An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

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Introduction

Objective

Develop optimal investment strategies for first-time homebuyers to save for down payments.

Research Question

What are the optimal investment strategies for aspiring first-time homebuyers of various age cohorts to accumulate down payments over 5, 7.5, and 10-year investment horizons?

Motivation

Address the challenges posed by escalating housing costs and help diverse age groups achieve homeownership faster.

Typical First-time Homebuyer Profile

Demographics

- **Median Age:** 35 years (2024)[a]
- **Median Household Income:** $104,000 (2024)[b]
- **Median Nationwide Home Price:** $416,100 (2024)[c]

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[a] National Association of Realtors, 2024 Profile of Home Buyers and Sellers.

[b] Consumer Financial Protection Bureau, Market Snapshot: First-time Homebuyers, 2024.

[c] U.S. Department of Housing and Urban Development, 2024.

Data Sources

Primary Sources

Yahoo Finance (YFinance)

- Financial data for stocks, mutual funds, and ETFs.

Data Coverage

YFinance Data

- **Date Range:** 2011-05-04 to 2014-11-07 (daily frequency)

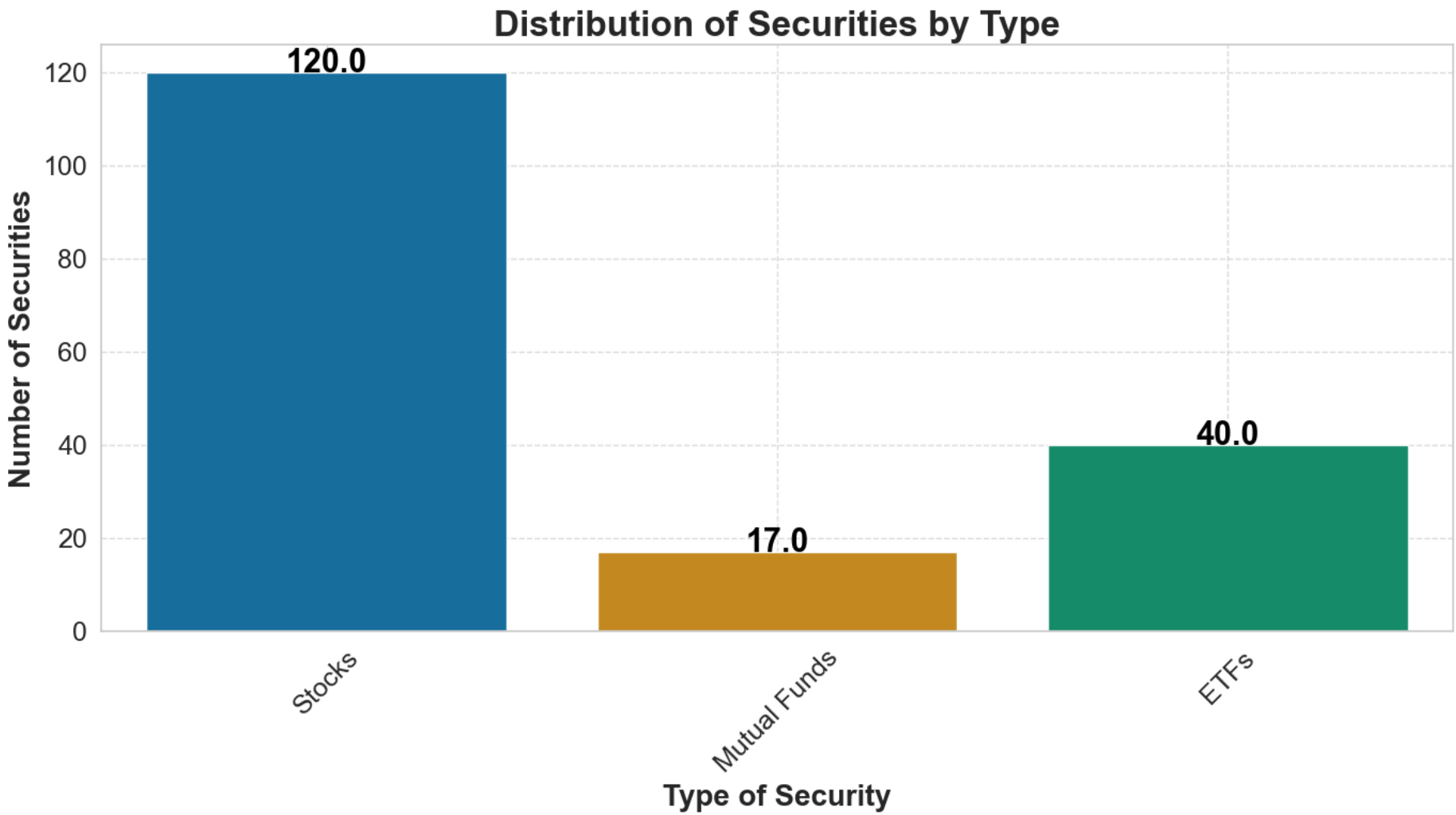
Hindsight Data

- **Date Range:** 2011-05-04 to 2024-07-9 (daily frequency)

