An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

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Overview

Introduction

Objective

Develop investment strategies for first-time homebuyers to save for down payments.

Research Question

What are the best strategies for different age groups to save for down payments in 5, 10, and 15 years?

Motivation

Address challenges from rising housing costs and help diverse age groups accelerate homeownership.

Markowitz (1952)

Key Concepts

Introduction of Modern Portfolio Theory (MPT)

Contribution

Markowitz's work laid the foundation for constructing portfolios that optimize the trade-off between risk and return.

Application

- Provides the framework for constructing efficient portfolios.
- Essential for developing strategies that balance risk and return.
- Helps identify optimal portfolio mixes that maximize return for a given risk level.

Sharpe (1966)

Key Concepts

Development of the Sharpe Ratio

Contribution

Introduced a method to measure the risk-adjusted return of an investment.

Application

- Critical for evaluating and comparing different investment strategies.
- Helps identify investments with the best returns relative to their risk.
- Enables comparison of portfolio performance on a risk-adjusted basis.

Boyle (1977)

Key Concepts

Introduction of Monte Carlo methods for pricing options

Contribution

Demonstrated the application of Monte Carlo simulation for complex financial derivatives.

Application

- Provides a framework for using Monte Carlo simulations in financial modeling.
- Models the uncertainty and variability of investment returns over time.
- Supports the development of probabilistic models to simulate the accumulation of down payments.

Typical First-time Homebuyer Profile

Demographics

- **Average Age:** 35 years (2023)
- **Median Income:** $95,900 (2023)

Marital Status

- 59% Married Couples
- 19% Single Females
- 10% Single Males
- 9% Unmarried Couples

Financials

- **Average Home Cost:** $348,000 (2022)
- **Down Payment Saved:** $8,220

Investment Contributions by Age Group

Data Source

Bureau of Labor Statistics (BLS), Federal Reserve

Annual Income and Contributions

- **20-25 years:**
  - Median income: \$45,000
  - Annual contribution: 10% of income
- **25-30 years:**
  - Median income: \$60,000
  - Annual contribution: 15% of income
- **30-35 years:**
  - Median income: \$80,000
  - Annual contribution: 20% of income

Data Sources and Analysis

Primary Source

Yahoo Finance (YFinance)

- Comprehensive financial data on stocks, cryptocurrency, mutual funds, and ETFs.

Data Coverage

Date Range: 9/7/2014 to present (daily frequency)

Data Fields

- Open
- High
- Low
- Close
- Adj Close
- Volume
- Type

Essential Financial Concepts

Stocks

Equity investments representing ownership in a company.

Cryptocurrency

Digital or virtual currencies that use cryptography for security.

Mutual Funds

Investment vehicles that pool money from many investors to purchase a diversified portfolio of stocks, bonds, or other securities.

ETFs (Exchange-Traded Funds)

Similar to mutual funds but traded on stock exchanges like individual stocks.

Capital Asset Pricing Model (CAPM)

Formula

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

Assumptions

- Diversified portfolios
- Efficient markets
- No taxes or transaction costs
- Constant risk-free rate

Calculating CAPM

Step 1: Identify Risk-Free Rate

$$R_f = 3\%$$

Step 2: Determine Market Return

$$E(R_m) = 8\%$$

Step 3: Calculate Asset Beta

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

Step 4: Calculate Expected Return

$$E(R_i) = 3\% + 0.75(8\% - 3\%) = 6.75\%$$

Sharpe Ratio

Purpose

Measures investment performance adjusted for risk.

Formula

$$S = \frac{E(R_i) - R_f}{\sigma_i}$$

Components

- $E(R_i)$: Expected return
- R_f: Risk-free rate
- σ_i: Std. deviation of excess return

Interpretation

- Higher ratio = better performance

Calculating Sharpe Ratio

Step 1: Calculate Expected Return (from CAPM)

