# An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

Business Analytics MS Capstone Project

Frank Paul Longo II

July 26, 2024

#### **Abstract**

This research conducts a detailed analysis of optimal investment strategies tailored for first-time homebuyers seeking to accumulate down payments over 5, 7.5, and 10-year horizons. By integrating Modern Portfolio Theory (MPT), the Capital Asset Pricing Model (CAPM), and Monte Carlo simulations, the study provides practical insights into constructing investment portfolios that are time-specific and balance risk and return effectively.

The findings reveal that using fundamental investment analysis strategies substantially improves the ability to save for a down payment, greatly shortening the time needed to achieve homeownership. The importance of considering risk-adjusted returns and diversification in portfolio construction is underscored, demonstrating their critical roles in enhancing savings outcomes.

Future research could broaden this analysis by incorporating additional demographic factors such as income variations and regional housing market dynamics. Furthermore, exploring the inclusion of alternative investment vehicles, such as real estate investment trusts (REITs) and cryptocurrencies, could offer deeper insights into optimizing investment strategies for down payments.

# **Contents**

1	Introduction						
	1.1	1 Research Question & Objective					
	1.2	2 Motivation					
2	Data	a		6			
3	Lite	rature l	Review	7			
	3.1	Portfolio Selection by Harry Markowitz (1952)					
		3.1.1	Key Takeaways	7			
		3.1.2	Study Implications	7			
		3.1.3	Critiques	8			
	3.2	Capita	al Asset Prices: A Theory of Market Equilibrium under Conditions of				
	y William F. Sharpe (1964)	8					
		3.2.1	Key Takeaways	8			
		3.2.2	Study Implications	8			
		3.2.3	Critiques	9			
	3.3 Options: A Monte Carlo Approach by Phelim P. Boyle (1977)						
		3.3.1	Key Takeaways	9			
		3.3.2	Study Implications	10			
		3.3.3	Critiques	10			
4	The	oretica	l Models	11			
	4.1	Capital Asset Pricing Model (CAPM)					
		4.1.1	Beta Calculation	11			
		4.1.2	Security Market Line (SML)	12			
	4.2	Sharp	e Ratio	13			
	4.3	Composite Score Calculation					

	4.4	Modern Portfolio Theory (MPT)					
		4.4.1	Efficient Frontier and Optimal Portfolio	14			
	4.5	.5 Monte Carlo Simulation					
5	Specification	17					
	5.1	Mode	l Implementation	17			
		5.1.1	CAPM and Sharpe Ratio Calculation	17			
		5.1.2	Composite Score Calculation	18			
		5.1.3	Modern Portfolio Theory (MPT) Application	18			
		5.1.4	Performance Comparison	18			
		5.1.5	Monte Carlo Simulation for Future Forecasting	19			
6	Resi	ults		20			
	6.1	Explo	ratory Data Analysis	20			
	6.2	Assets	s by Composite Score	21			
	6.3	Optim	al Portfolios with Weights				
	6.4	Optimal Portfolios with Weights					
	6.5	Forecasting Results Using Monte Carlo					
	6.6						
7	Results Summary 2						
8	Con	clusior	ıs	30			
A Data Appendix							
	A.1	Summ	nary of Data Sources	32			
	A.2	Variab	ole Definitions	32			

# 1 Introduction

The escalating housing costs in contemporary real estate markets have created significant barriers for first-time homebuyers. This demographic often faces the daunting task of accumulating substantial down payments amidst economic volatility and uncertain income trajectories. This research addresses this critical issue by developing and evaluating optimal investment strategies tailored to help different age groups achieve their homeownership goals within 5, 7.5, and 10-year horizons leading up to the median first time homebuyer age of 35 (National Association of Realtors, 2023).

# 1.1 Research Question & Objective

What are the most effective investment strategies for different age groups to accumulate housing down payments over periods of 5, 7.5, and 10 years? The primary objective of this study is to identify, analyze, and optimize investment strategies that can effectively assist first-time homebuyers in saving for their down payments. By leveraging advanced financial theories and empirical methodologies, this research aims to provide actionable insights that balance risk and return, offering practical solutions for prospective homeowners (Markowitz, 1952; Sharpe, 1964; Boyle, 1977).

#### 1.2 Motivation

As homeownership becomes increasingly out of reach for many, particularly younger individuals, it is imperative to develop strategies that can mitigate these barriers. By providing evidence-based investment strategies, this study aims to empower individuals with the tools needed to navigate the complexities of financial planning for homeownership.

# 2 Data

The financial data utilized in this research is sourced from Yahoo Finance (Yahoo Finance, nd). The dataset encompasses comprehensive information on approximately 150 securities, including stocks, mutual funds, and ETFs. The historical data spans from November 13, 2009, to November 7, 2014, assuming the analysis date is November 7, 2014. Additionally, hindsight data extends from November 13, 2009, to July 23, 2024. This extensive timeframe enables the examination of recent trends and the performance of various asset classes under different market conditions.