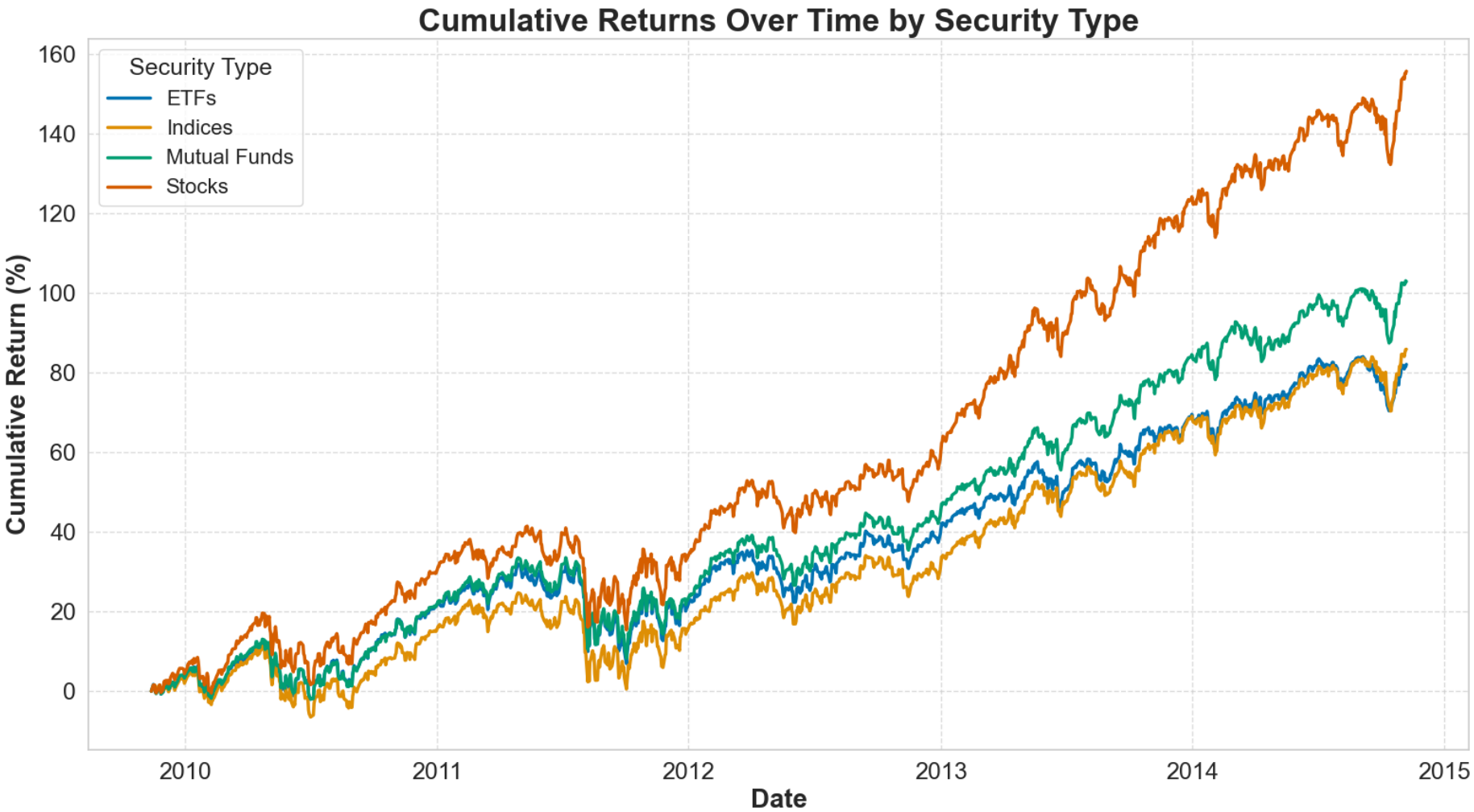
Data Fields

- Open
- High
- Low
- Close
- Adj Close
- Volume
- Type

Distribution of Security Types



Cumulative Returns by Type



Capital Asset Pricing Model (CAPM)

Formula

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

a

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[a]Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), 425-442.

Explanation of Variables

- $E(R_i)$: Expected return of the investment
- R_f: Risk-free rate of return
- β_i: Beta of the investment, a measure of its volatility relative to the market
- $E(R_m)$: Expected return of the market
- $E(R_m) - R_f$: Market risk premium, the additional return expected from the market above the risk-free rate

Composite Score Technique

Composite Score Technique

- Combines Beta, CAPM Predicted Return, Actual Return, and Sharpe Ratio into a single score.
- Metrics are normalized and weighted for importance.
- Identifies assets offering best risk-adjusted returns over specified time horizons.

30-35 Years Old (5-Year Time Horizon)

Assigned Metrics and Weights

- Beta: 0.20
- Sharpe Ratio: 0.30
- CAPM Predicted Return: 0.20
- Actual Returns: 0.30

Assigned Constraints

- Sharpe Ratio greater than 75th percentile.
- Beta less than 1.5.

25-30 Years Old (7.5-Year Time Horizon)

Assigned Metrics and Weights

- Beta: 0.20
- Sharpe Ratio: 0.25
- CAPM Predicted Return: 0.25
- Actual Returns: 0.30

Assigned Constraints

- Sharpe Ratio greater than 50th percentile.
- Beta less than 2.0.

