Radar Cross Section Data Compression

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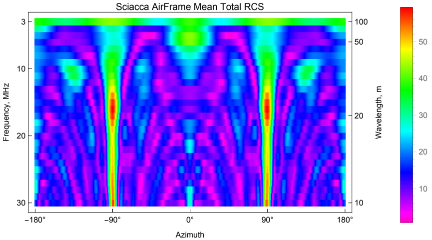
How to Condense Information?



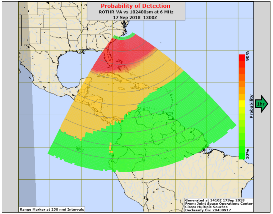


Table: Compression or reduction?

Data Compression Methods and Results

1. Modal Decomposition
   - 1.1 Fourier Series
   - 1.2 Taylor Series
2. Data Reduction
   - 2.1 Sampling
   - 2.2 Averaging

Radar as a Tool

1. Radar interrogation is a powerful tool
2. Reveal target details
3. Reveals environment details
4. Description of the complex electromagnetic field
    - 4.1 A: amplitude (strength)
    - 4.2 : i: complex unit modulus
    - 4.3 k: wavenumber (wavelength)
    - 4.4 r: position vector
    - 4.5 f: frequency
    - 4.6 t: time
    - 4.7 ϕ: phase

$$Ae^{i(k \cdot r - ft)} = Ae^{i\phi}$$

Extracting Information

Phase		Amplitude	
$\frac{\partial \phi}{\partial x}$	angle	$\frac{\partial A}{\partial x}$	shape
$\frac{\partial \phi}{\partial t}$	relative velocity	$\frac{\partial A}{\partial t}$	rotation
$\frac{\partial \phi}{\partial f}$	range	$\frac{\partial A}{\partial f}$	size

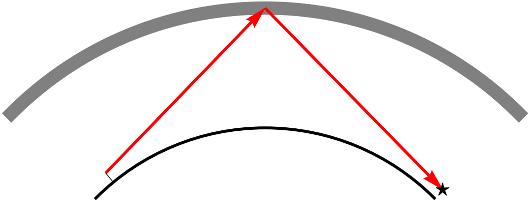
Table: *Intro. to Radar Systems*, M. I. Skolnik, §10.2

Exploit Radar Cross Section

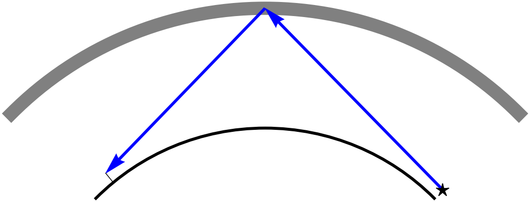
Exploit radar cross section to demonstrate how probability of detection varies with asset perspective.

Radar Cross Section: A Measure of Energy Difference

Incident ⇒



Scattered ⇐



Radar Cross Section: Discussion

- Rayleigh scattering
- Radar cross section is a far field phenomenon
- Assumes single polarization to and from target
- Target is completely metallic:
  E field results from surface currents
- Shape is quasi-dimensional
  - Dimensions in two known directions
  - Fuselage, wings
- Resonant scattering:
  Ratio of typical dimension to wavelength ≈ 1
- See Kolosov, §4.6

Effective Radar Cross Section: Definition

$$\sigma_{\star} = \frac{\text{power scattered per unit solid angle}}{\text{incident power density per } 4\pi} \tag{1.1}$$

Effective Radar Cross Section: Definition

$$\sigma_\star = 4\pi \lim_{r\to\infty} r^2 \left| \frac{E_{incident}}{E_{scattered}} \right|^2 \tag{1.2}$$

Early Progress: Report Delivered

Radar Cross Section Models for AFCAP Dashboard: Rapid Report 2020-02

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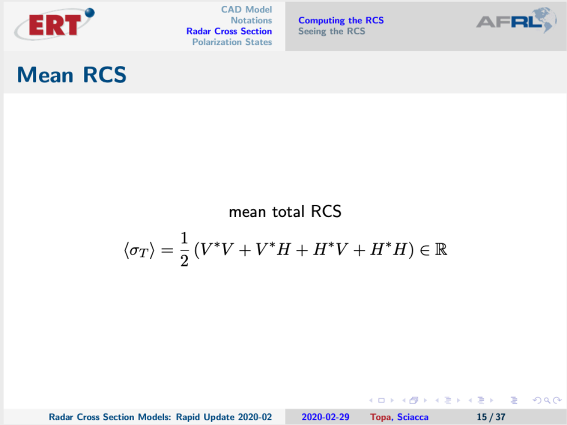
2020-02-29

From E Fields to RCS



Dramatis Personae: Phase I

Table: Sciacca data flow

FreeCAD		MATLAB		MM Viz		Hg MoM
*.obj	$\rightarrow$	*.facet	$\rightarrow$	*.geo	$\rightarrow$	*.4112.txt

Dramatis Personae: Phase II

Table: McGeorge data flow

FreeCAD		Python		Vim		Hg MoM
`*.obj`	→	`*.facet`	→	`*.geo`	→	`*.4112.txt`

Process Report: Delivered

Radar Cross Section Models for AFCAP Dashboard: Phase I Report

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2020-02-24

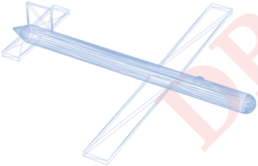
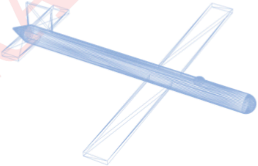
Mesh Mysteries Resolved: Report Delivered

Acceptable Meshes

Mesh Resolution Limit

Table: Resolution Edge in Pictures

PASS	FAIL
50 cm	10 cm
$T = 5.3$	$T = 6.5$

Acceptable Meshes

Mesh Resolution Limit

	Method	Mesh Resolution	Faces	Points	Spectral Radius
✓	Standard	1.0 m	626	315	5.3
✓	Standard	0.1 m	766	385	5.3
✓	Standard	0.05 m	1,198	601	5.3
✗	Standard	0.01 m	3,352	1,678	6.5
✗	Standard	0.001 m	28,394	14,199	8.7
✗	Mefisto	1.0 m	3,974	1,992	2.5
✗	Netgen	very fine	10,098	5,051	3.0

Table: One model, many meshes. How does Mercury MoM fare?

Communicating with MoM

Mercury Method of Moments Adjunct Visualization Tool: Trials and Tribulations

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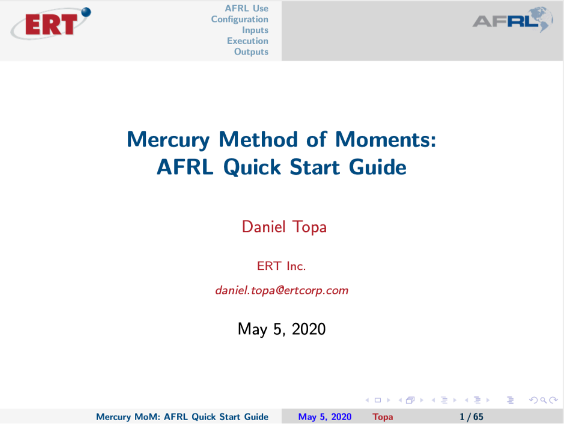
April 19, 2020

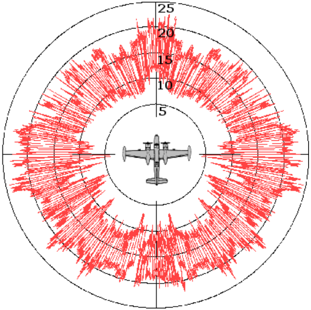
Abstract

While `Mercury MoM` has successfully modeled radar cross sections to test objects, the 64-bit adjunct tool, `MM Viz` has been problematic in the Windows 10 environment and most popular Linux distributions. One purpose of this report is to describe the purpose of the visualization tool as a pre– and post–processor for `Mercury MoM` to document the efforts to use it. Another purpose is to show that the intrinsic tool of mesh repair has a modern replacement and that the MATLAB scripts are replaced by Python scripts.

AFRL Quick Start Guide: Delivered



Current Dashboard Control



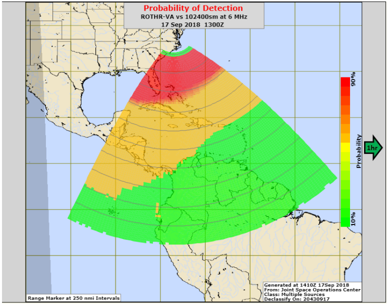


Table: Capture RCS Variability in the Dashboard

Nominative Dashboard Control

	current		enhanced
1.	select target type	1.	select asset (A, B, . . .)
2.	select RCS (2^k, $k \in \mathbb{Z}^+$)	2.	pick radar frequency in MHz $3 \leq \nu \leq 30$
		3.	pick yaw angle $\alpha \in [-\pi, \pi]$
		4.	pitch angle is fixed at $\beta = \pi/12$

Table: AFCAP Dashboard capability for RCS.

Config.xml

```
<Asset>
  <Label>16sm</Label>
  <ICONImage>Bald Eagle-sm.png</ICONImage>
  <crossSection>16</crossSection>
  <description>Aircraft</description>
  <nominalSpeed>400</nominalSpeed>
  <CIT>2.0</CIT>
</Asset>
```

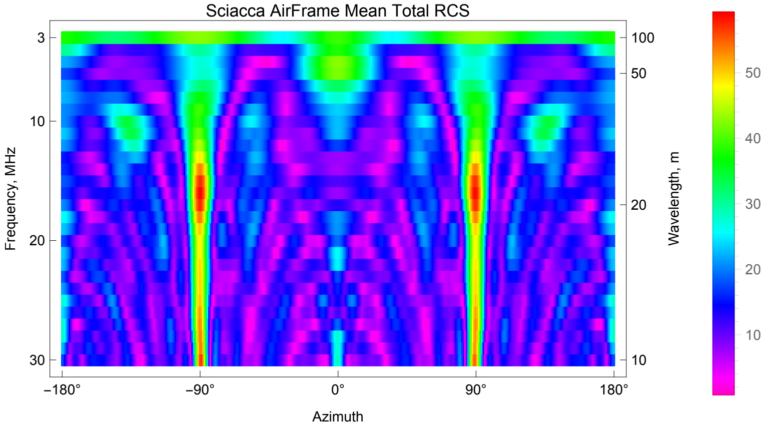
Probability of Detection

```
from frOPCclass.js
function plotProbability (ctx, jsonObj, jsonCoord, \
 xSection, assetCIT, nomSpeed) {
 ...
 var xSecRadius = Math.sqrt(xSection/Math.PI)
 var sphereArea = Math.PI * xSecRadius * xSecRadius;
 ...
 }
```

