Meshing: Lessons Learned

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Executive Summary

1. FreeCAD generates meshes
   1.1 Use **Standard**
   1.2 Do Not Use **Mefisto**, **Netgen**
2. FreeCAD fixes meshes
3. Mercury MoM's single precision limits resolution

Scope

1. Opposing requirements in mesh resolution
    1.1 Crude mesh: Finest resolution: $\lambda = 10$ m ($\nu = 30$ MHz)
    1.2 Fine mesh: Realistic objects are smooth
2. Survey of results
3. Understanding mesh resolution

Counterposing Requirements

- Use a crude mesh: radar resolution

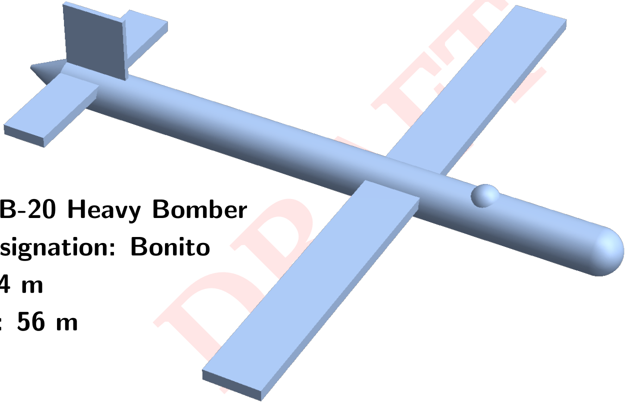
$$\lambda = 10 \text{ m at } \nu = 30 \text{ MHz}$$

- Use a fine mesh: avoid unnaturally flat features

Mercury MoM Precision Limit

- Mercury MoM is single precision
- Imposes stringent limit on mesh resolution
- Case study follows ...

B-20 Heavy Bomber



FreeCAD B-20 Heavy Bomber
NATO Designation: Bonito
Length: 54 m
Wingspan: 56 m

Mesh Resolution Limit

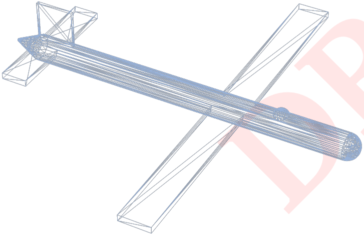
	Method	Mesh Resolution	Faces	Points	Spectral Radius
✓	Standard	1.0 m	626	315	5.3
✓	Standard	0.1 m	766	385	5.3
✓	Standard	0.05 m	1,198	601	5.3
✗	Standard	0.01 m	3,352	1,678	6.5
✗	Standard	0.001 m	28,394	14,199	8.7
✗	Mefisto	1.0 m	3,974	1,992	2.5
✗	Netgen	very fine	10,098	5,051	3.0

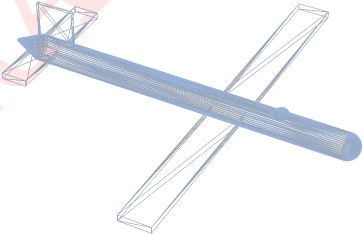
Table: One model, many meshes. How does Mercury MoM fare?

Mesh Resolution Limit

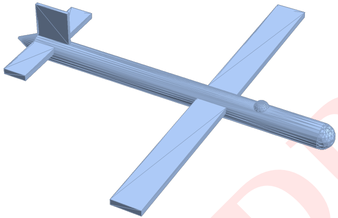
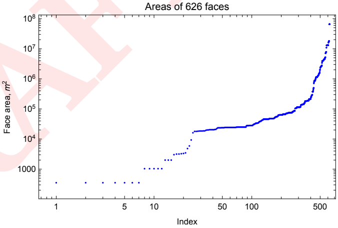
Table: Resolution Edge in Pictures

PASS	FAIL
50 cm	10 cm
$T = 5.3$	$T = 6.5$



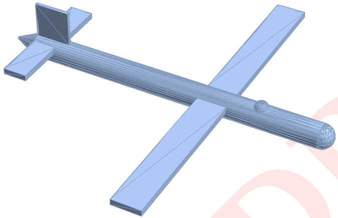
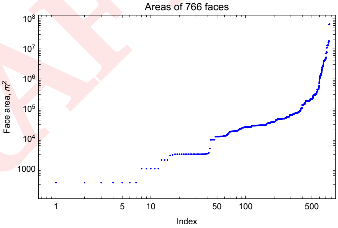