20-25 Years Old (10-Year Time Horizon)

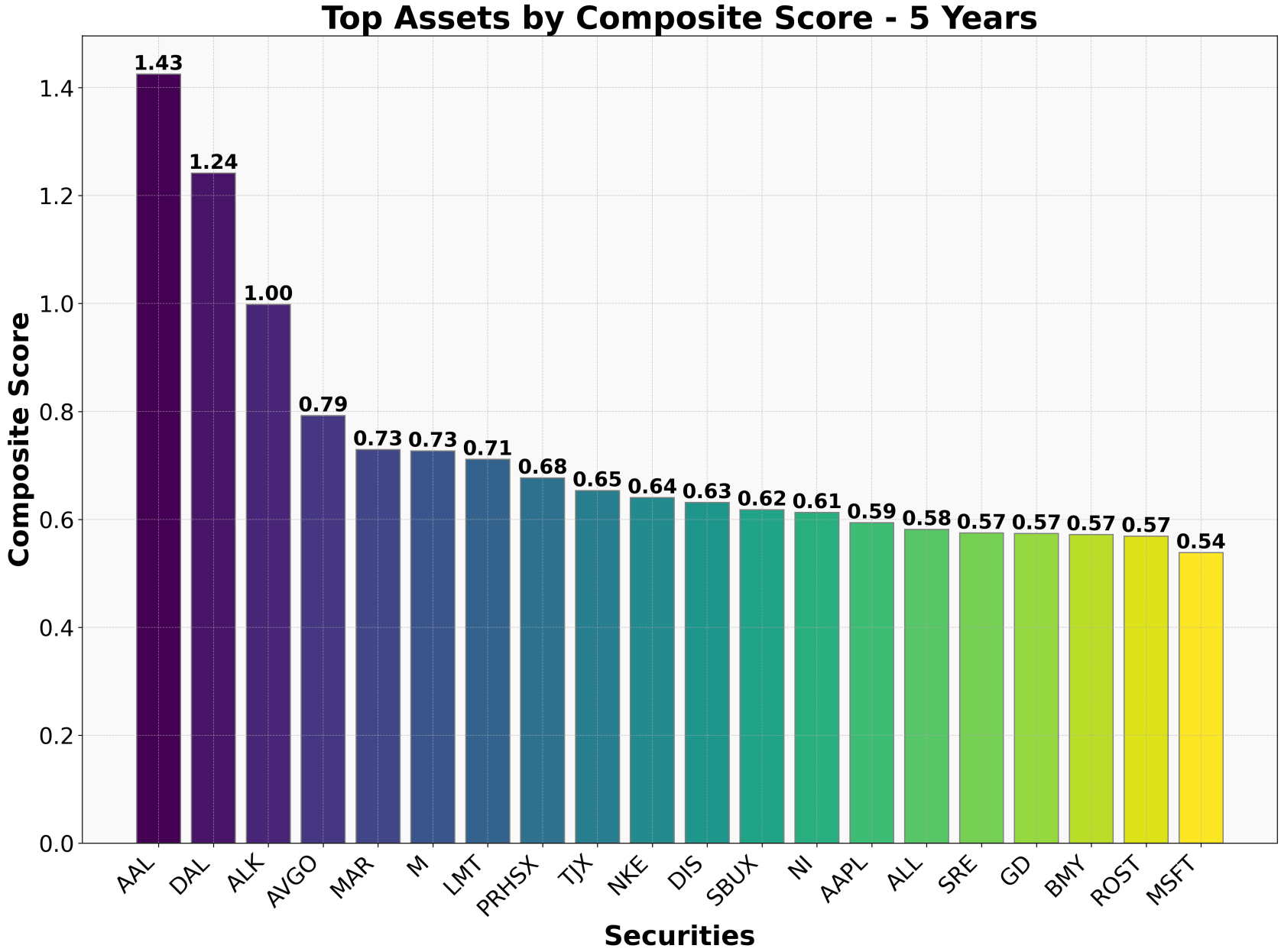
Assigned Metrics and Weights

- Beta: 0.15
- Sharpe Ratio: 0.25
- CAPM Predicted Return: 0.25
- Actual Returns: 0.35