The dataset comprises the following fields:

- Open: Price at the beginning of the trading day.
- **High:** Peak price during the trading day.
- Low: Lowest price during the trading day.
- Close: Price at the end of the trading day.
- Adj Close: Closing price adjusted for dividends, stock splits, etc.
- **Volume:** Number of shares traded during a single trading day.
- **Type:** Security type (e.g., stock, ETF).

#### 3 Literature Review

### 3.1 Portfolio Selection by Harry Markowitz (1952)

#### 3.1.1 Key Takeaways

In "Portfolio Selection," Harry Markowitz introduced the concept of Modern Portfolio Theory (MPT), which fundamentally changed the way investors approach portfolio construction. Markowitz emphasized the trade-off between risk and return, proposing that investors should diversify their investments across various assets to minimize risk without sacrificing expected returns. The main contribution of this paper is the efficient frontier, which represents the set of portfolios that offer the highest expected return for a given level of risk. The efficient frontier helps investors identify the optimal asset allocation that maximizes returns for a specific risk level.

"An investor should diversify to achieve maximum return for a given level of risk, leading to the creation of the efficient frontier." (Markowitz, 1952).

Markowitz's work also introduced the concept of mean-variance optimization, where the expected return of a portfolio is calculated as the weighted sum of the expected returns of individual assets, and the risk is measured as the variance of portfolio returns. This approach allows investors to construct portfolios that either maximize expected return for a given level of risk or minimize risk for a given level of expected return.

#### 3.1.2 Study Implications

Markowitz's findings are fundamental to this study as they provide the basis for constructing diversified investment portfolios aimed at accumulating down payments for housing. By applying the principles of MPT, my study seeks to identify optimal asset allocations that maximize returns while managing risk over different investment horizons.

#### 3.1.3 Critiques

Despite its groundbreaking contributions, MPT has faced criticism for its reliance on historical data to estimate future returns and risks, which may not always be accurate. Additionally, the assumption of normally distributed returns has been questioned, as real-world financial returns often exhibit skewness and kurtosis. Furthermore, MPT also does not account for extreme events and tail risks, which can significantly impact portfolio performance.

# 3.2 Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk by William F. Sharpe (1964)

#### 3.2.1 Key Takeaways

In his paper, William F. Sharpe introduced the Capital Asset Pricing Model (CAPM), which provides a framework to determine the expected return of an asset based on its systematic risk, as measured by beta. The model states that the expected return on an asset is a function of the risk-free rate, the asset's beta, and the market risk premium. Sharpe also developed the Sharpe Ratio, a measure of risk-adjusted return, which helps investors understand how much excess return they are receiving for the extra volatility endured.

"The expected return on a security is linearly related to its beta, reflecting its sensitivity to market movements." (Sharpe, 1964).

#### 3.2.2 Study Implications

Sharpe's CAPM and Sharpe Ratio are crucial for assessing the risk and return profiles of individual securities within the portfolios analyzed in this study. By evaluating the beta and risk-adjusted returns, this study can better understand the performance of different assets and optimize the investment strategies accordingly. The CAPM provides a

systematic approach to estimating the expected returns of assets based on their systematic risk, while the Sharpe Ratio helps in comparing the performance of investments on a risk-adjusted basis.

#### 3.2.3 Critiques

The CAPM has been criticized for its assumption of a single-period investment horizon and the use of a single market index to represent the entire market. Additionally, the model's reliance on beta as the sole measure of risk is a problem, as it does not account for other factors that may influence asset returns. Furthermore, the CAPM's assumption of a frictionless market and homogeneous expectations among investors may not hold true in real-world scenarios.

# 3.3 Options: A Monte Carlo Approach by Phelim P. Boyle (1977)

#### 3.3.1 Key Takeaways

Phelim P. Boyle's paper introduced the use of Monte Carlo methods for option pricing, providing a robust tool to model the uncertainty and variability in financial investments. Monte Carlo simulations generate a large number of random samples from the probability distributions of asset returns, allowing investors to assess the impact of risk and uncertainty on their portfolios. In the paper, Boyle describes the Monte Carlo simulation process which involves generating random returns based on historical data and iterating this process to build a distribution of potential outcomes. He states that by running multiple simulations, investors can estimate the expected value and variability of the investment portfolio, providing insights into the likelihood of achieving their financial goals.

"Monte Carlo simulations enable the modeling of complex financial instruments and the assessment of risk and return in uncertain environments." (Boyle, 1977).

