Exercises #2

[Compiled on September 5, 2017]

1. Let max be a function that returns the maximal number between two input numbers. Write a specification of max as precise as possible.

   - $\{?\}\,max(x, y)\,\{?\}$

2. Write the specification of a function that concatenates two integer lists. You may define other functions of list and use them in the specification.

   - List of integers is defined as $list ::= nil \mid cons(Int, list)$.

3. Complete the proof outline.

   $\{x \geq 0 \land y \geq 0 \land gcd(x, y) = gcd(m, n)\}$
   while $x \neq 0 \land y \neq 0$ **do**
      if $x < y$ **then**
         x, y := y, x
      fi ;
      x := x − y
   od
   $\{(x = 0 \land y \geq 0 \land y = gcd(x, y) = gcd(m, n))\lor$
   $(x \geq 0 \land y = 0 \land x = gcd(x, y) = gcd(m, n))\}$

4. Compute weakest preconditions.

   (a) $wp(x := x + 2; y := y - 2, x + y = 0)$

   (b) $wp(\textbf{if } x < y \textbf{ then } res := y \textbf{ else } res := x \textbf{ fi}, res \geq x \land res \geq y)$

5. Conside the following program.

```
x = nil;
i = 0;
while(i < n) {
   x = cons(i, x);
   i = i + 1;
}
j = 0
while(j < n) {
   assert(x != nil)
   x = del(x);
   j = j + 1;
}
```

   Assume $n \geq 0$ and

   - $list(0, x, x)$ for all x

- $list(0, x, z) \rightarrow x = z$
- $x = cons(a, b) \wedge list(n, b, z) \leftrightarrow list(n + 1, x, z)$
- $list(n, x, z) \wedge y = del(x) \wedge n > 0 \rightarrow list(n - 1, y, z)$
- $list(n, x, z) \wedge n > 0 \rightarrow x \neq nil$

Either show that the assertion wont be violated or find a counterexample that violates the assertion. ($list(n, x, y)$: x points to a list ended at y with length n.)