An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

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Overview

Introduction

Objective

Develop optimal investment strategies for first-time homebuyers to save for down payments.

Research Question

What are the most effective strategies for different age groups to save for down payments over 5, 10, and 15 years?

Motivation

Address the challenges posed by rising housing costs and assist diverse age groups in accelerating their path to homeownership.

Essential Financial Concepts

Stocks

Equity investments representing ownership in a company.

Mutual Funds

Investment vehicles that pool money from many investors to purchase a diversified portfolio of stocks, bonds, or other securities.

ETFs (Exchange-Traded Funds)

Similar to mutual funds but traded on stock exchanges like individual stocks.

Typical First-time Homebuyer Profile

Demographics

- **Average Age:** 35 years (2023)
- **Median Income:** $95,900 (2023)

Marital Status

- 59% Married Couples
- 19% Single Females
- 10% Single Males
- 9% Unmarried Couples

Financials

- **Average Home Cost:** $348,000 (2022)
- **Down Payment Saved:** $8,220

Investment Contributions by Age Group

Data Source

Bureau of Labor Statistics (BLS), Federal Reserve

Annual Income and Contributions

- **20-25 years:**
  - Median income: $45,000
  - Annual contribution: 10% of income
- **25-30 years:**
  - Median income: $60,000
  - Annual contribution: 15% of income
- **30-35 years:**
  - Median income: $80,000
  - Annual contribution: 20% of income

Data Sources

Primary Source

Yahoo Finance (YFinance)

- Comprehensive financial data on stocks, mutual funds, and ETFs.

Data Coverage

Date Range: 10/24/2012 to present (daily frequency)

Data Snapshot

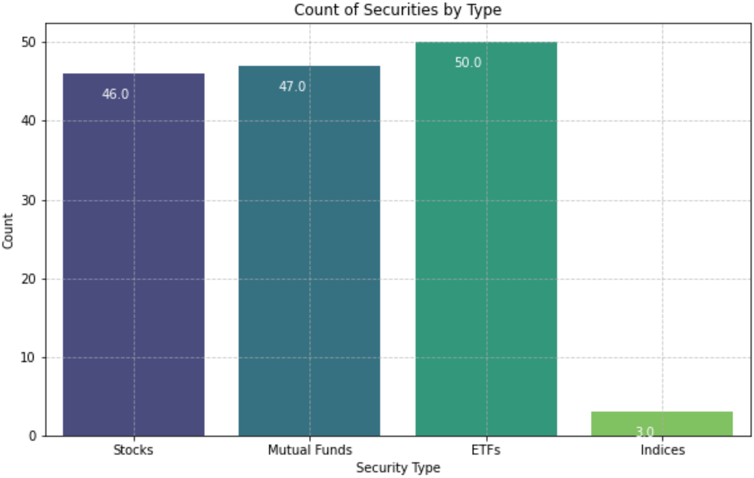
Date	Ticker	Open	High	Low	Close	Adj Close	Volume	Type	Daily Return
2012-10-24 00:00:00	AAPL	22.194286346435547	22.376785278320312	21.808570861816406	22.02964210510254	18.70499038696289	558527200.0	Stocks	nan
2012-10-24 00:00:00	ADBE	33.560001373291016	33.599998474121094	33.279998779296875	33.36000061035156	33.36000061035156	3381500.0	Stocks	nan
2012-10-24 00:00:00	AGTHX	33.29999923706055	33.29999923706055	33.29999923706055	33.29999923706055	14.800816535949707	0.0	Mutual Funds	nan
2012-10-24 00:00:00	AMD	2.1700000762939453	2.2200000286102295	2.059999942779541	2.0799999237060547	2.0799999237060547	37324400.0	Stocks	nan
2012-10-24 00:00:00	AMZN	11.793999671936035	11.79699993133545	11.397500038146973	11.424500465393066	11.424500465393066	73574000.0	Stocks	nan
2012-10-24 00:00:00	AVGO	33.529998779296875	33.59000015258789	32.59000015258789	32.72999954223633	24.057003021240234	1352200.0	Stocks	nan
2012-10-24 00:00:00	AXP	55.61000061035156	55.95000076293945	55.08000183105469	55.22999954223633	46.850711822509766	5726800.0	Stocks	nan
2012-10-24 00:00:00	BAC	9.449999809265137	9.489999771118164	9.300000190734863	9.3100004196167	7.564661502838135	120868300.0	Stocks	nan
2012-10-24 00:00:00	BLK	188.3000030517578	190.27999877929688	186.3300018310547	186.99000549316406	138.58750915527344	748900.0	Stocks	nan
2012-10-24 00:00:00	BMY	32.849998474121094	33.209999084472656	32.70000076293945	33.04999923706055	23.345792770385742	12524100.0	Stocks	nan