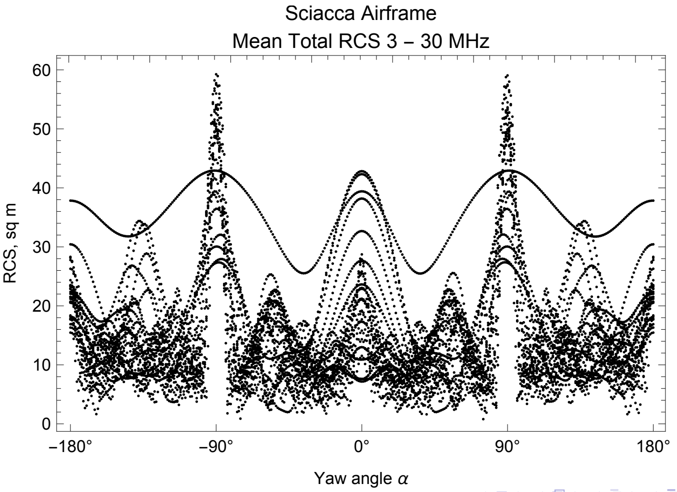
Rustic JavaScript Timing Loop

```
<html>
    <body>
        <button onclick='runDemo()'>Run</button>
        <label id='outputLabel'></label>
        <script type='text/javascript'>

            function calculate2(a0, a1, a2, alpha) {
                return a0/2 + (a1 * Math.cos(2.0 * alpha)) + (a2 * Math.cos(3.0 * alpha));
            }

            function getInterval(start) {
                return Date.now() - start;
            };

            function runDemo() {
                let start;

                const outputLabelElement = document.getElementById('outputLabel');
                var i;
                start = Date.now();
                for (i = 0; i < 10000; i++) {
                    outputLabelElement.innerText = 'Result: ' + calculate2(1, 2, 3, 0);
                }
                outputLabelElement.innerText = 'time for 10000, d = 2: ' + getInterval(start);
            }
        </script>
    </body>
</html>
```

RCS Data Table

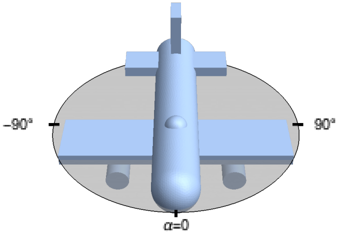


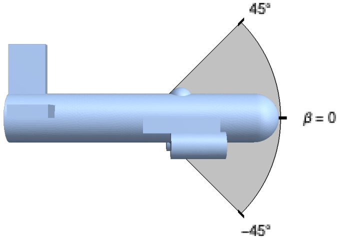
RCS Data Table



Defining Angles

Yaw α	Pitch β
$-180 \leq \alpha < 180$	$\beta_0 = 30°$





Best Way To Update Dashboard?

1. Compression
   - Fourier decomposition
   - Taylor decomposition
   - wavelets
2. Reduction
   - Average neighbors
   - Sampling

Best Way To Update Dashboard?

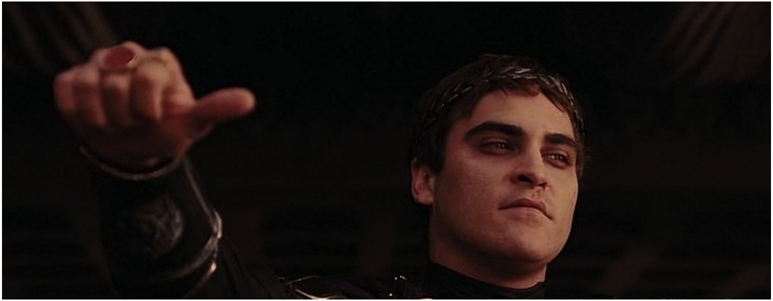


Figure: Criterion: Bonito Thumb Test

The Method of Least Squares

1. To find modal decompositions, use the method of least squares
2. Computes the amplitudes for each mode
3. Least squares is the most useful tool in applied mathematics
4. Elegant, powerful, too often misused

The Method of Least Squares

1. Provides best fit using familiar Euclidean norm
2. That is, uses orthogonal projection
3. Gauss–Markov theorem: best unbiased estimators
4. Quantifies quality of solution
5. Differentiable (unlike other $p-$norms)

Defining the Least Squares Problem

1. We focus on linear systems
2. Deep, and profound categorization:
3. The Fredholm Alternative

Start with Linear System

$$\mathbf{A}x = b \tag{2.1}$$

1. $\mathbf{A}$: system matrix (mesh)
   - m rows (measurements)
   - n columns (amplitudes)
   - ρ matrix rank (here $\rho = n$)
2. b: data vector (m measurements)
3. x: solution vector (n amplitudes)

Challenge: Inverse Problem

$$\mathbf{A}x = b$$

Given the matrix $\mathbf{A}$ and the data vector b,

Solve for the solution matrix x

Definition of Least Squares Solution

The set of least squares minimizers, x_{LS}, is defined as

$$x_{LS} = \arg\min_{x \in \mathbb{C}^n} \|\mathbf{A}\,x - b\|_2^2 \tag{2.2}$$

Least Squares: General Solution

1. particular + homogeneous solutions
2. That is, range + null space components
3. Usual story for solution of linear systems
4. We have existence, unclear on uniqueness

Least Squares: General Solution

$$
\begin{aligned}
x_{LS} &= x_p + x_h \\
&= \mathbf{A}^{\dagger} b + \left(\mathbf{I}_n - \mathbf{A}^{\dagger}\mathbf{A}\right) y, \qquad y \in \mathbb{C}^n \qquad (2.3) \\
&= \mathbf{A}^{\dagger} b + \mathbf{P}_{\mathcal{N}(\mathbf{A})} y, \qquad y \in \mathbb{C}^n
\end{aligned}
$$

Nomenclature

$\mathbf{A}^{\dagger}$: Moore-Penrose pseudoinverse matrix

$x_p = \mathbf{A}^{\dagger} b$: Least squares solution of minimum error norm

$\mathbf{P}_{\mathcal{N}(\mathbf{A})} = \left(\mathbf{I}_n - \mathbf{A}^{\dagger}\mathbf{A}\right)$: Projector on the null space $\mathcal{N}(\mathbf{A})$ (rows)

Error Propagation

$$\sigma_k^2 = \frac{\|\mathbf{A}\,x_p - b\|_2^2}{m-n}\,(\mathbf{A}^*\mathbf{A})_{k,k}^{-1} \tag{2.4}$$

Quantifies stability of amplitudes

compared to variation in measurements

Error Propagation

$$\sigma_k^2 = \frac{\|\mathbf{A}\, x_p - b\|_2^2}{m-n} \left(\mathbf{A}^* \mathbf{A}\right)^{-1}_{k,k}$$

Better measurements = more precise computations

Statement of Formal Results

$$\begin{gathered} a_0 \pm \sigma_0 \\ a_1 \pm \sigma_1 \\ \vdots \\ a_d \pm \sigma_d \end{gathered}$$

More concise vector notation: $a \pm \sigma$

Alert: Confusing Notation

Colliding canon:

σ_k = standard deviation or error in mathematics

$\sigma(\psi)$ = radar cross section in engineering

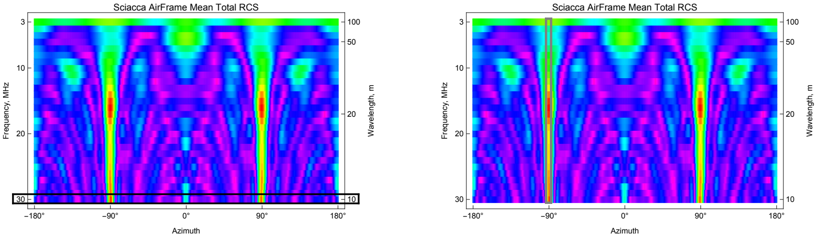
Two Paradigms for Decomposition

1. $\sigma(\alpha)$: RCS in yaw, frequency fixed
2. $\sigma(\nu)$: RCS in radar frequency, yaw angle fixed

Very different mathematical processes.

Two Paradigms for Decomposition

$\sigma(\nu)$ (row space)	$\sigma(\alpha)$ (column space)



Classic RCS: Radar Frequency Fixed

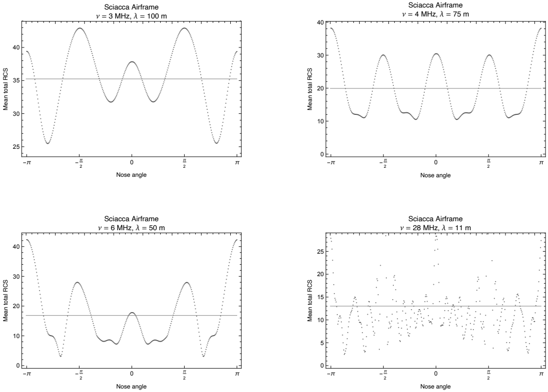


Table: A sampling of RCS computations for wavelengths of 100 m (top left), 75 m (top right), 50 m (bottom left), 11 m (bottom right).

Nelson RCS: Yaw Angle Fixed

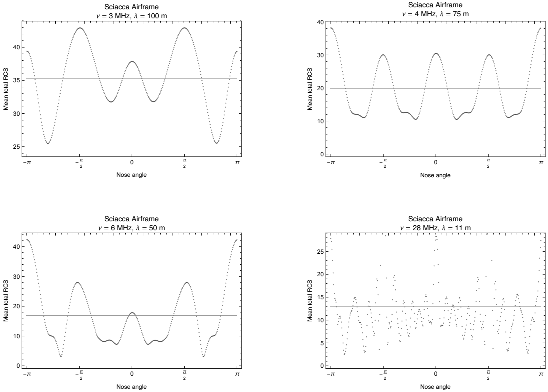


Table: A sampling of RCS computations for wavelengths of 100 m (top left), 75 m (top right), 50 m (bottom left), 11 m (bottom right).

Compare and Contrast the Results

1. Quality of fit
2. Expansion order
3. Aggregate performance

Why Fourier?

1. Problem is inherently 2π periodic

$$\sigma_\nu\left(\alpha, \beta\right) = \sigma_\nu\left(\alpha + 2\pi, \beta\right)$$

2. Problem is posed on unit circle
3. Problem screams Fourier!
4. Integral form of Maxwell's equation improves smoothness

Weierstrass Approximation Theorem

Polynomials are dense

in the space of continuous functions

with respect to the uniform norm.

Weierstrass Approximation Theorem

An amazingly powerful theorem...

