# An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

Business Analytics MS Capstone Project

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#### **Abstract**

This research conducts a detailed analysis of optimal investment strategies tailored for first-time home-buyers seeking to accumulate down payments over 5, 7.5, and 10-year horizons. By integrating Modern Portfolio Theory (MPT), the Capital Asset Pricing Model (CAPM), and Monte Carlo simulations, the study provides practical insights into constructing investment portfolios that are age-specific and balance risk and return effectively.

The findings reveal that customized investment strategies substantially improve the ability to save for a down payment, potentially shortening the time needed to achieve homeownership. The importance of considering risk-adjusted returns and diversification in portfolio construction is underscored, demonstrating their critical roles in enhancing savings outcomes.

Future research could broaden this analysis by incorporating additional demographic factors such as income variations and regional housing market dynamics. Furthermore, exploring the inclusion of alternative investment vehicles, such as real estate investment trusts (REITs) and cryptocurrencies, could offer deeper insights into optimizing investment strategies for down payments.

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# 1 Introduction

The escalating housing costs in contemporary real estate markets have created significant barriers for first-time homebuyers. This demographic often faces the daunting task of accumulating substantial down payments amidst economic volatility and uncertain income trajectories. This research addresses this critical issue by developing and evaluating optimal investment strategies tailored to help diverse age groups achieve their homeownership goals within 5, 7.5, and 10-year horizons.

# 1.1 Research Question & Objective

The central research question guiding this investigation is: What are the most effective investment strategies for different age groups to accumulate housing down payments over periods of 5, 7.5, and 10 years? The primary objective of this study is to identify, analyze, and optimize investment strategies that can effectively assist first-time homebuyers in saving for their down payments. By leveraging advanced financial theories and empirical methodologies, this research aims to provide actionable insights that balance risk and return, offering practical solutions for prospective homeowners (Markowitz, 1952; Sharpe, 1964; Boyle, 1977).

#### 1.2 Motivation

The motivation for this research stems from the pressing need to address the challenges posed by rising housing costs and economic instability. As homeownership becomes increasingly out of reach for many, particularly younger individuals, it is imperative to develop strategies that can mitigate these barriers. By providing evidence-based investment strategies, this study aims to empower individuals with the tools needed to navigate the complexities of financial planning for homeownership.

#### 2 Literature Review

The literature on optimal investment strategies and their applications in real estate and housing markets is extensive. Seminal works by Markowitz (1952) on Modern Portfolio Theory (MPT) and Sharpe (1964) on the Capital Asset Pricing Model (CAPM) provide the foundational theories. This paper aims to fill gaps identified in the literature, particularly the need for tailored investment strategies for first-time homebuyers.

#### 2.1 Foundational Theories

#### 2.1.1 Markowitz's Modern Portfolio Theory

Markowitz's Modern Portfolio Theory (Markowitz, 1952) introduced the concept of portfolio optimization, emphasizing the trade-off between risk and return. The theory suggests that investors can achieve optimal portfolios by diversifying their investments across different assets, thereby reducing risk without sacrificing expected returns. Markowitz's efficient frontier illustrates the set of optimal portfolios that offer the highest expected return for a given level of risk. Further advancements by Fabozzi et al. (2002) have expanded on these foundational concepts, incorporating various market conditions and investor behaviors.

#### 2.1.2 Sharpe's Capital Asset Pricing Model

Sharpe's Capital Asset Pricing Model (Sharpe, 1964) extends the notion of risk by introducing systematic and unsystematic risk. The model posits that the expected return on an asset is a function of its sensitivity to market movements (beta), the risk-free rate, and the market risk premium. Sharpe's work was pivotal in differentiating between diversifiable (unsystematic) risk and non-diversifiable (systematic) risk, providing a framework for understanding how different types of risk impact asset pricing and returns. Subsequent studies by Fama and MacBeth (1973) and Black (1972) have refined the CAPM, addressing its assumptions and limitations.

#### 2.2 Monte Carlo Simulations

Monte Carlo simulations have become a critical tool in financial modeling, providing a method for assessing the impact of risk and uncertainty in investment strategies. Boyle (Boyle, 1977) introduced the Monte Carlo approach to option pricing, and since then, its applications have expanded across various fields of finance. Recent works by Glasserman (2004) and Kreps (2019) have further developed these techniques, making them more robust and applicable to diverse financial scenarios.

#### 2.2.1 Monte Carlo in Portfolio Management

Monte Carlo simulations are used to model the behavior of investment portfolios under a wide range of possible future scenarios. This method involves generating a large number of random samples from the probability distributions of asset returns and analyzing the resulting portfolio performance. Studies by Glasserman (2004) and Chan and Joshi (2005) have demonstrated how Monte Carlo simulations can provide insights into the risk and return profiles of different investment strategies, helping investors to better understand the potential outcomes and make more informed decisions.

