Input Slides 2024-12: My Two Slides

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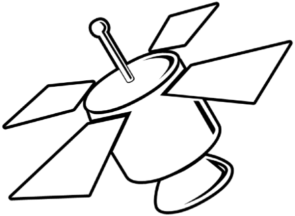
Models of Radar Cross Sections for Satellites



Spherical Harmonics Expansion

$$f(r,\theta,\phi) \approx a_{0,0}Y_0^0 + a_{1,-1}Y_1^{-1} + a_{1,0}Y_1^0 + a_{1,1}Y_1^1 + \dots$$

where

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}}P_n^m(\cos\theta)e^{im\phi}$$

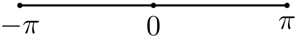
Overview

1. Writ Large
2. Radiation
3. Results in 2D
4. Meshing
5. Software Components

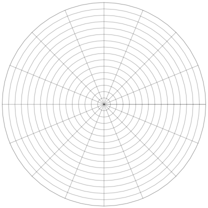
Goal: RCS Models in 3D

RCS models have been built in 2D.
Extend to 3D using the same MoM code.

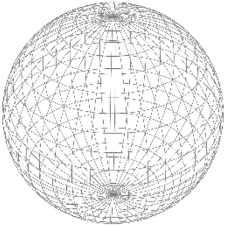
Fourier Domains in 1-, 2-, and 3D



$$\theta \in [-\pi, \pi]$$



$$\theta \in [-\pi, \pi]$$
$$r \in [0, 1]$$



$$\theta \in [-\pi, \pi]$$
$$r \in [0, 1]$$
$$\phi \in [0, \pi]$$

Why We Love Fourier for Smooth Functions

Smooth Functions, Beautiful Representations

- **Weierstrass Approximation Theorem:** Any continuous function on $[a, b]$ can be uniformly approximated by polynomials. Fourier provides a similar approximation, using trigonometric bases instead of polynomials.
- **Riesz-Fischer Theorem:** Hunting license - Fourier coefficients $(a_n, b_n) \in l^2$, guaranteeing convergence in the L^2 sense.
- **Uniform Convergence for Smooth Periodic Functions:** Smooth (C^∞ or C^k) functions, Fourier series converge uniformly, ensuring no oscillatory artifacts (Gibbs phenomenon disappears).

Fourier and Extensions to 2- and 3D

1D:
$$f(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta}$$

2D:
$$f(r,\theta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n,2} a_n^m R_n^m(r) e^{in\theta}$$

3D:
$$f(r,\theta,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_n^m \sqrt{\frac{(2m+1)(m-n)!}{4\pi(m+n)!}} P_l^m(\cos\theta) e^{in\theta}$$

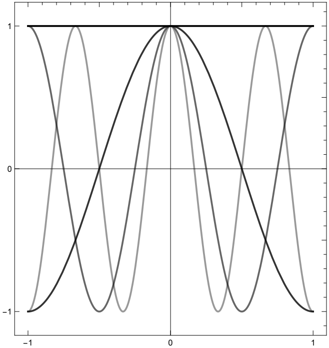
Fourier and Extensions to 2- and 3D

- **1D Fourier Series:** Decomposes a periodic function $f(\theta)$ into a sum of complex exponentials with coefficients a_n capturing the amplitudes of each frequency component.
- **2D Fourier-Bessel:** Extends Fourier analysis to two dimensions using radial functions $R_n^m(r)$, often employed in circular domains or optical applications.
- **3D Spherical Harmonics:** Represents functions on a sphere using harmonics $Y_l^m(\theta, \phi)$ and radial components $R_l(r)$, crucial in fields like quantum mechanics and gravitational modeling.