1. Existence!
2. Guarantees uniform convergence over entire domain
3. We can pick the maximum acceptable error
4. Notice connection between periodicity and smoothness

Fourier in the Continuum

An expansion to order d takes the form...

$$\sigma_\nu\left(\alpha, \beta_0 = \tfrac{\pi}{12}\right) \approx \frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha + b_k \sin k\alpha.$$

Fourier in the Continuum

The decomposition entails finding the amplitudes a and b:

$$\sigma_\nu\left(\alpha, \beta_0 = \tfrac{\pi}{12}\right) \approx \frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha + b_k \sin k\alpha.$$

Weierstrass Again

Uniform continuity implies:
Given $\epsilon > 0$, there exists $N \in \mathbb{Z}^+$ such that:

$$\int_{-\pi}^{\pi} \left(\sigma_\nu(\alpha) - \frac{a_0}{2} - \sum_{k=1}^{N} a_k \cos k\alpha + b_k \sin k\alpha \right) d\alpha < \epsilon$$

Even Functions

$$f(\theta) = f(-\theta)$$

1. When the asset has bilateral symmetry, we can use even functions (cosines) for the basis
2. Cut computation in half
3. However, Mercury MoM introduces an odd–valued noise component

Solution: Projection a.k.a Least Squares

$$
\begin{aligned}
\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx &= \pi\delta_n^m \\
\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx &= \pi\delta_n^m \\
\int_{-\pi}^{\pi} \cos(mx)\sin(nx)\,dx &= 0 \\
\int_{-\pi}^{\pi} \sin(mx)\,dx &= 0 \\
\int_{-\pi}^{\pi} \cos(mx)\,dx &= 0 \\
\int_{-\pi}^{\pi} 1dx &= 2\pi
\end{aligned}
\tag{3.1}
$$

Decoupled Linear System: Continuous Topology

Solution for orthogonal system with continuum topology

$$
\begin{aligned}
a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \\
a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx dx \\
b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kx dx
\end{aligned} \tag{3.2}
$$

where $k = 1, 2, 3, \ldots, d$.

Decoupled Linear System: Discrete Topology

Solution for orthogonal system with discrete topolog

$$\begin{aligned} a_0 &= m^{-1} \sum_{k=1}^{m} \sigma\left(\alpha_k\right) \\ a_k &= m^{-1} \sum_{k=1}^{m} \cos\left(k\alpha_j\right) \sigma\left(\alpha_j\right) \\ b_k &= m^{-1} \sum_{k=1}^{m} \sin\left(k\alpha_j\right) \sigma\left(\alpha_j\right) \end{aligned} \tag{3.3}$$

where $k = 1, 2, 3, \ldots, d$.

Derivation of Solution

1. Derivation provides deep insight about the results
2. Augment derivation with simple example
3. Look at best and worst cases

Pose Linear Equations

$$\begin{array}{ccccccc} \frac{a_0}{2} & + & a_1 \cos \alpha_1 & + & a_2 \cos 2\alpha_1 & = & \sigma\left(\alpha_1, \beta_0\right) \\ \vdots & & \vdots & & \vdots & & \vdots \\ \frac{a_0}{2} & + & a_1 \cos \alpha_m & + & a_2 \cos 2\alpha_m & = & \sigma\left(\alpha_m, \beta_0\right) \end{array} \tag{3.4}$$

Pose Linear System

$$\begin{array}{cccc} \mathbf{A} & a & = & b \\ \begin{bmatrix} 1 & \cos\alpha_1 & \cos 2\alpha_1 \\ \vdots & \vdots & \vdots \\ 1 & \cos\alpha_m & \cos 2\alpha_m \end{bmatrix} & \begin{bmatrix} \frac{a_0}{2} \\ a_1 \\ a_2 \end{bmatrix} & = & \begin{bmatrix} \sigma\left(\alpha_1\right) \\ \vdots \\ \sigma\left(\alpha_m\right) \end{bmatrix} \end{array} \tag{3.5}$$

Rows and Columns

Fun facts:

1. Gaussian elimination, Cholesky decomposition, etc. use row operations
2. The Method of Least Squares uses column operations
3. Let's cast the problem in terms of column vectors

Column Vectors

Define the $m-$vectors (column vectors):

$$(\cos\alpha) = \begin{bmatrix} \cos\alpha_1 \\ \cos\alpha_2 \\ \vdots \\ \cos\alpha_m \end{bmatrix}, \quad (\cos 2\alpha) = \begin{bmatrix} \cos 2\alpha_1 \\ \cos 2\alpha_2 \\ \vdots \\ \cos 2\alpha_m \end{bmatrix},$$

$$\mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad (\sigma(\alpha)) = \begin{bmatrix} \sigma(\alpha_1) \\ \sigma(\alpha_2) \\ \vdots \\ \sigma(\alpha_m) \end{bmatrix}$$

Revised Linear System

$$
\begin{array}{ccc}
\mathbf{A} & a & = & b \\
\begin{bmatrix} \mathbf{1} & (\cos\alpha) & (\cos 2\alpha) \end{bmatrix} & \begin{bmatrix} \frac{a_0}{2} \\ a_1 \\ a_2 \end{bmatrix} & = & \begin{bmatrix} (\sigma(\alpha)) \end{bmatrix}
\end{array} \tag{3.6}
$$

What Solution Method to Use?

To classify the solution method, use the Fredholm Alternative

Fredholm Alternative

1. Either the linear system has an exact solution a such that $\|\mathbf{A}a - b\|_2 = 0$
2. Or there exists a vector $y \in \mathbb{R}^m$ such that

$$\mathbf{A}^* y = 0 \text{ and } b \cdot y \neq 0$$

Fredholm Alternative

1. The Fredholm Alternative is a statement of about the residency of the data vector
2. Our problem is the latter case
3. There is no exact solution
4. The data vector straddles both range and null spaces

$$b = b_{\mathcal{R}(\mathbf{A})} + b_{\mathcal{N}(\mathbf{A}^*)} \in \mathcal{R}(\mathbf{A}) \oplus \mathcal{N}(\mathbf{A}^*)$$

5. Range space component is the signal
6. Null space component is noise
7. Signal to noise will limit fidelity of fit

Geometric View

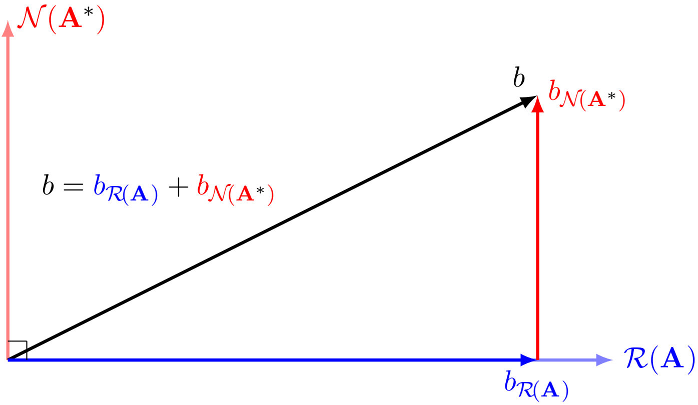


Figure: Decomposition of the data vector b in terms of the orthogonal range space and null space components.

Insight From Fredholm Alternative

1. Fundamental problem:
   data vector b in not in the column space of $\mathbf{A}$
2. That is, data vector resides in range and null space
3. Canonical Trick (Column manipulation):
   Premultiply left– and righthand sides by $\mathbf{A}^*$
4. This forces the righthand side, $\mathbf{A}^*b$,
   to be the column space of $\mathbf{A}^*$

Pose Normal Equations

$$\mathbf{A}^*\mathbf{A}a = \mathbf{A}^*b$$

$$\mathbf{A}^*\mathbf{A} = \left[\begin{array}{rrr} \mathbf{1}\cdot\mathbf{1} & \mathbf{1}\cdot(\cos\alpha) & \mathbf{1}\cdot(\cos 2\alpha) \\ (\cos\alpha)\cdot\mathbf{1} & (\cos\alpha)\cdot(\cos\alpha) & (\cos\alpha)\cdot(\cos 2\alpha) \\ (\cos 2\alpha)\cdot\mathbf{1} & (\cos 2\alpha)\cdot(\cos\alpha) & (\cos 2\alpha)\cdot(\cos 2\alpha) \end{array}\right],$$

$$\mathbf{A}^*b = \left[\begin{array}{r} \mathbf{1}\cdot\sigma(\alpha,\beta_0) \\ (\cos\alpha)\cdot\sigma(\alpha,\beta_0) \\ (\cos 2\alpha)\cdot\sigma(\alpha,\beta_0) \end{array}\right]. \tag{3.7}$$

Solve Normal Equations

For $d + 1 \geq m$, a unique solution exists:

$$\mathbf{A}^*\mathbf{A}a = \mathbf{A}^*b \tag{3.8}$$

$$\Downarrow$$

$$a_{LS} = \left(\mathbf{A}^*\mathbf{A}\right)^{-1}\mathbf{A}^*b \tag{3.9}$$

Solve Normal Equations

For $d + 1 \geq m$, a unique solution exists:

$$\mathbf{A}^*\mathbf{A}a = \mathbf{A}^*b \tag{3.10}$$

$$\Downarrow$$

$$a_{LS} = \left(\mathbf{A}^*\mathbf{A}\right)^{-1}\mathbf{A}^*b \tag{3.11}$$

Least Squares Solution

1. We have selected a value for d, and,
2. Computed the least squares solution a_{LS}
3. Task is not completed
4. Next: error quantification...

Square of Residual Error Vector

Foundation of error analysis: r^2, least total error

$$r^2 = r_k\left(a_{LS}\right) \cdot r_k\left(a_{LS}\right) \tag{3.12}$$

1. Foundation for error propagation
2. a.k.a. sum of the squares of the residual errors
3. Difference between measurement and prediction
4. Can determine best order of fit d
5. Used to compute uncertainty in amplitudes
6. Measures projection of data vector into null space
7. Measures noise component

Quantification of Fit Quality

1. Least squares quantifies quality of answer
    1.1 Provides error bounds
    1.2 Identifies significant digits
    1.3 Quantifies signal–to–noise
2. Too often ignored
3. Error in kth amplitude (eq. (42)):

$$\epsilon_k = \sqrt{\frac{r^2}{m-(d+1)} \left(\mathbf{A}^* \mathbf{A}\right)^{-1}_{kk}} \tag{3.13}$$

Quantification of Fit Quality

$$\epsilon_k = \sqrt{\frac{r^2}{m-(d+1)}\left(\mathbf{A}^*\mathbf{A}\right)^{-1}_{kk}} \tag{3.13}$$

1. Reduces to standard deviation for $d = 0$
2. r^2: least total error
3. m: number of measurements
4. $d+1$: number of fit parameters
5. $(\mathbf{A}^*\mathbf{A})^{-1}$: curvature matrix

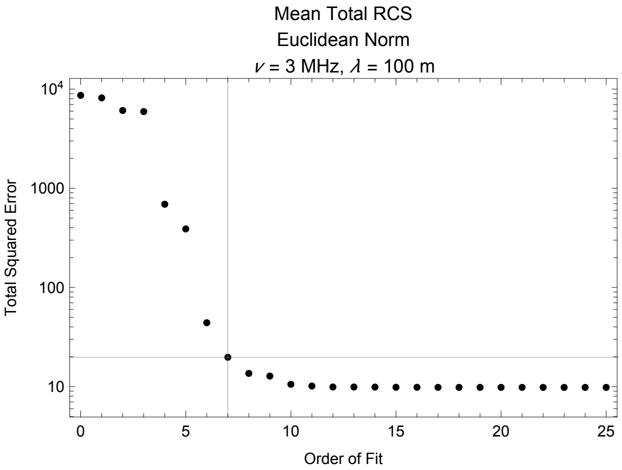
How Many Terms Do We Need?