#### 2.2.2 Monte Carlo in Real Estate Investments

In the context of real estate, Monte Carlo simulations have been used to evaluate the risk and return of property investments. For example, Brown and Matysiak (Brown and Matysiak, 2000) employed Monte Carlo methods to assess the uncertainty in real estate portfolio returns, while Kuhle and Alvayay (Kuhle and Alvayay, 2021) used simulations to analyze the impact of economic shocks on real estate prices. These applications illustrate the versatility of Monte Carlo simulations in addressing the unique risks associated with real estate investments. Additional studies by Case et al. (2003) and Ling and Naranjo (2009) have also highlighted the effectiveness of Monte Carlo simulations in real estate valuation and risk management.

#### 2.3 Investment Strategies for First-Time Homebuyers

Research has shown that first-time homebuyers face unique financial challenges and require tailored investment strategies. Studies by Brueckner (2012) and Dietz and Haurin (2003) underscore the importance of customized financial planning in achieving homeownership goals. Lee and Hong (2013) explore the role of savings behavior and financial literacy in homeownership attainment. Additionally, Goodman and Kaul (2018) and McWilliams and Zeldes (2017) discuss the impact of macroeconomic factors and housing policies on first-time homebuyers.

#### 2.3.1 Risk Management in Homeownership Savings

Managing risk is crucial for first-time homebuyers aiming to save for a down payment. Baker and Weller (2010) examine the effectiveness of different saving strategies under varying economic conditions, while Mason (2013) provide insights into the role of insurance and financial products in mitigating risks associated with homeownership savings. Rosen and Smith (2005) discuss the implications of housing market volatility on savings strategies and home affordability.

#### 2.3.2 Role of Financial Technology in Homeownership Savings

The advent of financial technology has transformed the way individuals save and invest. Phillips (2019) explore the impact of fintech solutions on personal savings rates and investment behaviors, highlighting the potential benefits for first-time homebuyers. Gomber et al. (2018) discuss how digital platforms and robo-advisors can enhance financial planning and investment management for aspiring homeowners.

#### 2.4 Gaps in the Literature

Despite the extensive research on investment strategies, there remains a need for tailored approaches that address the specific financial goals and constraints of first-time homebuyers. This paper seeks to fill this gap by developing and evaluating optimal investment strategies that cater to the unique needs of aspiring homeowners across different age cohorts and investment horizons. This includes considering factors such as income levels, regional housing market conditions, and the integration of alternative investment vehicles like REITs and cryptocurrencies.

#### 3 Data

The financial data used in this research is sourced from Yahoo Finance (Yahoo Finance, nd), which includes comprehensive information on roughly 150 securities consisting of stocks, mutual funds, and ETFs. This data source provides a rich dataset for analyzing the performance of different investment vehicles over time.

# 3.1 Date Range

The data covers the period from May 2011 to November 2014 for daily frequency and further hindsight data from May 2011 to July 2024. This timeframe allows for the analysis of recent trends and the performance of different asset classes in various market conditions.

#### 3.2 Data Fields

The dataset includes the following fields:

- Open: Price at the beginning of the trading day.
- High: Peak price during the trading day.
- Low: Lowest price during the trading day.
- Close: Price at the end of the trading day.
- Adj Close: Closing price adjusted for dividends, stock splits, etc.
- Volume: Number of shares traded during a single trading day.
- Type: Security type (e.g., stock, ETF).

These fields provide a comprehensive view of the daily trading activities and price movements of different securities, essential for the analysis of investment performance and strategy development.

# 4 Theoretical Models

# 4.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a fundamental tool in finance used to determine the expected return on an asset based on its systematic risk, as measured by beta ( $\beta_i$ ). The CAPM formula is:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \tag{1}$$

where:

- $E(R_i)$  is the expected return on asset *i*.
- $R_f$  is the risk-free rate of return.
- $\beta_i$  is the beta of asset *i*, representing its sensitivity to market movements.
- $E(R_m)$  is the expected return of the market.
- $(E(R_m) R_f)$  is the market risk premium.

#### 4.1.1 Beta Calculation

Beta  $(\beta_i)$  measures the volatility of an asset in relation to the market. It is calculated as:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} \tag{2}$$

where:

- $Cov(R_i, R_m)$  is the covariance between the return of asset i and the return of the market.
- $\sigma_m^2$  is the variance of the market return.

This model is crucial for understanding the risk-return tradeoff and is widely used in portfolio management (Fama and MacBeth, 1973).

