Logic: Homework

A Feast of Double Negation
12th Formosan Summer School on Logic, Language, and Computation, 2018

July 18, 2018

1. Derive $\vdash \neg\neg(\neg\neg A \to A)$ in NJ.

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            \cfrac{\overline{\neg(\neg\neg A \to A), \neg\neg A, A \vdash (\neg\neg A \to A) \to \bot}\ \text{(assum)} \quad \cfrac{\cfrac{\overline{\neg(\neg\neg A \to A), \neg\neg A, A \vdash A}\ \text{(assum)}}{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \neg\neg A}\ (\to\text{I})}{}}{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \bot}\ (\bot\text{E})
          }{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg A}\ (\bot\text{E}) \quad \overline{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg A}\ \text{(assum)}
        }{\neg(\neg\neg A \to A), \neg\neg A \vdash \bot}\ (\to\text{E})
      }{\neg(\neg\neg A \to A), \neg\neg A \vdash A}\ (\bot\text{E})
    }{\neg(\neg\neg A \to A) \vdash \neg\neg A \to A}\ (\to\text{I}) \quad \overline{\neg(\neg\neg A \to A) \vdash \neg(\neg\neg A \to A)}\ \text{(assum)}
  }{\neg(\neg\neg A \to A) \vdash \bot}\ (\to\text{E})
}{\vdash \neg\neg(\neg\neg A \to A)}\ (\to\text{I})
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1. Let $\Gamma := \neg\neg(A \lor B), \neg\neg A \to \neg\neg C, \neg\neg A \to \neg\neg C$. Derive $\Gamma \vdash \neg\neg C$ in NJ. (There are two ways of deriving this!)

(a)

(b)