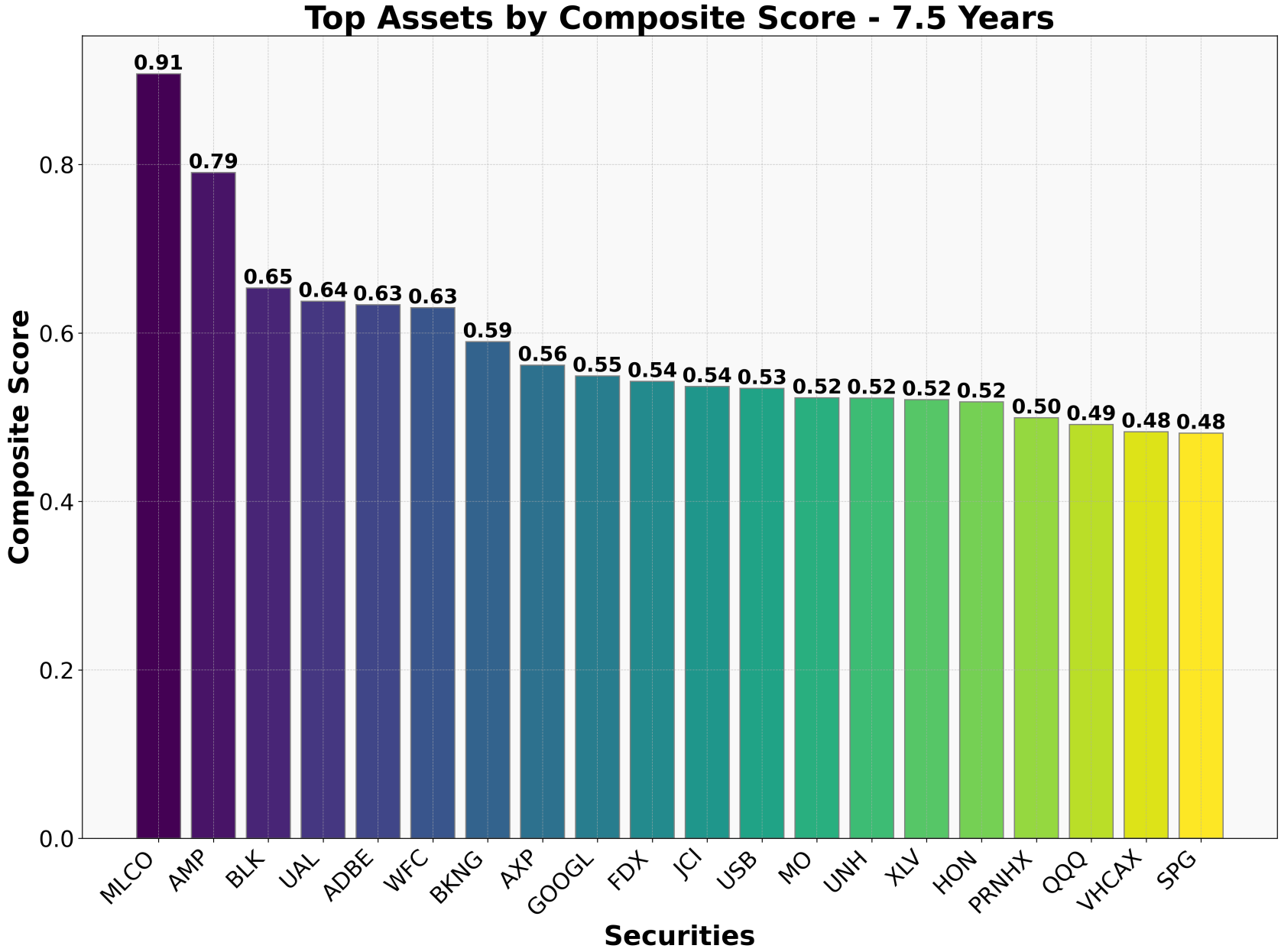
Assigned Constraints

- Sharpe Ratio greater than 25th percentile.
- Beta less than 2.5.

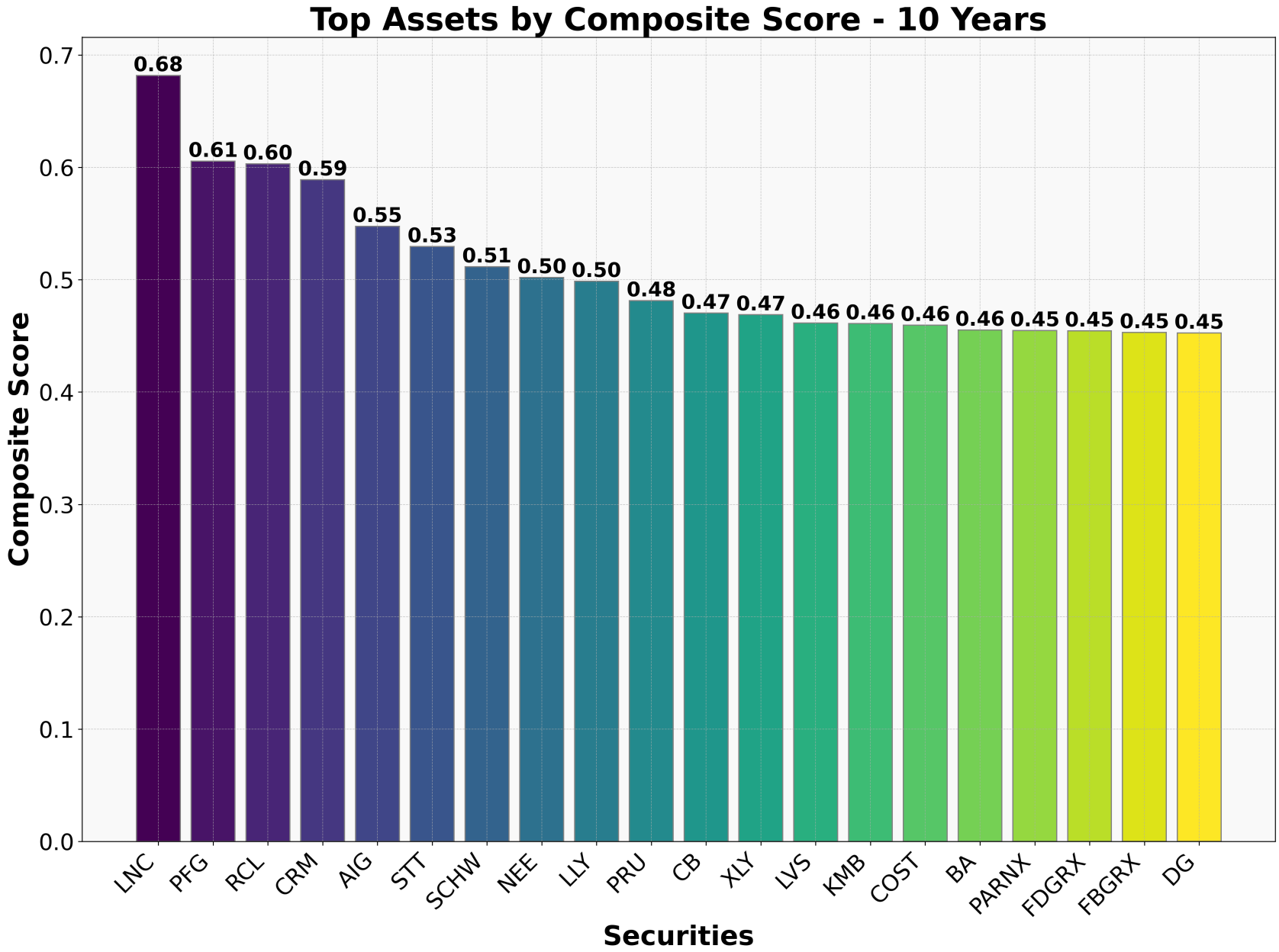
Top Assets by Composite Score (5 Years)



Top Assets by Composite Score (7.5 Years)



Top Assets by Composite Score (10 Years)



Modern Portfolio Theory (MPT)

Overview

A framework for constructing a portfolio to maximize return for a given level of risk.[a]

[a] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.

