Radar Cross Section Models for AFCAP Dashboard: Approximating the RCS

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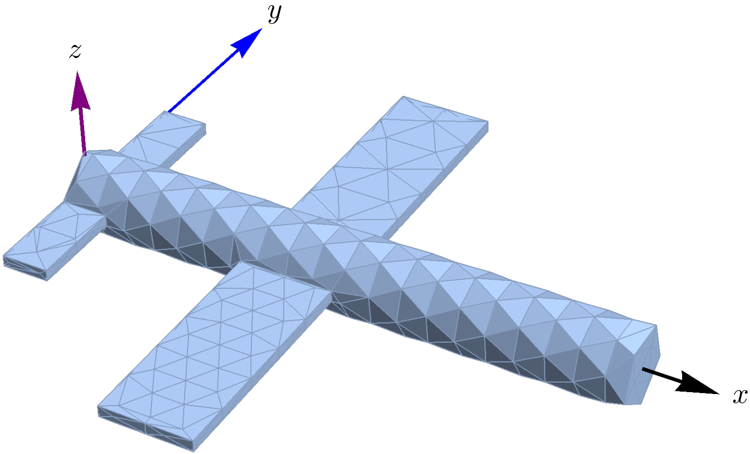
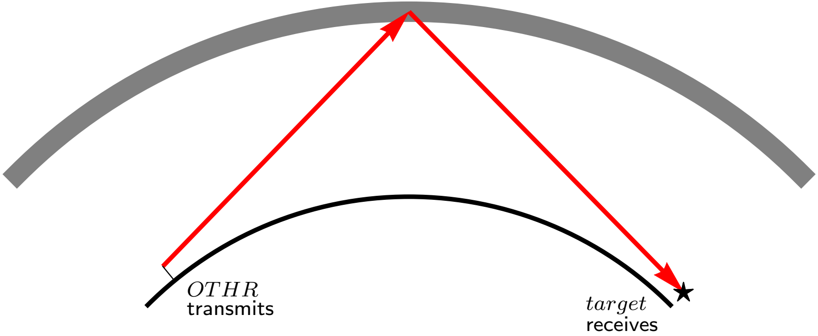
Scope

1. CAD Model
2. Notations

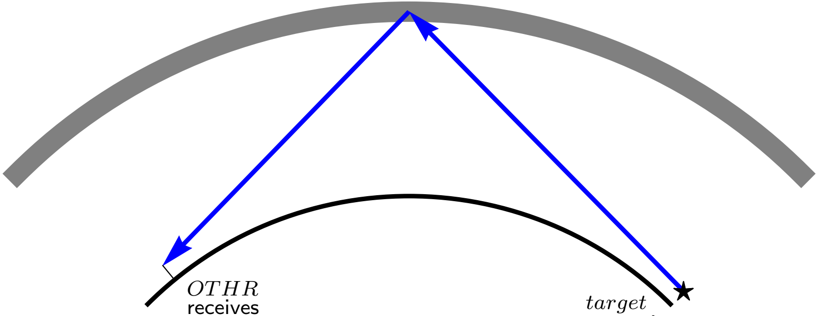
CAD Airframe: Coordinate System



Radar Illuminates Target



Radar Measures Return Energy



Exponential Function: Definition

Given $z \in \mathbb{C}$,

$$e^{z} := 1 + \sum_{k=1}^{\infty} \frac{z^k}{k!}$$

Exponential Function: Euler Formula

Let $z = i\theta$,

$$\begin{aligned} e^{i\theta} &= 1 + \sum_{k=1}^{\infty} \frac{(i\theta)^k}{k!} \\ &= 1 + i\theta - \tfrac{1}{2}\theta^2 - \tfrac{i}{6}\theta^3 + \tfrac{1}{24}\theta^4 + \tfrac{i}{720}\theta^6 + \cdots \end{aligned} \tag{4.1}$$

Collect real and imaginary terms...

A Necessary Lemma

Theorem

Polynomials are dense in the space $C[-\pi, \pi]$ *with respect to the uniform norm.*

Weierstrass Approximation Theorem Statement

Theorem

Polynomials are dense in the space of continuous functions with respect to the uniform norm.

Weierstrass Approximation Theorem Colloquially

Polynomials can approximate

any smooth function

to arbitrary accuracy.

Weierstrass Bounty

- Existence
- Uniform approximation

Karl Weierstrass

Uniform approximation

You pick the error.

Trigonometric Functions?

Does the Weierstrass Theorem apply for trigonometric series?

Trigonometric Functions?

Does the Weierstrass Theorem apply for trigonometric series?

Yes . . .

Riesz-Fischer Theorem Statement

Theorem (Riesz-Fischer)

Let $\{\phi_n\}$ *be an orthonormal sequence of functions on* Ω *and suppose* $\sum |a_n|^2$ *converges. Denote the partial sum as*

$$s_\tau = a_0\phi_0 + a_1\phi_1 + \cdots + a_\tau\phi_\tau.$$

There exists a function $F \in L^2(\Omega)$ *such that* $\{s_\tau\}$ *converges to* F *in* $L^2(\Omega)$*, and such that*

$$F = \sum_{k=0}^{\infty} a_k\phi_k,$$

almost everywhere.

Riesz-Fischer Theorem Colloquially

Absolute quadratic convergence

guarantees

approximation functions is $L^2(\Omega)$.

Riesz-Fischer Bounty

- Completeness of Lebesque space $L^2(\Omega)$
- Hunting license for $L^2(\Omega)$ functions

Frigyes Riesz

Ernst Fischer

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