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Survey on advances in orbital dynamics and control for libration point orbits

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ABSTRACT

Libration point orbits (LPOs) have drawn a lot of interest because of their great significance in deep space exploration. This paper summarizes the past developments and then presents the current state-of-art of LPOs including the dynamical structure of phase space, the transfer trajectories and homoclinic/heterclinic connections of LPOs, the station-keeping strategies, and some constellation deployments employing LPOs. Subsequently addressed are the applications of the LPO theory into the fields of lunar transfers, solar sail equilibria and formation flying. Finally, future research directions on LPOs are described from the aspects of the existence proof of Halo orbits, orbital design for the potential missions motivated by LPOs, and so on.

Contents

1. Introduction

Libration points are essentially the gravitational equilibria in celestial mechanics, where a spacecraft is able to maintain stationarity with respect to the primary and secondary bodies without fuel consumption. Therefore, libration points can motivate numerous space missions due to their special locations. Located between the Sun and Earth, the Sun-Earth libration point EL_1 has advantages in solar activity observation. For example, the solar wind can be observed and warned about an hour earlier [1]. If a spacecraft is placed on a Halo orbit or a Lissajous orbit around EL_1, it can accomplish observation with its communication signal not disturbed by solar wind. Similarly, EL_2 is also an ideal location for cosmos observation and astronomy studies, with few influences from atmospheres, space debris, Earth's infrared radiation and magnetic field around it.

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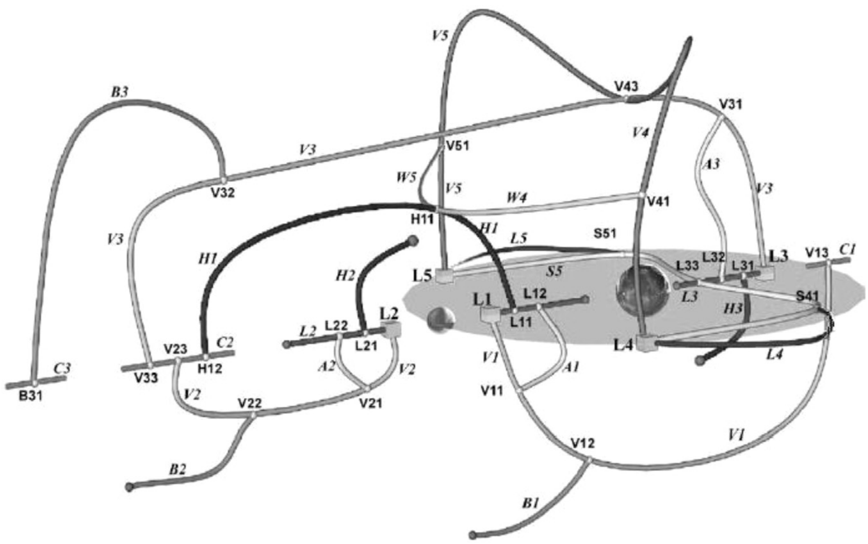

Fig 1. The bifurcations of periodic orbits in CR3BP [25].

The Earth–Moon libration point LL_1 is located between the Earth and Moon, which is expected to be a transfer station for future lunar exploration. Halo orbits around LL_1 point play an important role as the gateway to the conceptional 'Planetary Superhighway' [2]. The bounded orbits near LL_2 point enable the exploration of the far side of Moon [3], which is always unobservable for human beings [4].

So far, NASA and ESA have successfully launched several spacecraft into the vicinity near libration points, such as ISEE-3 (launched in 1978) [5], WIND (1994) [6], SOHO (1995) [7], ACE (1997) [8], MAP (2001) [9], Genesis (2001) [10], Herschel (2007) [11], Planck (2007) [11] and GAIA (2013) [12], and some underway missions. The Kuafu Project is being planned by China including Kuafu-A spacecraft on a Halo orbit near EL_1 point, and Kuafu-B1 and Kuafu-B2 on a polar orbit form a widely-used observation system in order to monitor the cislunar space environment change caused by solar activities [13].

Like the geosynchronous Earth orbit (GEO), the LPOs in both the Sun–Earth system and the Earth–Moon system are precious resources. It is an important issue in the astronautical research field how to take better advantages of these resources.

2. Survey on dynamics and control for libration point orbits

2.1. Historical review of libration point orbits

Euler and Lagrange [14] derived five libration points (also known as Lagrange points) when studying particular solutions of the circular restricted three-body problem (CR3BP) in 1767 and 1772, respectively. In the past, the study focused on two aspects, the stability of libration points and the bounded motions near libration points.

Collinear libration points (L_1, L_2 and L_3 points) are exponentially unstable. This conclusion can be derived from the linearized equations of CR3BP [15]. However, the stability of triangular libration points (L_4 and L_5 points) was still confusing to experts in the astronomical and mathematical societies until the emergence of the Kolmogorov-Arnold-Moser (KAM) theory in the 20th century [16].

The bounded motions near libration points consist of periodic and quasi-periodic orbits, i.e. quasi-periodic orbits span the center manifolds of a corresponding periodic orbit [17]. Therefore, a periodic orbit together with its invariant manifolds contributes to the understanding of the phase space structure near libration points. In fact, searching for periodic orbits was first carried out by Poincaré, the founder of nonlinear science, who once believed that the "periodic orbit provides the only access to understanding the behavior in the difficult three-body problem" [18]. The early research was limited to the planar case. Strömgren obtained some families of periodic orbits [14], including planar Lyapunov orbits and the distant retrograde orbit (DRO), which will be mentioned later; Moulton studied oscillating motions near libration points in a three-dimensional framework [14], and first derived the periodic solutions, namely 'Halo orbit', as well as vertical Lyapunov orbits. Hénon investigated the periodic orbits in the Hill's problem [19,20], and proposed 'vertical stability index' a_ν in order to quantize the ability of planar periodic orbits avoiding out-of-plane motions under perturbation. He also indicated that $|a_\nu|=1$ was the bifurcation point from planar orbits to spatial ones. Bobin and Markellos [21] presented several types of three-dimensional orbits with axis symmetry, planar symmetry, dual symmetry, etc., and illuminated the bifurcation of them from planar to spatial case. Zagouras and Kazantzis [22] investigated the three-dimensional motions near collinear libration points in the Sun-Jupiter system, and derived Halo orbits from the bifurcation of planar Lyapunov orbits. Howell and Campbell [23] concentrated on bifurcation given different periods of Halo orbits, and proved that n-period Halo orbits will appear as the Jacobi integral becomes large enough. Techniques such as continuation and bifurcation, greatly developed during a one-hundred-year investigation on periodic orbits are considered as powerful tools for nonlinear dynamics [24]. So far, the search for periodic orbits has yielded rich achievements with numerous families of orbits discovered, shown in Fig. 1.

