

Applying the Isabelle Insider Framework to Airplane Security

Florian Kammüller and Manfred Kerber

March 29, 2020

Abstract

Avionics is one of the fields in which verification methods have been pioneered and brought a new level of reliability to systems used in safety critical environments. Tragedies, like the 2015 insider attack on a German airplane, in which all 150 people on board died, show that safety and security crucially depend not only on the well functioning of systems but also on the way how humans interact with the systems. Policies are a way to describe how humans should behave in their interactions with technical systems, formal reasoning about such policies requires integrating the human factor into the verification process.

We model insider attacks on airplanes using logical modelling and analysis of infrastructure models and policies with actors to scrutinize security policies in the presence of insiders [1]. The Isabelle Insider framework framework has been first presented in [3]. Triggered by case studies, like the present one of airplane security, it has been greatly extended now formalizing Kripke structures and the temporal logic CTL to enable reasoning on dynamic system states. Furthermore, we illustrate that Isabelle modelling and invariant reasoning reveal subtle security assumptions: the formal development uses locales to model the assumptions on insider and their access credentials. Technically interesting is how the locale is interpreted in the presence of an abstract type declaration for actor in the Insider framework redefining this type declaration at a later stage like a “post-hoc type definition” as proposed in [4]. The case study and the application of the methodology are described in more detail in the preprint [2].

Contents

1	Fixpoint lemmas to support the definition of Kripke structures and CTL	2
2	Insider	8
3	Airplane case study	14

1 Fixpoint lemmas to support the definition of Kripke structures and CTL

```

theory MC
imports Main
begin

thm monotone-def
definition monotone :: ('a set  $\Rightarrow$  'a set)  $\Rightarrow$  bool
where monotone  $\tau \equiv (\forall p\ q. p \subseteq q \longrightarrow \tau\ p \subseteq \tau\ q)$ 

lemma monotoneE: monotone  $\tau \Longrightarrow p \subseteq q \Longrightarrow \tau\ p \subseteq \tau\ q$ 
  <proof>

lemma lfp1: monotone  $\tau \longrightarrow (\text{lfp } \tau = \bigcap \{Z. \tau\ Z \subseteq Z\})$ 
  <proof>

lemma gfp1: monotone  $\tau \longrightarrow (\text{gfp } \tau = \bigcup \{Z. Z \subseteq \tau\ Z\})$ 
  <proof>

primrec power :: ['a  $\Rightarrow$  'a, nat]  $\Rightarrow$  ('a  $\Rightarrow$  'a) ((-  $\wedge$  -) 40)
where
  power-zero: (f  $\wedge$  0) = ( $\lambda x. x$ ) |
  power-suc: (f  $\wedge$  (Suc n)) = (f o (f  $\wedge$  n))

lemma predtrans-empty:
  assumes monotone  $\tau$ 
  shows  $\forall i. (\tau \wedge i) (\{\}) \subseteq (\tau \wedge (i + 1)) (\{\})$ 
  <proof>

lemma ex-card: finite  $S \Longrightarrow \exists n::\text{nat}. \text{card } S = n$ 
  <proof>

lemma less-not-le:  $\llbracket (x::\text{nat}) < y; y \leq x \rrbracket \Longrightarrow \text{False}$ 
  <proof>

lemma infchain-outruns-all:
  assumes finite (UNIV :: 'a set)
  and  $\forall i::\text{nat}. (\tau \wedge i) (\{\}::'a\ \text{set}) \subset (\tau \wedge i + (1::\text{nat})) (\{\})$ 
  shows  $\forall j::\text{nat}. \exists i::\text{nat}. j < \text{card } ((\tau \wedge i) (\{\}))$ 
  <proof>

lemma no-infinite-subset-chain:
  assumes finite (UNIV :: 'a set)
  and monotone ( $\tau::('a\ \text{set} \Rightarrow 'a\ \text{set})$ )
  and  $\forall i::\text{nat}. ((\tau::'a\ \text{set} \Rightarrow 'a\ \text{set}) \wedge i) (\{\}) \subset (\tau \wedge i + (1::\text{nat})) (\{\}::'a\ \text{set})$ 
  shows False

```

idea: Since $UNIV$ is finite, we have from `ex_card` that there is an n with $card\ UNIV = n$. Now, use `infchain_outruns_all` to show as contradiction point that $\exists i. card\ UNIV < card\ ((\tau \wedge i)\ \{\})$. Since all sets are subsets of $UNIV$, we also have $card\ ((\tau \wedge i)\ \{\}) \leq card\ UNIV$: Contradiction!, i.e. proof of False

<proof>

lemma *finite-fixp*:

assumes *finite* ($UNIV :: 'a\ set$)
and *monotone* ($\tau :: ('a\ set \Rightarrow 'a\ set)$)
shows $\exists i. (\tau \wedge i)\ (\{\}) = (\tau \wedge (i + 1))(\{\})$

idea: with *predtrans-empty* we know $\forall i. (\tau \wedge i)\ \{\} \subseteq (\tau \wedge i + 1)\ \{\}$ (1). If we can additionally show $\exists i. (\tau \wedge i + 1)\ \{\} \subseteq (\tau \wedge i)\ \{\}$ (2), we can get the goal together with equality $I \subseteq + \supseteq \longrightarrow =$. To prove (1) we observe that $(\tau \wedge i + 1)\ \{\} \subseteq (\tau \wedge i)\ \{\}$ can be inferred from $\neg (\tau \wedge i)\ \{\} \subseteq (\tau \wedge i + 1)\ \{\}$ and (1). Finally, the latter is solved directly by `no_infinite_subset_chain`.

<proof>

lemma *predtrans-UNIV*:

assumes *monotone* τ
shows $\forall i. (\tau \wedge i)\ (UNIV) \supseteq (\tau \wedge (i + 1))(UNIV)$

<proof>

lemma *Suc-less-le*: $x < (y - n) \Longrightarrow x \leq (y - (Suc\ n))$

<proof>

lemma *card-univ-subtract*:

assumes *finite* ($UNIV :: 'a\ set$) **and** *monotone* ($\tau :: 'a\ set \Rightarrow 'a\ set$)
and $(\forall i :: nat. ((\tau :: 'a\ set \Rightarrow 'a\ set) \wedge i + (1 :: nat))\ (UNIV :: 'a\ set) \subset (\tau \wedge i)\ UNIV)$
shows $(\forall i :: nat. card((\tau \wedge i)\ (UNIV :: 'a\ set)) \leq (card\ (UNIV :: 'a\ set)) - i)$
<proof>

lemma *card-UNIV-tau-i-below-zero*:

assumes *finite* ($UNIV :: 'a\ set$) **and** *monotone* ($\tau :: 'a\ set \Rightarrow 'a\ set$)
and $(\forall i :: nat. ((\tau :: 'a\ set \Rightarrow 'a\ set) \wedge i + (1 :: nat))\ (UNIV :: 'a\ set) \subset (\tau \wedge i)\ UNIV)$
shows $card((\tau \wedge (card\ (UNIV :: 'a\ set)))\ (UNIV :: 'a\ set)) \leq 0$
<proof>

lemma *finite-card-zero-empty*: $\llbracket finite\ S; card\ S \leq 0 \rrbracket \Longrightarrow S = \{\}$

<proof>

lemma *UNIV-tau-i-is-empty*:

assumes *finite* ($UNIV :: 'a\ set$) **and** *monotone* ($\tau :: 'a\ set \Rightarrow 'a\ set$)
and $(\forall i :: nat. ((\tau :: 'a\ set \Rightarrow 'a\ set) \wedge i + (1 :: nat))\ (UNIV :: 'a\ set) \subset (\tau \wedge i)\ UNIV)$

shows $(\tau \wedge (\text{card } (UNIV :: 'a \text{ set}))) (UNIV :: 'a \text{ set}) = \{\}$
 $\langle \text{proof} \rangle$

lemma *down-chain-reaches-empty*:

assumes *finite* $(UNIV :: 'a \text{ set})$ **and** *monotone* $(\tau :: 'a \text{ set} \Rightarrow 'a \text{ set})$
and $(\forall i :: \text{nat}. ((\tau :: 'a \text{ set} \Rightarrow 'a \text{ set}) \wedge i + (1 :: \text{nat})) UNIV \subset (\tau \wedge i) UNIV)$
shows $\exists (j :: \text{nat}). (\tau \wedge j) UNIV = \{\}$
 $\langle \text{proof} \rangle$

lemma *no-infinite-subset-chain2*:

assumes *finite* $(UNIV :: 'a \text{ set})$ **and** *monotone* $(\tau :: ('a \text{ set} \Rightarrow 'a \text{ set}))$
and $\forall i :: \text{nat}. (\tau \wedge i) UNIV \supset (\tau \wedge i + (1 :: \text{nat})) UNIV$
shows *False*
 $\langle \text{proof} \rangle$

lemma *finite-fix2*:

assumes *finite* $(UNIV :: 'a \text{ set})$ **and** *monotone* $(\tau :: ('a \text{ set} \Rightarrow 'a \text{ set}))$
shows $\exists i. (\tau \wedge i) UNIV = (\tau \wedge (i + 1)) UNIV$
 $\langle \text{proof} \rangle$

