Simulation of Radar Profiles for Satellites

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Abstract

A brief survey of characterizing the three dimensional radar cross section of satellites. The process of finding the optimal Fourier expression for each band is explored and different success measures are presented.

Contents

1 Writ Large

Radar is an invaluable tool for space domain awareness which can characterize not only location, but also orientation of satellites in space. It can answer where antennas are pointed and how a space craft is maneuvering. The open question is one of interpreting the radar return section. The key value is the amount of energy returned by reflection, and this is the essence of the radar cross section (res).

Figure 1 shows a sample radar cross section measurement for a vintage A-26 Invader. The strength of the return signal varies over several orders of magnitude depending upon the viewing angle.

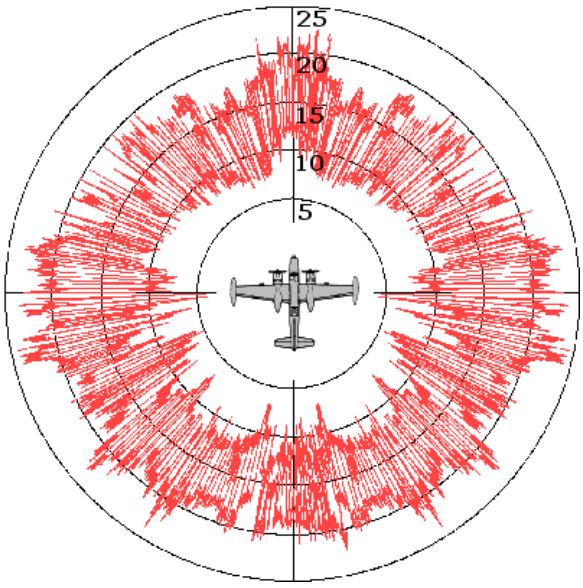


Figure 1: A radar cross section measurement for the A-26 Invader taken from the Wikipedia article on Radar Cross Section show energy change where 0 db represents 1 milliwatt return energy.

A simplified aircraft model in figure 2 presents requisite variation without over-complications and makes the point that even a stealthy aircraft can present large cross sections at proper view angles as seen in table 2.

Look angle

The purpose of this analysis is to start with a CAD model of a structure and simulate the radar cross section, a sample show in figure 3.

render the radar cross section into a form like

$$\sigma_3(\theta) = 35.237 \pm 0.012 + (1.675 \pm 0.018)\cos\theta + (-3.434 \pm 0.018)\cos 2\theta + (-0.866 \pm 0.018)\cos 3\theta + (5.386 \pm 0.018)\cos 4\theta + (-1.280 \pm 0.018)\cos 5\theta + (1.379 \pm 0.018)\cos 6\theta + (-0.675 \pm 0.018)\cos 7\theta$$

where θ is an azimuthal angle with $\theta = 0$ representing a nose-one view and $\pm\pi representing a tail - on view$

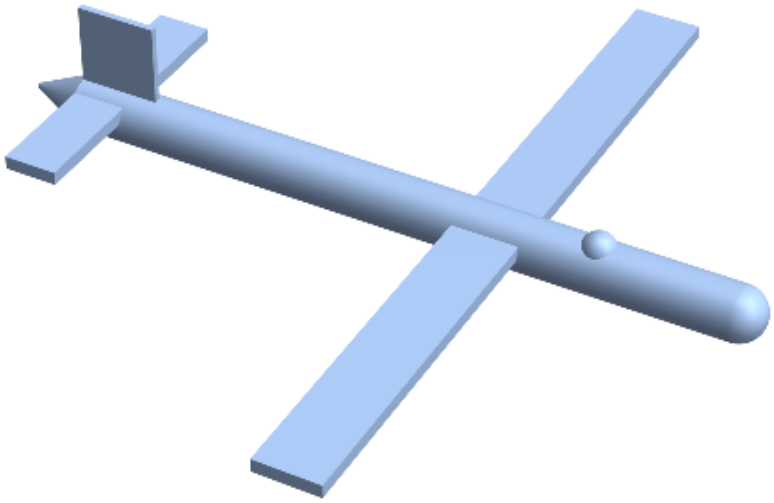


Figure 2: Toy model for aircraft. Although simplistically rendered, this representation is adequate for over the horizon radars.

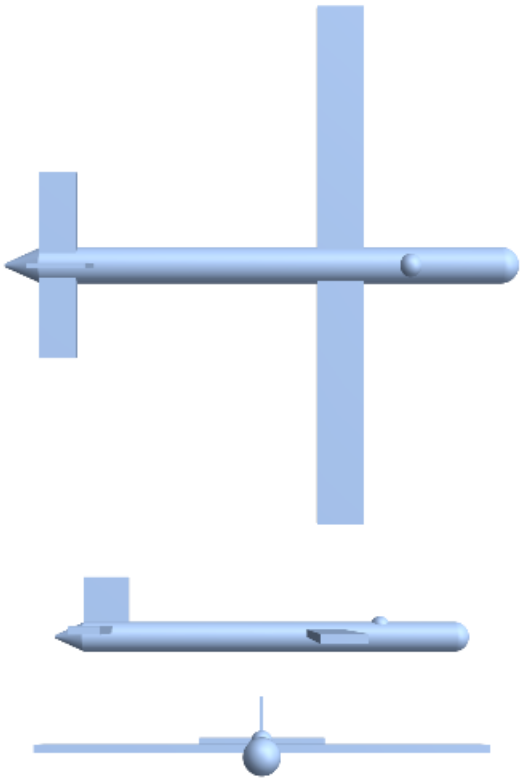


Table 1: Different aircraft views present very different cross-sectional areas.

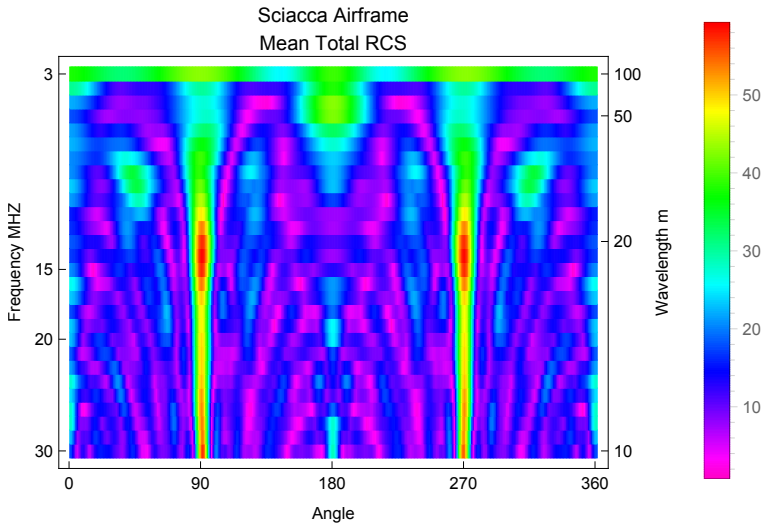


Figure 3: Output for a radar backscatter simulation for radar frequencies from $3 - 30$ MHz, corresponding to wavelengths of $10 - 100$ m.

1.1 Toy Satellite Model

A toy satellite model is much easier to start with for developing a data analysis can reduction chain. The model below (figure /reftab:toy-sat) features a spherical body and readily distinguishable solar panels.

2 Mathematical Precís

2.1 Models of Radar Cross Section

$$\sigma_\nu(\alpha) \approx \frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha + b_k \sin k\alpha \tag{1}$$

Amplitudes and Errors for $\nu = 3$ MHz and $d = 7$:

$$\begin{aligned}\sigma_3(\theta) = a_0 &+ a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta \\ &+ a_4 \cos 4\theta + a_5 \cos 5\theta + a_6 \cos 6\theta + a_7 \cos 7\theta\end{aligned}$$

$$\begin{aligned}\sigma_3(\theta) = {} & 35.237 \pm 0.012 + (1.675 \pm 0.018)\cos\theta + (-3.434 \pm 0.018)\cos 2\theta + (-0.866 \pm 0.018)\cos 3\theta \\ & + (5.386 \pm 0.018)\cos 4\theta + (-1.280 \pm 0.018)\cos 5\theta + (1.379 \pm 0.018)\cos 6\theta + (-0.675 \pm 0.018)\cos 7\theta\end{aligned}$$

2.2 Models of Increasing Fidelity

2.3 Running the Code

```
./MMoM_4.1.12 sample.geo
```

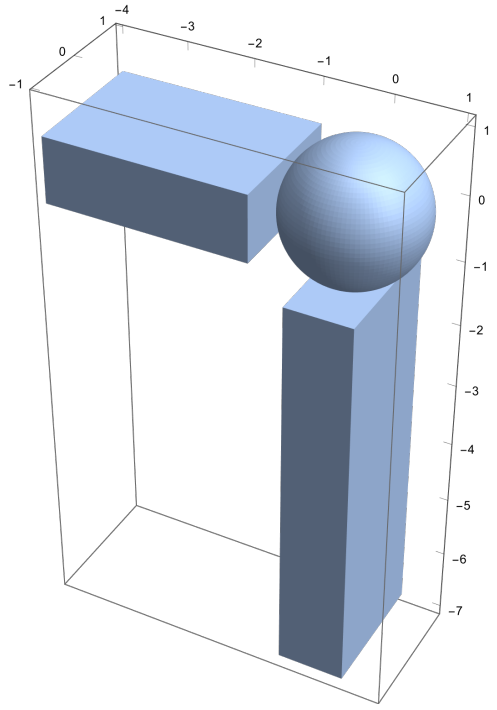
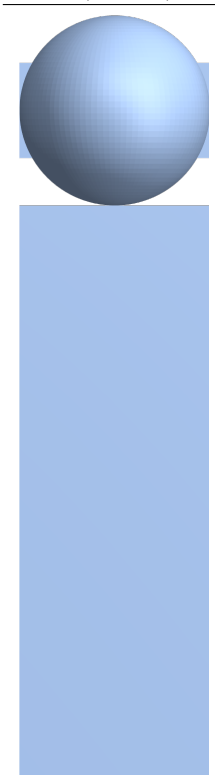
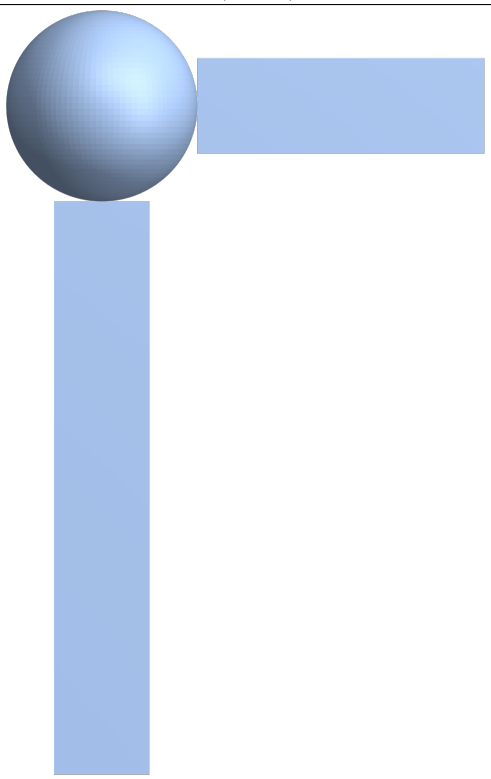
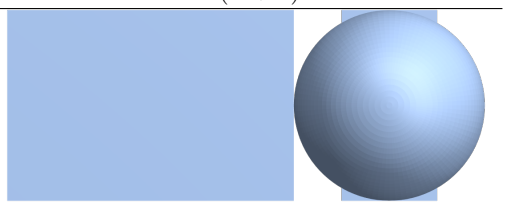


Figure 4: A toy satellite model designed to present a spherical backscatter and resonances at 6, 3, 2, and 1 meter wavelengths.

Table 2: Contrasting views of a toy satellite present very different cross sectional areas, A.

$A \approx (\pi + 12)\, r^2$	$A = (\pi + 9)\, r^2$	$A \approx (\pi + 6)\, r^2$
		

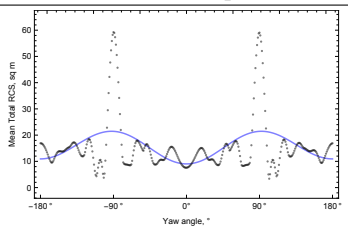
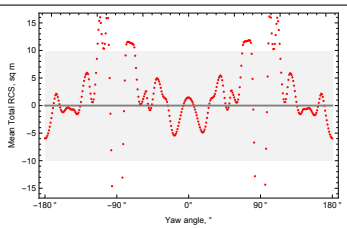
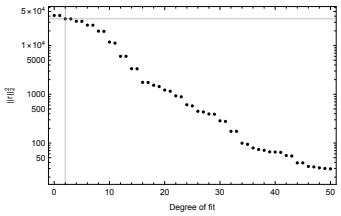
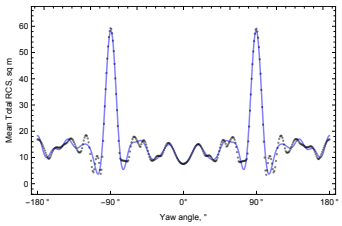
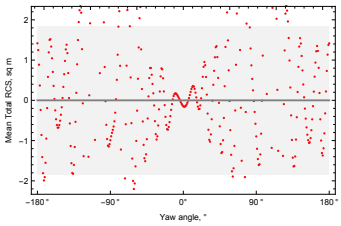
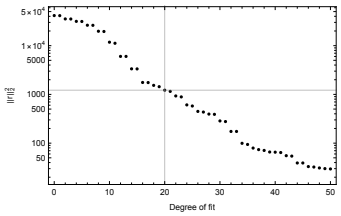
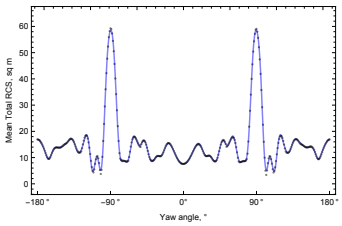
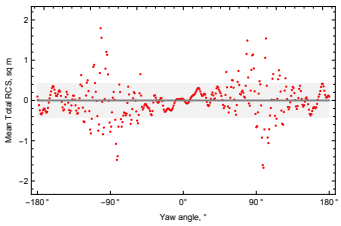
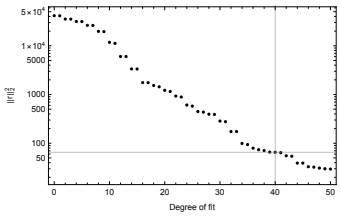
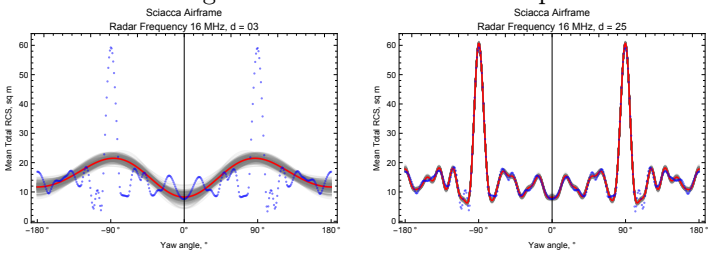
d	Data and Fourier Representation	Residual Fit Error	Total Error for Fit Sequence
$d = 2$			
$d = 20$			
$d = 40$			

Table 3: Seeing the uncertainties in the fit parameters.



2.4 Radar

[14] [14] Working with CAF files, producing output, compressing data. [15] Topa [15]

2.5 Process

1	Create CAD model	CAD software
2	Convert CAD to `*.obj`	CAD software
3	Convert `*.obj` to `*.facet`	Mathematica, Fortran
4	Input properties to `materials.lib`	`VIM`
5	Set radar frequencies	`VIM`
6	**Simulate radar irradiation**	`Mercury MoM`
7	Harvest reflection values from output	Mathematica, Fortran, Python
8	Describe RCS as a series of amplitudes	Not written

Table 4: Start with a CAD model and construct a Radar Cross Section model

3 Overview: Modeling Radar Cross Section

3.1 Radar

Wave speed equation

$$\lambda\nu = c \tag{2}$$

band	ν	λ
HF	$3-30$ MHz	$10-1$ m
UHF	$30-300$ MHz	$0.1-0.01$ m
VHF	$300-1000$ MHz	$0.01-0.03$ m
L	$1-2$ GHz	$30-15$ mm
S	$2-4$ GHz	$15-7.5$ mm
C	$4-8$ GHz	$7.5-3.7$ mm
X	$8-12$ GHz	$3.7-2.5$ mm
Ku	$12-18$ GHz	$2.5-1.7$ mm
K	$18-27$ GHz	$1.7-1.1$ mm
Ka	$27-40$ GHz	$1.1-0.75$ mm
V	$40-75$ GHz	$0.75-0.4$ mm
W	$75-110$ GHz	$0.4-0.27$ mm
mm	$110-300$ GHz	$0.27-0.1$ mm

Table 5: IEEE Standard Designations for Radar Bands ([2]).

(A) Build a CAD model of the satellite (*.cad)
(B) Seal the CAD mesh
(C) Create geometry file (*.geo)
(D) Irradiate object with Mercury MoM
(E) Harvest backscatter

ϵ_0	vacuum electric permittivity	8.854×10^{-12} F m^{-1}
μ_0	vacuum magnetic permeability	1.257×10^{-7} N A^{-2}

Table 6: Constants used in Maxwell's equations

(F) Construct RCS
(G) Resolve RCS measurements into spherical harmonics

3.2 Process

(A) Build a CAD model of the satellite (*.cad)
(B) Seal the CAD mesh
(C) Create geometry file (*.geo)
(D) Irradiate object with Mercury MoM
(E) Harvest backscatter
(F) Construct RCS
(G) Resolve RCS measurements into spherical harmonics

4 Computing the Radar Cross Section

The computation of the radar cross section is based on a simple scenario guided by Maxwell's equations. A radar illuminates the target with radar energy. The target, acting like an antenna, reradiates the energy and illuminates the radar receiver. The radar cross section is measurement of the difference between energy absorbed and energy radiated.