The astronautical experts' researches on libration points started in 1950, when Clarke [26] first proposed that the Earth-Moon LL_2 point was an ideal position for the communication and broadcast between the far side of the Moon and the Earth. Farquhar and Breakwell declared that the best way to accomplish the communication between the far side of the Moon and the Earth was to place a satellite on a periodic orbit near the LL_2 point, named Halo orbit due to its shape [27]. Farquhar and Kamel [28] investigated a higher order analytical formula (called Farquhar–Kamel expansion) of periodic and quasi-periodic orbits under solar gravity. Breakwell and Brown [29] derived the algorithm to generate Halo orbits in the Earth-Moon system; later, Popescu and Cardos [30] verified that this algorithm was highly effective to generate the initial values of Halo orbits. Based on the results, the far side of the Moon was once selected as the candidate landing site for Apollo-

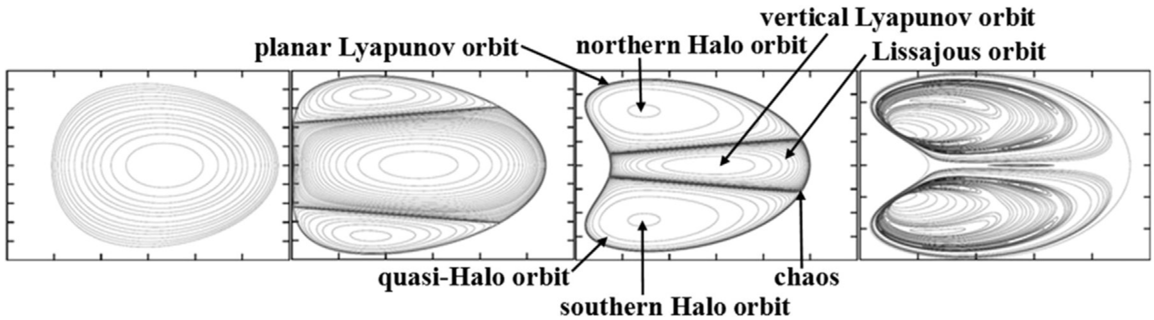

Fig. 2. Poincaré section of center manifolds near L_1 point (Jacobi energy increases from left to right) [44].

17. Meanwhile quasi-periodic orbits are utilized to solve the communication problem of lunar missions, but the project was aborted due to budget cuts [31]. The successful launch of the first LPO spacecraft ISEE-3 in 1978 indicated that the research interest was extended from the Earth-Moon to the Sun-Earth system [1]. By the Lindstedt-Poincaré method, Richardson [32] deduced the famous third order expansion of Halo orbits near collinear libration points of CR3BP, which was later named after him as a good approximation of Halo orbits. Howell [33] investigated the effect of mass ratio μ and orbital amplitude on Halo orbits, and concluded that larger amplitude leads to stronger stability. Howell and Pernicka developed the iterative algorithm to generate Lissajous orbits near collinear libration points of CR3BP [34], which was successfully extended into a more complicated ephemeris model by Howell et al. [35]. Employing the Lindstedt-Poincaré method and Fourier expansion, Gómez et al. [36] derived a 35th order expansion to calculate Lissajous orbits. In order to improve the computational efficiency of Halo orbits, Kechichian [37] introduced a variable transformation at the expense of more meaningless variables.

CR3BP is the simplest model to study the three-body problem, as well as the most useful one to investigate the motions and phase space structure near libration points. However, it is sometimes not accurate enough for astronautical applications. So far, the JPL ephemeris is regarded as the most accurate model except for the real gravitational field. The nominal trajectory is often designed based on the results derived from the simplified model, and then corrected in a more precise model. Besides CR3BP and the ephemeris model, there were five intermediate models proposed:

1. Hill's Model [38]: This model focuses on the motions near a secondary body, and can be regarded as the approximation of CR3BP. The model has a family of $2\pi m$-period orbits (m denotes a parameter of this family) which are called Hill's variational orbits.

2. Restricted Hill's Four-Body Model [39]: This model possesses two parameters, mass ratio μ and Hill's variational orbits parameter m; when m approaches to 0, the model degenerates to CR3BP, and when μ tends to 0, the model degenerates to Hill's model.

3. Elliptic Restricted Three-Body Model [23]: The primary and secondary bodies are no longer on a circular motion around each other, but on an elliptic motion. Different from CR3BP, this model has periodic perturbations.

4. Bicircular Model (BCM) [40]: The Earth-Moon system is considered as a whole system on a two-body circular motion around the Sun, and the Sun has no influence on the two-body circular motion of the Earth and Moon; hence, the bicircular model can be decomposed into two CR3BPs.

5. Quasi-Bicircular Model (QBCM) [41]: The drawback of BCM is that the motions of the Sun, Earth and Moon do not satisfy Newton's laws, thus some improvements in QBCM are to substitute this coherent motion by a natural solution of the Sun-Earth-Moon three body problem, which is expressed as a Fourier series.

2.2. Phase space structure near libration points

According to linear Hamiltonian system theory, the dynamical behaviors near the collinear libration points are of the topological type "center × center × saddle", which implies the motions near libration points have periodic and hyperbolic components [42]. Due to the difficulty separating periodic from hyperbolic components in Euclidean space, the Brikhoff normalization method is implemented to reduce the Hamiltonian function by orthogonalizing them [44] so that the remaining periodic components span a four-dimensional space named center manifolds after the elimination of uncoupled hyperbolic stable/unstable manifolds. Thus the phase space structure can be illustrated by mapping the remaining center manifolds onto a Poincaré section [43]. Fig. 2 shows the evolution with respect to the energy of the behavior of the Poincaré section for the L_1 point, and the L_2 and L_3 points have a similar structure.

In each subfigure of Fig. 2, the exterior curve is a planar Lyapunov orbit of the energy corresponding to the subfigure. Inside the region bounded by the planar Lyapunov orbit exist equilibrium points, the number of which increases from one to three or even more as the energy increases. In the first subfigure of the smallest energy, the center point indicates a vertical Lyapunov orbit and the motions around it are quasi-periodic orbits, called Lissajous orbits. By the Lyapunov center theorem, the planar and vertical Lyapunov orbits possess two fundamental frequencies respectively and Lissajous orbits involve these two fundamental motions. As energy increases, two bifurcation points appear located above and below to the center point in the second and third subfigures, indicating the northern and southern Halo orbits, respectively, surrounded by quasi-Halo orbits. However, the existence of Halo orbits still remains to be proven [45]. As energy further increases, more bifurcation points appear, which indicates n-period Halo orbits [46], presented in the fourth subfigure of Fig. 2. Lissajous or quasi-Halo orbits near periodic orbits constitute invariant tori, which however cannot fill out the entire reduced phase space. In other words, there is a gap between them, where there exist some chaotic trajectories led by stable/unstable manifolds of Lyapunov or Halo orbits [37,43]. The rigorous existence of these tori is more problematic. Because the fundamental frequencies of libration points are too close to resonant; the frequencies change with their amplitudes and they go across the resonances when the amplitudes are changed, leading to the appearance of a Cantor set of a tori [47]. Jorba and Villanueva applied Moser's iteration method to illustrate the persistence of the invariant tori of the quasi-periodic orbit [48].