lemma *mono-monotone*: *mono* $(\tau :: ('a \text{ set} \Rightarrow 'a \text{ set})) \Longrightarrow \text{monotone } \tau$
 $\langle \text{proof} \rangle$

lemma *monotone-mono*: *monotone* $(\tau :: ('a \text{ set} \Rightarrow 'a \text{ set})) \Longrightarrow \text{mono } \tau$
 $\langle \text{proof} \rangle$

lemma *power-power*: $((\tau :: ('a \text{ set} \Rightarrow 'a \text{ set})) \wedge \wedge n) = ((\tau :: ('a \text{ set} \Rightarrow 'a \text{ set})) \wedge n)$
 $\langle \text{proof} \rangle$

lemma *lfp-Kleene-iter-set*: *monotone* $(f :: ('a \text{ set} \Rightarrow 'a \text{ set})) \Longrightarrow$
 $(f \wedge \text{Suc}(n)) \{\} = (f \wedge n) \{\} \Longrightarrow \text{lfp } f = (f \wedge n) \{\}$
 $\langle \text{proof} \rangle$

lemma *lfp-loop*:

assumes *finite* $(UNIV :: 'b \text{ set})$ **and** *monotone* $(\tau :: ('b \text{ set} \Rightarrow 'b \text{ set}))$
shows $\exists n. \text{lfp } \tau = (\tau \wedge n) \{\}$
 $\langle \text{proof} \rangle$

These next two are produced as duals from the corresponding theorems in HOL/ZF/Nat.thy. Would make sense to have them in the HOL/Library

lemma *Kleene-iter-gfp*:

assumes *mono* f **and** $p \leq f p$ **shows** $p \leq (f \wedge k) (\text{top} :: 'a :: \text{order-top})$
 $\langle \text{proof} \rangle$

lemma *gfp-Kleene-iter*: **assumes** *mono* f **and** $(f \wedge \text{Suc } k) \text{ top} = (f \wedge k) \text{ top}$
shows $\text{gfp } f = (f \wedge k) \text{ top}$
 $\langle \text{proof} \rangle$

lemma *gfp-Kleene-iter-set*:
assumes *monotone* ($f :: ('a \text{ set} \Rightarrow 'a \text{ set})$)
and ($f \wedge \text{Suc}(n)$) $\text{UNIV} = (f \wedge n) \text{ UNIV}$
shows $\text{gfp } f = (f \wedge n) \text{ UNIV}$
 $\langle \text{proof} \rangle$

lemma *gfp-loop*:
assumes *finite* ($\text{UNIV} :: 'b \text{ set}$)
and *monotone* ($\tau :: ('b \text{ set} \Rightarrow 'b \text{ set})$)
shows $\exists n. \text{gfp } \tau = (\tau \wedge n)(\text{UNIV} :: 'b \text{ set})$
 $\langle \text{proof} \rangle$

Definitions of the generic type of state with state transition and CTL Operators

class *state* =
fixes *state-transition* :: [$'a :: \text{type}, 'a$] $\Rightarrow \text{bool}$ $((- \rightarrow_i -) \ 50)$

definition *AX* **where** $\text{AX } f \equiv \{s. \{f0. s \rightarrow_i f0\} \subseteq f\}$
definition *EX'* **where** $\text{EX}' f \equiv \{s. \exists f0 \in f. s \rightarrow_i f0\}$

definition *AF* **where** $\text{AF } f \equiv \text{lfp } (\lambda Z. f \cup \text{AX } Z)$
definition *EF* **where** $\text{EF } f \equiv \text{lfp } (\lambda Z. f \cup \text{EX}' Z)$
definition *AG* **where** $\text{AG } f \equiv \text{gfp } (\lambda Z. f \cap \text{AX } Z)$
definition *EG* **where** $\text{EG } f \equiv \text{gfp } (\lambda Z. f \cap \text{EX}' Z)$
definition *AU* **where** $\text{AU } f1 f2 \equiv \text{lfp } (\lambda Z. f2 \cup (f1 \cap \text{AX } Z))$
definition *EU* **where** $\text{EU } f1 f2 \equiv \text{lfp } (\lambda Z. f2 \cup (f1 \cap \text{EX}' Z))$
definition *AR* **where** $\text{AR } f1 f2 \equiv \text{gfp } (\lambda Z. f2 \cap (f1 \cup \text{AX } Z))$
definition *ER* **where** $\text{ER } f1 f2 \equiv \text{gfp } (\lambda Z. f2 \cap (f1 \cup \text{EX}' Z))$

Kripke and Modelchecking

datatype $'a \text{ kripke} =$
Kripke $'a \text{ set } 'a \text{ set}$

primrec *states* **where** $\text{states } (\text{Kripke } S \ I) = S$
primrec *init* **where** $\text{init } (\text{Kripke } S \ I) = I$

definition *check* $(- \vdash - \ 50)$
where $M \vdash f \equiv (\text{init } M) \subseteq \{s \in (\text{states } M). s \in f\}$

definition *state-transition-refl* $((- \rightarrow_{i*} -) \ 50)$
where $s \rightarrow_{i*} s' \equiv ((s, s') \in \{(x, y). \text{state-transition } x \ y\}^*)$

Support lemmas

lemma *EF-lem0*: $(x \in \text{EF } f) = (x \in f \cup \text{EX}' (\text{lfp } (\lambda Z :: ('a :: \text{state}) \text{ set}. f \cup \text{EX}' Z)))$
 $\langle \text{proof} \rangle$

lemma *EF-lem00*: $(\text{EF } f) = (f \cup \text{EX}' (\text{lfp } (\lambda Z :: ('a :: \text{state}) \text{ set}. f \cup \text{EX}' Z)))$

$\langle \text{proof} \rangle$

lemma *EF-lem000*: $(EF\ f) = (f \cup EX'\ (EF\ f))$
 $\langle \text{proof} \rangle$

lemma *EF-lem1*: $x \in f \vee x \in (EX'\ (EF\ f)) \implies x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-lem2b*:
 assumes $x \in (EX'\ (EF\ f))$
 shows $x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-lem2a*: **assumes** $x \in f$ **shows** $x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-lem2c*: **assumes** $x \notin f$ **shows** $x \in EF\ (\neg f)$
 $\langle \text{proof} \rangle$

lemma *EF-lem2d*: **assumes** $x \notin EF\ f$ **shows** $x \notin f$
 $\langle \text{proof} \rangle$

lemma *EF-lem3b*: **assumes** $x \in EX'\ (f \cup EX'\ (EF\ f))$ **shows** $x \in (EF\ f)$
 $\langle \text{proof} \rangle$

lemma *EX-lem0l*: $x \in (EX'\ f) \implies x \in (EX'\ (f \cup g))$
 $\langle \text{proof} \rangle$

lemma *EX-lem0r*: $x \in (EX'\ g) \implies x \in (EX'\ (f \cup g))$
 $\langle \text{proof} \rangle$

lemma *EX-step*: **assumes** $x \rightarrow_i y$ **and** $y \in f$ **shows** $x \in EX'\ f$
 $\langle \text{proof} \rangle$

lemma *EF-E[rule-format]*: $\forall f. x \in (EF\ (f :: ('a :: \text{state})\ \text{set})) \longrightarrow x \in (f \cup EX'\ (EF\ f))$
 $\langle \text{proof} \rangle$

lemma *EF-step*: **assumes** $x \rightarrow_i y$ **and** $y \in f$ **shows** $x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-step-step*: **assumes** $x \rightarrow_i y$ **and** $y \in EF\ f$ **shows** $x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-step-star*: $\llbracket x \rightarrow_i^* y; y \in f \rrbracket \implies x \in EF\ f$
 $\langle \text{proof} \rangle$

lemma *EF-induct-prep*:
 assumes $(a :: 'a :: \text{state}) \in \text{lfp } (\lambda Z. (f :: 'a :: \text{state}\ \text{set}) \cup EX'\ Z)$

and $\text{mono } (\lambda Z. (f :: 'a :: \text{state set}) \cup EX' Z)$
shows $(\bigwedge x :: 'a :: \text{state}.$
 $x \in ((\lambda Z. (f :: 'a :: \text{state set}) \cup EX' Z)(\text{lfp } (\lambda Z. (f :: 'a :: \text{state set}) \cup EX' Z) \cap$
 $\{x :: 'a :: \text{state}. (P :: 'a :: \text{state} \Rightarrow \text{bool}) x\})) \Rightarrow P x) \Rightarrow$
 $P a$
 $\langle \text{proof} \rangle$

lemma *EF-induct*: $(a :: 'a :: \text{state}) \in EF (f :: 'a :: \text{state set}) \Rightarrow$
 $\text{mono } (\lambda Z. (f :: 'a :: \text{state set}) \cup EX' Z) \Rightarrow$
 $(\bigwedge x :: 'a :: \text{state}.$
 $x \in ((\lambda Z. (f :: 'a :: \text{state set}) \cup EX' Z)(EF f \cap \{x :: 'a :: \text{state}. (P :: 'a :: \text{state} \Rightarrow$
 $\text{bool}) x\})) \Rightarrow P x) \Rightarrow$
 $P a$
 $\langle \text{proof} \rangle$

lemma *valEF-E*: $M \vdash EF f \Rightarrow x \in \text{init } M \Rightarrow x \in EF f$
 $\langle \text{proof} \rangle$

lemma *EF-step-star-rev[rule-format]*: $x \in EF s \Rightarrow (\exists y \in s. x \rightarrow_i^* y)$
 $\langle \text{proof} \rangle$

lemma *EF-step-inv*: $(I \subseteq \{sa :: 's :: \text{state}. (\exists i :: 's \in I. i \rightarrow_i^* sa) \wedge sa \in EF s\})$
 $\Rightarrow \forall x \in I. \exists y \in s. x \rightarrow_i^* y$
 $\langle \text{proof} \rangle$