One form of the radar cross section equation is

$$\sigma = 4\pi r^2 \frac{S_r}{S_t} \tag{3}$$

where r is the distance to the target, S_t is the energy *intercepted* by the target and S_r is the energy *radiated* by the target. The result has units of area and can be compared to an ideally reflecting sphere.

The areal measure σ is computed through numerical simulation by the program Mercury MoM.

4.1 Maxwell's Equations

The mathematical foundation for the radar cross section computation is the set of Maxwell's equations. Start with two conservative fields on a simply connected domain, the electric field $\mathbf{E}\colon \mathbb{R}^3 \mapsto \mathbb{R}^3$, and the magnetic field $\mathbf{B}\colon \mathbb{R}^3 \mapsto \mathbb{R}^3$. We can relax the requirements for the electric current $\mathbf{J}\colon \mathbb{R}^3 \mapsto \mathbb{R}^3$. The differential form of Maxwell's equations in SI units is

$$\begin{array}{rclcrcl} \nabla \cdot \mathbf{E} & = & \frac{\rho}{\epsilon_0} & & \nabla \times \mathbf{E} & = & -\partial_t \mathbf{B} \\ \nabla \cdot \mathbf{B} & = & 0 & & \nabla \times \mathbf{B} & = & \mu_0 \left(\mathbf{J} + \epsilon_0 \partial_t \mathbf{E} \right) \end{array} \tag{4}$$

The fundamental physical constant in SI units are given in table 6.

4.2 Method of Moments

There is a rich mathematical toolkit for the solution of Maxwell's equations. Mercury MoM uses the method of moments (MoM), also known as the boundary element method (BEM). Maxwell's equations in integral form

$$\begin{array}{rclcrcl} \nabla \cdot \mathbf{E} & = & \dfrac{\rho}{\epsilon_0} & \Rightarrow & \displaystyle\oint_{d\Omega} \mathbf{E} \cdot d\mathbf{S} & = & \displaystyle\int_{\Omega} \rho dV \\ \nabla \times \mathbf{E} & = & -\partial_t \mathbf{B} & \Rightarrow & \displaystyle\oint_{d\Omega} \mathbf{B} \cdot d\mathbf{S} & = & 0 \\ \nabla \cdot \mathbf{B} & = & 0 & \Rightarrow & \displaystyle\oint_{d\Sigma} \mathbf{E} \cdot d\mathbf{l} & = & -\dfrac{d}{dt}\mathbf{B} \cdot d\mathbf{S} \\ \nabla \times \mathbf{B} & = & \mu_0 \left(\mathbf{J} + \epsilon_0 \partial_t \mathbf{E}\right) & \Rightarrow & \displaystyle\oint_{d\Sigma} \mathbf{B} \cdot d\mathbf{l} & = & \mu_0 \displaystyle\int_{\Sigma} \left(\mathbf{J} + \epsilon_0 \dfrac{d}{dt}\mathbf{E}\right) \cdot d\mathbf{S} \end{array} \tag{5}$$

The method of moments creates dense matrices which implies quadratic growth in storage and computation time as the problem scales up.

4.3 Mercury MoM Example

The power of Mercury MoM is the ability to simultaneously solve equations with millions of unknowns using a patented low rank approximation method for the linear system.

Internally, the Mercury MoM package resolves the signal emitted from the target into two complex electric fields: θ for the vertical component and ϕ for the horizontal component. As these fields are complex, they each have orthogonal real and imaginary components.

The prescription for the mean total radar cross section is taken from the Sciacca report as the average of the total field energy

$$\langle \sigma_T \rangle = \tfrac{1}{2} \left(\theta^* \theta + \theta^* \phi + \phi^* \theta + \phi^* \theta\right), \tag{6}$$

and is assigned areal units of squared meters m^2.

5 Fourier Expansion

5.1 Why Fourier?

What is the best way to approximate information in the cross sections curves shown in figure **??**? Given that the data is periodic over the unit circle,

$$\sigma_\nu \left(\alpha, \beta\right) = \sigma_\nu \left(\alpha + 2\pi, \beta\right) \tag{7}$$

an obvious choice is the Fourier series. For example, a dth order approximation can be written as

$$\sigma_\nu \left(\alpha, \beta_0 = \tfrac{\pi}{12}\right) \approx \frac{a_0}{2} + \sum_{k=1}^{d} a_k \cos k\alpha + b_1 \sin k\alpha. \tag{8}$$

What does this series represent? It represents a sequence of oscillations about the mean value.

There are three powerful theoretical reasons which support this choice. The first is the convergence of the series. The Weierstrass Approximation Theorem states that the polynomial

approximation of smooth curves converges *uniformly*. That is, we can define a maximum error $\epsilon > 0$ and we are guaranteed the existence of $N \in \mathbb{Z}^+$ such that

$$\left\| \sigma_\nu \left(\alpha, \beta_0 \right) - \frac{a_0}{2} - \sum_{k=1}^{N} a_k \cos k\alpha + b_k \sin k\alpha \right\|_\infty \leq \epsilon. \tag{9}$$

The ability to choice a maximum error is both remarkable and useful. However, the application to trigonometric polynomials, while valid, is not immediate and left for another document.

The second reason is the Riesz–Fischer theorem. While often cited as evidence that the Lebesgue space L^2 is complete, it also establishes that an infinite series whose terms converge quadratically represents an L^2 function. And so the amplitudes, the a and bv values, will eventually converge at least quadratically.

The final reason is the often-overlooked fact that of all the orthogonal polynomials, only the trigonometric polynomials are orthogonal in both the continuous topology of L^2 and and the discrete orthogonality of l^2. Having an orthogonal basis decouples the problem and can be solved mode-by–mode, obviating the need to solve a large linear system.

5.2 Orthogonality

$$\int_{-\pi}^{\pi} e^{Im\theta} e^{In\theta} d\theta = \int_{-\pi}^{\pi} e^{Im+n\theta} d\theta = \pi \delta_n^m \tag{10}$$

Given non-zero integers m and n, the expression of orthogonality in L^2 can be written as

$$\begin{aligned}
\int_{-\pi}^{\pi} \sin\left(mx\right) \sin\left(nx\right) dx &= \pi \delta_n^m \\
\int_{-\pi}^{\pi} \cos\left(mx\right) \cos\left(nx\right) dx &= \pi \delta_n^m \\
\int_{-\pi}^{\pi} \cos\left(mx\right) \sin\left(nx\right) dx &= 0 \\
\int_{-\pi}^{\pi} \sin\left(mx\right) dx &= 0 \\
\int_{-\pi}^{\pi} \cos\left(mx\right) dx &= 0
\end{aligned} \tag{11}$$

where δ_n^m is the Kronecker delta tensor. To complete the toolkit to solve the linear system, recall that

$$\int_{-\pi}^{\pi} 1 dx = 2\pi. \tag{12}$$

The linear system decouples and amplitudes for each mode can be computed independently:

$$\begin{aligned}
a_0 &= \tfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \\
a_k &= \tfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx dx \\
b_k &= \tfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx dx
\end{aligned} \tag{13}$$

where $k = 1, 2, 3, \ldots$.

5.3 Projection

Consider the low–order approximation

$$f(\varphi) \approx \frac{a_0}{2} + a_1 \cos\varphi + a_2 \cos 2\varphi. \tag{14}$$

To compute the amplitudes $a = \{a_0, a_1, a_2\}$, use Hilbert's projection theorem, which is tantamount to using the method of least squares. Multiply both side of the equation by $\cos j\varphi$ and integrate over the domain to isolate the contribution of a_j. For example, to isolate the component a_2,

$$\begin{aligned} \int_{-\pi}^{\pi} f(\varphi) \cos 2\varphi d\varphi &\approx \int_{-\pi}^{\pi} \left(\frac{a_0}{2} + a_1 \cos\varphi + a_2 \cos 2\varphi \right) \cos 2\varphi d\varphi. \\ &= a_2 \int_{-\pi}^{\pi} \cos^2 2\varphi d\varphi. \end{aligned} \tag{15}$$

The result is the rather general formula

$$a_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\varphi) \cos 2\varphi d\varphi. \tag{16}$$

Because the basis functions are orthogonal, the modes are decoupled and the amplitudes can be computed independently.

5.4 Discrete Topology

The continuum formulation in L^2 is useful for understanding the critical components of the theory and makes a natural starting point for discussing the discrete topology of l^2. The moment we discretize the problem in a computer code, we switch from the continuum to a discrete space. Out of all the known orthogonal polynomials, only two are orthogonal in both L^2 and l^2, and these are the sine and cosine functions.

However, because of sampling mesh that was used in Mercury MoM, the data vectors are *not* orthogonal. But resolution is deferred until the end.

5.5 Special Case: The Mean

The Fourier series resolves a function into successively higher oscillations about a constant value. This constant value is the mean, and it is a least squares solution and in this effort, is a valuable bellwether.

5.5.1 The Method of Least Squares

$$r^2 = \sum_{k=1}^{m} \left(\sigma(\alpha_k, \beta_0) - \mu \right)^2 \tag{17}$$

$$D_\mu r^2 = -2 \left(\sigma(\alpha_k, \beta_0) - \mu \right) = 0 \tag{18}$$

The least squares solution for the parameter μ is simply the average of the data:

$$\mu = m^{-1} \sigma(\alpha_k, \beta_0). \tag{19}$$

Compare to a_0

5.5.2 Continuum to discrete

The continuum formulation translates to discrete spaces using the idea of the integral as a Riemann sum:

$$\int_{-\pi}^{\pi} \cos j\alpha d\alpha \quad \Rightarrow \quad \sum_{k=1}^{m} \cos\left(j\alpha_k\right) \Delta_k. \tag{20}$$

Here Δ_k, the interval length, corresponds to the Riemann integration measure $d\alpha$. A basic result from the calculus is that the average $\langle f\left(x\right)\rangle$ of an integrable function over a finite, ordered interval (a, b) is

$$\langle f\left(x\right)\rangle = \frac{\int_b^a f(x)dx}{b-a}, \tag{21}$$

where the interval length $L = b - a$. The connection to the discrete case is clear with the realization that the Mercury MoM results are spaced at regular intervals, that is,

$$\Delta_k = \Delta.$$

Then the interval length is

$$L = \sum_{k=1}^{m} \Delta_k = \sum_{k=1}^{m} \Delta = m\Delta. \tag{22}$$

$$\frac{\int_b^a f(x)dx}{L} \quad \Rightarrow \quad \frac{\sum_{k=1}^{m} f(x)\Delta}{m\Delta} = \frac{\sum_{k=1}^{m} f(x)}{m} \tag{23}$$

5.6 Linear Independence

5.7 Quality of Fit

6 Spherical Harmonics

Discovery and Origin

Spherical harmonics were first introduced in the 18th century by mathematicians such as *Pierre-Simon Laplace* in the context of solving Laplace's equation in spherical coordinates. They are solutions to the angular part of the Laplace equation and form a complete orthonormal set of functions on the unit sphere.

Definition and Properties

The spherical harmonics $Y_\ell^m(\theta, \phi)$ are complex-valued functions defined on the sphere and are parameterized by two integers:

- ℓ: the degree, representing the total number of nodal lines,
- m: the order, representing the number of nodal lines that intersect the equator.

These functions are expressed as:

$$Y_\ell^m(\theta, \phi) = \sqrt{\frac{(2\ell+1)(\ell-|m|)!}{4\pi(\ell+|m|)!}} P_\ell^{|m|}(\cos\theta) e^{im\phi},$$

where P_ℓ^m are the associated Legendre polynomials, θ is the polar angle, and ϕ is the azimuthal angle.

Spherical harmonics satisfy the orthogonality conditions:

$$\int_0^{2\pi} \int_0^{\pi} Y_\ell^m(\theta, \phi) Y_{\ell'}^{m'*}(\theta, \phi) \sin\theta \, d\theta \, d\phi = \delta_{\ell\ell'}\delta_{mm'}.$$

Applications

Spherical harmonics are extensively used in physics, engineering, and mathematics, particularly in fields involving spherical domains or symmetry:

- **Quantum Mechanics:** Solutions to the angular part of the Schrödinger equation for atoms.
- **Geophysics and Astrophysics:** Modeling gravitational and magnetic fields, cosmic microwave background radiation, and planetary shape analysis.
- **Signal Processing:** Analysis on spherical data, such as 3D object representations in computer graphics.
- **Partial Differential Equations:** Solutions in spherical coordinate systems.
- **Harmonic Analysis:** Representing functions on the sphere.

Recent Developments

Recent studies have focused on extensions of spherical harmonics, including generalizations to higher dimensions and applications in machine learning, such as spherical convolutional neural networks for analyzing spherical data.

Examples

The spherical harmonics up to degree $\ell = 2$ are shown in Table **??**.

Table 7: Spherical harmonics up to degree $\ell = 3$.

Order	**Symbol**	Spherical Harmonic Function		
0	Y_0^0	$\frac{1}{\sqrt{4\pi}}$	1	
1	Y_1^{-1}	$\sqrt{\frac{3}{8\pi}}$	$\sin\theta$	$e^{-i\phi}$
	Y_1^0	$\sqrt{\frac{3}{4\pi}}$	$\cos\theta$	
	Y_1^1	$-\sqrt{\frac{3}{8\pi}}$	$\sin\theta$	$e^{i\phi}$
2	Y_2^{-2}	$\sqrt{\frac{15}{32\pi}}$	$\sin^2\theta$	$e^{-2i\phi}$
	Y_2^{-1}	$\sqrt{\frac{15}{8\pi}}$	$\sin(2\theta)$	$e^{-i\phi}$
	Y_2^0	$\sqrt{\frac{5}{16\pi}}$	$3\cos^2\theta - 1$	
	Y_2^1	$-\sqrt{\frac{15}{8\pi}}$	$\sin(2\theta)$	$e^{i\phi}$
	Y_2^2	$\sqrt{\frac{15}{32\pi}}$	$\sin^2\theta$	$e^{2i\phi}$
3	Y_3^{-3}	$\sqrt{\frac{35}{64\pi}}$	$\sin^3\theta$	$e^{-3i\phi}$
	Y_3^{-2}	$\sqrt{\frac{105}{32\pi}}$	$\sin^2\theta\cos\theta$	$e^{-2i\phi}$
	Y_3^{-1}	$\sqrt{\frac{21}{64\pi}}$	$\sin(2\theta)\sin\theta$	$e^{-i\phi}$
	Y_3^0	$\sqrt{\frac{7}{16\pi}}$	$5\cos^3\theta - 3\cos\theta$	
	Y_3^1	$-\sqrt{\frac{21}{64\pi}}$	$\sin(2\theta)\sin\theta$	$e^{i\phi}$
	Y_3^2	$\sqrt{\frac{105}{32\pi}}$	$\sin^2\theta\cos\theta$	$e^{2i\phi}$
	Y_3^3	$-\sqrt{\frac{35}{64\pi}}$	$\sin^3\theta$	$e^{3i\phi}$

6.1 Spherical Harmonic Decomposition

This document provides a detailed procedure to resolve a list of function values $f(\theta, \phi)$ into amplitudes of spherical harmonic functions $Y_\ell^m(\theta, \phi)$.

6.2 Spherical Harmonic Expansion

The function $f(\theta, \phi)$ can be expanded as:

$$f(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta, \phi),$$

where the coefficients $a_{\ell m}$ are computed by:

$$a_{\ell m} = \int_0^{2\pi} \int_0^{\pi} f(\theta, \phi) Y_\ell^m(\theta, \phi)^* \sin\theta \, d\theta \, d\phi.$$

Here, $Y_\ell^m(\theta, \phi)^*$ is the complex conjugate of the spherical harmonics.

6.3 Mathematica Code

6.3.1 Define the Function

Define the function $f(\theta, \phi)$ in Mathematica:

```
(* Function definition *)
f[theta_, phi_] := Sin[theta] * Cos[phi]
```

6.3.2 Compute the Coefficients

The following Mathematica code computes the spherical harmonic coefficients $a_{\ell m}$ for a given function $f(\theta, \phi)$ up to a maximum ℓ value:

```
(* Define the maximum value of l for expansion *)
maxL = 5;

(* Create a table of coefficients *)
coefficients = Table[
   NIntegrate[
     f[theta, phi] Conjugate[SphericalHarmonicY[l, m, theta, phi]]
    Sin[theta],
     {theta, 0, Pi}, {phi, 0, 2 Pi}
   ],
   {l, 0, maxL}, {m, -l, l}
];
```

6.3.3 Reconstruct the Function

The reconstructed function is computed as:

```
(* Reconstruct the function *)
reconstructedF[theta_, phi_] := Sum[
  coefficients[[l + 1, m + l + 1]] SphericalHarmonicY[l, m, theta, phi],
  {l, 0, maxL}, {m, -l, l}
];
```

6.3.4 Visualization

Compare the original and reconstructed functions using plots:

```
(* Plot original function *)
SphericalPlot3D[Abs[f[theta, phi]], {theta, 0, Pi}, {phi, 0, 2 Pi}]

(* Plot reconstructed function *)
SphericalPlot3D[Abs[reconstructedF[theta, phi]], {theta, 0, Pi}, {phi, 0, 2
    Pi}]
```

6.4 Notes

- Ensure the resolution of the θ and ϕ sampling is sufficient for accurate representation.
- The numerical integration step (`NIntegrate`) can be adjusted for higher precision if required.