1. Discuss criteria for fit truncation
2. $\|r\|_2$
3. $\|r\|_\infty$
4. Signal–to–Noise
5. Cross hairs on solution point

Total Error

1. Ensuing scatter plot shows total error for $d = 0, 1, 2, \ldots, 25$
2. Promise of Weierstrass blunted by noise in data
3. Demonstrates an improvement plateau
4. Evinces diminishing returns for computational burden

Total Error


Mean Total RCS
Euclidean Norm
ν = 3 MHz, λ = 100 m
Total Squared Error
10^4
1000
100
10
0
5
10
15
20
25
Order of Fit

Weierstrass Criteria

1. Ensuing scatter plot shows total error for $d = 0, 1, 2, \ldots, 25$
2. Promise of Weierstrass blunted by noise in data
3. Demonstrates an improvement plateau
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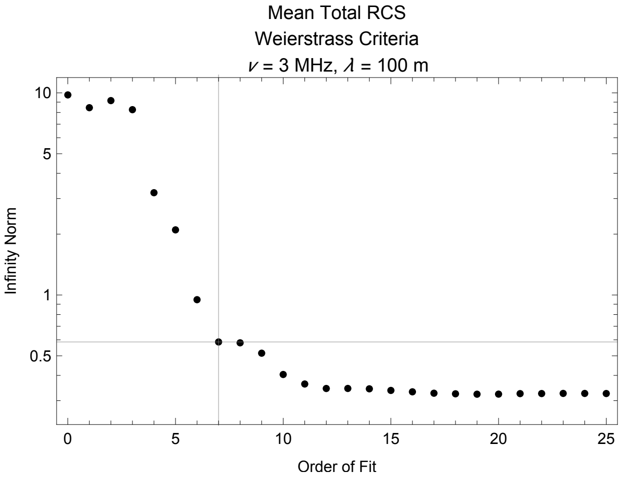
5. Comparable to total squared error

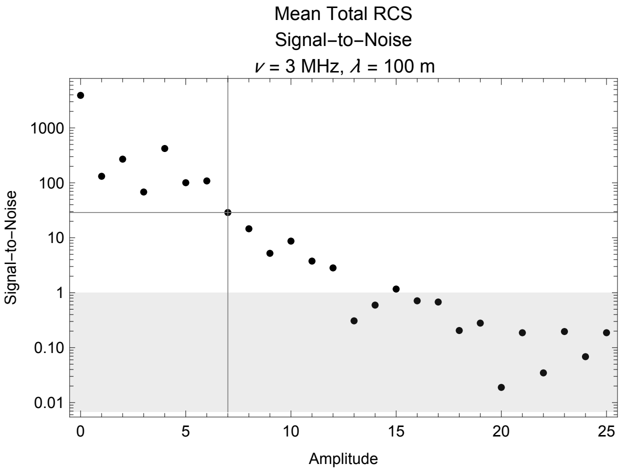
Weierstrass Criteria



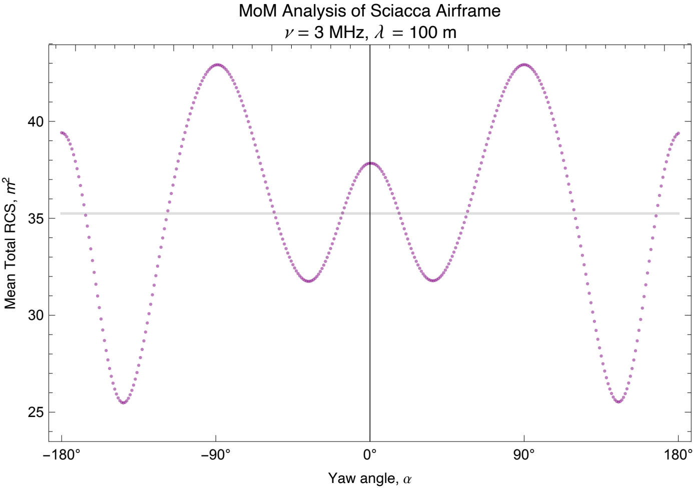
Signal–to–Noise

1. Where is the point of computational futility?
2. That is, when does error dominate signal?
3. Look at the highest order term for each degree of fit
4. Chart shows $\left|\frac{a_d}{\epsilon_d}\right|$, $d = 0, 2, \ldots, 25$
5. Evinces diminishing returns for computational burden
6. Comparable to total squared error

Signal–to–Noise



Demonstration Problem: $\nu = 3$ MHz $(\lambda = 100$ m$)$



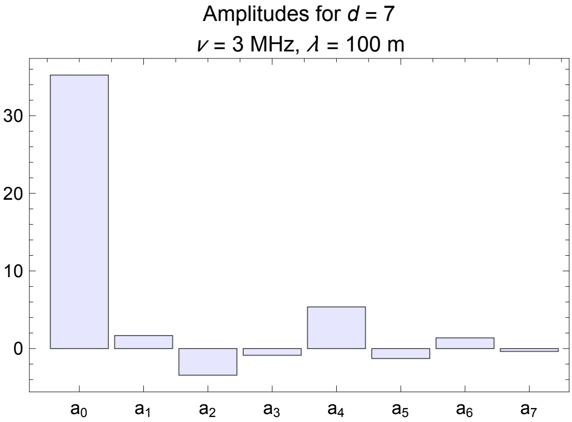
Mission Accomplished

1. Solution function: use $d = 7$
2. Made in consideration of:
   2.1 Basic agreement to data (slide 101)
   2.2 Strength of amplitude (slide 76), including
   2.3 Signal–to–Noise performance (slide 83)
   2.4 Total error performance ($\|r\|_2^2$, slide 79; $\|r\|_\infty$, slide 81)
3. Let's examine the solution...

Dominant Terms

1. Ensuing bar chart shows amplitudes visually
2. Mean $(d = 0)$ term dominates
3. Shows relative contributions by each mode
4. Even terms dominate odd terms:
5. $|a_2| > |a_1|$, $|a_4| > |a_3|$, $|a_6| > |a_5|$
6. Within the even terms:
7. $|a_0| > |a_4| > |a_2| > |a_6|$
8. Shows decreasing contribution of higher order modes
9. Advantage of using Fourier decomposition

Dominant Terms

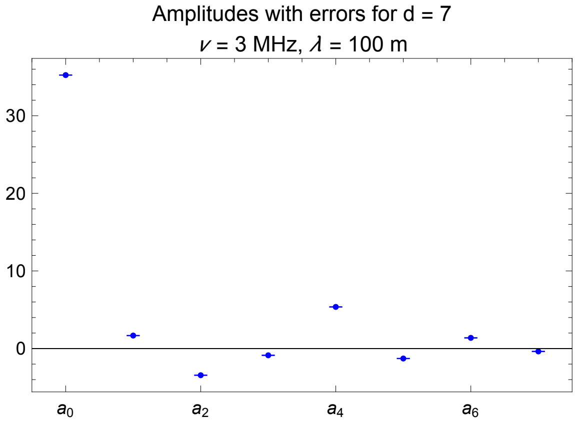


Error Propagation

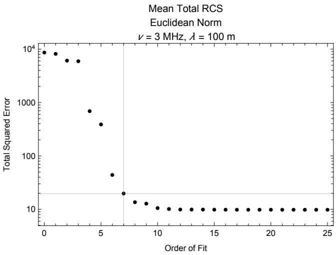
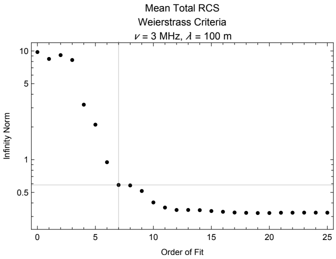
About the following graph of the amplitudes with error bars:

1. Error bars indicate stability against perturbations in the data
2. Quantifies quality of amplitude calculation
3. Advantage of using Least Squares
4. Signal–to–Noise

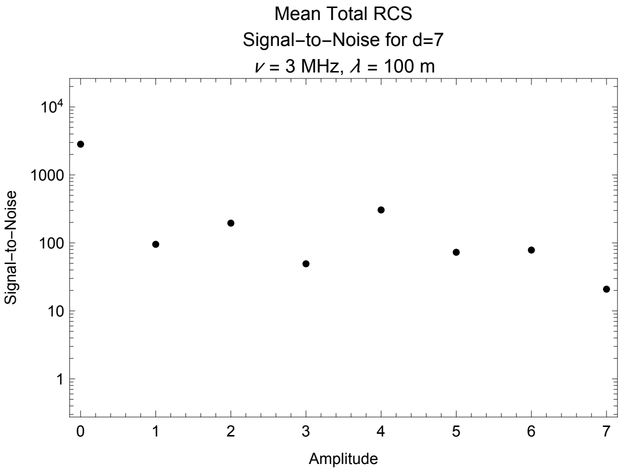
Error Propagation



Truncation Criteria: Error Norms

$\|r\|_2^2$	$\|r\|_\infty$
	
Least Squares	Weierstrass

Truncation Criteria: Error Norms



The Intuition of Fourier

1. The Fourier series lends itself to intuition
2. Present the solution as an accumulation of corrections
3. $d = 0, 1, 2, 3, 4, 5, 6, 7$
4. Sequence shows refinement of solution

Behavior of the Residual Errors

1. Looking at data vs fit is a great start
2. But also a terrible place to stop
3. Examine residual errors
4. Examine amplitude errors

Fit and Residual Errors for $d = 0$

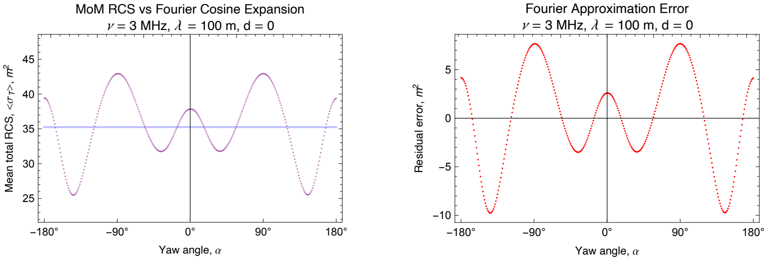


Table: Lowest order term: the mean value

Fit and Residual Errors for $d = 1$

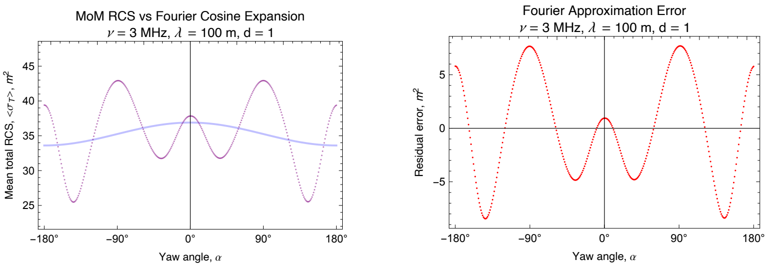
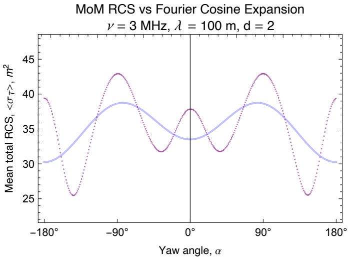


Table: Add contribution of $\cos\alpha$ term.

