An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

Frank Paul Longo II

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Introduction

Objective

Develop optimal investment strategies for first-time homebuyers to save for down payments.

Research Question

What are the optimal investment strategies for aspiring first-time homebuyers of various age cohorts to accumulate down payments over 5, 10, and 15-year investment horizons?

Motivation

Address the challenges posed by escalating housing costs and assist diverse age groups in expediting their path to homeownership.

Essential Financial Concepts

Stocks

Equity investments representing proprietorship in a company.

Mutual Funds

Investment vehicles that aggregate capital from numerous investors to purchase a diversified portfolio of stocks, bonds, or other securities.

ETFs (Exchange-Traded Funds)

Analogous to mutual funds but traded on stock exchanges like individual stocks.

Typical First-time Homebuyer Profile

Demographics

- **Median Age:** 35 years (2024)[a]
- **Median Household Income:** $104,000 (2024)[b]
- **Median Nationwide Home Cost:** $416,100 (2024)[c]

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[a] National Association of Realtors, 2024 Profile of Home Buyers and Sellers.

[b] Consumer Financial Protection Bureau, Market Snapshot: First-time Homebuyers, 2024.

[c] U.S. Department of Housing and Urban Development, 2024.

Investment Contributions by Age Group

20-25 Years Old (15-Year Time Horizon)

- **Median Gross Income:** \$45,000[a]
- **Annual Contribution:** 10%[b]
- **Risk Tolerance (Beta):** 1.2[c]
- **Equity Breakdown:**
  - Stocks: 50%[d]
  - Mutual Funds: 25%[e]
  - ETFs: 25%[f]

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[a] Bureau of Labor Statistics, 2024.
[b] Federal Reserve, 2024.
[c] Morgan Stanley, 2024.
[d] Empower, 2024.
[e] Charles Schwab, 2024.
[f] J.P. Morgan, 2024.

Investment Contributions by Age Group

25-30 Years Old (10-Year Time Horizon)

- **Median Gross Income:** \$60,000[a]
- **Annual Contribution:** 15%[b]
- **Risk Tolerance (Beta):** 1.0[c]
- **Equity Breakdown:**
  - Stocks: 45%[d]
  - Mutual Funds: 30%[e]
  - ETFs: 25%[f]

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[a] Bureau of Labor Statistics, 2024.
[b] Federal Reserve, 2024.
[c] Morgan Stanley, 2024.
[d] Morgan Stanley, 2024.
[e] Charles Schwab, 2024.
[f] J.P. Morgan, 2024.

Investment Contributions by Age Group

30-35 Years Old (5-Year Time Horizon)

- **Median Gross Income:** \$80,000[a]
- **Annual Contribution:** 20%[b]
- **Risk Tolerance (Beta):** 0.8[c]
- **Equity Breakdown:**
  - Stocks: 40%[d]
  - Mutual Funds: 35%[e]
  - ETFs: 25%[f]

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[a] Bureau of Labor Statistics, 2024.
[b] Federal Reserve, 2024.
[c] Morgan Stanley, 2024.
[d] Empower, 2024.
[e] Charles Schwab, 2024.
[f] J.P. Morgan, 2024.

Data Sources

Primary Source

Yahoo Finance (YFinance)

- Comprehensive financial data on stocks, mutual funds, and ETFs.

