Radar Cross Section Models for AFCAP Dashboard: Rapid Report 2020-02: Corrected

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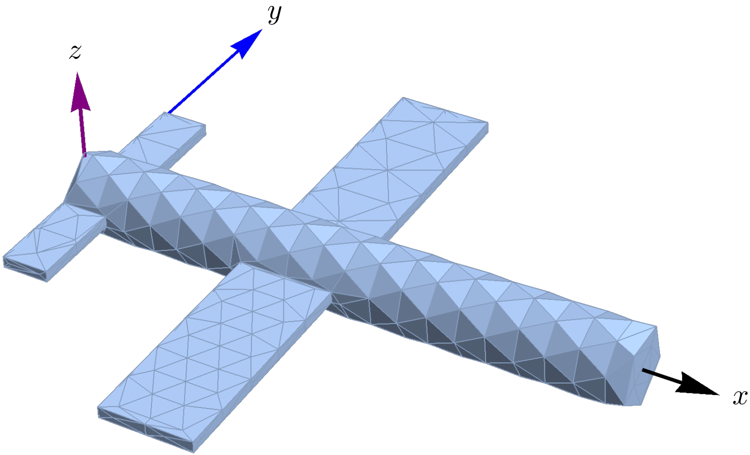
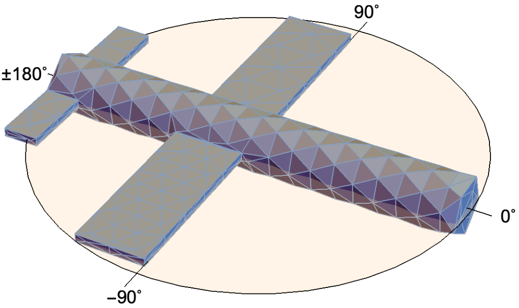
Scope - Sciacca Reformulation

1. CAD Model
2. Notations
3. Radar Cross Section
4. Polarization States

CAD Airframe: Coordinate System



CAD Airframe: Nose Angle



MoM Input Files

1. geometry file: `*.geo` which points to:
2. mesh file: `*.facet`
3. material properties: `*.lib`, `*.inhomo`

Sample Output: `PTW.4112.txt`

```
  Freq = 3.00E+00 MHz
  Lambda = 99.93E+00 m
  k = 62.88E-03 m-1

BACKSCATTER RCS RESULTS .....

  Theta, Phi, Theta-Theta (complex efield), Phi-Theta (complex efield), ...
  90.000, 0.000,( -0.7852032E+00, 0.5591055E+00),( -0.1204033E-02, -0.1038521E-02),
  90.000, 1.000,( -0.7855634E+00, 0.5595894E+00),( -0.6291079E-03, -0.1558805E-03),
  90.000, 2.000,( -0.7866909E+00, 0.5610294E+00),( -0.5536337E-04, 0.7288436E-03),
  90.000, 3.000,( -0.7885820E+00, 0.5634323E+00),( 0.5208306E-03, 0.1614471E-02),
  90.000, 4.000,( -0.7912294E+00, 0.5668129E+00),( 0.1098664E-02, 0.2495263E-02),
  90.000, 5.000,( -0.7946265E+00, 0.5711926E+00),( 0.1677461E-02, 0.3367479E-02),
        .
        .
        .
  90.000, 355.000,( -0.7948560E+00, 0.5711109E+00),( -0.4070709E-02, -0.5369121E-02),
  90.000, 356.000,( -0.7914131E+00, 0.5667469E+00),( -0.3494872E-02, -0.4525486E-02),
  90.000, 357.000,( -0.7887191E+00, 0.5633824E+00),( -0.2919853E-02, -0.3667939E-02),
  90.000, 358.000,( -0.7867830E+00, 0.5609957E+00),( -0.2347211E-02, -0.2797580E-02),
  90.000, 359.000,( -0.7856095E+00, 0.5595730E+00),( -0.1773704E-02, -0.1923206E-02),
  90.000, 360.000,( -0.7852034E+00, 0.5591057E+00),( -0.1203678E-02, -0.1037296E-02),
```

