Logic: Homework

A Feast of Double Negation

12th Formosan Summer School on Logic, Language, and Computation, 2018

July 18, 2018

1. Derive $\vdash \neg \neg (\neg \neg A \rightarrow A)$ in NJ. $\frac{\neg(\neg\neg A \to A), \neg\neg A, A \vdash A}{\neg(\neg\neg A \to A), \neg\neg A, A \vdash \neg\neg A \to A} \xrightarrow{(\to I)} (\bot E)$ $\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash \neg\neg A}{\neg(\to E)} \text{ (assum)}$ $\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash \bot}{\neg(\neg\neg A \to A), \neg\neg A \vdash A} (\bot E)$ $\frac{\neg(\neg\neg A \to A), \neg\neg A \vdash A}{\neg(\neg\neg A \to A) \vdash \neg\neg A \to A} (\to I)$ $\frac{\neg(\neg\neg A \to A) \vdash \neg(\neg\neg A \to A)}{\neg(\neg A \to A)} \xrightarrow{\text{(assum)}}$ $\frac{\neg(\neg\neg A \to A) \vdash \bot}{\vdash \neg\neg(\neg\neg A \to A)} \ (\to I)$ 1. Let $\Gamma := \neg \neg (A \lor B), \neg \neg A \to \neg \neg C, \neg \neg A \to \neg \neg C$. Derive $\Gamma \vdash \neg \neg C$ in NJ. (There are two ways of deriving this!) $\frac{\overline{\Gamma, \neg C, A \lor B, B, \neg B \vdash \neg B} \text{ (assum)}}{\Gamma, \neg C, A \lor B, B, \neg B \vdash B} \frac{\Gamma, \neg C, A \lor B, B, \neg B \vdash B}{(\rightarrow E)}$ $\frac{\Gamma, \neg C, A \lor B, A, \neg A \vdash \neg A}{\Gamma, \neg C, A \lor B, A, \neg A \vdash A} \xrightarrow{\text{(assum)}} \frac{\Gamma, \neg C, A \lor B, A, \neg A \vdash A}{(\rightarrow E)} \xrightarrow{\text{(assum)}}$ $\frac{\Gamma, \neg C, A \lor B, A, \neg A \vdash \bot}{\Gamma, \neg C, A \lor B, A \vdash \neg \neg A} \; (\to I)$ $(\to E)$ $\frac{\Gamma, \neg C, A \lor B, B, \neg B \vdash \bot}{\Gamma, \neg C, A \lor B, B \vdash \neg \neg B} \; (\to I)$ $(\to E)$ $\overline{\Gamma, \neg C, A \lor B, A} \vdash \neg \neg A \to \neg \neg C$ (assum) $\overline{\Gamma, \neg C, A \lor B, B \vdash \neg \neg B \to \neg \neg C} \text{ (assum)}$ $\frac{\Gamma, \neg C, A \lor B, A \vdash C \to \bot}{\Gamma, \neg C, A \lor B, A \vdash C \to \bot}$ (assum) $\frac{\Gamma, \neg C, A \lor B, B \vdash \neg C}{\Gamma, \neg C, A \lor B, B \vdash \neg C} \text{ (assum)}$ $\Gamma, \neg C, A \lor B, B \vdash \neg \neg C$ $\Gamma, \neg C, A \lor B, A \vdash \neg \neg C$ $\overline{\Gamma, \neg C, A \lor B \vdash A \lor B} \text{ (assum)}$ $\Gamma, \neg C, A \lor B, B \vdash \bot \quad (\lor E)$ (a) $\Gamma, \neg C, A \lor B, A \vdash \bot$ $\frac{\Gamma, \neg C, A \lor B \vdash \bot}{\Gamma, \neg C \vdash \neg (A \lor B)} \xrightarrow{(\to I)} (\to I)$ $\overline{\Gamma, \neg C \vdash \neg \neg (A \lor B)} \text{ (assum)}$ $\frac{\Gamma, \neg C \vdash \bot}{\Gamma \vdash \neg C \to \bot} \ (\to I)$ $\frac{\overline{\Gamma, \neg C, \neg A, \neg B, A \lor B, A \vdash A} \text{ (assum)} \qquad \overline{\Gamma, \neg C, \neg A, \neg B, A \lor B, A \vdash \neg A} \text{ (assum)}}{\Gamma, \neg C, \neg A, \neg B, A \lor B, A \vdash \bot} \text{ (assum)}} \qquad \frac{\overline{\Gamma, \neg C, \neg A, \neg B, A \lor B, B \vdash B} \text{ (assum)}}{\Gamma, \neg C, \neg A, \neg B, A \lor B, B \vdash \bot} \text{ (assum)}}{\Gamma, \neg C, \neg A, \neg B, A \lor B, B \vdash \bot} \text{ (be)}} \qquad (\rightarrow E)$ $\overline{\Gamma, \neg C, \neg A, \neg B, A \lor B \vdash A \lor B} \text{ (assum)}$ $\frac{\Gamma, \neg C, \neg A, \neg B, A \lor B \vdash \bot}{\Gamma, \neg C, \neg A, \neg B \vdash \neg (A \lor B)} (\to I)$ $(\to E)$ $\underline{\Gamma, \neg C, \neg A, \neg B \vdash \neg (A \lor B) \to \bot} \text{ (assum)}$ $\frac{\Gamma, \neg C, \neg A, \neg B \vdash \bot}{\Gamma, \neg C, \neg A \vdash \neg \neg B} (\rightarrow I)$ $\frac{\Gamma, \neg C, \neg A \vdash \neg \neg C}{\Gamma, \neg C, \neg A \vdash \neg \neg C} (\rightarrow E)$ (b) $\overline{\Gamma, \neg C, \neg A \vdash \neg \neg B \to \neg \neg C} \text{ (assum)}$ $\frac{\Gamma, \neg C \vdash \neg C \text{ (assum)}}{\Gamma, \neg C \vdash \neg C}$ $\frac{\Gamma, \neg C, \neg A \vdash \bot}{\Gamma, \neg C \vdash \neg \neg A} (\rightarrow I)$ $(\rightarrow E)$

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