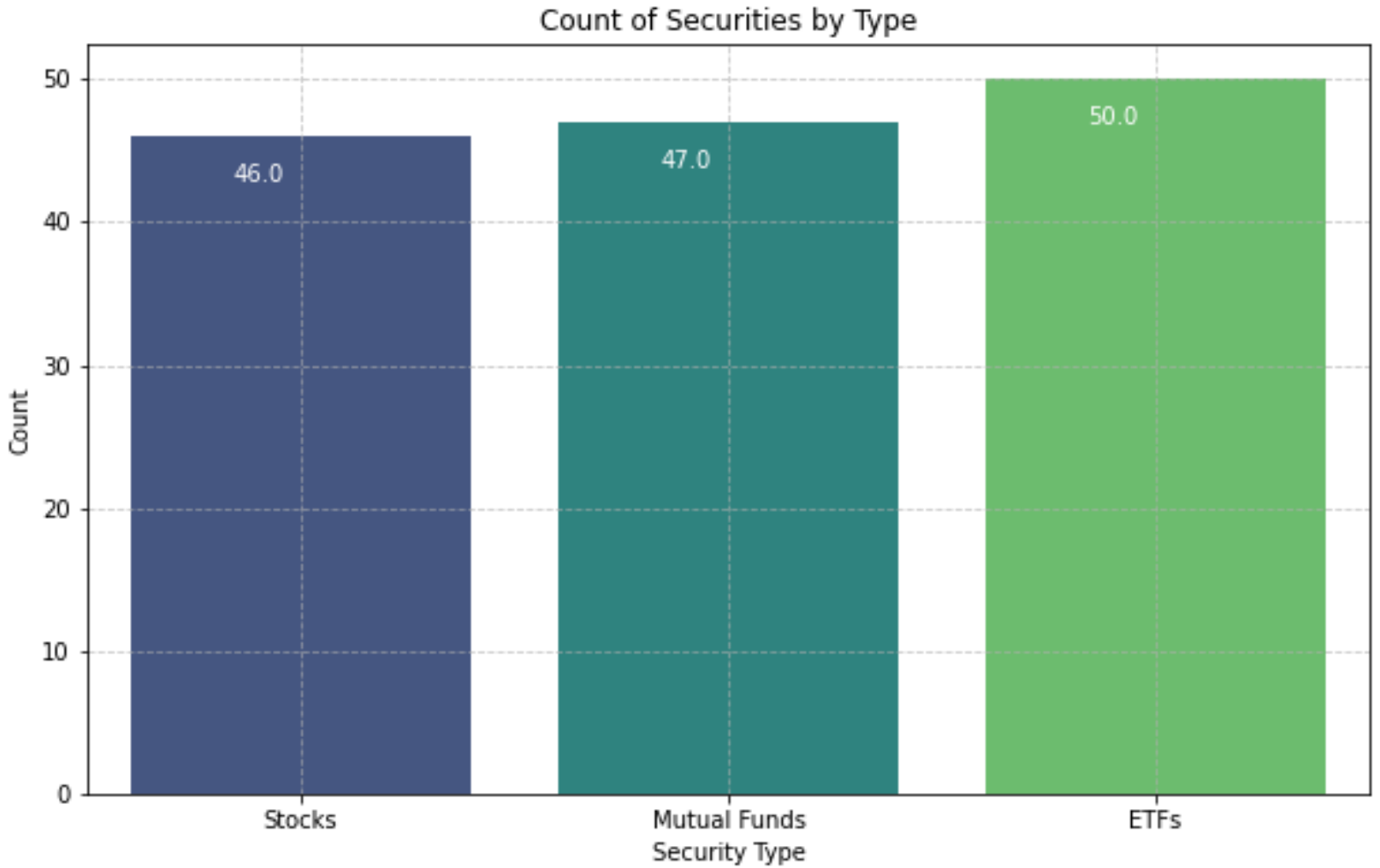
Data Coverage

Date Range: 10/24/2012 to present (daily frequency)

