Logic: Homework (Solution Updated)

A Feast of Double Negation
12th Formosan Summer School on Logic, Language, and Computation, 2018

July 19, 2018

1. Derive $\vdash \neg\neg(\neg\neg A \to A)$ in NJ.

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{
          \cfrac{\neg(\neg\neg A \to A), \neg\neg A, A \vdash (\neg\neg A \to A) \to \bot}{} \text{(assum)}
          \quad
          \cfrac{
            \cfrac{\neg(\neg\neg A \to A), \neg\neg A, A \vdash A}{} \text{(assum)}
          }{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \neg\neg A \to A} (\to\text{I})
        }{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \bot} (\to\text{E})
      }{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg A} (\to\text{I})
      \quad
      \cfrac{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg\neg A}{} \text{(assum)}
    }{\neg(\neg\neg A \to A), \neg\neg A \vdash \bot} (\to\text{E})
  }{
    \cfrac{\neg(\neg\neg A \to A), \neg\neg A \vdash A}{\neg(\neg\neg A \to A) \vdash \neg\neg A \to A} (\to\text{I})
  } (\bot\text{E})
  \quad
  \cfrac{\neg(\neg\neg A \to A) \vdash \neg(\neg\neg A \to A)}{} \text{(assum)}
}{
  \cfrac{\neg(\neg\neg A \to A) \vdash \bot}{\vdash \neg\neg(\neg\neg A \to A)} (\to\text{I})
} (\to\text{E})
$$

1. Let $\Gamma := \neg\neg(A \lor B), \neg\neg A \to \neg\neg C, \neg\neg A \to \neg\neg C$. Derive $\Gamma \vdash \neg\neg C$ in NJ.

$$
\cfrac{
  \cfrac{\Gamma, \neg C \vdash \neg\neg(A \lor B)}{} \text{(assum)}
  \quad
  \cfrac{
    \cfrac{\Gamma, \neg C, A \lor B \vdash A \lor B}{} \text{(assum)}
    \quad
    \cfrac{\cdots}{\Gamma, \neg C, A \lor B, A \vdash \bot}
    \quad
    \cfrac{\cdots}{\Gamma, \neg C, A \lor B, B \vdash \bot}
  }{
    \cfrac{\Gamma, \neg C, A \lor B \vdash \bot}{\Gamma, \neg C \vdash \neg(A \lor B)} (\to\text{I})
  } (\lor\text{E})
}{
  \cfrac{\Gamma, \neg C \vdash \bot}{\Gamma \vdash \neg\neg C \to \bot} (\to\text{I})
} (\to\text{E})
$$