Data Import and Preparation

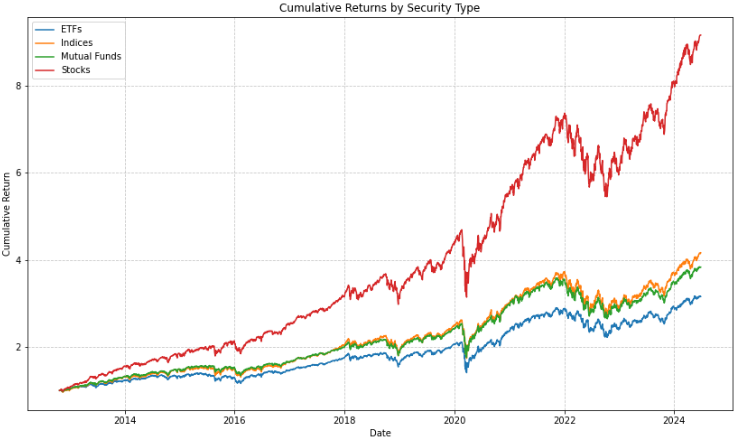
Loading Data

```
# import yfinance as yf

# Download data from Yahoo Finance
tickers = ['AAPL', 'MSFT', 'GOOG', 'AMZN',.....]
data = yf.download(tickers, start='2012-10-24',
end='2024-06-26')
# Save data to CSV
data.to_csv('yfinance_data.csv')
# Load data from CSV
data = pd.read_csv('yfinance_data.csv', index_col='Date')
```

Visualizing Security Count



Visualizing Returns Distribution



Capital Asset Pricing Model (CAPM)

Formula

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

[a]

[a] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), 425-442.

Assumptions

- Diversified portfolios
- Efficient markets
- No taxes or transaction costs
- Constant risk-free rate

Calculating Beta

Market Return Calculation

```
# Market return (e.g., S&P 500)
market = yf.download('^GSPC', start='2012-10-24',
end='2024-09-26')
market_returns = market['Adj Close'].pct_change().dropna()

# Calculate beta for each stock
beta = {}
for ticker in tickers:
    cov_matrix = np.cov(returns[ticker], market_returns)
    beta[ticker] = cov_matrix[0, 1] / cov_matrix[1, 1]
```

Calculating Expected Return using CAPM

Expected Return Calculation

```
# Assuming a risk-free rate of 3% and market return of 8%
risk_free_rate = 0.03
market_return = 0.08

# Calculate expected return for each stock
capm_returns = {}
for ticker in tickers:
    capm_returns[ticker] = risk_free_rate + beta[ticker] *
    (market_return - risk-free-rate)
```

Sharpe Ratio

Purpose

Measures investment performance adjusted for risk.