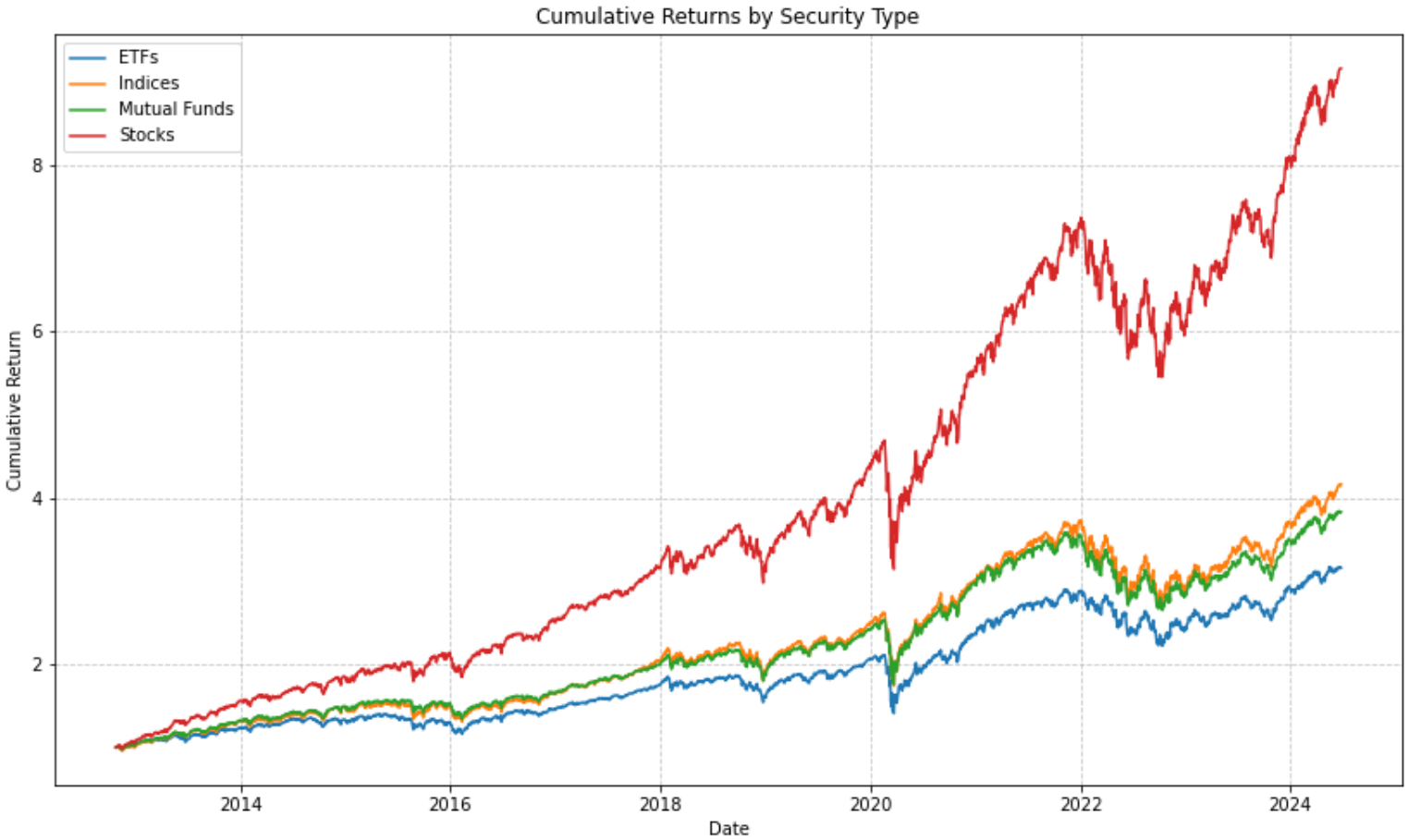
Data Fields

- Open
- High
- Low
- Close
- Adj Close
- Volume
- Type

Visualizing Security Count



Visualizing Returns Distribution



Capital Asset Pricing Model (CAPM)

Formula

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

[a]

[a] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), 425-442.

Assumptions

- Diversified portfolios
- Efficient markets
- No taxes or transaction costs
- Constant risk-free rate

Calculating Beta

Market Return Calculation

```
# Market return (e.g., S&P 500)
market = yf.download('^GSPC', start='2012-10-24', end='2024-09
market_returns = market['Adj Close'].pct_change().dropna()

# Calculate beta for each stock
beta = {}
for ticker in tickers:
    cov_matrix = np.cov(returns[ticker], market_returns)
    beta[ticker] = cov_matrix[0, 1] / cov_matrix[1, 1]
```