Inheriting from libration points, the periodic orbits near them also have hyperbolic components, which implies that both libration points and periodic orbits have stable and unstable manifolds. The geometric structure of stable and unstable manifolds enables the global nonlinear investigation on the entire phase space. Stable and unstable manifolds of Halo orbits can span two-dimensional manifolds in the phase space, which is called Conley–McGehee tube (C–M tube). Howell et al. [49] attempted to project these manifolds onto three-dimensional configuration space to obtain the structure of the two-dimensional C–M tube. They proposed two approaches to plot and store invariant manifolds, namely the Splines and Cells mode. Howell employed the equal-time interval method to document the distribution of the C–M tube. Because the velocity of each point on the tube varies, the C–M tube inevitably wrinkles heavily. Hirani and Lo [50] utilized the equal-space interval method to document the distribution of the C–M tube, and revealed that when the C–M tube is unfolded to some degree, its inner surface would be exposed completely.

2.3. Transfers from/to Halo orbits and homoclinic/heteroclinic connections

Farquhar et al. [51] first studied the transfer problem of the libration point orbits and employed the shooting method to obtain the transfer orbits of ISEE-3 by several iterations. Gómez et al. [52] investigated the hyperbolic behaviors of a Halo orbit, and discovered that some branches of its stable manifolds passed through near-Earth space, based on which they designed transfer trajectories from Earth to the Halo orbit. Howell et al. [53] derived Halo orbit transfer trajectories in the real gravitational environment by combining invariant manifolds with differential correction, and concluded that as the amplitude of the Halo orbit increases, the minimum distance between the invariant manifolds and the Earth's surface decreases.

According to Gómez et al., a spacecraft can be inserted into a Halo orbit without any orbital maneuver with the assistance of its natural stable manifolds [45]. Such orbital transfer design is based on a deep understanding of orbital dynamics, which is very hard to achieve by traditional optimization theory. Therefore, invariant manifolds theory is becoming more attractive to astronautical experts.

The two advances in orbital design technique, the low-thrust propulsion and gravity-assist transfer techniques have been applied to driving a spacecraft from Earth to Halo orbits in the Sun–Earth system. For instance, the lunar fly-by (or swing-by) technique was employed in space missions, such as SOHO, WIND, etc. [54]. The low-thrust propulsion technique is essentially the gradually-changed impulse technique. During the long journey of escaping from the Earth, the spacecraft gradually adjusts the insert window by controlling the low thrust, and eventually inserts into the stable manifolds of a Halo orbit [55]. In the escape segment from Earth, the control law is derived from the maximum gradient direction of the Jacobi energy. However, if continuous thrust is implemented in the entire process, a small term of continuous thrust control will be added to the right side of the nonlinear dynamics equations, resulting in slow evolvement of invariant manifolds, namely controlled gradual manifolds [56]. The distance between the Halo orbit and Earth is about four times the Earth–Moon distance. Hence there will be enough time and space to reduce the launch cost by the lunar gravity assist technique. A spacecraft will directly insert into the stable manifolds of a Halo orbit after it flies by the Moon. When conceiving the initial guess, Howell et al. [57] used the patched-conic method to obtain the first-order approximation of the gravity assist segment, then further corrected the flyby segment by the pseudo-status theory in the four-body model, finally employed the optimization algorithms to derive real trajectories, satisfying various constraints.

Under the influence of launch deviation, navigation error and propulsion error, a spacecraft's trajectory will deviate from the designated one. Moreover, the duration to Halo orbits is very long, usually 8–12 months. Therefore, several halfway corrections are necessary. Serban et al. [58] believed that launch deviation was the main reason leading to the deviation from the designated trajectory. They provided possible solutions using optimization algorithms and obtained the optimal solutions of halfway corrections for the Genesis mission. Based on the sensibility of stable/unstable manifolds of Halo orbits, Gómez et al. [59] analyzed the influence of different insertion point, error magnitude and time of flight on corrected results, and put forward a halfway correction strategy. Although Gómez et al. employed no optimization computation, they derived the same results as Serban et al. Serban et al. and Gómez et al. [58,59] proposed a method to eliminate the launch error, but the correction must be obtained from the ground station, causing a time delay and loss of autonomy of spacecraft. Jenkin et al. [60] put forward a scheduled closed-loop correction strategy, and analyzed the error distribution after correction, but their strategy was too simple to deal with the significant navigation error. Aiming at a type of single-impulse transfer which was often ignored, Xu et al. [61] designed a scheduled halfway correction strategy by stochastic optimal control theory and Monte-Carlo simulations and demonstrated that this strategy could still yield results similar to those of Jenkin et al. considering errors in launch, navigation and propulsion, etc. [60].

McGehee [62] noticed that on a periodic orbit near collinear libration points in CR3BP there exist homoclinic connections. Later, Llibre et al. [63] studied the transversal of invariant manifolds of planar Lyapunov orbits near L_1 point, and achieved the mathematical proof of homoclinic connections, which had not been completed by McGehee. Koon et al. [64] investigated how to solve the homoclinic orbits by mapping invariant manifolds onto a Poincaré section, and numerically proved the existence of homoclinic orbits based on all intersection points of mapped closed curves. The homoclinic connections can be derived from Koon's results not only in the interior region, but also in the exterior region. Koon's results successfully explain the unusual trajectories of Oterma orbiting Jupiter [65]. Attracted by the same problem, Howell et al. [66] utilized a numerical approach to explain the temporary capture of Oterma by Jupiter. Koon employed the analogous method to study the heteroclinic orbit in CR3BP and showed that there might exist (1,1)-, (2,2)-heteroclinic connections for periodic orbits near L_1 and L_2 points of the same Jacobi energy [64]. By computer-aided proof technique, Wilczak et al. [67] and Kirchgraber et al. [68] verified the conclusion that Koon et al. drew about transversal homoclinic/heteroclinic connections, respectively. Gidea and Masdemont [69] followed Koon's thought to research the Sitnikov problem when the mapped invariant manifolds on the Poincaré section intersect with each other many times and employed symbolic dynamics to classify the homoclinic orbits based on bi-infinite sequence. Gómez et al. [70] discovered more complicated homoclinic/heteroclinic connections as follows: the heteroclinic connections between Lissajous orbit near the L_1 point and quasi-Halo orbits near the L_2 point, the homoclinic connections of center manifolds between Lissajous orbits and quasi-Halo orbits near the same libration point.

