

setup

overhead

tag

```
In[383]:= home = "ert/stc/algorithms/";
   Get["utility modules.m", Path → dirPack];
   stamp1;
   maximum memory: 0.210887 GB
   seed file: /Users/dantopa/Mathematica_files/nb/seed 19_12.nb
   user: dantopa, CPU: Xiuhcoatl, MM v. 12.0.0 for Mac OS X x86
   date: Mar 30, 2020, time: 16:16:46
   nb: /Users/dantopa/Mathematica_files/nb/ert/stc/algorithms/log-01.nb

modules, functions, settings, ...
```

2 spreadsheet

```
ln[411]:= (* column B *)
                              n = 10^{14} Exp \left[ -\frac{x^2}{5000} \right];
                               % // N
Out[412]= \{1. \times 10^{14}, 9.55997 \times 10^{13}, 8.3527 \times 10^{13}, 6.66977 \times 10^{13}, 4.86752 \times 10^{13}, 3.24652 \times 10^{13}, 6.66977 \times 10^{13}, 4.86752 \times 10^{13}, 3.24652 \times 10^{13}, 6.66977 \times 10^{13}, 6.6697
                                     1.97899 \times 10^{13}, 1.10251 \times 10^{13}, 5.61348 \times 10^{12}, 2.61214 \times 10^{12}, 1.1109 \times 10^{12},
                                     4.31784 \times 10^{11}, 1.53381 \times 10^{11}, 4.97955 \times 10^{10}, 1.47748 \times 10^{10}, 4.00653 \times 10^{9},
                                     9.9295 \times 10^{8}, 2.24906 \times 10^{8}, 4.65572 \times 10^{7}, 8.80818 \times 10^{6}, 1.523 \times 10^{6}, 240672, 34758.9,
                                     4587.96, 553.461, 61.0194, 6.1484, 0.5662, 0.047653, 0.00366543, 0.000257676,
                                     0.0000165552, 9.72099 \times 10^{-7}, 5.21674 \times 10^{-8}, 2.55859 \times 10^{-9}, 1.14688 \times 10^{-10},
                                     4.69835 \times 10^{-12}, 1.75909 \times 10^{-13}, 6.01928 \times 10^{-15}, 1.88241 \times 10^{-16}, 5.38019 \times 10^{-18}
   In[409]:= (* column c *)
                              nx = \frac{-2 \times n}{5000};
Out[410]= \{0., -5.73598 \times 10^{11}, -1.00232 \times 10^{12}, -1.20056 \times 10^{12}, -1.16821 
                                     -9.73957 \times 10^{11}, -7.12435 \times 10^{11}, -4.63052 \times 10^{11}, -2.69447 \times 10^{11}, -1.41056 \times 10^{11},
                                     -6.6654 \times 10^{10}, -2.84977 \times 10^{10}, -1.10434 \times 10^{10}, -3.88405 \times 10^{9}, -1.24109 \times 10^{9},
                                     -3.60588 \times 10^{8}, -9.53232 \times 10^{7}, -2.29404 \times 10^{7}, -5.02817 \times 10^{6}, -1.00413 \times 10^{6},
                                     -182760., -30324.7, -4588.18, -633.139, -79.6984, -9.15291, -0.95915,
                                     -0.0917243, -0.00800571, -0.000637785, -0.0000463816, -3.07927 \times 10^{-6},
                                     -1.86643 \times 10^{-7}, -1.03291 \times 10^{-8}, -5.21953 \times 10^{-10}, -2.40844 \times 10^{-11}, -1.01484 \times 10^{-12},
                                     -3.90518 \times 10^{-14}, -1.3724 \times 10^{-15}, -4.40484 \times 10^{-17}, -1.29124 \times 10^{-18}}
   In[407]:= (* column d *)
                               nx1 = Table
                                                   n[[k+1]] - n[[k-1]]
                                                   x[[k+1]] - x[[k-1]]
                                                  , {k, 2, Length[x] - 1}];
Out[408]= \{-5.49099 \times 10^{11}, -9.63402 \times 10^{11}, -1.16173 \times 10^{12}, -1.14108 \times 10
                                     -9.62845 \times 10^{11}, -7.14673 \times 10^{11}, -4.72546 \times 10^{11}, -2.8043 \times 10^{11}, -1.50086 \times 10^{11},
                                     -7.26786 \times 10^{10}, -3.19173 \times 10^{10}, -1.27329 \times 10^{10}, -4.62021 \times 10^{9}, -1.5263 \times 10^{9},
                                     -4.59396 \times 10^{8}, -1.26054 \times 10^{8}, -3.15464 \times 10^{7}, -7.20325 \times 10^{6}, -1.50114 \times 10^{6},
                                     -285584., -49608., -7869.48, -1140.18, -150.898, -18.2438, -2.01511,
                                     -0.203358, -0.0187511, -0.00157985, -0.000121629, -8.55679 \times 10^{-6},
                                     -5.50102 \times 10^{-7}, -3.2318 \times 10^{-8}, -1.73509 \times 10^{-9}, -8.51298 \times 10^{-11},
                                     -3.81706 \times 10^{-12}, -1.56411 \times 10^{-13}, -5.85736 \times 10^{-15}, -2.00463 \times 10^{-16}}
```

```
In[437]:= (* column e *)
                               relerror1 = Table