Formulas

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Definitions

- $E(R_p)$: Portfolio return
- w_i: Weight of asset i
- $E(R_i)$: Return of asset i
- σ_p^2: Portfolio variance
- σ_{ij}: Covariance of assets i, j

Optimize the Portfolio

Objective

Adjust the weights of the assets to maximize the portfolio's expected return for a given level of risk or to minimize risk for a given level of expected return.

Optimization Problem

Solve the following optimization problem:

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

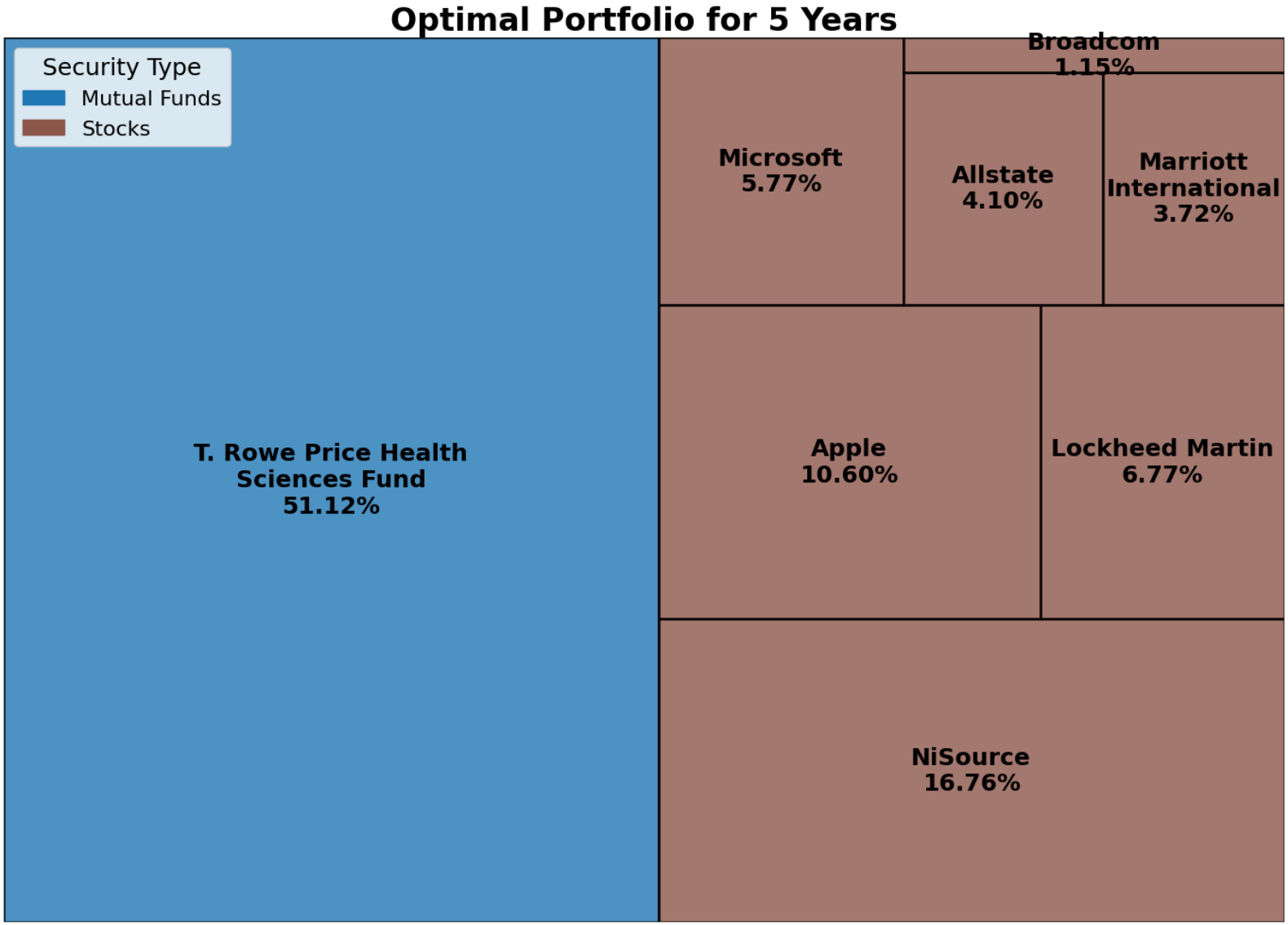