Parsing field values

reader

```
In[∘]:= nAngles = 361;

In[∘]:= Do[
    myLine = census[[k]] + 1;
    fields = Table[
      str = StringSplit[strmList[[myLine + j]]];
      str = StringReplace[#, "," → ""] & /@ str;
      str = StringReplace[#, "(" → ""] & /@ str;
      str = StringReplace[#, ")" → ""] & /@ str;
      str = StringReplace[#, "{" → ""] & /@ str;
      str = StringReplace[#, "}" → ""] & /@ str;
      tbl = Read[StringToStream[#], Number] & /@ Flatten[str]
      , {j, nAngles}];
    AppendTo[θθ, fields[[All, 3]] + i fields[[All, 4]]];
    AppendTo[θϕ, fields[[All, 5]] + i fields[[All, 6]]];
    AppendTo[ϕθ, fields[[All, 7]] + i fields[[All, 8]]];
    AppendTo[ϕϕ, fields[[All, 9]] + i fields[[All, 10]]];
    , {k, m}];
```

Circular polarization – nomenclature

MoM computes four complex electric fields:

1. VV: vertical in – vertical out
2. VH: vertical in – horizontal out
3. HV: horizontal in – vertical out
4. HH: horizontal in – horizontal out

Sciacca uses *linear combinations* of these fields

Circular polarization states

$$
\begin{aligned}
RR &= \tfrac{1}{2}\left(V^*V - H^*V - i\left(V^*H + H^*V\right)\right) \\
RL &= \tfrac{1}{2}\left(V^*V + H^*V - i\left(V^*H - H^*V\right)\right) \\
LR &= \tfrac{1}{2}\left(V^*V + H^*V + i\left(V^*H - H^*V\right)\right) \\
LL &= \tfrac{1}{2}\left(V^*V - H^*V + i\left(V^*H + H^*V\right)\right)
\end{aligned}
\qquad (1)
$$

Circular polarization – matrix form $\mathbb{C}^4 \mapsto \mathbb{R}^4$

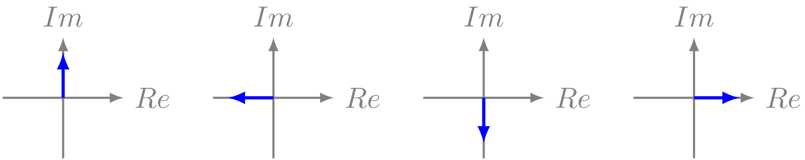
$$\begin{bmatrix} RR \\ RL \\ LR \\ LL \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -i & 1 \\ 1 & 1 & -i & -1 \\ 1 & 1 & i & -1 \\ 1 & -1 & i & 1 \end{bmatrix} \begin{bmatrix} V^*V \\ V^*H \\ H^*V \\ H^*H \end{bmatrix} \tag{2}$$

Circular polarization – matrix form $\mathbb{R}^4 \mapsto \mathbb{C}^4$

$$\begin{bmatrix} V^*V \\ V^*H \\ H^*V \\ H^*H \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & 2 & 2 & 2 \\ -1 & 1 & 1 & -1 \\ 2i & 2i & -2i & -2i \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} RR \\ RL \\ LR \\ LL \end{bmatrix} \tag{3}$$

Complex Plane $\mathbb{C}$

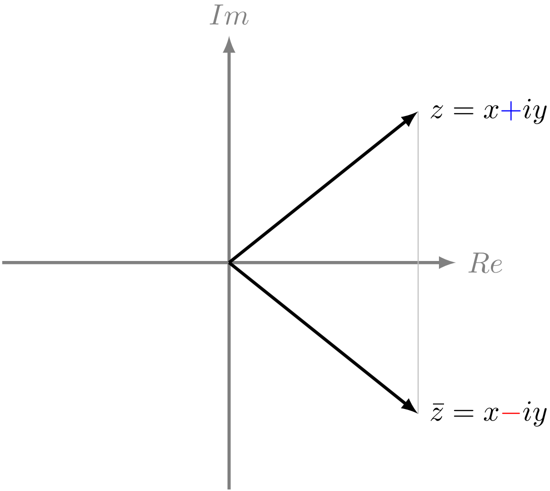
Table: Why $\mathbb{C}$ instead of $\mathbb{R}^2$



