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THE OBSERVED TROJANS AND THE GLOBAL DYNAMICS AROUND THE LAGRANGIAN POINTS OF THE SUN–JUPITER SYSTEM

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Abstract. In this paper, we make a systematic study of the global dynamical structure of the Sun–Jupiter L_4 tadpole region. The results are based on long-time simulations of the Trojans in the Sun, Jupiter, Saturn system and on the frequency analysis of these orbits. We give some initial results in the description of the resonant structure that guides the long-term dynamics of this region. Moreover, we are able to connect this global view of the phase space with the observed Trojans and identify resonances in which some of the real bodies are located.

Key words: Arnold web, Frequency Map analysis, Trojan asteroids, resonances

1. Introduction

The long-term stability of the Jovian Trojan asteroids is a classical problem of dynamical astronomy. In the literature, this question is usually approached using analytical (see Giorgilli et al., 1989; Celletti and Giorgilli, 1991; Giorgilli and Skokos, 1997; Skokos and Dokoumetzidis, 2000; Gabern and Jorba, 2001) or numerical methods. Among the numerical studies, two points of view are prominent:

(1) *Local approach*. Definition and computation of proper elements and proper frequencies. A synthetic theory for the proper elements was first established by Milani (1993, 1994). Later on, Beaugé and Roig developed a semi-analytical theory for Trojan proper elements (Beaugé and Roig, 2001). The question of Trojan proper frequencies was also tackled by (Gabern et al., 2004).
(2) *Global approach*. Since the work by (Levison et al., 1997), where the spatial distribution of the escape times was studied, different attempts to describe the global dynamics of the co-orbital region have been done: (Michtchenko et al., 2001; Nesvorny and Dones, 2002 and Tsiganis et al., 2005).

In this paper, we try to combine these two different approaches by placing several hundreds of observed Trojans in the corresponding global dynamical background.

First, we show some initial results of a global and systematic study of the tadpole region near the Jupiter co-orbital L_4 point (Robutel et al., in preparation). We describe some of the families of resonances that are fundamental to understand the complicated structure underlying the 1:1 mean motion resonance between Jupiter and a Trojan. The computations are based on the Frequency Map analysis (Laskar, 1999) and the results rely on an estimate of the chaoticity of some relevant slices of the phase space. These global pictures of the dynamical structure around the L_4 co-orbital region and the knowledge of the specific values of the fundamental frequencies allow us to identify the resonances that arrange the main structures of the phase space.

Once these global dynamical pictures are obtained, a natural question is what information they provide concerning the real Trojan asteroids. In this regard, we explain a way of superimposing the observed Trojans in these dynamical maps and we are able to identify actual asteroids inside some of the main resonances of the global pictures.

The simulations are based on a direct numerical integration of the Restricted 4-Body Problem defined by Sun, Jupiter, Saturn and the asteroid (SJS model). Gabern et al. (2004) already showed that for studying the Trojan problem, restricted 3-body models are not enough. Moreover, in (Gabern, 2003), some relatively simple semi-analytic four and five body models (3-body models with 2-dimensional quasi-periodic forcing (Gabern and Jorba, 2004) were used to study this problem and also proved not accurate enough for the complete description of the fundamental frequencies of the Trojan asteroids. Thus, more planetary frequencies have to be taken into account.

In (Robutel et al., in preparation), it is shown that the SJS model already captures the main global dynamical structures of the co-orbital regions. Actually, the addition of Uranus and Neptune to the problem, does not affect the main features of the phase space, but just shifts slightly the location of the resonances and makes everything a little bit more unstable (Robutel et al., in preparation).

2. Frequency Map and Global Structure of the Phase Space

In Figure 1, we show a dynamical map of the tadpole region of the leading Lagrangian point L_4 (similar pictures are obtained in the L_5 case). This picture is generated by an integration of 32,000 fictitious Trojans. Their