Simplex approximation

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Suppose we have a function $f(x, y)$ defined at the three vertices $\vec{a}, \vec{b}, \vec{c}$ of a triangle. Any point $\vec{r} = (x, y)$ inside the triangle has a unique representation as

$$\vec{r} = (x, y) = \rho_a \vec{a} + \rho_b \vec{b} + \rho_c \vec{c} \tag{1}$$

in which

$$\rho_a + \rho_b + \rho_c = 1. \tag{2}$$

The vector equation (1) and the scalar equation (2) are three linear equations in the three unkowns ρ_a, ρ_b, ρ_c. So the solution is unique and is given by a 2×2 matrix equation

$$\begin{pmatrix} x - c_x \\ y - c_y \end{pmatrix} = \begin{pmatrix} a_x - c_x & b_x - c_x \\ a_y - c_y & b_y - c_y \end{pmatrix} \begin{pmatrix} \rho_a \\ \rho_b \end{pmatrix} \tag{3}$$

which gives ρ_a and ρ_b. Then $\rho_c = 1 - \rho_a - \rho_b$.

So the simplex approximation is

$$f(x, y) = \rho_a f(\vec{a}) + \rho_b f(\vec{b}) + \rho_c f(\vec{c}). \tag{4}$$

But check this.