(\underbrace{-c - (b - c) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right)}_{-C \nearrow} \right. \\
 & \left. + \underbrace{\frac{Q_{\text{in}} - Q_{\text{out}}}{1 - Q_{\text{out}}}}_{R \searrow} \left[\underbrace{b - (b - c)(n - 1) \left(\frac{(1-m)^2}{n} + \frac{m^2}{n(N_d - 1)} \right)}_{B \nearrow} \right] \right)
 \end{aligned}$$

Effect of the emigration probability m on the expected proportion of altruists

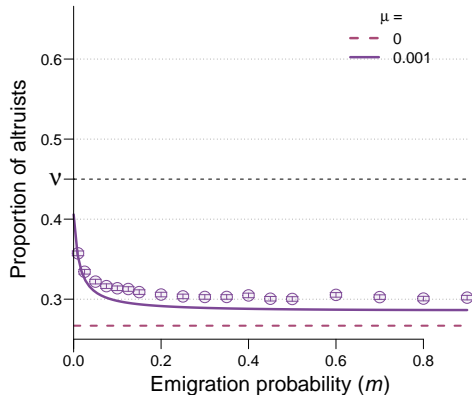
Wright-Fisher (N deaths & N births)



$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$

Effect of the emigration probability m on the expected proportion of altruists

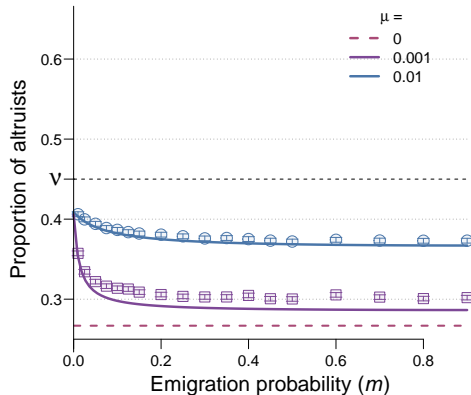
Wright-Fisher (N deaths & N births)



($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

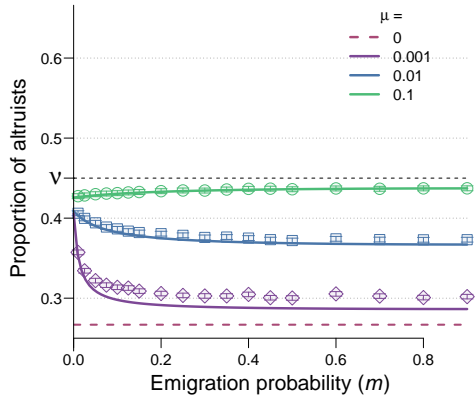
Wright-Fisher (N deaths & N births)



($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

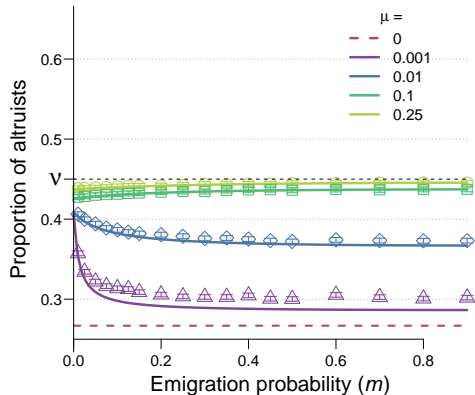
Wright-Fisher (N deaths & N births)



($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

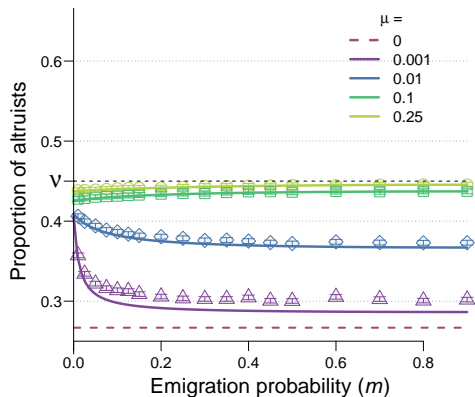
Wright-Fisher (N deaths & N births)