Standard meshing, 1 m resolution

Mesh	Spectrum
	


```
-------------------------------------------------------------------------------
---| Mercury MOM Completed Sucessfully |---
-------------------------------------------------------------------------------
```

Standard meshing, 0.1 m resolution

Mesh	Spectrum
	

```
--------------------------------------------------------------------------------
---| Mercury MOM Completed Sucessfully |---
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```
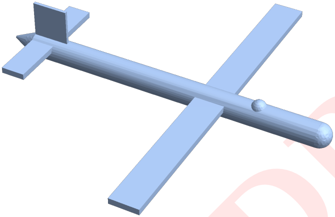
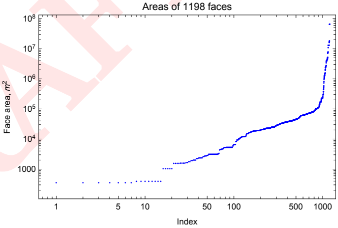
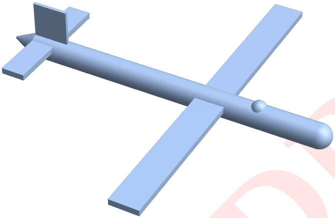
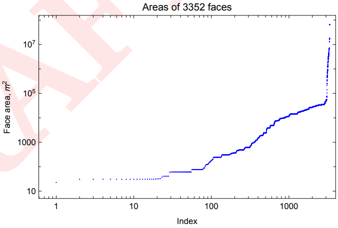

Standard meshing, 0.05 m resolution

Mesh	Spectrum
	

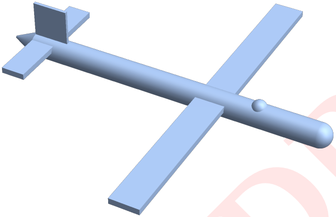
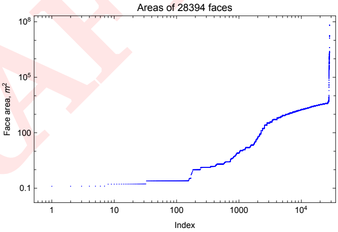
```
-------------------------------------------------------------------------------
                 ---| Mercury MOM Completed Sucessfully |---
-------------------------------------------------------------------------------
```

Standard meshing, 0.01 m resolution

Mesh	Spectrum
	

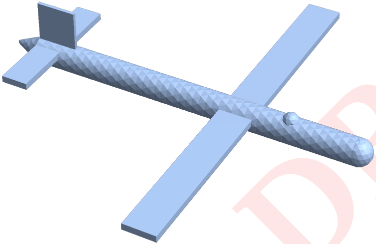
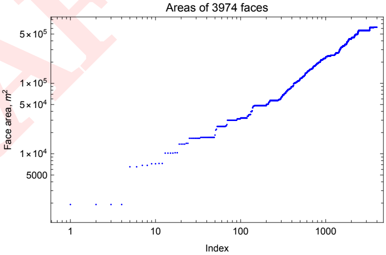

```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
 subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =          0
```

Standard meshing, 0.001 m resolution

Mesh	Spectrum
	

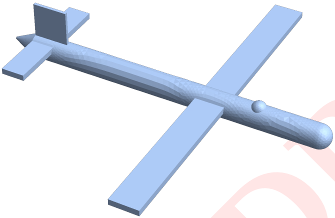
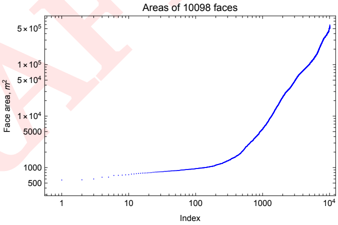
```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine Geometry_TRI_Compute( Tris, tol ) :Have Triangles with effective zero area
nTris_With_Zero_Area =         60
```

Mefisto meshing, 1 m resolution

Mesh	Spectrum
	