Lowest Order Fourier Basic Functions

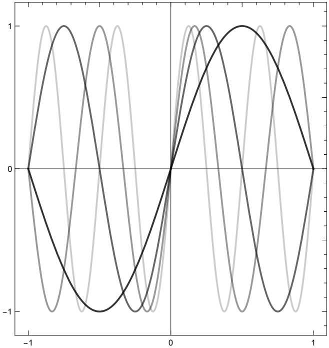
$$f(-\theta) = f(\theta)$$

Even Parity

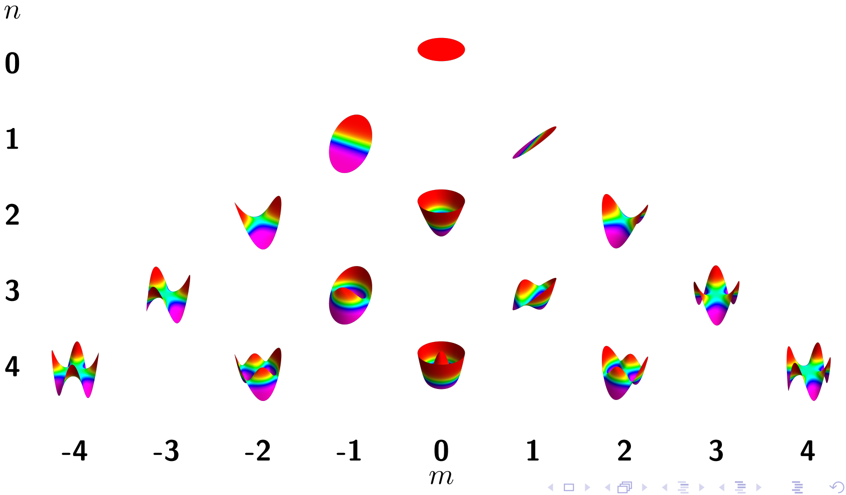


$$f(-\theta) = -f(\theta)$$

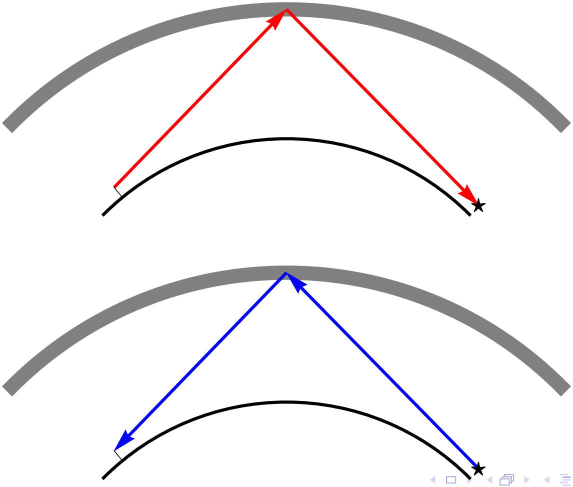
Odd Parity



Lowest Order Zernike Disk Polynomials



Energy Out, Energy In



Energy Out, Energy In

1. radar (left) irradiates target (star)
2. backscatter travels from target to radar

Radar Cross Section: Definition

$$\sigma_{\star} = 4\pi \lim_{r\to\infty} r^2 \left| \frac{E_{\text{incident}}}{E_{\text{scattered}}} \right|^2 \tag{2.1}$$

Skolnik 1962, (2.36)

Radar Cross Section: Discussion

- Radar cross section is a far field phenomenon
- Assumes single polarization to and from target
- Target is completely metallic:
  E field results from surface currents
- Shape is quasi-dimensional
  - Dimensions in two known directions
  - Dish antennae, solar panels, booms
- Resonant scattering:
  Ratio of typical dimension to wavelength ≈ 1
- Kolosov 1987, §4.6

Radar Cross Section: Conceptual Overview

- **Radar Basics:**
  - Transmit energy (Skolnik 1962, p. 21).
  - Receive scattered signal.
  - Direction, strength $\rightarrow$ object properties (Knott, Schaeffer, and Tulley 2004, p. 45).
- **What is RCS?**
  - Measures object "visibility" to radar Lab 2002, Section 2.
  - Depends on:
    - Material
    - Geometry
    - Orientation (Peebles 2007, pp. 3-4).
- **Key Question:** Power reflected vs. power transmitted.

Factors Influencing Radar Cross Section (1/2)

- **Shape:**
  - Smooth → directional reflection (Knott, Schaeffer, and Tulley 2004, p. 47).
  - Complex → scattered energy.
- **Material:**
  - Metal → strong reflection.
  - Absorbers → reduced RCS (Knott, Schaeffer, and Tulley 2004, Section 3.2).
- **Size vs. Wavelength:**
  - Large → high scattering.
  - Small → "invisible" (Rayleigh scattering) (Kolosov 1987, p. 188).