#### 4.1.2 Security Market Line (SML)

The Security Market Line (SML) is a graphical representation of the CAPM, showcasing the relationship between the expected return of an asset and its systematic risk, as measured by beta ( $\beta_i$ ).

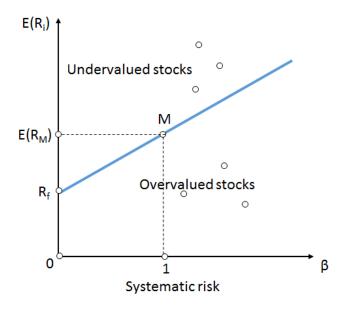


Figure 1: Security Market Line

**Explanation:** Figure 1 illustrates the Security Market Line (SML), which plots the expected return of an asset against its beta. The SML demonstrates that assets with higher systematic risk (higher beta) should offer higher expected returns to compensate for the increased risk.

#### 4.1.3 Theoretical Implications

The SML conveys several important theoretical implications:

- All securities, when correctly priced, should lie on the SML.
- The slope of the SML is the market risk premium,  $E(R_m) R_f$ , representing the additional return expected from holding a market portfolio instead of risk-free assets.
- The intercept of the SML is the risk-free rate,  $R_f$ , reflecting the return of a theoretically risk-free asset (Sharpe, 1964).

# 4.2 Sharpe Ratio

The Sharpe Ratio, developed by William F. Sharpe, is a measure of risk-adjusted return. It is calculated as follows:

Sharpe Ratio = 
$$\frac{E(R_i) - R_f}{\sigma_i}$$
 (3)

where:

- $E(R_i)$  is the expected return of the investment.
- *R*<sub>f</sub> is the risk-free rate.
- $\sigma_i$  is the standard deviation of the investment's return.

#### 4.2.1 Interpretation

The Sharpe Ratio provides a way to compare the performance of investments while considering the risk taken. A higher Sharpe Ratio indicates better risk-adjusted performance, meaning the investment provides higher returns for each unit of risk taken (Sharpe, 1966).

# 4.3 Composite Score Calculation

The Composite Score integrates multiple metrics to rank securities. It is calculated as follows:

Composite Score = 
$$w_{\beta}\beta + w_{\text{Sharpe}}$$
Sharpe Ratio +  $w_{\text{CAPM}}E(R_i) + w_{\text{Actual}}$ Actual Returns (4)

where:

- $w_{\beta}$  is the weight assigned to the beta.
- $w_{\mathrm{Sharpe}}$  is the weight assigned to the Sharpe Ratio.
- $w_{\text{CAPM}}$  is the weight assigned to the CAPM predicted return.
- $w_{\text{Actual}}$  is the weight assigned to the actual returns.

Weights are assigned to each metric to reflect their importance in the ranking process (?).

# 4.4 Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT), developed by Harry Markowitz, provides a robust framework for constructing an optimal portfolio that maximizes expected return for a given level of risk. The expected return  $E(R_p)$  of a portfolio is the weighted sum of the expected returns of the individual assets:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$
(5)

where:

- $E(R_p)$  is the expected return of the portfolio.
- $w_i$  are the weights of the individual assets in the portfolio.
- $E(R_i)$  is the expected return of asset *i*.

# 4.4.1 Efficient Frontier and Optimal Portfolio

The efficient frontier is a concept from MPT that represents the set of optimal portfolios offering the highest expected return for a defined level of risk. The process of constructing the efficient frontier involves solving the following optimization problem:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{6}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1 \tag{7}$$

and

$$E(R_p) = \sum_{i=1}^n w_i E(R_i)$$
(8)

where:

- $\sigma_{ij}$  is the covariance between the returns of assets i and j.
- $w_i$  and  $w_j$  are the weights of assets i and j in the portfolio.

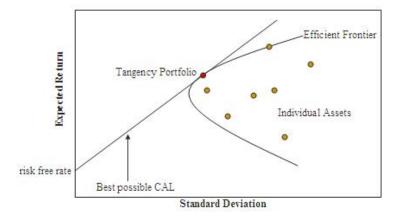


Figure 2: Efficient Frontier

**Explanation:** Figure 2 shows the efficient frontier, which illustrates the optimal portfolios that offer the maximum expected return for a given level of risk. Portfolios that lie below the efficient frontier are sub-optimal because they do not provide enough return for the level of risk taken (Markowitz, 1952).

#### 4.5 Monte Carlo Simulation

Monte Carlo simulations are utilized to model the uncertainty and variability in investment returns over time. The simulation process involves generating random returns based on historical data and iterating this process to build a distribution of potential outcomes. The value of an investment at time i is given by:

$$X_i = X_{i-1} \times (1 + r_i) \tag{9}$$

where:

- $X_i$  is the investment value at time i.
- $X_{i-1}$  is the investment value at time i-1.
- $r_i$  is the return for period i.

