An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

Frank Paul Longo II

University of Central Florida

June 21, 2024

1 Introduction and Motivation

The escalating housing costs in contemporary real estate markets have created significant barriers for first-time homebuyers. This demographic often faces the daunting task of accumulating substantial down payments amidst economic volatility and uncertain income trajectories. This research addresses this critical issue by developing and evaluating optimal investment strategies tailored to help diverse age groups achieve their homeownership goals within 5, 10, and 15-year horizons.

1.1 Research Question

The central research question guiding this investigation is: What are the most effective investment strategies for different age groups to accumulate housing down payments over periods of 5, 10, and 15 years?

1.2 Objective

The primary objective of this study is to identify, analyze, and optimize investment strategies that can effectively assist first-time homebuyers in saving for their down payments. By leveraging advanced financial theories and empirical methodologies, this research aims to provide actionable insights that balance risk and return, offering practical solutions for prospective homeowners.

1.3 Motivation

The motivation for this research stems from the pressing need to address the challenges posed by rising housing costs and economic instability. As homeownership becomes increasingly out of reach for many, particularly younger individuals, it is imperative to develop strategies that can mitigate these barriers. By providing evidence-based investment strategies, this study aims to empower individuals with the tools needed to navigate the complexities of financial planning for homeownership.

2 Literature Review

2.1 Modern Portfolio Theory (Markowitz, 1952)

Harry Markowitz's Modern Portfolio Theory (MPT) revolutionized the way investors approach portfolio construction by introducing the concept of diversification to optimize the trade-off between risk and return. MPT posits that an efficient portfolio is one that offers the maximum possible return for a given level of risk, or equivalently, the minimum risk for a given level of return. This theory underpins the portfolio optimization models used in this study to develop effective investment strategies for down payment accumulation.

2.2 Sharpe Ratio (Sharpe, 1966)

William Sharpe's introduction of the Sharpe Ratio provided a pivotal tool for assessing the risk-adjusted performance of investment portfolios. The Sharpe Ratio is calculated by dividing the difference between the portfolio return and the risk-free rate by the standard deviation of the portfolio returns. This measure allows investors to compare the performance of different portfolios on a standardized basis, adjusting for the risk taken to achieve those returns. This study employs the Sharpe Ratio to evaluate and compare the effectiveness of various investment strategies.

2.3 Monte Carlo Methods (Boyle, 1977)

Phelim Boyle's application of Monte Carlo methods to financial modeling marked a significant advancement in the ability to simulate and understand the behavior of complex financial instruments under uncertainty. Monte Carlo simulations generate random variables based on historical return distributions to model the potential future performance of investments. This approach is crucial for developing probabilistic models that can predict the accumulation of down payments over time, accounting for the inherent uncertainty in financial markets.

3 Theoretical Models

3.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is employed to determine the expected return on an asset based on its systematic risk, as measured by beta (β_i). The CAPM formula is:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where R_f is the risk-free rate, $E(R_m)$ is the expected return of the market portfolio, and β_i is the asset's beta, reflecting its sensitivity to market movements. This model facilitates the estimation of expected returns required for the portfolio optimization process.

3.1.1 Beta Calculation

Beta (β_i) measures the volatility of an asset in relation to the market. It is calculated as:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

where $\text{Cov}(R_i, R_m)$ is the covariance between the asset return R_i and the market return R_m, and σ_m^2 is the variance of the market return.

3.1.2 Security Market Line (SML)

The Security Market Line (SML) is a graphical representation of the Capital Asset Pricing Model (CAPM), showcasing the relationship between the expected return of an asset and its systematic risk, as measured by beta (β_i). The SML illustrates the expected return of a security at different levels of systematic risk. The equation of the SML is given by CAPM:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where:

- $E(R_i)$ is the expected return of the asset,

- R_f is the risk-free rate,
- β_i is the beta of the asset,
- $E(R_m)$ is the expected return of the market portfolio.