Besides the transfers from Earth to the Halo orbits, the research on this issue also includes transfers between different LPOs. Gómez et al. [71] investigated the transfer problem between two Halo orbits of the same family, and gave the first- and second-order approximations of the transfer trajectories based on the phase space structure near LPOs. However, Howell et al. [72] selected a Lissajous orbit as the transfer trajectory connecting different Halo orbits, but they consumed 9% more fuel than Gómez et al. Analogous to Gómez, Cobos and Masdemont [73] studied the transfers between Lissajous orbits of the same family, but only gave the first-order approximation of the transfer trajectories, which essentially are linear motions near libration points. For transfers between Halo orbits of different families, the theoretical foundation is the heteroclinic connections between them [74,75]. During the journey back to Earth from the Sun-Earth EL_1 point, the Genesis spacecraft was guided to fly by the LL_2 point, which can be regarded as a successful example employing heteroclinic connections as zero-consumption transfer trajectory [76]. Similarly, lunar sample return missions have adopted such transfer trajectories as well [77].

Therefore, according to the conclusions of Koon [64], Gómez [45] et al., space manifold dynamics (SMD) theory is referred to as nonlinear methods or techniques developed in CR3BP, including the phase space structure near collinear libration points, generating algorithms of bounded orbits (periodic or quasi-periodic orbits), center/stable/unstable manifolds near LPOs as well as homoclinic/heteroclinic connections between them.

2.4. Station-keeping strategies of Halo orbits

As is well known, Halo orbits near libration point are unstable, thus orbital station-keeping maneuvers are necessary to maintain a spacecraft on them. Ever since libration point missions were proposed, the station-keeping problem of Halo orbits has always been a research hotspot. So far, all the station-keeping strategies of Halo orbits can be categorized as Target Point mode and Floquet mode approaches [78]. The Target Point mode strategies lead the spacecraft to the nominal trajectory by the feedback of position deviation from the nominal one. The Floquet mode strategies aim at keeping a spacecraft away from the possible deviation caused by unstable manifolds so that the spacecraft can be guided onto the nominal trajectory [78].

The Floquet mode is based on the Floquet theory and the dynamical structure of invariant manifolds requiring less fuel consumption, but it is not welcome among engineers due to its complicated mathematical formula. In accordance with the general structure of control systems, the Target Point mode can be easily used together with various advanced control theories and is therefore preferred by engineers.

The Floquet mode was put forward by Gómez et al. and initiated by the SOHO mission. The Target Point mode was proposed by Farquhar [31] and Breakwell [79] and later Howell et al. [80] proposed a linear control strategy balancing the control precision and fuel consumption. Giamberardino [81] et al. devised an asymptotically stable nonlinear controller, tracking a nominal Halo orbit by perturbation compensation. Cielaszyk et al. [82] put forward numerical iterative algorithms to generate Halo orbits based on the idea of the LQR method, which also works as a station-keeping controller. Dunham and Roberts [83] elaborated station-keeping strategies for practical libration missions such as ISEE-3, SOHO, ACE, etc. Infeld et al. [84] treated station-keeping for Halo orbits as a two-point boundary problem (TPBP) and solved it by the sequential quadratic programming method. Rahmani et al. [85] solved the same problem by the variation method in the optimal control theory. Kulkarni et al. [86] achieved the asymptotic stability control for Halo orbits utilizing the H_∞ theory, and extended the conclusions to other fields such as formation flying control near LPOs. To derive a suboptimal station-keeping strategy of Halo orbits, Xin et al. [87] put forward the approximation solutions to the difficult HJB equations in the optimal control problem, which is called the θ-D method. Although this method cannot yield the optimal solutions, it is promising for station-keeping the Halo orbits because of its high real-time capability. Wong et al. [88] stabilized a spacecraft near the Sun-Earth EL_2 point by an adaptive learning controller as well as filter and then proved the stability of the controller. Xu et al. [89] applied the back-stepping procedure to deriving an adaptive robust Halo-orbit station-keeping controller with globally asymptotical stability. Enlightened by the idea of linear feedback, Lin et al. [90] proposed two station-keeping strategies of Halo orbits near the EL_1 point for the impulse and the low-thrust engines, respectively. Taking advantage of the periodicity of Halo orbits, Xu et al. [91] designed a linear periodic strategy in order to stabilize the Halo orbits near the EL_1 point.

The attitude controller system makes the spacecraft point at an expected orientation by exchanging angular momentum with some actuators such as a flywheel or a gyroscope. Analogously, the tether-based controller can be used for station-keeping by the exchange of momentum with the bodies connected by the tether. The first research on station-keeping on a libration point by the tether system was presented in Farquhar's doctoral dissertation [92], where a spacecraft at the EL_2 point is stabilized by adjusting the tether's length, and the PD control law was verified to be an effective way to maintain stabilization. However, he only discussed the feasibility of a specific tether system without detailed simulations, which were later accomplished by Misra [93]. Wong et al. [94] allocated a tether system in a spoke-shaped configuration near the EL_2 point, trying to motivate NASA's SPECS mission, and constructed feedback controllers such as an LQR one to stabilize the system.

2.5. Attitude description of libration point orbits

A spacecraft moving in multi-body gravity fields (MBGF) necessitates the investigation of not only its barycenter's motion, but also its attitude orientation. The former is known as the amply documented restricted multi-body problem. In contrast research on the latter problem is limited mostly to discussing the attitude stability of a rigid body in MBGF, plotting unstable regions, solving the equilibrium configuration of coupled orbit-attitude dynamics in MBGF, and detecting the local stability of the configurations.

Kane et al. [95] studied the attitude dynamics of an axisymmetric spinning spacecraft, and numerically sketched its unstable regions when it is placed at the five libration points. Robinson [96] explained the attitude dynamics of a dumbbell-shaped spacecraft located at a triangular libration point, and then investigated the attitude stability of an arbitrarily shaped spacecraft at five libration points. In order to investigate the stability of the axisymmetric spinning spacecraft moving on two stable periodic orbits in the planar CR3BP, Hitzl et al. [97] plotted the unstable regions by Floquet theory, derived the approximation of these regions analytically using the averaging method, and revealed the relationship between coupled orbit-attitude resonance and attitude stability. Michalakis et al. [98] discussed the coupled orbit-attitude motion of two hinge–connected rigid bodies in a two-body gravity field (referred to as a restricted $2+2$ rigid body problem), and derived several equilibrium configurations for this problem. Kalvouridis et al. [99] obtained the equilibrium configurations of coupled orbit-attitude motion for a three-axis stabilized spacecraft in MBGF, which are distributed symmetrically in a circle (referred to as the restricted $N+1$ rigid body problem) and analyzed the local stability of such configurations. Tsogas et al. [100] considered a spacecraft equipped with a gyroscope in MBGF, and derived the equilibrium configurations in this scenario. Mavraganis et al. [101] studied the coupled orbit-attitude motion of N hinged-connected bodies in two-body gravity fields (referred to as restricted $2+N$ rigid body problem), and obtained its equilibrium configurations. In all the aforementioned researches, the spacecraft is located on libration points or on stable periodic orbits in planar CR3BP. Wong et al. [102] studied the attitude of the spacecraft moving on a spatial Halo orbit.