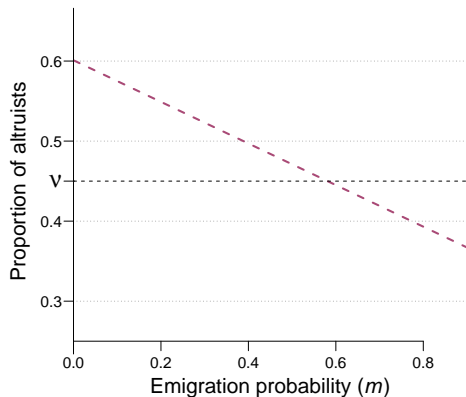
($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

Wright-Fisher (N deaths & N births)



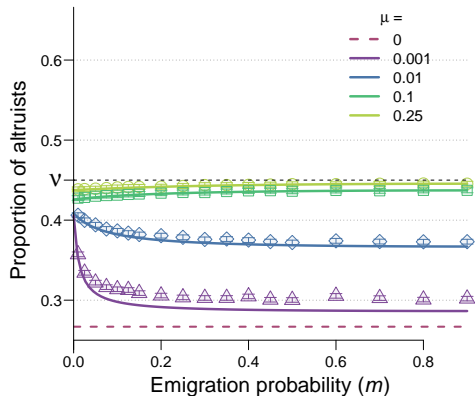
Moran Death-Birth (1 death & 1 birth)



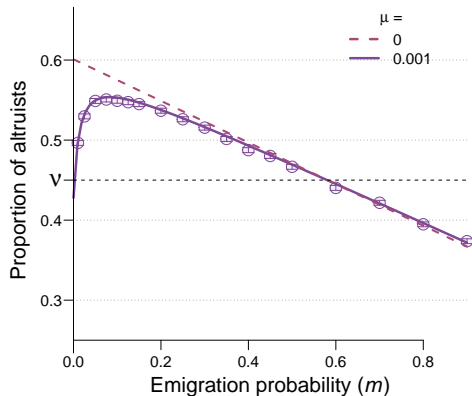
($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

Wright-Fisher (N deaths & N births)



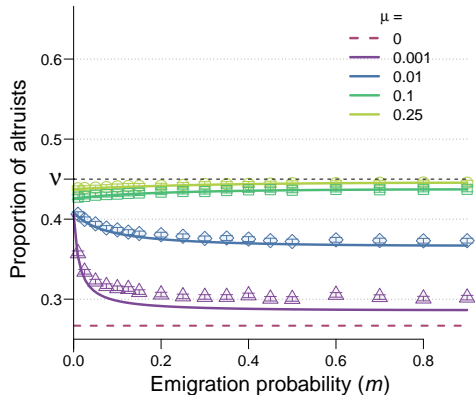
Moran Death-Birth (1 death & 1 birth)



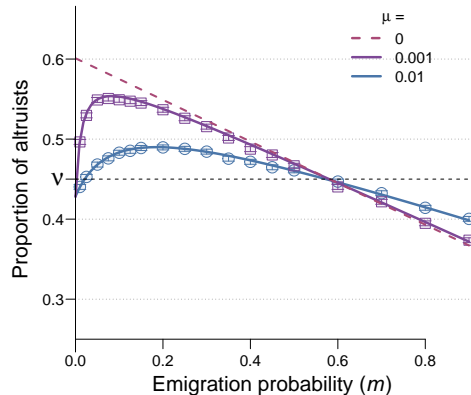
($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

Wright-Fisher (N deaths & N births)



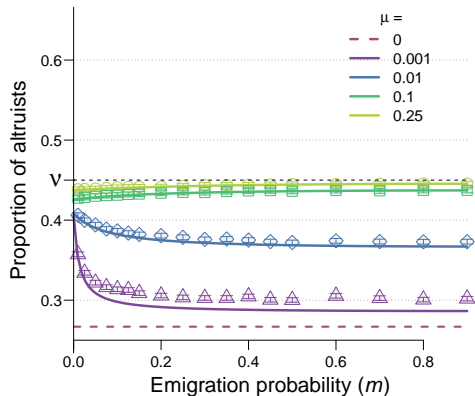
Moran Death-Birth (1 death & 1 birth)