The SML provides a benchmark for evaluating the performance of individual securities. Securities plotted above the SML are considered undervalued as they offer higher returns for a given level of risk. Conversely, securities below the SML are considered overvalued as they offer lower returns for the same level of risk.

3.1.3 Theoretical Implications

The SML conveys several important theoretical implications:

- All securities, when correctly priced, should lie on the SML.
- The slope of the SML is the market risk premium, $E(R_m) - R_f$, representing the additional return expected from holding a market portfolio instead of risk-free assets.
- The intercept of the SML is the risk-free rate, R_f, reflecting the return of a theoretically risk-free asset.

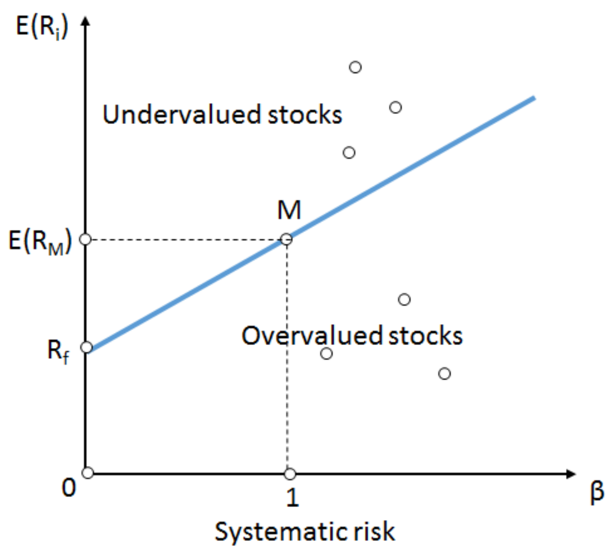

Figure 1: Security Market Line. Source: Wikipedia

The SML is a fundamental concept in our next topic of modern portfolio theory, aiding investors in making informed decisions about asset allocation based on expected returns and systematic risks.

3.2 Modern Portfolio Theory

Modern Portfolio Theory provides a robust framework for constructing an optimal portfolio that maximizes expected return for a given level of risk. The expected return $E(R_p)$ of a portfolio is the weighted sum of the expected returns of the individual assets:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

where w_i is the weight of asset i in the portfolio, and $E(R_i)$ is the expected return of asset i. The portfolio's variance σ_p^2, representing its risk, is given by:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

where σ_{ij} is the covariance between the returns of assets i and j. This formulation allows for the identification of efficient portfolios that lie on the efficient frontier, representing the optimal trade-offs between risk and return.

3.2.1 Efficient Frontier and Optimal Portfolio

The efficient frontier is a concept from MPT that represents the set of optimal portfolios offering the highest expected return for a defined level of risk. The process of constructing the efficient frontier involves solving the following optimization problem:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1$$

and

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

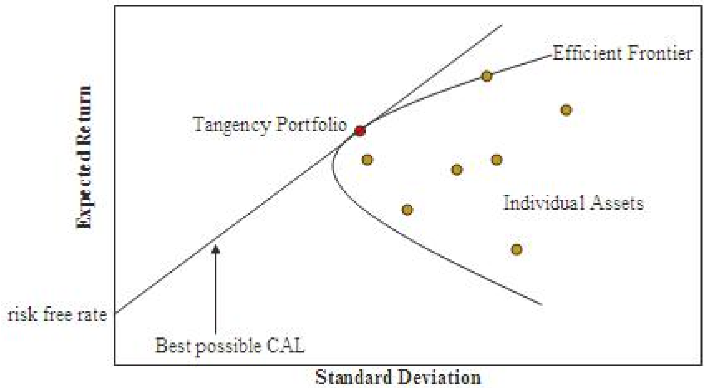

Figure 2: Efficient Frontier of Optimal Portfolios. Source: Wikipedia

3.3 Monte Carlo Simulation

Monte Carlo simulations are utilized to model the uncertainty and variability in investment returns over time. The simulation process involves generating random returns based on historical data and iterating this process to build a distribution of potential outcomes. The value of an investment at time i is given by:

$$X_i = X_{i-1} \times (1 + r_i)$$

where X_i is the investment value at time i and r_i is the return for period i. By running multiple simulations, we can estimate the expected value and variability of the investment portfolio, providing insights into the likelihood of achieving the desired down payment amount within the specified time horizon.