                                                  nx1[[k]] - nx[[k+1]]
                                                                           nx[[k+1]]
                                                 , {k, Length[nx1]}];
                              % // N
(0.01438) = \{-0.0427114, -0.0388318, -0.0323447, -0.0232187, -0.0114093, 0.00314109, 0.0205036, -0.0114093, -0.0114093, -0.00314109, -0.0205036, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0114093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144093, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.0144094, -0.
                                     0.0407632, 0.0640192, 0.090386, 0.119993, 0.152988, 0.189533, 0.22981, 0.274021,
                                    0.322386, 0.37515, 0.432577, 0.494961, 0.562617, 0.635893, 0.715164, 0.80084,
                                    0.893365, 0.99322, 1.10093, 1.21706, 1.34222, 1.47708, 1.62236, 1.77883,
                                    1.94735, 2.12882, 2.32423, 2.53464, 2.76122, 3.00522, 3.26798, 3.55098}
                                (* column f *)
                               ln = Log[n];
                              % // N
Out[421] = \{32.2362, 32.1912, 32.0562, 31.8312, 31.5162, 31.1112, 30.6162, 30.0312, 29.3562, 31.8312, 31.5162, 31.1112, 30.6162, 30.0312, 29.3562, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.5162, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312, 31.8312
                                     28.5912, 27.7362, 26.7912, 25.7562, 24.6312, 23.4162, 22.1112, 20.7162, 19.2312,
                                    17.6562, 15.9912, 14.2362, 12.3912, 10.4562, 8.43119, 6.31619, 4.11119, 1.81619,
                                    -0.568809, -3.04381, -5.60881, -8.26381, -11.0088, -13.8438, -16.7688,
                                    -19.7838, -22.8888, -26.0838, -29.3688, -32.7438, -36.2088, -39.7638
   In[428]:= (* column g *)
                              nx2 = Table
                                               n[[k+1]] \ \frac{ln[[k]] - ln[[k+2]]}{x[[k]] - x[[k+2]]}
                                                , {k, Length[nx1]}];
                              % // N
Out[429]= \{-5.73598 \times 10^{11}, -1.00232 \times 10^{12}, -1.20056 \times 10^{12}, -1.16821 \times 10
                                    -9.73957 \times 10^{11}, -7.12435 \times 10^{11}, -4.63052 \times 10^{11}, -2.69447 \times 10^{11}, -1.41056 \times 10^{11},
                                    -6.6654 \times 10^{10}, -2.84977 \times 10^{10}, -1.10434 \times 10^{10}, -3.88405 \times 10^{9}, -1.24109 \times 10^{9},
                                    -3.60588 \times 10^{8}, -9.53232 \times 10^{7}, -2.29404 \times 10^{7}, -5.02817 \times 10^{6}, -1.00413 \times 10^{6},
                                    -182760., -30324.7, -4588.18, -633.139, -79.6984, -9.15291, -0.95915,
                                    -0.0917243, -0.00800571, -0.000637785, -0.0000463816, -3.07927 \times 10^{-6},
                                    -1.86643 \times 10^{-7}, -1.03291 \times 10^{-8}, -5.21953 \times 10^{-10}, -2.40844 \times 10^{-11},
                                    -1.01484 \times 10^{-12}, -3.90518 \times 10^{-14}, -1.3724 \times 10^{-15}, -4.40484 \times 10^{-17}}
```

```
 \begin{array}{l} \text{relerror2 = Table} \Big[ \\ & \underbrace{ \begin{array}{l} \text{nx2[[k]] - nx[[k+1]]} \\ \text{nx[[k+1]]} \\ \text{nx[[k+1]]} \\ \\ & \\ \end{array} \Big] \\ & \underbrace{ \begin{array}{l} \text{nx[[k+1]]} \\ \text{nx[[k+1]]} \\ \text{nx[[443] = } \Big\{ -1.4897 \times 10^{-15}, -1.58323 \times 10^{-15}, 4.88054 \times 10^{-15}, -1.6719 \times 10^{-15}, -1.50401 \times 10^{-15}, \\ 1.88477 \times 10^{-15}, 1.31811 \times 10^{-15}, -1.58564 \times 10^{-15}, -1.51446 \times 10^{-15}, 3.43388 \times 10^{-16}, \\ 2.67719 \times 10^{-16}, 1.72713 \times 10^{-16}, 0., -1.92105 \times 10^{-16}, -1.65299 \times 10^{-16}, 9.37935 \times 10^{-16}, \\ 6.49561 \times 10^{-16}, 0., 0., -4.77739 \times 10^{-16}, 3.59903 \times 10^{-16}, 1.98226 \times 10^{-16}, \\ 1.79561 \times 10^{-16}, 0., -1.94076 \times 10^{-16}, 0., 0., -2.16686 \times 10^{-16}, -1.69995 \times 10^{-16}, \\ 0., -2.75076 \times 10^{-16}, 7.09102 \times 10^{-16}, 4.80493 \times 10^{-16}, -3.96195 \times 10^{-16}, \\ -4.02481 \times 10^{-16}, -3.97989 \times 10^{-16}, 4.84809 \times 10^{-16}, -4.31104 \times 10^{-16}, -5.59655 \times 10^{-16} \Big\} \end{array}
```