The aforementioned work involves the coupled orbit-attitude dynamics, which is of great importance in theoretical research. Given a spacecraft moving near the EL_1 point in the Sun–Earth system, the angular acceleration generated by the Sun's and the

Earth's gravitational moment is only of the order $10^{-28}(°)/s^2$ [102]. Besides, as the Sun-spacecraft distance decreases, the moment of solar radiation pressure becomes the main perturbation factor. For a specific sensor (Sun sensor, Earth sensor and star sensor) assembled on a three-axis stabilized spacecraft, Xu et al. [103] provided two different schemes to define the orbital coordinate frame for it.

2.6. Constellation design of libration point orbits

Since the 1960s, many experts have studied the design of the communication constellation using different types of orbits meeting specific coverage requirements. For example, three spacecraft are allocated equal-spaced on GEO to achieve a full coverage over Earth's surface, and Walker designed a Walker-δ constellation to meet the requirements of multiple coverges over specific ground points.

The classical work of constellation design is constrained within the Kepler orbits near Earth in the context of two-body dynamics. The current trend is to devise constellations in the framework of the three-body model in order to meet the demands of interplanetary exploration. Bhasin and Hayden [104] suggested that communication in deep space can be accomplished through LPO constellation, which supports NASA's future space exploration and science missions. Klkarni et al. [105] studied the communication from/to Moon, Mars and beyond, and furtherly proposed a deep space communication network consisting of spacecraft on Halo orbits in the Earth–Moon system, the Sun–Earth system, the Sun–Mars system, respectively, as well as two spacecraft on the heliocentric orbits near Earth. Lee et al. [106] investigated the communication network only in the Earth–Moon system, and suggested to construct a triangle configuration by allocating spacecraft on the LL_3, LL_4, LL_5 points as a replacement of the geostationary orbit to achieve global coverage. However, he did not provide the feasibility and coverage analysis of this constellation. Its altitude is so high that the disadvantages, such as high launch costs, severe communication delay, and high communication power, cannot be avoided if it is employed as communication infrastructure [106]. Nevertheless, this constellation contributes to dealing with the limited available places of GEO, which draws more attention from astronautical experts. Xu et al. [107] proposed a modified constellation in cislunar space with four spacecraft, respectively, three of which are placed at LL_3, LL_4, LL_5 points to achieve Earth coverage, and one on Halo or Lissajous orbit near the LL_2 point to accomplish Moon coverage with the aid of those at the LL_4 and LL_5 points, and then analyzed the station-keeping cost, communication and launch consumption. Xu et al. [108] investigated the tracking laws relative to Earth and Moon of spacecraft on the Lissajous orbits near the LL_2 point in the constellation, and evaluated the opportunities of being out of the lunar shadow in tracking Earth. In addition, they analyzed the relationship between the amplitude and phase of the Lissajous orbits and the tracking law, established a schedule for tracking relay to avoid communication loss during lunar shadow, and studied the influence of the antenna beam angle on the requirements for pointing accuracy by tracking laws.

3. Surveys on application of libration point orbits

Despite the complicated SMD theory of LPOs, all the nonlinear dynamical behaviors have amazing applications for practical orbital design, such as the phase space structure to generate bounded orbits, or stable/unstable manifolds to construct zero-cost transfer trajectories. Furthermore, the research on LPOs has been extended to many astronomical problems, such as the weak stability boundary (WSB) low-energy cislunar trajectory employed by the Hiten spacecraft, and resonant transit orbits followed by the comet Oterma from the exterior region to the interior region of the Sun-Jupiter system. Hence, the methods developed in LPOs have potential applications in other fields of orbital dynamics to overcome the drawback of traditional techniques.

3.1. Application of libration point orbits into cislunar transfers

The investigators of the cislunar transfers in the two-body problem believed that the hyperbolic velocity was the necessary condition for escaping from Earth's gravity. However, when studying it in the context of the three-body problem, the hyperbolic velocity is no longer essential for cislunar transfers. Compared with Hohmann transfers, the ballistic capture in the three-body problem, known as low-energy transfer trajectories, is characterized by reducing energy consumption but longer time of flight [109].

In 1968, Conley [110] investigated the phase space structure near libration points, and categorized the motions as: (1) periodic orbits, (2) stable/unstable manifolds of periodic orbits, (3) transit orbits and (4) non-transit orbits. Additionally, he thought that the invariant manifolds of a periodic orbit separate transit orbits from non-transit orbits, and the former can be used to construct low-energy cislunar transfer trajectories. The same conclusions were obtained by McGehee [62]. The stable/unstable manifolds of a periodic orbit can span a two-dimensional C–M tube, inside which the transit orbits are packed [63]. Recently, the research shows [111]: the tube will distort even under small perturbation, but most transit orbits still exist inside the tube, which guarantees that the concept of C–M tubes of CR3BP is still meaningful in other complicated models.

Ever since Conley utilized the LL_1 point to achieve low-energy cislunar transfers, a great number of experts have investigated such transfers. For instance, Bolt et al., [112] employed shooting methods in chaotic dynamics to obtain a low-energy cislunar transfer trajectory with the perilune altitude of $h_M =$ 12,232 km, perigee altitude of $h_E =$ 53,291 km, fuel consumption of $\Delta v = 750$ m/s and time of flight of $\Delta t = 748$ days. Schroer et al., [113] improved the shooting methods, and obtained the transfer trajectories, of which the fuel consumption is similar to the result of Bolt et al., but the time of flight was reduced by half i.e. $\Delta t = 377.5$ days. Macau [114] obtained a similar trajectory, whose fuel consumption was slightly more than in the work of Schroer et al., but the time of flight was significantly reduced ($\Delta v = 767$ m/s, $\Delta t = 284$ days). By optimization techniques, Ross et al. [115] obtained new results $\Delta t = 860$ m/s, $\Delta t = 65$ days. Solving the Lambert equation in CR3BP, Topputo et al. [116] transformed the cislunar transfer problem into TPBPs obtaining similar results [115]. Xu et al. [117] focused on the conditions of ballistic capture concluding that the transition through the LL_1 point are more suitable for constructing cislunar low-thrust trajectories, and then obtained semi-analytically a similar trajectory to SMART-1, which avoids heavy computations compared with the numerical optimization carried out for the SMART-1 mission.

