In[368]:=
```
A = {{1, 1}, {-1/√2, 1/√2}, {0, 0}};
% // MatrixForm
%% // N // MatrixForm
```

Out[369]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}$$

Out[370]//MatrixForm=

$$\begin{pmatrix} 1. & 1. \\ -0.707107 & 0.707107 \\ 0. & 0. \end{pmatrix}$$

In[364]:=
```
PseudoInverse[A] // MatrixForm
% // N // MatrixForm
```

Out[364]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Out[365]//MatrixForm=

$$\begin{pmatrix} 0.5 & -0.707107 & 0. \\ 0.5 & 0.707107 & 0. \end{pmatrix}$$

In[381]:=
```
{U, Σ, V} = SingularValueDecomposition[A];
Print["U = ", U // MatrixForm];
Print["V = ", V // MatrixForm];
Print["Σ = ", Σ // MatrixForm];
```

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

matrix rank :

In[388]:=
```
ρ = 2;
```

verify SVD: does A = U Σ V*? YES

In[386]:=
```
U.Σ.V^H // MatrixForm
```
Out[386]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}$$

In[390]:=
```
S = Σ[[1 ;; ρ, 1 ;; ρ]];
% // MatrixForm
```
Out[391]//MatrixForm=

$$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

In[403]:=
```
PseudoInverse[Σ];
% // MatrixForm
```
Out[404]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

compute pseudoinverse matrix Ap = $V\Sigma^{-1}$ U^*

In[401]:=
```
Ap = V.PseudoInverse[Σ].U^H;
% // MatrixForm
```
Out[402]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$