Formula

$$S = \frac{E(R_i) - R_f}{\sigma_i}$$

Components

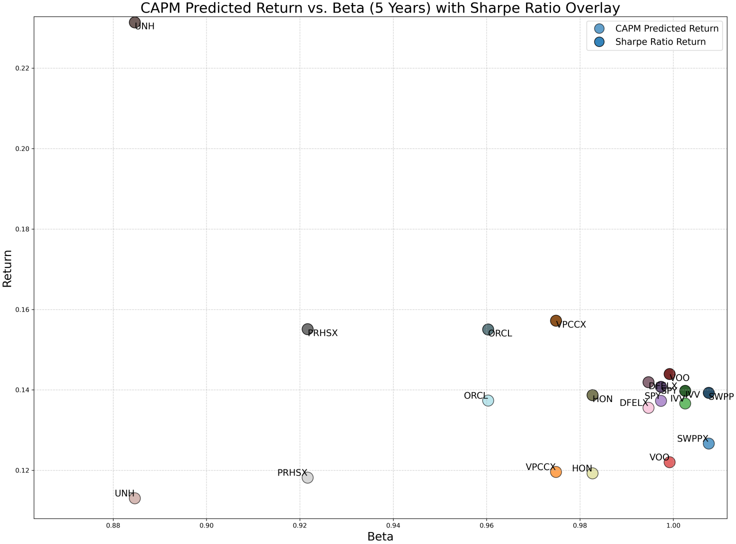
- $E(R_i)$: Expected return
- R_f: Risk-free rate
- σ_i: Std. deviation of excess return

Calculating the Sharpe Ratio

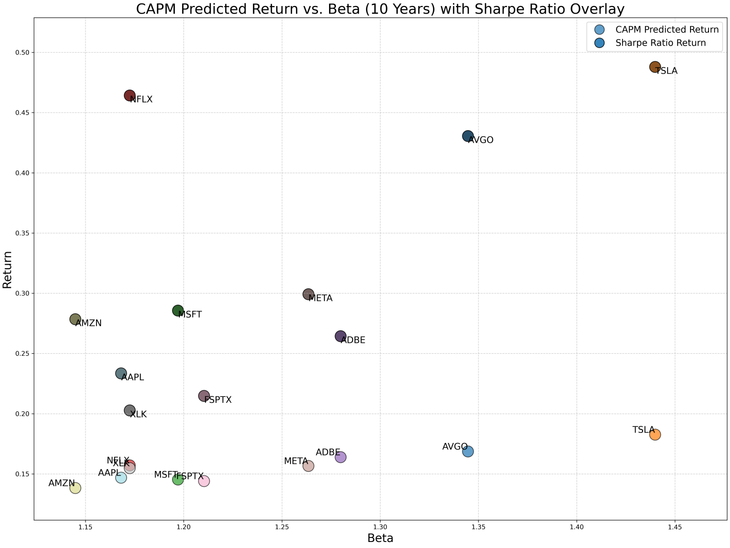
Formula and Calculation

```
# Sharpe Ratio Calculation
def sharpe_ratio(expected_return, risk_free_rate, stddev):
    return (expected_return - risk_free_rate) / stddev

# Example Calculation
expected_return = 0.0675  # Example from CAPM
risk_free_rate = 0.03
stddev = 0.1  # Example standard deviation

sharpe = sharpe_ratio(expected_return, risk_free_rate,
stddev)
print(f'Sharpe Ratio: {sharpe}')
```

Security Market Line (SML)



Security Market Line (SML)