Factors Influencing Radar Cross Section (2/2)

- **Orientation:**
  - Aligned → max RCS (Knott, Schaeffer, and Tulley 2004, Eq. 2.1).
  - Tilted → min RCS (Skolnik 1962, p. 22).
- **Radar Parameters:**
  - Frequency, polarization, incidence angle (Lab 2002, p. 45).

What is Mercury Method of Moments (MoM)?

- **Mercury MoM solves Maxwell's equations in integral form.**
- **Targets:**
  - **Perfect Electric Conductors (PECs).**
  - **Far-field monostatic radar configurations.**
- **Applications:**
  - **Electromagnetic scattering simulations.**
  - **Radar Cross Section (RCS) analysis.**
- **Benefits:**
  - **Standalone Linux execution.**

Mercury MoM Setup Workflow

1. **Model Geometry:**
   - **Create CAD model using** `FreeCAD`**.**
   - **Export mesh as** `*.obj` **file.**
2. **Convert Mesh:**
   - **Use Python tool to convert** `*.obj` **to** `*.facet`**.**
3. **Prepare Input Files:**
   - `B-20.geo` **(Simulation control).**
   - `B-20.facet` **(Geometry).**
   - `Materials.lib` **(Material properties).**
4. **Run Simulation:**

Running Mercury MoM Simulations

- Extract the Mercury MoM package:

```
tar -xvf Hg-MoM-4.1.12.tar.gz
```

- Run the example bash script:

```
./run_mom_example.sh
```

- Simulation:
  - Reads input files (`*.geo`, `*.facet`, `*.lib`).
  - Configures radar frequency, sampling angles, and conditions.

Key Notes

- **System Requirements:**
  - **Linux environment.**
  - **Minimal dependencies (e.g., `libpthread`, `libc`).**
- **Radar Parameters:**
  - **Frequency range: 3–30 MHz.**
  - **Azimuth cuts and sampling.**
- **Applications:**
  - **Analyze far-field scattering patterns.**
  - **Perform monostatic radar cross-section simulations.**

Summary: How to Use Mercury MoM

1. **Model CAD geometry in `FreeCAD`.**
2. **Convert mesh to `*.facet`.**
3. **Prepare input files (`geo`, `facet`, `lib`).**
4. **Extract and execute Mercury MoM package.**
5. **Analyze results.**

Mercury MoM: Accurate, reliable, and ready for PEC far-field radar simulations.

Input Files (1/2)

1. `B-20.geo`
    1. **Points to facet file**
    2. **Configure linear algebra solver**
    3. **Radar frequency range**
    4. **Angular sampling ranges**
    5. **Boundary conditions**
    6. **Mono– or Bistatic**
    7. **Surface or Volume integral elements**
    8. **Length units**

Input Files (2/2)

2. `B-20.facet`
    1. **Vertex list**
    2. **Face list**
3. `Materials.lib`
    1. **Permiability**
    2. **Permittivity**

Linear algebra (don't alter)

```
&MM_MOM
 bUseACA = .TRUE.,
 bSolve_ACA = .TRUE.,
 bOutOfCore = .TRUE.,
 bNormalizeToWaveLength = .FALSE.,
 bNormalize = .FALSE.,
 dCloseLambda = 0.100000,
 ACA_Factor_Tol = 0.000010,
 ACA_RHS_Tol = 0.000100,
 Point_Tolerance = 0.001000,
 nLargestBlockSize = -1,
 MemorySize_GB = -1.000000,
 stackSize_GB = -1.000000,
 nFillThreads = -1,
 nFillMKLThreads = 1,
 nLUThreads = -1,
 nLUMKLThreads = 1,
 nRHSThreads = 1,
 nRHSMKLThreads = 1,
 bOutputACAGrouping = .FALSE.,
 bOutputRankFraction = .FALSE.,
 bLimitLUColumns = .FALSE.,
 Lop_Admissibility = WEAK,
 Kop_Admissibility = CLOSE
```