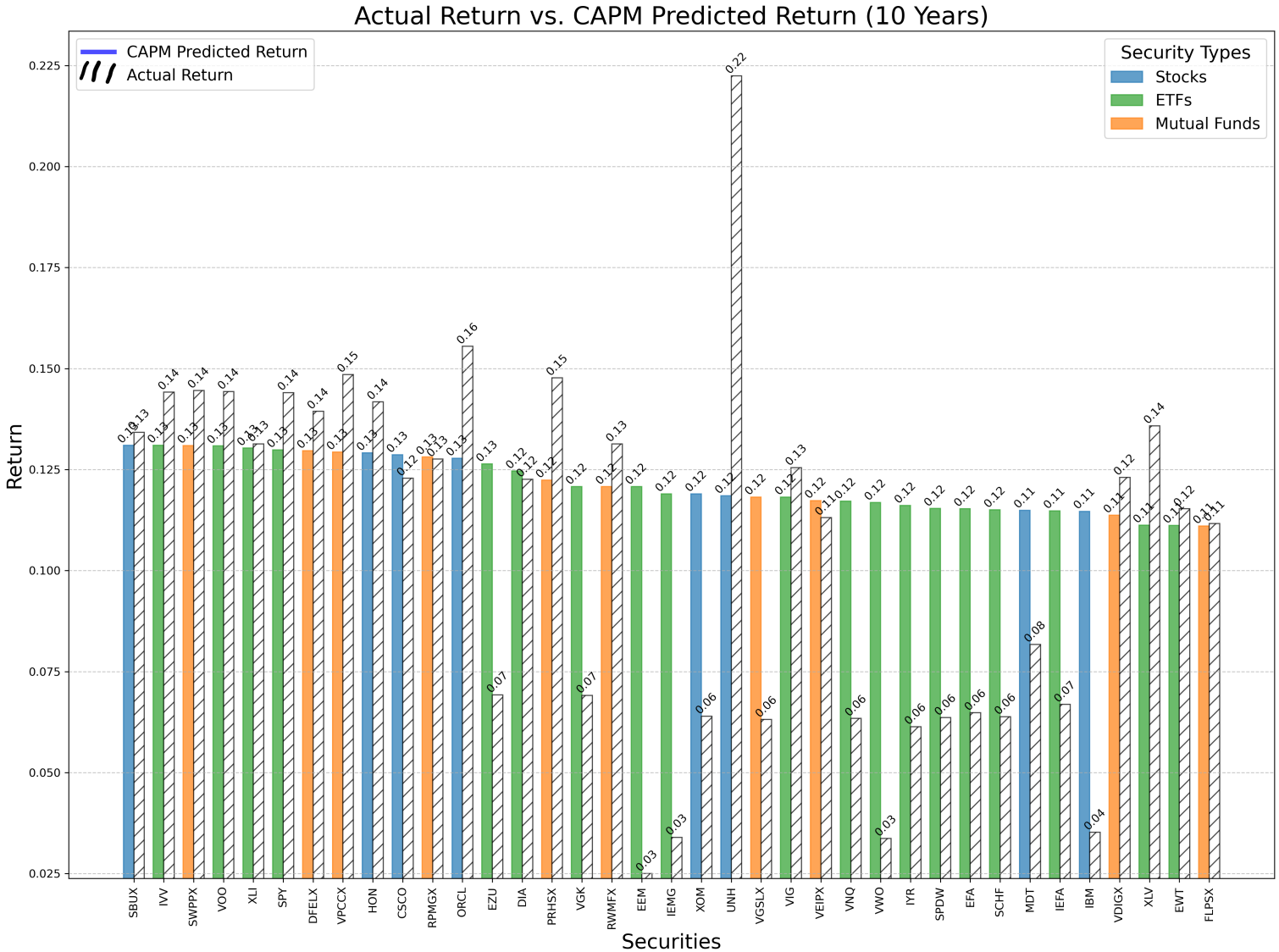
Calculating Expected Return using CAPM

Expected Return Calculation

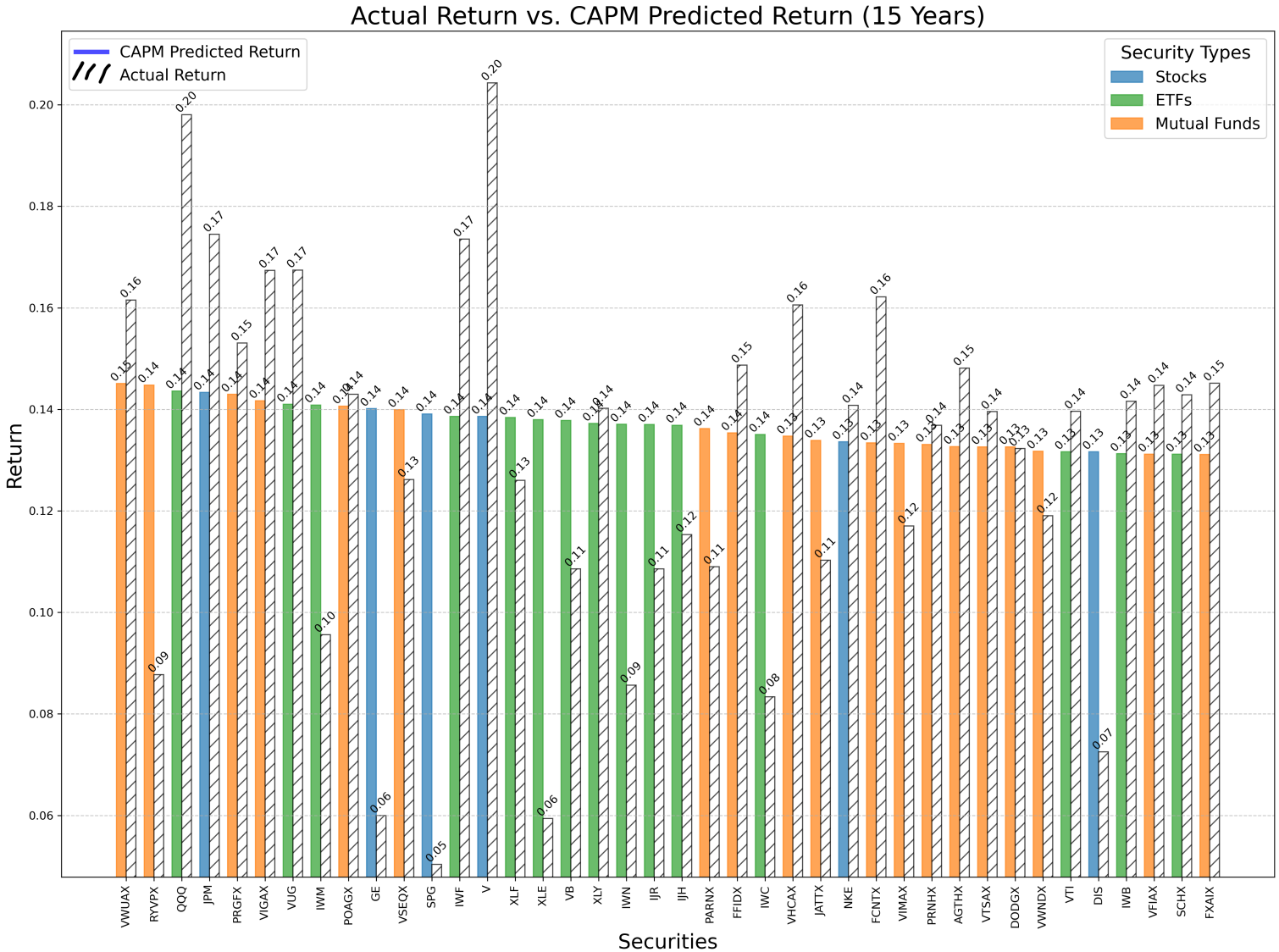
```
# Assuming a risk-free rate of 3% and market return of 8%
risk_free_rate = 0.03
market_return = 0.08

# Calculate expected return for each stock
capm_returns = {}
for ticker in tickers:
    capm_returns[ticker] = risk_free_rate + beta[ticker] *
    (market_return - risk-free_rate)
```

Actual Return vs. CAPM Expected Return (5 Year Time Horizon)

Actual Return vs. CAPM Expected Return (10 Year Time Horizon)



Actual Return vs. CAPM Expected Return (15 Year Time Horizon)



Resulting Filtered Data Generated (For Each Time Horizon)

Data Fields

- Ticker
- Beta
- CAPM
- Predicted Return
- Actual Return
- Volatility
- Sharpe Ratio
- Type

Modern Portfolio Theory (MPT)

Overview

A framework for constructing a portfolio to maximize return for a given level of risk.[a]

[a] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.

Formulas

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Definitions

- $E(R_p)$: Portfolio return
- w_i: Weight of asset i
- $E(R_i)$: Return of asset i
- σ_p^2: Portfolio variance
- σ_{ij}: Covariance of assets i, j

Optimize the Portfolio

Objective

Adjust the weights of the assets to maximize the portfolio's expected return for a given level of risk or to minimize risk for a given level of expected return.