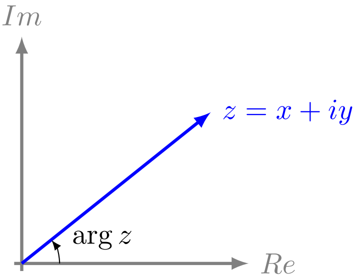
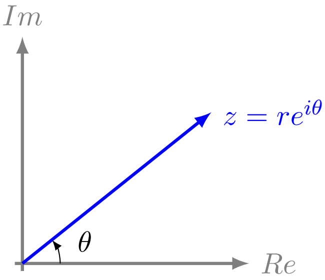
i $\quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

Multiplication by complex unit modulus i = rotation by $\frac{\pi}{2}$

Complex Numbers



Complex Numbers: Representations

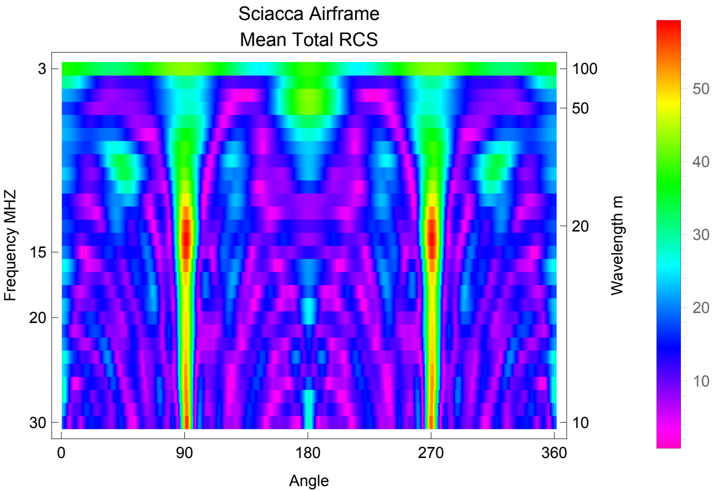
Cartesian	Polar
	
length $= \sqrt{z\bar{z}}$	$r = \sqrt{x^2 + y^2}$
angle $= \arg z$	$\theta = \arctan \frac{y}{x}$

Mean RCS

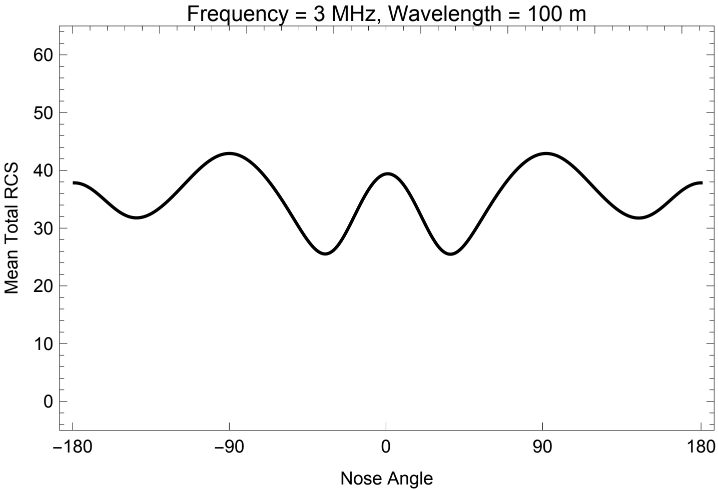
mean total RCS

$$\langle \sigma_T \rangle = \frac{1}{2} \left(V^*V + V^*H + H^*V + H^*H \right) \in \mathbb{R}$$

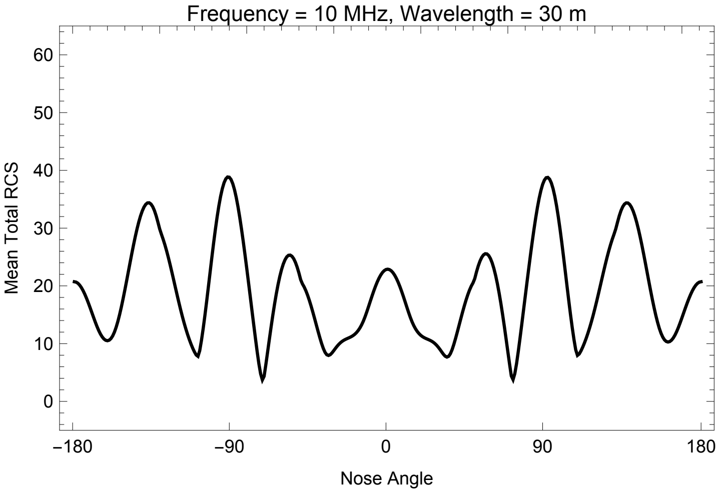
Sciacca Airframe : mean total RCS, $\langle \sigma_T \rangle$



