Logic: Homework (Solution Updated)

A Feast of Double Negation 12th Formosan Summer School on Logic, Language, and Computation, 2018

July 19, 2018

1. Derive $\vdash \neg \neg (\neg \neg A \rightarrow A)$ in NJ.

| $\frac{\neg(\neg\neg A \to A), \neg\neg A, A \vdash (\neg\neg A \to A) \to \bot}{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \neg A \to A} \xrightarrow{\text{(assum)}} \frac{\neg(\neg\neg A \to A), \neg\neg A, A \vdash A}{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \neg\neg A \to A} \xrightarrow{\text{(\to I })} (\to \text{E})$ | | | |
|---|--|--|--|
| $\neg(\neg\neg A \rightarrow A) \neg \neg A A \vdash \bot$ | $\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg\neg A}{\neg(\to E)} $ (assum) | | |
| $\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash \bot}{\neg(\neg\neg A \to A), \neg\neg A \vdash A} \text{ (\botE)}}{\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash A}{\neg(\neg\neg A \to A) \vdash \neg\neg A \to A} \text{ (\toI)}}$ | (→E) | ${\neg(\neg\neg A \to A) \vdash \neg(\neg\neg A \to A)} \text{ (assum)}$ $(\to E)$ | |
| $\frac{((+21-7H)++41-7H)}{}$ | $\frac{\neg(\neg\neg A \to A) \vdash \bot}{\vdash \neg\neg(\neg\neg A \to A)} \ (\to I)$ | $\frac{(+1/1 - 7/1) + (+1/1 - 7/1)}{(\rightarrow E)} (\rightarrow E)$ | |
| . Let $\Gamma \coloneqq \neg \neg (A \lor B), \neg \neg A \to \neg \neg C, \neg \neg A \to \neg \neg C$. Derive $\Gamma \vdash \neg \neg C$ in NJ. | | | |

| | | $\overline{\Gamma, \neg C, A \lor B,}$ | $A, \neg A \vdash \neg A$ (assum) $\Gamma, \neg C, A \lor B, A, \neg A \vdash A$ (assum) $(A, \neg A, \neg$ | m) | | $\Gamma, \neg C, A \lor B, B,$ | $ \overline{\neg B \vdash \neg B} \text{ (assum)} \qquad \overline{\Gamma, \neg C, A \lor B, B, \neg B \vdash B} \text{ (assum)} $ $ (\rightarrow E) $ | |
|--|---|---|---|---|---|--|---|--|
| | | $\overline{\Gamma, \neg C, A \lor B, A \vdash \neg \neg A \to \neg \neg C} \text{ (assum)}$ | $\frac{\Gamma, \neg C, A \lor B, A, \neg A \vdash \bot}{\Gamma, \neg C, A \lor B, A \vdash \neg \neg A} (\to I)$ | () | $\overline{\Gamma, \neg C, A \lor B, B \vdash \neg \neg B \to \neg \neg C}$ | (assum) | $\frac{\Gamma, \neg C, A \lor B, B, \neg B \vdash \bot}{\Gamma, \neg C, A \lor B, B \vdash \neg \neg B} (\to I)$ | |
| | | $\Gamma, \neg C, A \lor B, A \vdash \neg \neg C$ | $(\rightarrow E)$ | $\Gamma, \neg C, A \lor B, A \vdash C \to \bot \text{ (assum)}$ | | $\Gamma, \neg C, A \lor B, B \vdash \neg \neg C$ | $(\rightarrow E)$ | $\overline{\Gamma, \neg C, A \lor B, B \vdash \neg C} \text{ (assum)}$ |
| | $\Gamma, \neg C, A \lor B \vdash A \lor B $ (assum) | | $\Gamma, \neg C, A \lor B, A \vdash \bot$ | (/ L) | | | $\Gamma, \neg C, A \lor B, B \vdash \bot$ (VE) | (, L) |
| (assum) | | | $\Gamma, \neg C, A \lor A$ | \longrightarrow (\rightarrow) | | | (VE) | |
| ${\Gamma, \neg C \vdash \neg \neg (A \lor B)} \text{ (assum)}$ | | | $\Gamma, \neg C \vdash \neg (A)$ | $A \vee B$) $\begin{pmatrix} & & & & & & & & & & & & & & & & & & $ | | | | |
| | | $\Gamma \neg C \vdash \bot$ | | \longrightarrow $(\rightarrow$ E) | | | | |

 $\frac{\Gamma, \neg C \vdash \bot}{\Gamma \vdash \neg C \to \bot} \ (\to I)$