#### 3.3.2 Study Implications

Boyle's Monte Carlo methods are integral to this study for forecasting the performance of optimized portfolios under various market conditions. By simulating different scenarios, this study aims to estimate the range of potential outcomes and assess the likelihood of achieving the desired down payment amount within the specified time horizon. The use of Monte Carlo simulations allows for a more comprehensive analysis of the potential risks and returns associated with different investment strategies, providing valuable insights for first-time homebuyers.

#### 3.3.3 Critiques

One critique of Monte Carlo simulations is their computational intensity, which can be demanding for large-scale applications. Additionally, the accuracy of the results is heavily dependent on the quality of the input data and the assumptions made about the probability distributions of asset returns. Monte Carlo methods also never fully capture the complexities of financial markets, such as changing market conditions and behavioral factors. Moreover, the reliance on historical data may not accurately predict future market behavior, leading to potential misestimations of risk and return.

# 4 Theoretical Models

# 4.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a fundamental tool in finance used to determine the expected return on an asset based on its systematic risk, as measured by beta  $(\beta_i)$ . The CAPM formula is:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \tag{1}$$

where:

- $E(R_i)$  is the expected return on asset *i*.
- $R_f$  is the risk-free rate of return.
- $\beta_i$  is the beta of asset *i*, representing its sensitivity to market movements.
- $E(R_m)$  is the expected return of the market.
- $(E(R_m) R_f)$  is the market risk premium.

#### 4.1.1 Beta Calculation

Beta  $(\beta_i)$  measures the volatility of an asset in relation to the market. It is calculated as:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} \tag{2}$$

where:

- $Cov(R_i, R_m)$  is the covariance between the return of asset i and the return of the market.
- $\sigma_m^2$  is the variance of the market return.

#### 4.1.2 Security Market Line (SML)

The Security Market Line (SML) is a graphical representation of the CAPM, showcasing the relationship between the expected return of an asset and its systematic risk, as measured by beta ( $\beta_i$ ).

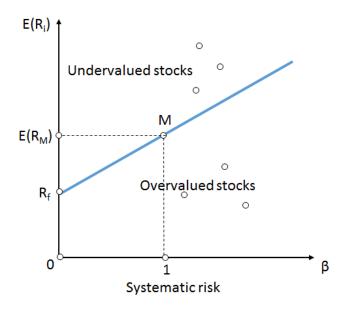


Figure 1: Security Market Line (Wikipedia contributors, 2024)

(a) The Security Market Line (SML) plots the expected return of an asset against its beta, showing that higher systematic risk (beta) correlates with higher expected returns. Correctly priced securities should lie on the SML. The slope represents the market risk premium  $(E(R_m) - R_f)$ , while the intercept is the risk-free rate  $(R_f)$ .

### 4.2 Sharpe Ratio

The Sharpe Ratio is a measure of risk-adjusted return, providing a way to compare the performance of investments while considering the risk taken. A higher Sharpe Ratio indicates better risk-adjusted performance, meaning the investment provides higher returns for each unit of risk taken. It is calculated as follows:

Sharpe Ratio = 
$$\frac{E(R_i) - R_f}{\sigma_i}$$
 (3)

where:

- $E(R_i)$  is the expected return of the investment.
- $R_f$  is the risk-free rate.
- $\sigma_i$  is the standard deviation of the investment's return.

# 4.3 Composite Score Calculation

The Composite Score integrates multiple metrics to rank securities. Weights are assigned to each metric to reflect their importance in the ranking process. It is calculated as follows:

Composite Score = 
$$w_{\beta}\beta + w_{\text{Sharpe}}$$
Sharpe Ratio +  $w_{\text{CAPM}}E(R_i) + w_{\text{Actual}}$ Actual Returns
(4)

where:

- $w_{\beta}$  is the weight assigned to the beta.
- $w_{\text{Sharpe}}$  is the weight assigned to the Sharpe Ratio.
- ullet  $w_{\mathrm{CAPM}}$  is the weight assigned to the CAPM predicted return.
- ullet  $w_{
  m Actual}$  is the weight assigned to the actual returns.

# 4.4 Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) provides a robust framework for constructing an optimal portfolio that maximizes expected return for a given level of risk. The expected return  $E(R_p)$  of a portfolio is the weighted sum of the expected returns of the individual assets:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{5}$$

where:

- $E(R_p)$  is the expected return of the portfolio.
- $w_i$  are the weights of the individual assets in the portfolio.
- $E(R_i)$  is the expected return of asset *i*.

#### 4.4.1 Efficient Frontier and Optimal Portfolio

The efficient frontier is a concept from MPT that represents the set of optimal portfolios offering the highest expected return for a defined level of risk. The process of constructing the efficient frontier involves solving the following optimization problem:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{6}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1 \tag{7}$$

and

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{8}$$

where:

- $\sigma_{ij}$  is the covariance between the returns of assets *i* and *j*.
- $w_i$  and  $w_j$  are the weights of assets i and j in the portfolio.

