

Simple Model of Energetic Reactions

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Motivation

Throughout modern history, scientists and engineers are continually coming up with new and interesting ways to produce energy. Of the most recent ways, nuclear reactions which harness the energy holding protons and neutrons together provide the most energy released per amount of material. It is conceivable that, since protons and neutrons are not the smallest constituents of matter, even more energy can be produced from sub-nucleonic processes. What follows is an attempt to provide a simple framework to address this question; although, we unfortunately already know the answer. The Standard Model of particle physics places fundamental limitations on our ability to produce energy. This model, which has proven to be highly successful, has no known metastable states for sub-nucleonic material. As we will discuss shortly, a metastable state is needed to produce energy in an energetic reaction. But, let's not let the Standard Model or a limited understanding of particle physics spoil our discussion.

The Model

Physicists prefer to deal with energies (kinetic energy, gravitational potential energy, chemical potential energy, etc.) because, although the concept of energy is abstract, energy obeys some relatively straightforward rules. Energy is always conserved (except of course when it's not) and a state with lower energy is preferable to a state with higher energy. A state is a particular ensemble of smaller constituents (like nucleons, atoms, or molecules) that corresponds to a certain energy level. Understanding various states with their corresponding energies is one of the basic drives of physics. We can graphically depict a simple model for energetic reactions using this energy-state framework (Fig 1). Physicists use and abuse energy level diagrams for just about everything.

Definitions

Stable State: For our purposes, a stable state refers to a substance in solid or liquid form that is not going to spontaneously change (into a gas) anytime soon. For atomic nuclei, this means the element has a relatively long half-life. In this stable state, the substance stores intrinsic energy as chemical, electrical, or nuclear bonds holding parts of the substance together. Thanks to Einstein's mass-energy equivalence, we know that the energy stored in these bonds will add mass to the substance ($\text{mass} = \text{bond energy} / c^2$). For individual protons and neutrons, most of their mass is a result of the internal binding energy between their quarks.

Energy Absorbed: This is the energy that needs to be put into the stable-state system to induce a reaction. In chemistry, this is referred to as the activation energy which we will quantify as thermal energy. Thermal energy is the averaged kinetic energy of the particles in a substance either vibrating in a lattice or whizzing around in a gas. We will define the thermal energy (slightly imprecisely) to be the Boltzmann constant times the absolute temperature in Kelvin. As the temperature increases, individual molecules only have so many internal ways to store that increasing energy (rotationally, vibrationally, etc.). Past a certain point, the increase in temperature literally shakes the molecules apart and we have a reaction. In nuclear processes, the energy absorbed can come in the form of electromagnetic radiation (usually gamma rays) or neutrons of various kinetic energies. If these neutrons can penetrate the dense nucleus held together by the strong force and be absorbed, then the nucleus could be promoted to an unstable excited state that will decay into more stable fission products.

Excited State: This is the metastable state referred to in the motivation paragraph of this text. The absorbed energy promotes the substance in the stable state to an excited state. This excited state is called "metastable" because it will last for only a short time before it decays to a lower state and releases energy. This term makes less sense for chemical reactions where the excited state is merely the original substance heated to a certain temperature to induce the chemical bonds to break and release energy. But for a quantum mechanical system like an atom or a nucleus, an excited state is well defined because the stable system can actually absorb energy and store that energy momentarily before releasing it.

Energy Released: When the bonds that held the substance together in the stable state break, the broken pieces of the original substance will seek to form new bonds with each other. For example, when a molecule breaks apart violently the individual atoms will quickly seek to recombine and might form gases. If the new bonds have less energy than the old ones, then that energy difference can be radiated away in the form of electromagnetic radiation (light) and kinetic energy of the new substance (heat). In chemistry, this energy is called the change in enthalpy between the products and the reactants. In thermodynamics, this energy is called the difference in the Gibbs free energy of the reactants and the products.

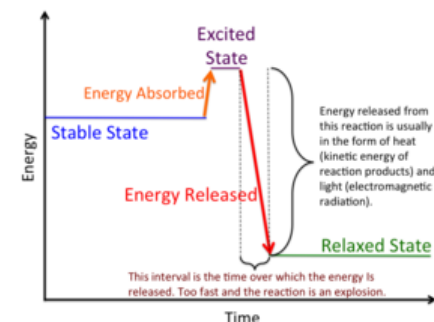


Fig. 1: A simple model of an energetic reaction using an energy level diagram. The vertical axis is increasing energy and the horizontal axis is increasing time. Each step in the reaction is explained in the definition part of the text. Table 1 provides examples of this depicted reaction.