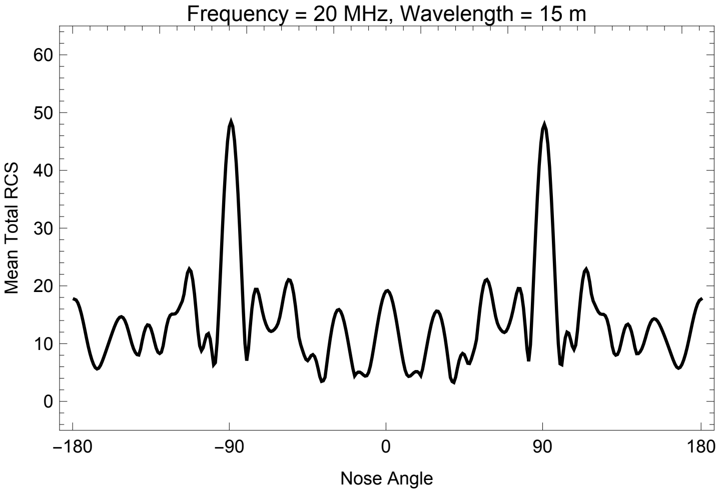
Sciacca Airframe : mean total RCS at 3 MHz



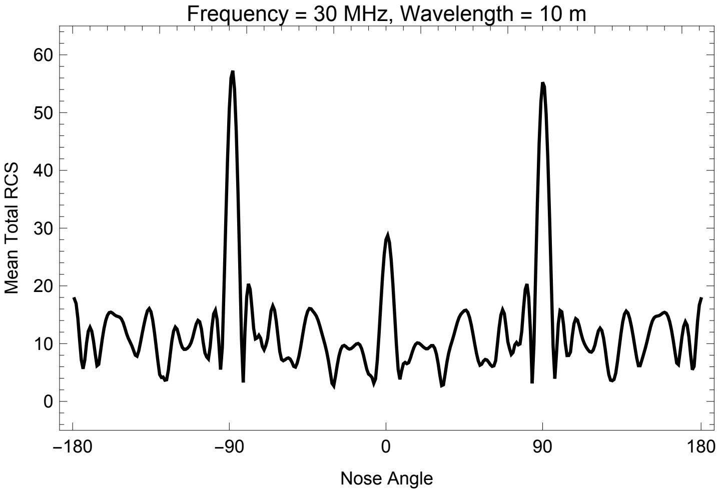
Sciacca Airframe : mean total RCS at 10 MHz



Sciacca Airframe : mean total RCS at 20 MHz



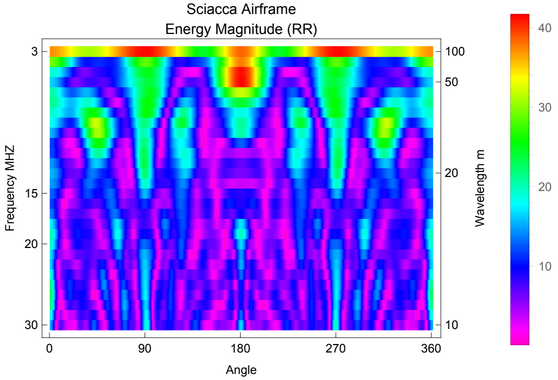
Sciacca Airframe : mean total RCS at 30 MHz



Seeing Complex Values

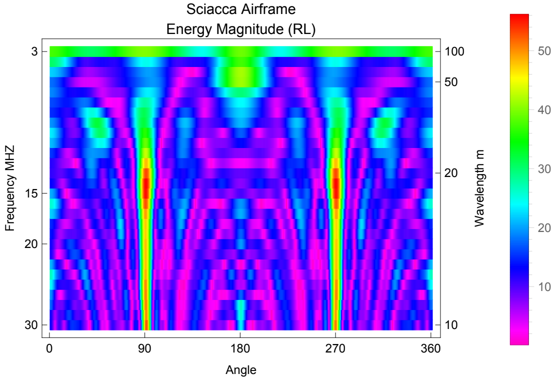
1. Magnitude: Length $\sqrt{x^2 + y^2}$
2. Argument: Angle $\in [-\pi, \pi)$
3. Real Part: x component
4. Imaginary Part: y component

