An Economic Analysis of Optimal Investment Strategies for Accumulating Housing Down Payments

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Agenda

Introduction

- **Objective:** To develop tailored investment strategies for different age groups of first-time homebuyers to accumulate funds for down payments.
- **Motivation:** Rising housing costs present significant challenges for many first-time homebuyers, particularly younger individuals. This research aims to identify optimal investment strategies to help people in various age groups save effectively for a down payment, thereby accelerating their path to homeownership.

Typical First-time Homebuyer Profile

- **Average Age:** 35 years (as of 2023)
- **Median Income:** $95,900 (as of 2023)
- **Marital Status:**
  - 59% Married Couples
  - 19% Single Females
  - 10% Single Males
  - 9% Unmarried Couples
- **Challenges:**
  - High student loan debt (national average: $37,172 in 2023)
  - Rising housing costs exceeding income growth
  - Difficulty saving for a down payment
- **Average Home Cost:** $348,000 (2022 data)
- **Down Payment:**
  - **Average Saved:** $8,220 (6% of average home price in 2023)

Data Sources Utilized

- **Yahoo Finance (YFinance):**
  - Comprehensive financial data on stocks, cryptocurrency, mutual funds, and ETFs.

Overview of Financial Data from Yahoo Finance

- **Date Range:** 9/7/2014 to present (daily frequency)
- **Data Fields:**
  - **Open:** Price at the beginning of the trading day
  - **High:** Peak price during the trading day
  - **Low:** Lowest price during the trading day
  - **Close:** Price at the end of the trading day
  - **Adj Close:** Closing price adjusted for dividends, stock splits, etc.
  - **Volume:** Number of shares traded during a single trading day
  - **Type:** Security type

Essential Financial Concepts

- **Stocks:**
  - Equity investments representing ownership in a company.
  - **Example:** Purchasing shares of Apple Inc. (AAPL).
- **Cryptocurrency:**
  - Digital or virtual currencies that use cryptography for security.
  - **Example:** Bitcoin (BTC) and Ethereum (ETH).

Essential Financial Concepts (cont.)

- **Mutual Funds:**
  - Investment vehicles that pool money from many investors to purchase a diversified portfolio of stocks, bonds, or other securities.
  - Managed by professional fund managers.
  - **Example:** Vanguard 500 Index Fund.
- **ETFs (Exchange-Traded Funds):**
  - Similar to mutual funds but traded on stock exchanges like individual stocks.
  - Provide diversification and are typically more cost-effective.
  - **Example:** SPDR S&P 500 ETF (SPY).

Understanding Time Value of Money (TVM)

Objective

To understand the concept that money available today is worth more than the same amount in the future due to its potential earning capacity.

Key Concepts

- **Present Value (PV):** The current value of a future sum of money.
- **Future Value (FV):** The value of a current sum of money at a future date, based on an assumed rate of growth.
- **Discount Rate:** The rate used to discount future cash flows to their present value.

Time Value of Money (TVM) - Formulas

Future Value Formula

$$FV = PV \times [1 + (i/n)]^{n \times t}$$

where:

- FV = Future Value
- PV = Present Value
- i = Interest rate per period
- n = Number of compounding periods per year
- t = Number of years

Explanation

This formula calculates the amount of money that an investment will grow to over a period of time when interest is compounded periodically.

Time Value of Money (TVM) - Present Value Formula

Present Value Formula

$$PV = \frac{FV}{[1 + (i/n)]^{n \times t}}$$

where:

- PV = Present Value
- FV = Future Value
- i = Interest rate per period
- n = Number of compounding periods per year
- t = Number of years

Explanation

This formula determines the current worth of a sum of money to be received in the future, given a specific interest rate and compounding frequency.

Time Value of Money (TVM) - Continuous Compounding

Continuous Compounding Formula

$$FV = PV \times e^{rt}$$

where:

- FV = Future Value
- PV = Present Value
- e = Euler's number (approximately 2.71828)
- r = Annual interest rate
- t = Time in years

Explanation

This formula is used when interest is compounded continuously, as opposed to periodically.

