WRITTEN ASSIGNMENT 1

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Question 1

Given: historical data of result of soccer matches of team playing against Ajax
Goal : predict whether a tea, will win, lose or draw against Ajax.
Learning task: Supervised learning, classification. I choose classification because *y* can only take a small number of discrete values (i.e. win, lose or draw).

Opponent	**Date of Match**	**Lose (0), Draw (1), Win (2)**
PSV	06-06-06	2
Feyenoord	18-06-06	1
FC Utrecht	20-06-06	2
$\vdots$	$\vdots$	$\vdots$

Question 2

(A)

$m = 3$
$h_\theta(x) = \theta_0 + \theta_1 x$
$h_\theta(x) = x$

FIRST ITERATION:

$\theta_0 := 0 - 0.1 * \frac{1}{3} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$
$\theta_0 := -0.1 * \frac{1}{3} * [(3 - 6) + (5 - 7) + (6 - 10)]$

$\theta_0 := -0.1 * \frac{1}{3} * -9$
$\theta_0 := 0.3$

$\theta_1 := 1 - 0.1 * \frac{1}{3} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) * x^{(i)}$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * [(3-6)*3 + (5-7)*5 + (6-10)*6]$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * [(-9) + (-10) + (-24)]$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * -43$
$\theta_1 := 2.433$

SECOND ITERATION:

$h_\theta(x) = 0.3 + 2.433x$

X	Y	h_θ
3	6	7.6
5	7	12.4667
6	10	14.6

$\theta_0 := 0.3 - 0.1 * \frac{1}{3} * [(7.6-6) + (12.467-7) + (14.6-10)]$
$\theta_0 := 0.3 - 0.1 * \frac{1}{3} * 11.667$
$\theta_0 := -0.11667$

$\theta_1 := 2.433 - 0.1 * \frac{1}{3} * [(7.6-6)*3 + (12.467-7)*5 + (14.6-10)*6]$
$\theta_1 := 2.433 - 0.1 * \frac{1}{3} * 59.7335$
$\theta_1 := -0.4418$

MEAN SQUARED ERROR:

$h_\theta(x) = -0.11667 + -0.4418x$
$MSE = \frac{1}{m} \sum_{i=1}^{m} (y^{(i)} - h_\theta(x^{(i)}))^2$

X	Y	h_θ
3	6	-1.44
5	7	-2.326
6	10	-2.767

$MSE = \frac{1}{m} * [(6 - -1.44)^2 + (7 - -2.326)^2 + (10 - -2.767)^2]$

$MSE = \frac{1}{3} * 305.324$
$MSE = 101.774$

(B)

X	Y
3	6
5	7
6	10

$n = 3$
$\mu_x = \frac{3+5+6}{3}$
$\mu_x = 4.667$

$\mu_y = \frac{6+7+10}{3}$
$\mu_y = 7.667$

$\sigma_x = \sqrt{\frac{1}{n-1} * \sum(x^i - \mu_x)}$
$\sigma_x = \sqrt{\frac{1}{2} * [(3-4.667)^2 + (5-4.667)^2 + (6-4.667)^2]}$
$\sigma_x = \sqrt{4.667}$
$\sigma_x = 2.160$

$\sigma_y = \sqrt{\frac{1}{n-1} * \sum(y^i - \mu_y)}$
$\sigma_y = \sqrt{\frac{1}{2} * [(6-7.667)^2 + (7-7.667)^2 + (10-7.667)^2]}$
$\sigma_y = \sqrt{4.335}$
$\sigma_y = 2.082$

$Z_x = \frac{x^{(i)} - \mu_x}{\sigma_x}$

$Z_y = \frac{y^{(i)} - \mu_y}{\sigma_y}$

X	Z_x	Y	Z_y
3	-0.772	6	-0.8
5	0.154	7	-0.32
6	0.617	10	1.120

$h_\theta(x) = \theta_0 + \theta_1 x$
$h_\theta(x) = x$

$\theta_0 := 0 - 0.1 * \frac{1}{3} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$
$\theta_0 := -0.1 * \frac{1}{3} * [(-0.772 - -0.8) + (0.154 - -0.32) + (0.617 - 1.120)]$
$\theta_0 := -0.1 * \frac{1}{3} * -0.001$
$\theta_0 := 0$

$\theta_1 := 1 - 0.1 * \frac{1}{3} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) * x^{(i)}$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * [(-0.772 - -0.8) * -0.772 + (0.154 - -0.32) * 0.154 + (0.617 - 1.120) * 0.617]$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * -0.259$
$\theta_1 := 1 - 0.1 * \frac{1}{3} * -0.259$
$\theta_1 := 1.0086$

Even though both the values for θ_1 and θ_0 are positive in both the original data set and the z-scores, the new values for θ_1 and θ_0 for the z-scores data set haven't changed compared to the original valued for both theta's.

Question 3

(A)

$Y = \alpha * X_1 + \beta * X_2$
$Y = \alpha * X_1 + \beta * (a + b * X_1)$
$Y = (\alpha + b * \beta) * X_1 + a * \beta$

Therefore the new introduced variable will affect the MSE linearly.

(B)

$Y = \alpha * X_1 + \beta * X_2$
$Y = \alpha * X_1 + \beta * (a + b * X_1^2)$
$Y = \alpha * X_1 + b * \beta * X_1^2 + a * \beta$

Therefore the new introduced variable will affect the MSE quadratically.

Question 4

$\theta_1 = \frac{\partial}{\partial\theta_1} J(\theta_0, \theta_1) := \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) * x^{(i)}$

$0 := \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) * x^{(i)}$

$0 := \sum_{i=1}^{m} (\theta_0 + \theta_1 * x^{(i)} - y^{(i)}) * x^{(i)}$

$0 := \sum_{i=1}^{m} (\theta_0 * x^{(i)} + \theta_1 * x^{2(i)} - x^{(i)} * y^{(i)})$

$0 := \sum_{i=1}^{m} (\theta_0 * x^{(i)}) + \sum_{i=1}^{m} (\theta_1 * x^{2(i)}) - \sum_{i=1}^{m} (x^{(i)} * y^{(i)})$

$\sum_{i=1}^{m} (\theta_1 * x^{2(i)}) = \sum_{i=1}^{m} (\theta_0 * x^{(i)}) - \sum_{i=1}^{m} (x^{(i)} * y^{(i)}) \sum_{i=1}^{m} \theta_1 = \sum_{i=1}^{m} \frac{(\theta_0 * x^{(i)}) - (x^{(i)} * y^{(i)})}{x^{2(i)}}$

$m * \theta_1 = \sum_{i=1}^{m} \frac{(\theta_0 * x^{(i)}) - (x^{(i)} * y^{(i)})}{x^{2(i)}}$

$\theta_1 = \frac{1}{m} \sum_{i=1}^{m} \frac{(\theta_0 * x^{(i)}) - (x^{(i)} * y^{(i)})}{x^{2(i)}}$

$\theta_1 = \frac{1}{m} \sum_{i=1}^{m} \frac{\theta_0 - y^{(i)}}{x^{(i)}}$