AG lemmas

lemma *AG-in-lem*: $x \in AG s \Rightarrow x \in s$
 $\langle \text{proof} \rangle$

lemma *AG-lem1*: $x \in s \wedge x \in (AX (AG s)) \Rightarrow x \in AG s$
 $\langle \text{proof} \rangle$

lemma *AG-lem2*: $x \in AG s \Rightarrow x \in (s \cap (AX (AG s)))$
 $\langle \text{proof} \rangle$

lemma *AG-lem3*: $AG s = (s \cap (AX (AG s)))$
 $\langle \text{proof} \rangle$

lemma *AG-step*: $y \rightarrow_i z \Rightarrow y \in AG s \Rightarrow z \in AG s$
 $\langle \text{proof} \rangle$

lemma *AG-all-s*: $x \rightarrow_i^* y \Rightarrow x \in AG s \Rightarrow y \in AG s$
 $\langle \text{proof} \rangle$

lemma *AG-imp-notnotEF*:
 $I \neq \{\} \Rightarrow ((\text{Kripke } \{s :: ('s :: \text{state}). \exists i \in I. (i \rightarrow_i^* s)\} (I :: ('s :: \text{state}) \text{set})$
 $\vdash AG s)) \Rightarrow$
 $(\neg(\text{Kripke } \{s :: ('s :: \text{state}). \exists i \in I. (i \rightarrow_i^* s)\} (I :: ('s :: \text{state}) \text{set}) \vdash EF (-$
 $s)))$

<proof>

lemma *check2-def*: (*Kripke S I* \vdash *f*) = (*I* \subseteq *S* \cap *f*)
<proof>

end

2 Insider

theory *AirInsider*
imports *MC*
begin
datatype *action* = *get* | *move* | *eval* | *put*

We use an abstract type declaration *actor* that can later be instantiated by a more concrete type.

typedecl *actor*
consts *Actor* :: *string* \Rightarrow *actor*

Alternatives to the type declaration do not work.

context fixes Abs Rep actor assumes td: "type_definition Abs Rep actor"
begin definition Actor where "Actor = Abs" ...doesn't work for replacing the actor typedecl because in "type_definition" above the "actor" is a set not a type! So can't be used for our purposes. Trying a locale instead for polymorphic type Actor locale ACT = fixes Actor :: "string \Rightarrow 'actor" begin ... That is a nice idea and works quite far but clashes with the generic state.transition later (it's not possible to instantiate within a locale and outside it we cannot instantiate "a infrastructure" to state (clearly an abstract thing as an instance is strange)

type-synonym *identity* = *string*
type-synonym *policy* = ((*actor* \Rightarrow *bool*) * *action set*)

definition *ID* :: [*actor*, *string*] \Rightarrow *bool*
where *ID a s* \equiv (*a* = *Actor s*)

datatype *location* = *Location nat*

datatype *igraph* = *Lgraph (location * location)set location* \Rightarrow *identity list*
actor \Rightarrow (*string list* * *string list*) *location* \Rightarrow *string list*

datatype *infrastructure* =
Infrastructure ighraph
[igraph, location] \Rightarrow *policy set*

primrec *loc* :: *location* \Rightarrow *nat*
where *loc(Location n)* = *n*
primrec *gra* :: *igraph* \Rightarrow (*location* * *location*)*set*
where *gra(Lgraph g a c l)* = *g*


```

primrec agra :: igraph  $\Rightarrow$  (location  $\Rightarrow$  identity list)
where agra(Lgraph g a c l) = a
primrec cgra :: igraph  $\Rightarrow$  (actor  $\Rightarrow$  string list * string list)
where cgra(Lgraph g a c l) = c
primrec lgra :: igraph  $\Rightarrow$  (location  $\Rightarrow$  string list)
where lgra(Lgraph g a c l) = l

definition nodes :: igraph  $\Rightarrow$  location set
where nodes g == { x. (? y. ((x,y): gra g) | ((y,x): gra g)) }

definition actors-graph :: igraph  $\Rightarrow$  identity set
where actors-graph g == { x. ? y. y : nodes g  $\wedge$  x  $\in$  set(agra g y) }

primrec graphI :: infrastructure  $\Rightarrow$  igraph
where graphI (Infrastructure g d) = g
primrec delta :: [infrastructure, igraph, location]  $\Rightarrow$  policy set
where delta (Infrastructure g d) = d
primrec tspace :: [infrastructure, actor]  $\Rightarrow$  string list * string list
where tspace (Infrastructure g d) = cgra g
primrec lspace :: [infrastructure, location]  $\Rightarrow$  string list
where lspace (Infrastructure g d) = lgra g

definition credentials :: string list * string list  $\Rightarrow$  string set
where credentials lxl  $\equiv$  set (fst lxl)
definition has :: [igraph, actor * string]  $\Rightarrow$  bool
where has G ac  $\equiv$  snd ac  $\in$  credentials(cgra G (fst ac))
definition roles :: string list * string list  $\Rightarrow$  string set
where roles lxl  $\equiv$  set (snd lxl)
definition role :: [igraph, actor * string]  $\Rightarrow$  bool
where role G ac  $\equiv$  snd ac  $\in$  roles(cgra G (fst ac))

definition isin :: [igraph, location, string]  $\Rightarrow$  bool
where isin G l s  $\equiv$  s  $\in$  set(lgra G l)

datatype psy-states = happy | depressed | disgruntled | angry | stressed
datatype motivations = financial | political | revenge | curious | competitive-advantage
| power | peer-recognition

datatype actor-state = Actor-state psy-states motivations set
primrec motivation :: actor-state  $\Rightarrow$  motivations set
where motivation (Actor-state p m) = m
primrec psy-state :: actor-state  $\Rightarrow$  psy-states
where psy-state (Actor-state p m) = p

definition tipping-point :: actor-state  $\Rightarrow$  bool where
  tipping-point a  $\equiv$  ((motivation a  $\neq$  { })  $\wedge$  (happy  $\neq$  psy-state a))

```

UasI and UasI' are the central predicates allowing to specify Insiders. They define which identities can be mapped to the same role by the Actor function.

For all other identities, Actor is defined as injective on those identities.

definition $UasI :: [identity, identity] \Rightarrow bool$

where $UasI\ a\ b \equiv (Actor\ a = Actor\ b) \wedge (\forall\ x\ y. x \neq a \wedge y \neq a \wedge Actor\ x = Actor\ y \longrightarrow x = y)$

definition $UasI' :: [actor \Rightarrow bool, identity, identity] \Rightarrow bool$

where $UasI'\ P\ a\ b \equiv P\ (Actor\ b) \longrightarrow P\ (Actor\ a)$

Two versions of Insider predicate corresponding to UasI and UasI'. Under the assumption that the tipping point has been reached for a person a then a can impersonate all b (take all of b's "roles") where the b's are specified by a given set of identities

definition $Insider :: [identity, identity\ set, identity \Rightarrow actor\ state] \Rightarrow bool$

where $Insider\ a\ C\ as \equiv (tipping\ point\ (as\ a) \longrightarrow (\forall\ b \in C. UasI\ a\ b))$

definition $Insider' :: [actor \Rightarrow bool, identity, identity\ set, identity \Rightarrow actor\ state] \Rightarrow bool$

where $Insider'\ P\ a\ C\ as \equiv (tipping\ point\ (as\ a) \longrightarrow (\forall\ b \in C. UasI'\ P\ a\ b \wedge inj\ on\ Actor\ C))$

definition $atI :: [identity, igrph, location] \Rightarrow bool\ (-\ @_{(-)}\ -\ 50)$

where $a\ @_G\ l \equiv a \in set(agra\ G\ l)$

enables is the central definition of the behaviour as given by a policy that specifies what actions are allowed in a certain location for what actors

definition $enables :: [infrastructure, location, actor, action] \Rightarrow bool$

where

$enables\ I\ l\ a\ a' \equiv (\exists\ (p,e) \in delta\ I\ (graphI\ I)\ l. a' \in e \wedge p\ a)$

behaviour is the good behaviour, i.e. everything allowed by policy

definition $behaviour :: infrastructure \Rightarrow (location * actor * action) set$

where $behaviour\ I \equiv \{(t,a,a').\ enables\ I\ t\ a\ a'\}$

misbehaviour is the complement of behaviour

definition $misbehaviour :: infrastructure \Rightarrow (location * actor * action) set$

where $misbehaviour\ I \equiv -(behaviour\ I)$

basic lemmas for enable

lemma $not\ enableI: (\forall\ (p,e) \in delta\ I\ (graphI\ I)\ l. (\sim(h : e) \mid (\sim(p(a)))) \implies \sim(enables\ I\ l\ a\ h)$

