

Input Slides 2024-12: My Two Slides

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Overview

1 Writ Large

2 Radiation

3 Results

Sealing the Mesh

Boo

Fourier and Extensions to 2- and 3-D

$e^{in\theta}$ From periodic functions in 1D to radial and angular
decompositions in 2D and 3D.

Why We Love Fourier for Smooth Functions

Smooth Functions, Beautiful Representations

- **Weierstrass Approximation Theorem:** Any continuous function on $[a, b]$ can be uniformly approximated by polynomials. Fourier provides a similar approximation, using trigonometric bases instead of polynomials.
- **Riesz-Fischer Theorem:** Fourier coefficients (a_n, b_n) belong to l^2 space, guaranteeing convergence in the L^2 sense. This bridges the gap between smoothness and square-integrability.
- **Uniform Convergence for Smooth Periodic Functions:** For sufficiently smooth functions (C^∞ or C^k), Fourier series converge uniformly, ensuring no oscillatory artifacts (Gibbs phenomenon disappears).

Fourier and Extensions to 2- and 3D

1D:

$$f(\theta) = \sum_{n=-\infty}^{\infty} a_m e^{in\theta}$$

2D:

$$f(r, \theta) = \sum_{n=1}^{\infty} \sum_{m=-n}^n R_n^m(r) f_{\text{fourier}}$$

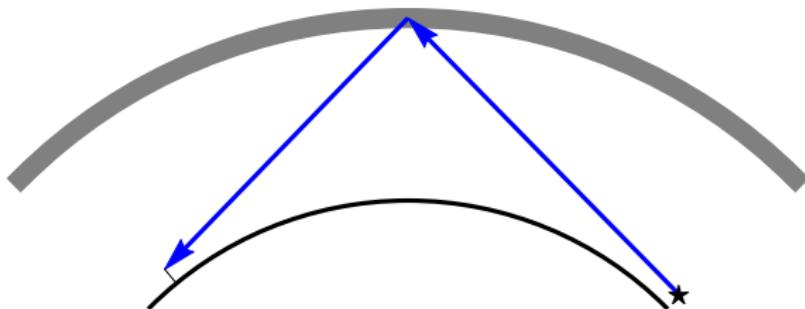
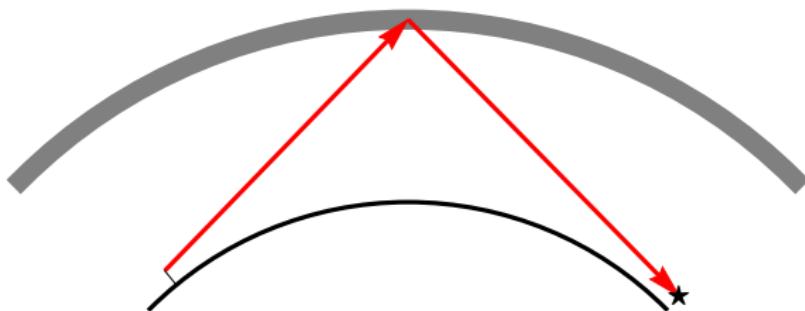
3D:

$$f(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^n \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta) e^{in\theta}$$

Fourier and Extensions to 2- and 3D

- **1D Fourier Series:** Decomposes a periodic function $f(\theta)$ into a sum of complex exponentials with coefficients a_n capturing the amplitudes of each frequency component.
- **2D Fourier-Bessel:** Extends Fourier analysis to two dimensions using radial functions $R_n^m(r)$, often employed in circular domains or optical applications.
- **3D Spherical Harmonics:** Represents functions on a sphere using harmonics $Y_l^m(\theta, \phi)$ and radial components $R_l(r)$, crucial in fields like quantum mechanics and gravitational modeling.

Energy Out, Energy In



Energy Out, Energy In

- ① radar (left) **irradiates** target (star)
- ② **backscatter** travels from target to radar

Radar Cross Section: Definition

$$\sigma_* = 4\pi \lim_{r \rightarrow \infty} r^2 \left| \frac{E_{\text{incident}}}{E_{\text{scattered}}} \right|^2 \quad (2.1)$$

Skolnik 1962, (2.36)

Radar Cross Section: Discussion

- Radar cross section is a **far field** phenomenon
- Assumes **single polarization** to and from target
- Target is **completely metallic**:
 E field results from surface currents
- Shape is **quasi-dimensional**
 - Dimensions in two known directions
 - Dish antennae, solar panels, booms
- **Resonant scattering**:
Ratio of typical dimension to wavelength ≈ 1
- **Kolosov 1987, §4.6**

