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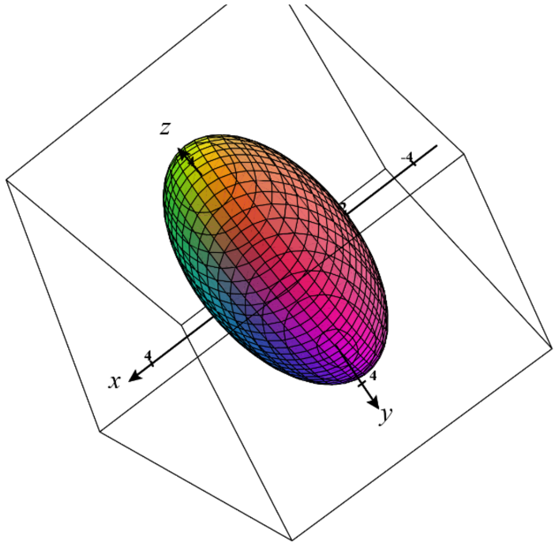
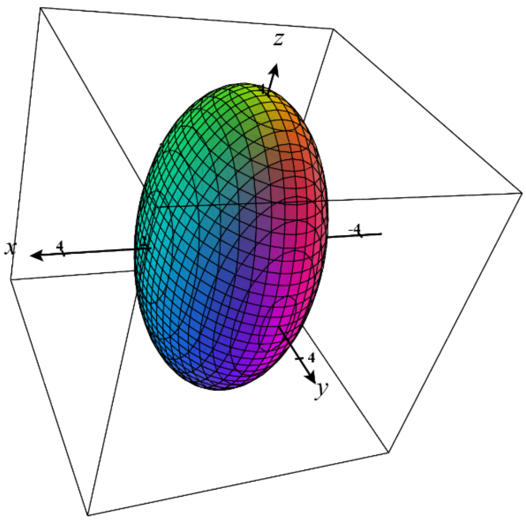
Math projet

step 3

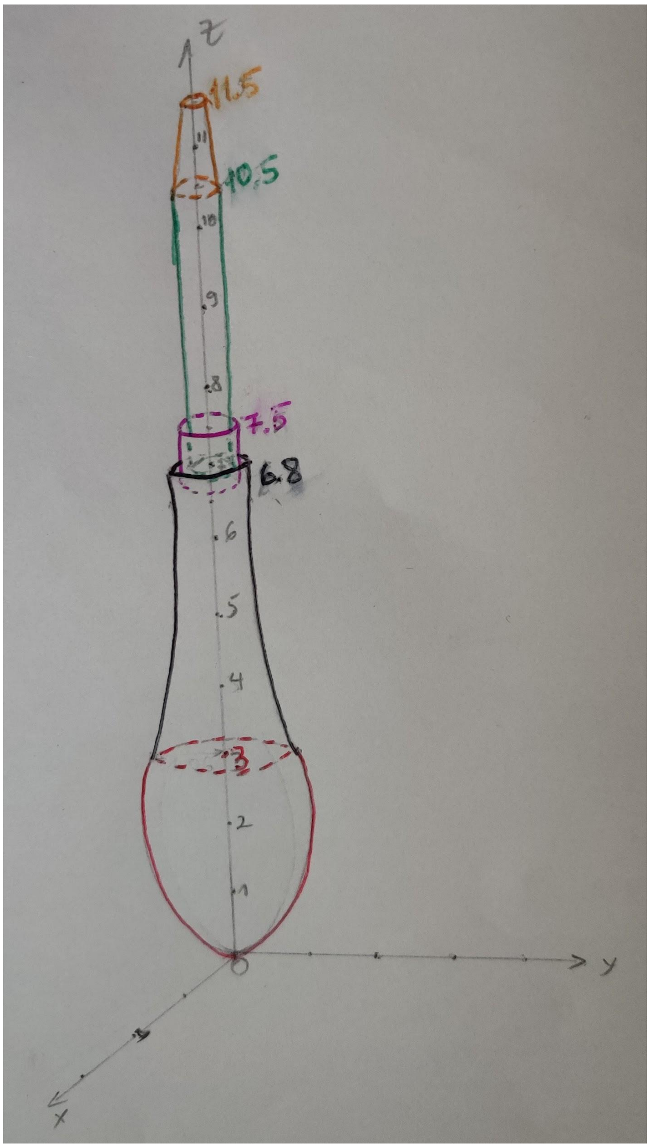
$$\frac{x^2}{2^2} + \frac{y^2}{3^2} + \frac{z^2}{4^2} = 1$$