Least Squares

The goal of this subsection is to find the amplitudes $a_{\ell m}$ of the spherical harmonic expansion that best approximate the function $f(\theta, \phi)$ sampled at N discrete points.

Spherical Harmonic Expansion

The spherical harmonic expansion of the function $f(\theta, \phi)$ is:

$$f(\theta, \phi) \approx \sum_{\ell=0}^{L_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta, \phi),$$

where $Y_\ell^m(\theta, \phi)$ are the spherical harmonics and $a_{\ell m}$ are the expansion coefficients.

Discrete Sampling

Assuming the function $f(\theta, \phi)$ is sampled at N discrete points (θ_i, ϕ_i), we denote the sampled values as $f_i = f(\theta_i, \phi_i)$ for $i = 1, 2, \ldots, N$. At these sampled points, the spherical harmonic expansion becomes:

$$f_i \approx \sum_{\ell=0}^{L_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta_i, \phi_i), \quad i = 1, 2, \ldots, N.$$

Matrix Formulation

The system of equations can be expressed in matrix form:

$$\mathbf{f} \approx \mathbf{Y}\mathbf{a},$$

where:

- $\mathbf{f} \in \mathbb{R}^N$ is the vector of sampled function values:

$$\mathbf{f} = [f_1, f_2, \ldots, f_N]^\top,$$

- $\mathbf{a} \in \mathbb{R}^M$ is the vector of unknown coefficients:

$$\mathbf{a} = [a_{00}, a_{1,-1}, a_{10}, a_{11}, \ldots, a_{L_{\max},L_{\max}}]^\top,$$

  where $M = (L_{\max} + 1)^2$ is the total number of coefficients,
- $\mathbf{Y} \in \mathbb{R}^{N \times M}$ is the design matrix:

$$Y_{ij} = Y_\ell^m(\theta_i, \phi_i),$$

  where j indexes the pair (ℓ, m) in lexicographic order.

Least Squares Solution

To find the best-fit coefficients $\mathbf{a}$, we minimize the residual:

$$\|\mathbf{f} - \mathbf{Y}\mathbf{a}\|^2.$$

The least-squares solution is obtained by solving the normal equations:

$$\mathbf{Y}^\top \mathbf{Y}\mathbf{a} = \mathbf{Y}^\top \mathbf{f}.$$

Key Components of the Linear System

1. **Matrix $\mathbf{Y}$:**
   - Rows correspond to the sampled points (θ_i, ϕ_i).
   - Columns correspond to the spherical harmonics $Y_\ell^m(\theta, \phi)$ for each (ℓ, m).
2. **Vector $\mathbf{a}$:**
   - Contains the amplitudes $a_{\ell m}$ of the spherical harmonic expansion.
3. **Vector $\mathbf{f}$:**
   - Contains the sampled function values f_i.

Summary

The least squares fit constructs a linear system:

$$\mathbf{Y}^\top \mathbf{Y}\mathbf{a} = \mathbf{Y}^\top \mathbf{f}.$$

Solving this system yields the amplitudes $a_{\ell m}$ that approximate the function $f(\theta, \phi)$ in terms of spherical harmonics.

6.5 Vector formulation

Let $\mathbf{a}$ represent the vector of amplitudes and $\mathbf{b}(\theta, \phi)$ denote the vector of basis functions evaluated at a given point (θ, ϕ). The function $f(\theta, \phi)$ is approximated as the dot product of these two vectors:

$$f(\theta, \phi) \approx \mathbf{a} \cdot \mathbf{b}(\theta, \phi).$$

Components

- **Vector of Amplitudes (a):**

$$\mathbf{a} = [a_{00}, a_{1,-1}, a_{10}, a_{11}, \ldots, a_{L_{\max},L_{\max}}]^\top,$$

  where $a_{\ell m}$ are the coefficients of the spherical harmonic expansion.
- **Vector of Basis Functions ($\mathbf{b}(\theta, \phi)$):**

$$\mathbf{b}(\theta, \phi) = [Y_0^0(\theta, \phi), Y_1^{-1}(\theta, \phi), Y_1^0(\theta, \phi), Y_1^1(\theta, \phi), \ldots, Y_{L_{\max}}^{L_{\max}}(\theta, \phi)]^\top,$$

  where $Y_\ell^m(\theta, \phi)$ are the spherical harmonics.
- **Approximation Function ($f(\theta, \phi)$):** The approximation of $f(\theta, \phi)$ is expressed as:

$$f(\theta, \phi) \approx \mathbf{a} \cdot \mathbf{b}(\theta, \phi) = \sum_{\ell=0}^{L_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta, \phi).$$

Discrete Sampling

For N discrete sampled points (θ_i, ϕ_i), the function values $f_i = f(\theta_i, \phi_i)$ can be written as:

$$f_i \approx \mathbf{a} \cdot \mathbf{b}(\theta_i, \phi_i), \quad i = 1, 2, \ldots, N.$$

This yields the linear system:

$$\mathbf{f} \approx \mathbf{B}\mathbf{a},$$

where:

- $\mathbf{f} = [f_1, f_2, \ldots, f_N]^\top$ is the vector of sampled function values.
- $\mathbf{B} \in \mathbb{R}^{N \times M}$ is the matrix of basis functions, with:

$$\mathbf{B}_{ij} = b_j(\theta_i, \phi_i) = Y_\ell^m(\theta_i, \phi_i),$$

  where j indexes the basis functions Y_ℓ^m.
- $\mathbf{a}$ is the vector of amplitudes as defined earlier.

Least Squares Solution

To determine $\mathbf{a}$, the squared residual is minimized:

$$\|\mathbf{f} - \mathbf{B}\mathbf{a}\|^2.$$

The least squares solution is given by:

$$\mathbf{a} = (\mathbf{B}^\top \mathbf{B})^{-1} \mathbf{B}^\top \mathbf{f}.$$

Interpretation

- The rows of $\mathbf{B}$ correspond to the sampled points (θ_i, ϕ_i).
- The columns of $\mathbf{B}$ correspond to the spherical harmonic basis functions $Y_\ell^m(\theta, \phi)$.
- The vector $\mathbf{a}$ contains the amplitudes $a_{\ell m}$ that weight the contribution of each basis function.

7 File Types

Standard file types

1. `*.obj`
2. `*.txt`

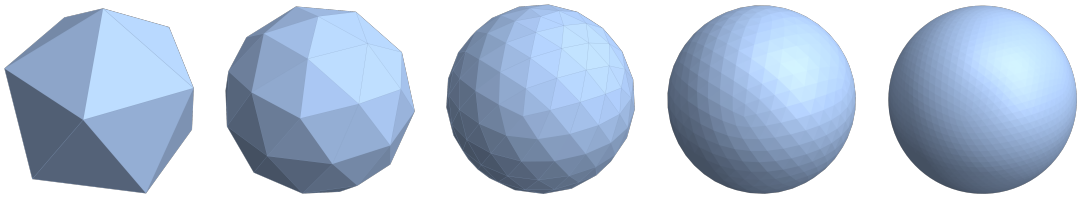
Intrinsic file types

1. `*.geo`
2. `*.facet`

7.1 Geometry Files `*.obj`

The geometry files are the primary input to Hg MoM.

1. Mathematica: Import/Export format OBJ
2. Wikipedia: Wavefront .obj file
3. All3DP: The OBJ File Format – Simply Explained



20 80 320 1280 5120

Table 8: A sphere resolved into facets in an `*.obj` file. The application irradiates each facet and aggregates the output.

Listing 1: The `*.obj` file for the sphere with 20 facets.

```
# Created with the Wolfram Language : www.wolfram.com
mtllib sphere-d050-01.mtl

# 12 vertex positions
v  13.81966018676758 42.53253936767578 22.36067962646484
v  -13.81966018676758 42.53253936767578 -22.36067962646484
v  -36.18033981323242 26.28655624389648 22.36067962646484
v  -36.18033981323242 -26.28655624389648 22.36067962646484
v  -13.81966018676758 -42.53253936767578 -22.36067962646484
v  13.81966018676758 -42.53253936767578 22.36067962646484
v  36.18033981323242 26.28655624389648 -22.36067962646484
v  -44.72135925292969 0 -22.36067962646484
v  44.72135925292969 0 22.36067962646484
v  36.18033981323242 -26.28655624389648 -22.36067962646484
v  0 0 50
v  0 0 -50

# 0 UV coordinates

# 0 vertex normals
```

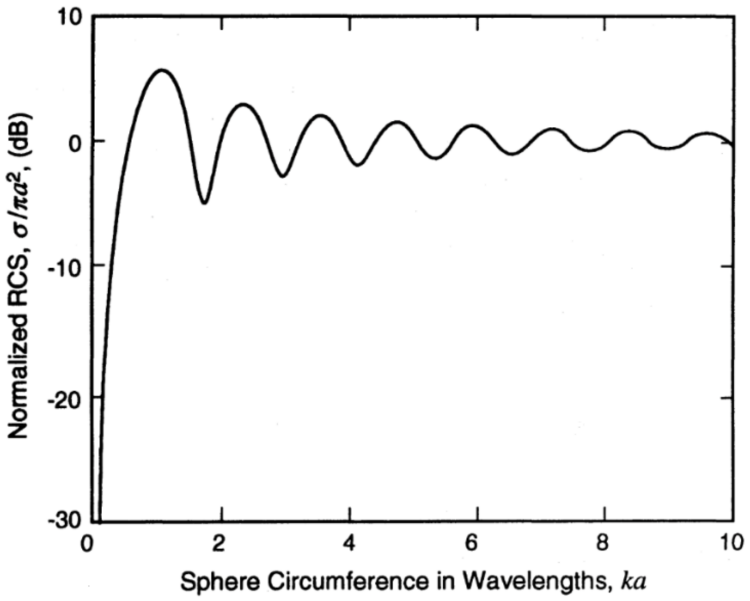


Figure 4.2. Sphere RCS backscatter as a function of *ka* as computed by the algorithm shown in Listing 4.1.

Table 9: Closed form solution for spherical scattering taken from [7, p. 123].

```
21
22 # Mesh '' with 20 faces
23 usemtl DefaultMaterial
24 f  1/ 2/ 3/
25 f  4/ 5/ 6/
26 f  2/ 1/ 7/
27 f  4/ 3/ 8/
28 f  9/ 6/ 10/
29 f  3/ 4/ 11/
30 f  2/ 7/ 12/
31 f  3/ 2/ 8/
32 f  2/ 12/ 8/
33 f  12/ 5/ 8/
34 f  7/ 1/ 9/
35 f  5/ 4/ 8/
36 f  6/ 5/ 10/
37 f  5/ 12/ 10/
38 f  12/ 7/ 10/
39 f  7/ 9/ 10/
40 f  4/ 6/ 11/
41 f  6/ 9/ 11/
42 f  9/ 1/ 11/
43 f  1/ 3/ 11/
```

Listing 2: The `*.facet` file for the sphere with 20 facets.

```
1  facimusFacet.f08  2020-06-25 11:34:36
2      1
3  <partName>
4  0
5       12
6       13.819660      42.532539      22.360680
7      -13.819660      42.532539     -22.360680
8      -36.180340      26.286556      22.360680
9      -36.180340     -26.286556      22.360680
10     -13.819660     -42.532539     -22.360680
11      13.819660     -42.532539      22.360680
12      36.180340      26.286556     -22.360680
13     -44.721359       0.000000     -22.360680
14      44.721359       0.000000      22.360680
15      36.180340     -26.286556     -22.360680
16       0.000000       0.000000      50.000000
17       0.000000       0.000000     -50.000000
18     1
19 <partName>
20       3     20      0      0      0      0      0
21       1      2      3      0
22       4      5      6      0
23       2      1      7      0
24       4      3      8      0
25       9      6     10      0
```

```
      3      4     11      0
      2      7     12      0
      3      2      8      0
      2     12      8      0
     12      5      8      0
      7      1      9      0
      5      4      8      0
      6      5     10      0
      5     12     10      0
     12      7     10      0
      7      9     10      0
      4      6     11      0
      6      9     11      0
      9      1     11      0
      1      3     11      0
```

7.2 New Efforts

8 Running Mercury Method of Moments

8.1 Inputs

Consider an example with the sphere.

Listing 3: "tabula-rasa.geo"

```
ego

!Mercury MoM input file, VIE/SIE Version 4.x compatible (VIE/Dual Sided SIE)

&MM_MOM
  bUseACA = .TRUE.,
  bSolve_ACA = .TRUE.,
  bOutOfCore = .TRUE.,
  bNormalizeToWaveLength = .FALSE.,
  bNormalize             = .FALSE.,
  dCloseLambda  = 0.100000,
  ACA_Factor_Tol = 0.000010,
  ACA_RHS_Tol = 0.000100,
  Point_Tolerance = 0.001000,
  nLargestBlockSize = 400,
  MemorySize_GB = -1.000000,
  stackSize_GB = -1.000000,
  nFillThreads = -1,
  nFillMKLThreads = 1,
  nLUThreads = -1,
  nLUMKLThreads = 1,
  nRHSThreads = 1,
  nRHSMKLThreads = 1,
  bOutputACAGrouping     = .FALSE.,
```

```
  bOutputRankFraction    = .FALSE.,
  bLimitLUColumns        = .FALSE.,
  Lop_Admissibility = WEAK,
  Kop_Admissibility = CLOSE
/

&Scratch_Memory
  Scratch_RankFraction_Z     = 0.300000,
  Scratch_RankFraction_LU    = 0.600000,
  Scratch_RankFraction_RHS   = 0.500000,
  Scratch_RankFraction_Solve = 1.000000,
  MemoryFraction_Z           = 0.950000,
  MemoryFraction_Scratch_LU  = 0.500000,
  MemoryFraction_LU          = 1.000000,
  MemoryFraction_RHS         = 0.500000,
  MemoryFraction_Solve       = 0.900000,
/

&QUADRATURE
  NTRISELF    = 7,
  NTRINEAR    = 3,
  NTRIFAR     = 3,
  NTETSELF    = 11,
  NTETNEAR    = 4,
  NTETFAR     = 4,
  NQGAUSS  = 4,
/

FREQUENCY
  mhz
  nu-mhz  nu-mhz  1   !Freq Start, Freq Stop, Number of Frequencies

Excitation
  MONOSTATIC

Angle Cut
  1
  0.000000  359.000000  360
  AZIMUTH
  90.000000

Boundary Conditions
PEC-Materials.lib
4
V_FREE_SPACE => Free_Space
V_PEC => PEC
V_PMC => PMC
V_NULL => NULL
1
0 BC_PEC V_FREE_SPACE
```

```

SIE
myFacet.facet
m

Geometry_End

! Fiducial run
```

Listing 4: A simple `*.facet` file

```
facimusFacet.f08  2020-06-25 11:34:36
    1
<partName>
0
     12
     13.819660      42.532539      22.360680
    -13.819660      42.532539     -22.360680
    -36.180340      26.286556      22.360680
    -36.180340     -26.286556      22.360680
    -13.819660     -42.532539     -22.360680
     13.819660     -42.532539      22.360680
     36.180340      26.286556     -22.360680
    -44.721359       0.000000     -22.360680
     44.721359       0.000000      22.360680
     36.180340     -26.286556     -22.360680
      0.000000       0.000000      50.000000
      0.000000       0.000000     -50.000000
    1
<partName>
      3     20      0      0      0      0      0
      1      2      3      0
      4      5      6      0
      2      1      7      0
      4      3      8      0
      9      6     10      0
      3      4     11      0
      2      7     12      0
      3      2      8      0
      2     12      8      0
     12      5      8      0
      7      1      9      0
      5      4      8      0
      6      5     10      0
      5     12     10      0
     12      7     10      0
      7      9     10      0
      4      6     11      0
      6      9     11      0
      9      1     11      0
      1      3     11      0
```