3.3.1 Simulation Algorithm

The Monte Carlo simulation algorithm follows these steps:

1. Initialize the investment value X_0.

2. For each time step i, generate a random return r_i from the historical return distribution.

3. Update the investment value: $X_i = X_{i-1} \times (1 + r_i)$.

4. Repeat steps 2 and 3 for the desired number of time steps.

5. Aggregate the results to build a distribution of potential outcomes.

3.3.2 Expected Value and Variance Calculation

The expected value $E(X)$ and variance $\sigma^2(X)$ of the investment outcomes are calculated as:

$$E(X) = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$$\sigma^2(X) = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - E(X))^2$$

These calculations provide a quantitative measure of the potential outcomes of the investment strategies, allowing for the assessment of risk and return profiles.

3.4 Implications for the Business Problem

The theoretical models described above provide a structured approach to constructing and evaluating investment portfolios aimed at accumulating housing down payments. The Modern Portfolio Theory ensures that the portfolios are optimized for maximum returns at a given risk level, while the CAPM helps in understanding the risk-return trade-off specific to each asset. The Monte Carlo simulations add a layer of probabilistic analysis, offering insights into the range of possible outcomes and their associated probabilities.

The implications for the business problem are significant. By employing these models, first-time homebuyers can design their own personalized investment strategies that cater to their unique risk tolerance and time horizons. This can lead to more effective savings plans, ultimately making homeownership more attainable.

4 Empirical Specification

4.1 Model Implementation

The empirical analysis proceeds by implementing the theoretical models using the historical data. The process involves the following steps:

1. **Data Cleaning and Preparation**: Ensuring the data is free from errors, outliers, and missing values. Adjustments are made for stock splits and dividends to maintain consistency in the price

data.

2. **Calculation of Daily Returns**: Daily returns are computed as the percentage change in closing prices, which serves as the basis for further analysis.

3. **Estimation of Expected Returns and Variances**: Using historical data, the expected returns and variances for each asset are estimated. These estimates are inputs for the portfolio optimization process.

4. **Portfolio Optimization**: Applying Modern Portfolio Theory to construct efficient portfolios that maximize expected return for a given level of risk. The optimization problem is solved using quadratic programming.

5. **Simulation of Investment Scenarios**: Using Monte Carlo simulations to model the accumulation of down payments over different investment horizons. Multiple scenarios are simulated to capture the range of possible outcomes.

4.2 CAPM and Sharpe Ratio Calculation

For each security, the Capital Asset Pricing Model (CAPM) and Sharpe Ratio are calculated. The CAPM is used to estimate the expected return of each security, while the Sharpe Ratio measures the risk-adjusted return. The steps involved are:

1. Calculate the average return of the market index (e.g., a broad market index like the S&P 500).

2. Determine the risk-free rate (e.g., the yield on 10-year U.S. Treasury bonds).

3. Compute the beta (β) of each security by regressing its returns against the market returns.

4. Use the CAPM formula to estimate the expected return for each security.

5. Calculate the Sharpe Ratio using the formula:

$$S = \frac{E(R_i) - R_f}{\sigma_i}$$

   where $E(R_i)$ is the expected return, R_f is the risk-free rate, and σ_i is the standard deviation of the security's excess return.

The securities are then ranked by their Sharpe Ratios to identify those with the best risk-adjusted returns.