Modern Portfolio Theory (MPT) - Overview

Overview

Modern Portfolio Theory (MPT), introduced by Harry Markowitz, is a framework for constructing a portfolio of assets such that the expected return is maximized for a given level of risk. It emphasizes diversification to reduce risk.

Modern Portfolio Theory (MPT) - Key Assumptions

Key Assumptions

- Investors are rational and risk-averse.
- Markets are efficient, and all investors have access to the same information.
- Asset returns are normally distributed.
- There are no transaction costs or taxes.

Step 1: Define Assets and Expected Returns

Expected Returns

Identify the assets you want to include in the portfolio and estimate their expected returns ($E(R_i)$). This involves analyzing historical data, considering economic conditions, and using financial models.

$$E(R_i) = \text{Expected return of asset } i$$

Example:

- Asset A: $E(R_A) = 10\%$
- Asset B: $E(R_B) = 15\%$

Step 2: Determine Asset Weights

Weights

Decide on the proportion (w_i) of the total investment to allocate to each asset. The sum of the weights should equal 1.

$$w_i = \text{Weight of asset } i$$

Example:

- Weight of Asset A: $w_A = 60\%$
- Weight of Asset B: $w_B = 40\%$

Step 3: Calculate Portfolio's Expected Return

Expected Return of Portfolio

The expected return of the portfolio ($E(R_p)$) is the weighted sum of the expected returns of the individual assets.

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

where:

- $E(R_p)$ = Expected return of the portfolio
- w_i = Weight of asset i in the portfolio
- $E(R_i)$ = Expected return of asset i

Example Calculation:

$$E(R_p) = (0.60 \times 0.10) + (0.40 \times 0.15) = 0.12 \text{ or } 12\%$$

Step 4: Calculate Covariances Between Assets

Covariance

Covariance measures how two assets move together. A positive covariance means that the assets tend to move in the same direction, while a negative covariance means they move in opposite directions.

$$\sigma_{ij} = \mathrm{Cov}(R_i, R_j) = \mathbb{E}[(R_i - \mathbb{E}[R_i])(R_j - \mathbb{E}[R_j])]$$

Example:

- Covariance between Asset A and Asset B: $\sigma_{AB} = 0.02$

Step 5: Calculate Portfolio's Variance (Risk)

Variance of Portfolio

The variance (σ_p^2) of the portfolio's return is determined by the variances of the individual assets and the covariances between them.

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

where:

- σ_p^2 = Variance of the portfolio's return
- w_i = Weight of asset i in the portfolio
- σ_{ij} = Covariance between asset i and asset j

Example Calculation:

$$\sigma_p^2 = (0.60)^2 \times 0.04 + (0.40)^2 \times 0.09 + 2 \times 0.60 \times 0.40 \times 0.02 = 0.0384$$

Step 6: Calculate Correlation Between Assets

Correlation

Correlation is a standardized measure of covariance that ranges from -1 to 1.

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

where:

- ρ_{ij} = Correlation coefficient between asset i and asset j
- σ_{ij} = Covariance between asset i and asset j
- σ_i = Standard deviation of asset i
- σ_j = Standard deviation of asset j

Example:

- Correlation between Asset A and Asset B: $\rho_{AB} = \frac{0.02}{\sqrt{0.04} \cdot \sqrt{0.09}} = 0.333$

Step 7: Optimize the Portfolio

Optimization

Adjust the weights of the assets to maximize the portfolio's expected return for a given level of risk or to minimize risk for a given level of expected return. This is done by solving the optimization problem:

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Example:

- Adjust weights w_i to find the optimal portfolio that minimizes risk for a given expected return.

Step 8: Construct the Efficient Frontier

Efficient Frontier

The efficient frontier represents the set of optimal portfolios that offer the highest expected return for a given level of risk. By solving the optimization problem repeatedly for different levels of expected return, you can plot the efficient frontier.

- Plot the portfolios on a graph with risk (standard deviation) on the x-axis and expected return on the y-axis.

Example:

- The curve shows the optimal portfolios, with points above the curve being unachievable and points below the curve being inefficient.

Transition from MPT to CAPM

From MPT to CAPM

While Modern Portfolio Theory (MPT) focuses on the construction of optimal portfolios based on diversification and risk-return trade-offs, the Capital Asset Pricing Model (CAPM) builds on this by explaining the relationship between systematic risk and expected return for individual assets.