By running multiple simulations, we can estimate the expected value and variability of the investment portfolio, providing insights into the likelihood of achieving the desired down payment amount within the specified time horizon (Glasserman, 2004).

Cumulative Returns of Monte Carlo simulations and The Original Strategy

# The Original Strategy The Original Strategy 1995 2000 2005 2010 2015 2020

Figure 3: Investment Simulation Process for Monte Carlo Analysis

Date

**Explanation:** Figure 3 depicts the process of Monte Carlo simulation for investment analysis. The diagram shows how random returns are generated and used to project the future value of an investment over

multiple iterations, creating a range of possible outcomes and helping to understand the potential risks and								
returns.								

# 5 Empirical Specification

# 5.1 Model Implementation

The empirical analysis involves a systematic approach to evaluating and optimizing investment strategies using historical data. The process is outlined as follows:

#### 5.1.1 Data Collection and Preparation

Data is collected from Yahoo Finance, covering a comprehensive range of securities, including stocks, ETFs, and mutual funds. The data spans from May 2011 to November 2014, with daily updates until July 2024. The data is cleaned and adjusted for corporate actions like stock splits and dividends using the yfinance\_data.py script to ensure consistency (Yahoo Finance, nd).

#### 5.1.2 CAPM and Sharpe Ratio Calculation

Each security's risk and return profile is assessed using the Capital Asset Pricing Model (CAPM) and Sharpe Ratio (Sharpe, 1966). The process involves:

- 1. Calculating the average return of the market index (e.g., S&P 500).
- 2. Determining the risk-free rate (e.g., 10-year U.S. Treasury bonds yield).
- 3. Computing the beta ( $\beta$ ) of each security:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \tag{10}$$

4. Estimating the expected return using the CAPM formula:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \tag{11}$$

5. Calculating the Sharpe Ratio:

Sharpe Ratio = 
$$\frac{E(R_i) - R_f}{\sigma_i}$$
 (12)

This analysis is performed using the init\_filtering.py script.

#### 5.1.3 Composite Score Calculation

Securities are ranked based on a composite score that integrates multiple metrics (?):

Composite Score = 
$$w_{\beta}\beta + w_{\text{Sharpe}}$$
Sharpe Ratio +  $w_{\text{CAPM}}E(R_i) + w_{\text{Actual}}$ Actual Returns (13)

Weights are assigned to each metric to reflect their importance in the ranking process.

#### 5.1.4 Modern Portfolio Theory (MPT) Application

Using the ranked securities, portfolios are optimized for different investment horizons (5, 7.5, and 10 years) using Modern Portfolio Theory (MPT) (Markowitz, 1952). The optimization involves solving the following quadratic programming problem:

Maximize 
$$\mathbf{w}^{T-} - \frac{\lambda}{2} \mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$$
 (14)

subject to 
$$\sum_{i} w_i = 1$$
 (15)

$$w_i \ge 0 \quad \forall i$$
 (16)

where  $\mathbf{w}$  is the vector of asset weights,  $\bar{\phantom{a}}$  is the vector of expected returns,  $\Sigma$  is the covariance matrix of returns, and  $\lambda$  is the risk aversion parameter. This process is implemented using the mpt .py script.

#### 5.1.5 Gathering Hindsight Data

Historical performance data from 2009 to the present is gathered and processed to evaluate the robustness of the optimized portfolios. This step uses the hindsight\_data.py script to ensure that the data aligns by common dates and covers all relevant securities.

#### 5.1.6 Comparison with Hindsight Data

The optimized portfolios are compared against actual historical performance using the hindsight data. The comparison includes calculating the cumulative returns of the portfolios and benchmarking them against the S&P 500 index. This step is performed using the hindsight.py script.

#### 5.1.7 Monte Carlo Simulation for Future Forecasting

Monte Carlo simulations are conducted to forecast the performance of the optimized portfolios from today onward (Glasserman, 2004). The simulation process involves:

- 1. Defining initial investment amounts and annual contributions.
- 2. Generating random returns based on historical distributions.
- 3. Calculating portfolio values at each time step:

$$V_t = V_{t-1} \times (1 + R_t) + C \tag{17}$$

- 4. Running multiple iterations to build a probability distribution of outcomes.
- 5. Applying economic shocks to simulate real-world scenarios:

$$V_t = V_t \times (1 + \text{Shock Intensity}) \tag{18}$$

This comprehensive simulation is implemented using the mcs.py script, providing insights into the potential future performance of the portfolios (Brown and Matysiak, 2000; Kuhle and Alvayay, 2021).