4.3 Modern Portfolio Theory (MPT) Application

Using the ranked securities, portfolios are constructed for different age groups (5, 10, and 15 years from the average first-time homebuyer age of 35). Modern Portfolio Theory (MPT) is applied to optimize these portfolios, balancing the trade-off between risk and return. The steps include:

1. Define the assets and their expected returns and covariances.
2. Determine the weights of the assets in the portfolio to maximize the expected return for a given level of risk.
3. Construct the efficient frontier to visualize the optimal portfolios.

4.4 Monte Carlo Simulation for Down Payment Accumulation

Monte Carlo simulations are conducted to assess the variability and uncertainty in the investment returns. This involves generating random returns based on historical distributions and running numerous simulations to build a probability distribution of potential outcomes. The simulation process helps in understanding the range of possible values for the investment portfolio and the likelihood of achieving the target down payment within the specified timeframe.

4.4.1 Simulation Process

The simulation process involves the following steps:

1. Define the initial investment amount and the annual contribution based on age-specific income data.
2. Generate a series of random returns for each asset in the portfolio using historical return distributions.
3. Compute the investment value at each time step by applying the generated returns.
4. Repeat the simulation for a large number of iterations to obtain a distribution of possible outcomes.
5. Analyze the distribution to determine the probability of achieving the down payment target.

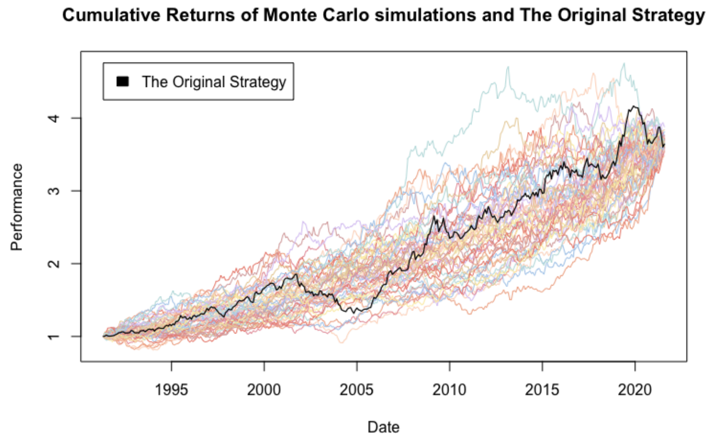

Figure 3: Investment Simulation Process for Monte Carlo Analysis. Source: Quantpedia

5 Data

5.1 Data Sources

The financial data used in this research is sourced from Yahoo Finance, which includes comprehensive information on roughly 150 securities consisting of stocks, cryptocurrencies, mutual funds, and ETFs. This data source provides a rich dataset for analyzing the performance of different investment vehicles over time.

5.2 Date Range

The data covers the period from September 2014, to the present, with a daily frequency. This timeframe allows for the analysis of recent trends and the performance of different asset classes in various market conditions.

5.3 Data Fields

The dataset includes the following fields:

- **Open**: Price at the beginning of the trading day.
- **High**: Peak price during the trading day.
- **Low**: Lowest price during the trading day.

- **Close**: Price at the end of the trading day.
- **Adj Close**: Closing price adjusted for dividends, stock splits, etc.
- **Volume**: Number of shares traded during a single trading day.
- **Type**: Security type (e.g., stock, ETF, cryptocurrency).

These fields provide a comprehensive view of the daily trading activities and price movements of different securities, essential for the analysis of investment performance and strategy development.

5.4 Data Collection and Processing

The data collection process involves using Python and the Yahoo Finance API to download historical price data for selected securities. The data is loaded into a DataFrame and indexed by date, ensuring that all securities are aligned temporally. This DataFrame serves as the basis for further calculations and analyses.

5.5 Data Limitations

While the dataset is comprehensive, there are limitations to consider. The data is limited to publicly traded securities, which may not capture the full range of investment opportunities available to first-time homebuyers. Additionally, the historical data may not fully account for future market conditions and economic events. These limitations will be addressed in the analysis and interpretation of the results.