- MPT provides the foundation for understanding the benefits of diversification.
- CAPM extends this by quantifying the expected return of an asset given its systematic risk.

Capital Asset Pricing Model (CAPM) - Key Assumptions

Key Assumptions

- Investors hold diversified portfolios.
- Markets are efficient, and all investors have access to the same information.
- There are no taxes or transaction costs.
- The risk-free rate is constant.

Step 1: Identify Risk-Free Rate

Risk-Free Rate (R_f)

The risk-free rate is the return on an investment with zero risk, typically represented by government bonds.

$$R_f = \text{Risk-free rate}$$

Example:

- Assume the risk-free rate is 3%.

Step 2: Determine Market Return

Market Return ($E(R_m)$)

The expected return of the market is the average return of the market portfolio, which includes all investable assets.

$$E(R_m) = \text{Expected return of the market}$$

Example:

- Assume the expected market return is 8%.

Step 3: Calculate Asset Beta

Beta (β)

Beta is a measure of an asset's volatility relative to the overall market. It indicates the asset's systematic risk.

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

where:

- $\text{Cov}(R_i, R_m)$ = Covariance of asset i with the market
- σ_m^2 = Variance of the market returns

Example Calculation:

$$\beta_i = \frac{0.015}{0.02} = 0.75$$

Step 4: Calculate Expected Return Using CAPM Formula

CAPM Formula

The CAPM formula calculates the expected return of an asset based on its systematic risk (beta).

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where:

- $E(R_i)$ = Expected return of asset i
- R_f = Risk-free rate
- β_i = Beta of asset i
- $E(R_m)$ = Expected return of the market

Example Calculation:

$$E(R_i) = 3\% + 0.75(8\% - 3\%) = 6.75\%$$

Step 5: Plot the Security Market Line (SML)

Security Market Line (SML)

The Security Market Line (SML) is a graphical representation of the CAPM. It plots the expected return of an asset against its beta.

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

- The y-intercept represents the risk-free rate (R_f).
- The slope represents the market risk premium ($E(R_m) - R_f$).

Explanation

The SML illustrates the trade-off between risk and return for efficient portfolios. Assets plotted above the SML are considered undervalued, while those below the SML are considered overvalued.

Step 6: Interpret the Results

Interpreting the CAPM

Use the CAPM results to make informed investment decisions.

- Compare the expected return calculated by CAPM with the actual return of the asset.
- Determine if the asset is fairly valued, undervalued, or overvalued based on its position relative to the SML.

Example:

- If the actual return of an asset is 8% but the CAPM expected return is 6.75%, the asset may be undervalued.

Transition from CAPM to Monte Carlo Simulation

From CAPM to Monte Carlo Simulation

While the Capital Asset Pricing Model (CAPM) helps in understanding the relationship between systematic risk and expected return, Monte Carlo Simulation provides a robust method for modeling the probability of different outcomes by incorporating randomness and uncertainty.

- CAPM gives a deterministic expected return for a given risk level.
- Monte Carlo Simulation introduces stochastic processes to evaluate the range of possible outcomes, providing a more comprehensive risk assessment.

Monte Carlo Simulation - Formula

Formula

$$E(X) = \frac{1}{N} \sum_{i=1}^{N} f(X_i)$$

where:

- $E(X)$ = Expected value of the outcome
- N = Number of simulations
- $f(X_i)$ = Function of the simulated variable X_i

Explanation

Monte Carlo Simulation uses repeated random sampling to obtain numerical results. The function $f(X_i)$ represents the process being simulated. $E(X)$ is the average result of all simulations.

Conclusion

- **Future Directions:** Explore optimal investment strategies tailored for first-time homebuyers to accumulate housing down payments, incorporating modern financial theories and data-driven insights.
- Effective strategies include diversified portfolios, the application of MPT, and lifecycle investing to navigate the unique financial challenges faced by first-time homebuyers.
- Ongoing research will focus on refining these strategies and exploring their practical applications to further assist first-time homebuyers in achieving their homeownership goals.

Q&A

- **Clarifications:** Please feel free to ask for any clarifications or additional details regarding the presented research and findings.

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