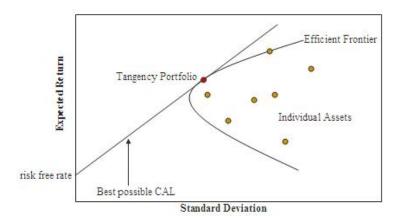


Figure 2: Efficient Frontier

(a) This figure shows the efficient frontier, illustrating optimal portfolios that offer the highest expected return for a given level of risk. Portfolios below the efficient frontier are sub-optimal as they do not provide sufficient return for the risk taken.

#### 4.5 Monte Carlo Simulation

Monte Carlo simulations are utilized to model the uncertainty and variability in investment returns over time. The simulation process involves generating random returns based on historical data and iterating this process to build a distribution of potential outcomes. By running multiple simulations, we can estimate the expected value and variability of the investment portfolio, providing insights into the likelihood of achieving the desired down payment amount within the specified time horizon. The value of an investment at time i is given by:

$$X_i = X_{i-1} \times (1 + r_i) \tag{9}$$

where:

•  $X_i$  is the investment value at time i.

- $X_{i-1}$  is the investment value at time i-1.
- $r_i$  is the return for period i.

#### **Cumulative Returns of Monte Carlo simulations and The Original Strategy**

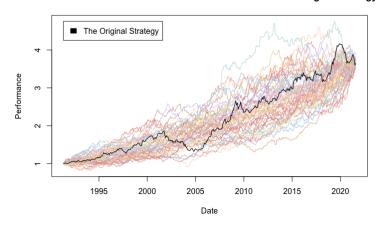


Figure 3: Investment Simulation Process for Monte Carlo Analysis

(a) The diagram illustrates the generation of random returns used to project the future value of an investment over multiple iterations, creating a range of possible outcomes to understand potential risks and returns.

# 5 Empirical Specification

# 5.1 Model Implementation

The empirical analysis involves a systematic approach to evaluating and optimizing investment strategies using historical data. The data is cleaned and adjusted for corporate actions like stock splits and dividends using the yfinance\_data.py script to ensure consistency, resulting in the creation of a file named yfinance\_data.csv.

#### 5.1.1 CAPM and Sharpe Ratio Calculation

Each security's risk and return profile is assessed using the Capital Asset Pricing Model (CAPM) and Sharpe Ratio. This analysis is performed using the init\_filtering.py script.

- 1. Calculating the average return of the market index (e.g., S&P 500).
- 2. Determining the risk-free rate (e.g., 10-year U.S. Treasury bonds yield).
- 3. Computing the beta ( $\beta_i$ ) of each security:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \tag{10}$$

4. Estimating the expected return using the CAPM formula:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f)$$
(11)

5. Calculating the Sharpe Ratio:

Sharpe Ratio = 
$$\frac{E(R_i) - R_f}{\sigma_i}$$
 (12)

#### 5.1.2 Composite Score Calculation

Securities are ranked based on a composite score that integrates multiple metrics:

Composite Score = 
$$w_{\beta}\beta + w_{\text{Sharpe}}$$
Sharpe Ratio +  $w_{\text{CAPM}}E(R_i) + w_{\text{Actual}}$ Actual Returns
(13)

Weights are assigned to each metric to reflect their importance in the ranking process. A file is then produced for each time horizon with the ranked securities in order, named top\_assets\_composite\_score.csv.

#### 5.1.3 Modern Portfolio Theory (MPT) Application

Using the ranked securities, portfolios are optimized for different investment horizons (5, 7.5, and 10 years) using Modern Portfolio Theory (MPT). The optimization involves solving the following quadratic programming problem:

Maximize 
$$\mathbf{w}^{T-} - \frac{\lambda}{2} \mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$$
 (14)

subject to 
$$\sum_{i} w_i = 1$$
 (15)

$$w_i \ge 0 \quad \forall i$$
 (16)

where  $\mathbf{w}$  is the vector of asset weights,  $\bar{\phantom{a}}$  is the vector of expected returns,  $\Sigma$  is the covariance matrix of returns, and  $\lambda$  is the risk aversion parameter. This process is implemented using the mpt .py script, which produces a data file for each time horizon named optimal\_weights.csv.

#### 5.1.4 Performance Comparison

The optimized portfolios are compared against actual historical performance using the hindsight data created by hindsight\_data.py. The comparison includes calculating

the cumulative returns of the portfolios and benchmarking them against the S&P 500 index. This step is performed using the hindsight.py script to create a data file named hindsight\_data.csv.

#### 5.1.5 Monte Carlo Simulation for Future Forecasting

Monte Carlo simulations are conducted to forecast the performance of the optimized portfolios. This comprehensive simulation is implemented using the mcs.py script, providing insights into the potential future performance of the portfolios.