Fit and Residual Errors for $d = 2$



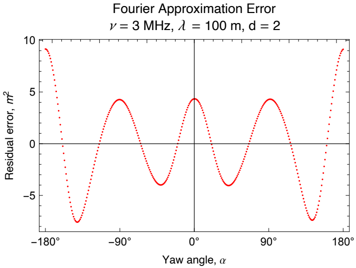
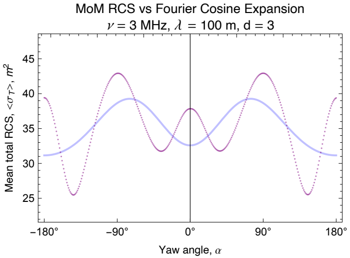


Table: Add contribution of $\cos 2\alpha$ term.

Fit and Residual Errors for $d = 3$



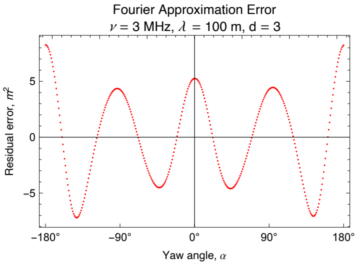
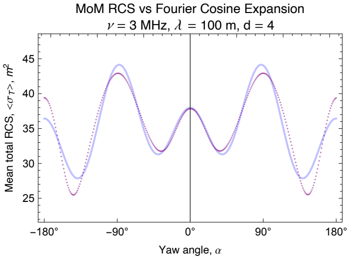


Table: Add contribution of $\cos 3\alpha$ term.

Fit and Residual Errors for $d = 4$



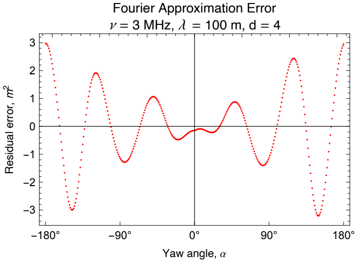
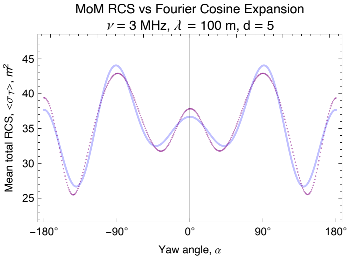


Table: Add contribution of $\cos 4\alpha$ term.

Fit and Residual Errors for $d = 5$



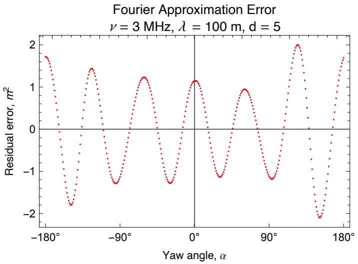
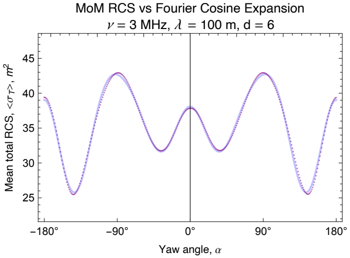


Table: Add contribution of $\cos 5\alpha$ term.

Fit and Residual Errors for $d = 6$



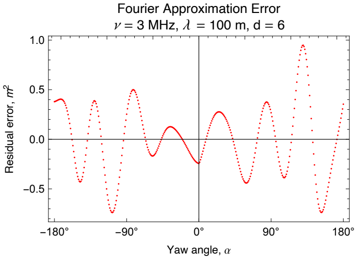


Table: Add contribution of $\cos 6\alpha$ term.

Fit and Residual Errors for $d = 7$

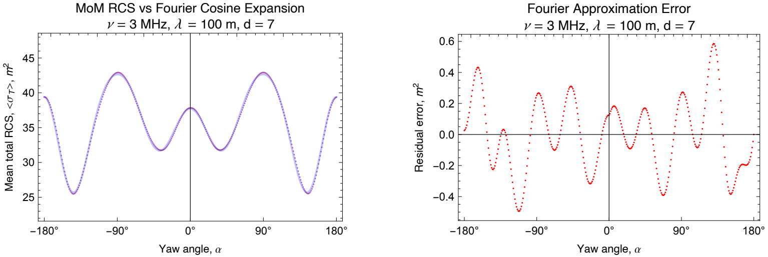


Table: Highly correlated residual errors reveal higher harmonic structures and contributions from odd functions.

Solutions: Amplitudes for Each Order d

	d	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
	0	35.2488							
	1	35.2533	1.64293						
	2	35.2626	1.62429	-3.38356					
	3	35.2601	1.62928	-3.38855	-0.912138				
	4	35.2455	1.65863	-3.41790	-0.882795	5.38447			
	5	35.2420	1.66561	-3.42488	-0.875809	5.37749	-1.28893		
	6	35.2383	1.67306	-3.43233	-0.868366	5.37004	-1.28148	1.38079	
$\Rightarrow$	7	35.2373	1.67502	-3.43429	-0.866400	5.36808	-1.27952	1.37883	-0.366535

Amplitudes and Errors for $\nu = 3$ MHz and $d = 7$

$$\sigma_3(\theta) = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta + a_4 \cos 4\theta + a_5 \cos 5\theta + a_6 \cos 6\theta + a_7 \cos 7\theta$$

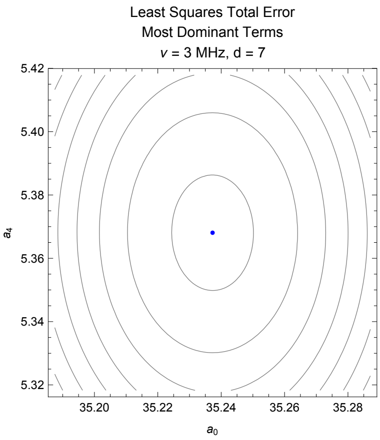
$$\begin{aligned}
a_0 &= 35.237 \pm 0.012\\
a_1 &= 1.675 \pm 0.018\\
a_2 &= -3.434 \pm 0.018\\
a_3 &= -0.866 \pm 0.018\\
a_4 &= 5.368 \pm 0.018\\
a_5 &= -1.280 \pm 0.018\\
a_6 &= 1.379 \pm 0.018\\
a_7 &= -0.367 \pm 0.018
\end{aligned}$$

Solution Function for $d = 7$

Fourier representation for the RCS at $\nu = 3$ MHz:

$$\boxed{\begin{aligned}\sigma_{\nu=3}(\alpha) \approx 35.237 &+ 1.675\cos\alpha - 3.434\cos 2\alpha \\ &- 0.866\cos 3\alpha + 5.368\cos 4\alpha - 1.280\cos 5\alpha \\ &+ 1.379\cos 6\alpha - 0.367\cos 7\alpha\end{aligned}} \tag{3.14}$$

Stability of the Solution: Dominant Terms



Stability of the Solution: Dominant Terms

Following slide shows merit function minimized the least squares fit:

$$\|r\|_2^2 = \sum_{j=1}^{m} \left(\frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha_j - \sigma_\nu \left(\alpha_j\right) \right)^2$$

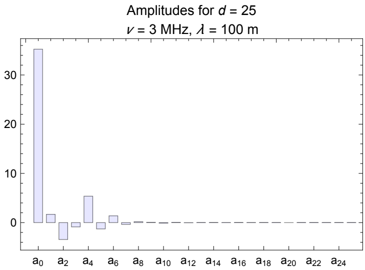
1. Visually establishes minimum value is attained
   1.1 Well, for a_0 and a_4
   1.2 Inspect remaining 6 parameters in 3 pairs
2. Objective measure of stability against perturbations of data
   2.1 Scale is $1-1$
   2.2 Consider a small change δ
   2.3 Substitution $a_0 \pm \delta$ will cause a greater error than $a_4 \pm \delta$

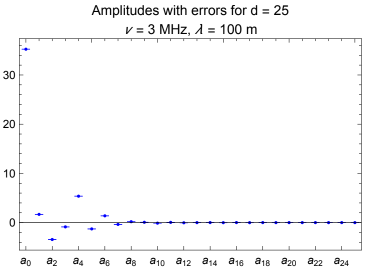
Higher Order Fits

About the two graphs on the next slide:

1. High order fit with $d = 25$
2. Rapid diminution of amplitudes (Riesz-Fischer)
3. Second graph shows amplitudes with error
4. Let's zoom in on the error bars...

Higher Order Fits



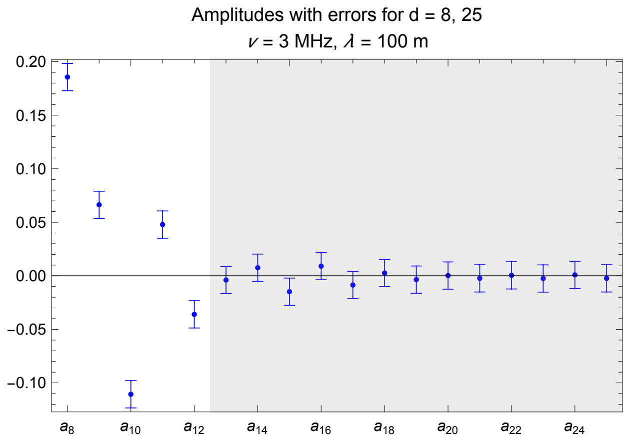


Higher Order Fits

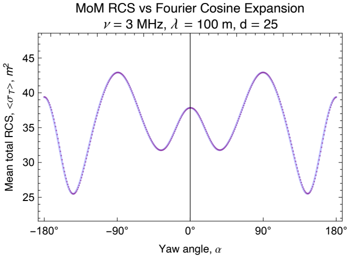
Zoom in on the plot with error bars:

1. These terms were discarded
2. $d \geq 13$: No computational value
3. $d = 8 - 13$: Well characterized, but...
4. Small contribution

Higher Order Fits



Higher Order Fits



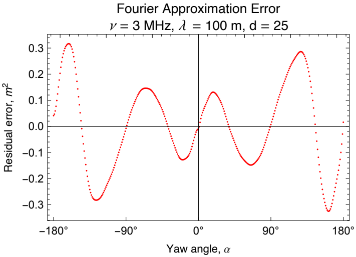
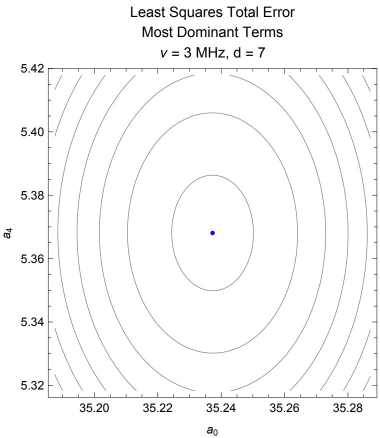


Table: The remaining error is an odd function. Even contributions are removed.