Optimization Problem

Solve the following optimization problem:

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Setting Up the Optimization

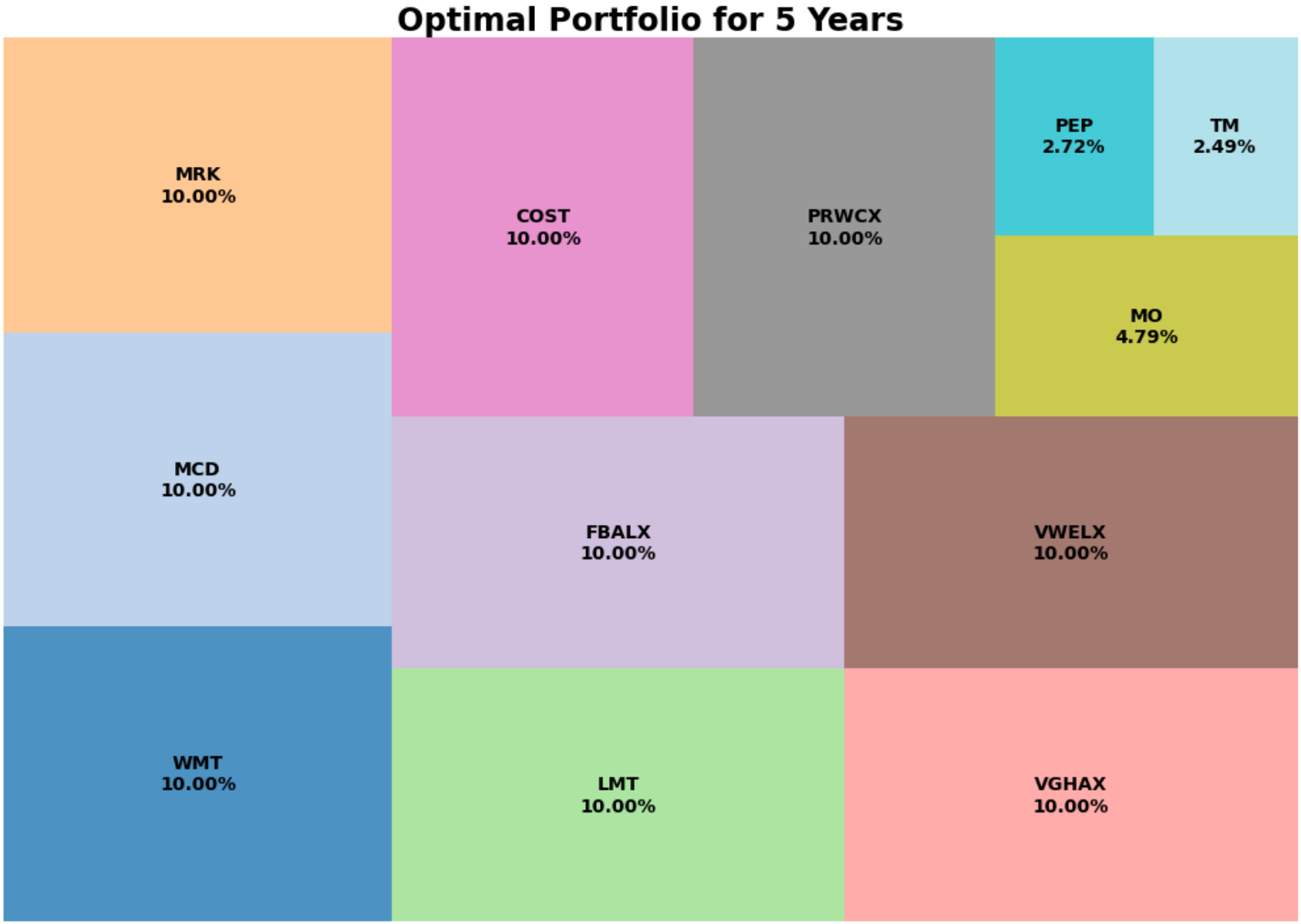
Bounds and Optimization Process

```
# Bounds to ensure diversification
bounds = tuple((0.0, 0.1) for _ in range(num_assets))

# Optimize the portfolio
result = minimize(negative_sharpe_ratio, init_guess,
                  args=(mean_returns, cov_matrix, risk_free_rate),
                  method='SLSQP', bounds=bounds, constraints=constraints)
```