Relaxed State: After breaking apart, the individual constituents of the substance (formally in the stable state) will seek to form new bonds that are energetically favorable. In the case of deflagration (a chain reaction of combustion propagating at less than the speed of sound) and detonation (a chain reaction of explosions propagating through a substance faster than sound), the relaxed state is mostly gases and solid carbon. In nuclear processes, the relaxed state is a myriad of smaller elements (like Ba, Cs, Zr, Kr, Xe, I, Sr, etc.) that will themselves eventually decay (just not fast enough to be economically useful).

Stable State	Energy Absorbed	Excited State	Energy Released	Relaxed State	Detonation Velocity
Black Powder	500°C or 0.07 eV	$10 \text{ KNO}_3 + 8 \text{ C} + 3 \text{ S} + \text{Energy}$	2 - 3 MJ/kg or about 30 eV per reaction	Mostly gases: N_2 , CO_2 , CO, etc.	600 m/sec
Gasoline	530°C or 0.07 eV	$2 \text{ C}_8\text{H}_{18} + 25 \text{ O}_2 + \text{Energy}$	10 MJ/kg (with O_2) or about 86 eV per reaction	$16 \text{ CO}_2 + 18 \text{ H}_2\text{O}$	17 m/sec
TNT	240°C or 0.04 eV	$2 \text{ C}_7\text{H}_5\text{N}_3\text{O}_6 + \text{Energy}$	4184 kJ/kg (NIST) or 20 eV per reaction	Mostly energetic gases	6900 m/sec
Uranium 235	Neutron of any energy or ~ 6 MeV to induce fission	Uranium 236	205 MeV	Fission fragments, Neutrons, Radiation	NA
Plutonium 239	Neutron of any energy or ~ 6 MeV to induce fission	Plutonium 240	207 MeV	Fission fragments, Neutrons, Radiation	NA
Quark Matter	Need about 290 MeV to excite quarks.	?	Binding energies on the order of 100s of MeV	?	NA

Table 1: Table of various examples of the reaction depicted in Fig. 1. The information for black powder, gasoline, and TNT come from Cooper, Kubota and Akhavan. [1-3] The information for U-235, Pu-239, and quarks come from Basdevant *et al.* and Cottingham and Greenwood. [4,5] An MeV is one million electron volts.

Examples

Table 1 is a chart of various materials undergoing the aforementioned process. The materials are listed in their smallest quantity that will undergo the energetic reaction. These processes are not useful in generating energy if the atom or molecule does not release enough energy to induce its neighbor to undergo the same process. If enough energy is released, then a chain reaction will spread through the entire substance releasing energy. Another key consideration in this process is the rate at which this energy is released. Energy that is released slowly might provide an effective way to create electricity or drive a car, but energy that is released too quickly will result in an explosion.

Science Fiction

Now, armed with this simple model of an energetic reaction, we can fantasize about more exotic processes. There is an obvious trend that the smaller the constituents and tighter the bonds holding them together, the more energy we can get out in a reaction. Moving from chemical bonds to nuclear bonds gives us several orders of magnitude in energy density. Where does this stop? Can we get energy from the decay of induced excited states of sub-nucleonic particles like quarks and gluons? While we are at it, what about string theory -- are there excited states of strings that we can induce which will release useful energy?... Well, we can ignore string theory since the energy scales are effectively impossible to reach. But, quark-ish matter (hadrons) might provide some possibilities. Some possible metastable states of hadronic matter are as follows: quark-antiquark systems, quark-gluon plasmas, tetraquarks, pentaquarks, etc. [6-9] So as long as you follow all the esoteric conservation rules dictated by Quantum Chromodynamics (QCD), you could theoretically derive a quark-based energy level system to extract energy. Unfortunately, there are currently no known stable or excited states of quark matter that will release more energy than is absorbed. So for now, quark reactors and quark bombs will have to remain in the realm of science fiction.

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