Memory management (don't alter)

```
&Scratch_Memory
 Scratch_RankFraction_Z = 0.300000,
 Scratch_RankFraction_LU = 0.600000,
 Scratch_RankFraction_RHS = 2.000000,
 Scratch_RankFraction_Solve = 1.000000,
 MemoryFraction_Z = 0.950000,
 MemoryFraction_Scratch_LU = 0.500000,
 MemoryFraction_LU = 1.000000,
 MemoryFraction_RHS = 0.500000,
 MemoryFraction_Solve = 0.900000,
```

Quadrature (don't alter)

```
&QUADRATURE
 NTRISELF = 7,
 NTRINEAR = 3,
 NTRIFAR = 3,
 NTETSELF = 11,
 NTETNEAR = 4,
 NTETFAR = 4,
 NQGAUSS = 4
```

Radar frequencies

```
FREQUENCY
 ghz
 0.003000 0.030000 28 !Freq Start, Freq Stop, Num Frequencies
```

Sampling

```
Angle Cut
 1
 0.000000 359.000000 360
 AZIMUTH
 90.000000
```

Monostatic or bistatic

```
Excitation
 MONOSTATIC
```

Boundary Conditions

```
Boundary Conditions
B-20-Materials.lib
4
V_FREE_SPACE => Free_Space
V_PEC => PEC
V_PMC => PMC
V_NULL => NULL
1
0 BC_PEC V_FREE_SPACE
```

Final settings

`SIE`	surface integral elements
`B-20A.facet`	CAD description
`m`	meters

Mercury MoM is Single Precision

Example: 8 MHz
Despite exact binary representation

$$8_{10} = 1000_2$$

```
Start Frequency = 7.9999994E-03GHz
```

Run sequence - launch

```
$./MMoM_4.1.12 b20.geo
--------------------------------------------------------------------------------
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
------------ Reporting output in MB from Linux command:  vmstat -s -S M ------------
 53113 M free memory
```

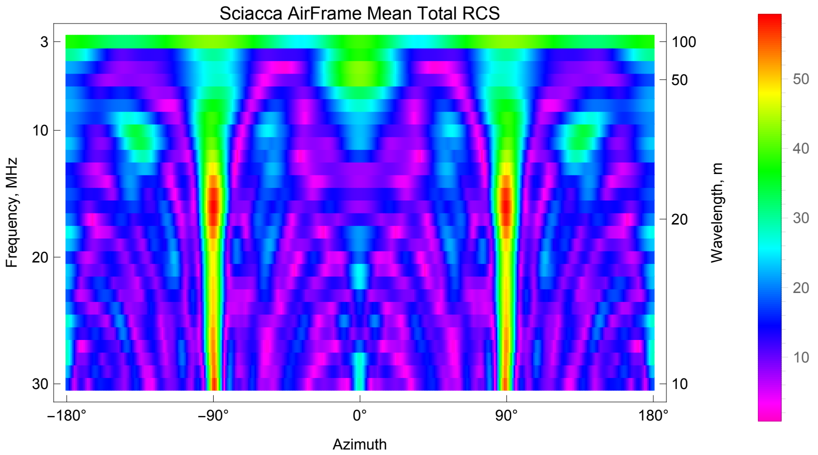
Run sequence - sample output

```
 Freq   = 30.00E+00 MHz
 Lambda = 9.99E+00 m
 k      = 628.75E-03 m-1
subroutine Solve_SetUp( Surface, bk, pSys, pD, Nodes ) :  ...Finished
--------------------------------------------------------------------------------
---| Time :  Time total for RHS solve
---| Twall = 0.0004168 ; Tcpu = 0.0002319 ; Ratio = 1.80
--------------------------------------------------------------------------------
---| Out Of Core Times:  Diagonal Blocks
---|
---| nWrites.........................:  2.
---| GigaBytes Write.................:  0.
---| Write Time (Hr).................:  0.00
---| Average Write Rate (MBytes/sec).:  19.
---| nReads..........................:  5.
---| GigaBytes Read..................:  0.
---| Read Time (Hr)..................:  0.0002
---| Average Read Rate (MBytes/sec)..:  48.
---|
--------------------------------------------------------------------------------
Z Column Summary IO 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
```

