1. Establish the Foundation

- **Function Signatures** describe how data is modeled and represented.
- Foundation: Start with **Fourier (1D)**, extend to **Zernike (2D)**, and then **Spherical Harmonics (3D)**.

Domain	**Basis Functions**	**Example Input**
1D	Sines/Cosines	$f(x)$
2D (disk)	Zernike Polynomials	$f(x, y)$
3D (sphere)	Spherical Harmonics	$f(\theta, \phi)$

2. Trivial Example: Constant Sphere

- Input: $f(\theta, \phi) = 1$
- Output: Only $c_{00} \neq 0$ in the SH expansion.

uniform_sphere.png

l	m	c_{lm}
0	0	1.0
1	-1, 0, 1	0.0

3. Perturbed Sphere: Higher Modes

- A small localized **bump** perturbs the sphere.
- Higher l-modes are excited in the SH expansion.

`bumped_sphere.png`

l	m	c_{lm} (Uniform)	c_{lm} (Bumped)
0	0	1.0	0.95
1	-1, 0, 1	0.0	Small nonzero

- Pick an angle (θ, ϕ), e.g., $(90^\circ, 0^\circ)$.
- Value $f(\theta, \phi)$ determines the color/height at that point.
- **Higher SH coefficients c_{lm}** add detail to the surface.

rcs_visual.png

5. Function Signature

$$\sigma(\theta, \phi) \approx \sum_{l=0}^{L} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi)$$

- **Inputs**: Angles (θ, ϕ).
- **Outputs**: Scalar values $f(\theta, \phi)$ (RCS, etc.).
- **Spherical Harmonics Coefficients c_{lm}** describe the function.