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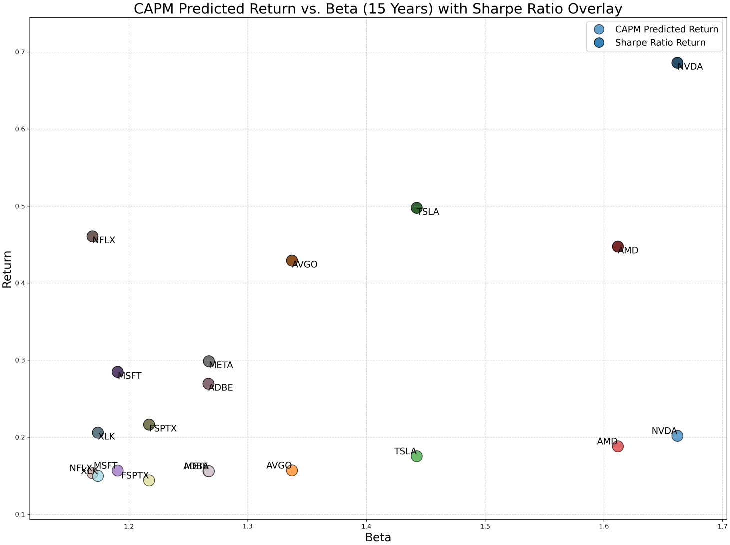


Modern Portfolio Theory (MPT)

Overview

Framework for constructing a portfolio to maximize return for a given level of risk.[a]

[a] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.

Formulas

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Definitions

- $E(R_p)$: Portfolio return
- w_i: Weight of asset i
- $E(R_i)$: Return of asset i
- σ_p^2: Portfolio variance
- σ_{ij}: Covariance of assets i, j

Optimize the Portfolio

Objective

Adjust the weights of the assets to maximize the portfolio's expected return for a given level of risk or to minimize risk for a given level of expected return.

Optimization Problem

Solve the following optimization problem:

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Subject to:

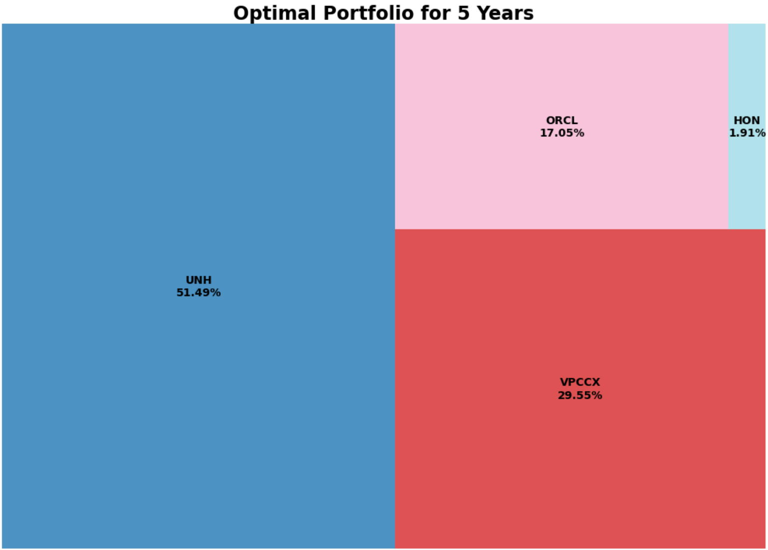
$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Defining the Optimization Function