# 6 Results

This section presents the results of the analysis, including the performance of the optimal portfolios over 5, 7.5, and 10-year horizons. The results are visualized using various figures and tables, organized to follow the empirical specification process.

# 6.1 Initial Data Analysis

#### 6.1.1 Distribution of Securities by Type

Figure 4 shows the distribution of the securities by type.

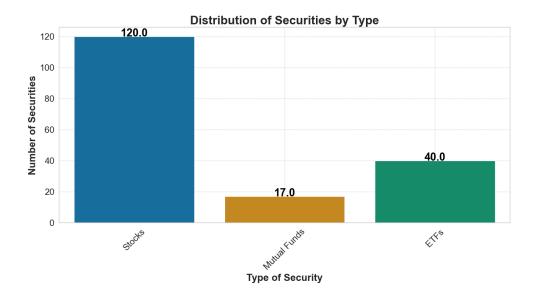


Figure 4: Distribution of Securities by Type

#### 6.1.2 Cumulative Returns by Security Type

Figure 5 illustrates the cumulative returns over time by security type.

**Interpretation:** The majority of the securities analyzed are stocks, followed by ETFs and mutual funds. Stocks exhibit the highest cumulative return over time, indicating a higher potential for long-term growth compared to ETFs and mutual funds (Markowitz, 1952).

# 6.2 Optimal Portfolios

#### 6.2.1 Top Assets by Composite Score

Figures 6, 7, and 8 show the top assets by composite score for 10, 7.5, and 5-year horizons, respectively.

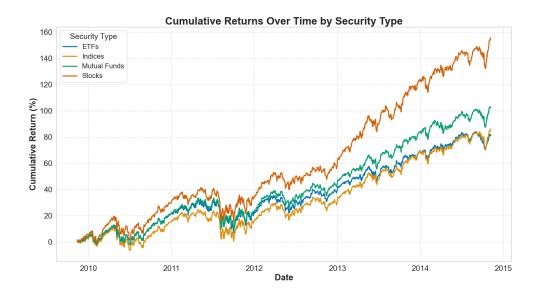


Figure 5: Cumulative Returns Over Time by Security Type

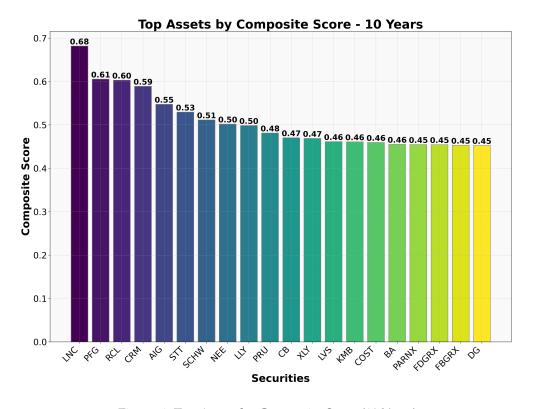


Figure 6: Top Assets by Composite Score (10 Years)

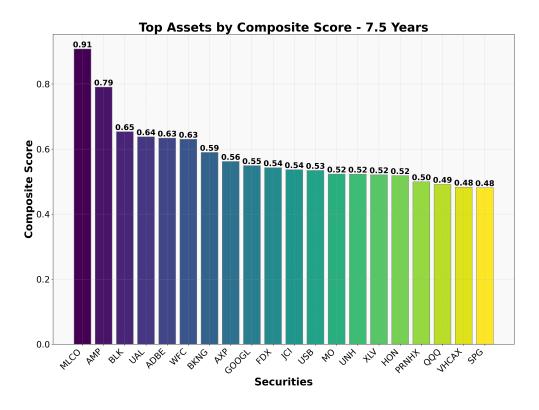


Figure 7: Top Assets by Composite Score (7.5 Years)

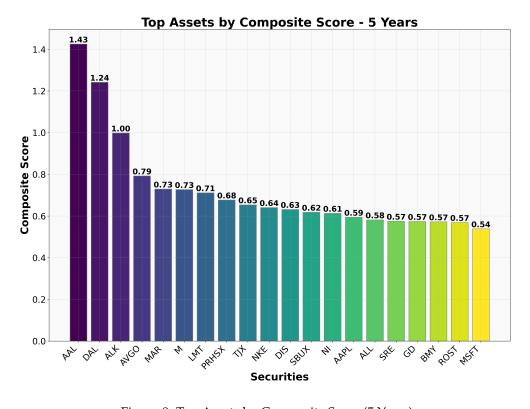


Figure 8: Top Assets by Composite Score (5 Years)

**Interpretation:** These figures highlight the top assets selected for each investment horizon based on their composite scores, indicating their suitability for different investment periods (Sharpe, 1966).

