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1. What can you say about the sign of the expression  $(2u+1)\ln\left(1+\frac{1}{u}\right)-2$  for  $u > 0$ ?
  2. Given a  $M \times N$  table with numbers from 1 to  $P$  written in each cell and each number is written in at least one cell. A pair of numbers  $(a, b)$ ,  $1 \leq a < b \leq P$  is called a neighbor, if there is at least one pair of cells sharing an edge such that the cells are marked with these numbers. Which low bound for the number of neighbors can you provide? Is it exact?
  3. In some city a car starts moving from the central square to some direction. The driver turns right at every crossroad. More precisely, the car turns to the nearest right if many available. Assume that all roads are two-ways (it's legal to drive in both directions) and any number of roads may cross at each intersection. Besides that, there can be multilevel roads so that you cannot assume that roads over the same point  $(x,y)$  intersect. What are the conditions under which the driver ever returns to the starting point?
  4. Maximize the function  $F(x) = \sum_{i=1}^n \log(\alpha_i + x_i)$  with  $\alpha_i > 0, i = 1, \dots, n$  subject to constraints:  $x_i \geq 0, i = 1, \dots, n$  and  $x_1 + \dots + x_n = 1$ . Prove that your solution is the global optimum.
  5. Military tanks ride only by asphalt bi-directional roads on the planet **BARSOOM**. While moving, tanks destroy roadbed under themselves. A very modern tank **Armata** made a trip from the **CAPITAL** to **ZODANGA** city. Denote by  $rb$  and  $ra$  the minimal road width on the **BARSOOM** before and after this trip correspondingly. We know that  $rb=ra$ . What values can **Armata**'s width take?

The width of every road is known. We expect that you provide a C++ code for the solution of this problem. Also we expect algorithm explanation (with prove of correctness and complexity estimation). You should take into account that

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**BARSOOM** planet can have a rich number of roads.

The format of data file with roadmap is up to you. You don't need to use any libraries except for input/output, random and sorting.

Roads width are real numbers; number of cities  $\leq 10^6$ ; number of roads  $\leq 10^6$ .

Provide generation of a sample input with any number of cities and roads in your code (choice of **CAPITAL** and **ZODANGA**, roads' placement and their capacities (width) should be generated randomly). The code should also contain a test function, which reads the data file, calls your solution method, and returns the result.

The result of compilation should be console application (e.g. opt.exe).

Console input	What does it mean
opt.exe -g 20 100 bars.txt	Generate graph with 20 nodes and 100 roads and write to bars.txt <i>Note: if number of roads <math>\geq</math> number of nodes then graph should be connected</i>
opt.exe -t 3 4 mars.txt	Test graph from mars.txt with ID of Capital equal to 3 and ID of Zodanga equal to 4