Radar Cross Section: Conceptual Overview

- **Radar Basics:**
 - Transmit energy Skolnik 1962, p. 21.
 - Receive scattered signal.
 - Direction, strength → object properties Knott, Schaeffer, and Tulley 2004, p. 45.
- **What is RCS?**
 - Measures object "visibility" to radar Lab 2002, Section 2.
 - Depends on:
 - Material
 - Geometry
 - Orientation Peebles 2007, pp. 3-4.
- **Key Question:** Power reflected vs. power transmitted.

Factors Influencing Radar Cross Section

- **Shape:**
 - Smooth → directional reflection Knott, Schaeffer, and Tulley 2004, p. 47.
 - Complex → scattered energy.
- **Material:**
 - Metal → strong reflection.
 - Absorbers → reduced RCS Knott, Schaeffer, and Tulley 2004, Section 3.2.
- **Size vs. Wavelength:**
 - Large → high scattering.
 - Small → "invisible" (Rayleigh scattering) Kolosov 1987, p. 188.
- **Orientation:**
 - Aligned → max RCS Knott, Schaeffer, and Tulley 2004,



Input files

① B-20.geo

- ① Points to facet file
- ② Configure linear algebra solver
- ③ Radar frequency range
- ④ Angular sampling ranges
- ⑤ Boundary conditions
- ⑥ Mono- or Bistatic
- ⑦ Surface or Volume integral elements
- ⑧ Length units

② B-20.facet

- ① Vertex list

Linear algebra (don't alter)

```
&MM_MOM
bUseACA = .TRUE.,
bSolve_ACA = .TRUE.,
bOutOfCore = .TRUE.,
bNormalizeToWaveLength = .FALSE.,
bNormalize = .FALSE.,
dCloseLambda = 0.100000,
ACA_Factor_Tol = 0.000010,
ACA_RHS_Tol = 0.000100,
Point_Tolerance = 0.001000,
nLargestBlockSize = -1,
MemorySize_GB = -1.000000,
stackSize_GB = -1.000000,
nFillThreads = -1,
nFillMKLThreads = 1,
nLUThreads = -1,
nLUMKLThreads = 1,
nRHSThreads = 1,
nRHSMKLThreads = 1,
bOutputACAGrouping = .FALSE.,
bOutputRankFraction = .FALSE.,
bLimitLUColumns = .FALSE.,
Lop_Admissibility = WEAK,
Kop_Admissibility = CLOSE
```

Memory management (don't alter)

```
&Scratch_Memory
  Scratch_RankFraction_Z = 0.300000,
  Scratch_RankFraction_LU = 0.600000,
  Scratch_RankFraction_RHS = 2.000000,
  Scratch_RankFraction_Solve = 1.000000,
  MemoryFraction_Z = 0.950000,
  MemoryFraction_Scratch_LU = 0.500000,
  MemoryFraction_LU = 1.000000,
  MemoryFraction_RHS = 0.500000,
  MemoryFraction_Solve = 0.900000,
```

Quadrature (don't alter)

```
&QUADRATURE
NTRISELF = 7,
NTRINEAR = 3,
NTRIFAR = 3,
NTETSELF = 11,
NTETNEAR = 4,
NTETFAR = 4,
NQGAUSS = 4
```

Radar frequencies

FREQUENCY

ghz

0.003000 0.030000 28 !Freq Start, Freq Stop, Num Frequencies

Sampling

Angle Cut

1

0.000000 359.000000 360

AZIMUTH

90.000000

Monostatic or bistatic

Excitation
MONOSTATIC

Boundary Conditions

Boundary Conditions

B-20-Materials.lib

4

V_FREE_SPACE => Free_Space

V_PEC => PEC

V_PMC => PMC

V_NULL => NULL

1

0 BC_PEC V_FREE_SPACE

Final settings

SIE	surface integral elements
B-20A.facet	CAD description
m	meters

Mercury MoM is Single Precision

Example: 8 MHz
Despite exact binary representation

$$8_{10} = 1000_2$$

Start Frequency = 7.9999994E-03GHz

Run sequence - launch

```
$./MMoM.4.1.12 b20.geo
-----
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
----- Reporting output in MB from Linux command: vmstat -s -S M -----
53113 M free memory
```