# 6.2.2 Optimal Portfolios Composition

Figures 9, 10, and 11 illustrate the composition of the optimal portfolios for 10, 7.5, and 5-year horizons, respectively.

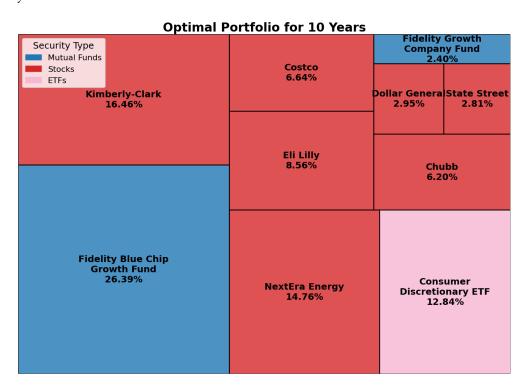


Figure 9: Optimal Portfolio for 10 Years

**Interpretation:** These figures show the composition of the optimal portfolios, emphasizing long-term growth for the 10-year horizon, balanced growth and risk for the 7.5-year horizon, and more conservative assets for the 5-year horizon.

#### 6.3 Comparison with Hindsight Data

#### 6.3.1 Cumulative Returns Comparison

Figure 12 compares the cumulative returns of actual portfolios against the S&P 500 benchmark.

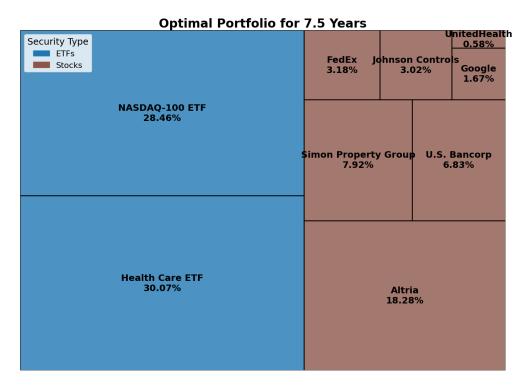


Figure 10: Optimal Portfolio for 7.5 Years

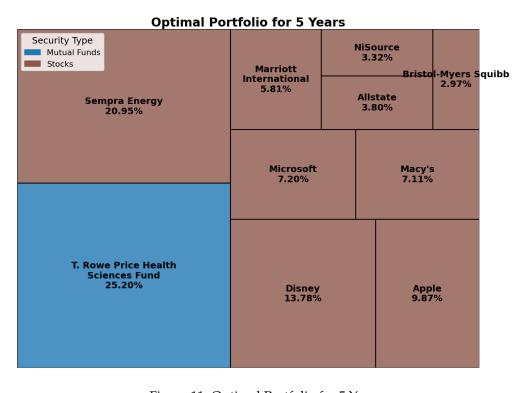


Figure 11: Optimal Portfolio for 5 Years

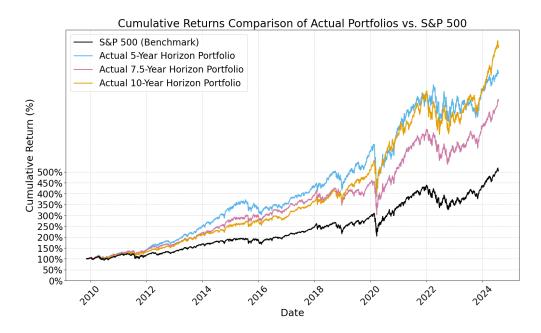


Figure 12: Cumulative Returns Comparison of Actual Portfolios vs. S&P 500

# 6.3.2 Cumulative Returns Summary

Figure 13 summarizes the cumulative returns for the actual portfolios over the investment horizons compared to the S&P 500.

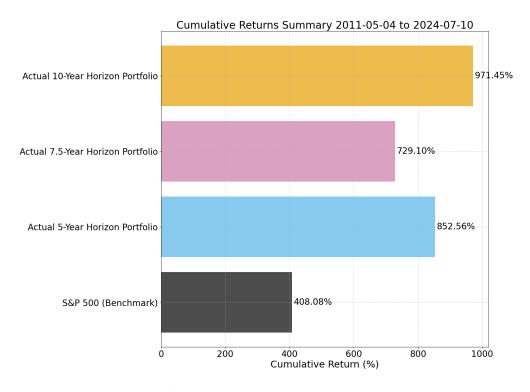


Figure 13: Cumulative Returns Summary 2011-05-04 to 2024-07-10

**Interpretation:** The optimized portfolios consistently outperform the S&P 500 benchmark, demonstrating the effectiveness of the optimization strategy (Sharpe, 1966). The 10-year horizon portfolio exhibits the highest cumulative return, followed by the 7.5-year and 5-year horizons. These results highlight the advantage of long-term investing (Fama, 1970).