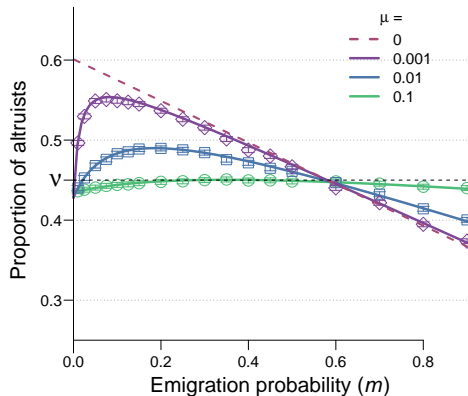
($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

Wright-Fisher (N deaths & N births)



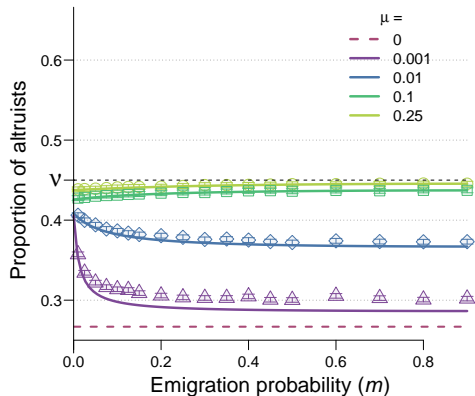
Moran Death-Birth (1 death & 1 birth)



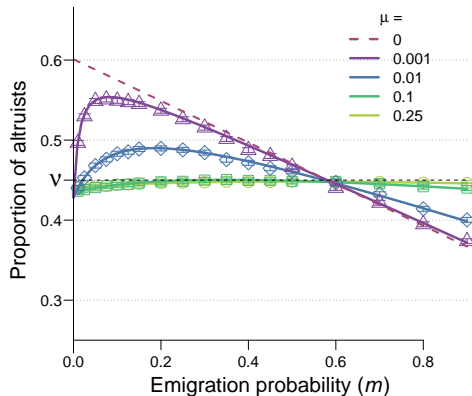
($b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005$)

Effect of the emigration probability m on the expected proportion of altruists

Wright-Fisher (N deaths & N births)



Moran Death-Birth (1 death & 1 birth)



$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$

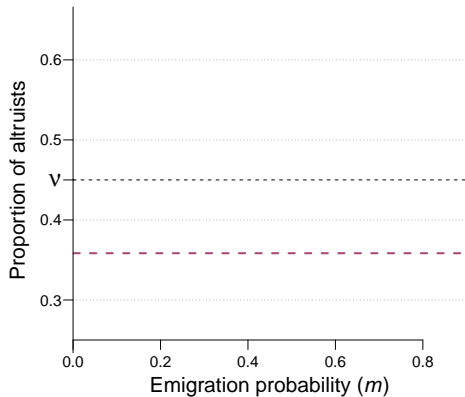
Is the result robust?

Another life-cycle

Moran Birth-Death (1 birth & 1 death)

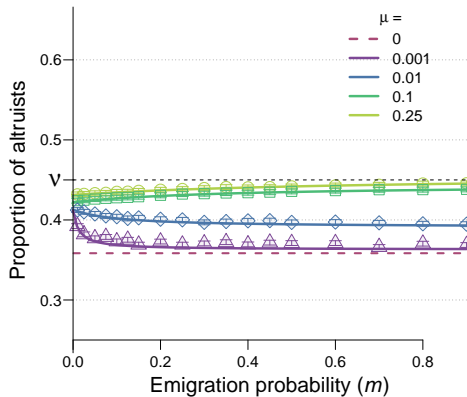
$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$

Moran Birth-Death (1 birth & 1 death)



$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$

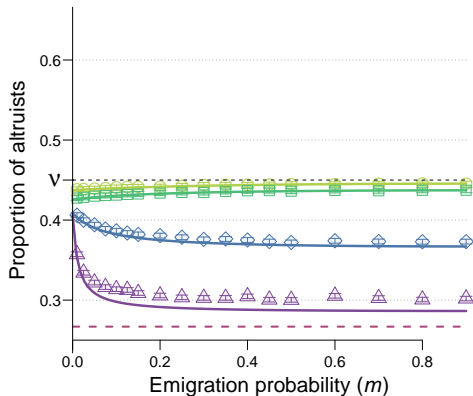
Moran Birth-Death (1 birth & 1 death)