(a) Using CalcPlot3D, I was able to get the following views of the hyperboloid of one sheet.





(b) The cross-section of the hyperboloid of one sheet with a plane parallel to the yz-plane is a hyperbola.



(b) The solution is not unique. This is what I decided to do. The bottom part of the screwdriver consists of a ellipsoid of height 3cm in the z direction, and circular cross section at z=3 in the xy plane of radius $r = \sqrt{1.125}cm \approx 1.06cm$, the part above the ellipsoidal part can be described as a truncated hyperboloid of one sheet of height 4 cm and a top cross section at z=7 cm that is a circle in the xy plane of radius $r = \sqrt{\frac{19}{60}}cm \approx 0.56cm$, . Above the hyperboloid part there is a shorter cylinder of height 0.7cm of radius $r = \sqrt{0.2}cm \approx 0.45cm$. We also have a longer cylinder of height 3.5cm and radius $r = \sqrt{0.1}cm \approx 0.32cm$ that passes through the shorter cylinder all the way out. Finally, after the longer cylindrical part, there is the top part which is can be described as a truncated hyperboloid of height 1 cm and a top cross section at z=11.5 in the xy plane of radius $r = \sqrt{0.02}cm \approx 0.14cm$.

c) The ellipsoidal part in the bottom can be described by $\frac{x^2}{1.5} + \frac{y^2}{1.5} + \frac{(z-2)^2}{4} = 1$ centered at (0,0,2) with 0 ≤ z ≤ 3. At z=3, the cross section is a circle in the xy plane

$$\frac{x^2}{1.5} + \frac{y^2}{1.5} = 1 - \frac{(3-2)^2}{4}$$

$$\frac{x^2}{1.5} + \frac{y^2}{1.5} = \frac{3}{4}$$

$$x^2 + y^2 = 1.125$$

$$r = \sqrt{1.125}cm \approx 1.06cm$$

For the hyperboloid, we can describe it by $\frac{x^2}{0.3} + \frac{y^2}{0.3} - \frac{(z-6.5)^2}{c^2} = 1$ centered at (0,0,6.5) with 3 ≤ z ≤ 7. At z=3, the cross section is a circle in the xy plane of radius $r = \sqrt{1.125}cm \approx 1.06cm$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{(3-6.5)^2}{c^2}$$

$$a^2 = b^2 = 0.3$$

$$\Rightarrow 0.3 \times (1 + \frac{(-3.5)^2}{c^2}) = 1.125$$

$$\Rightarrow \frac{0.3 \times 12.25}{c^2} = 1.125 - 0.3$$

$$\Rightarrow c = \sqrt{\frac{3.675}{0.825}} = 2.11$$

$$\frac{x^2}{0.3} + \frac{y^2}{0.3} - \frac{(z-6.5)^2}{4.5} = 1$$

At z=7, the cross section is a circle in the xy plane of radius

$$\frac{x^2}{0.3} + \frac{y^2}{0.3} = 1 + \frac{(7-6.5)^2}{4.5}$$

$$\frac{x^2}{0.3} + \frac{y^2}{0.3} = \frac{19}{18}$$

$$x^2 + y^2 = \frac{19}{60}$$

$$r = \sqrt{\frac{19}{60}}cm \approx 0.56cm$$

For the shorter cylinder, I made its overlapped a little bit with the above hyperboloid, and its radius is smaller than the radius of the cross section of the hyperboloid at z=7.
The shorter cylinder can be described by
$x^2 + y^2 = 0.2$ with radius $r = \sqrt{0.2}cm \approx 0.45cm$ and 6.8 ≤ z ≤ 7.5