On the other hand, Belbruno proposed a so-called WSB transit trajectory through the Earth–Moon LL_2 point [118,119], which was utilized to rescue the lunar spacecraft Hiten in 1991. Belbruno indicated that permanent and ballistic captures are equivalent on a special set called WSB, which gave opportunities for translunar transfers into Moon-centered orbits from Earth-centered orbits [118]. Further investigations have been conducted by Circi [120], Yagasaki [121], Parker [122] and Gómez [123] among others.

Koon et al. studied extended C–M tubes in the restricted Sun–Earth–Moon–Spacecraft four body model, and noticed that the C–M tubes transiting through the LL_2 point intersected with the one through the EL_1 (or EL_2) point, which may result in lunar ballistic

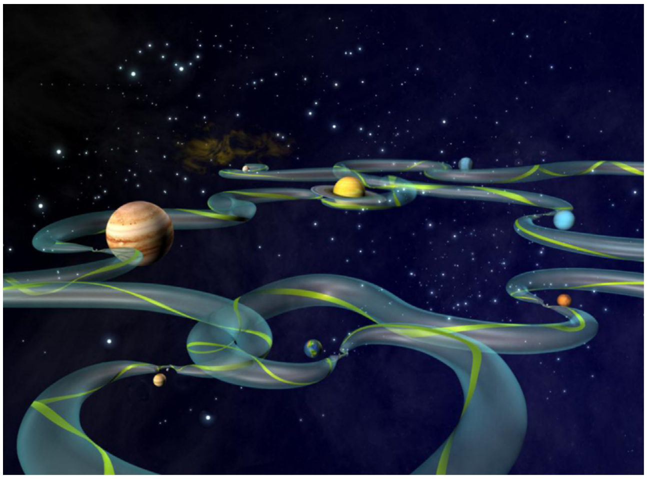

Fig. 3. Artist's drawing for InterPlanetary Superhighway [130].

capture mentioned by Belbruno [119]. Enlightened by the patched-conic methods constructing the direct transfer trajectories in the Apollo missions, Koon patched the invariant manifolds by decomposing the restricted four-body problem into two CR3BPs i.e. Sun–Earth–Spacecraft and Earth–Moon–Spacecraft systems and intersecting the aforementioned two C–M tubes through an appropriate Poincaré section. The patched-trajectory with continuous positions and velocities at intersection points is essentially the WSB-type transfer trajectory yielded by Belbruno [64]. Koon et al. explained WSB transfers by seemingly irrelevant C–M tubes in SMD, which is regarded as a great success of SMD theory. Not only has it solved the theoretical problem having troubled the astronomical society for decades, but also provides a new design method for low-energy WSB transfers.

Note that the success of Koon lies in exploiting the topological property of planar CR3BP. Because of four-dimensional phase space, three-dimensional energy space, two-dimensional invariant manifolds of Lyapunov orbit, the energy space can obviously be divided into the internal (corresponding to transit orbits) and external region (corresponding to non-transit orbits) by invariant manifolds. However, the problem is completely different in the spatial case, i.e. the dimension of the invariant manifolds of a Halo orbit remains two, while the energy space is five-dimensional. In other words, the two-dimensional invariant manifolds cannot partition five-dimensional energy space completely into interior and exterior regions. It is why the stability of the L_4 and L_5 points has been solved in the two-dimensional case, but not in the three-dimensional one. The KAM theory can be used to prove the planar stability of the L_4 and L_5 points due to two-dimensional KAM tori splitting up the three-dimensional energy space while KAM tori cannot partition five-dimensional energy space, which may cause Arnold diffusion in the spatial case [124]. Moreover, the patched manifolds utilized by Koon were only feasible from the viewpoint of kinetics. However, the patched manifolds in the sense of dynamics takes into consideration the solar gravity upon the manifolds of the LL_2 Lyapunov orbit as well as the lunar gravity upon the manifolds of the EL_1 (or EL_2) Lyapunov orbits. Therefore, a practical approach to yield the real transfer trajectory is to use the patched manifolds in kinetics as the iterative initialization and then correct it through shooting methods [125].

Meanwhile, Lo and Chung [77] developed an orbital design tool, named LTool, to design the round-trip trajectories for lunar sample return missions based on the relationship between WSB and the Halo or Lissajous orbits. Utilizing a special geometry that certain Lyapunov orbits are tangential to a distant retrograde orbit (DRO), Xu et al. [126] investigated two transfer orbits from Earth to a DRO, i.e. using the invariant manifolds of the LL_1 Lyapunov orbit to construct a fast transfer to a DRO, and employing the LL_2 Lyapunov orbits as the gateway of WSB transfer to build up a low-energy transfer to a DRO. Topputo et al. [127] patched the manifolds of the Halo orbits of the EL_2 point with those of the JL_2 point in the Sun–Jupiter system, with the ML_2 point in the Sun–Mars system, or with the VL_2 point in the Sun–Venus system, respectively, and obtained the interplanetary transfers between Earth and Jupiter, Earth and Mars, or Earth and Venus. Similarly to smooth heteroclinic connections between Halo orbits near the EL_1 and EL_2 points, a branch of invariant manifolds of a Halo orbit near the LL_2 point can be connected smoothly to the one near the EL_1 or EL_2 point by appropriate intersections. Howell et al. [128] applied both smooth heteroclinic and patched connections in the JPL ephemeris model and achieved transfer trajectories with little fuel consumption between the aforementioned Halo orbits. Thus the LL_1 point can act as a service station for future space missions [129], i.e., spacecraft working on Halo orbits near the EL_1 or EL_2 point can return back to the one near the LL_1 point to be maintained after breakdown. Serving as a springboard for interplanetary missions, the LL_1 point enables transfers to Mars, Venus, etc. with enlarged launch windows and decreased launch cost [130]. Meanwhile it is also a natural intermediate port in the journey to Moon. Summarizing the previous researches, Lo further conceived the C–M tubes of each planet in the solar system, proposed the “InterPlanetary Superhighway” (IPS) [131] presented in Fig. 3, and regarded the LL_1 point in the Earth-Moon system as the gateway of these low-energy transfers.

3.2. *Artificial libration point orbits by solar sails*

The libration points no longer exist in non-CR3BP models, however, some equivalent libration points can be defined geometrically. Howell et al. [132], Gómez et al. [133] studied the equivalent libration points to the EL_2 point in the Sun–Earth system under perturbations. Meanwhile Andreu [41] systematically investigated the equivalent libration points to collinear ones in the Earth–Moon system. For the restricted Sun–Earth–sail problem perturbed by solar radiation pressure, there exist equivalent libration points defined by both the spatial position and the orientation of the solar sail, which are called the artificial libration points in engineering [134]. Geometrically, the family of artificial libration points is an enveloping surface parameterized by the area-mass ratio β of the solar sail [135]. Originating from the generating conditions of Halo orbits, some periodic or quasi-periodic orbits appear near the artificial libration points as well. By classical libration point theory, McInnes A. [136] found Halo orbits near the libration points on the x-axis and subsequently Baoyin et al. [137] created two different kinds of Halo orbits by orienting the sail parallel to the Sun–Earth or Sun–sail direction, respectively. For the libration points off x-axis, McInnes A. [136] failed to generate Halo orbits near these points. To improve it, Waters et al. [138] used the irregular Monodromy matrix method to allocate Halo orbits in a specific region off x-axis near libration points.