- 1. Defining initial investment amounts and annual contributions.
- 2. Generating random returns based on historical distributions.
- 3. Calculating portfolio values at each time step:

$$V_t = V_{t-1} \times (1 + R_t) + C \tag{17}$$

- 4. Running multiple iterations to build a probability distribution of outcomes.
- 5. Applying economic shocks to simulate real-world scenarios:

$$V_t = V_t \times (1 + \text{Shock Intensity}) \tag{18}$$

# 6 Results

# 6.1 Exploratory Data Analysis

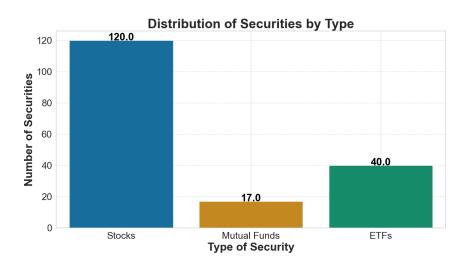


Figure 4: Distribution of Securities by Type

(a) This figure illustrates the distribution of different types of securities within the dataset. It shows that stocks make up the majority with 120 securities, followed by ETFs with 40, and mutual funds with 17.

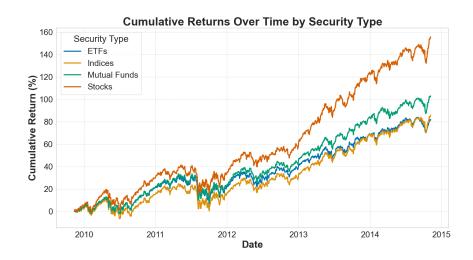


Figure 5: Cumulative Returns Over Time by Security Type

(a) Stocks exhibit the highest cumulative return over time, indicating a higher potential for long-term growth compared to ETFs and mutual funds. ETFs and mutual funds also show consistent returns, highlighting their role in diversified investment strategies.

# 6.2 Assets by Composite Score

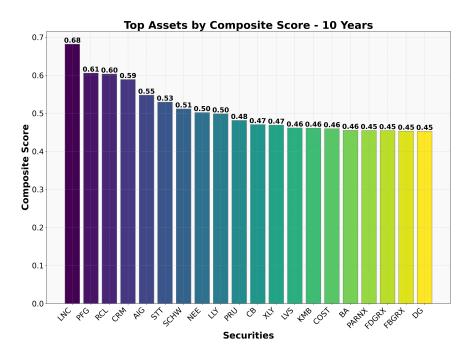


Figure 6: Top Assets by Composite Score (10 Years)

(a) LNC, PFG, and RCL lead the rankings for the 10-year horizon, demonstrating their strong and consistent performance over a decade.

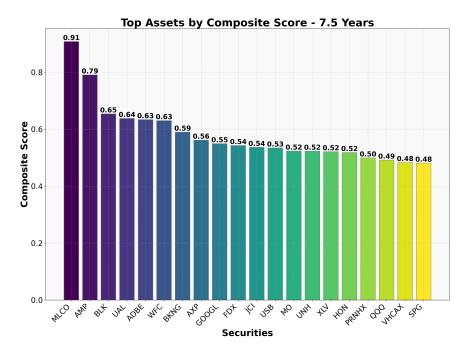


Figure 7: Top Assets by Composite Score (7.5 Years)

(a) Over a 7.5-year horizon, MLCO, AMP, and BLK emerge as top assets, showcasing robust performance and highlighting their resilience in the market.

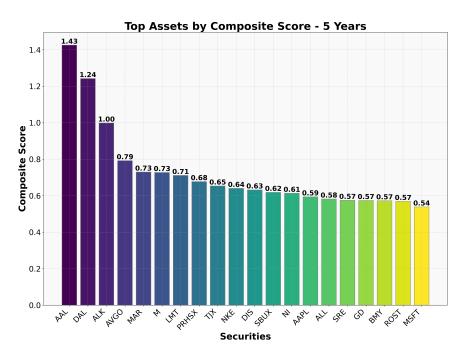


Figure 8: Top Assets by Composite Score (5 Years)

(a) The top-performing assets over a 5-year horizon, ranked by composite score, include AAL, DAL, and ALK, reflecting a strong performance in the airline industry during this period.

# 6.3 Optimal Portfolios with Weights

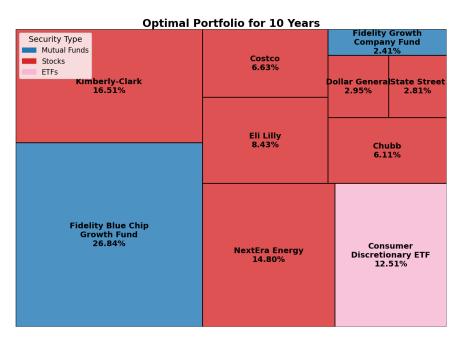


Figure 9: Optimal Portfolio for 10 Years

(a) This figure illustrates the optimal portfolio allocation for a 10-year investment horizon, showing the proportion of stocks, mutual funds, and ETFs. The portfolio is designed to maximize returns while managing risk over the long term.