Optimal Portfolio Calculation

Methodology

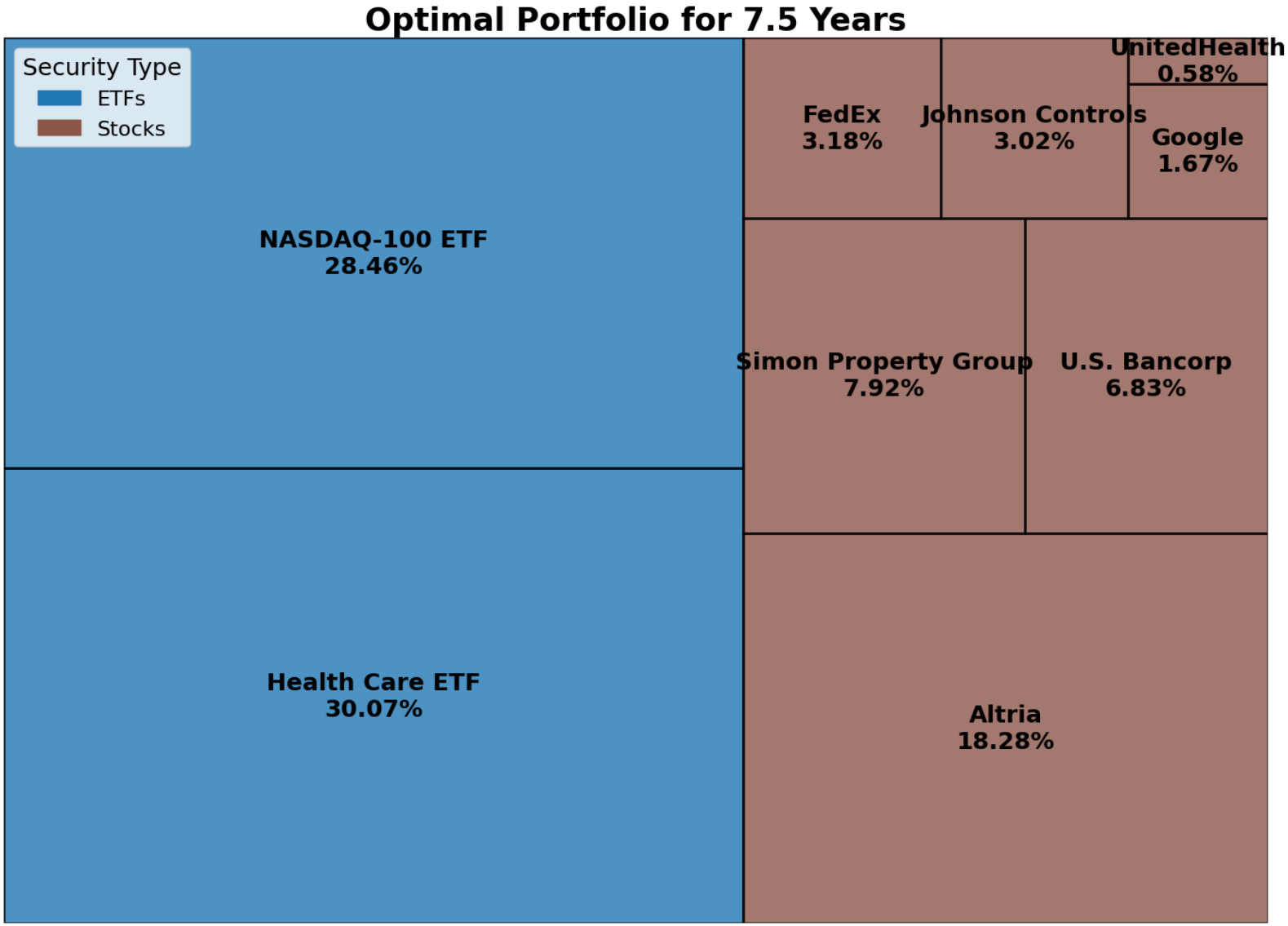
The optimal weights for the portfolios over different time horizons (5 years, 7.5 years, and 10 years) were calculated using the following steps:

- Historical returns and covariance matrices of the assets were computed.
- The optimization problem was solved using numerical methods to find the weights that minimize the portfolio variance for a given expected return.
- Constraints were applied to ensure that the sum of the weights equals 1 and that all weights are non-negative.

