Logic: Homework 1

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Sixth Formosan Summer School on Logic, Language, and Computation, 2012

Please submit your solutions to me before 9:10AM on Wednesday, 29 August 2012.

1. Show that $\vdash_{\mathrm{NJ}} (\mathtt{A} \wedge \mathtt{B}) \wedge \mathtt{C} \to \mathtt{A} \wedge (\mathtt{B} \wedge \mathtt{C})$.

2. Show that $\vdash_{\mathrm{NJ}} (\mathtt{A} \wedge \mathtt{B} \to \mathtt{C}) \leftrightarrow (\mathtt{A} \to \mathtt{B} \to \mathtt{C})$.

3. Show that $\vdash_{\mathrm{NJ}} \mathtt{A} \vee (\mathtt{B} \wedge \mathtt{C}) \leftrightarrow (\mathtt{A} \vee \mathtt{B}) \wedge (\mathtt{A} \vee \mathtt{C})$.
   (Assume that '$\leftrightarrow$' has the same, lowest precedence as '$\to$'.)

4. Show that $\vdash_{\mathrm{NJ}} \neg(\mathtt{A} \vee \mathtt{B}) \leftrightarrow \neg\mathtt{A} \wedge \neg\mathtt{B}$.

5. Show that $\vdash_{\mathrm{NJ}} \neg\neg(\mathtt{A} \vee \neg\mathtt{A})$.

6. Show that the following deduction rule is admissible in NJ.

$$\frac{\Gamma \vdash \varphi \wedge \psi \qquad \Gamma, \varphi, \psi \vdash \vartheta}{\Gamma \vdash \vartheta} \ (\wedge\mathrm{E}')$$

7. Show that $\vdash_{\mathrm{NJ}} (\forall \mathtt{x}.\ \mathtt{P}\,\mathtt{x} \wedge \mathtt{Q}\,\mathtt{x}) \leftrightarrow (\forall \mathtt{x}.\ \mathtt{P}\,\mathtt{x}) \wedge (\forall \mathtt{x}.\ \mathtt{Q}\,\mathtt{x})$.

8. Show that $\vdash_{\mathrm{NJ}} ((\exists \mathtt{x}.\ \mathtt{P}\,\mathtt{x}) \to \mathtt{Q}) \leftrightarrow (\forall \mathtt{x}.\ \mathtt{P}\,\mathtt{x} \to \mathtt{Q})$.

9. Show that $\vdash_{\mathrm{NJ}} (\exists \mathtt{x}.\ \mathtt{P}\,\mathtt{x}) \to \neg(\forall \mathtt{x}.\ \neg(\mathtt{P}\,\mathtt{x}))$.

10. Show that $\mathbf{HA} \vdash_{\mathrm{NJ}} \forall \mathtt{x}.\ \mathtt{x} + \mathtt{zero} \equiv \mathtt{x}$. (Hint: induction on $\mathtt{x}$.)