$\langle proof \rangle$

lemma $not\ enableI2: [\bigwedge\ p\ e. (p,e) \in delta\ I\ (graphI\ I)\ l \implies (\sim(t : e) \mid (\sim(p(a))))] \implies \sim(enables\ I\ l\ a\ t)$

$\langle proof \rangle$

lemma $not\ enableE: [\sim(enables\ I\ l\ a\ t); (p,e) \in delta\ I\ (graphI\ I)\ l] \implies$

$$\langle \text{proof} \rangle \quad \Longrightarrow (\sim(t : e) \mid (\sim(p(a))))$$

lemma not-enableE2: $\llbracket \sim(\text{enables } I \ l \ a \ t); (p, e) \in \text{delta } I \ (\text{graphI } I) \ l; \\ t : e \rrbracket \Longrightarrow (\sim(p(a)))$
 $\langle \text{proof} \rangle$

some constructions to deal with lists of actors in locations for the semantics of action move

primrec $\text{del} :: ['a, 'a \text{ list}] \Rightarrow 'a \text{ list}$

where

$\text{del-nil}: \text{del } a \ [] = [] \mid$

$\text{del-cons}: \text{del } a \ (x \# ls) = (\text{if } x = a \text{ then } ls \text{ else } x \# (\text{del } a \ ls))$

primrec $\text{jonce} :: ['a, 'a \text{ list}] \Rightarrow \text{bool}$

where

$\text{jonce-nil}: \text{jonce } a \ [] = \text{False} \mid$

$\text{jonce-cons}: \text{jonce } a \ (x \# ls) = (\text{if } x = a \text{ then } (a \notin (\text{set } ls)) \text{ else } \text{jonce } a \ ls)$

primrec $\text{nodup} :: ['a, 'a \text{ list}] \Rightarrow \text{bool}$

where

$\text{nodup-nil}: \text{nodup } a \ [] = \text{True} \mid$

$\text{nodup-step}: \text{nodup } a \ (x \# ls) = (\text{if } x = a \text{ then } (a \notin (\text{set } ls)) \text{ else } \text{nodup } a \ ls)$

definition $\text{move-graph-a} :: [\text{identity}, \text{location}, \text{location}, \text{igraph}] \Rightarrow \text{igraph}$

where $\text{move-graph-a } n \ l \ l' \ g \equiv \text{Lgraph } (\text{gra } g)$

$(\text{if } n \in \text{set } ((\text{agra } g) \ l) \ \& \ n \notin \text{set } ((\text{agra } g) \ l') \text{ then}$

$((\text{agra } g)(l := \text{del } n \ (\text{agra } g \ l)))(l' := (n \# (\text{agra } g \ l'))))$

$\text{else } (\text{agra } g))(cgra \ g)(lgra \ g)$

State transition relation over infrastructures (the states) defining the semantics of actions in systems with humans and potentially insiders *)

inductive $\text{state-transition-in} :: [\text{infrastructure}, \text{infrastructure}] \Rightarrow \text{bool} \ ((- \rightarrow_n -)$
 $50)$

where

$\text{move}: \llbracket G = \text{graphI } I; a @_G l; l \in \text{nodes } G; l' \in \text{nodes } G;$

$(a) \in \text{actors-graph}(\text{graphI } I); \text{enables } I \ l' \ (\text{Actor } a) \ \text{move};$

$I' = \text{Infrastructure } (\text{move-graph-a } a \ l \ l' \ (\text{graphI } I))(\text{delta } I) \rrbracket \Longrightarrow I \rightarrow_n I'$

$\mid \text{get} : \llbracket G = \text{graphI } I; a @_G l; a' @_G l; \text{has } G \ (\text{Actor } a, z);$

$\text{enables } I \ l \ (\text{Actor } a) \ \text{get};$

$I' = \text{Infrastructure}$

$(\text{Lgraph } (\text{gra } G)(\text{agra } G)$

$((cgra \ G)(\text{Actor } a' :=$

$(z \# (\text{fst}(cgra \ G \ (\text{Actor } a'))), \text{snd}(cgra \ G \ (\text{Actor } a')))))$

$(lgra \ G))$

$(\text{delta } I)$

$\rrbracket \Longrightarrow I \rightarrow_n I'$

$\mid \text{put} : \llbracket G = \text{graphI } I; a @_G l; \text{enables } I \ l \ (\text{Actor } a) \ \text{put};$

$I' = \text{Infrastructure}$

$$\begin{aligned}
& (Lgraph \ (gra \ G)(agra \ G)(cgra \ G) \\
& \quad ((lgra \ G)(l := [z]))) \\
& \quad (delta \ I) \] \\
\implies & I \rightarrow_n I' \\
| \text{ put-remote} : & \llbracket G = graphI \ I; \text{ enables } I \ l \ (\text{Actor } a) \ \text{put}; \\
& I' = Infrastructure \\
& \quad (Lgraph \ (gra \ G)(agra \ G)(cgra \ G) \\
& \quad \quad ((lgra \ G)(l := [z]))) \\
& \quad (delta \ I) \] \\
\implies & I \rightarrow_n I'
\end{aligned}$$

show that this infrastructure is a state as given in MC.thy

instantiation *infrastructure* :: *state*
begin

definition

state-transition-infra-def: $(i \rightarrow_i i') = (i \rightarrow_n (i' :: infrastructure))$

instance

<proof>

definition *state-transition-in-refl* $((- \rightarrow_n^* -) \ 50)$

where $s \rightarrow_n^* s' \equiv ((s, s') \in \{(x, y). \text{ state-transition-in } x \ y\}^*)$

lemma *del-del*[*rule-format*]: $n \in \text{set } (del \ a \ S) \longrightarrow n \in \text{set } S$

<proof>

lemma *del-dec*[*rule-format*]: $a \in \text{set } S \longrightarrow \text{length } (del \ a \ S) < \text{length } S$

<proof>

lemma *del-sort*[*rule-format*]: $\forall \ n. (\text{Suc } n :: \text{nat}) \leq \text{length } (l) \longrightarrow n \leq \text{length } (del \ a \ (l))$

<proof>

lemma *del-jonce*: $\text{jonce } a \ l \longrightarrow a \notin \text{set } (del \ a \ l)$

<proof>

lemma *del-nodup*[*rule-format*]: $\text{nodup } a \ l \longrightarrow a \notin \text{set } (del \ a \ l)$

<proof>

lemma *nodup-up*[*rule-format*]: $a \in \text{set } (del \ a \ l) \longrightarrow a \in \text{set } l$

<proof>

lemma *del-up* [*rule-format*]: $a \in \text{set } (del \ aa \ l) \longrightarrow a \in \text{set } l$

<proof>

lemma *nodup-notin*[*rule-format*]: $a \notin \text{set } list \longrightarrow \text{nodup } a \ list$

<proof>

lemma *nodup-down*[*rule-format*]: $\text{nodup } a \ l \longrightarrow \text{nodup } a \ (\text{del } a \ l)$
 $\langle \text{proof} \rangle$

lemma *del-notin-down*[*rule-format*]: $a \notin \text{set } list \longrightarrow a \notin \text{set } (\text{del } aa \ list)$
 $\langle \text{proof} \rangle$

lemma *del-not-a*[*rule-format*]: $x \neq a \longrightarrow x \in \text{set } l \longrightarrow x \in \text{set } (\text{del } a \ l)$
 $\langle \text{proof} \rangle$

lemma *nodup-down-notin*[*rule-format*]: $\text{nodup } a \ l \longrightarrow \text{nodup } a \ (\text{del } aa \ l)$
 $\langle \text{proof} \rangle$

lemma *move-graph-eq*: $\text{move-graph-a } a \ l \ l \ g = g$
 $\langle \text{proof} \rangle$