8.2 Outputs

```
 Freq   =    10.00E+00 MHz
 Lambda =    29.98E+00 m
 k      =   209.58E-03 m-1

BACKSCATTER RCS RESULTS .....

 Theta, Phi, Theta-Theta (complex efield), Phi-Theta (complex efield), Theta-Phi (complex efield), Phi-Phi (complex efield)

     90.0000,      0.0000,(  0.2333228E-01,  0.1410688E-03),(  0.8868390E-04,  0.5353989E-05),(  0.8868393E-04,  0.5354086E-05),(  0.2275015E-01,  0.1270486E-03)
     90.0000,      1.0000,(  0.2333244E-01,  0.1402642E-03),(  0.8605096E-04,  0.5259093E-05),(  0.8604993E-04,  0.5248534E-05),(  0.2274918E-01,  0.1264947E-03)
     90.0000,      2.0000,(  0.2333259E-01,  0.1393928E-03),(  0.8339065E-04,  0.5144119E-05),(  0.8338990E-04,  0.5151524E-05),(  0.2274821E-01,  0.1259348E-03)
     90.0000,      3.0000,(  0.2333274E-01,  0.1384604E-03),(  0.8070502E-04,  0.5039024E-05),(  0.8070488E-04,  0.5032761E-05),(  0.2274726E-01,  0.1253780E-03)
     90.0000,      4.0000,(  0.2333289E-01,  0.1374878E-03),(  0.7799496E-04,  0.4922949E-05),(  0.7799467E-04,  0.4916130E-05),(  0.2274633E-01,  0.1248239E-03)
     90.0000,      5.0000,(  0.2333302E-01,  0.1364364E-03),(  0.7526141E-04,  0.4795374E-05),(  0.7526139E-04,  0.4787100E-05),(  0.2274541E-01,  0.1242470E-03)
     90.0000,      6.0000,(  0.2333315E-01,  0.1353522E-03),(  0.7250279E-04,  0.4655677E-05),(  0.7250353E-04,  0.4652742E-05),(  0.2274450E-01,  0.1236636E-03)
     90.0000,      7.0000,(  0.2333328E-01,  0.1342022E-03),(  0.6972432E-04,  0.4524583E-05),(  0.6972397E-04,  0.4519492E-05),(  0.2274361E-01,  0.1230949E-03)
     90.0000,      8.0000,(  0.2333339E-01,  0.1330091E-03),(  0.6692425E-04,  0.4379594E-05),(  0.6692352E-04,  0.4374137E-05),(  0.2274274E-01,  0.1225020E-03)
     90.0000,      9.0000,(  0.2333351E-01,  0.1317574E-03),(  0.6410291E-04,  0.4236832E-05),(  0.6410385E-04,  0.4231747E-05),(  0.2274190E-01,  0.1219020E-03)
     90.0000,     10.0000,(  0.2333361E-01,  0.1304482E-03),(  0.6126419E-04,  0.4088875E-05),(  0.6126271E-04,  0.4079980E-05),(  0.2274107E-01,  0.1213053E-03)
     90.0000,     11.0000,(  0.2333371E-01,  0.1290931E-03),(  0.5840381E-04,  0.3920618E-05),(  0.5840345E-04,  0.3921868E-05),(  0.2274026E-01,  0.1207031E-03)
     90.0000,     12.0000,(  0.2333380E-01,  0.1276849E-03),(  0.5552738E-04,  0.3757901E-05),(  0.5552594E-04,  0.3756007E-05),(  0.2273948E-01,  0.1200838E-03)
     90.0000,     13.0000,(  0.2333388E-01,  0.1262230E-03),(  0.5263346E-04,  0.3593899E-05),(  0.5263261E-04,  0.3591580E-05),(  0.2273871E-01,  0.1194466E-03)
     90.0000,     14.0000,(  0.2333395E-01,  0.1247289E-03),(  0.4972452E-04,  0.3426847E-05),(  0.4972324E-04,  0.3422775E-05),(  0.2273798E-01,  0.1188115E-03)
     90.0000,     15.0000,(  0.2333402E-01,  0.1231622E-03),(  0.4679928E-04,  0.3249850E-05),(  0.4679913E-04,  0.3247067E-05),(  0.2273727E-01,  0.1181715E-03)
     90.0000,     16.0000,(  0.2333408E-01,  0.1215549E-03),(  0.4386077E-04,  0.3066160E-05),(  0.4386053E-04,  0.3071021E-05),(  0.2273658E-01,  0.1175054E-03)
     90.0000,     17.0000,(  0.2333414E-01,  0.1199063E-03),(  0.4090846E-04,  0.2884632E-05),(  0.4090883E-04,  0.2891672E-05),(  0.2273593E-01,  0.1168209E-03)
     90.0000,     18.0000,(  0.2333418E-01,  0.1182040E-03),(  0.3794509E-04,  0.2701195E-05),(  0.3794405E-04,  0.2696849E-05),(  0.2273530E-01,  0.1161288E-03)
     90.0000,     19.0000,(  0.2333422E-01,  0.1164673E-03),(  0.3497020E-04,  0.2512710E-05),(  0.3497056E-04,  0.2501005E-05),(  0.2273469E-01,  0.1154252E-03)
     90.0000,     20.0000,(  0.2333425E-01,  0.1146772E-03),(  0.3198408E-04,  0.2319652E-05),(  0.3198424E-04,  0.2317885E-05),(  0.2273412E-01,  0.1146969E-03)
     90.0000,     21.0000,(  0.2333428E-01,  0.1128494E-03),(  0.2898948E-04,  0.2112308E-05),(  0.2898854E-04,  0.2120017E-05),(  0.2273357E-01,  0.1139418E-03)
     90.0000,     22.0000,(  0.2333429E-01,  0.1109812E-03),(  0.2598479E-04,  0.1921074E-05),(  0.2598468E-04,  0.1917728E-05),(  0.2273306E-01,  0.1131798E-03)
     90.0000,     23.0000,(  0.2333430E-01,  0.1090673E-03),(  0.2297422E-04,  0.1725964E-05),(  0.2297450E-04,  0.1716165E-05),(  0.2273258E-01,  0.1123858E-03)
     90.0000,     24.0000,(  0.2333430E-01,  0.1071143E-03),(  0.1995689E-04,  0.1524590E-05),(  0.1995638E-04,  0.1524156E-05),(  0.2273213E-01,  0.1115781E-03)
     90.0000,     25.0000,(  0.2333429E-01,  0.1051199E-03),(  0.1693341E-04,  0.1322952E-05),(  0.1693271E-04,  0.1316243E-05),(  0.2273170E-01,  0.1107367E-03)
     90.0000,     26.0000,(  0.2333428E-01,  0.1030911E-03),(  0.1390601E-04,  0.1110616E-05),(  0.1390536E-04,  0.1106821E-05),(  0.2273132E-01,  0.1098675E-03)
     90.0000,     27.0000,(  0.2333426E-01,  0.1010452E-03),(  0.1087428E-04,  0.9074643E-06),(  0.1087365E-04,  0.9026098E-06),(  0.2273096E-01,  0.1089735E-03)
     90.0000,     28.0000,(  0.2333423E-01,  0.9894501E-04),(  0.7839484E-05,  0.7072128E-06),(  0.7838623E-05,  0.7006824E-06),(  0.2273064E-01,  0.1080477E-03)
     90.0000,     29.0000,(  0.2333420E-01,  0.9680697E-04),(  0.4801968E-05,  0.4874063E-06),(  0.4802196E-05,  0.4862423E-06),(  0.2273035E-01,  0.1070995E-03)
     90.0000,     30.0000,(  0.2333416E-01,  0.9465464E-04),(  0.1765148E-05,  0.2732763E-06),(  0.1764301E-05,  0.2821041E-06),(  0.2273009E-01,  0.1061108E-03)
     90.0000,     31.0000,(  0.2333411E-01,  0.9246088E-04),( -0.1273034E-05,  0.7069519E-07),( -0.1272663E-05,  0.7093635E-07),(  0.2272987E-01,  0.1050739E-03)
     90.0000,     32.0000,(  0.2333405E-01,  0.9023387E-04),( -0.4309802E-05, -0.1458179E-06),( -0.4310374E-05, -0.1394553E-06),(  0.2272967E-01,  0.1040102E-03)
     90.0000,     33.0000,(  0.2333399E-01,  0.8799594E-04),( -0.7344976E-05, -0.3539628E-06),( -0.7344815E-05, -0.3522355E-06),(  0.2272952E-01,  0.1029183E-03)
     90.0000,     34.0000,(  0.2333392E-01,  0.8569709E-04),( -0.1037675E-04, -0.5588603E-06),( -0.1037787E-04, -0.5538839E-06),(  0.2272940E-01,  0.1017700E-03)
     90.0000,     35.0000,(  0.2333385E-01,  0.8340531E-04),( -0.1340611E-04, -0.7677730E-06),( -0.1340637E-04, -0.7624512E-06),(  0.2272931E-01,  0.1005938E-03)
     90.0000,     36.0000,(  0.2333376E-01,  0.8107661E-04),( -0.1643024E-04, -0.9759825E-06),( -0.1643008E-04, -0.9753808E-06),(  0.2272925E-01,  0.9937227E-04)
     90.0000,     37.0000,(  0.2333368E-01,  0.7871235E-04),( -0.1944779E-04, -0.1176726E-05),( -0.1944765E-04, -0.1178540E-05),(  0.2272923E-01,  0.9810377E-04)
     90.0000,     38.0000,(  0.2333359E-01,  0.7633654E-04),( -0.2246019E-04, -0.1377676E-05),( -0.2246095E-04, -0.1376291E-05),(  0.2272925E-01,  0.9677992E-04)
     90.0000,     39.0000,(  0.2333349E-01,  0.7394435E-04),( -0.2546455E-04, -0.1582781E-05),( -0.2546509E-04, -0.1583517E-05),(  0.2272929E-01,  0.9543045E-04)
     90.0000,     40.0000,(  0.2333338E-01,  0.7153424E-04),( -0.2846110E-04, -0.1786374E-05),( -0.2846093E-04, -0.1782039E-05),(  0.2272937E-01,  0.9402471E-04)
     90.0000,     41.0000,(  0.2333328E-01,  0.6909868E-04),( -0.3144751E-04, -0.1982437E-05),( -0.3144809E-04, -0.1981662E-05),(  0.2272949E-01,  0.9257005E-04)
     90.0000,     42.0000,(  0.2333316E-01,  0.6664295E-04),( -0.3442489E-04, -0.2177858E-05),( -0.3442387E-04, -0.2174608E-05),(  0.2272963E-01,  0.9106311E-04)
     90.0000,     43.0000,(  0.2333303E-01,  0.6418363E-04),( -0.3739010E-04, -0.2378527E-05),( -0.3739017E-04, -0.2372635E-05),(  0.2272981E-01,  0.8950324E-04)
     90.0000,     44.0000,(  0.2333291E-01,  0.6171301E-04),( -0.4034362E-04, -0.2561433E-05),( -0.4034395E-04, -0.2563795E-05),(  0.2273003E-01,  0.8788771E-04)
     90.0000,     45.0000,(  0.2333278E-01,  0.5922416E-04),( -0.4328624E-04, -0.2751016E-05),( -0.4328505E-04, -0.2742658E-05),(  0.2273027E-01,  0.8622059E-04)
     90.0000,     46.0000,(  0.2333264E-01,  0.5670952E-04),( -0.4621203E-04, -0.2930050E-05),( -0.4621215E-04, -0.2936943E-05),(  0.2273055E-01,  0.8449983E-04)
     90.0000,     47.0000,(  0.2333250E-01,  0.5419380E-04),( -0.4912426E-04, -0.3104662E-05),( -0.4912537E-04, -0.3112095E-05),(  0.2273086E-01,  0.8272303E-04)
     90.0000,     48.0000,(  0.2333236E-01,  0.5167195E-04),( -0.5202224E-04, -0.3285702E-05),( -0.5202286E-04, -0.3284841E-05),(  0.2273119E-01,  0.8090130E-04)
     90.0000,     49.0000,(  0.2333221E-01,  0.4914165E-04),( -0.5490336E-04, -0.3457679E-05),( -0.5490287E-04, -0.3454552E-05),(  0.2273156E-01,  0.7901491E-04)
     90.0000,     50.0000,(  0.2333206E-01,  0.4659708E-04),( -0.5776730E-04, -0.3628157E-05),( -0.5776656E-04, -0.3622614E-05),(  0.2273197E-01,  0.7707268E-04)
     90.0000,     51.0000,(  0.2333190E-01,  0.4405807E-04),( -0.6061237E-04, -0.3786502E-05),( -0.6061319E-04, -0.3788289E-05),(  0.2273240E-01,  0.7505807E-04)
     90.0000,     52.0000,(  0.2333174E-01,  0.4150420E-04),( -0.6343987E-04, -0.3945118E-05),( -0.6344026E-04, -0.3940577E-05),(  0.2273285E-01,  0.7301238E-04)
```

9 Additional Information

9.1 YouTube Videos

YouTube offers useful didactic presentations and simulations.

1. The Radar cross-section of backscattering objects
2. Basic Concepts of Radar Cross Section (RCS)
3. Mie scattering
4. Mie theory (BME51 Lecture 5)
5. Mie Scattering

9.2 Further Reading

Radar rudiments

1. D. K. Barton and H.R. Ward. *Handbook of Radar Measurement*. New York, NY: Penguin Random House, 1969
2. Andrei A. Kolosov. *Over the Horizon Radar*. Artech House, 1987. ISBN: 9780890062333. URL: `https://us.artechhouse.com/Over-the-Horizon-Radar-P254.aspx`
3. Peyton Z Peebles. *Radar principles*. John Wiley & Sons, 2007

Radar cross section

1. JW Jr Crispin. *Methods of radar cross-section analysis*. Elsevier, 2013
2. Allen E Fuhs. *Radar cross section lectures*. Monterey, California, Naval Postgraduate School, 1982. URL: `https://calhoun.nps.edu/server/api/core/bitstreams/9e69ec48-4628-4243-9f9b-7e879521f7f8/content`
3. Eugene F Knott, John F Schaeffer, and Michael T Tulley. *Radar cross section*. SciTech Publishing, 2004
4. M Madheswaran and P Suresh Kumar. "Estimation of wide band radar cross section (RCS) of regular shaped objects using method of moments (MOM)". in: *Ictact Journal on Communication Tech-nology* 3.2 (2012), pp. 536–541

Method of Moments

1. Walton C Gibson. *The method of moments in electromagnetics*. Chapman and Hall/CRC, 2021
2. Roger F Harrington. "The method of moments in electromagnetics". In: *Journal of Electromagnetic waves and Applications* 1.3 (1987), pp. 181–200
3. Cai-Cheng Lu and Chong Luo. "Comparison of iteration convergences of SIE and VSIE for solving electromagnetic scattering problems for coated objects". In: *Radio Science* 38.2 (2003), pp. 11–1
4. Jiade Yuan, Changqing Gu, and Guodong Han. "Efficient generation of method of moments matrices using equivalent dipole-moment method". In: *IEEE Antennas and Wireless Propagation Letters* 8 (2009), pp. 716–719

Using Mercury MoM and post-processing

1. Daniel Topa. *Radar Cross Section Models for AFCAP Dashboard: Rapid Report 2020-02: Corrected*. Briefing. Mar. 2020
2. Daniel Topa. *Mercury Method of Moments Adjunct Visualization Tool: Trials and Tribulations*. Tech. rep. ARFL/RVB, Apr. 2020
3. Daniel Topa. *Radar Cross Section: Phase 1 Summary Report*. Tech. rep. ARFL/RVB, Apr. 2020
4. Daniel Topa. *Mercury Method of Moments: AFRL Quick Start Guide*. Tech. rep. AFRL, 2020

A Mercury Method of Moments: Data Formats

A.1 Numeric Results

The MoM RCS data is delivered in a matrix with m rows and n columns (standard matrix addressing).

$1 \leq m \leq 28$ MHz (integer steps)
$1 \leq n \leq 90$ degrees (integer steps)

The matrix is WIDE (more columns than rows)

Frequency partition: row 1: 3 MHz row 2: 4 MHz . . . . row 28: 30 Mhz
Let r index the rows. Then frequency ν is in row $= \nu - 2$
Angular partition col 1: 0 col 2: 1 . . . col 181: 180
col 1 col 2 col 3 col 181 0 1 2 . . . 180
Let c be the column index. The measurement for angle alpha is in column $c = alpha + 1$

The test asset is symmetric: $\sigma(\alpha) = \sigma(-\alpha)$

But the matrix can easily be delivered in other forms, such as the transpose (interchange rows and columns), or packed into a linear array.

Sample:

4.16411, 4.14247, 4.07319, 3.95637, 3.79263, 3.58287, 3.32827, 3.0303, . . .
18.2776, 18.2369, 18.1199, 17.9248, 17.6523, 17.3041, 16.8817, 16.3876, . . .
25.6306, 25.5886, 25.463, 25.2538, 24.9618, 24.5882, 24.1346, 23.6028, . . .
. . .

B Mercury Method of Moments: Software Toolkit

Mercury MoM produces thousands of lines of output to a `*.4112.txt` file, a mix of numbers and strings. Once the data portions are located, they can be harvested straightaway. However, the text messages include debug information and the text patterns are varied.

Data analysis on data sets with a large number of facets can take several hours.

B.1 `rcsharvester.f08`

```
!  harvest the electric field values from the ASCII file *.4112.txt mixed text and numeric
lines
!  compute the mean total RCS and write these values
```

B.1.1 Class Electric Fields: `m-class-electric-fields.f08`

B.1.2 `m-class-electric-fields.f08`

The primary output of the simulation are the electric fields. Lines 17-24 define the class; the remainder of the codes is for methods. The input electric field is resolved into two polarization axes: horizontal and vertical. Each of these fields are resolved into horizontal and vertical components creating four complex vectors (line 21) whose length matches the angular sample size.