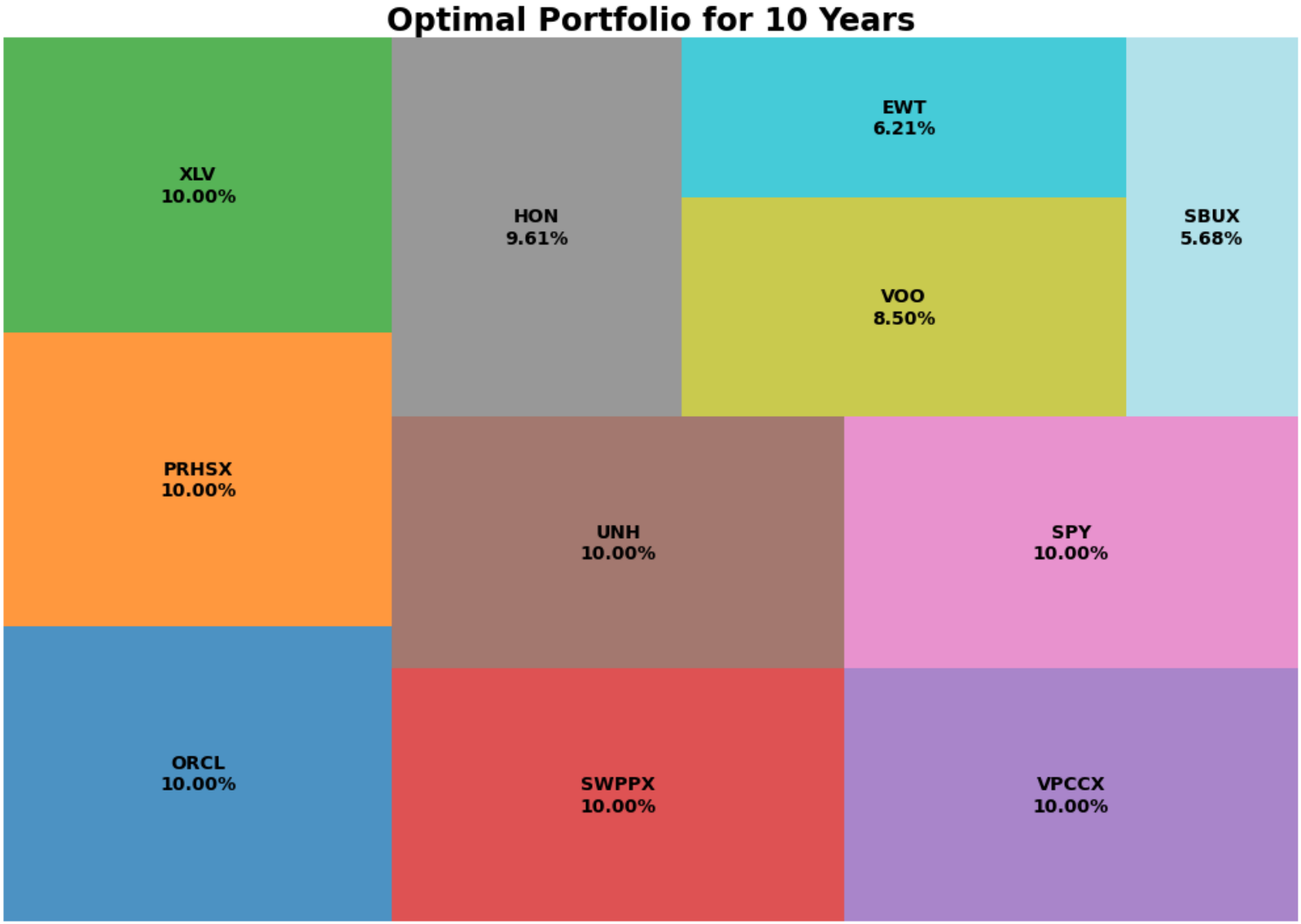
Explanation

- **Bounds**: Limit the weight of any single asset to a maximum of 10
- **Optimization**: Use the SLSQP method to find the optimal weights that maximize the Sharpe ratio by minimizing its negative.