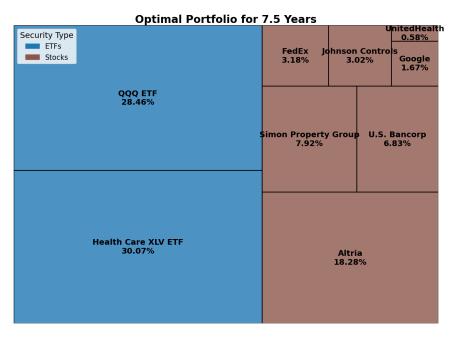


Figure 10: Optimal Portfolio for 7.5 Years

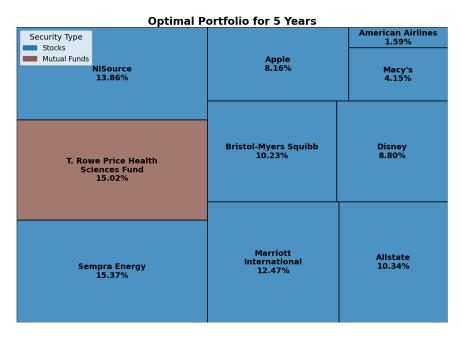


Figure 11: Optimal Portfolio for 5 Years

# 6.4 Comparison with Hindsight Data

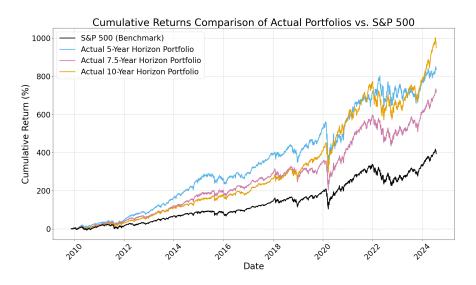


Figure 12: Cumulative Returns Comparison of Actual Portfolios vs. S&P 500

(a) This figure compares the cumulative returns of actual portfolios with the S&P 500 benchmark. The results show that the actual portfolios for 5, 7.5, and 10-year horizons consistently outperform the S&P 500, highlighting the effectiveness of the optimization strategy.

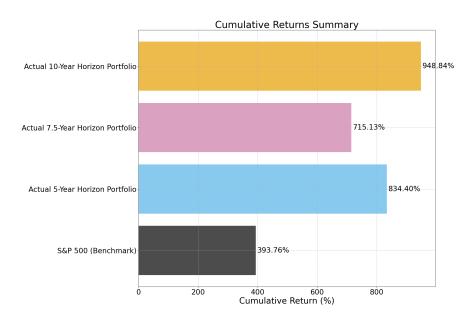


Figure 13: Cumulative Returns Summary

(a) This summary illustrates the cumulative returns for the 5, 7.5, and 10-year horizon portfolios compared to the S&P 500 benchmark.

# 6.5 Forecasting Results Using Monte Carlo

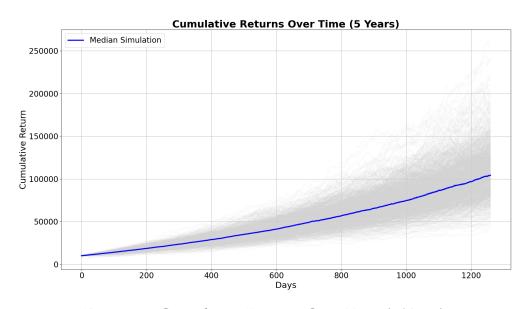


Figure 14: Cumulative Returns Over Time (5 Years)

(a) This figure shows the cumulative returns over a 5-year period. The blue line represents the median simulation of portfolio returns, with the gray lines indicating the range of possible outcomes from the Monte Carlo simulations.

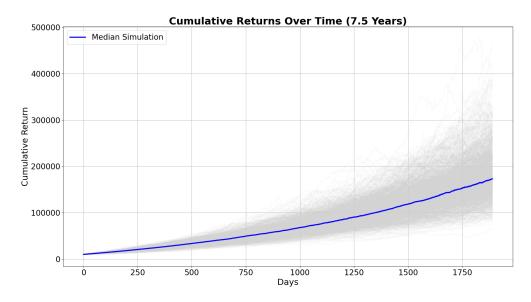


Figure 15: Cumulative Returns Over Time (7.5 Years)