A solar sail is a type of ideal spacecraft for LPOs and interplanetary exploration missions without fuel consumption [134]. Baoyin et al. [139] utilized solar radiation pressure to offset the nonlinear terms in the Sun–Earth–sail dynamics, located the solar sail on the periodic orbits derived only from the linear terms, and then investigated the stability and the controllability of the controlled periodic orbits. Lawrence et al. [140] studied the controllability of the artificial libration points by only two attitude angles, and employed the Gramian method to quantitatively estimate the upper boundary of allowable largest initial deviations. Treating a quasi-periodic Lissajous orbit as a nominal trajectory of a solar sail, Bookless et al. [141] derived an optimal station-keeping controller from the simplified first-order approximation of the nonlinear dynamics using the orientation and the area–mass ratio of the sail as control inputs. Furthermore, he also obtained the transfer trajectories based on the invariant manifolds of this Lissajous orbit. Ariadna Farrés et al. [142] investigated the station-keeping strategy of a solar sail employing the invariant manifolds of a libration point to build up a controller, and maintained the sail on a family of artificial libration points, rather than on a specific point. Xu et al. [143] utilized the invariant manifolds of hyperbolic equilibrium points to build up a structure-preserving controller, which can arbitrarily allocate poles along the imaginary axis in order to change the topology of artificial libration point from hyperbolic to elliptic so that stable Lissajous orbits near off-x-axis libration points can be generated.

For the transfers to Halo or Lissajous orbits near an artificial libration point, Bookless et al. [141] concluded that the transfer trajectories can also be constructed through invariant manifolds of the Lissajous orbits derived from the aforementioned first-order approximation dynamics. Qin et al. [144] devised a sail angle control strategy to guide the sail successfully into stable manifolds of the Halo orbit, which is generated by orienting the sail parallel to the Sun-sail direction.

The solar sail displaced orbits seem to be completely distinguished from the LPOs in CR3BP. However, Xu et al. [145] performed a series of coordinate transformations obtaining similar nonlinear dynamical behaviors between these two kinds of orbits. It was found that there are two equilibria for the displaced orbit system, one hyperbolic (saddle) and one elliptic (center), except for the degenerate and saddle-node bifurcation points. Then the necessary and sufficient conditions for stability of motion near the equilibria were determined by means of Birkhoff normal form and dynamical system techniques. Moreover, the critical KAM torus was proven to be filled with the (1,1)-homoclinic orbits of the Lyapunov orbits, which span the whole invariant manifolds of Lyapunov orbits as well.

3.3. Formation flying on libration point orbits

Formation flying has drawn much interest in recent years and many missions have taken advantage of spacecraft formation such as the GRACE mission (launched in 2002) [146] and the TerraSAR-X and TanDEM-X missions (launched in 2007 and 2010, respectively) [147]. The formation flying technologies can also be applied to LPOs in order to accomplish high-resolution deep space observation missions, such as NASA's planning missions MAXIM [148] and TPF [149], as well as the DARWIN mission developed by NASA and ESA together [150]. The formation can not only enhance the functions, but also is more advanced than a single spacecraft in terms of launch and update. Howell et al. [151] utilized Halo and Lissajous orbits to accomplish formation flying and classified the formation types as natural and non-natural ones. The latter requires orbital maneuvers to force a spacecraft stay on an artificial but nearly natural trajectory near a Halo orbit, while the former is aiming to place the formation spacecraft on the invariant tori of a Halo orbit. Apparently, the natural formation takes full advantage of the phase space structure near libration points with extra fuel consumption, but the formation is constrained on two-dimensional manifolds due to the geometric shape of the invariant tori [152]. The non-natural formation can achieve an arbitrary configuration at the expense of fuel consumption in station-keeping, which is in general acceptable for missions because of few perturbations existing near libration points. Scheeres et al. [153] investigated the station-keeping control problem of non-natural formation, and constructed a Hamiltonian-structure preserving controller by feed-backing the stable/unstable manifolds of LPOs, which can keep the formation spacecraft flying in a relative configuration near a Halo orbit. Grufil et al. [154] employed the LQR method and adaptive neuron control to perform formation station-keeping on a Halo orbit. Gong et al. [155] investigated double-impulse orbital maneuvers to achieve the reconfiguration of spacecraft formation near a Halo orbit. Aiming at station-keeping the relative orientation of spacecraft formation flying in CR3BP, Qi et al. [156] proposed an efficient two-level differential correction algorithm, namely the tangent targeting method to control the Chief/Deputy architecture to maintain a prespecified orientation. Li et al. [157] investigated the optimal control of loose leader-following spacecraft formations near libration points and developed an effective penalty function to avoid collision of follower spacecraft as well as a symplectic penalty iteration to obtain the optimal solutions with minimization of energy cost. Héritier et al. [158] explored the natural regions near quasi-periodic Lissajous trajectories near the EL_2 point and determined where the large-scale formation can be maintained. Cai et al. [159] studied the tethered formation system near libration points based on Hill's approximation and checked numerically the dynamical stability of this system. Peng et al. [160] proposed an optimal periodic station-keeping controller to stabilize the formation spacecraft along a Halo orbit in the Sun–Earth system by solving the periodic Riccati differential equation numerically. Taking into account the perturbations from the Moon's eccentricity and Sun's gravity, Salazar et al. [161] analyzed the best regions to place a formation flying close to a bounded solution near the LL_4 point, and designed a fuel-optimal controller for formation maintenance. Infeld et al. [162] investigated the concurrent problem of orbit design and formation control near a libration point. Contrary to quadratic measure costs, the L^1 norm of the control acceleration was used to measure fuel consumption. The Legendre pseudospectral method was employed to generate quick solutions facilitating redesign for the early stages of formation design. This approach did not use linearizations in modeling the dynamics nor analytical results, but it was presented for both a large separation constraint (about a third to half of orbit size), and a small separation constraint (about a millionth of orbit size).

Furthermore, the relative motions on low-Earth orbits (LEO) are regarded as a limitation of CR3BP, where the secondary body's mass is zero [163], i.e., the libration points degenerating into a chief spacecraft in formation, and the Halo orbits degenerating into a relative configuration. Hence, some conclusions developed in LEO formation flying also are extended to LPOs. Actually, many experts from astronomical and astronautical societies have made great contributions to both aforementioned fields. For example, Alfriend et al. [164] conducted both researches on the stability of libration points as well as LEO formation flying, Richardson [165] derived third-order expansions for both Halo orbits and LEO formation flying, and Gómez et al. [166] applied their Fourier algorithm to higher-order expansion of Lissajous orbits and LEO formation flying.