Run sequence - completion

```
$./MMoM_4.1.12 b20.geo
--------------------------------------------------------------------------------
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
------------ Reporting output in MB from Linux command:  vmstat -s -S M ------------
 53113 M free memory
```

Mercury MoM Sweeps Frequencies and Orientation



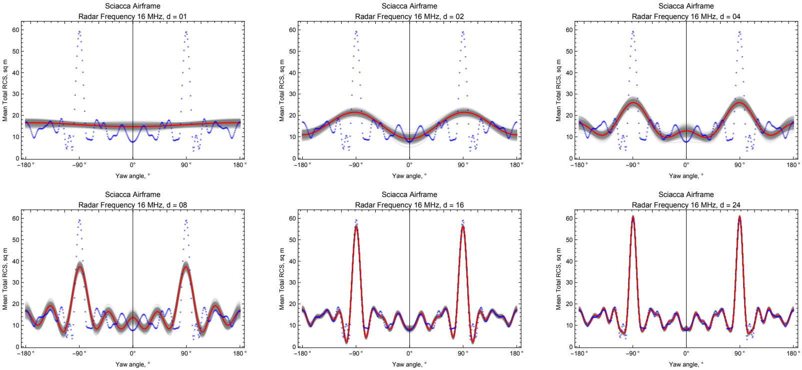
Follow-on Notes: Mercury MoM Sweeps

- **Sweep Overview:**
  - **OTHR Frequency range:** $[3, 30]$ **MHz.**
  - **Wavelength:** $[100, 10]$ **m.**
  - **Color represents return energy levels**
- **Azimuth Details:**
  - **Azimuth range:** $[0°, 360°]$**.**
    - $0°$**: Nose on.**
    - $-90°$**: Copilot side.**
    - $90°$**: Pilot side.**
    - $180°$**: Tail on.**

Frequency Notes and Plot Interpretation

- Frequency Notes:
  - Resonance peaks identified at key frequencies in red.
  - Highlighted transitions between modes for particular azimuths.
- Plot Interpretation:
  - MoM will sweep frequencies.
  - MoM will sweep orientation.
  - MoM is ready for 3D.
- Resolution
  - At 3 Mhz (100 m) target is fuzzy.

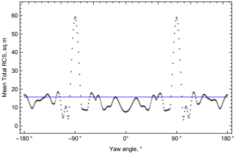
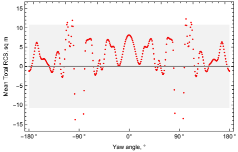
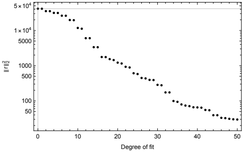
Fourier Transform Visualizations at 16 MHz



Note

Blue: Data **Red: Approximation** **Gray: Error**

A Closer Look A Fits and Errors 1/3

<table>
<tr><th>d</th><th>Data Fit</th><th>Residual Error</th><th>Total Error</th></tr>
<tr><td>0</td><td>
</td><td>
</td><td>
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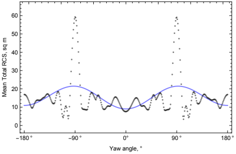
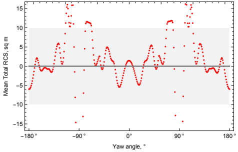
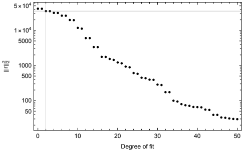
<tr><td>2</td><td>
</td><td>
</td><td>
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A Closer Look A Fits and Errors 2/3

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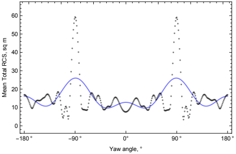
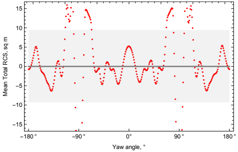
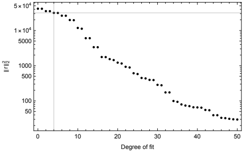
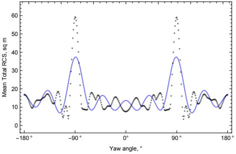
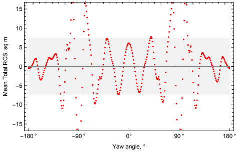
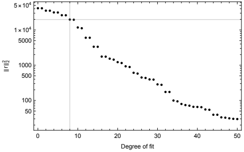
<tr><td>4</td><td></td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td><td></td></tr>
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A Closer Look A Fits and Errors 3/3