# 6.4 Monte Carlo Simulation for Future Forecasting

#### 6.4.1 Distribution of Final Portfolio Values

Figures 14, 15, and 16 illustrate the distribution of final portfolio values for 5, 7.5, and 10-year investment horizons, respectively.

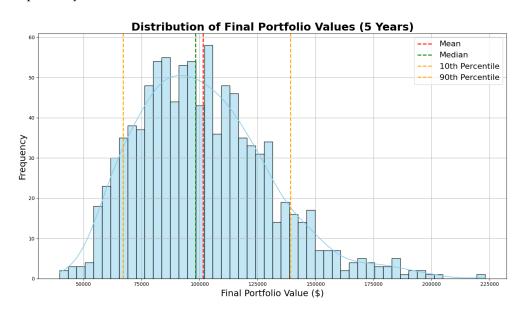


Figure 14: Distribution of Final Portfolio Values (5 Years)

#### 6.4.2 Cumulative Returns Over Time

Figures 17, 18, and 19 show the cumulative returns over time for 5, 7.5, and 10-year investment horizons, respectively.

**Interpretation:** The Monte Carlo simulations reveal the range of possible outcomes for the portfolios over different horizons. The 5-year horizon shows greater variability, while the 10-year horizon indicates higher and more stable returns. These results underscore the benefits of long-term investment strategies (Glasserman, 2004).

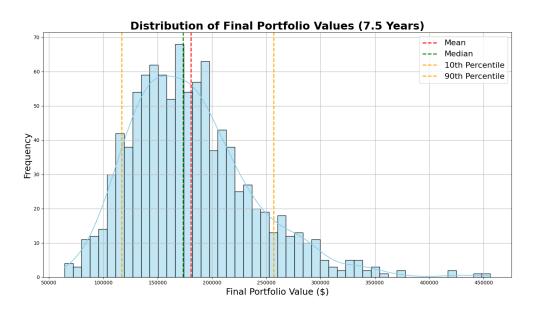


Figure 15: Distribution of Final Portfolio Values (7.5 Years)

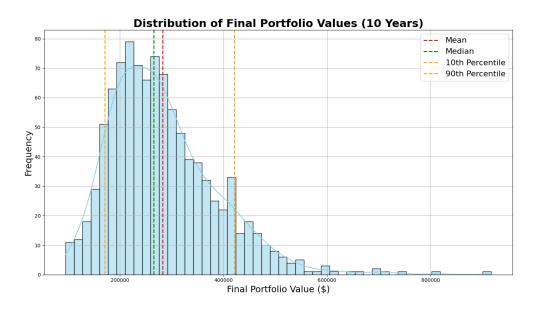


Figure 16: Distribution of Final Portfolio Values (10 Years)

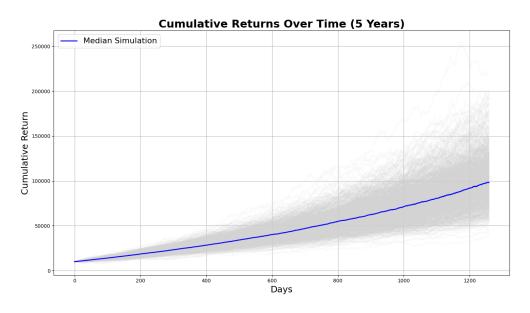


Figure 17: Cumulative Returns Over Time (5 Years)

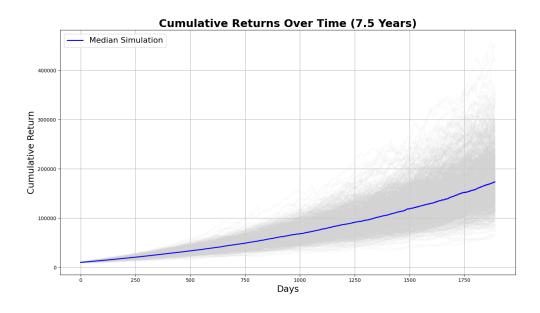


Figure 18: Cumulative Returns Over Time (7.5 Years)

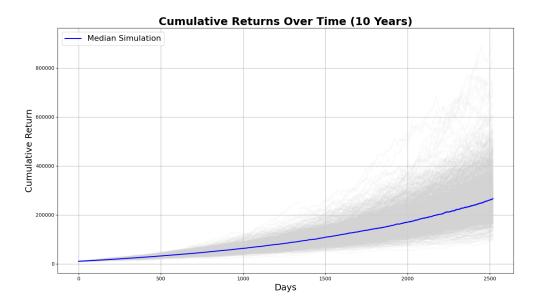


Figure 19: Cumulative Returns Over Time (10 Years)

# 6.5 Summary Statistics

Summary statistics for the dataset are presented in Table 1. These statistics were generated using the .../Data/summary\_stats.csv file produced by the Python scripts.