Stability of the Solution: Dominant Terms



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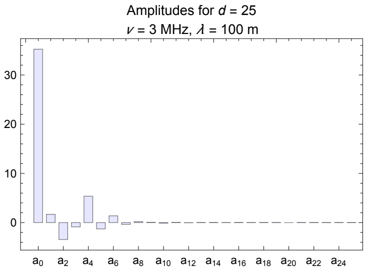
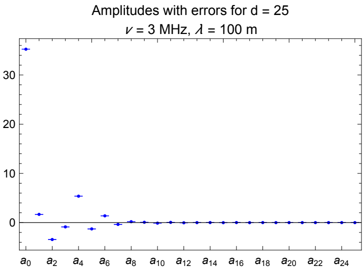
Higher Order Fits





Higher Order Fits

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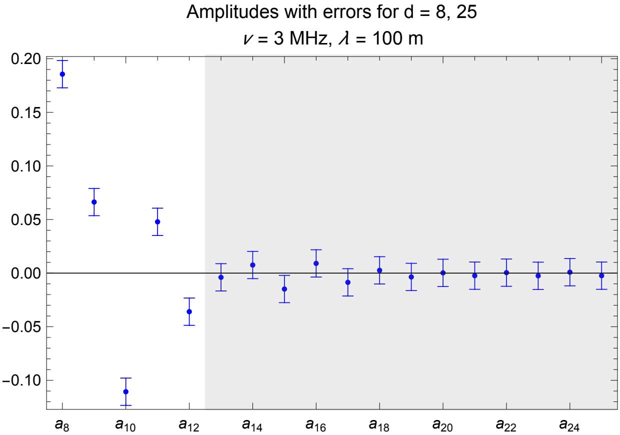
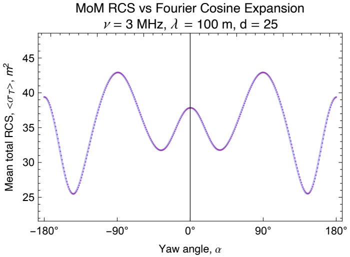
Higher Order Fits



Higher Order Fits



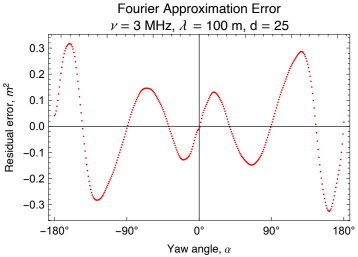


Table: The remaining error is an odd function. Even contributions are removed.

Whoops ⚠

Inspection of the amplitude sequence fits reveals a defect

Whoops ⚠

d	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
0	35.2488							
1	35.2533	1.64293						
2	35.2626	1.62429	−3.38356					
3	35.2601	1.62928	−3.38855	−0.912138				
4	35.2455	1.65863	−3.41790	−0.882795	5.38447			
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7	35.2373	1.67502	−3.43429	−0.866400	5.36808	−1.27952	1.37883	−0.366535
	⇑							

1. There is a problem
2. If we are using orthogonal functions, the amplitudes for each order α_k must be constant
3. That is, the numbers within each column are the same value
4. For example: a_0: there are 8 values for the mean!

Whoops ⚠

Sciacca formulation destroys orthogonality

1. System no longer decoupled
2. Can't solve mode–by–mode
3. Must use linear independence
4. Must solve dense linear system
5. Details to follow...

Whoops ⚠

Consequences:

1. Product matrix $\mathbf{A}^*\mathbf{A}$ not diagonal
2. Linear system is not decoupled
3. Must solve dense linear system
4. Must compute all nodes at once
5. Example follows...

Consequences: Product Matrix

$$\text{orthogonal: } \mathbf{A}^*\mathbf{A} = 180 \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.15}$$

$$\text{not orthogonal: } \mathbf{A}^*\mathbf{A} = \begin{bmatrix} 361 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 181 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 181 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 181 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 181 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 181 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 181 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 181 \end{bmatrix} \tag{3.16}$$

Moral of the Story

$$\mathbf{A}^*\mathbf{A} = \begin{bmatrix} 361 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 181 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 181 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 181 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 181 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 181 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 181 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 181 \end{bmatrix} \tag{3.16}$$

⚠ You don't invert this matrix by pretending it is diagonal

Consequences: Product Matrix Inverse

$$\text{orthogonal:} \quad (\mathbf{A}^*\mathbf{A})^{-1} = \frac{1}{180}\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.17}$$

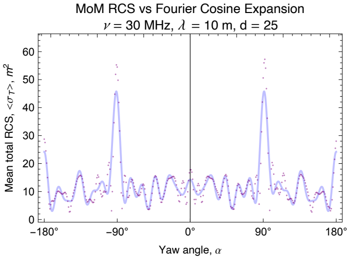
$$\text{not orthogonal:} \quad (\mathbf{A}^*\mathbf{A})^{-1} = \begin{bmatrix} 361 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 181 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 181 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 181 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 181 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 181 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 181 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 181 \end{bmatrix} \tag{3.18}$$

Worst Case

We saw the fit for the best ($\nu = 3$ MHz, $\lambda = 100$ m).

Let's look at the worst ($\nu = 30$ MHz, $\lambda = 10$ m).

Worst Case: $\nu = 30$ MHz, $\lambda = 10$ m



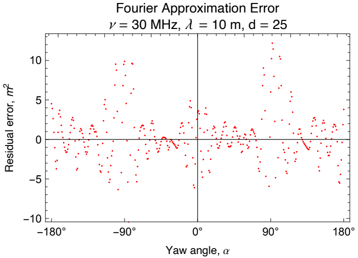


Table: Worst case is the highest physical resolution.

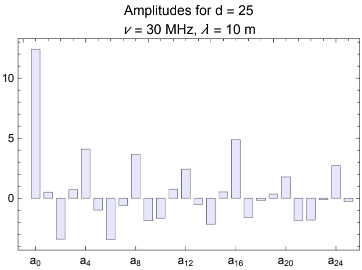
Amplitudes and Errors for $\nu = 30$ MHz and $d = 25$

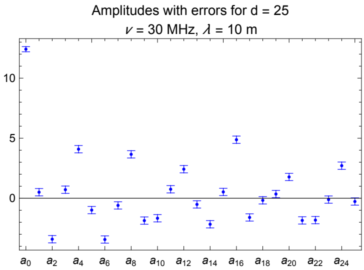
$$
\begin{aligned}
\sigma_{30}(\theta) = {} & a_0 + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta + a_4 \cos 4\theta + a_5 \cos 5\theta \\
& + a_6 \cos 6\theta + a_7 \cos 7\theta + a_8 \cos 8\theta + a_9 \cos 9\theta + a_{10} \cos(10\theta) \\
& + a_{11} \cos(11\theta) + a_{12} \cos(12\theta) + a_{13} \cos(13\theta) + a_{14} \cos(14\theta) \\
& + a_{15} \cos(15\theta) + a_{16} \cos(16\theta) + a_{17} \cos(17\theta) + a_{18} \cos(18\theta) \\
& + a_{19} \cos(19\theta) + a_{20} \cos(20\theta) + a_{21} \cos(21\theta) + a_{22} \cos(22\theta) \\
& + a_{23} \cos(23\theta) + a_{24} \cos(24\theta) + a_{25} \cos(25\theta)
\end{aligned}
$$

Amplitudes and Errors for $\nu = 30$ MHz and $d = 25$

$$
\begin{array}{rcrcl}
a_0 & = & 12.41 & \pm & 0.21 \\
a_1 & = & 0.50 & \pm & 0.30 \\
a_2 & = & -3.41 & \pm & 0.30 \\
a_3 & = & 0.71 & \pm & 0.30 \\
a_4 & = & 4.90 & \pm & 0.30 \\
a_5 & = & -0.99 & \pm & 0.30 \\
a_6 & = & -3.44 & \pm & 0.30 \\
a_7 & = & -0.60 & \pm & 0.30 \\
a_8 & = & 3.66 & \pm & 0.30 \\
a_9 & = & -1.86 & \pm & 0.30 \\
a_{10} & = & -1.67 & \pm & 0.30 \\
a_{11} & = & 0.75 & \pm & 0.30 \\
a_{12} & = & 2.42 & \pm & 0.30
\end{array}
\qquad
\begin{array}{rcrcl}
a_{13} & = & -0.51 & \pm & 0.30 \\
a_{14} & = & -2.17 & \pm & 0.30 \\
a_{15} & = & 0.53 & \pm & 0.30 \\
a_{16} & = & 4.87 & \pm & 0.30 \\
a_{17} & = & -1.60 & \pm & 0.30 \\
a_{18} & = & -0.17 & \pm & 0.30 \\
a_{19} & = & 0.35 & \pm & 0.30 \\
a_{20} & = & 1.77 & \pm & 0.30 \\
a_{21} & = & -1.85 & \pm & 0.30 \\
a_{22} & = & -1.82 & \pm & 0.30 \\
a_{23} & = & -0.11 & \pm & 0.30 \\
a_{24} & = & 2.71 & \pm & 0.30 \\
a_{25} & = & -0.27 & \pm & 0.30
\end{array}
$$

Truncation Criteria





Truncation Criteria

Find the minimum order of fit d such that

$$\|\text{measurement} - \text{prediction}\|_2 \leq 5 \text{ sq m} \tag{3.19}$$

at 99.7% confidence level (3σ) for the entire data set.