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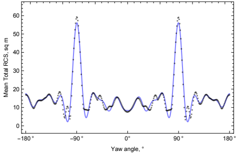
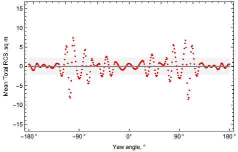
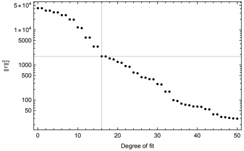
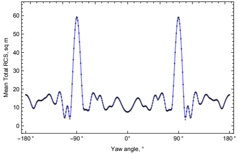
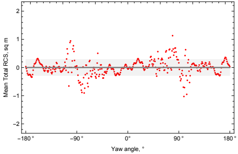
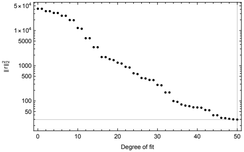
16			
50			

Fourier Transform Visualizations at 16 MHz

- **Approximation Order (d):** The number of terms in the Fourier approximation increases as $d = 1, 2, 4, 8, 16, 24$.
- **Low-Order Approximations:** Approximations with smaller d (e.g., $d = 1, 2, 4$) fail to capture fine structures, leading to significant residual error.
- **Higher-Order Approximations:** As d grows, the approximation better resolves finer features, and the error (gray) shrinks significantly, especially at smooth regions.
- **Error Behavior:** The error decreases non-uniformly—large errors persist near abrupt changes or peaks due to Gibbs phenomena, but smooth regions converge faster.

Fourier Transform Visualizations at 16 MHz

- **Key Insight:** Fourier approximations demonstrate trade-offs: fidelity improves with d, as does computational cost.
- **General Notes:**
  - Fourier series resolve functions as sums of sines and cosines.
  - Low-frequency terms: broad trends; high-frequency: fine details.
  - Convergence is faster for smooth functions but slower for discontinuities or sharp changes.
  - More terms improves fidelity, but can introduce numerical artifacts.

Meshing Schemes

	Method	Mesh Resolution	Faces	Points	Spectral Radius
✓	Standard	1.0 m	626	315	5.3
✓	Standard	0.1 m	766	385	5.3
✓	Standard	0.05 m	1,198	601	5.3
✗	Standard	0.01 m	3,352	1,678	6.5
✗	Standard	0.001 m	28,394	14,199	8.7
✗	Mefisto	1.0 m	3,974	1,992	2.5
✗	Netgen	very fine	10,098	5,051	3.0

Table: One model, many meshes. How does Mercury MoM fare?

Achilles Heel: Minimum triangle size

Mercury MoM is very sensitive to Spectral radius

Linux Environment

```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine Geometry_TRI_Compute( Tris, tol ) :Have Triangles with effective zero area
nTris_With_Zero_Area = 15244
```

Mesh Resolution

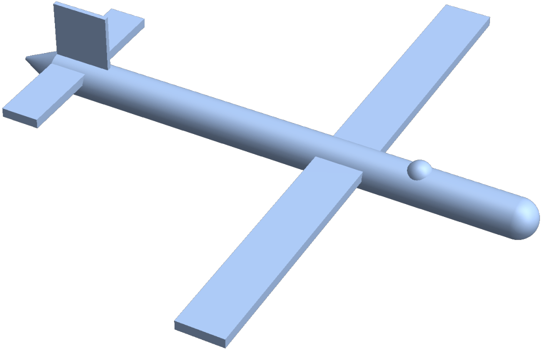
We can quantify mesh limits by looking at the spectrum of triangle areas

Spectral Radius

1. We know *a priori* which mesh resolutions will fail
2. MMoM is single precision
3. Key idea: difference between largest and smallest triangle

