

1. The following Census Bureau figures give the population of the United States by year.

Year	Population
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298

Fit these data in a least squares sense by second and fourth degree polynomials, use the normal equations approach. (You may use matlab, lapack or clapack routines to solve the equations.) What is the ‘predicted’ population by 1990? Approximately what is the condition number of the system you solved? How did you do the approximation?

2. Repeat problem 1 using QR factorization. You may use matlab, lapack or clapack routines. Extra credit: write your own QR factorization.
3. Fit the data by a function of the form

$$y(t) = c_1 + c_2(t - 1900) + c_3 \exp(d(t - 1900))$$

This function involves four parameters, c_1 , c_2 , c_3 and d . The problem is linear in c_1 , c_2 , c_3 , but nonlinear in d . Let y be the 8-vector of population data, \mathbf{c} be the unknown 3-vector of coefficients. Let $A(d)$ be the 8×3 coefficient matrix of \mathbf{c} obtained by plugging in the 8 years of data from the table. Minimize the two norm of $\|A(d)\mathbf{c} - \mathbf{y}\|$ with respect to \mathbf{c} and d . You should use a combination of QR routines and a routine to compute the minimum of a function of one variable. Find the values of \mathbf{c} and d that give the best fit. Predict the 1980 and 1990 population.

4. Make tables summarizing your results and discuss the merits of each approach. What is your best prediction of the 1990 population?
5. Extra Credit: Repeat 2-3 using matlab, lapack or clapack routines for the SVD in place of QR factorization. What are the singular values of the coefficient matrices?