```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
 subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =          0
```

Netgen meshing, *very fine* resolution

Mesh	Spectrum
	


```
-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =          0
```

Mesh Resolution

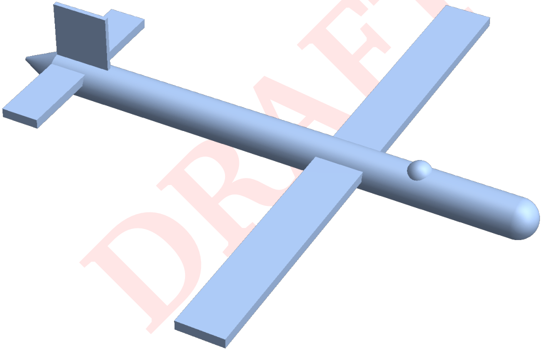
We can quantify mesh limits by looking at the spectrum of triangle areas

Spectral Radius

1. We know *a priori* which mesh resolutions will fail
2. MMoM is single precision
3. Key idea: difference between largest and smallest triangle

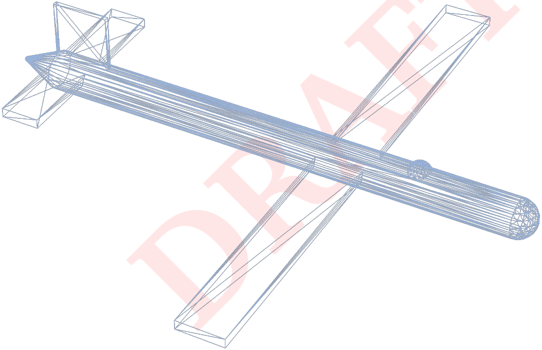
Spectral Radius

Start with Ω, a closed, simply connected surface:

$\Omega =$ 

Defining the Spectrum

Let Ω_P, be a triangular partition of Ω

$\Omega_P =$  $= \bigcup_{k=1}^{m} \tau_k$

Defining the Spectrum

Properties of the triangular partition Ω_P

1. $\Omega_P = \bigcup\limits_{k=1}^{m} \tau_k$
2. $\text{Area}\left(\Omega_P\right) = \text{Area}\left(\bigcup_{k=1}^{m} \tau_k\right)$
3. $\text{Area}\left(\tau_k\right) > 0, \quad k = 1 : m$
4. $\tau_k \cap \tau_j = \delta_k^j \times \text{Area}\ \left(\tau_j\right)$

Defining the Spectrum

Colloquially, it's a good mesh:

1. The mesh is watertight (sealed)
2. No triangles overlap
3. No triangles underlap

Defining the Spectrum

Define α_k as the area of triangle τ_k

1. Define α_k as the area of triangle τ_k
2. The sequence is in ascending order:

$$0 < \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_m$$

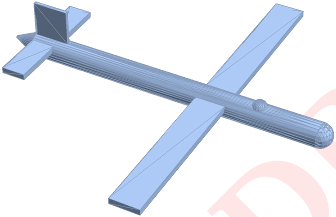
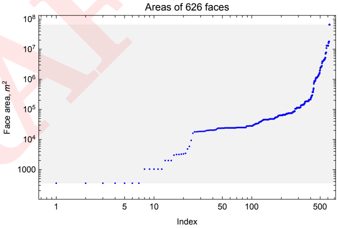
3. The spectrum is the sequence $\{\alpha_k\}_{k=1}^{m}$

Spectral Radius

Define T, the spectral radius as

$$T = \ln \tau_m - \ln \tau_1 = \ln \frac{\tau_m}{\tau_1}$$

Seeing the Spectral Radius

Mesh	Spectrum
	

Think of the height of the gray box as the spectral radius

Spectral Radius

Compare to κ the matrix condition number in the $2-$norm:

Given a matrix $\mathbf{A}$ of rank ρ with singular value spectrum $\{\sigma_k\}_{k=1}^{\rho}$
The matrix condition number is defined as

$$\kappa_2 = \frac{\|\mathbf{A}\|_2}{\|\mathbf{A}^{-1}\|_2} = \frac{\sigma_k}{\sigma_1}$$

$$T = \ln \frac{\tau_m}{\tau_1}$$

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