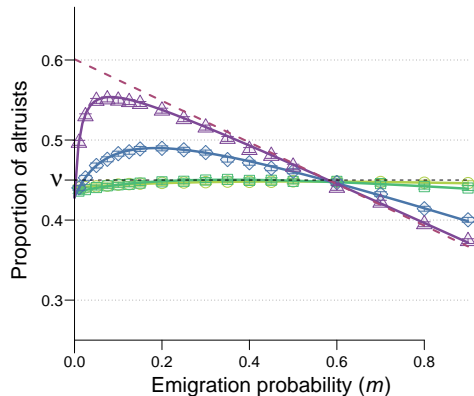
$$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$$

Strong selection

Wright-Fisher, weak selection



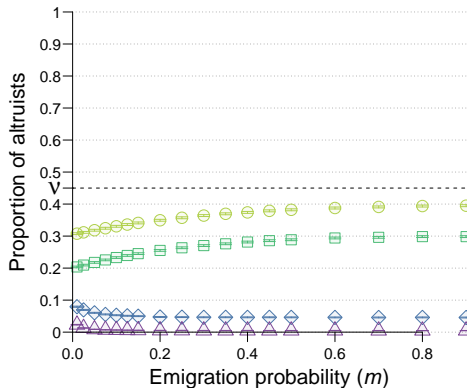
Moran Death-Birth, weak selection



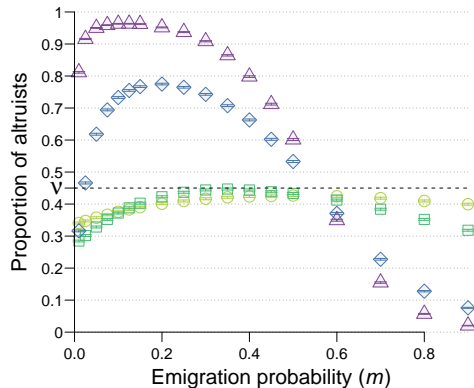
$$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.005)$$

Strong selection

Wright-Fisher, strong selection



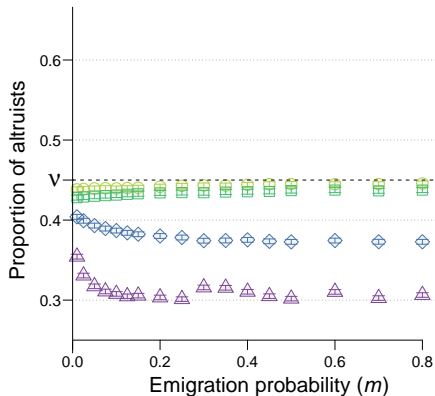
Moran Death-Birth, strong selection



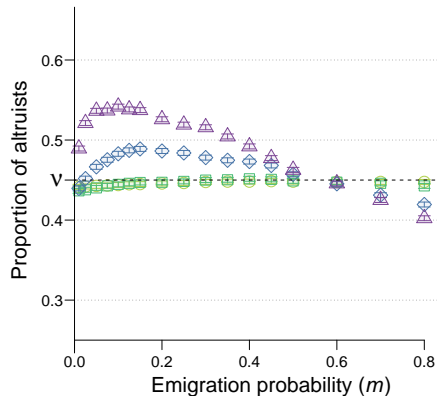
$$(b = 15, c = 1, n = 4, N_d = 15, \delta = 0.1)$$

Heterogeneous deme sizes ($\bar{n} = 4$ as before, but $2 \leq n \leq 5$)

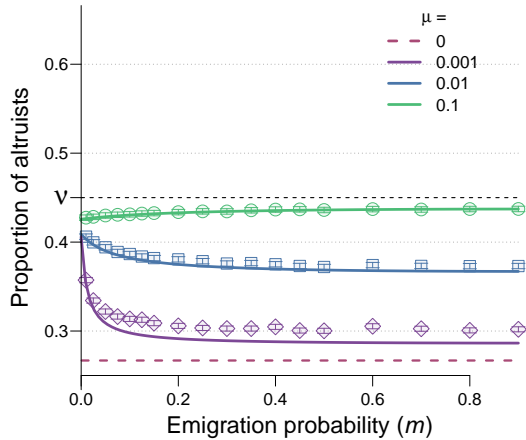
Wright-Fisher



Moran Death-Birth



($b = 15, c = 1, \bar{n} = 4, N_d = 15, \delta = 0.005$)



bfjdklf

fjdksl fjj fjkds lj