The class `m-class-electric-fields.f08` reads the text file and harvests the electric fields eventually passing back a composite value (lines 65-66) for all four components of the scattering return.

```
1  ! Parses alphanumeric line from MoM *.4112.txt and extracts electric field
       values
2  module mClassElectricFields
3
4      use mFormatDescriptors,              only : fmt_stat, fmt_iomsg
5      use mLibraryOfConstants,             only : cZero, MoMlineLength,
       messageLength
6      use mPrecisionDefinitions,           only : ip, rp
7
8      implicit none
9
10     integer ( ip ) :: left = 0, right = 0
11     integer :: io_stat = 0
12     character ( len = messageLength ) :: io_msg = ""
13     character ( len = 15 )            :: number = ""
14
15     ! theta = azimuth
16     ! phi = elevation (North Pole = 0, equator = 90)
17     type :: electricFields
18         real     ( rp ) :: meanTotalRCS = 0.0_rp
19         real     ( rp ) :: dBsm  = 0.0_rp
20         real     ( rp ) :: theta = 0.0_rp, phi = 0.0_rp
21         complex  ( rp ) :: thetaTheta = cZero, thetaPhi = cZero, phiTheta =
       cZero, phiPhi = cZero
22     contains
23         procedure, public :: gather_mean_total_rcs =>
       gather_mean_total_rcs_sub
24     end type electricFields
25
26     private :: gather_mean_total_rcs_sub
27     private :: compute_mean_total_rcs_sub, compute_dbsm_sub,
       extract_electric_fields_sub
28     private :: gather_complex_field_sub, gather_real_field_sub
29
30     ! parameters
31     integer ( ip ), parameter :: mll = MomlineLength
32     ! finger print of data line: start and stop positions for each numerical
       field
33     ! load matrix as columns
34     ! sample data line:
35     !      90.0000,      0.0000,( -0.4572920E+05,  0.8350829E+05),(
       0.2034567E+06, -0.9493007E+05),(  0.2034813E+06, -0.9492184E+05),(
       -0.1727375E+06,  0.3787291E+05)
36     integer, parameter :: endpoints ( 1 : 10, 1 : 2 ) = &
37         reshape ( [ [  1, 14, 28, 44, 62, 78,  96, 112, 130, 146 ], &
38     [ 12, 25, 42, 58, 76, 92, 110, 126, 144, 160 ] ], [ 10, 2 ] )
39     ! constructor
40     type ( electricFields ), parameter :: electricFields0 = &
```

```
41          electricFields ( meanTotalRCS = 0.0, theta = 0.0, phi = 0.0, &
42          thetaTheta = cZero, thetaPhi = cZero, phiTheta = cZero, phiPhi =
    cZero )
43  contains
44
45      ! master routine: only exposed procedure
46      subroutine gather_mean_total_rcs_sub ( me, textLine )
47          class ( electricFields ),       target :: me
48          character ( len = mll ), intent ( in ) :: textLine
49              call extract_electric_fields_sub ( me, textLine )
50              call compute_mean_total_rcs_sub  ( me )
51              call compute_dbsm_sub            ( me )
52          return
53      end subroutine gather_mean_total_rcs_sub
54
55      ! Sciacca prescription
56      subroutine compute_dbsm_sub ( me )
57          class ( electricFields ), target :: me
58              me % dBsm = 10.0_rp * log10 ( me % meanTotalRCS )
59          return
60      end subroutine compute_dbsm_sub
61
62      ! Sciacca prescription
63      subroutine compute_mean_total_rcs_sub ( me )
64          class ( electricFields ), target :: me
65              me % meanTotalRCS = abs ( me % thetaTheta ) + abs ( me %
    thetaPhi ) &
66                                + abs ( me % phiTheta ) + abs ( me % phiPhi )
67              me % meanTotalRCS = me % meanTotalRCS / real ( 2, kind = rp )
68          return
69      end subroutine compute_mean_total_rcs_sub
70
71      subroutine extract_electric_fields_sub ( me, textLine )
72          class ( electricFields ),       target :: me
73          character ( len = mll ), intent ( in ) :: textLine
74          integer ( ip ) :: position = 0
75              ! move across text line gathering numeric values
76              position = 1
77              call gather_real_field_sub  &
78                  ( position = position, real_value = me % theta, textLine =
    textLine, fmt = "( f12.4 )" )
79              call gather_real_field_sub  &
80                  ( position = position, real_value = me % phi,   textLine =
    textLine, fmt = "( f12.4 )" )
81              call gather_complex_field_sub  &
82                  ( position = position, complex_value = me % thetaTheta,
    textLine = textLine )
83              call gather_complex_field_sub  &
84                  ( position = position, complex_value = me % thetaPhi,
    textLine = textLine )
```

```
85              call gather_complex_field_sub  &
86                  ( position = position, complex_value = me % phiTheta,
    textLine = textLine )
87              call gather_complex_field_sub  &
88                  ( position = position, complex_value = me % phiPhi,
    textLine = textLine )
89          return
90      end subroutine extract_electric_fields_sub
91
92      subroutine gather_real_field_sub ( position, real_value, textLine, fmt )
93          real ( rp ),             intent ( out )   :: real_value
94          integer ( ip ),          intent ( inout ) :: position
95          character ( len = mll ), intent ( in )    :: textLine
96          character ( len =   9 ), intent ( in )    :: fmt
97              left  = endpoints ( position, 1 )
98              right = endpoints ( position, 2 )
99              write ( number, fmt = 100 ) textLine ( left : right )
100                 if ( io_stat /= 0 ) then
101                     write ( * , fmt = '( 3g0 )' ) "Failure to WRITE string
    value '", trim ( textLine ( left : right ) ) , "'."
102                     write ( * , fmt = fmt_stat  ) io_stat
103                     write ( * , fmt = fmt_iomsg ) trim ( io_msg )
104                     stop "Error occured in module 'mClassElectricFields',
    subroutine 'gather_real_field_sub'."
105                 end if
106             read  ( number, fmt = fmt ) real_value
107                 if ( io_stat /= 0 ) then
108                     write ( * , fmt = '( 5g0 )' ) "Failure to READ string
    value '", trim ( textLine ( left : right ) ) , &
109                         "' as a REAL number using format descriptor ", fmt,
    "."
110                     write ( * , fmt = fmt_stat  ) io_stat
111                     write ( * , fmt = fmt_iomsg ) trim ( io_msg )
112                     stop "Error occured in module 'mClassElectricFields',
    subroutine 'gather_real_field_sub'."
113                 end if
114             position = position + 1
115         return
116         100 format ( g0 )
117     end subroutine gather_real_field_sub
118
119     subroutine gather_complex_field_sub ( position, complex_value, textLine )
120         complex ( rp ),          intent ( out )   :: complex_value
121         integer ( ip ),          intent ( inout ) :: position
122         character ( len = mll ), intent ( in )    :: textLine
123         real ( rp ) :: x = 0.0_rp, y = 0.0_rp
124             call gather_real_field_sub ( position = position, real_value =
    x, textLine = textLine, fmt = "( e15.7 )" )
125             call gather_real_field_sub ( position = position, real_value =
    y, textLine = textLine, fmt = "( e15.7 )" )
```

```
126             complex_value = cmplx ( x, y )
127         return
128     end subroutine gather_complex_field_sub
129 
130 end module mClassElectricFields
```

B.1.3 Class Data File: `m-class-data-file.f08`

```
1  module mClassDataFile
2  
3      use, intrinsic :: iso_fortran_env,  only : iostat_end
4      ! classes
5      use mClassAverages,                 only : average, average0
6      use mClassElectricFields,           only : electricFields,
       electricFields0
7      use mClassMesh,                     only : meshReal
8      use mAllocations,                   only : allocationToolKit,
       allocationToolKit0
9      use mAllocationsSpecial,            only : allocate_rank_one_averages_sub
10     ! utilities
11     use mLibraryOfConstants,            only : fileNameLength,
       messageLength, MoMlineLength
12     ! use mBulkRCS,                       only : BulkRCS, BulkRCS0
13     use mFileHandling,                  only : safeopen_readonly,
       safeopen_writereplace
14     use mFormatDescriptors,             only : fmt_one, fmt_stat, fmt_iomsg,
       fmt_shape2
15     use mPrecisionDefinitions,          only : ip, rp
16     use mTextFileUtilities,             only : count_lines_sub,
       mark_frequencies_sub,read_text_lines_sub, file_closer_sub, &
17                                                iostat_check_sub
18     ! use mTextFileUtilities,             only : count_lines_sub,
       file_closer_sub, iostat_check_sub, mark_frequencies_sub, &
19     !                                           parse_name_sub,
       read_text_lines_sub
20     implicit none
21 
22     ! parameters
23     integer ( ip ), parameter :: fnl = fileNameLength, msgl = messageLength,
       mll = MoMlineLength
24     character ( len = 9 ), parameter :: strAzimuth   = "azimuth  "
25     character ( len = 9 ), parameter :: strElevation = "elevation"
26     character ( len = * ), parameter :: moduleCrash  = "Program crashed in
       module 'mClassDataFile', "
27 
28     integer :: io_stat = 0
29     character ( len = msgl ) :: io_msg = ""
30 
31     type :: dataFile4112
32         ! rank 2
```

```
33          real ( rp ),             allocatable :: rcs_table  ( : , : ) ! angle
    mesh length x nu mesh length
34          real ( rp ),             allocatable :: dbsm_table ( : , : ) ! angle
    mesh length x nu mesh length
35          ! ! rank 1
36          integer ( ip ),          allocatable :: lineNumbersFrequency ( : )!,
    &
37  !                                              lineNumbersFinished  ( : )
38          type ( average ),        allocatable :: perFrequencyAverage  ( : )
    ! nu mesh length
39          type ( average )                     :: globalAverage
40          character ( len = mll ), allocatable :: lines4112Text ( : )        !
    length numlines4112Text
41          ! rank 0
42          type ( electricFields ) :: eFields = electricFields0
43          type ( meshReal ) :: meshFrequency, &
44                               meshFreeAngle
45          integer ( ip ) :: numFrequencies   = 0, &
46                            numFixedAngles   = 0, &
47                            numFreeAngles    = 0, &
48                            numMeasurements  = 0, &
49                            numLines4112Text = 0
50          integer ( ip ) :: io_unit = 0
51          character ( len =   9 ) :: angleFixedType  = "", angleFreeType = ""
52          character ( len = fnl ) :: file4112Name = "", fileRCStxtName = "",
    fileRCSbinaryName  = "", &
53                                                     filedBsmTxtName = "",
    filedBsmBinaryName = ""
54          ! allocation tools
55          type ( allocationToolKit ) :: myKit = allocationToolKit0
56      contains
57          procedure, public :: allocate_rcs_tables               =>
    allocate_rcs_tables_sub, &
58                               allocate_rcsAverages              =>
    allocate_rcsAverages_sub, &
59                               characterize_rcs_by_frequency     =>
    characterize_rcs_by_frequency_sub, &
60                               check_rcs_table_structure         =>
    check_rcs_table_structure_sub, &
61                               establish_free_angle_mesh         =>
    establish_free_angle_mesh_sub, &
62                               establish_frequency_mesh          =>
    establish_frequency_mesh_sub, &
63                               extract_rcs_from_4112_file        =>
    extract_rcs_from_4112_file_sub, &
64                               harvest_frequencies               =>
    harvest_frequencies_sub, &
65                               set_file_names                    =>
    set_file_names_sub, &
```

```
66                              set_free_angle_azimuth            =>
    set_free_angle_azimuth_sub, &
67                              set_free_angle_elevation          =>
    set_free_angle_elevation_sub, &
68                              write_rcs_file_set                =>
    write_rcs_file_set_sub, &
69                              write_rcs_binary                  =>
    write_rcs_binary_sub, &
70                              write_rcs_csv                     =>
    write_rcs_csv_sub, &
71                              write_dBsm_binary                 =>
    write_dBsm_binary_sub, &
72                              write_dBsm_csv                    =>
    write_dBsm_csv_sub, &
73                              write_summary_by_frequency        =>
    write_summary_by_frequency_sub, &
74                              write_summary_for_all_frequencies =>
    write_summary_for_all_frequencies_sub
75      end type dataFile4112
76
77      private :: allocate_rcs_tables_sub, allocate_rcsAverages_sub, &
78                 establish_free_angle_mesh_sub, establish_frequency_mesh_sub,
    extract_rcs_from_4112_file_sub, &
79                 harvest_frequencies_sub, &
80                 set_file_names_sub, set_free_angle_azimuth_sub,
    set_free_angle_elevation_sub, &
81                 write_summary_by_frequency_sub,
    write_summary_for_all_frequencies_sub
82
83  contains
84
85      subroutine characterize_rcs_by_frequency_sub ( me )
86          class ( dataFile4112 ), target :: me
87          type ( average ), pointer :: p => null ( )
88          integer ( ip ) :: kFrequency = 0
89
90              sweep_frequencies: do kFrequency = 1, me % numFrequencies
91                  p => me % perFrequencyAverage ( kFrequency )
92                      call p % find_max_and_min          ( vector = me %
    rcs_table ( 1 : me % numFreeAngles, kFrequency ) )
93                      call p % compute_mean_and_variance ( vector = me %
    rcs_table ( 1 : me % numFreeAngles, kFrequency ), &
94                                                           one = me %
    meshFreeAngle % one )
95                  p => null ( )
96              end do sweep_frequencies
97
98          return
99      end subroutine characterize_rcs_by_frequency_sub
100
```

```
101     module subroutine write_summary_for_all_frequencies_sub ( me )
102         class ( dataFile4112 ), target :: me
103         integer ( ip ) :: kFrequency = 0, first = 0, last = 0,
    numConvolution = 0
104         real ( rp ), allocatable :: global_rcs ( : ), one ( : )
105             ! allocate memory for all RCS measurements
106             numConvolution = me % numFrequencies * me % numFreeAngles
107             call me % myKit % allocate_rank_one_reals ( real_array =
    global_rcs, index_min = 1, index_max = numConvolution )
108             call me % myKit % allocate_rank_one_reals ( real_array = one,
        index_min = 1, index_max = numConvolution )
109             ! load data vector
110             sweep_frequencies: do kFrequency = 1, me % meshFrequency %
    numMeshElements
111                 first = ( kFrequency - 1 ) * me % numFreeAngles + 1
112                 last  = first + me % numFreeAngles - 1
113                 global_rcs ( first : last ) = me % rcs_table ( 1 : me %
    numFreeAngles, kFrequency )
114             end do sweep_frequencies
115             ! compute extrema
116             one ( : ) = global_rcs ( : ) - global_rcs ( : ) + 1.0_rp
117             call me % globalAverage % find_max_and_min          ( vector =
    global_rcs ( 1 : numConvolution ) )
118             call me % globalAverage % compute_mean_and_variance ( vector =
    global_rcs ( 1 : numConvolution ), one = one )
119             write ( * , * )
120             write ( * , fmt = 100 ) me % globalAverage % mean, &
121                                     me % globalAverage % standardDeviation, &
122                                     me % globalAverage % extrema % minValue,
    &
123                                     me % globalAverage % extrema % maxValue
124         return
125         100 format ( "Aggregate for all RCS measurements: mean = ", g0, "
    +/- ", g0, ", min = ", g0, ", max = ", g0 )
126     end subroutine write_summary_for_all_frequencies_sub
127
128     module subroutine write_summary_by_frequency_sub ( me )
129         class ( dataFile4112 ), target :: me
130         integer ( ip ) :: kFrequency = 0
131             write ( * , * )
132             sweep_frequencies: do kFrequency = 1,   me % meshFrequency %
    numMeshElements
133                 write ( * , fmt = 100 ) kFrequency, me % meshFrequency %
    meshValues ( kFrequency ), &
134                                                     me % perFrequencyAverage
    ( kFrequency ) % mean, &
135                                                     me % perFrequencyAverage
    ( kFrequency ) % standardDeviation, &
136                                                     me % perFrequencyAverage
    ( kFrequency ) % extrema % minValue, &
```

```
                                                    me % perFrequencyAverage ( kFrequency ) % extrema % maxValue
            end do sweep_frequencies
        return
        100 format ( I3.3, ". nu = ", g0, ", mean RCS = ", g0, " +/- ", g0, ", min = ", g0, ", max = ", g0 )
    end subroutine write_summary_by_frequency_sub

    module subroutine write_rcs_file_set_sub ( me )
        class ( dataFile4112 ), target :: me
            call me % write_rcs_csv     ( )
            call me % write_rcs_binary  ( )
            call me % write_dBsm_csv    ( )
            call me % write_dBsm_binary ( )
        return
    end subroutine write_rcs_file_set_sub

    module subroutine write_rcs_binary_sub ( me )
        class ( dataFile4112 ), target :: me
        integer ( ip )           :: io_rcs = 0
        character ( len = msgl ) :: crashChain = ""

            crashChain = moduleCrash // "subroutine 'write_rcs_binary_sub'."

            open ( newunit = io_rcs, file = me % fileRCSbinaryName, action = 'WRITE', status = 'REPLACE', form = 'UNFORMATTED', &
                                    iostat = io_stat, iomsg = io_msg )
            call iostat_check_sub ( action = "UNFORMATTED OPENING", fileName = me % fileRCSbinaryName, crashChain = crashChain, &
                                    iostat = io_stat, iomsg = io_msg )

            write ( io_rcs, iostat = io_stat,  iomsg = io_msg ) me % rcs_table ( 1 : me % meshFreeAngle  % numMeshElements, &

        1 : me % meshFrequency  % numMeshElements )
            call iostat_check_sub ( action = "UNFORMATTED WRITE to", fileName = me % fileRCSbinaryName, crashChain = crashChain, &
                                    iostat = io_stat, iomsg = io_msg )
            call file_closer_sub ( io_unit = io_rcs, fileName = me % fileRCSbinaryName, crashChain = crashChain )

        return
    end subroutine write_rcs_binary_sub

    module subroutine write_rcs_csv_sub ( me )
        class ( dataFile4112 ),         target :: me
        integer ( ip ) :: kFrequency = 0, kFreeAngle = 0, &
                          io_out = 0
        character ( len = msgl ) :: crashChain = ""