Sciacca Airframe : magnitude of energy, RR



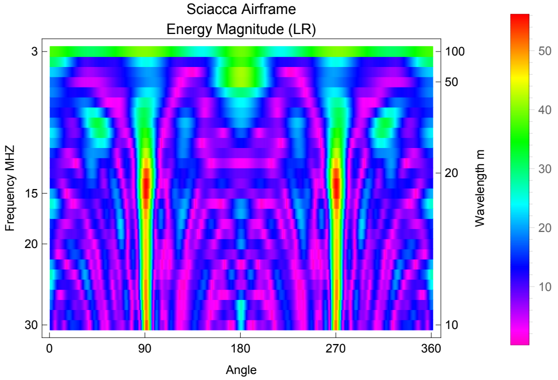
$$|RR| = \frac{1}{2}\left|V^*V - H^*V - i\left(V^*H + H^*V\right)\right|$$

Sciacca Airframe : magnitude of energy, RL



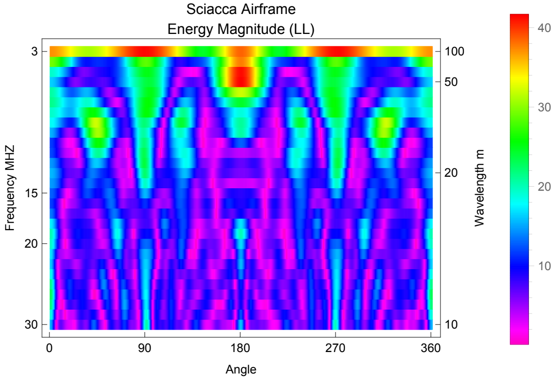
$$|RL| = \frac{1}{2}\left|V^*V + H^*V - i\left(V^*H - H^*V\right)\right|$$

Sciacca Airframe : magnitude of energy, LR



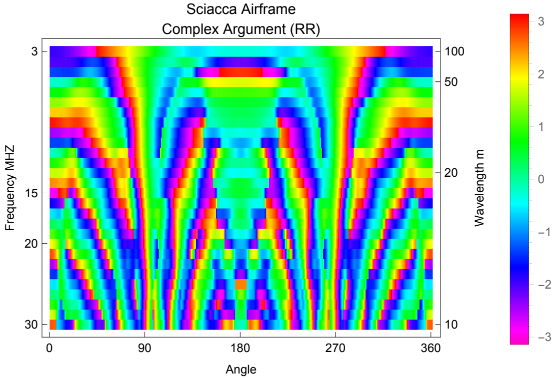
$$|LR| = \frac{1}{2} \left| V^*V + H^*V + i\left(V^*H - H^*V\right) \right|$$

Sciacca Airframe : magnitude of energy, LL



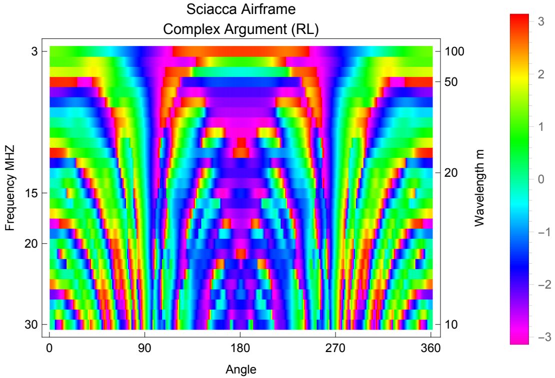
$$|LL| = \frac{1}{2}\left|V^*V - H^*V + i\left(V^*H + H^*V\right)\right|$$