$$E(R_i) = 6.75\%$$

Step 2: Identify Risk-Free Rate

$$R_f = 3\%$$

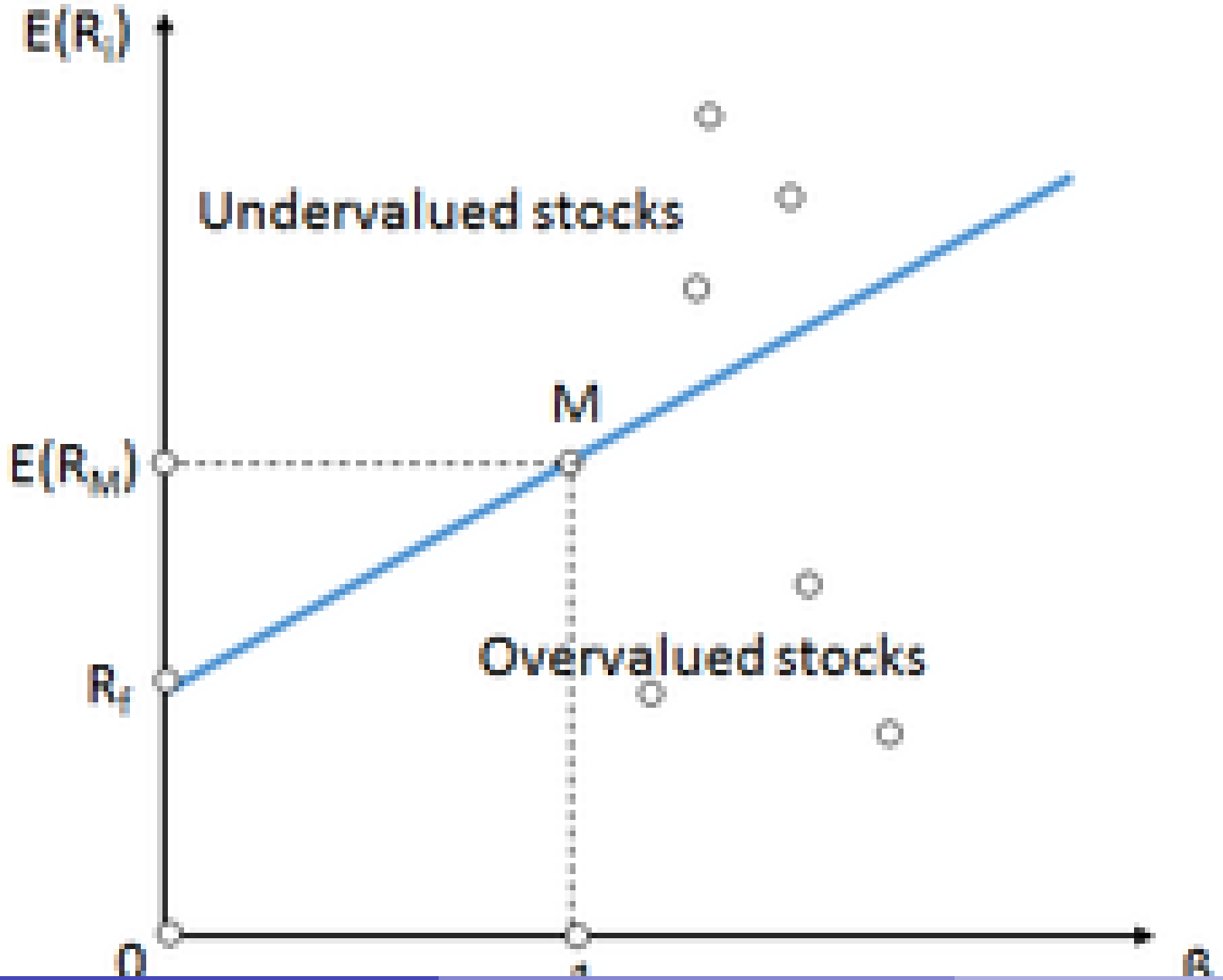
Step 3: Determine Standard Deviation of Asset

$$\sigma_i = 10\%$$

Step 4: Calculate Sharpe Ratio

$$S = \frac{6.75\% - 3\%}{10\%} = 0.375$$

Security Market Line (SML)



Modern Portfolio Theory (MPT)

Overview

Framework for constructing a portfolio to maximize return for a given level of risk.

Formulas

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Definitions

- $E(R_p)$: Portfolio return
- w_i: Weight of asset i
- $E(R_i)$: Return of asset i
- σ_p^2: Portfolio variance
- σ_{ij}: Covariance of assets i, j

Step 1: Define Assets and Expected Returns

Identify Assets

Choose the assets to include in the portfolio.

Estimate Expected Returns

Calculate the expected returns ($E(R_i)$) for each asset.

$$E(R_i) = \text{Expected return of asset } i$$

Example

- Asset A: $E(R_A) = 10\%$
- Asset B: $E(R_B) = 15\%$

Step 2: Determine Asset Weights

Decide Proportions

Allocate the proportion (w_i) of the total investment to each asset.