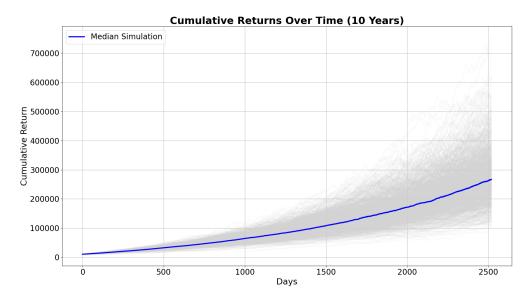


Figure 16: Cumulative Returns Over Time (10 Years)

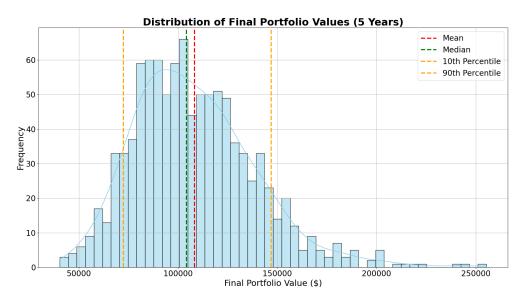


Figure 17: Distribution of Final Portfolio Values (5 Years)

(a) The histogram displays the distribution of final portfolio values after a 5-year investment horizon. Vertical lines indicate the mean, median, 10th percentile, and 90th percentile values.

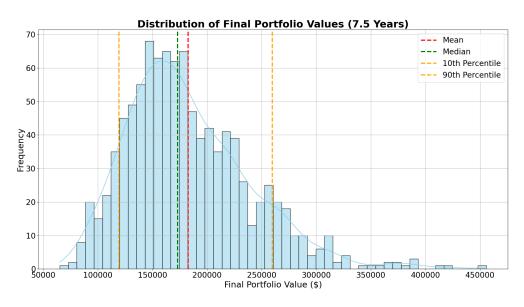


Figure 18: Distribution of Final Portfolio Values (7.5 Years)

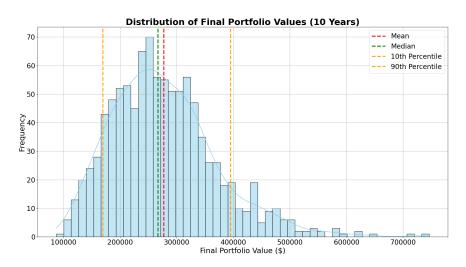


Figure 19: Distribution of Final Portfolio Values (10 Years)

#### 6.6 Monte Carlo Forecast Summary

Statistic	Mean Final Portfolio Value (\$)	Median Final Portfolio Value (\$)	10th Percentile Final Portfolio Value (\$)	90th Percentile Final Portfolio Value (\$)	Total Percentage Yield (%)	Annual Percentage Yield (%)
	rornono varue (\$)	r ortiono varue (\$)	r ortiono varue (φ)	r ortiono varue (φ)	11eiu (70)	Heid (70)
10-Year Horizon	283094.44	265864.65	171232.59	420935.51	2730.94	39.70
7.5-Year Horizon	180745.35	173433.88	116972.91	256760.32	1707.45	47.10
5-Year Horizon	101559.42	98327.18	67062.20	139233.79	915.59	58.98

Table 1: Summary statistics of final portfolio values across different investment horizons (5, 7.5, and 10 years). The table includes mean, median, 10th percentile, and 90th percentile final portfolio values, as well as the total and annual percentage yields for each investment horizon.

# 7 Results Summary

The analysis of optimal investment strategies for first-time homebuyers provides several key insights. Stocks consistently outperform ETFs and mutual funds, underscoring their long-term growth potential. This is confirmed by the exploratory data analysis. In the 5-year horizon, top assets include airlines like AAL, DAL, and ALK, indicating strong short-term growth potential. The 7.5-year horizon features a diverse mix of technology, finance, and industrial sectors. In contrast, the 10-year horizon is dominated by financial and industrial stocks such as Fidelity Blue Chip Growth Fund, Kimberly-Clark, and NextEra Energy, showcasing significant long-term growth potential.

Optimal portfolio allocation using Modern Portfolio Theory (MPT) mirrors these trends. The 5-year portfolio balances stocks and mutual funds, favoring T. Rowe Price Health Sciences Fund and Sempra Energy. The 7.5-year portfolio is mainly composed of ETFs containing NASDAQ-100 and healthcare securities, along with key stocks such as Altria. For the 10-year horizon, the portfolio is primarily stocks, with significant holdings in Fidelity Blue Chip Growth Fund, Kimberly-Clark, and NextEra Energy, demonstrating the effectiveness of a long-term strategy.

The cumulative returns comparison indicates that the 5-year horizon portfolio achieves the highest returns, significantly outperforming the S&P 500. The 10-year horizon, while still achieving substantial gains, comes in second place, demonstrating the benefits of long-term investment. The 7.5-year horizon, although yielding considerable returns, ranks third. This highlights the remarkable performance of the 5-year portfolio, even at lower percentiles, showcasing its robustness. The mean final portfolio values further confirm these findings, with the 5-year horizon showing impressive returns, the 10-year horizon reflecting substantial growth potential, and the 7.5-year horizon indicating better performance over a medium-term horizon.

