Excursions in linear least squares

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ABSTRACT.

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CHAPTER 1

Least Squares

1.1. Linear Systems

This story begins with the archetypal matrix-vector equation

$$\mathbf{A}x = b.$$

The matrix $\mathbf{A}$ has m rows, n columns, and has rank ρ; the vector b encodes m measurements. The solution vector x represents the n free parameters in the model. In mathematical shorthand,

$$\mathbf{A} \in \mathbb{C}_{\rho}^{m\times n}, \quad b \in \mathbb{C}^{m}, \quad x \in \mathbb{C}^{n}$$

with $\mathbb{C}$ representing the field of complex numbers. The matrix $\mathbf{A}$ and the vector b are given, and the task is to find the vector x.

$$\left[\begin{array}{rrr} -\frac{1}{L_1} & \frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} & \frac{1}{L_2} \end{array}\right] \left[\begin{array}{c} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{array}\right] = \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right]$$

1.2. Least Squares Solution

The solutions for the linear system in (1.1)

$$\left[\begin{array}{c} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{array}\right] = \left[\begin{array}{rr} -2L_1 & -L_2 \\ L_1 & -L_2 \\ L_1 & 2L_2 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right] + \alpha \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right], \qquad \alpha \in \mathbb{C}.$$

Given $\mathbf{A} \in \mathbb{C}^{m\times n}$

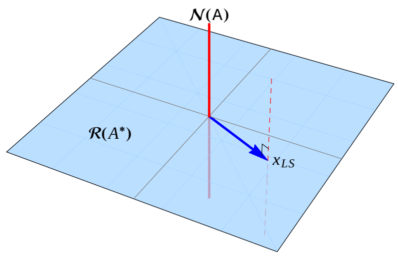
1.3. Fundamental Theorem of Linear Algebra

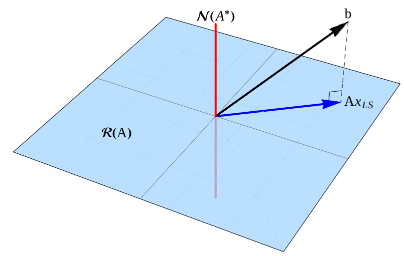
Table 1.1. The Fundamental Theorem of Linear Algebra

domain:	$\mathbb{C}^n$	$=$	$\mathcal{R}(\mathbf{A}^*)$	$\oplus$	$\mathcal{N}(\mathbf{A})$
codomain:	$\mathbb{C}^m$	$=$	$\mathcal{R}(\mathbf{A})$	$\oplus$	$\mathcal{N}(\mathbf{A}^*)$

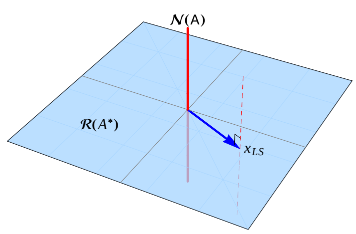
TABLE 1.2. The Fundamental Theorem of Linear Algebra in pictures

Domain	Codomain
$\mathbb{C}^n$	$\mathbb{C}^m$





$$
\begin{array}{rcl}
\mathbf{A}\colon \mathbb{C}^n & \mapsto & \mathbb{C}^m \\
\mathbb{C}^n & \leftarrow\!\shortmid & \mathbb{C}^m : \mathbf{A}^*
\end{array}
$$



$\mathbf{A}\colon \mathbb{C}^n \mapsto \mathbb{C}^m$