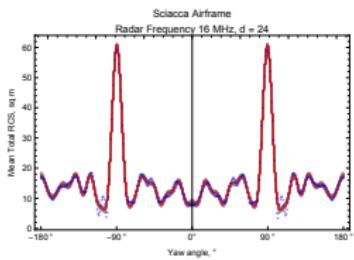
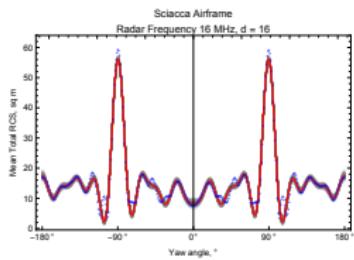
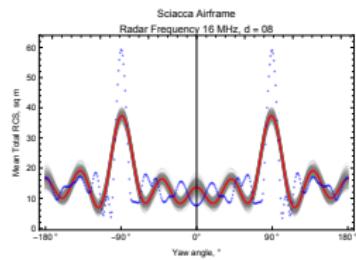
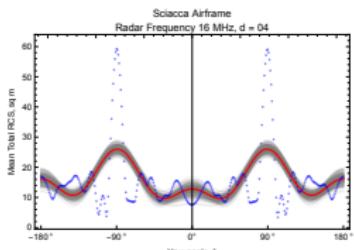
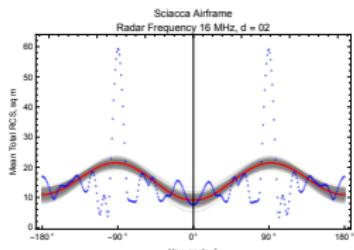
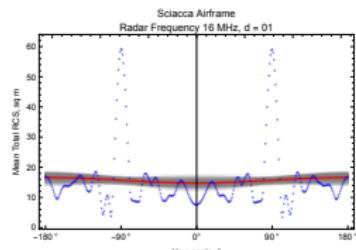
Run sequence - sample output

```
Freq    = 30.00E+00 MHz
Lambda = 9.99E+00 m
k       = 628.75E-03 m-1
subroutine Solve_SetUp( Surface, bk, pSys, pD, Nodes ) : ...Finished
-----
---| Time : Time total for RHS solve
---| Twall = 0.0004168 ; Tcpu = 0.0002319 ; Ratio = 1.80
-----
---| Out Of Core Times: Diagonal Blocks
---|
---| nWrites.....: 2.
---| GigaBytes Write.....: 0.
---| Write Time (Hr).....: 0.00
---| Average Write Rate (MBytes/sec)..: 19.
---| nReads.....: 5.
---| GigaBytes Read.....: 0.
---| Read Time (Hr).....: 0.0002
---| Average Read Rate (MBytes/sec)..: 48.
---|
-----
Z Column Summary IO 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
```

Run sequence - completion

```
$./MMoM.4.1.12 b20.geo
-----
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
----- Reporting output in MB from Linux command: vmstat -s -S M -----
53113 M free memory
```

Fourier Transform Visualizations at 16 MHz



Note

Blue: Data

Red: Approximation

Gray: Error

Fourier Transform Visualizations at 16 MHz

- **Approximation Order (d):** The number of terms in the Fourier approximation increases as $d = 1, 2, 4, 8, 16, 24$.
- **Low-Order Approximations:** Approximations with smaller d (e.g., $d = 1, 2, 4$) fail to capture fine structures, leading to significant residual error.
- **Higher-Order Approximations:** As d grows, the approximation better resolves finer features, and the error (gray) shrinks significantly, especially at smooth regions.
- **Error Behavior:** The error decreases non-uniformly—large errors persist near abrupt changes or peaks due to Gibbs phenomena, but smooth regions converge faster.
- **Key Insight:** Fourier approximations demonstrate trade-offs: computational complexity increases with d , but fidelity improves.
- **General Notes:**

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Fourier Transform Visualizations at 16 MHz

- **Key Insight:** Fourier approximations demonstrate trade-offs: fidelity improves with d , as does computational cost.
- **General Notes:**
 - Fourier series resolve functions as sums of sines and cosines.
 - Low-frequency terms: broad trends; high-frequency: fine details.
 - Convergence is faster for smooth functions but slower for discontinuities or sharp changes.
 - More terms improves fidelity, but can introduce numerical artifacts.

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7e879521f7f8/content](https://calhoun.nps.edu/server/api/core/bitstreams/9e69ec48-4628-4243-9f9b-7e879521f7f8/content).
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