Truncation Criteria: Delivered

Choosing Fourier Representations
For $\nu = 3, 30$ MHz

Daniel Topa

May 4, 2020

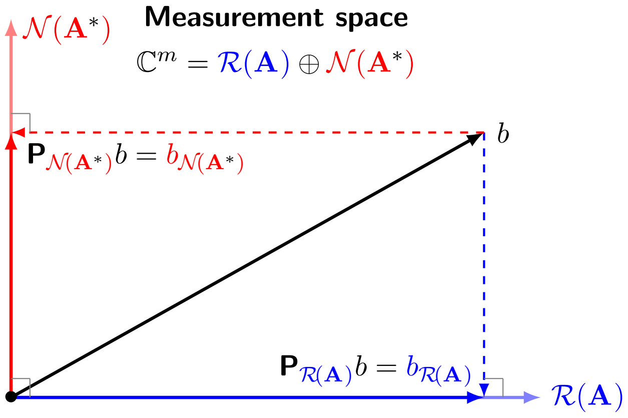
1 Truncation Criteria

Having decided on the Fourier representation basis for the Mercury MoM output data, the immediate question becomes where to truncate the approximation:

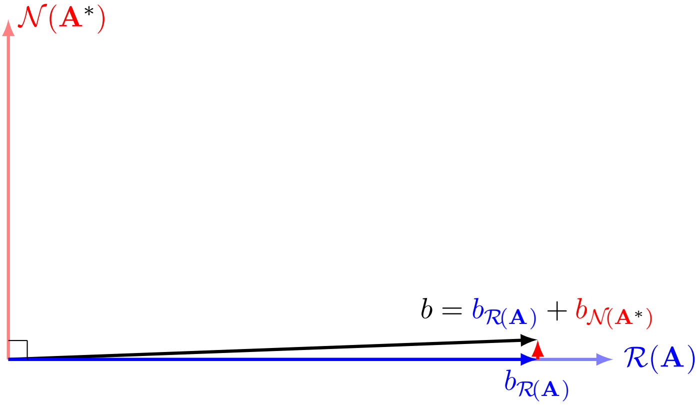
$$f(\alpha) \approx \frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha \tag{1}$$

This is tantamount to finding a degree d such that a global constraint is satisfied. Due to operational constraints, there is a real pressure to minimize the impact of computation in the host environment. And so the the challenge is to strike a balance between computation complexity and fidelity of representation.

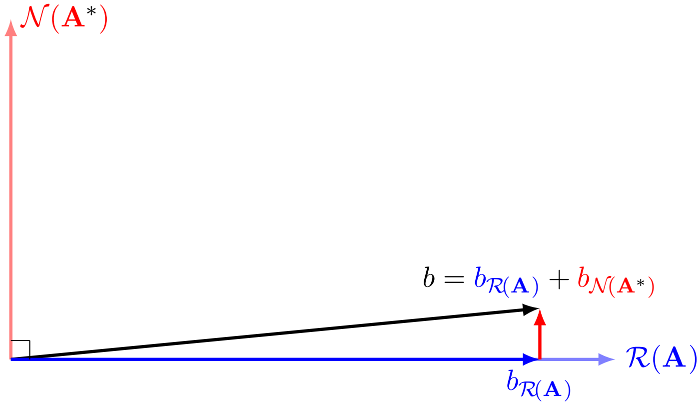
Truncation Criteria and Fredholm Diagram



Best Case: $\nu = 3$ MHz, order of fit $d = 4$

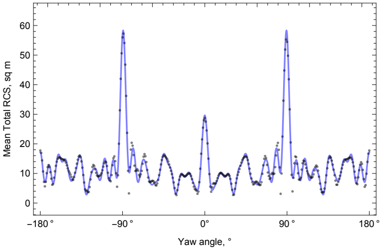


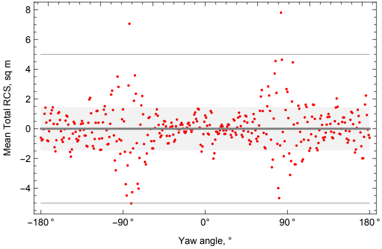
Worst Case: $\nu = 30$ MHz, order of fit $d = 40$



Truncation Criteria: Need For Statistical Criteria

Outlier points for $\nu = 30$ MHz, $d = 40$.





Why Taylor?

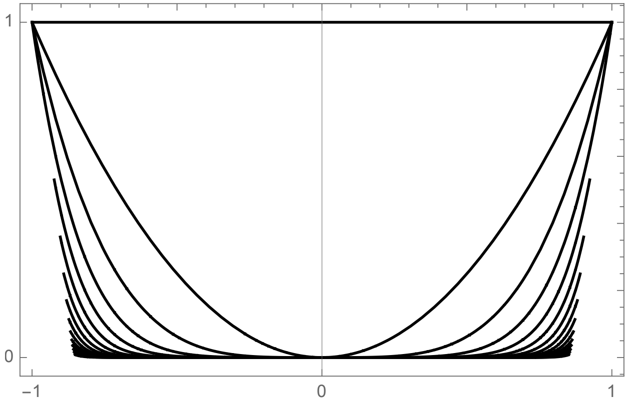
1. Problem is inherently discontinuous
2. Measurements don't manifest periodic structures
3. Monomial basis is affine-equivalent to orthogonal polynomial functions

Monomials in the Continuum

An expansion to order d takes the form...

$$\sigma_\alpha\left(\nu\right) \approx c_0 + \sum_{k=1}^{d} c_k \nu^k$$

Monomials: The Highway to Ill Conditioning



Rescale Coordinate System

$$[-180, 180) \mapsto [-1, 1] \tag{4.1}$$

Reduce matrix condition number κ_2

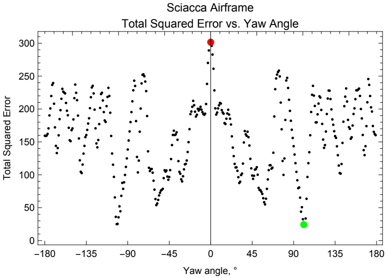
Pose Linear Equations

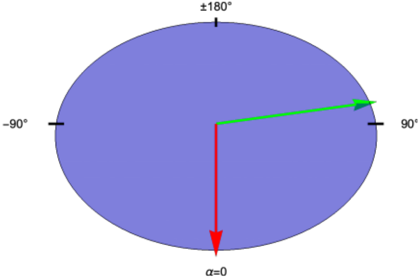
$$\begin{array}{ccccccc} c_0 & + & c_1\nu_1 & + & c_2\nu_1^2 & = & \sigma(\nu_1) \\ \vdots & & \vdots & & \vdots & & \vdots \\ c_0 & + & c_1\nu_m & + & c_2\nu_m^2 & = & \sigma(\nu_m) \end{array} \tag{4.2}$$

Pose Linear System

$$\begin{array}{ccc} \mathbf{A} & a & = & N \\ \begin{bmatrix} 1 & \nu_1 & \nu_1^2 \\ \vdots & \vdots & \vdots \\ 1 & \nu_m & \nu_m^2 \end{bmatrix} & \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} & = & \begin{bmatrix} \sigma\left(\nu_1\right) \\ \vdots \\ \sigma\left(\nu_m\right) \end{bmatrix} \end{array} \tag{4.3}$$

Extremal Values of Total Error





The Best Fit and the Worst Fit

1. Data with Best Fit
   1.1 Total squared error $r^2 = 24.4$
   1.2 Yaw angle $\alpha = 101°$
   1.3 Only amplitude with signal–to–noise < 1: linear term, a_0
2. Data with Worst Fit
   2.1 Total squared error $r^2 = 301.5$
   2.2 Yaw angle $\alpha = 0°$: nose on
   2.3 Only amplitude with signal–to–noise < 2: a_4, a_6

Best Case: $\alpha = 101°$

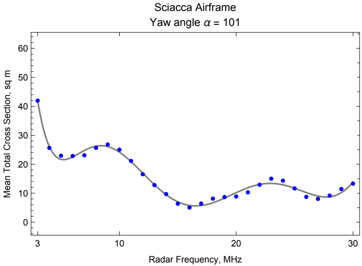
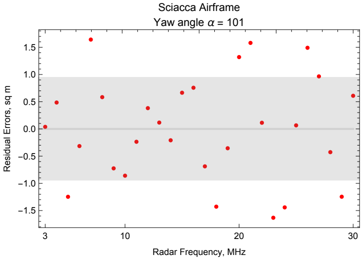
Data v. Fit	Residual Error
	

Table: Best fit occurs at yaw angle $\alpha = 101°$

Best Case: $\alpha = 101^\circ$, Amplitudes

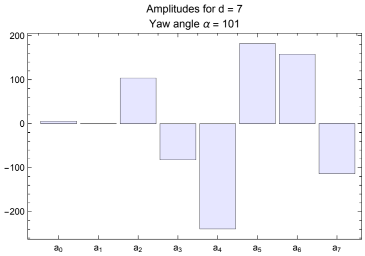
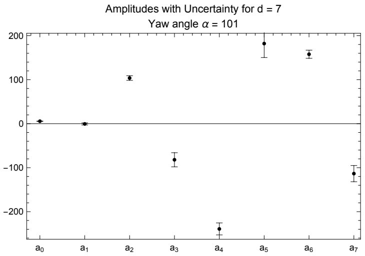
Dominance	Uncertainty
	

Table: Best fit occurs at yaw angle $\alpha = 101^\circ$

Amplitudes and Errors for $\alpha = 101°$

$$\sigma_{101^\circ}(\nu) = c_0 + c_1\nu + c_2\nu^2 + c_3\nu^3 + c_4\nu^4 + c_5{}^5 + c_6\nu^6 + c_7\nu^7$$

$$\begin{array}{lcrcr}
c_0 & = & 5.61 & \pm & 0.46 \\
c_1 & = & 0.5 & \pm & 2.3 \\
c_2 & = & 103.7 & \pm & 5.3 \\
c_3 & = & -82. & \pm & 19. \\
c_4 & = & -239. & \pm & 14. \\
c_5 & = & 182. & \pm & 32. \\
c_6 & = & 157.8 & \pm & 9.3 \\
c_7 & = & -114. & \pm & 18.
\end{array}$$

Stability of Amplitudes

1. How stable are the results against perturbations to the data?
2. Look at a sample of 250 solution curves
3. Use normal distribution
4. Here $\mu = a_k$, $\sigma = \sigma_k$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma}} \tag{4.4}$$

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma_k\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-a_k)^2}{\sigma_k}} \tag{4.5}$$

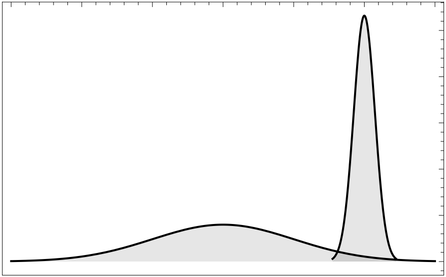


Figure: Width parameter is σ_k, center is a_k

Best Case: $\alpha = 101^\circ$, Seeing the Uncertainties

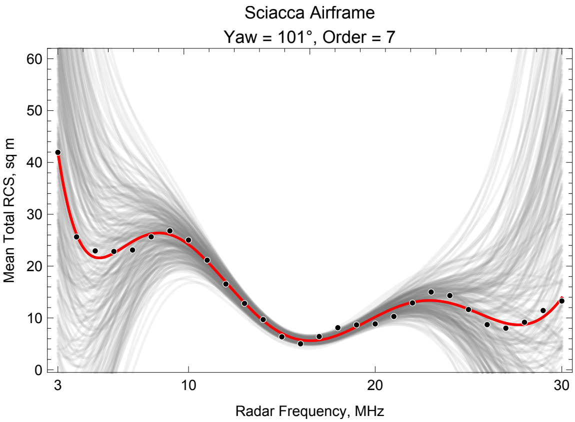


Figure: Sampling 250 solutions.