Spectral Radius

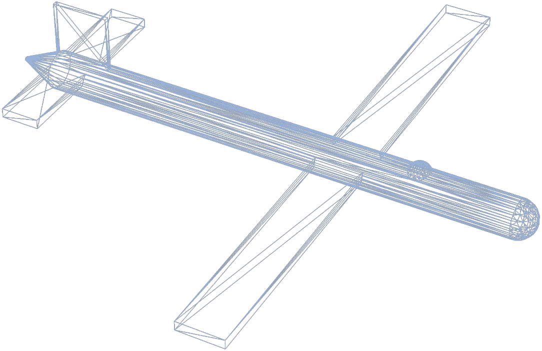
Start with Ω, a closed, simply connected surface:

$\Omega =$ 

Defining the Spectrum

Let Ω_P, be a triangular partition of Ω

$\Omega_P =$



$= \bigcup_{k=1}^{m} \tau_k$

Defining the Spectrum

Properties of the triangular partition Ω_P

1. $$\Omega_P = \bigcup_{k=1}^{m} \tau_k$$
2. $\textbf{Area}\left(\Omega_P\right) = \textbf{Area}\left(\bigcup_{k=1}^{m} \tau_k\right)$
3. $\textbf{Area}\left(\tau_k\right) > 0, \quad k = 1 : m$
4. $\tau_k \cap \tau_j = \delta_k^j \times \textbf{Area}\ \left(\tau_j\right)$

Defining the Spectrum

Colloquially, it's a good mesh:

1. **The mesh is watertight (sealed)**
2. **No triangles overlap**
3. **No triangles underlap**

Defining the Spectrum

Define α_k as the area of triangle τ_k

1. Define α_k as the area of triangle τ_k
2. The sequence is in ascending order:

$$0 < \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_m$$

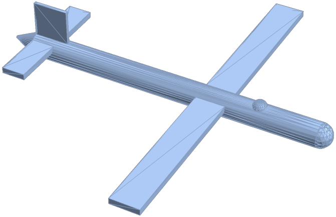
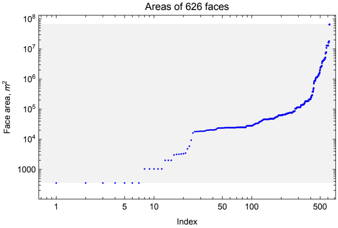
3. The spectrum is the sequence $\{\alpha_k\}_{k=1}^{m}$

Spectral Radius

Define T, the spectral radius as

$$T = \ln \tau_m - \ln \tau_1 = \ln \frac{\tau_m}{\tau_1}$$

Seeing the Spectral Radius

Mesh	Spectrum
	

Think of the height of the gray box as the spectral radius

Spectral Radius

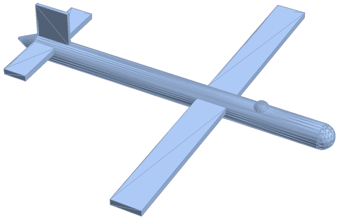
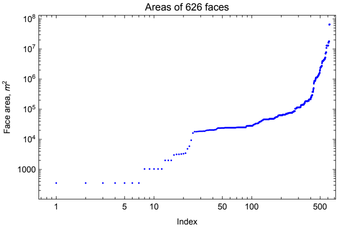
Compare to κ the matrix condition number in the $2-$norm:

Given a matrix A of rank ρ with singular value spectrum $\{\sigma_k\}_{k=1}^{\rho}$
The matrix condition number is defined as

$$\kappa_2 = \frac{\|\mathbf{A}\|_2}{\|\mathbf{A}^{-1}\|_2} = \frac{\sigma_k}{\sigma_1}$$

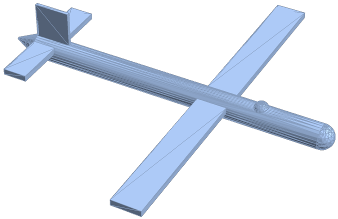
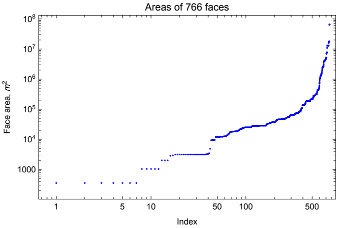
$$T = \ln \frac{\tau_m}{\tau_1}$$