Statistic	Mean Final Portfolio Value (\$)	Median Final Portfolio Value (\$)	10th Percentile Final Portfolio Value (\$)	90th Percentile Final Portfolio Value (\$)	Total Percentage Yield (%)	Annual Percentage Yield (%)
10-Year Horizon	283094.44	265864.65	171232.59	420935.51	2730.94	39.70
7.5-Year Horizon	180745.35	173433.88	116972.91	256760.32	1707.45	47.10
5-Year Horizon	101559.42	98327.18	67062.20	139233.79	915.59	58.98

Table 1: Summary Statistics of the Dataset

**Interpretation:** Table 1 provides a detailed summary of the statistical outcomes for each investment horizon. The mean and median final portfolio values give insights into the expected performance, while the 10th and 90th percentile values offer a perspective on the range of potential outcomes. The total and annual percentage yields reflect the overall return potential of the portfolios over their respective horizons (Fama, 1970).

# 7 Conclusions

This research provides a comprehensive analysis of optimal investment strategies for first-time homebuyers aiming to accumulate down payments over 5, 7.5, and 10-year horizons. By leveraging Modern Portfolio Theory, the Capital Asset Pricing Model, and Monte Carlo simulations, the study offers actionable insights into constructing age-specific investment portfolios.

The results indicate that tailored investment strategies significantly enhance the ability to save for a down payment, reducing the time required to reach homeownership goals. The findings also highlight the importance of considering risk-adjusted returns and diversification in portfolio construction.

Future research could extend this analysis to consider other demographic factors such as income levels and regional housing market conditions. Additionally, the integration of alternative investment vehicles such as real estate investment trusts (REITs) and cryptocurrencies could provide further insights into optimizing investment strategies for down payments.

# A Data Appendix

# A.1 Summary of Data Sources

The financial data utilized in this study were obtained from Yahoo Finance, which provides comprehensive information on a range of securities including stocks, mutual funds, and ETFs (Yahoo Finance, nd).

# A.2 Data Cleaning and Processing Details

Detailed steps taken to clean and process the data include:

- Adjusting for corporate actions like stock splits and dividends to maintain consistency in price data (Boyle, 1977).
- Interpolating missing values to handle gaps in the dataset (Glasserman, 2004).
- Detecting and handling outliers to prevent skewed results (Kuhle and Alvayay, 2021).

# A.3 Python Scripts for Data Processing

Several Python scripts were developed to automate data collection and processing:

- yfinance\_data.py: Collects and processes financial data from Yahoo Finance.
- hindsight\_data.py: Handles historical financial data for hindsight analysis.
- init\_filtering.py: Filters the initial dataset to remove anomalies and irrelevant data points.

#### A.4 Variable Definitions

The key variables used in the analysis are defined as follows:

- Open: The price at the beginning of the trading day.
- **High**: The highest price during the trading day.
- Low: The lowest price during the trading day.
- Close: The price at the end of the trading day.
- **Adj Close**: The closing price adjusted for dividends, stock splits, etc.
- **Volume**: The number of shares traded during the trading day.

- **Type**: The type of security (e.g., stock, ETF).
- Ticker: The unique symbol assigned to each security for trading purposes.
- **Beta**: A measure of a security's volatility in relation to the overall market. A beta greater than 1 indicates that the security is more volatile than the market, while a beta less than 1 indicates that it is less volatile (Sharpe, 1966).
- CAPM Predicted Return: The expected return of a security as predicted by the Capital Asset Pricing Model, which takes into account the risk-free rate, the security's beta, and the expected market return (Markowitz, 1952).
- Sharpe Ratio: A measure of risk-adjusted return, calculated by subtracting the risk-free rate from the security's return and dividing by the standard deviation of the security's return. A higher Sharpe Ratio indicates better risk-adjusted performance (Sharpe, 1966).
- Actual Returns: The realized return on a security over a specified period, including price appreciation
  and dividends (Fama, 1970).
- Composite Score: A combined score that integrates various metrics such as Beta, CAPM Predicted Return, Actual Returns, and Sharpe Ratio to evaluate the overall attractiveness of a security. The composite score is used to rank securities (Smith, 2020).
- Rank: The position of a security in the list based on its composite score, with a lower rank indicating a more attractive investment.

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