Worst Case: $\alpha = 0^\circ$

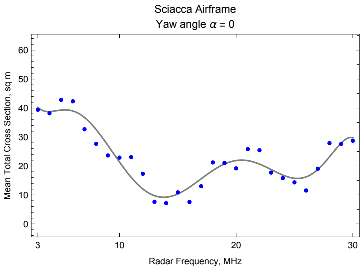
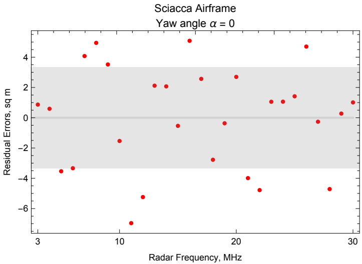
Data v. Fit	Residual Error
	

Table: Worst fit occurs at yaw angle $\alpha = 0^\circ$

Worst Case: $\alpha = 0°$, Amplitudes

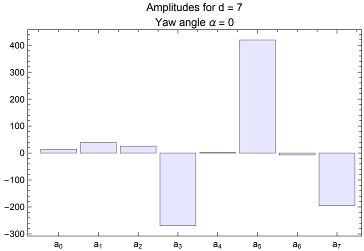
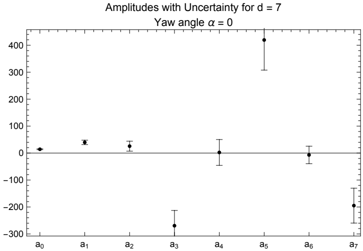
Dominance	Uncertainty
	

Table: Worst fit occurs at yaw angle $\alpha = 0°$

Worst Case: $\alpha = 0°$, Seeing the Uncertainties

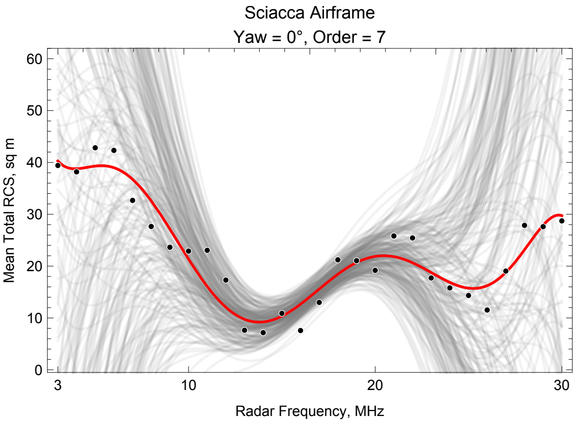


Figure: Sampling 250 solutions.

Worst Case: Amplitudes and Errors for $\alpha = 0^\circ$

$$\sigma_{101^\circ}(\nu) = c_0 + c_1\nu + c_2\nu^2 + c_3\nu^3 + c_4\nu^4 + c_5{}^5 + c_6\nu^6 + c_7\nu^7$$

$$\begin{array}{lcrcr}
c_0 & = & 14.0 & \pm & 1.6 \\
c_1 & = & 39.7 & \pm & 8.2 \\
c_2 & = & 26. & \pm & 19. \\
c_3 & = & -269. & \pm & 57. \\
c_4 & = & 2. & \pm & 48. \\
c_5 & = & 420. & \pm & 110. \\
c_6 & = & -7. & \pm & 33. \\
c_7 & = & -195. & \pm & 65.
\end{array}$$

Comparing Best and Worst Cases: Monomials

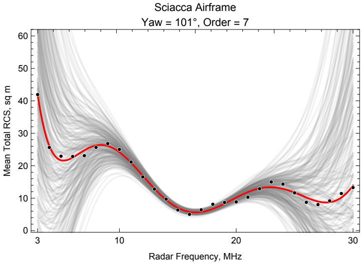
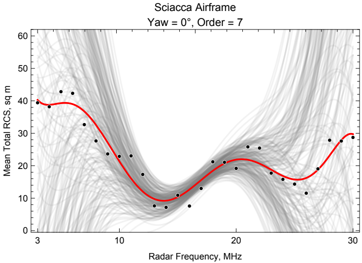
Best Fit	Worst Fit
	

Table: The harsh world outside the aegis of Weierstrass.

Comparing Best and Worst Cases: Fourier

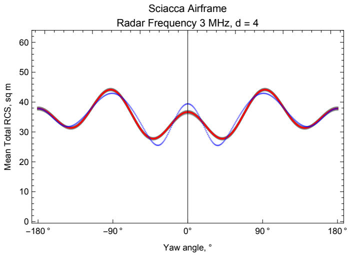
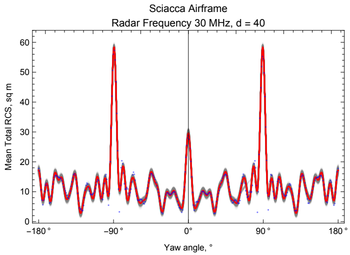
Best Fit	Worst Fit
	

Table: The joy of uniform convergence.

Detailed Report Delivered

Radar Cross Section:
Phase 1 Summary Report

Daniel Topa, ERT Inc.
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Albuquerque, NM

April 20, 2020

Abstract

Currently the AFCAP dashboard represents just the mean radar cross section, the dominant term. The initiative is to expand the representation of the radar cross section for a more realistic characterization. Several reports have been submitted which record the progress and deliverables on the radar cross section analysis for HF radar using Mercury Method of Moments package. A brief synthesis of results follows, presenting an overview.

Bonito Information Budget

Count	method
10,080	raw data
5,040	Average 2 consecutive measurements (180 angles)
3,360	Average 3 consecutive measurements (120 angles)
2,880	Naive **monomial** fit at every degree
2,520	Average 4 consecutive measurements (90 angles)
2,016	Average 5 consecutive measurements (72 angles)
1,680	Average 6 consecutive measurements (60 angles)
1,440	Average 8 consecutive measurements (45 angles)
1,440	**Monomial** fit every two degrees
1,260	Average 9 consecutive measurements (40 angles)
1,120	Average 10 consecutive measurements (36 angles)
1,008	Average 12 consecutive measurements (30 angles)
840	Average 15 consecutive measurements (24 angles)
720	**Monomial** fit every four degrees
672	Average 18 consecutive measurements (20 angles)
560	Average 20 consecutive measurements (18 angles)
504	Average 20 consecutive measurements (18 angles)
360	**Monomial** fit every eight degrees
244	**Fourier representation** with eight terms
360	**Monomial** fit every 16 degrees

Two Options

1. Sampling (eliminate data)
2. Averaging (combine data)

Mean Total RCS, Sciacca Airframe

	$\alpha = -180°$	$\alpha = -179°$	$\alpha = -178°$	$\alpha = -177°$
$\nu = 3$ MHz	37.8366	37.823	37.7868	37.728
$\nu = 4$ MHz	30.4298	30.396	30.2916	30.1166
$\nu = 5$ MHz	22.0446	22.0216	21.9476	21.8229
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\nu = 28$ MHz	28.3074	27.6088	25.4786	22.1958
$\nu = 29$ MHz	22.945	22.1855	19.8479	16.2697
$\nu = 30$ MHz	17.7572	16.942	14.405	10.6723

Averaging Retains Information

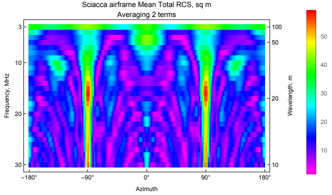
	$\alpha = -180°$
$\nu = 3$ MHz	$\frac{1}{4}(37.8366 + 37.823 + 37.7868 + 37.728)$
$\nu = 4$ MHz	$\frac{1}{4}(30.4298 + 30.396 + 30.2916 + 30.1166)$
$\nu = 5$ MHz	$\frac{1}{4}(22.0446 + 22.0216 + 21.9476 + 21.8229)$
	$\vdots$
$\nu = 28$ MHz	$\frac{1}{4}(28.3074 + 27.6088 + 25.4786 + 22.1958)$
$\nu = 29$ MHz	$\frac{1}{4}(22.945 + 22.1855 + 19.8479 + 16.2697)$
$\nu = 30$ MHz	$\frac{1}{4}(17.7572 + 16.942 + 14.405 + 10.6723)$

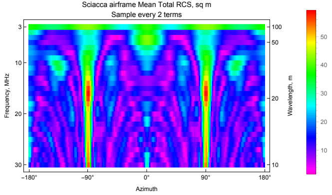
Sampling Ignores Information

	$\alpha = -180°$	$\alpha = -179°$	$\alpha = -178°$	$\alpha = -177°$
$\nu = 3$ MHz	37.8366	37.823	37.7868	37.728
$\nu = 4$ MHz	30.4298	30.396	30.2916	30.1166
$\nu = 5$ MHz	22.0446	22.0216	21.9476	21.8229
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\nu = 28$ MHz	28.3074	27.6088	25.4786	22.1958
$\nu = 29$ MHz	22.945	22.1855	19.8479	16.2697
$\nu = 30$ MHz	17.7572	16.942	14.405	10.6723

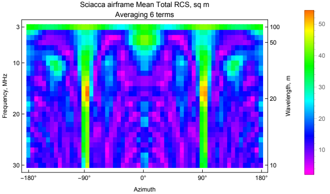
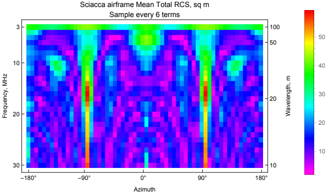
Reduction by 2

Averaging	Sampling

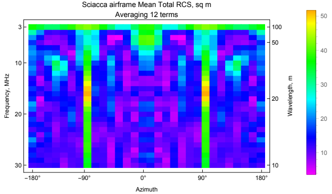
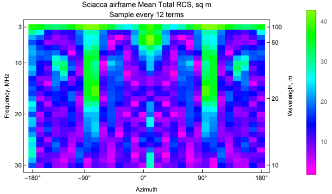




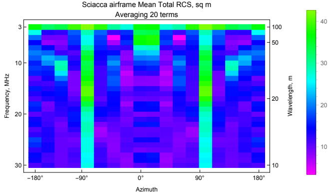
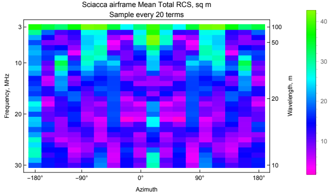
Reduction by 6

Averaging	Sampling
	

Reduction by 12

Averaging	Sampling
	

Reduction by 20

Averaging	Sampling
	

Conclusion

Conclusion: For the same reduction factor, averaging provides superior performance over sampling.

References

- **Handbook of Radar Measurement**
  D. K. Barton, H.R. Ward
  1969 ISBN 13-380683-9
- **Introduction to Radar Systems**
  Merrill I. Skolnik
  1962 LoC CCN ISBN 61-17675
- **Over-The-Horizon Radar**
  A. A. Kolosov, et al.
  1987 ISBN 0-890006-233-1
- **Radar Cross Section**
  E. F. Knott, M. T. Tuley, J. F. Shaeffer
  1993 ISBN 9780890066188

Radar Cross Section Data Compression

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May 31, 2020