3 plots

```
In[436]:= ListLogPlot[{x, n}<sup>T</sup>,

PlotStyle \rightarrow Black,

PlotLabel \rightarrow "Gaussian function n(x)",

FrameLabel \rightarrow {"x", "n"},

Frame \rightarrow True]

Gaussian function n(x)

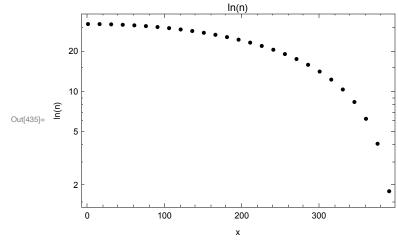
10^{15}
10^{5}

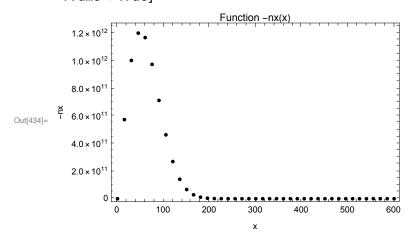
Out[436]=

0 \quad 100 \quad 200 \quad 300 \quad 400 \quad 500 \quad 600

x
```

```
In[435]:= ListLogPlot[{x, ln}<sup>T</sup>,
    PlotStyle → Black,
    PlotLabel → "ln(n)",
    FrameLabel → {"x", "ln(n)"},
    Frame → True]
```





```
ln[432]:= x = Drop[Drop[x, -1], 1];
         ListPlot[\{x, Abs[nx1]\}^{\mathsf{T}},
           PlotStyle \rightarrow Blue, PlotLabel \rightarrow "Function -nx1(x)",
           FrameLabel → {"x", "-nx1"},
           PlotRange → All,
           Frame → True]
                                              Function -nx1(x)
             1.2 \times 10^{12}
             1.0 \times 10^{12}
             8.0 \times 10^{11}
         ₹ 6.0 × 10<sup>11</sup>
Out[433]=
             4.0 \times 10^{11}
             2.0\times10^{11}
                                100
                                                      300
                                                                 400
                                                                            500
ln[431]:= ListPlot[{x, Abs[nx2]}^{\mathsf{T}},
           PlotStyle → Blue,
           PlotLabel \rightarrow "Function -nx2(x)",
           FrameLabel → {"x", "-nx2"},
           PlotRange → All,
           Frame → True]
                                              Function -nx2(x)
             1.2 \times 10^{12}
             1.0 \times 10^{12}
             8.0 \times 10^{11}
            6.0 \times 10^{11}
Out[431]=
             4.0 \times 10^{11}
             2.0 \times 10^{11}
                      0
                                100
                                                      300
                                                                 400
                                                                            500
```

```
ln[447]:= ListPlot[{x, Abs[relerror2 // N]}^{T},
            PlotStyle → Blue,
            PlotLabel → "Function |relerror2|",
            FrameLabel → {"x", "|relerror2|"},
            PlotRange → All,
            Frame → True]
                                                Function |relerror2|
              5. × 10<sup>-15</sup>
              4. \times 10^{-15}
Out[447]= \frac{\overline{Q}}{\underline{Q}} 3. \times 10^{-15}
0 = \frac{1}{2} 2. \times 10^{-15}
              2. \times 10^{-15}
              1. \times 10^{-15}
                                  100
                                              200
                                                                     400
                                                                                 500
                                                          300
```

relerror2

end