4. Future research directions

Based on the systematical summaries on the past developments and current state-of-art of LPOs, the final section addresses the possible future research directions as follows.

Despite the rich implemented or planned missions employing the periodic Halo orbits, the rigorous proof of the existence of this type of orbits has not yet been certified theoretically. For the planar case, the Lyapunov center theorem guarantees the existence of Lyapunov orbits, but it is not applicable for Halo orbits bifurcated from them. It is not only a dream for academic scholars to achieve the mathematical proof, but also necessary for aerospace engineers to examine whether the Halo orbit is suitable as a nominal working orbit for potential missions. Recently, a very useful approach, the variational method was developed to demonstrate the existences of some periodic orbits by searching for the critical point (or minima) of the chosen Lagrangian action. A famous result of this approach was to derive the existence theorem of a periodic eight-shaped solution of the three-body problem in the case of equal masses in 2000 [167]. A closer proposal to the existence condition was reached by Ambrosetti and Rabinowitz (named as Mountain Pass Theorem) [168], which deduced the criteria for minima existence as the Palais-Smale condition and three others. Thus, a mathematical proof on the existence of Halo orbits is expected to be achieved by the minimization of action referred above or other advanced mathematical tools.

Although some helpful tools are utilized to investigate LPOs, such as Poincaré mapping to illustrate the quasi- and periodic orbits, the KAM theory to determine the stability of triangular points (i.e., L_4 and L_5 points), other methodologies or techniques developed by nonlinear science theory also have potential applications to solving specific problems. Recently, a successful application was to introduce the Lagrangian coherent structure (LCS) originating from fluid dynamics to describe the phase space structure for the elliptic restricted three-body problem (ER3BP). Previous approaches were very weak in dealing with the time-periodic terms in ER3BP; while the newly introduced LCS can overcome their drawbacks by adopting Finite Time Lyapunov Exponents as an indicator [169]. Besides, other advanced methods in the theories of dynamical systems, nonlinear dynamics and chaos, and others, such as symbolic dynamics and Melnikov method, contribute to the researches on chaotic behaviors of LPOs. For example, a subset of low-energy homoclinic trajectories connecting different LPOs are discovered by the technique of symbolic dynamics [15], and the chaos in the sense of Smale horseshoes is indicated by the Melnikov function. Although it has been shown that some authors have already begun considering employing these methods, the further investigation of LPOs is in its infancy. Thus increasing the knowledge base in this area and further investigating and employing these advanced theories will be important in revealing, illustrating, and understanding the dynamics and control issues of LPOs.

Another research direction is to extend the methodologies developed in LPOs to other fields of astrodynamics, such as the topics of bounded trajectories around asteroids [170], tethered-satellite formation (TSF) [171], displaced orbits above a planet [172], and relative motions on a J_2-perturbed orbit [173]. Most of them obey the equations of motion as $\ddot{x} + 2n\mathbf{J}\dot{\boldsymbol{x}} = \partial U/\partial x$ where $\boldsymbol{x}$ is the variable (e.g., the position vector for CR3BP or the angle between tethers for TSF), $\mathbf{J}$ the symplectic matrix, n the factor measuring the Coriolis term, and U the effective potential function. Thus, one potential research direction of the extended works deals with U in different mathematical formulations, such as the perturbed potential of asteroids [170], the Szebehely–Giacaglia potential for formation flying [174], and the solar sail's potential in the reduced space [175]. By reducing the center manifolds to Birkhoff normal form and then mapping them onto Poincaré sections, all the bounded (periodic or quasi-periodic) trajectories near equilibrium points in all the dynamical models can be categorized by the aforementioned dynamical structure, just as was done for CR3BP shown in Fig. 2. The transit behaviors in neck regions are expected to be investigated for other topics with the help of homoclinic/heteroclinic connections to invariant manifolds. Besides these common conclusions, some specific ones may be drawn as U varies. For example, in the gravity field of a contact binary asteroid, the collinear libration point L_1 located between the primary and secondary bodies will disappear so that there exist only four libration points (L_2, L_3, L_4, L_5 points) of the same type with those in CR3BP. Furthermore, the parameter n that emerged in the dynamical structure can be considered as another extended direction as well. For example, compared with n set to be one in CR3BP, it is regarded as a controllable variable in the theory of Hamiltonian-structure preserving stabilization [176], and the relationship between n and transit behaviors in neck regions is investigated for the displaced orbits. In summary, potential extensions to other related fields are expected to achieve some innovative results beyond their traditional results.

The most appealing work related to LPOs is the design of an orbit to generate some potential missions based on deep understanding of the mechanism of LPO dynamics and control, such as the initial and extended missions of ISEE-3 following the Halo orbit and its transfer trajectories to comets calculated by Farquhar et al. [5], the sample-return mission Genesis with its amazing heteroclinic-based reentry trajectory. Thus, some future missions may be motivated by LPOs, such as the Halo-type orbits around the collinear points of a dumbbell-shape body acting as the mission orbit for asteroid exploration, a crazy mission capturing a mineral asteroid by driving it along the patched manifolds from the exterior region into cislunar space, and a deep-space distributed telescope mission allocating formation spacecraft on LPOs. Moreover, the onboard parts of LPO spacecraft demand constraints on flight trajectories for orbit designers, such as a new orbital coordinate system in the LPO region required by the attitude and determination control system, the thermal control schemes for LPOs in the Sun-Earth or Earth-Moon system that are similar to Sun-synchronous orbit (SSO) and GEO, respectively. Furthermore, different from uncontrollable celestial bodies, the engine firings can generate a series of intricate orbits to meet some unprecedented missions. The latest advances in modern control theories, such as Hamiltonian-structure-preserving control for station-keeping or formation flying, optimal stochastic control for trajectory transfer, are believed to be quite beneficial to the selection of valuable maneuver strategies from the viewpoints of minimizing the fuel or time or their combination.

5. Conclusion

Due to their potential for useful applications in deep-space exploration, LPOs have recently drawn much attention in academic and industrial circles. A review of the technical developments in LPOs has been presented, including the theoretical advances in the phase space structure near libration points and homoclinic/heteroclinic connections, the technical progress in design methodologies of transfer trajectories from/to Halo orbits and station-keeping strategies, and some applications of LPO constellation and formation flying.

Furthermore, some potential research directions are addressed from the aspects of the mathematical verification of Halo orbits in CR3BP, applications of the advanced methods in the theories of dynamical systems, nonlinear dynamics and chaos, and others into the field of LPOs, the extension of the methodologies developed in LPOs to other fields of astrodynamics, and some practical requirements for proposed LPO missions.

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