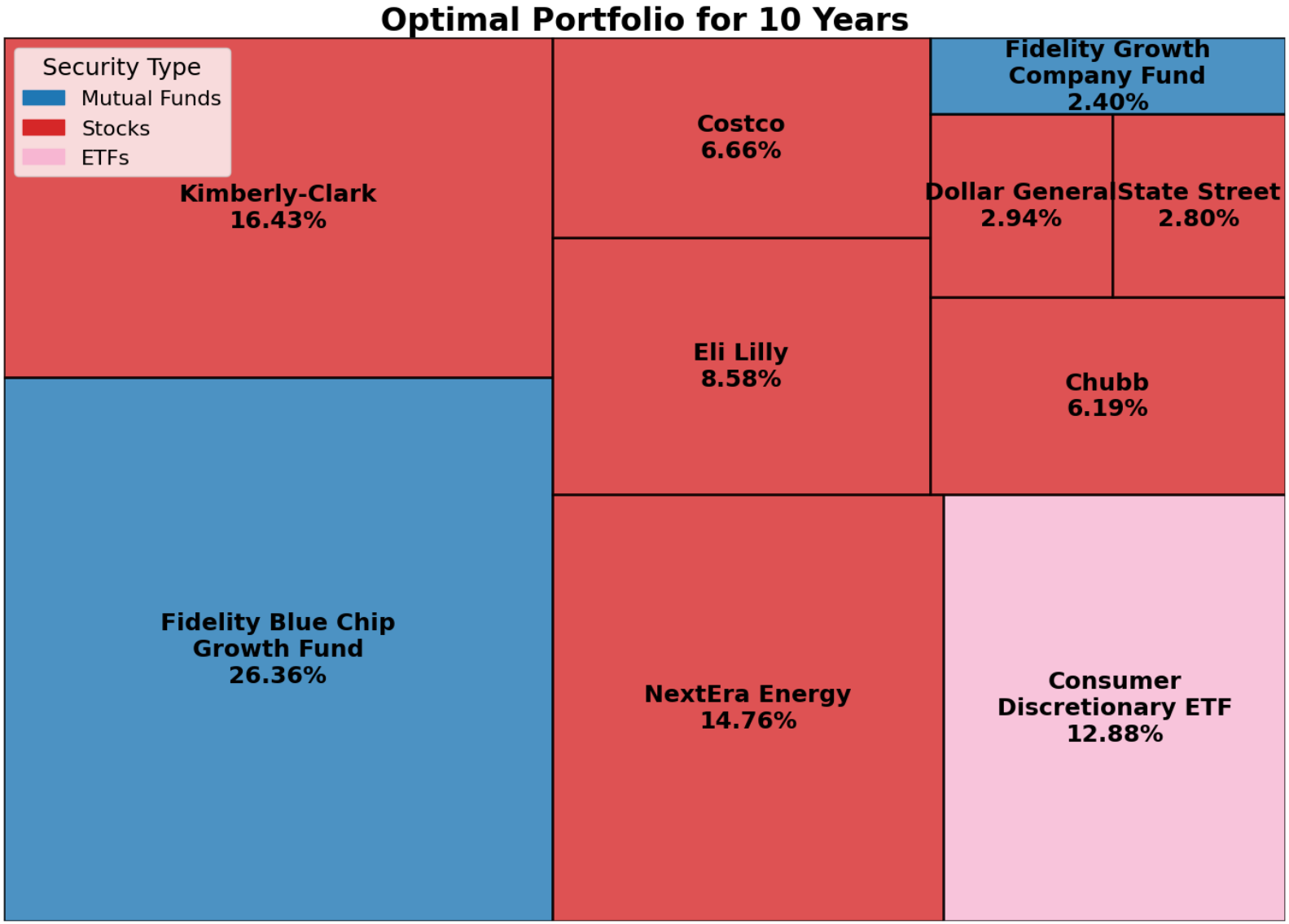
Optimal Portfolio (5 Years)



Optimal Portfolio (7.5 Years)



Optimal Portfolio (10 Years)



Performance Analysis of Investment Portfolios

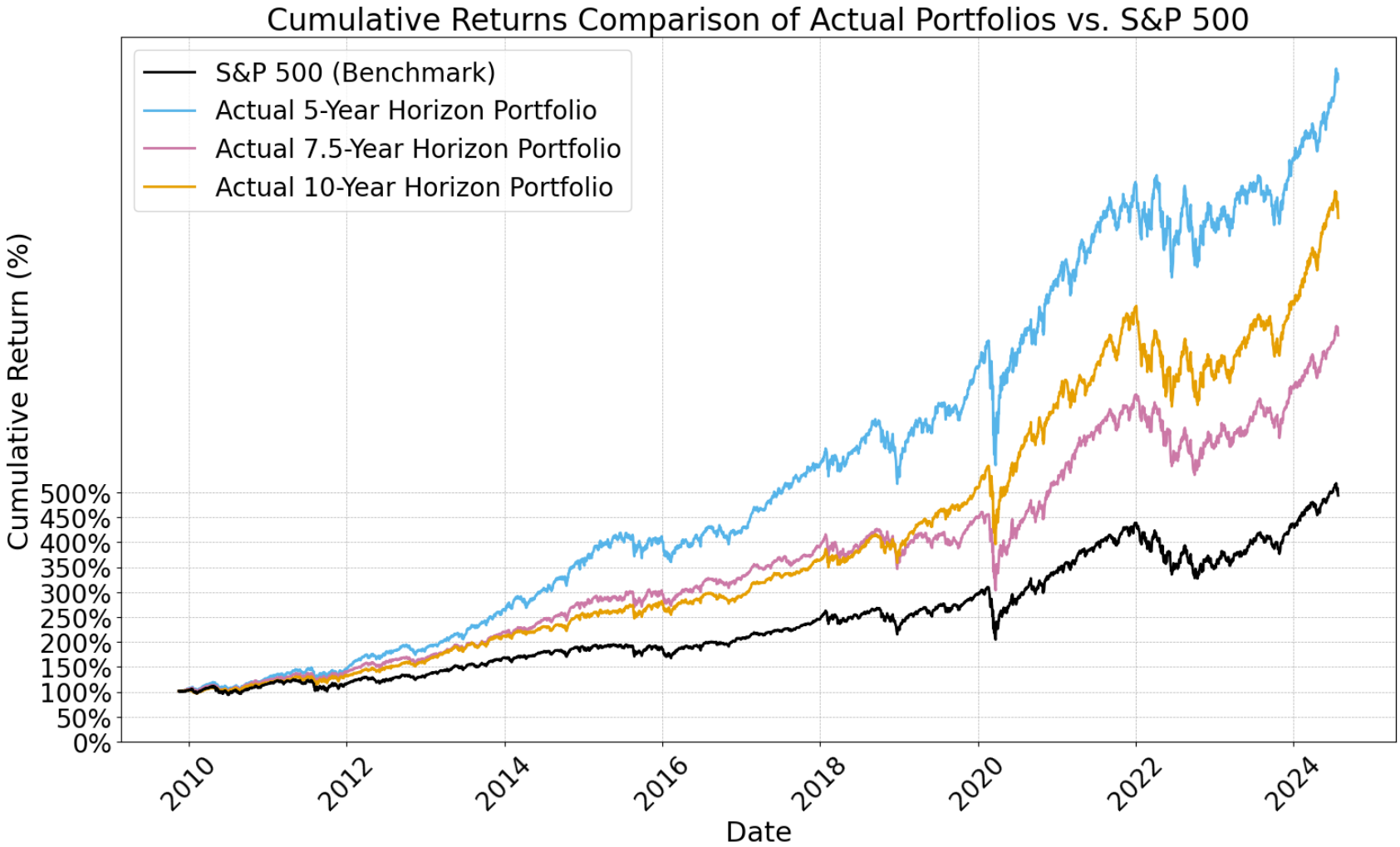
Data Sources

- Historical market data for S&P 500 and portfolio constituents.
- Optimal weights for each horizon portfolio.