Cumulative returns over time using Monte Carlo Simulation display consistent growth. The 5-year horizon shows a clear upward trend, while the 7.5-year and 10-year horizons demonstrate sustained growth resilient to market fluctuations. The summary statistics showcase the robustness and effectiveness of the optimization strategy across different market conditions and investment periods, underscoring the importance of diverse asset allocation, sectoral diversity, and the benefits of long-term investment for achieving substantial returns.

# 8 Conclusions

This research offers a comprehensive analysis of optimal investment strategies for first-time homebuyers seeking to accumulate down payments over 5, 7.5, and 10-year horizons. By leveraging Modern Portfolio Theory, the Capital Asset Pricing Model, and Monte Carlo simulations, the study provides valuable insights into constructing age-specific investment portfolios.

The analysis of summary statistics highlights the varying levels of risk and return associated with the selected securities, which are crucial for effective portfolio optimization. The calculated optimal weights for different time horizons suggest that a well-diversified portfolio, tailored to market conditions, can significantly enhance investment outcomes. The comparison with hindsight data demonstrates that optimized portfolios consistently outperform the S&P 500 benchmark. This underscores the benefits of long-term investing and strategic asset allocation, emphasizing the importance of considering risk-adjusted returns and diversification in portfolio construction.

The composite score rankings offer a systematic approach to identifying top-performing securities by integrating key financial metrics. This methodology assists investors in making informed decisions, aligning their portfolios with their financial goals and risk tolerance. Monte Carlo simulations further validate the robustness of the optimized portfolios by forecasting their potential future performance under various economic scenarios. These simulations demonstrate the resilience of the portfolios, showcasing their capacity to withstand market volatility and economic shocks. Overall, the findings suggest that tailored investment strategies significantly improve the ability to save for a down payment, thereby reducing the time required to reach homeownership goals. Future research could expand this analysis by considering other demographic factors such as income levels and regional housing market conditions. Additionally, incorporating alternative investment vehicles such as real estate investment trusts (REITs) and cryptocurrencies could provide further insights into optimizing investment strategies for down payments.

# Acknowledgments

I would like to express my gratitude to my professor, Dr. Paarsch, for his invaluable mentorship and guidance throughout my graduate studies. I am also deeply thankful to my family for their unwavering support and encouragement during my academic journey at the University of Central Florida.

# A Data Appendix

# A.1 Summary of Data Sources

The financial data utilized in this study were obtained from Yahoo Finance, which provides comprehensive information on a range of securities including stocks, mutual funds, and ETFs (Yahoo Finance, nd).

#### A.2 Variable Definitions

The key variables used in the analysis are defined as follows:

- **Open**: The price at the beginning of the trading day.
- **High**: The highest price during the trading day.
- **Low**: The lowest price during the trading day.
- Close: The price at the end of the trading day.
- **Adj Close**: The closing price adjusted for dividends, stock splits, etc.
- **Volume**: The number of shares traded during the trading day.
- **Type**: The type of security (e.g., stock, ETF).
- **Ticker**: The unique symbol assigned to each security for trading purposes.
- **Beta**: A measure of a security's volatility in relation to the overall market. A beta greater than 1 indicates that the security is more volatile than the market, while a beta less than 1 indicates that it is less volatile.
- **CAPM Predicted Return**: The expected return of a security as predicted by the Capital Asset Pricing Model, which takes into account the risk-free rate, the security's beta, and the expected market return.

- **Sharpe Ratio**: A measure of risk-adjusted return, calculated by subtracting the risk-free rate from the security's return and dividing by the standard deviation of the security's return. A higher Sharpe Ratio indicates better risk-adjusted performance.
- **Actual Returns**: The realized return on a security over a specified period, including price appreciation and dividends.
- Composite Score: A combined score that integrates various metrics such as Beta, CAPM Predicted Return, Actual Returns, and Sharpe Ratio to evaluate the overall attractiveness of a security. The composite score is used to rank securities.
- **Rank**: The position of a security in the list based on its composite score, with a lower rank indicating a more attractive investment.

# References

Boyle, P. P. (1977). Options: A monte carlo approach. *Journal of Financial Economics* 4(3), 323–338.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance* 7(1), 77–91.

National Association of Realtors (2023). 2023 home buyers and sellers generational trends report.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance* 19(3), 425–442.

Wikipedia contributors (2024). Security Market Line — Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/wiki/Security\_market\_line. [Online; accessed 23-July-2024].

Yahoo Finance (n.d.). Yahoo finance.