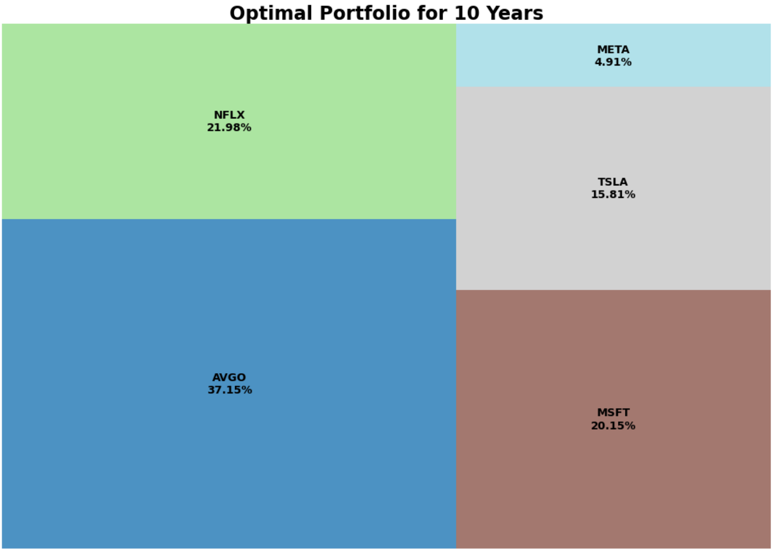
Sharpe Ratio Optimization

```
# Define the objective function to minimize
def sharpe_ratio(weights, returns, risk_free_rate=0.03):
    portfolio_return = np.sum(returns.mean() * weights) * 252
    portfolio_stddev = np.sqrt(np.dot(weights.T,
                                      np.dot(returns.cov() * 252, weights)))
    return - (portfolio_return - risk_free_rate) / portfolio_stddev

# Initial guess for weights
num_assets = len(tickers)
init_guess = num_assets * [1. / num_assets,]

# Constraints and bounds
constraints = ({'type': 'eq', 'fun': lambda weights: np.sum(weights) - 1})
bounds = tuple((0, 1) for asset in range(num_assets))

# Optimize
opt_results = minimize(sharpe_ratio, init_guess,
                       args=(returns,), method='SLSQP', bounds=bounds,
                       constraints=constraints)
```

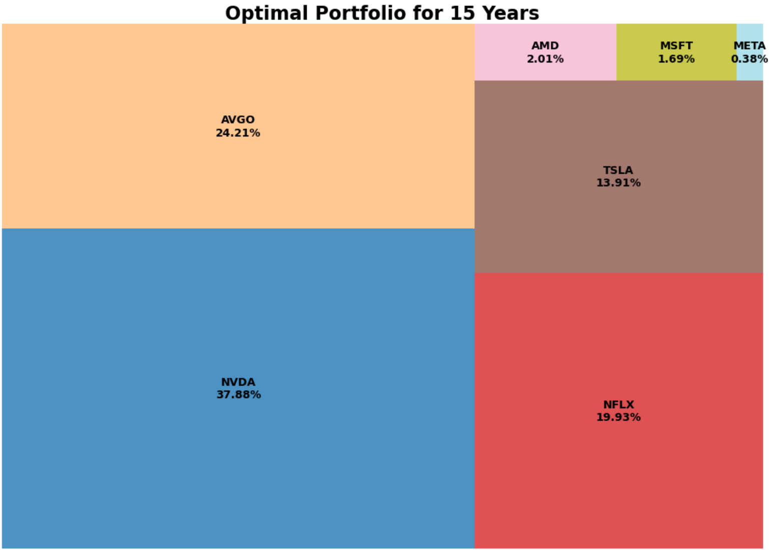
Optimal Portfolio (5 Years)



Optimal Portfolio (10 Years)



Optimal Portfolio (15 Years)



Results Interpretation

Findings

Through this analysis using the Capital Asset Pricing Model (CAPM) alongside the Sharpe Ratio for calculating risk-adjusted return, and Modern Portfolio Theory (MPT), we successfully identified the optimal weights for the selected securities. The analysis revealed that diversification across different asset classes and strategic allocation can significantly enhance the potential for accumulating sufficient down payments over varying time horizons.

Implications

These findings underscore the importance of tailored investment strategies for different age groups and income levels. By leveraging these models, first-time homebuyers can make informed decisions that align with their financial goals and risk tolerance.

Future Work and Limitations

Future Work

In future research, I plan to incorporate Monte Carlo Simulation to forecast future trends for the optimized portfolio using Modern Portfolio Theory. This will provide a more robust understanding of potential investment outcomes under various market conditions.

Limitations

This study's limitations include the assumption of constant risk-free rates and market returns, as well as the exclusion of transaction costs and taxes. Future work should aim to address these limitations to enhance the model's accuracy and applicability.

Q&A

Questions and Clarifications

Please feel free to ask for any clarifications or additional details regarding the presented research and findings.

Thank you for your attention!

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