The NIST Reference on Constants, Units, and Uncertainty

Uncertainty of Measurement Results

Return to Uncertainty home page

Uncertainty Topics:
Basic
Evaluating
Combining
Expanded/ coverage
Examples
Background

Uncertainty Bibliography

Constants, Units & Uncertainty home page

Combining uncertainty components

Calculation of combined standard uncertainty

The **combined standard uncertainty** of the measurement result y, designated by $u_c(y)$ and taken to represent the estimated standard deviation of the result, is the positive square root of the estimated variance $u_c^2(y)$ obtained from

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) \qquad (6)$$

Equation (6) is based on a first-order Taylor series approximation of the measurement equation $Y = f(X_1, X_2, \ldots, X_N)$ given in equation (1) and is conveniently referred to as the *law of propagation of uncertainty*. The partial derivatives of f with respect to the X_i (often referred to as *sensitivity coefficients*) are equal to the partial derivatives of f with respect to the X_i evaluated at $X_i = x_i$; $u(x_i)$ is the standard uncertainty associated with the input estimate x_i; and $u(x_i, x_j)$ is the estimated covariance associated with x_i and x_j.

Simplified forms

Equation (6) often reduces to a simple form in cases of practical interest. For example, if the input estimates x_i of the input quantities X_i can be assumed to be uncorrelated, then the second term vanishes. Further, if the input estimates are uncorrelated and the measurement equation is one of the following two forms, then equation (6) becomes simpler still.

Measurement equation:

A sum of quantities X_i multiplied by constants a_i.

$$Y = a_1X_1 + a_2X_2 + \ldots a_NX_N$$

Measurement result:

$$y = a_1x_1 + a_2x_2 + \ldots a_Nx_N$$

Combined standard uncertainty:

$$u_c^2(y) = a_1^2 u^2(x_1) + a_2^2 u^2(x_2) + \ldots a_N^2 u^2(x_N)$$

Measurement equation:

A product of quantities X_i, raised to powers $a, b, \ldots p$, multiplied by a constant A.

$Y = AX_1^a X_2^b \ldots X_N^p$

Measurement result:

$$y = Ax_1^a x_2^b \ldots x_N^p$$

Combined standard uncertainty:

$$u_{c,r}^2(y) = a^2 u_r^2(x_1) + b^2 u_r^2(x_2) + \ldots p^2 u_r^2(x_N)$$

Here $u_r(x_i)$ is the **relative standard uncertainty** of x_i and is defined by $u_r(x_i) = u(x_i) / |x_i|$, where $|x_i|$ is the absolute value of x_i and x_i is not equal to zero; and $u_{c,r}(y)$ is the **relative combined standard uncertainty** of y and is defined by $u_{c,r}(y) = u_c(y) / |y|$, where $|y|$ is the absolute value of y and y is not equal to zero.

Meaning of uncertainty

If the probability distribution characterized by the measurement result y and its combined standard uncertainty $u_c(y)$ is approximately normal (Gaussian), and $u_c(y)$ is a reliable estimate of the standard deviation of y, then the interval $y \;\; u_c(y)$ to $y + u_c(y)$ is expected to encompass approximately 68 % of the distribution of values that could reasonably be attributed to the value of the quantity Y of which y is an estimate. This implies that it is believed with an approximate level of confidence of 68 % that Y is greater than or equal to $y \;\; u_c(y)$, and is less than or equal to $y + u_c(y)$, which is commonly written as $Y = y \pm u_c(y)$.

[Top](#)

Continue to [Expanded uncertainty and coverage factor](#)