Standard meshing, 1 m resolution

Mesh	Spectrum
	

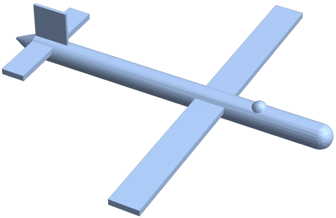
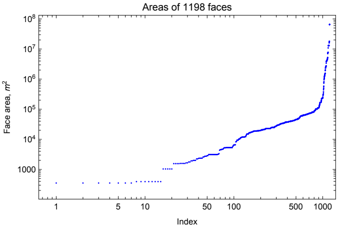

```
--------------------------------------------------------------------------------
---| Mercury MOM Completed Sucessfully |---
--------------------------------------------------------------------------------
```

Standard meshing, 0.1 m resolution

Mesh	Spectrum
	

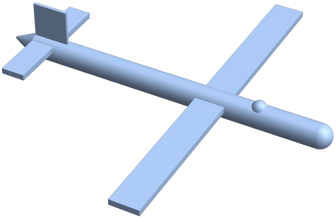
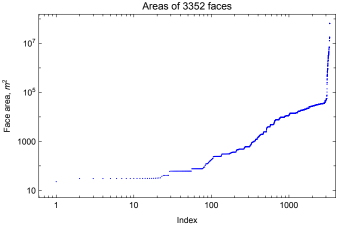
```
-----------------------------------------------------------------------------
---| Mercury MOM Completed Sucessfully |---

-----------------------------------------------------------------------------
```

Standard meshing, 0.05 m resolution

Mesh	Spectrum
	

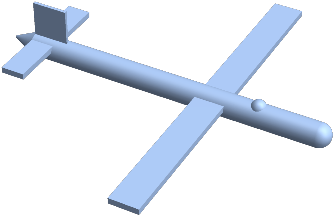
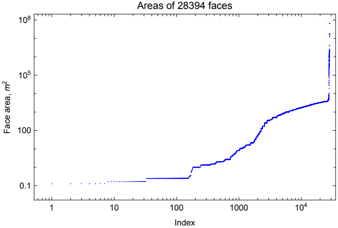

```
--------------------------------------------------------------------------------
                ---| Mercury MOM Completed Sucessfully |---

--------------------------------------------------------------------------------
```

Standard meshing, 0.01 m resolution

Mesh	Spectrum
	

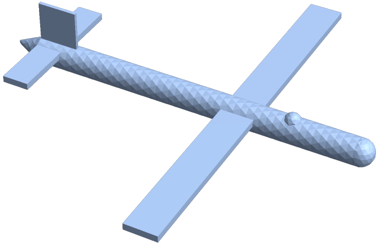
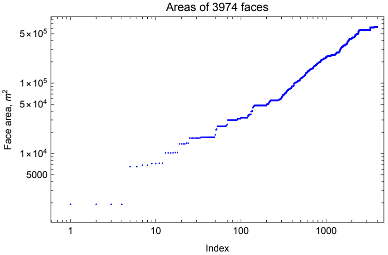
```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
 subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =           0
```

Standard meshing, 0.001 m resolution

Mesh	Spectrum
	

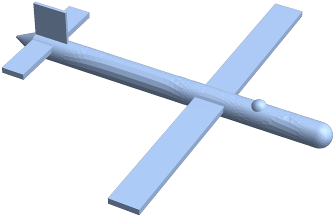
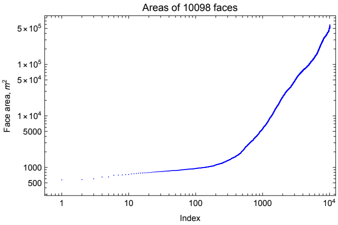

```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine Geometry_TRI_Compute( Tris, tol ) :Have Triangles with effective zero area
nTris_With_Zero_Area =           60
```

Mefisto meshing, 1 m resolution

Mesh	Spectrum
	

```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
 subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =          0
```

Netgen meshing, very fine resolution

Mesh	Spectrum
	


```
-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =           0
```

Code Conversion Overview

MATLAB and ALPINE Codes Converted

- Python
- Fortran
- Shell Scripts

Survey and Details Follow.

Primary Components

Fortran:

1. toolkit 1

Python:

1. script 1

Sealing the Mesh

Show code structure

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Input Slides 2024-12: My Two Slides

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