Some useful properties about the invariance of the nodes, the actors, and the policy with respect to the state transition

lemma *delta-invariant*: $\forall z \ z'. z \rightarrow_n z' \longrightarrow \text{delta}(z) = \text{delta}(z')$
 $\langle \text{proof} \rangle$

lemma *init-state-policy0*:
assumes $\forall z \ z'. z \rightarrow_n z' \longrightarrow \text{delta}(z) = \text{delta}(z')$
and $(x, y) \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^*$
shows $\text{delta}(x) = \text{delta}(y)$
 $\langle \text{proof} \rangle$

lemma *init-state-policy*: $\llbracket (x, y) \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^* \rrbracket \implies$
 $\text{delta}(x) = \text{delta}(y)$
 $\langle \text{proof} \rangle$

lemma *same-nodes0*[*rule-format*]: $\forall z \ z'. z \rightarrow_n z' \longrightarrow \text{nodes}(\text{graphI } z) = \text{nodes}(\text{graphI } z')$
 $\langle \text{proof} \rangle$

lemma *same-nodes*: $(I, y) \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^*$
 $\implies \text{nodes}(\text{graphI } y) = \text{nodes}(\text{graphI } I)$
 $\langle \text{proof} \rangle$

lemma *same-actors0*[*rule-format*]: $\forall z \ z'. z \rightarrow_n z' \longrightarrow \text{actors-graph}(\text{graphI } z) = \text{actors-graph}(\text{graphI } z')$
 $\langle \text{proof} \rangle$

lemma *same-actors*: $(I, y) \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^*$
 $\implies \text{actors-graph}(\text{graphI } I) = \text{actors-graph}(\text{graphI } y)$
 $\langle \text{proof} \rangle$

end
end

3 Airplane case study

```
theory Airplane
imports AirInsider
begin
datatype doorstate = locked | norm | unlocked
datatype position = air | airport | ground

locale airplane =

fixes airplane-actors :: identity set
defines airplane-actors-def: airplane-actors  $\equiv$  {"Bob", "Charly", "Alice"}

fixes airplane-locations :: location set
defines airplane-locations-def:
  airplane-locations  $\equiv$  {Location 0, Location 1, Location 2}

fixes cockpit :: location
defines cockpit-def: cockpit  $\equiv$  Location 2
fixes door :: location
defines door-def: door  $\equiv$  Location 1
fixes cabin :: location
defines cabin-def: cabin  $\equiv$  Location 0

fixes global-policy :: [infrastructure, identity]  $\Rightarrow$  bool
defines global-policy-def: global-policy I a  $\equiv$  a  $\notin$  airplane-actors
   $\longrightarrow \neg(\text{enables } I \text{ cockpit (Actor } a) \text{ put})$ 

fixes ex-creds :: actor  $\Rightarrow$  (string list * string list)
defines ex-creds-def: ex-creds  $\equiv$ 
  ( $\lambda$  x. (if x = Actor "Bob"
    then ([ "PIN" ], [ "pilot" ])
    else (if x = Actor "Charly"
      then ([ "PIN" ], [ "copilot" ])
      else (if x = Actor "Alice"
        then ([ "PIN" ], [ "flightattendant" ])
        else ([ ], [ ]))))))

fixes ex-locs :: location  $\Rightarrow$  string list
defines ex-locs-def: ex-locs  $\equiv$  ( $\lambda$  x. if x = door then [ "norm" ] else
  (if x = cockpit then [ "air" ] else [ ]))

fixes ex-locs' :: location  $\Rightarrow$  string list
defines ex-locs'-def: ex-locs'  $\equiv$  ( $\lambda$  x. if x = door then [ "locked" ] else
  (if x = cockpit then [ "air" ] else [ ]))

fixes ex-graph :: igraph
defines ex-graph-def: ex-graph  $\equiv$  Lgraph
  {(cockpit, door), (door, cabin)}
  ( $\lambda$  x. if x = cockpit then [ "Bob", "Charly" ]
```

```

      else (if x = door then []
            else (if x = cabin then ["Alice'"] else [])))
ex-creds ex-locs

```

```

fixes aid-graph :: igrph
defines aid-graph-def: aid-graph ≡ Lgraph
  {(cockpit, door),(door,cabin)}
  (λ x. if x = cockpit then ["Charly'"]
        else (if x = door then []
               else (if x = cabin then ["Bob", "Alice'"] else [])))
ex-creds ex-locs'

```

```

fixes aid-graph0 :: igrph
defines aid-graph0-def: aid-graph0 ≡ Lgraph
  {(cockpit, door),(door,cabin)}
  (λ x. if x = cockpit then ["Charly'"]
        else (if x = door then ["Bob'"]
               else (if x = cabin then ["Alice'"] else [])))
ex-creds ex-locs

```

```

fixes agid-graph :: igrph
defines agid-graph-def: agid-graph ≡ Lgraph
  {(cockpit, door),(door,cabin)}
  (λ x. if x = cockpit then ["Charly'"]
        else (if x = door then []
               else (if x = cabin then ["Bob", "Alice'"] else [])))
ex-creds ex-locs

```

```

fixes local-policies :: [igrph, location] ⇒ policy set
defines local-policies-def: local-policies G ≡
  (λ y. if y = cockpit then
    {(λ x. (? n. (n @G cockpit) ∧ Actor n = x), {put}),
      (λ x. (? n. (n @G cabin) ∧ Actor n = x ∧ has G (x, "PIN")
              ∧ isin G door "norm"),{move})
    }
    else (if y = door then {(λ x. True, {move}),
                             (λ x. (? n. (n @G cockpit) ∧ Actor n = x), {put})}
          else (if y = cabin then {(λ x. True, {move})}
                                   else {})))

```

```

fixes local-policies-four-eyes :: [igrph, location] ⇒ policy set
defines local-policies-four-eyes-def: local-policies-four-eyes G ≡
  (λ y. if y = cockpit then
    {(λ x. (? n. (n @G cockpit) ∧ Actor n = x) ∧
        2 ≤ length(agra G y) ∧ (∀ h ∈ set(agra G y). h ∈ airplane-actors),
      {put}),
      (λ x. (? n. (n @G cabin) ∧ Actor n = x ∧ has G (x, "PIN") ∧
              isin G door "norm"),{move})
    }
  )

```

```

    }
    else (if y = door then
      {(\lambda x. ((? n. (n @_G cockpit) \wedge Actor n = x) \wedge 3 \leq length(agra G
cockpit)), {move})})
      else (if y = cabin then
        {(\lambda x. ((? n. (n @_G door) \wedge Actor n = x)), {move})})
        else {})))

```

fixes *Airplane-scenario* :: *infrastructure* (**structure**)
defines *Airplane-scenario-def*:
Airplane-scenario \equiv *Infrastructure ex-graph local-policies*

fixes *Airplane-in-danger* :: *infrastructure*
defines *Airplane-in-danger-def*:
Airplane-in-danger \equiv *Infrastructure aid-graph local-policies*

fixes *Airplane-getting-in-danger0* :: *infrastructure*
defines *Airplane-getting-in-danger0-def*:
Airplane-getting-in-danger0 \equiv *Infrastructure aid-graph0 local-policies*

fixes *Airplane-getting-in-danger* :: *infrastructure*
defines *Airplane-getting-in-danger-def*:
Airplane-getting-in-danger \equiv *Infrastructure agid-graph local-policies*

fixes *Air-states*
defines *Air-states-def*: *Air-states* \equiv { *I. Airplane-scenario* $\rightarrow_n^* I$ }

fixes *Air-Kripke*
defines *Air-Kripke* \equiv *Kripke Air-states* {*Airplane-scenario*}

fixes *Airplane-not-in-danger* :: *infrastructure*
defines *Airplane-not-in-danger-def*:
Airplane-not-in-danger \equiv *Infrastructure aid-graph local-policies-four-eyes*

fixes *Airplane-not-in-danger-init* :: *infrastructure*
defines *Airplane-not-in-danger-init-def*:
Airplane-not-in-danger-init \equiv *Infrastructure ex-graph local-policies-four-eyes*

fixes *Air-tp-states*
defines *Air-tp-states-def*: *Air-tp-states* \equiv { *I. Airplane-not-in-danger-init* $\rightarrow_n^* I$ }
}

fixes *Air-tp-Kripke*
defines *Air-tp-Kripke* \equiv *Kripke Air-tp-states* {*Airplane-not-in-danger-init*}


```

fixes Safety :: [infrastructure, identity]  $\Rightarrow$  bool
defines Safety-def: Safety I a  $\equiv$  a  $\in$  airplane-actors
     $\longrightarrow$  (enables I cockpit (Actor a) move)

fixes Security :: [infrastructure, identity]  $\Rightarrow$  bool
defines Security-def: Security I a  $\equiv$  (isin (graphI I) door "locked")
     $\longrightarrow$   $\neg$ (enables I cockpit (Actor a) move)

fixes foe-control :: [location, action]  $\Rightarrow$  bool
defines foe-control-def: foe-control l c  $\equiv$ 
    (! I :: infrastructure. (? x :: identity.
    x @graphI I l  $\wedge$  Actor x  $\neq$  Actor "Eve")
     $\longrightarrow$   $\neg$ (enables I l (Actor "Eve") c))

fixes astate :: identity  $\Rightarrow$  actor-state
defines astate-def: astate x  $\equiv$  (case x of
    "Eve"  $\Rightarrow$  Actor-state depressed {revenge, peer-recognition}
    | -  $\Rightarrow$  Actor-state happy {})

assumes Eve-precipitating-event: tipping-point (astate "Eve")
assumes Insider-Eve: Insider "Eve" {"Charly"} astate
assumes cockpit-foe-control: foe-control cockpit put

begin

lemma ex-inv: global-policy Airplane-scenario "Bob"
  <proof>

lemma ex-inv2: global-policy Airplane-scenario "Charly"
  <proof>

lemma ex-inv3:  $\neg$ global-policy Airplane-scenario "Eve"
  <proof>

show Safety for Airplane_scenario

lemma Safety: Safety Airplane-scenario ("Alice")
  <proof>

show Security for Airplane_scenario

lemma inj-lem:  $\llbracket \text{inj } f; x \neq y \rrbracket \implies f\ x \neq f\ y$ 
  <proof>

lemma inj-on-lem:  $\llbracket \text{inj-on } f\ A; x \neq y; x \in A; y \in A \rrbracket \implies f\ x \neq f\ y$ 
  <proof>

lemma inj-lemma': inj-on (isin ex-graph door) {"locked", "norm"}
  <proof>