```

```
179             crashChain = moduleCrash // "subroutine 'write_rcs_csv_sub'."
180             io_out = safeopen_writereplace ( me % fileRCStxtName )
181             ! write RCS values one row (frequency) at a time
182             sweep_frequencies: do kFrequency = 1, me % meshFrequency %
    numMeshElements
183                 write ( io_out, fmt = me % meshFreeAngle %
    valuesFormatDescriptor ) ( me % rcs_table ( kFreeAngle, kFrequency ), &
184                         kFreeAngle = 1, me % meshFreeAngle % numMeshElements
    )
185                 call iostat_check_sub ( action = "WRITE to", fileName = me %
    fileRCStxtName, crashChain = crashChain, &
186                                             iostat = io_stat, iomsg = io_msg
    )
187             end do sweep_frequencies
188             ! close io handle
189             call file_closer_sub ( io_unit = io_out, fileName =  me %
    fileRCStxtName, crashChain = crashChain )
190
191         return
192     end subroutine write_rcs_csv_sub
193
194     module subroutine write_dBsm_binary_sub ( me )
195         class ( dataFile4112 ), target :: me
196         integer ( ip )           :: io_rcs = 0
197         character ( len = msgl ) :: crashChain = ""
198
199             crashChain = moduleCrash // "subroutine write_dBsm_binary_sub'."
200
201             open ( newunit = io_rcs, file = me % filedBsmBinaryName, action
    = 'WRITE', status = 'REPLACE', form = 'UNFORMATTED', &
202                                 iostat = io_stat, iomsg = io_msg )
203             call iostat_check_sub ( action = "UNFORMATTED OPENING", fileName
    = me % fileRCSbinaryName, crashChain = crashChain, &
204                                 iostat = io_stat, iomsg = io_msg )
205
206             write ( io_rcs, iostat = io_stat,  iomsg = io_msg ) me %
    dBsm_table ( 1 : me % meshFreeAngle  % numMeshElements, &
207
         1 : me % meshFrequency  % numMeshElements )
208             call iostat_check_sub ( action = "UNFORMATTED WRITE to",
    fileName = me % fileRCSbinaryName, crashChain = crashChain, &
209                                 iostat = io_stat, iomsg = io_msg )
210             call file_closer_sub ( io_unit = io_rcs, fileName = me %
    filedBsmBinaryName, crashChain = crashChain )
211
212         return
213     end subroutine write_dBsm_binary_sub
214
215     module subroutine write_dBsm_csv_sub ( me )
216         class ( dataFile4112 ),         target :: me
```

```
217         integer ( ip ) :: kFrequency = 0, kFreeAngle = 0, &
218                           io_out = 0
219         character ( len = msgl ) :: crashChain = ""
220 
221             crashChain = moduleCrash // "subroutine 'write_dBsm_csv_sub'."
222             io_out = safeopen_writereplace ( me % filedBsmTxtName )
223             ! write RCS values one row (frequency) at a time
224             sweep_frequencies: do kFrequency = 1, me % meshFrequency %
    numMeshElements
225                 write ( io_out, fmt = me % meshFreeAngle %
    valuesFormatDescriptor ) ( me % dBsm_table ( kFreeAngle, kFrequency ), &
226                         kFreeAngle = 1, me % meshFreeAngle % numMeshElements
    )
227                 call iostat_check_sub ( action = "WRITE to", fileName = me %
    filedBsmTxtName, crashChain = crashChain, &
228                                             iostat = io_stat, iomsg = io_msg
    )
229             end do sweep_frequencies
230             ! close io handle
231             call file_closer_sub ( io_unit = io_out, fileName =  me %
    fileRCStxtName, crashChain = crashChain )
232 
233         return
234     end subroutine write_dBsm_csv_sub
235 
236     subroutine set_file_names_sub ( me, file4112Name )
237         class ( dataFile4112 ),         target :: me
238         character ( len = fnl ), intent ( in ) :: file4112Name
239         integer ( ip ) :: nameLength = 0
240 
241             nameLength =        len ( trim ( file4112Name ) )
242             me % file4112Name       = trim ( file4112Name )
243             me % fileRCStxtName     = trim ( file4112Name ( 1 : nameLength -
    4 ) ) // ".rcs.txt"
244             me % fileRCSbinaryName  = trim ( file4112Name ( 1 : nameLength -
    4 ) ) // ".rcs.r32"
245             me % filedBsmTxtName    = trim ( file4112Name ( 1 : nameLength -
    4 ) ) // ".dBsm.txt"
246             me % filedBsmBinaryName = trim ( file4112Name ( 1 : nameLength -
    4 ) ) // ".dBsm.r32"
247 
248         return
249     end subroutine set_file_names_sub
250 
251     subroutine allocate_rcsAverages_sub ( me )
252         class ( dataFile4112 ), target :: me
253             call allocate_rank_one_averages_sub ( rank_1_average = me %
    perFrequencyAverage, &
254                                                   index_min = 1,
    index_max = me % numFrequencies )
```

```
255         return
256     end subroutine allocate_rcsAverages_sub
257 
258     subroutine allocate_rcs_tables_sub ( me )
259         class ( dataFile4112 ), target :: me
260             call me % myKit % allocate_rank_two_reals (  rank_2_real_array =
    me % rcs_table,           &
261                                                          dim1_index_min =
    1, dim1_index_max = me % numFreeAngles, &
262                                                          dim2_index_min =
    1, dim2_index_max = me % numFrequencies )
263             call me % myKit % allocate_rank_two_reals (  rank_2_real_array =
    me % dBsm_table,           &
264                                                          dim1_index_min =
    1, dim1_index_max = me % numFreeAngles, &
265                                                          dim2_index_min =
    1, dim2_index_max = me % numFrequencies )
266         return
267     end subroutine allocate_rcs_tables_sub
268 
269     subroutine establish_frequency_mesh_sub ( me )
270         class ( dataFile4112 ), target :: me
271             ! count lines in MoM file (e.q. 14844)
272             call count_lines_sub ( fullFileName = me % file4112Name,
    numLines = me % numLines4112Text )
273             ! allocate object to hold text of MoM file as a collection of
    text lines
274             call me % myKit % allocate_rank_one_characters ( character_array
    = me % lines4112Text, &
275                                                              index_min = 1,
    index_max = me % numLines4112Text )
276             ! load MoM text into memory to count frequencies and angles
277             call read_text_lines_sub ( fileName = me % file4112Name,
    linesText = me % Lines4112Text )
278             ! sift through text lines for " Freq ="
279             call me % harvest_frequencies ( )
280         return
281     end subroutine establish_frequency_mesh_sub
282 
283     ! sweep through character array looking for "  Freq"
284     ! store these values in a temporary array until nuMesh is allocated
285     subroutine harvest_frequencies_sub ( me )
286         class ( dataFile4112 ), target :: me
287         ! pointers
288         !character ( len =  mll ),   pointer :: p => null ( )
289         type ( meshReal ),          pointer :: q => null ( )
290         type ( allocationToolKit ), pointer :: s => null ( )
291         ! temp arrays
292         real    ( ip ) :: tempFrequencyValues   ( 1 : 500 )
293         integer ( ip ) :: tempLineNumsFrequency ( 1 : 500 )
```

```
294         ! scalars
295         integer ( ip ) :: numfrequencies = 0, kFrequency = 0
296 
297             ! find lines containing " Freq ="
298             call mark_frequencies_sub ( lines4112Text = me % lines4112Text,
      &
299                                         numLines4112Text = me %
    numLines4112Text, &
300                                   tempFrequencyValues = tempFrequencyValues,
      &
301                                  tempLineNumsFrequency =
    tempLineNumsFrequency, &
302                                       numfrequencies = numfrequencies )
303 
304             ! record what we have learned about the mesh
305             q => me % meshFrequency
306                 q % numMeshElements = numfrequencies
307                 ! allocate data objects
308                 call q % allocate_mesh_real (  )
309                 s => me % myKit
310                     call s % allocate_rank_one_integers ( integer_array = me
    % lineNumbersFrequency, index_min = 1, &
311                                                                 index_max = q
    % numMeshElements )
312                 s => null ( )
313                 ! move temporary array data into data object
314                 do kFrequency = 1, q % numMeshElements
315                      q % meshValues           ( kFrequency ) =
    tempFrequencyValues   ( kFrequency )
316                     me % lineNumbersFrequency ( kFrequency ) =
    tempLineNumsFrequency ( kFrequency )
317                 end do
318                 me % numFrequencies = q % numMeshElements
319                 call q % analyze_mesh_values ( )
320             q => null ( )
321         return
322     end subroutine harvest_frequencies_sub
323 
324     subroutine extract_rcs_from_4112_file_sub ( me )
325         class ( dataFile4112 ),         target :: me
326         ! locals
327         !real    ( rp )            :: sigma = 0.0_rp
328         integer ( ip )            :: kFrequency = 0, kFreeAngle = 0,
    linePosition = 0
329         character ( len = mll )   :: textLine
330 
331             ! open *.4112.txt file, read text lines into memory
332             call read_text_lines_sub ( fileName = me % file4112Name,
    linesText = me % lines4112Text )
333             ! sweep and harvest RCS value
```

```
334             sweep_frequencies: do kFrequency = 1, me % numFrequencies
335                 linePosition = me % lineNumbersFrequency ( kFrequency ) + 8
336                 sweep_free_angles: do kFreeAngle = 1, me % numFreeAngles
337                     textLine = me % lines4112Text ( linePosition )
338                     call me % eFields % gather_mean_total_rcs ( textLine =
    textLine )
339                         me % rcs_table  ( kFreeAngle, kFrequency ) = me %
    eFields % meanTotalRCS
340                         me % dBsm_table ( kFreeAngle, kFrequency ) = me %
    eFields % dBsm
341                     linePosition = linePosition + 1
342                 end do sweep_free_angles
343             end do sweep_frequencies
344
345         return
346     end subroutine extract_rcs_from_4112_file_sub
347
348     subroutine set_free_angle_elevation_sub ( me )
349         class ( dataFile4112 ), target :: me
350             me % angleFreeType  = strElevation
351             me % angleFixedType = strAzimuth
352         return
353     end subroutine set_free_angle_elevation_sub
354
355     subroutine set_free_angle_azimuth_sub ( me )
356         class ( dataFile4112 ), target :: me
357             me % angleFreeType  = strAzimuth
358             me % angleFixedType = strElevation
359         return
360     end subroutine set_free_angle_azimuth_sub
361
362     subroutine establish_free_angle_mesh_sub ( me, angle_min, angle_max,
    angle_count )
363         class ( dataFile4112 ), target :: me
364         real    ( rp ), intent ( in ) :: angle_min, angle_max
365         integer ( ip ), intent ( in ) :: angle_count
366
367             me % meshFreeAngle % meshAverage % extrema % minValue = angle_min
368             me % meshFreeAngle % meshAverage % extrema % maxValue = angle_max
369             me % meshFreeAngle % numMeshElements = angle_count
370             me % numFreeAngles                   = angle_count
371
372             call me % meshFreeAngle % allocate_mesh_real (  )
373             call me % meshFreeAngle % compute_real_mesh_length ( )
374             call me % meshFreeAngle % compute_real_mesh_interval ( )
375             call me % meshFreeAngle % populate_real_mesh ( )
376             call me % meshFreeAngle % populate_integer_mesh ( )
377
378         return
379     end subroutine establish_free_angle_mesh_sub
```

```
380
381     subroutine check_rcs_table_structure_sub ( me )
382         class ( dataFile4112 ), target :: me
383             write ( * , * )
384             write ( * , fmt = '(  g0 )' ) "#  #  Dimensions for RCS data
    container  #  #"
385             write ( * , * )
386             write ( * , fmt = '(  g0 )' ) "# Expected dimensions:"
387             write ( * , fmt = '( 2g0 )' ) "# Number of radar frequencies
    scanned by MoM:   ",                me % numFrequencies
388             write ( * , fmt = '( 4g0 )' ) "# Number of ", me %
    angleFreeType, " angles scanned by MoM:  ",   me % numFreeAngles
389             write ( * , * )
390             write ( * , fmt = '( 2g0 )' ) "# Container MoM 4112.txt file:
    rcs_table"
391             write ( * , fmt = '( 6g0 )' ) "# Free angle dimension = ",  size
      ( me % rcs_table, 1 ), &
392                                                 " indices run from ",
    lbound ( me % rcs_table, 1 ), &
393                                                 " to ",
    ubound ( me % rcs_table, 1 )
394             write ( * , fmt = '( 6g0 )' ) "# Frequency dimension  = ",  size
      ( me % rcs_table, 2 ), &
395                                                 " indices run from ",
    lbound ( me % rcs_table, 2 ), &
396                                                 " to ",
    ubound ( me % rcs_table, 2 )
397             write ( * , * )
398         return
399     end subroutine check_rcs_table_structure_sub
400
401 end module mClassDataFile
```

C Mercury Method of Moments: Distribution and Rights

C.1 Distribution Letter for Software

The subsequent distribution letter was signed by Randy J. Petyak of the NASA Software Release Authority and describes terms for distribution, Government rights, and the ITAR status of the software.

December 11, 2019

Air Force Research Laboratory
RVB
3550 Aberdeen Ave SE
Kirtland Air Force Base, NM 87117-5776
Attn: Mr. Nelson Bonito

Subject: Transmittal of Mercury MoM version 4.1.12, MM_Viz Code.

This distribution letter details the terms for distribution, the Government rights in the software, and the ITAR status of the software. The software usage agreement you signed covers Mercury MoM and MMViz executable codes on both Linux 64 bit and Windows 64 bit. The Mercury MoM software is copyrighted by Matrix Compression, LLC. of which the Government retains certain rights to the software, and must be controlled as outlined in the signed Software Usage Agreement.

NASA furnishes this software under the condition that no further dissemination of the software shall be made without prior written permission of the NASA Langley Research Center. Additionally, this software has been designated as ITAR and needs appropriate protection while on the DVD or on an installed machine.

Note: The software falls under the purview of the U.S. Munitions List (USML), as defined in the International Traffic in Arms Regulations (ITAR), 22 CFR 120-130, and is export controlled. It shall not be taken out of the U.S. nor transferred to foreign nationals in the U.S. or abroad, without specific approval of a knowledgeable export control official, and/or unless an export license/license exemption is obtained/available from the United States Department of State. Violation of these regulations is punishable by fine, imprisonment, or both.

We are interested in your use of this software and the results you obtain. Please include us on your mailing list for any publications that may result from your use of this code.

If you have any additional questions related to your request, please contact me.

Randy J. Petyak
NASA Software Release Authority
(202) 358-4387

C.2 Copyright Statement by the Author

C.3 Legal Statement

MERCURY MOM™
Copyrighted
US Patents: 7,742,886; 7,844,407; 8,209,138; 8,725,464

This software was developed under NASA Contracts NAS1-02057, NAS1-02117, NNL08AA00B, and NNL13AA08B, and the U.S. Government retains certain rights.

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==========

C.4 Obtaining Software and Documentation

For more information regarding this document contact the following:

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Figure 5: Contact information to request Mercury MoM Software and Documentations

C.5 Distribution Contents

C.5.1 Executables

1. Linux 64-bit
2. Windows 64-bit

C.5.2 Documentation

The distribution includes four documents in PDF which are marked as CUI:

1. User's Guide
2. Pill Tutorial
3. Code Validation Report
4. Benchmark Tests

D Using Python to Create Spreadsheets

Some users preferred to digest the radar data in spreadsheet form. The Python toolkit xlswriter simplifies moving the data into `*.xlsx` form.

The spreadsheets eschew row-column notation (e.g. `A4`) and makes use of variables and named ranges to simplify the interpretation of the results.

D.1 Inputs

D.1.1 Main

Main module:

```
#! /usr/bin/python3

# # Daniel Topa

# # Excel tools
# xl_new_workbook( workbook_title )
# xl_sheet_requirements( this_workbook )
# xl_sheet_generate( this_workbook, title_sheet )
# xl_s( this_workbook )
# xl_sheet_header_footer( this_worksheet )

# # imports
import os               # probe, change directories
import sys              # python version
import datetime         #
    https://stackoverflow.com/questions/415511/how-to-get-the-current-time-in-python
import numpy as np
import pandas as pd
import xlsxwriter       # API for Excel
from xlsxwriter.utility import xl_rowcol_to_cell
import numpy as np
import pandas as pd

import cls_TestObject

# # modules
def xl_new_workbook( testObject ):

    MoMresults = xlsxwriter.Workbook( testObject.outputFile )
    print( "output file %s" % testObject.outputFile )
    print( "source file %s" % testObject.sourceFile )

    xl_sheet_master( MoMresults, testObject ) # MoM summary
    xl_add_data_sheets( MoMresults, testObject ) # MoM summary
    xl_sheet_provenance( MoMresults ) # provenance sheet

    return MoMresults;

#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

def xl_add_data_sheets( this_workbook, testObject ):

    format_MoM_title = this_workbook.add_format( )
    format_MoM_title.set_bold( )
    format_MoM_title.set_font_color( "red" )