For the longer cylinder, I made its radius smaller than the radius of the shorter cylinder and it overlapped with the shorter cylinder.
The longer cylinder can be described by
$x^2 + y^2 = 0.1$ with radius $r = \sqrt{0.1}cm \approx 0.32cm$ and 7 ≤ z ≤ 10.5

For the top part of the object, which I described as a truncated hyperboloid, we can describe it as
$\frac{x^2}{0.02} + \frac{y^2}{0.02} - \frac{(z-11.5)^2}{c^2} = 1$ centered at (0,0,11.5) and 10.5 ≤ z ≤ 11.5
At z=10.5, the cross section is a circle in the xy plane of radius
$r = \sqrt{0.1}cm \approx 0.32cm$

$$\frac{x^2}{0.02} + \frac{y^2}{0.02} = 1 + \frac{(10.5-11.5)^2}{c^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{(10.5-14)^2}{c^2}$$

$$a^2 = b^2 = 0.02$$

$$\Rightarrow 0.02 \times (1 + \frac{(-1)^2}{c^2}) = 0.1$$

$$\Rightarrow \frac{0.02}{c^2} = 0.1 - 0.02$$

$$\Rightarrow c = \sqrt{\frac{0.02}{0.1-0.02}} = 0.5$$

$$\frac{x^2}{0.02} + \frac{y^2}{0.02} - \frac{(z-14)^2}{0.25} = 1$$

Its cross section at z=11.5 is a circle in the xy plane of radius

$$\frac{x^2}{0.02} + \frac{y^2}{0.02} - \frac{(11.5-11.5)^2}{0.25} = 1$$

$$\frac{x^2}{0.02} + \frac{y^2}{0.02} = 1$$

$$x^2 + y^2 = 0.02$$

$$r = \sqrt{0.02}cm \approx 0.14cm$$

d) Plot the functions:

$\frac{x^2}{1.5} + \frac{y^2}{1.5} + \frac{(z-2)^2}{4} = 1$, with $0 \le z \le 3$

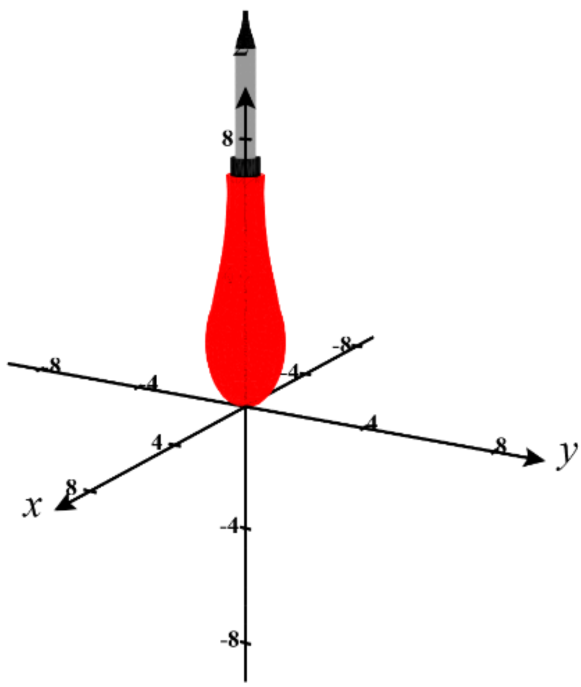
$\frac{x^2}{0.3} + \frac{y^2}{0.3} - \frac{(z-6.5)^2}{4.5} = 1$, with $3 \le z \le 7$