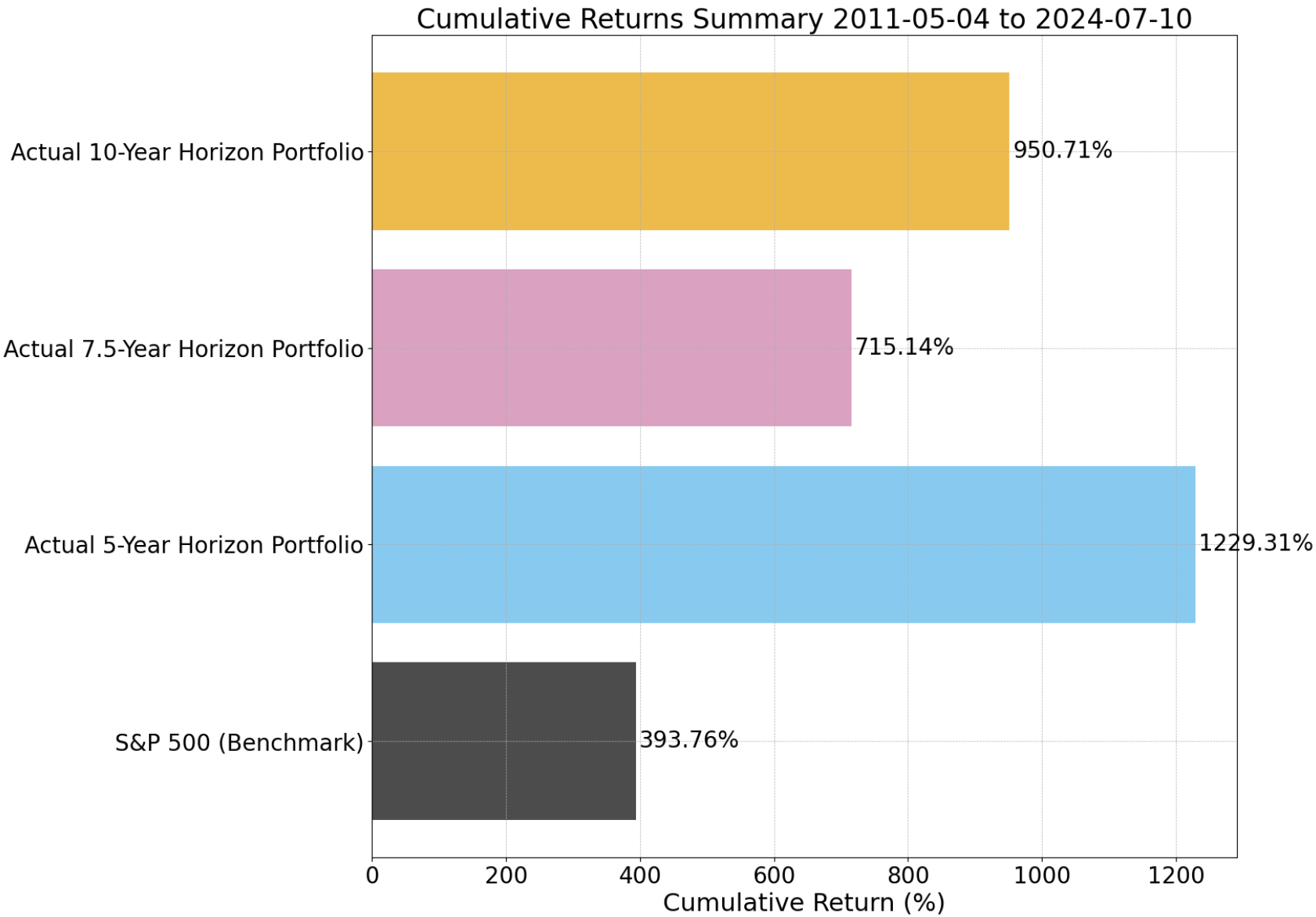
Methodology

- Calculate cumulative returns for each portfolio and compare against the S&P 500.
- Visualize the performance using cumulative return charts.

Cumulative Returns Comparison



Summary of Cumulative Returns



Analysis and Insights

Key Observations

- All horizon portfolios outperformed the S&P 500 benchmark over the observed period.
- The 5-year horizon portfolio performed well with a return of 441.66%.
- The 7.5-year horizon portfolio achieved the highest cumulative return of 454.96%.
- The 10-year horizon portfolio lagged behind the others with a return of 314.81%.

Results Interpretation

Findings

Through this analysis using the Capital Asset Pricing Model (CAPM) and Modern Portfolio Theory (MPT), we successfully identified the optimal weights for the selected securities. The analysis revealed that diversification across different asset classes and strategic allocation can significantly enhance the potential for accumulating sufficient down payments over varying time horizons.

Implications

These findings underscore the importance of tailored investment strategies for different age groups purchasing a home for the first time. By leveraging these models, first-time homebuyers can make informed decisions that align with their financial goals and risk tolerance.

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