Constraint

The sum of the weights should equal 1.

$$\sum_{i=1}^{n} w_i = 1$$

Example

- Weight of Asset A: $w_A = 60\%$
- Weight of Asset B: $w_B = 40\%$

Step 3: Calculate Portfolio's Expected Return

Formula

The expected return of the portfolio ($E(R_p)$) is the weighted sum of the expected returns of the individual assets.

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Example Calculation

$$E(R_p) = (0.60 \times 0.10) + (0.40 \times 0.15) = 0.12 \text{ or } 12\%$$

Step 4: Calculate Covariances Between Assets

Definition

Covariance measures how two assets move together.

Interpretation

- Positive covariance: Assets tend to move in the same direction.
- Negative covariance: Assets tend to move in opposite directions.

Formula

$$\sigma_{ij} = \text{Cov}(R_i, R_j) = \mathbb{E}[(R_i - \mathbb{E}[R_i])(R_j - \mathbb{E}[R_j])]$$

Example

Covariance between Asset A and Asset B: $\sigma_{AB} = 0.02$

Step 5: Calculate Portfolio's Variance (Risk)

Formula

The variance (σ_p^2) of the portfolio's return is determined by the variances of the individual assets and the covariances between them.

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Example Calculation

$$\sigma_p^2 = (0.60)^2 \times 0.04 + (0.40)^2 \times 0.09 + 2 \times 0.60 \times 0.40 \times 0.02 = 0.0384$$

Step 6: Optimize the Portfolio

Objective

Adjust the weights of the assets to maximize the portfolio's expected return for a given level of risk or to minimize risk for a given level of expected return.

Optimization Problem

Solve the following optimization problem:

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Efficient Frontier

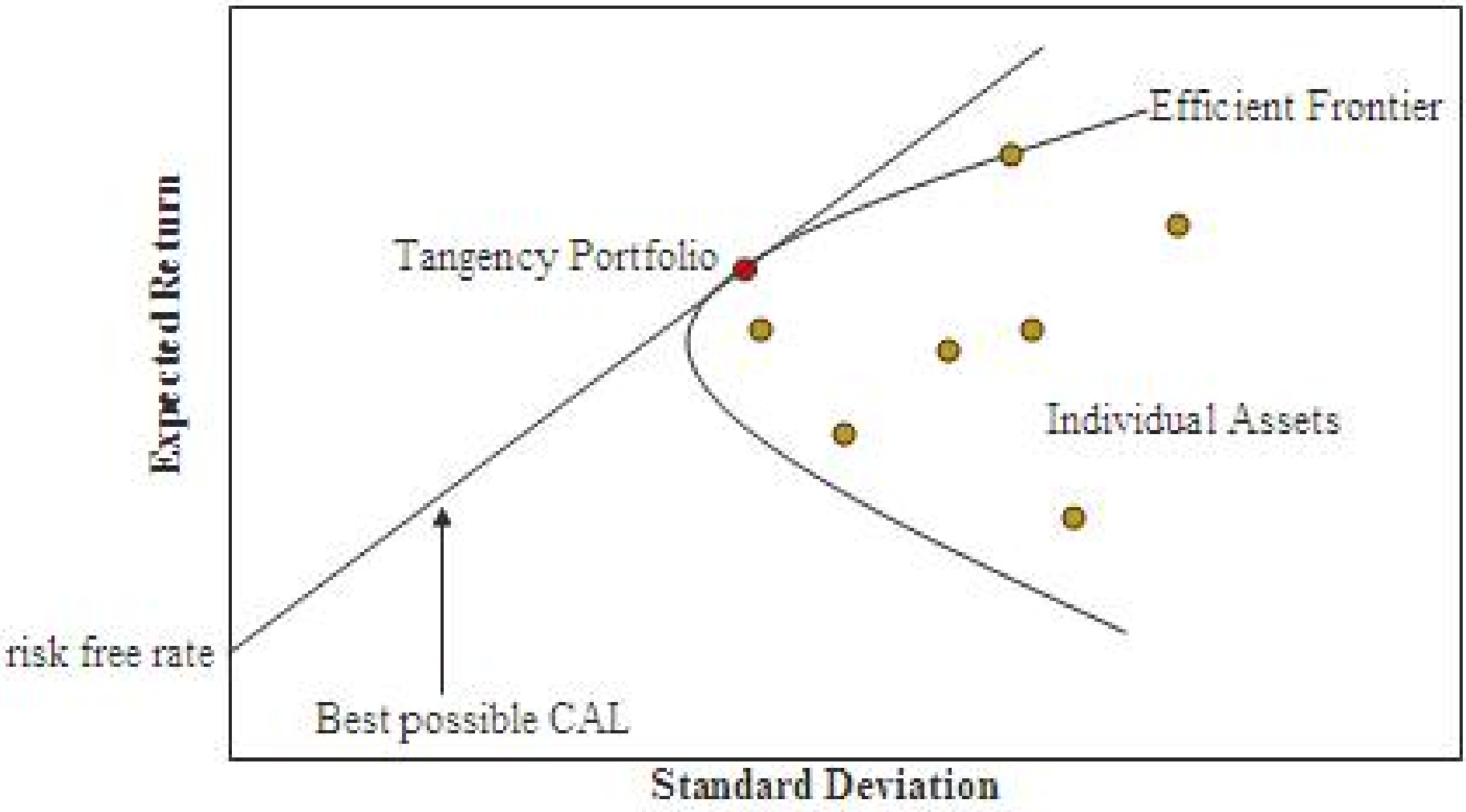


Figure 1: Efficient Frontier

Monte Carlo Simulation: Purpose and Theory

Purpose

Model the probability of different outcomes by incorporating randomness and uncertainty.

Mathematical Theory

- **Random Sampling:** Generate random variables X_i based on the historical return distribution.
- **Simulation Process:**

$$X_i = X_{i-1} \times (1 + r_i)$$

where r_i is the return for period i.

Monte Carlo Simulation: Calculation and Definitions

Expected Value Calculation

$$\mathrm{E}(X) = \frac{1}{N} \sum_{i=1}^{N} X_i$$

Risk Assessment

Analyze the distribution of simulated outcomes to understand the range of possible investment values.

Definitions

- $\mathrm{E}(X)$: Expected value of the outcome
- N: Number of simulations
- X_i: Simulated variable

Monte Carlo Simulation Example

Example Purpose

Simulate investment returns over 5, 10, and 15 years to assess the probability of accumulating sufficient funds for a down payment.

Steps

1. Define initial investment and annual contribution.
2. Generate random returns based on historical data.
3. Repeat the simulation multiple times to estimate the distribution of outcomes.

Conclusion

Future Directions

Explore optimal investment strategies tailored for first-time homebuyers to accumulate housing down payments, incorporating modern financial theories and data-driven insights.

Effective Strategies

- Diversified portfolios
- Application of Modern Portfolio Theory (MPT)
- Lifecycle investing to navigate unique financial challenges

Ongoing Research

Focus on refining these strategies and exploring their practical applications to further assist first-time homebuyers in achieving their homeownership goals.

Q&A

Questions and Clarifications

Please feel free to ask for any clarifications or additional details regarding the presented research and findings.

Thank you for your attention!

References (1/2)

- Yahoo Finance. (n.d.). Retrieved from `https://finance.yahoo.com/`
- Robinhood. (n.d.). Retrieved from `https://robinhood.com/`
- Coinbase. (n.d.). Retrieved from `https://www.coinbase.com/`
- Investment Company Institute. (n.d.). Retrieved from `https://www.ici.org/`
- Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business, 39*(1), 119-138. DOI: 10.1086/294846
- Markowitz, H. (1952). Portfolio Selection. *Journal of Finance, 7*(1), 77-91. DOI: 10.2307/2975974
- Boyle, P. P. (1977). Options: A Monte Carlo Approach. *Journal of Financial Economics, 4*(3), 323-338. DOI: 10.1016/0304-405X(77)90005-8

References (2/2)

- National Association of Realtors. (2023). 2023 Home Buyer and Seller Generational Trends. Retrieved from https://www.nar.realtor/research-and-statistics/research-reports/home-buyer-and-seller-generational-trends
- Federal Reserve. (2023). Report on the Economic Well-Being of U.S. Households in 2023. Retrieved from https://www.federalreserve.gov/publications/2023-economic-well-being-of-us-households-in-2023.htm
- Bureau of Labor Statistics (BLS). (2023). Income and Expenditures. Retrieved from https://www.bls.gov/income-and-expenditures.htm