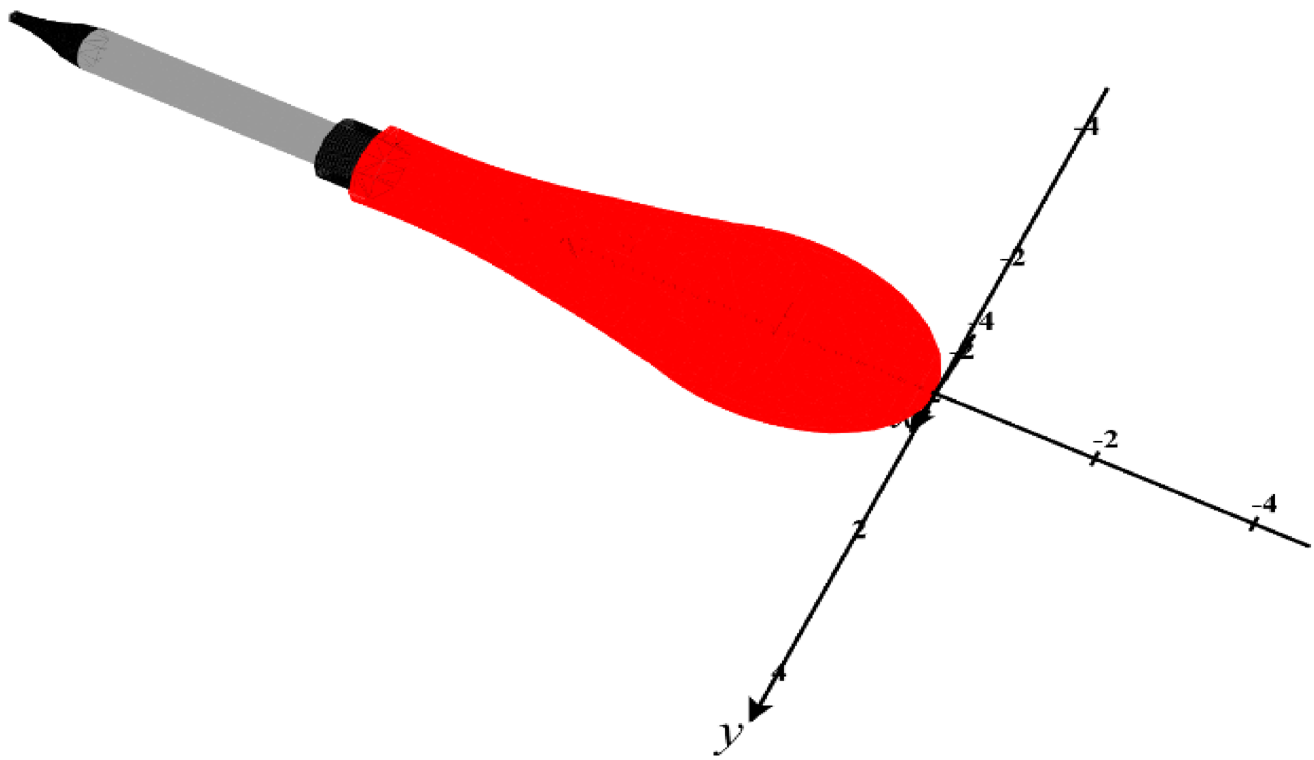
$x^2 + y^2 = 0.2$, with $6.8 \le z \le 7.5$

$x^2 + y^2 = 0.1$, with 7 ≤ z ≤ 10.5

$\frac{x^2}{0.02} + \frac{y^2}{0.02} - \frac{(z-11.5)^2}{0.25} = 1$, with 10.5 ≤ z ≤ 11.5

on CalcPlot3D, we get the following figure.





Step 5:
Reflect:
The most interesting thing i learned when completing this assignment was how to make a smooth transition between surfaces. The most challenging thing was to match the calculations with the 3D model of the object. At first, i was putting the equations corresponding to the form of each surface then i was proceeding to a trial of the values of a,b, and c, until the object looks smooth. However, when it came to translate the surfaces into calculations of the equations, it was a bit confusing, but when I understood how to match the radius of the cross section of the previous surface with the radius of the next surface in order to find the corresponding value of a, b, and c, it became easier and it helped me improve my 3D model to better transition of surfaces.