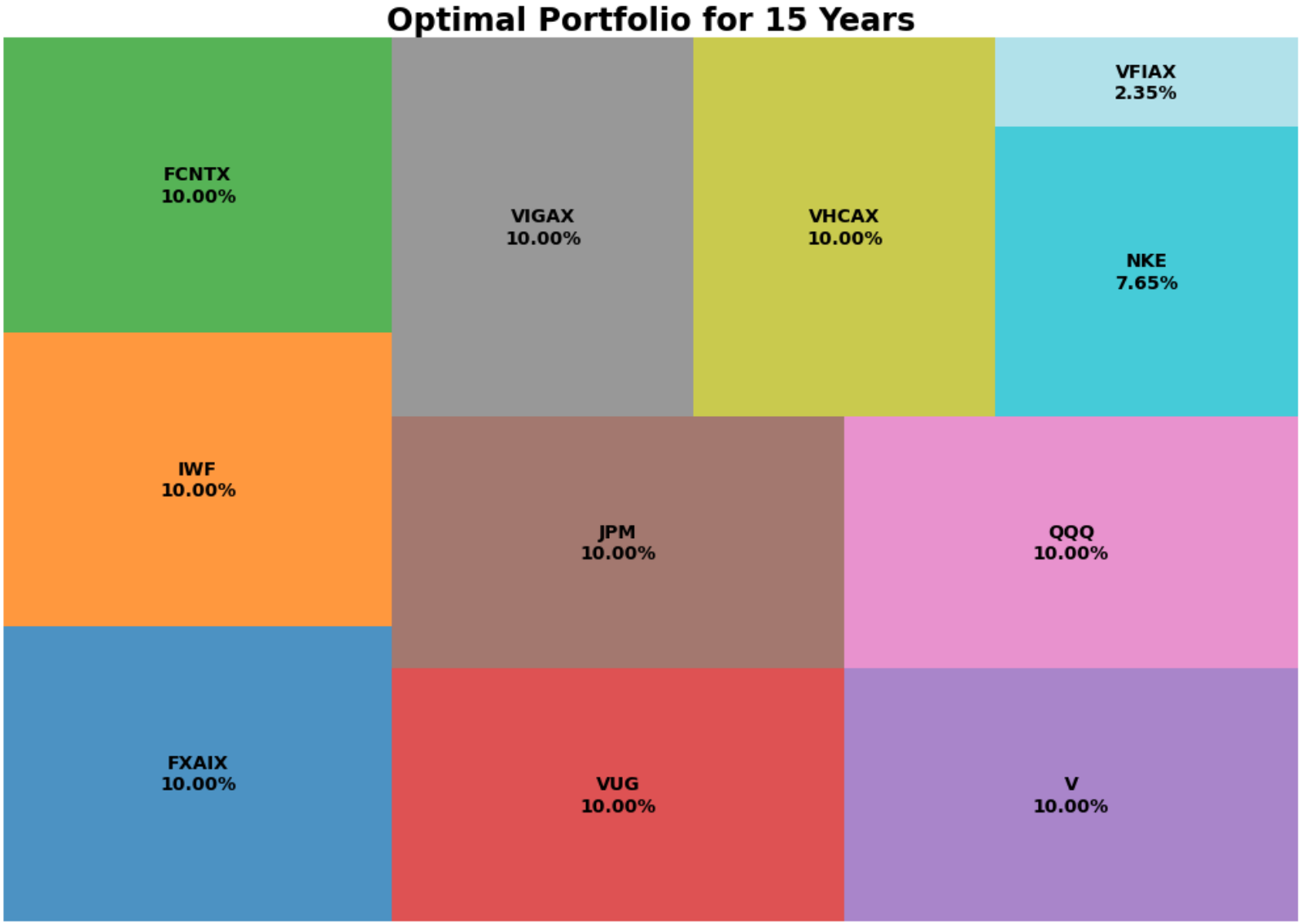
Optimal Portfolio (5 Years)



Optimal Portfolio (10 Years)



Optimal Portfolio (15 Years)



Results Interpretation

Findings

Through this analysis using the Capital Asset Pricing Model (CAPM) and Modern Portfolio Theory (MPT), we successfully identified the optimal weights for the selected securities. The analysis revealed that diversification across different asset classes and strategic allocation can significantly enhance the potential for accumulating sufficient down payments over varying time horizons.

Implications

These findings underscore the importance of tailored investment strategies for different age groups and income levels. By leveraging these models, first-time homebuyers can make informed decisions that align with their financial goals and risk tolerance.

Future Work and Limitations

Future Work

In future research, I plan to incorporate Monte Carlo Simulations to forecast future trends for the optimized portfolio. This will provide a more robust understanding of potential investment outcomes under various market conditions.

Limitations

This study's limitations include the assumption of constant risk-free rates and market returns, as well as the exclusion of transaction costs and taxes. Future work should aim to address these limitations to enhance the model's accuracy and applicability.

Q&A

Questions and Clarifications

Please feel free to ask for any clarifications or additional details regarding the presented research and findings.

Thank you for your attention!

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