```

```
    format_MoM_head = this_workbook.add_format( )
    format_MoM_head.set_bold( )

    format_MoM_polarization = this_workbook.add_format( )
    format_MoM_polarization.set_bold( )

    number_format = this_workbook.add_format({'num_format': '#,##0.000'})

    # https://xlsxwriter.readthedocs.io/format.html#set_center_across
    cell_format = this_workbook.add_format()
    cell_format.set_center_across()

    for index in range( 1, 29 ):
        # add sheet and tag header and footer
        title = str( index + 2 ) + ' MHz'
        print ( 'adding sheet %s' % title )
        s = xl_sheet_generate( this_workbook, title )
        xl_sheet_header_footer( s )
        s.write( "A1", "MoM 4.1.12 output (*.dat)", format_MoM_title )
        #
        s.write( "A3", "azimuth, °", format_MoM_head )
        s.write( "B3", "HH, dBsm",   format_MoM_head )
        s.write( "C3", "VV, dBsm",   format_MoM_head )
        s.write( "D3", "HV, dBsm",   format_MoM_head )
        s.write( "E3", "VH, dBsm",   format_MoM_head )
        #
        s.write( "H3", "mean", format_MoM_head )
        s.write( "J3", "standard deviation", format_MoM_head )
        #
        s.write( "G4", "HH", format_MoM_polarization )
        s.write( "G5", "VV", format_MoM_polarization )
        #
    #    AttributeError: 'str' object has no attribute '_get_xf_index'
    #    s.write( "I4", "HH", u"\u00B1" )
        s.write( "I4", "±", cell_format )
        s.write( "I5", "±", cell_format )
        s.set_column( "I:I", 3 )

        # = AVERAGE( B5:B364)
        # = STDEV( B5:B364 )
        s.write( "H4", '= AVERAGE( B5:B364)', number_format )
        s.write( "H5", '= AVERAGE( C5:C364)', number_format )
        s.write( "J4", '= STDEV( B5:B364 )',  number_format )
        s.write( "J5", '= STDEV( B5:B364 )',  number_format )

        # read in data file
        filename = './data/sphere-005-' + testObject.resolution + '-' + str(
    index + 2 ).zfill(2) + '.4112.dat.txt'
        s.write_string( "D1", filename )
        data = pd.read_csv( filename, delimiter=r"\s+", header = None )
```

```
95          data_np = data.values
96          row = 3
97          col = 0
98          for line in range( 0, len ( data_np ) ):
99              cell = xl_rowcol_to_cell ( row, col )
100             s.write( row, col, data_np[ line ][ 0 ], number_format )
101             s.write( row, col + 1, data_np[ line ][ 1 ], number_format )
102             s.write( row, col + 2, data_np[ line ][ 2 ], number_format )
103             s.write( row, col + 3, data_np[ line ][ 3 ], number_format )
104             s.write( row, col + 4, data_np[ line ][ 4 ], number_format )
105             row += 1
106
107     return
108
109 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
110
111 def xl_sheet_generate( this_workbook, title_sheet ):
112
113     # insure every worksheet has a header and footer
114     mySheet = this_workbook.add_worksheet( title_sheet )
115     xl_sheet_header_footer( mySheet )
116
117     return mySheet;
118
119 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
120
121 def xl_sheet_provenance( this_workbook ):
122
123     # Define some global names.
124     this_workbook.define_name( 'c_', '=299792458' )
125     # forensic info
126     s = xl_sheet_generate( this_workbook, "provenance" )
127     #  # special formats
128     # https://xlsxwriter.readthedocs.io/format.html?highlight=bold
129
130     # method 1
131     # setting the property as a dictionary of key/value pairs in the
        constructor
132     format_title = this_workbook.add_format( )
133     format_title.set_bold( )
134     format_title.set_font_color( "blue" )
135
136     # method 2
137     # passing a dictionary of properties to the add_format() constructor
138     format_time = this_workbook.add_format( {'num_format': 'yy/mm/dd hh:mm'}
        ) # https://xlsxwriter.readthedocs.io/working_with_dates_and_time.html
139
140     # widen first columns
141     s.set_column( "A:A", 15 )
142     s.set_column( "B:B", 13 )
```

```
143
144     # https://xlsxwriter.readthedocs.io/worksheet.html
145     s.write_url( "A1",
        "https://en.wikipedia.org/wiki/Computational_electromagnetics", string =
        "Radar Cross Section Measurements" )
146
147     #  #  provencance
148     s.write( "A3", "Workbook created by", format_title )
149     #s.write( "A1", tip, "boo" )
150
151     # python notebook which creates workbook
152     s.write( "A4", "python source" )
153     s.write( "B4", os.path.basename( __file__ ) ) # charlie.py
154
155     # current working directory
156     s.write( "A5", "directory" )
157     s.write( "B5", os.getcwd( ) ) #
        /Volumes/Tlaltecuhtli/repos/GitHub/topa-development/python/xlsx
158
159     # python version
160     s.write( "A6", "python version" )
161     s.write( "B6", sys.version ) # "3.7.0 (default, Jun 28 2018, 07:39:16)
        [Clang 4.0.1 (tags/RELEASE_401/final)]"
162
163     #  #  environment variables
164     # practise row, col notation
165     col = 0 # starting column
166     row = 7 # starting row
167     s.write( row, col, "Environment variables", format_title ); row += 1
168
169     s.write( row, col, "$USER" ) # l127914
170     s.write( row, col + 1, os.environ[ "USER" ] ); row += 1
171
172     s.write( row, col, "$HOSTNAME" ) # Cauchy.Schwarz
173     s.write( row, col + 1, os.environ[ "HOSTNAME" ] ); row += 1
174
175     s.write( row, col, "$HOME" ) # /Users/l127914
176     s.write( row, col + 1, os.environ[ "HOME" ] ); row += 1
177
178     s.write( row, col, "timestamp" ) # 11/21/18 16:18
179     s.write( row, col + 1, datetime.datetime.now( ), format_time ); row += 1
180
181     #  #  Excel info routines
182     # https://xlsxwriter.readthedocs.io/working_with_formulas.html
183
184     row += 1 # jump
185     s.write( row, col, "XL info function", format_title ); row += 1
186
187     s.write( row, col, "platform" ) # mac
188     s.write_formula( row, col + 1, '= INFO( "system" )' ); row += 1
```

```
189
190     s.write( row, col, "recalculation mode" ) # Automatic
191     s.write_formula( row, col + 1, '= INFO( "recalc" )' ); row += 1
192
193     s.write( row, col, "active sheets" ) # 1
194     s.write_formula( row, col + 1, '= INFO( "numfile" )' ); row += 1
195
196     s.write( row, col, "cursor" ) # $A:$A$1
197     s.write_formula( row, col + 1, '= INFO( "origin" )' ); row += 1
198
199     s.write( row, col, "XL release" ) # 16.16
200     s.write_formula( row, col + 1, '= INFO( "release" )' ); row += 1
201
202     s.write( row, col, "application directory" ) #
        /Users/dantopa/Library/Containers/com.microsoft.Excel/Data/Documents/
203     s.write_formula( row, col + 1, '= INFO( "directory" )' ); row += 1
204
205     s.write( row, col, "operating systems" ) # Macintosh (Intel) Version
        10.13.3 (Build 17D47)
206     s.write_formula( row, col + 1, '= INFO( "osversion" )' ); row += 1
207
208     return
209
210 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
211
212 def xl_sheet_header_footer( this_worksheet ):
213
214     # header: sheet name (center)
215     # footer: date/time, page number, path/file
216
217     myheader = "&C&12&A" # fontsize 12
218     myfooter = "&L&8&T\n&8&D" + "&C &P / &N" + "&R&8&Z\n&8&F" # fontsize 8
219
220     this_worksheet.set_header( myheader )
221     this_worksheet.set_footer( myfooter )
222
223     return
224
225 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
226
227 def xl_sheet_master( this_workbook, testObject ):
228
229     number_format = this_workbook.add_format({'num_format': '#,##0.000'})
230
231     masterRow = 0
232     masterCol = 0
233     xl_set_label_column ( this_workbook, testObject, masterRow, masterCol )
234
235     dataRow = 8
236     dataCol = 0
```

```
237     s = this_workbook.get_worksheet_by_name( testObject.masterSheet )
238     for index in range(1, 29):
239         dataCol += 1
240         nu = index + 2
241         xl_computation ( s, dataRow, dataCol, nu, number_format )
242 
243     return
244 
245 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
246 
247 # https://xlsxwriter.readthedocs.io/working_with_cell_notation.html
248 def xl_computation ( wsheet, row, col, nu, number_format ):
249 
250     # frequency
251     wsheet.write_number ( row, col, nu )
252 
253     # wavelength =  c_ / ( B11 * 1000000 )
254     cell = xl_rowcol_to_cell ( row, col )
255     wsheet.write ( row + 1, col, '= c_ / ( ' + cell + ' * 1000000 )',
        number_format ); row += 1
256 
257     # = radius / wavelength
258     cell = xl_rowcol_to_cell ( row, col )
259     wsheet.write ( row + 1, col, '= radius / ' + cell, number_format ); row
        += 3
260 
261     # MoM average dBsm  = '30 MHz'!$H4
262     wsheet.write_formula(row, col, "= '" + str( nu ) + " MHz'!$H$4",
        number_format ); row += 1
263     # relative error dBsm
264     cell = xl_rowcol_to_cell ( row - 1, col )
265     wsheet.write_formula ( row, col, '= 1 - size_optical_dbsm / ' + cell,
        number_format ); row += 2
266 
267     # rcs, sq m  = 10^( B15 / 10 )
268     cell = xl_rowcol_to_cell ( row - 3, col )
269     wsheet.write_formula ( row, col, '= 10^( ' + cell + ' / 10 )',
        number_format ); row += 1
270     # rel error (sq m)  =  1 - size_optical_sq_m / B18
271     cell = xl_rowcol_to_cell ( row - 1, col )
272     wsheet.write_formula ( row, col, '= 1 - size_optical_sq_m / ' + cell,
        number_format )
273 
274     return
275 
276 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
277 
278 def xl_set_label_column( wbook, testObject, row, col ):
279 
280     # method 1
```

```
281     # setting the property as a dictionary of key/value pairs in the
        constructor
282     format_title = wbook.add_format( )
283     format_title.set_bold( )
284     format_title.set_font_color( "blue" )
285
286     format_label = wbook.add_format( )
287     format_label.set_bold( )
288
289     # https://xlsxwriter.readthedocs.io/example_defined_name.html
290     # https://docs.python.org/2.0/ref/strings.html
291     wbook.define_name( 'c_', '=299792458' )
292     #string = '\'=' + str( testObject.sizeValue / 2 ) + '\''
293     #print( 'string = %s' % string )
294     wbook.define_name( 'radius', '=5' )
295     wbook.define_name( 'size_optical_sq_m', '=\'' + testObject.masterSheet +
        '\'!$B$6' )
296     wbook.define_name( 'size_optical_dbsm', '=\'' + testObject.masterSheet +
        '\'!$B$7' )
297
298     # sheet operations
299     s = xl_sheet_generate( wbook, testObject.masterSheet )
300     s.set_first_sheet( )
301
302     # widen first columns
303     s.set_column( "A:A", 17 )
304     s.set_column( "B:B", 10  )
305
306     # column of labels
307     s.write_string( row, col, 'INPUT', format_title ); row += 2
308
309     s.write( row, col, 'MoM output:', format_label )
310     s.write( row, col + 1, testObject.sourceFile ); row += 2
311
312     s.write( row, col, testObject.sizeName, format_label );
313     s.write( row, col + 1, testObject.sizeValue )
314     s.write( row, col + 2, 'm' );  row += 1
315
316     s.write( row, col, 'optical size', format_label )
317     s.write( row, col + 1, '= pi() * radius^2' )
318     s.write_string( row, col + 2, testObject.areaUnits ); row += 1
319     s.write_formula( row, col + 1, '= 10 * LOG10( size_optical_sq_m )' );
320     s.write( row, col + 2, 'dB area' ); row += 2
321
322     s.write( row, col, 'frequency (MHz)', format_label ); row += 1
323     s.write( row, col, 'wavelength (m)', format_label ); row += 1
324     s.write( row, col, 'radius / lambda', format_label ); row += 2
325
326     s.write( row, col, 'MoM average (dbSm)', format_label ); row += 1
327     s.write( row, col, 'rel error (dbSm)', format_label ); row += 2
```

```
328
329     s.write( row, col, 'rcs, sq m', format_label ); row += 1
330     s.write( row, col, 'rel error (sq m)', format_label )
331
332     xl_sheet_header_footer( s )
333
334
335     return
336
337 # root@f21d93a5a2e9:sphere $ python tools_xl.py
338 #
339 # root@f21d93a5a2e9:sphere $ date
340 # Wed Jun 24 01:19:38 MDT 2020
341 #
342 # root@f21d93a5a2e9:sphere $ pwd
343 # /Tlaloc/python/sphere
344
```

D.1.2 Class Test Object

Radar return data.

```
1  #!/usr/bin/python3
2
3  # # Daniel Topa
4
5  # imports
6  import math                 # pi
7  import uuid                 # Universal Unique IDentifier
8  #from pathlib import Path   # rename file
9
10 class TestObject( object ):
11     def __init__( self ):
12
13         self._descriptor      = None    # sphere
14         self._sizeName        = None    # diameter
15         self._sizeValue       = None    # 10
16         self._sizeUnits       = None    # m
17         self._areaValue       = None    # pi r^2
18         self._areaUnits       = None    # m^2
19         self._resolution      = None    # 04
20         self._mastersheet     = None    # sphere, d = 10 m
21         self._sourceFile      = None    # *.dat
22         self._sourcePath      = None    # absolute path to *.dat
23         self._sourcePathFile  = None    # path + source file name
24         self._outputFile      = None    # *.xlsx
25         self._outputPath      = None    # absolute path to *.xlsx
26         self._outputPathFile  = None    # path + *.xlsx
27         self._uuid            = uuid.uuid4( ) # de facto time stamp
28
29 #   P R O P E R T I E S   #
```

```
30
31      @property
32      def descriptor( self ):
33          """Descriptor (sphere, cube, etc.)"""
34          return self._descriptor
35
36      @property
37      def sizeName( self ):
38          """Name of size parameter (edge, radius, etc.)"""
39          return self._sizeName
40
41      @property
42      def sizeValue( self ):
43          """Length parameter"""
44          return self._sizeValue
45
46      @property
47      def sizeUnits( self ):
48          """Units (m, mm, etc.)"""
49          return self._sizeUnits
50
51      @property
52      def areaValue( self ):
53          """Area"""
54          return self._areaValue
55
56      @property
57      def areaUnits( self ):
58          """Area units (m^2, mm, etc.)"""
59          return self._areaUnits
60
61      @property
62      def masterSheet( self ):
63          """Name of master sheet"""
64          return self._masterSheet
65
66      @property
67      def sourcePath( self ):
68          """Path (absolute) to source file"""
69          return self._sourcePath
70
71      @property
72      def sourceFile( self ):
73          """Path + Name for input file"""
74          return self._sourceFile
75
76      @property
77      def outputFile( self ):
78          """Name of output file"""
79          return self._outputFile
```

```
80
81      @property
82      def outputPath( self ):
83          """Path (absolute) to output file"""
84          return self._outputPath
85
86      @property
87      def outputPathFile( self ):
88          """Path + Name for output file"""
89          return self._outputPathFile
90
91      @property
92      def uuid( self ):
93          """Universal unique identifier: connects requirements to source
        document"""
94          return self._uuid
95
96  #   S E T T E R S   #
97
98      @descriptor.setter
99      def descriptor( self, value ):
100         self._descriptor = value
101
102     @sizeName.setter
103     def sizeName( self, value ):
104         self._sizeName = value
105
106     @sizeValue.setter
107     def sizeValue( self, value ):
108         self._sizeValue = value
109
110     @sizeUnits.setter
111     def sizeUnits( self, value ):
112         self._sizeUnits = value
113
114     @areaValue.setter
115     def areaValue( self, value ):
116         self._areaValue = value
117
118     @areaUnits.setter
119     def areaUnits( self, value ):
120         self._areaUnits = value
121
122     @masterSheet.setter
123     def masterSheet( self, value ):
124         self._masterSheet = value
125
126     @sourcePath.setter
127     def sourcePath( self, value ):
128         self._sourcePath = value
```

```
129
130     @sourceFile.setter
131     def sourceFile( self, value ):
132         self._sourceFile = value
133
134     @outputFile.setter
135     def outputFile( self, value ):
136         self._outputFile = value
137
138     @outputPath.setter
139     def outputPath( self, value ):
140         self._outputPath = value
141
142     @outputPathFile.setter
143     def outputPathFile( self, value ):
144         self._outputPathFile = value
145
146 #   D E L E T E R S   #
147
148     @descriptor.deleter
149     def descriptor( self ):
150         del self._descriptor
151
152     @sizeName.deleter
153     def sizeName( self ):
154         del self._sizeName
155
156     @sizeValue.deleter
157     def sizeValue( self ):
158         del self._sizeValue
159
160     @sizeUnits.deleter
161     def sizeUnits( self ):
162         del self._sizeUnits
163
164     @areaValue.deleter
165     def areaValue( self ):
166         del self._areaValue
167
168     @areaUnits.deleter
169     def areaUnits( self ):
170         del self._areaUnits
171
172     @masterSheet.deleter
173     def masterSheet( self ):
174         del self._masterSheet
175
176     @sourcePath.deleter
177     def sourcePath( self ):
178         del self._sourcePath
```

```
179
180     @sourceFile.deleter
181     def sourceFile( self ):
182         del self._sourceFile
183
184     @outputFile.deleter
185     def outputFile( self ):
186         del self._outputFile
187
188     @outputPath.deleter
189     def outputPath( self ):
190         del self._outputPath
191
192     @outputPathFile.deleter
193     def outputPathFile( self ):
194         del self._outputPathFile
195
196     @uuid.deleter
197     def uuid( self ):
198         del self._uuid
199
200 #   M E T H O D S   #
201
202     def print_attributes( self ):
203         print('\nSource attributes:')
204         print( 'descriptor     = %s' % self.descriptor )
205         print( 'sizeName       = %s' % self.sizeName )
206         print( 'sizeValue      = %s' % self.sizeValue )
207         print( 'sizeUnits      = %s' % self.sizeUnits )
208         print( 'sourcePath     = %s' % self.sourcePath )
209         print( 'sourceFile     = %s' % self.sourceFile )
210         print( 'sourcePathFile = %s' % self.sourcePathFile )
211         print( 'outputFile     = %s' % self.outputFile )
212         print( 'outputPath     = %s' % self.outputPath )
213         print( 'outputPathFile = %s' % self.outputPathFile )
214         print( 'uuid           = %s' % self.uuid )
215         return
216
217 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
218
219     def scenario( self ):
220         self.setup_io( ) # establish outut file
221         #self.read_MoM_file( )
222         self.area_circular( ) # compute area for given geometry
223         return
224
225 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
226
227     def read_MoM_file( self ):
228         ## ## read source file
```

```
229         print ( "reading source file %s" % self.sourceFile )
230         #
    https://stackoverflow.com/questions/3277503/in-python-how-do-i-read-a-file-line-by-line-into-a-lis
231         with open( self.sourceFile ) as f:
232             self.col_lines = f.read( ).splitlines( )
233             self.numLines = len( self.col_lines )
234         return
235
236 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
237
238     def setup_io( self ):
239         # combine path and file name
240         self.sourcePathFile  = self.sourcePath + self.sourceFile
241         self.outputPathFile  = self.outputPath + self.outputFile
242         self.masterSheet     = self.descriptor + ', ' + self.sizeName[0] + '
    = ' + str( self.sizeValue ) + ' ' + self.sizeUnits
243         return
244
245 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
246
247     def area_circular( self ):
248         # combine path and file name
249         self.areaValue = math.pi * ( self.sizeValue / 2 )**2
250         return
251
252 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
253
```