```

lemma *inj-lemma'*: *inj-on (isin aid-graph door) {"locked","norm"}*
 ⟨proof⟩

lemma *locl-lemma2*: *isin ex-graph door "norm" ≠ isin ex-graph door "locked"*
 ⟨proof⟩

lemma *locl-lemma3*: *isin ex-graph door "norm" = (¬ isin ex-graph door "locked")*
 ⟨proof⟩

lemma *locl-lemma2a*: *isin aid-graph door "norm" ≠ isin aid-graph door "locked"*
 ⟨proof⟩

lemma *locl-lemma3a*: *isin aid-graph door "norm" = (¬ isin aid-graph door "locked")*
 ⟨proof⟩

lemma *Security*: *Security Airplane-scenario s*
 ⟨proof⟩

show that pilot can't get into cockpit if outside and locked = Airplane_in_danger

lemma *Security-problem*: *Security Airplane-scenario "Bob"*
 ⟨proof⟩

show that pilot can get out of cockpit

lemma *pilot-can-leave-cockpit*: *(enables Airplane-scenario cabin (Actor "Bob") move)*
 ⟨proof⟩

show that in Airplane_in_danger copilot can still do put = put position to ground

lemma *ex-inv4*: *¬global-policy Airplane-in-danger ("Eve")*
 ⟨proof⟩

lemma *Safety-in-danger*:
 fixes *s*
 assumes *s* ∈ *airplane-actors*
 shows *¬(Safety Airplane-in-danger s)*
 ⟨proof⟩

lemma *Security-problem'*: *¬(enables Airplane-in-danger cockpit (Actor "Bob") move)*
 ⟨proof⟩

show that with the four eyes rule in Airplane_not_in_danger Eve cannot crash plane, i.e. cannot put position to ground

lemma *ex-inv5*: *a* ∈ *airplane-actors* → *global-policy Airplane-not-in-danger a*
 ⟨proof⟩

lemma *ex-inv6*: *global-policy Airplane-not-in-danger a*

$\langle \text{proof} \rangle$

lemma *step0*: $\text{Airplane-scenario} \rightarrow_n \text{Airplane-getting-in-danger0}$
 $\langle \text{proof} \rangle$

lemma *step1*: $\text{Airplane-getting-in-danger0} \rightarrow_n \text{Airplane-getting-in-danger}$
 $\langle \text{proof} \rangle$

lemma *step2*: $\text{Airplane-getting-in-danger} \rightarrow_n \text{Airplane-in-danger}$
 $\langle \text{proof} \rangle$

lemma *step0r*: $\text{Airplane-scenario} \rightarrow_{n*} \text{Airplane-getting-in-danger0}$
 $\langle \text{proof} \rangle$

lemma *step1r*: $\text{Airplane-getting-in-danger0} \rightarrow_{n*} \text{Airplane-getting-in-danger}$
 $\langle \text{proof} \rangle$

lemma *step2r*: $\text{Airplane-getting-in-danger} \rightarrow_{n*} \text{Airplane-in-danger}$
 $\langle \text{proof} \rangle$

theorem *step-allr*: $\text{Airplane-scenario} \rightarrow_{n*} \text{Airplane-in-danger}$
 $\langle \text{proof} \rangle$

theorem *aid-attack*: $\text{Air-Kripke} \vdash EF (\{x. \neg \text{global-policy } x \text{ "Eve"}\})$
 $\langle \text{proof} \rangle$

Invariant: actors cannot be at two places at the same time

lemma *actors-unique-loc-base*:

assumes $I \rightarrow_n I'$
and $(\forall l l'. a @_{\text{graph} I} l \wedge a @_{\text{graph} I} l' \longrightarrow l = l') \wedge$
 $(\forall l. \text{nodup } a (\text{agra } (\text{graph } I) l))$
shows $(\forall l l'. a @_{\text{graph} I'} l \wedge a @_{\text{graph} I'} l' \longrightarrow l = l') \wedge$
 $(\forall l. \text{nodup } a (\text{agra } (\text{graph } I') l))$

$\langle \text{proof} \rangle$

lemma *actors-unique-loc-step*:

assumes $(I, I') \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^*$
and $\forall a. (\forall l l'. a @_{\text{graph} I} l \wedge a @_{\text{graph} I} l' \longrightarrow l = l') \wedge$
 $(\forall l. \text{nodup } a (\text{agra } (\text{graph } I) l))$
shows $\forall a. (\forall l l'. a @_{\text{graph} I'} l \wedge a @_{\text{graph} I'} l' \longrightarrow l = l') \wedge$
 $(\forall l. \text{nodup } a (\text{agra } (\text{graph } I') l))$

$\langle \text{proof} \rangle$

lemma *actors-unique-loc-aid-base*:

$\forall a. (\forall l l'. a @_{\text{graph} I} \text{Airplane-not-in-danger-init } l \wedge$
 $a @_{\text{graph} I} \text{Airplane-not-in-danger-init } l' \longrightarrow l = l') \wedge$
 $(\forall l. \text{nodup } a (\text{agra } (\text{graph } I) \text{Airplane-not-in-danger-init } l))$

$\langle \text{proof} \rangle$

lemma *actors-unique-loc-aid-step*:

$(Airplane-not-in-danger-init, I) \in \{(x::infrastructure, y::infrastructure). x \rightarrow_n y\}^*$
 $\implies \forall a. (\forall l l'. a @_{graphI I} l \wedge a @_{graphI I} l' \longrightarrow l = l') \wedge$
 $(\forall l. nodup a (agra (graphI I) l))$
 $\langle proof \rangle$

Using the state transition, Kripke structure and CTL, we can now also express (and prove!) unreachability properties which enable to formally verify security properties for specific policies, like two-person rule.

lemma *Anid-airplane-actors*: $actors-graph (graphI Airplane-not-in-danger-init) = airplane-actors$
 $\langle proof \rangle$

lemma *all-airplane-actors*: $(Airplane-not-in-danger-init, y) \in \{(x::infrastructure, y::infrastructure). x \rightarrow_n y\}^*$
 $\implies actors-graph(graphI y) = airplane-actors$
 $\langle proof \rangle$

lemma *actors-at-loc-in-graph*: $\llbracket l \in nodes(graphI I); a @_{graphI I} l \rrbracket$
 $\implies a \in actors-graph (graphI I)$
 $\langle proof \rangle$

lemma *not-en-get-Apnid*:

assumes $(Airplane-not-in-danger-init, y) \in \{(x::infrastructure, y::infrastructure). x \rightarrow_n y\}^*$
shows $\sim(enables y l (Actor a) get)$
 $\langle proof \rangle$

lemma *Apnid-tsp-test*: $\sim(enables Airplane-not-in-danger-init cockpit (Actor "Alice") get)$
 $\langle proof \rangle$

lemma *Apnid-tsp-test-gen*: $\sim(enables Airplane-not-in-danger-init l (Actor a) get)$
 $\langle proof \rangle$

lemma *test-graph-atI*: $"Bob" @_{graphI Airplane-not-in-danger-init cockpit}$
 $\langle proof \rangle$

Invariant: number of staff in cockpit never below 2

lemma *two-person-inv*:

fixes $z z'$
assumes $(2::nat) \leq length (agra (graphI z) cockpit)$
and $nodes(graphI z) = nodes(graphI Airplane-not-in-danger-init)$
and $delta(z) = delta(Airplane-not-in-danger-init)$
and $(Airplane-not-in-danger-init, z) \in \{(x::infrastructure, y::infrastructure). x \rightarrow_n y\}^*$
and $z \rightarrow_n z'$

shows $(2::nat) \leq \text{length } (\text{agra } (\text{graphI } z') \text{ cockpit})$
 $\langle \text{proof} \rangle$

lemma *two-person-inv1*:

assumes $(\text{Airplane-not-in-danger-init}, z) \in \{(x::\text{infrastructure}, y::\text{infrastructure}).$
 $x \rightarrow_n y\}^*$
shows $(2::nat) \leq \text{length } (\text{agra } (\text{graphI } z) \text{ cockpit})$
 $\langle \text{proof} \rangle$

The version of *two_person_inv* above we need, uses cardinality of lists of actors rather than length of lists. Therefore first some equivalences and then a restatement of *two_person_inv* in terms of sets

proof idea: show since there are no duplicates in the list $\text{agra } (\text{graphI } z)$ cockpit therefore then $\text{card}(\text{set}(\text{agra } (\text{graphI } z))) = \text{length}(\text{agra } (\text{graphI } z))$

lemma *nodup-card-insert*:

$a \notin \text{set } l \longrightarrow \text{card } (\text{insert } a \text{ (set } l)) = \text{Suc } (\text{card } (\text{set } l))$
 $\langle \text{proof} \rangle$

lemma *no-dup-set-list-num-eq*[*rule-format*]:

$(\forall a. \text{nodup } a \text{ } l) \longrightarrow \text{card } (\text{set } l) = \text{length } l$
 $\langle \text{proof} \rangle$

lemma *two-person-set-inv*:

assumes $(\text{Airplane-not-in-danger-init}, z) \in \{(x::\text{infrastructure}, y::\text{infrastructure}).$
 $x \rightarrow_n y\}^*$
shows $(2::nat) \leq \text{card } (\text{set } (\text{agra } (\text{graphI } z) \text{ cockpit}))$
 $\langle \text{proof} \rangle$

lemma *Pred-all-unique*: $\llbracket ? x. P x; (! x. P x \longrightarrow x = c) \rrbracket \Longrightarrow P c$

$\langle \text{proof} \rangle$

lemma *Set-all-unique*: $\llbracket S \neq \{\}; (\forall x \in S. x = c) \rrbracket \Longrightarrow c \in S$

$\langle \text{proof} \rangle$

lemma *airplane-actors-inv0*[*rule-format*]:

$\forall z z'. (\forall h::\text{char list} \in \text{set } (\text{agra } (\text{graphI } z) \text{ cockpit}). h \in \text{airplane-actors}) \wedge$
 $(\text{Airplane-not-in-danger-init}, z) \in \{(x::\text{infrastructure}, y::\text{infrastructure}). x$
 $\rightarrow_n y\}^* \wedge$
 $z \rightarrow_n z' \longrightarrow (\forall h::\text{char list} \in \text{set } (\text{agra } (\text{graphI } z') \text{ cockpit}). h \in$
 $\text{airplane-actors})$
 $\langle \text{proof} \rangle$

lemma *airplane-actors-inv*:

assumes $(\text{Airplane-not-in-danger-init}, z) \in \{(x::\text{infrastructure}, y::\text{infrastructure}).$
 $x \rightarrow_n y\}^*$
shows $\forall h::\text{char list} \in \text{set } (\text{agra } (\text{graphI } z) \text{ cockpit}). h \in \text{airplane-actors}$
 $\langle \text{proof} \rangle$

lemma *Eve-not-in-cockpit*: (*Airplane-not-in-danger-init*, *I*)
 $\in \{(x::\text{infrastructure}, y::\text{infrastructure}). x \rightarrow_n y\}^* \implies$
 $x \in \text{set } (\text{agra } (\text{graphI } I) \text{ cockpit}) \implies x \neq \text{"Eve"}$
 $\langle \text{proof} \rangle$

2 person invariant implies that there is always some x in cockpit x not equal Eve

lemma *tp-imp-control*:
assumes (*Airplane-not-in-danger-init*, *I*) $\in \{(x::\text{infrastructure}, y::\text{infrastructure}).$
 $x \rightarrow_n y\}^*$
shows ($? x :: \text{identity. } x @_{\text{graphI } I} \text{cockpit} \wedge \text{Actor } x \neq \text{Actor "Eve"}$)
 $\langle \text{proof} \rangle$

lemma *Fend-2*: (*Airplane-not-in-danger-init*, *I*) $\in \{(x::\text{infrastructure}, y::\text{infrastructure}).$
 $x \rightarrow_n y\}^* \implies$
 $\neg \text{enables } I \text{ cockpit } (\text{Actor "Eve"}) \text{ put}$
 $\langle \text{proof} \rangle$

theorem *Four-eyes-no-danger*: *Air-tp-Kripke* $\vdash AG (\{x. \text{global-policy } x \text{ "Eve"}\})$
 $\langle \text{proof} \rangle$

end

In the following we construct an instance of the locale airplane and proof that it is an interpretation. This serves the validation.

definition *airplane-actors-def'*: *airplane-actors* $\equiv \{\text{"Bob"}, \text{"Charly"}, \text{"Alice"}\}$

definition *airplane-locations-def'*:

airplane-locations $\equiv \{\text{Location } 0, \text{Location } 1, \text{Location } 2\}$

definition *cockpit-def'*: *cockpit* $\equiv \text{Location } 2$

definition *door-def'*: *door* $\equiv \text{Location } 1$

definition *cabin-def'*: *cabin* $\equiv \text{Location } 0$

definition *global-policy-def'*: *global-policy* *I a* $\equiv a \notin \text{airplane-actors}$
 $\longrightarrow \neg(\text{enables } I \text{ cockpit } (\text{Actor } a) \text{ put})$

definition *ex-creds-def'*: *ex-creds* \equiv
 $(\lambda x. (\text{if } x = \text{Actor "Bob"}$
 $\text{then } ([\text{"PIN"}], [\text{"pilot"}])$
 $\text{else } (\text{if } x = \text{Actor "Charly"}$
 $\text{then } ([\text{"PIN"}], [\text{"copilot"}])$
 $\text{else } (\text{if } x = \text{Actor "Alice"}$
 $\text{then } ([\text{"PIN"}], [\text{"flightattendant"}])$
 $\text{else } ([], []))))$

definition *ex-locs-def'*: *ex-locs* $\equiv (\lambda x. \text{if } x = \text{door then } [\text{"norm"}] \text{ else}$
 $(\text{if } x = \text{cockpit then } [\text{"air"}] \text{ else } []))$

definition *ex-locs'-def'*: *ex-locs'* $\equiv (\lambda x. \text{if } x = \text{door then } [\text{"locked"}] \text{ else}$
 $(\text{if } x = \text{cockpit then } [\text{"air"}] \text{ else } []))$

definition *ex-graph-def'*: $ex-graph \equiv Lgraph$
 $\{(cockpit, door), (door, cabin)\}$
 $(\lambda x. \text{if } x = cockpit \text{ then } ["Bob", "Charly"]$
 $\quad \text{else } (\text{if } x = door \text{ then } []$
 $\quad \quad \text{else } (\text{if } x = cabin \text{ then } ["Alice"] \text{ else } [])))$
ex-creds ex-locs

definition *aid-graph-def'*: $aid-graph \equiv Lgraph$
 $\{(cockpit, door), (door, cabin)\}$
 $(\lambda x. \text{if } x = cockpit \text{ then } ["Charly"]$
 $\quad \text{else } (\text{if } x = door \text{ then } []$
 $\quad \quad \text{else } (\text{if } x = cabin \text{ then } ["Bob", "Alice"] \text{ else } [])))$
ex-creds ex-locs'

definition *aid-graph0-def'*: $aid-graph0 \equiv Lgraph$
 $\{(cockpit, door), (door, cabin)\}$
 $(\lambda x. \text{if } x = cockpit \text{ then } ["Charly"]$
 $\quad \text{else } (\text{if } x = door \text{ then } ["Bob"]$
 $\quad \quad \text{else } (\text{if } x = cabin \text{ then } ["Alice"] \text{ else } [])))$
ex-creds ex-locs

definition *agid-graph-def'*: $agid-graph \equiv Lgraph$
 $\{(cockpit, door), (door, cabin)\}$
 $(\lambda x. \text{if } x = cockpit \text{ then } ["Charly"]$
 $\quad \text{else } (\text{if } x = door \text{ then } []$
 $\quad \quad \text{else } (\text{if } x = cabin \text{ then } ["Bob", "Alice"] \text{ else } [])))$
ex-creds ex-locs

definition *local-policies-def'*: $local-policies\ G \equiv$
 $(\lambda y. \text{if } y = cockpit \text{ then}$
 $\quad \{(\lambda x. (? n. (n @_G cockpit) \wedge Actor\ n = x), \{put\}),$
 $\quad \quad (\lambda x. (? n. (n @_G cabin) \wedge Actor\ n = x \wedge has\ G\ (x, "PIN")$
 $\quad \quad \quad \wedge isin\ G\ door\ "norm"), \{move\})$
 $\quad \}$
 $\quad \text{else } (\text{if } y = door \text{ then } \{(\lambda x. True, \{move\}),$
 $\quad \quad (\lambda x. (? n. (n @_G cockpit) \wedge Actor\ n = x), \{put\})\}$
 $\quad \quad \text{else } (\text{if } y = cabin \text{ then } \{(\lambda x. True, \{move\})\}$
 $\quad \quad \quad \text{else } \{\})\})$

definition *local-policies-four-eyes-def'*: $local-policies-four-eyes\ G \equiv$
 $(\lambda y. \text{if } y = cockpit \text{ then}$
 $\quad \{(\lambda x. (? n. (n @_G cockpit) \wedge Actor\ n = x) \wedge$
 $\quad \quad 2 \leq length(agra\ G\ y) \wedge (\forall h \in set(agra\ G\ y). h \in airplane-actors),$
 $\quad \quad \{put\}),$
 $\quad \quad (\lambda x. (? n. (n @_G cabin) \wedge Actor\ n = x \wedge has\ G\ (x, "PIN") \wedge$
 $\quad \quad \quad isin\ G\ door\ "norm"), \{move\})$
 $\quad \}$
 $\quad \text{else } (\text{if } y = door \text{ then}$
 $\quad \quad \{(\lambda x. ((? n. (n @_G cockpit) \wedge Actor\ n = x) \wedge 3 \leq length(agra\ G$
 $\quad \quad cockpit)), \{move\})\}$

else (if $y = \text{cabin}$ then
 $\{(\lambda x. ((? n. (n @_G \text{door}) \wedge \text{Actor } n = x)), \{\text{move}\})\}$
 else $\{\}\}$)))

definition *Airplane-scenario-def'*:

Airplane-scenario \equiv *Infrastructure ex-graph local-policies*

definition *Airplane-in-danger-def'*:

Airplane-in-danger \equiv *Infrastructure aid-graph local-policies*

Intermediate step where pilot left cockpit but door still in norm position

definition *Airplane-getting-in-danger0-def'*:

Airplane-getting-in-danger0 \equiv *Infrastructure aid-graph0 local-policies*

definition *Airplane-getting-in-danger-def'*:

Airplane-getting-in-danger \equiv *Infrastructure agid-graph local-policies*

definition *Air-states-def'*: *Air-states* $\equiv \{ I. \text{Airplane-scenario} \rightarrow_n^* I \}$

definition *Air-Kripke-def'*: *Air-Kripke* \equiv *Kripke Air-states* $\{\text{Airplane-scenario}\}$

definition *Airplane-not-in-danger-def'*:

Airplane-not-in-danger \equiv *Infrastructure aid-graph local-policies-four-eyes*

definition *Airplane-not-in-danger-init-def'*:

Airplane-not-in-danger-init \equiv *Infrastructure ex-graph local-policies-four-eyes*

definition *Air-tp-states-def'*: *Air-tp-states* $\equiv \{ I. \text{Airplane-not-in-danger-init} \rightarrow_n^* I \}$

definition *Air-tp-Kripke-def'*:

Air-tp-Kripke \equiv *Kripke Air-tp-states* $\{\text{Airplane-not-in-danger-init}\}$

definition *Safety-def'*: *Safety* $I a \equiv a \in \text{airplane-actors}$

$\longrightarrow (\text{enables } I \text{ cockpit } (\text{Actor } a) \text{ move})$

definition *Security-def'*: *Security* $I a \equiv (\text{isin } (\text{graph } I) \text{ door } \text{"locked"})$

$\longrightarrow \neg(\text{enables } I \text{ cockpit } (\text{Actor } a) \text{ move})$

definition *foe-control-def'*: *foe-control* $l c \equiv$

($! I :: \text{infrastructure}. (? x :: \text{identity}.$
 $x @_{\text{graph } I} l \wedge \text{Actor } x \neq \text{Actor } \text{"Eve"})$
 $\longrightarrow \neg(\text{enables } I l (\text{Actor } \text{"Eve"}) c)$)

definition *astate-def'*: *astate* $x \equiv$

(case x of
 $\text{"Eve"} \Rightarrow \text{Actor-state depressed } \{\text{revenge, peer-recognition}\}$
 $| - \Rightarrow \text{Actor-state happy } \{\}$)

print-interps *airplane*

The additional assumption identified in the case study needs to be given as an axiom

axiomatization where

cockpit-foe-control': foe-control cockpit put

(The following addresses the issue of redefining an abstract type. We experimented with suggestion given here: Makarius Wenzel, Re: [isabelle] typedecl versus explicit type parameters, Isabelle users mailing list, 2009, <https://lists.cam.ac.uk/pipermail/cl-isabelle-users/2009-July/msg00111.html>.) We furthermore need axiomatization to add the missing semantics to the abstractly declared type actor and thereby be able to redefine consts Actor. Since the function Actor has also been defined as a consts :: identity =, actor as an abstract function without a definition, we now also now add its semantics mimicking some of the concepts of the conservative type definition of HOL. The alternative method of using a Locale to replace the abstract type_decl actor in the AirInsider is a more elegant method for representing an abstract type actor but it is not working properly for our framework since it necessitates introducing a type parameter 'actor into infrastructures which then makes it impossible to instantiate them to the typeclass state in order to use CTL and Kripke and the generic state transition. Therefore, we go the former way of a post-hoc axiomatic redefinition of the abstract type actor by using axiomatization of the existing Locale "type_definition". This is done in the following. It allows to abstractedly assume as an axiom that there is a type definition for the abstract type actor. Adding a suitable definition of a representation for this type then additionally enables to introduce a definition for the function Actor (again using axiomatization to enforce the new definition).

definition *Actor-Abs* :: identity \Rightarrow identity option

where

Actor-Abs $x \equiv$ (if $x \in \{"Eve", "Charly"\}$ then None else Some x)

lemma *UasI-ActorAbs*: *Actor-Abs* "Eve" = *Actor-Abs* "Charly" \wedge

$(\forall (x::char\ list)\ y::char\ list. x \neq "Eve" \wedge y \neq "Eve" \wedge \text{Actor-Abs } x = \text{Actor-Abs } y \longrightarrow x = y)$

<proof>

lemma *Actor-Abs-ran*: *Actor-Abs* $x \in \{y :: identity\ option. y \in \text{Some } ' \{x :: identity. x \notin \{"Eve", "Charly"\}\} \mid y = \text{None}\}$

<proof>

With the following axiomatization, we can simulate the abstract type actor and postulate some unspecified Abs and Rep functions between it and the simulated identity option subtype.

axiomatization where *Actor-type-def*:

type-definition (*Rep* :: actor \Rightarrow identity option)(*Abs* :: identity option \Rightarrow actor)

$\{y :: \text{identity option}. y \in \text{Some } ' \{x :: \text{identity}. x \notin \{"Eve", "Charly"\}\} | y = \text{None}\}$

lemma *Abs-inj-on*: $\bigwedge \text{Abs Rep} :: \text{actor} \Rightarrow \text{char list option}. x \in \{y :: \text{identity option}. y \in \text{Some } ' \{x :: \text{identity}. x \notin \{"Eve", "Charly"\}\} | y = \text{None}\} \Rightarrow y \in \{y :: \text{identity option}. y \in \text{Some } ' \{x :: \text{identity}. x \notin \{"Eve", "Charly"\}\} | y = \text{None}\} \Rightarrow (\text{Abs} :: \text{char list option} \Rightarrow \text{actor}) x = \text{Abs } y \Rightarrow x = y$
 $\langle \text{proof} \rangle$

lemma *Actor-td-Abs-inverse*:
 $(y \in \{y :: \text{identity option}. y \in \text{Some } ' \{x :: \text{identity}. x \notin \{"Eve", "Charly"\}\} | y = \text{None}\}) \Rightarrow (\text{Rep} :: \text{actor} \Rightarrow \text{identity option})(\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) y = y$
 $\langle \text{proof} \rangle$

Now, we can redefine the function Actor using a second axiomatization

axiomatization *where Actor-redef*: $\text{Actor} = (\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) o \text{Actor-Abs}$

need to show that $\text{Abs } (\text{Actor-Abs } x) = \text{Abs } (\text{Actor-Abs } y) \longrightarrow \text{Actor-Abs } x = \text{Actor-Abs } y$, i.e. *injective Abs*. Generally, Abs is not injective but *injective-on* the type predicate. So, need to show that for any x, *Actor-Abs* x is in the type predicate, then it would follow. What is the type predicate? $\{y. y \in \text{Some } ' \{x. x \notin \{"Eve", "Charly"\}\} \vee y = \text{None}\}$

lemma *UasI-Actor-redef*:
 $\bigwedge \text{Abs Rep} :: \text{actor} \Rightarrow \text{char list option}.$
 $((\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) o \text{Actor-Abs}) \text{"Eve"} = ((\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) o \text{Actor-Abs}) \text{"Charly"} \wedge$
 $(\forall (x :: \text{char list}) y :: \text{char list}. x \neq \text{"Eve"} \wedge y \neq \text{"Eve"} \wedge$
 $((\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) o \text{Actor-Abs}) x = ((\text{Abs} :: \text{identity option} \Rightarrow \text{actor}) o \text{Actor-Abs}) y \longrightarrow x = y)$
 $\langle \text{proof} \rangle$

Finally all of this allows us to show the last assumption contained in the Insider Locale assumption needed for the interpretation of airplane.

lemma *UasI-Actor*: $\text{UasI } \text{"Eve"} \text{"Charly"}$
 $\langle \text{proof} \rangle$

interpretation *airplane airplane-actors airplane-locations cockpit door cabin global-policy*

ex-creds ex-locs ex-locs' ex-graph aid-graph aid-graph0 agid-graph
local-policies local-policies-four-eyes Airplane-scenario Airplane-in-danger
Airplane-getting-in-danger0 Airplane-getting-in-danger Air-states
Air-Kripke
Airplane-not-in-danger Airplane-not-in-danger-init Air-tp-states
Air-tp-Kripke Safety Security foe-control astate

$\langle proof \rangle$

end

References

- [1] F. Kammüller and M. Kerber. Investigating airplane safety and security against insider threats using logical modeling. In *IEEE Security and Privacy Workshops, Workshop on Research in Insider Threats, WRIT'16*. IEEE, 2016.
- [2] F. Kammüller and M. Kerber. Applying the isabelle insider framework to airplane security, 2020. arxiv preprint 2003.11838.
- [3] F. Kammüller and C. W. Probst. Modeling and verification of insider threats using logical analysis. *IEEE Systems Journal, Special issue on Insider Threats to Information Security, Digital Espionage, and Counter Intelligence*, 11(2):534–545, 2017.
- [4] M. Wenzel. Re: [isabelle] typedecl versus explicit type parameters, 2009. Isabelle users mailing list.