Sciacca Airframe : argument, RR



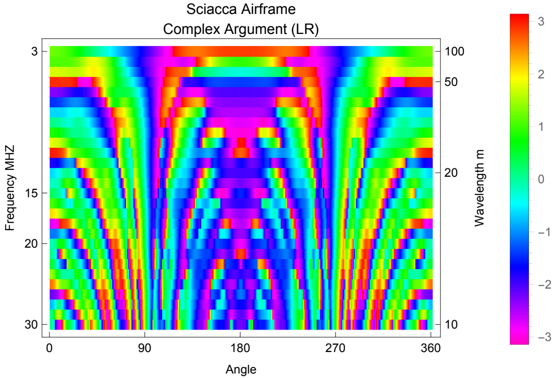
$$\mathrm{Arg}\,(RR) = \mathrm{Arg}\left(\frac{1}{2}\left(V^*V - H^*V - i\left(V^*H + H^*V\right)\right)\right)$$

Sciacca Airframe : argument, RL



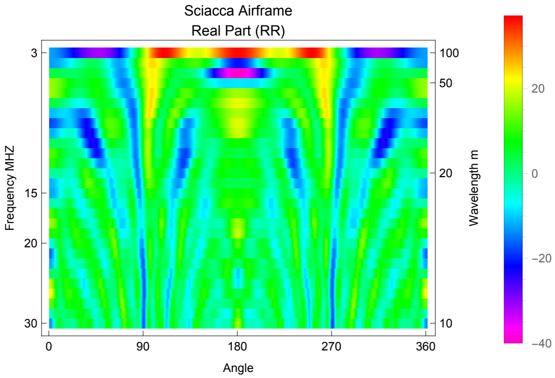
$$\text{Arg}\left(RL\right) = \text{Arg}\left(\frac{1}{2}\left(V^*V + H^*V - i\left(V^*H - H^*V\right)\right)\right) = \text{Arg}\left(LR\right)$$

Sciacca Airframe : argument, LR



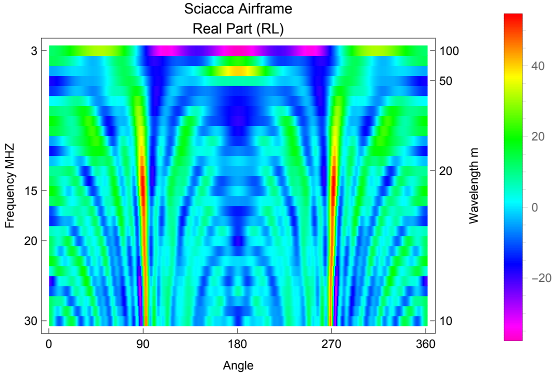
$$\text{Arg}\left(LR\right) = \text{Arg}\left(\frac{1}{2}\left(V^*V + H^*V + i\left(V^*H - H^*V\right)\right)\right) = \text{Arg}\left(RL\right)$$

Sciacca Airframe : real part, RR



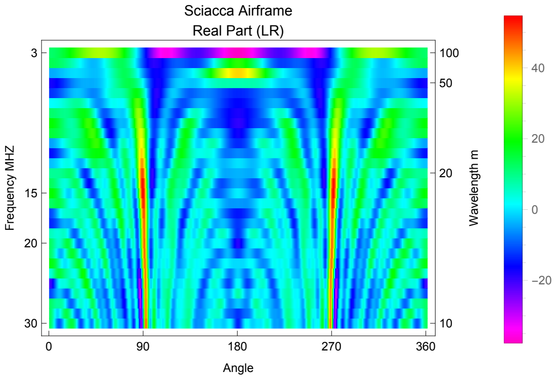
$$\mathrm{Re}\,(RR) = \frac{1}{2}\,(V^*V - H^*V) = \mathrm{Re}\,(LL)$$

Sciacca Airframe : real part, RL



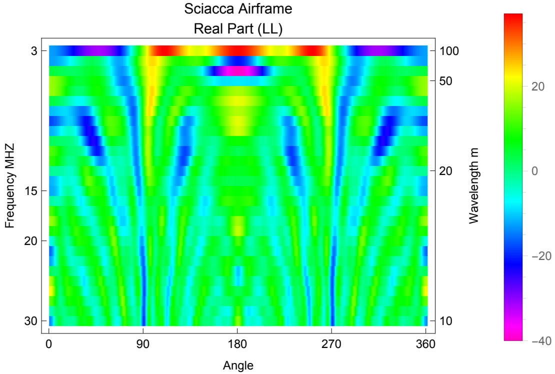
$$\text{Re}\,(RL) = \frac{1}{2}\,(V^*V + H^*V) = \text{Re}\,(LR)$$