D.1.3 Excel Details

Toolkit for writing to spreadsheets.

```
1  #! /usr/bin/python3
2
3  # # Daniel Topa
4
5  # # Excel tools
6  # xl_new_workbook( workbook_title )
7  # xl_sheet_requirements( this_workbook )
8  # xl_sheet_generate( this_workbook, title_sheet )
9  # xl_s( this_workbook )
10 # xl_sheet_header_footer( this_worksheet )
11
12 # # imports
13 import os               # probe, change directories
14 import sys              # python version
15 import datetime         #
       https://stackoverflow.com/questions/415511/how-to-get-the-current-time-in-python
16 import numpy as np
17 import pandas as pd
18 import xlsxwriter       # API for Excel
```

```
from xlsxwriter.utility import xl_rowcol_to_cell
import numpy as np
import pandas as pd

import cls_TestObject

# # modules
def xl_new_workbook( testObject ):

    MoMresults = xlsxwriter.Workbook( testObject.outputFile )
    print( "output file %s" % testObject.outputFile )
    print( "source file %s" % testObject.sourceFile )

    xl_sheet_master( MoMresults, testObject ) # MoM summary
    xl_add_data_sheets( MoMresults, testObject ) # MoM summary
    xl_sheet_provenance( MoMresults ) # provenance sheet

    return MoMresults;

#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

def xl_add_data_sheets( this_workbook, testObject ):

    format_MoM_title = this_workbook.add_format( )
    format_MoM_title.set_bold( )
    format_MoM_title.set_font_color( "red" )

    format_MoM_head = this_workbook.add_format( )
    format_MoM_head.set_bold( )

    format_MoM_polarization = this_workbook.add_format( )
    format_MoM_polarization.set_bold( )

    number_format = this_workbook.add_format({'num_format': '#,##0.000'})

    # https://xlsxwriter.readthedocs.io/format.html#set_center_across
    cell_format = this_workbook.add_format()
    cell_format.set_center_across()

    for index in range( 1, 29 ):
        # add sheet and tag header and footer
        title = str( index + 2 ) + ' MHz'
        print ( 'adding sheet %s' % title )
        s = xl_sheet_generate( this_workbook, title )
        xl_sheet_header_footer( s )
        s.write( "A1", "MoM 4.1.12 output (*.dat)", format_MoM_title )
        #
        s.write( "A3", "azimuth, °", format_MoM_head )
        s.write( "B3", "HH, dBsm",   format_MoM_head )
        s.write( "C3", "VV, dBsm",   format_MoM_head )
```

```
69          s.write( "D3", "HV, dBsm",   format_MoM_head )
70          s.write( "E3", "VH, dBsm",   format_MoM_head )
71          #
72          s.write( "H3", "mean", format_MoM_head )
73          s.write( "J3", "standard deviation", format_MoM_head )
74          #
75          s.write( "G4", "HH", format_MoM_polarization )
76          s.write( "G5", "VV", format_MoM_polarization )
77          #
78      #    AttributeError: 'str' object has no attribute '_get_xf_index'
79      #    s.write( "I4", "HH", u"\u00B1" )
80          s.write( "I4", '±', cell_format )
81          s.write( "I5", '±', cell_format )
82          s.set_column( "I:I", 3 )
83
84          # = AVERAGE( B5:B364)
85          # = STDEV( B5:B364 )
86          s.write( "H4", '= AVERAGE( B5:B364)', number_format )
87          s.write( "H5", '= AVERAGE( C5:C364)', number_format )
88          s.write( "J4", '= STDEV( B5:B364 )',  number_format )
89          s.write( "J5", '= STDEV( B5:B364 )',  number_format )
90
91          # read in data file
92          filename = './data/sphere-005-' + testObject.resolution + '-' + str(
    index + 2 ).zfill(2) + '.4112.dat.txt'
93          s.write_string( "D1", filename )
94          data = pd.read_csv( filename, delimiter=r"\s+", header = None )
95          data_np = data.values
96          row = 3
97          col = 0
98          for line in range( 0, len ( data_np ) ):
99              cell = xl_rowcol_to_cell ( row, col )
100             s.write( row, col, data_np[ line ][ 0 ], number_format )
101             s.write( row, col + 1, data_np[ line ][ 1 ], number_format )
102             s.write( row, col + 2, data_np[ line ][ 2 ], number_format )
103             s.write( row, col + 3, data_np[ line ][ 3 ], number_format )
104             s.write( row, col + 4, data_np[ line ][ 4 ], number_format )
105             row += 1
106
107     return
108
109 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
110
111 def xl_sheet_generate( this_workbook, title_sheet ):
112
113     # insure every worksheet has a header and footer
114     mySheet = this_workbook.add_worksheet( title_sheet )
115     xl_sheet_header_footer( mySheet )
116
117     return mySheet;
```

```
118
119 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
120
121 def xl_sheet_provenance( this_workbook ):
122
123     # Define some global names.
124     this_workbook.define_name( 'c_', '=299792458' )
125     # forensic info
126     s = xl_sheet_generate( this_workbook, "provenance" )
127     #  # special formats
128     # https://xlsxwriter.readthedocs.io/format.html?highlight=bold
129
130     # method 1
131     # setting the property as a dictionary of key/value pairs in the
        constructor
132     format_title = this_workbook.add_format( )
133     format_title.set_bold( )
134     format_title.set_font_color( "blue" )
135
136     # method 2
137     # passing a dictionary of properties to the add_format() constructor
138     format_time = this_workbook.add_format( {'num_format': 'yy/mm/dd hh:mm'}
        ) # https://xlsxwriter.readthedocs.io/working_with_dates_and_time.html
139
140     # widen first columns
141     s.set_column( "A:A", 15 )
142     s.set_column( "B:B", 13 )
143
144     # https://xlsxwriter.readthedocs.io/worksheet.html
145     s.write_url( "A1",
        "https://en.wikipedia.org/wiki/Computational_electromagnetics", string =
        "Radar Cross Section Measurements" )
146
147     #  #  provencance
148     s.write( "A3", "Workbook created by", format_title )
149     #s.write( "A1", tip, "boo" )
150
151     # python notebook which creates workbook
152     s.write( "A4", "python source" )
153     s.write( "B4", os.path.basename( __file__ ) ) # charlie.py
154
155     # current working directory
156     s.write( "A5", "directory" )
157     s.write( "B5", os.getcwd( ) ) #
        /Volumes/Tlaltecuhtli/repos/GitHub/topa-development/python/xlsx
158
159     # python version
160     s.write( "A6", "python version" )
161     s.write( "B6", sys.version ) # "3.7.0 (default, Jun 28 2018, 07:39:16)
        [Clang 4.0.1 (tags/RELEASE_401/final)]"
```

```
162
163     #  #  environment variables
164     # practise row, col notation
165     col = 0 # starting column
166     row = 7 # starting row
167     s.write( row, col, "Environment variables", format_title ); row += 1
168
169     s.write( row, col, "$USER" ) # l127914
170     s.write( row, col + 1, os.environ[ "USER" ] ); row += 1
171
172     s.write( row, col, "$HOSTNAME" ) # Cauchy.Schwarz
173     s.write( row, col + 1, os.environ[ "HOSTNAME" ] ); row += 1
174
175     s.write( row, col, "$HOME" ) # /Users/l127914
176     s.write( row, col + 1, os.environ[ "HOME" ] ); row += 1
177
178     s.write( row, col, "timestamp" ) # 11/21/18 16:18
179     s.write( row, col + 1, datetime.datetime.now( ), format_time ); row += 1
180
181     #  #  Excel info routines
182     # https://xlsxwriter.readthedocs.io/working_with_formulas.html
183
184     row += 1 # jump
185     s.write( row, col, "XL info function", format_title ); row += 1
186
187     s.write( row, col, "platform" ) # mac
188     s.write_formula( row, col + 1, '= INFO( "system" )' ); row += 1
189
190     s.write( row, col, "recalculation mode" ) # Automatic
191     s.write_formula( row, col + 1, '= INFO( "recalc" )' ); row += 1
192
193     s.write( row, col, "active sheets" ) # 1
194     s.write_formula( row, col + 1, '= INFO( "numfile" )' ); row += 1
195
196     s.write( row, col, "cursor" ) # $A:$A$1
197     s.write_formula( row, col + 1, '= INFO( "origin" )' ); row += 1
198
199     s.write( row, col, "XL release" ) # 16.16
200     s.write_formula( row, col + 1, '= INFO( "release" )' ); row += 1
201
202     s.write( row, col, "application directory" ) #
        /Users/dantopa/Library/Containers/com.microsoft.Excel/Data/Documents/
203     s.write_formula( row, col + 1, '= INFO( "directory" )' ); row += 1
204
205     s.write( row, col, "operating systems" ) # Macintosh (Intel) Version
        10.13.3 (Build 17D47)
206     s.write_formula( row, col + 1, '= INFO( "osversion" )' ); row += 1
207
208     return
209
```

```
#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

def xl_sheet_header_footer( this_worksheet ):

    # header: sheet name (center)
    # footer: date/time, page number, path/file

    myheader = "&C&12&A" # fontsize 12
    myfooter = "&L&8&T\n&8&D" + "&C &P / &N" + "&R&8&Z\n&8&F" # fontsize 8

    this_worksheet.set_header( myheader )
    this_worksheet.set_footer( myfooter )

    return

#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

def xl_sheet_master( this_workbook, testObject ):

    number_format = this_workbook.add_format({'num_format': '#,##0.000'})

    masterRow = 0
    masterCol = 0
    xl_set_label_column ( this_workbook, testObject, masterRow, masterCol )

    dataRow = 8
    dataCol = 0
    s = this_workbook.get_worksheet_by_name( testObject.masterSheet )
    for index in range(1, 29):
        dataCol += 1
        nu = index + 2
        xl_computation ( s, dataRow, dataCol, nu, number_format )

    return

#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

# https://xlsxwriter.readthedocs.io/working_with_cell_notation.html
def xl_computation ( wsheet, row, col, nu, number_format ):

    # frequency
    wsheet.write_number ( row, col, nu )

    # wavelength =  c_ / ( B11 * 1000000 )
    cell = xl_rowcol_to_cell ( row, col )
    wsheet.write ( row + 1, col, '= c_ / ( ' + cell + ' * 1000000 )',
    number_format ); row += 1

    # = radius / wavelength
    cell = xl_rowcol_to_cell ( row, col )
```

```
259     wsheet.write ( row + 1, col, '= radius / ' + cell, number_format ); row
        += 3
260
261     # MoM average dBsm  = '30 MHz'!$H4
262     wsheet.write_formula(row, col, "= '" + str( nu ) + " MHz'!$H$4",
        number_format ); row += 1
263     # relative error dBsm
264     cell = xl_rowcol_to_cell ( row - 1, col )
265     wsheet.write_formula ( row, col, '= 1 - size_optical_dbsm / ' + cell,
        number_format ); row += 2
266
267     # rcs, sq m  = 10^( B15 / 10 )
268     cell = xl_rowcol_to_cell ( row - 3, col )
269     wsheet.write_formula ( row, col, '= 10^( ' + cell + ' / 10 )',
        number_format ); row += 1
270     # rel error (sq m)  =  1 - size_optical_sq_m / B18
271     cell = xl_rowcol_to_cell ( row - 1, col )
272     wsheet.write_formula ( row, col, '= 1 - size_optical_sq_m / ' + cell,
        number_format )
273
274     return
275
276 #  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #
277
278 def xl_set_label_column( wbook, testObject, row, col ):
279
280     # method 1
281     # setting the property as a dictionary of key/value pairs in the
        constructor
282     format_title = wbook.add_format( )
283     format_title.set_bold( )
284     format_title.set_font_color( "blue" )
285
286     format_label = wbook.add_format( )
287     format_label.set_bold( )
288
289     # https://xlsxwriter.readthedocs.io/example_defined_name.html
290     # https://docs.python.org/2.0/ref/strings.html
291     wbook.define_name( 'c_', '=299792458' )
292     #string = '\'=' + str( testObject.sizeValue / 2 ) + '\''
293     #print( 'string = %s' % string )
294     wbook.define_name( 'radius', '=5' )
295     wbook.define_name( 'size_optical_sq_m', '=\'' + testObject.masterSheet +
        '\'!$B$6' )
296     wbook.define_name( 'size_optical_dbsm', '=\'' + testObject.masterSheet +
        '\'!$B$7' )
297
298     # sheet operations
299     s = xl_sheet_generate( wbook, testObject.masterSheet )
300     s.set_first_sheet( )
```

```
301
302     # widen first columns
303     s.set_column( "A:A", 17 )
304     s.set_column( "B:B", 10  )
305
306     # column of labels
307     s.write_string( row, col, 'INPUT', format_title ); row += 2
308
309     s.write( row, col, 'MoM output:', format_label )
310     s.write( row, col + 1, testObject.sourceFile ); row += 2
311
312     s.write( row, col, testObject.sizeName, format_label );
313     s.write( row, col + 1, testObject.sizeValue )
314     s.write( row, col + 2, 'm' );  row += 1
315
316     s.write( row, col, 'optical size', format_label )
317     s.write( row, col + 1, '= pi() * radius^2' )
318     s.write_string( row, col + 2, testObject.areaUnits ); row += 1
319     s.write_formula( row, col + 1, '= 10 * LOG10( size_optical_sq_m )' );
320     s.write( row, col + 2, 'dB area' ); row += 2
321
322     s.write( row, col, 'frequency (MHz)', format_label ); row += 1
323     s.write( row, col, 'wavelength (m)', format_label ); row += 1
324     s.write( row, col, 'radius / lambda', format_label ); row += 2
325
326     s.write( row, col, 'MoM average (dbSm)', format_label ); row += 1
327     s.write( row, col, 'rel error (dbSm)', format_label ); row += 2
328
329     s.write( row, col, 'rcs, sq m', format_label ); row += 1
330     s.write( row, col, 'rel error (sq m)', format_label )
331
332     xl_sheet_header_footer( s )
333
334
335     return
336
337 # root@f21d93a5a2e9:sphere $ python tools_xl.py
338 #
339 # root@f21d93a5a2e9:sphere $ date
340 # Wed Jun 24 01:19:38 MDT 2020
341 #
342 # root@f21d93a5a2e9:sphere $ pwd
343 # /Tlaloc/python/sphere
344
345
```

D.2 Outputs

The main module arranges the output data by creating a tab for each separate radar frequency in the range $3 - 30$ MHz. Other tabs contain aggregate information and diagnostics.

```
#! /usr/bin/python3

# # Daniel Topa

# # imports
import datetime             # timestamps
import os                   # opeating system
import sys                  # python version
from pathlib import Path    # rename file
import xlsxwriter           # API for Excel
import tools_xl             # spreadsheet authoring tools
# home brew
# classes
import cls_TestObject

#  ==   ==   == ==   ==   == ==   ==   == ==   ==   ==  #

if __name__ == "__main__":

    series = '050'
    object = cls_TestObject.TestObject( )     # instantiate TestObject
    # populate object properties
    object.descriptor = "sphere"
    object.sizeName   = "diameter"
    object.sizeValue  = 10
    object.sizeUnits  = "m"
    object.areaUnits  = "m^2"
    object.resolution = "03"
    object.sourceFile = "sphere-" + series + "-" + object.resolution
    object.sourcePath = "/Tlaloc/python/sphere/"
    object.outputFile = 'sphere-d' + series + '-res' + object.resolution +
    '.xlsx'
    object.outputPath = "/Tlaloc/python/sphere/"
    object.setup_io( )
    object.area_circular( )

    # container for MoM data and results
    MoMresults = tools_xl.xl_new_workbook( object )

    # close MoMresults
    MoMresults.close( )
    print( "\n", datetime.datetime.now( ) )
    print( "source: %s/%s" % ( os.getcwd( ), os.path.basename( __file__ ) ) )
    print( "python version %s" % sys.version )

# root@f21d93a5a2e9:sphere $ date
# Wed Jun 24 01:20:41 MDT 2020
#
# root@f21d93a5a2e9:sphere $ pwd
```

```
# /Tlaloc/python/sphere
#
# root@f21d93a5a2e9:sphere $ python MoM.py
# output file RCS-sphere-10.xlsx
# source file sphere-050-01
# adding sheet 3 MHz
# adding sheet 4 MHz
# adding sheet 5 MHz
# adding sheet 6 MHz
# adding sheet 7 MHz
# adding sheet 8 MHz
# adding sheet 9 MHz
# adding sheet 10 MHz
# adding sheet 11 MHz
# adding sheet 12 MHz
# adding sheet 13 MHz
# adding sheet 14 MHz
# adding sheet 15 MHz
# adding sheet 16 MHz
# adding sheet 17 MHz
# adding sheet 18 MHz
# adding sheet 19 MHz
# adding sheet 20 MHz
# adding sheet 21 MHz
# adding sheet 22 MHz
# adding sheet 23 MHz
# adding sheet 24 MHz
# adding sheet 25 MHz
# adding sheet 26 MHz
# adding sheet 27 MHz
# adding sheet 28 MHz
# adding sheet 29 MHz
# adding sheet 30 MHz
#
#  2020-06-24 01:20:46.349142
# source: /Tlaloc/python/sphere/MoM.py
# python version 3.7.7 (default, Jun 22 2020, 22:42:46)
# [GCC 10.1.0]

# $ lsb_release -a
# LSB Version:  :core-4.1-amd64:core-4.1-noarch
# Distributor ID:   CentOS
# Description:  CentOS Linux release 7.8.2003 (Core)
# Release:  7.8.2003
# Codename: Core
```

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