Sciacca Airframe : real part, LR



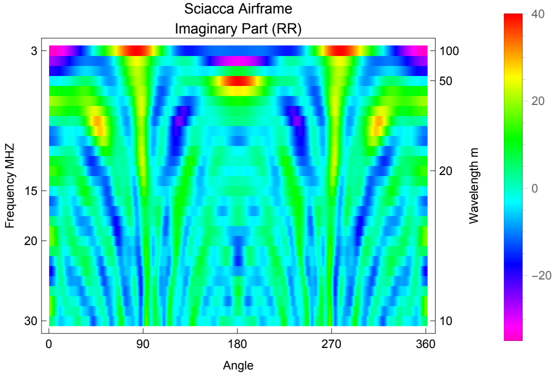
$$\mathrm{Re}\left(LR\right) = \frac{1}{2}\left(V^{*}V + H^{*}V\right) = \mathrm{Re}\left(RL\right)$$

Sciacca Airframe : real part, LL



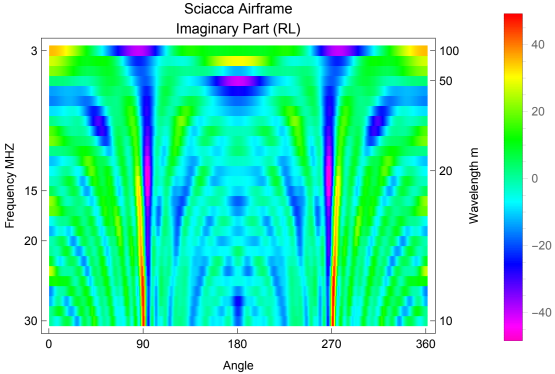
$$\mathrm{Re}\,(LL) = \frac{1}{2}\,(V^*V - H^*V) = \mathrm{Re}\,(RR)$$

Sciacca Airframe : imaginary part, LL



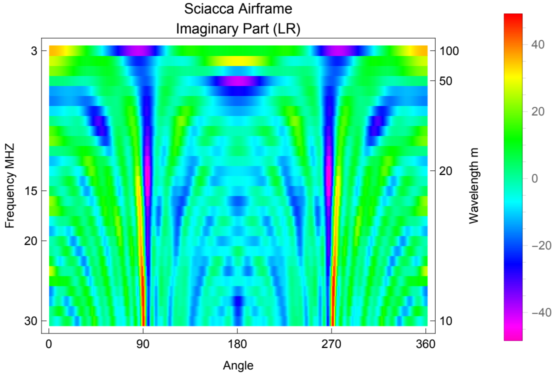
$$\mathrm{Im}\,(RR) = -\frac{1}{2}\,(V^*H + H^*V) = -\mathrm{Im}\,(LL)$$

Sciacca Airframe : imaginary part, RL



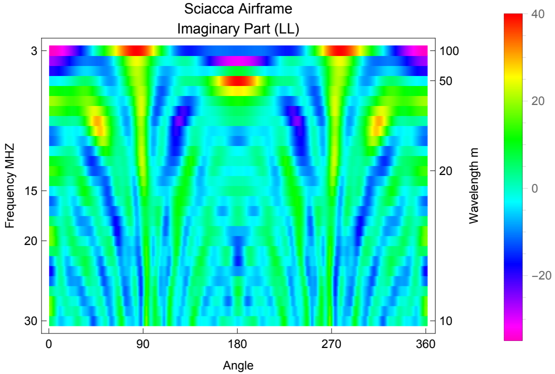
$$\mathrm{Im}\left(RL\right) = \frac{1}{2}\left(-V^{*}H + H^{*}V\right) = -\mathrm{Im}\left(LR\right)$$

Sciacca Airframe : imaginary part, LR



$$\operatorname{Im}(LR) = \frac{1}{2}(V^*H - H^*V) = -\operatorname{Im}(RL)$$

Sciacca Airframe : imaginary part, LL



$$\mathrm{Im}\,(LL) = \frac{1}{2}\,(V^*H + H^*V) = -\mathrm{Im}\,(RR)$$

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