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New GEO paradigm: Re-purposing satellite components from the GEO graveyard

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ABSTRACT

The rising production rate of space debris poses an increasingly severe threat of collision to satellites in the crowded Geostationary Orbit (GEO). It also presents a unique opportunity to make use of a growing supply of in-space resources for the benefit of the satellite community. "The Recycler" is a mission proposed to source replacements for failed components in GEO satellites by extracting functioning components from non-operational spacecraft in the GEO graveyard. This paper demonstrates a method of analyzing in-space re-purposing missions such as the Recycler, using real satellite data to provide a strong platform for accurate performance estimates. An inventory of 1107 satellites in the extended GEO region is presented, and a review into past GEO satellite anomalies is conducted to show that solar arrays would be in the greatest demand for re-purposing. This inventory is used as an input to a greedy selection algorithm and trajectory simulation to show that the Recycler spacecraft could harvest components for 67 client satellites with its allotted fuel budget. This capacity directly meets the levels of customer demand estimated from the GEO satellite anomaly data, placing the Recycler as a strong contender in a future second-hand satellite-component industry. Propellant mass is found to be a greater restriction on the Recycler mission than its 15-year lifetime — a problem which could be solved by on-orbit refueling.

1. Introduction

The Geostationary Orbit (GEO) is home to one of the most profitable and expensive sectors of the satellite industry. With some GEO communications satellites costing on the order of €400 million [1], each unit is a significant investment and is expected to operate to specifications over a 10–20 year lifetime. Premature failure of a GEO satellite can lead to a lengthy and expensive replacement process, as well as significant revenue losses during operational downtime before a new satellite can be procured and launched. Rather than replacing the satellite completely, or suffering a reduced lifetime, existing resources in space could be used to repair or replace the failed components in a more sustainable and cost-effective manner. Research into this field will allow the technology and infrastructure required for in-space re-purposing to be developed, potentially making it less expensive and more feasible than alternative solutions such as launching replacement components in the future.

It is this need that the Recycler is designed to address. The Recycler would be a commercial service available to GEO satellite owners, who would submit requests for their failed satellite components, and receive replacements harvested by the Recycler spacecraft from old, non-operational satellites in the GEO graveyard[3] and surrounding areas. Components would be harvested from non-operational satellites only after a contractual agreement with the owner. The legal aspects of such an agreement are beyond the scope of this study, and are discussed in Refs. [2,3]. Preliminary designs of the Recycler predict an operational

Abbreviations: GEO, Geostationary Orbit; OOS, On-Orbit Servicing; DARPA, Defense Advanced Research Projects Agency; EGO, Extended GEO; EOL, End Of Life; NORAD, North American Aerospace Defense Command; SATCAT, Satellite Catalog; TLE, Two-Line Element; COSPAR, Committee on Space Research; RDV, Rendezvous; ECI, Earth-Centered Inertial; RAAN, Right Ascension of the Ascending Node

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[3] The GEO graveyard is the region approximately ± 250 km from the GEO altitude of 35 786 km. The upper zone is definitively more used, and is the focus of this study.

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Long-term orbit dynamics of decommissioned geostationary satellites

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ABSTRACT

In nominal mission scenarios, geostationary satellites perform end-of-life orbit maneuvers to reach suitable disposal orbits, where they do not interfere with operational satellites. This research investigates the long-term orbit evolution of decommissioned geostationary satellite under the assumption that the disposal maneuver does not occur and the orbit evolves with no control. The dynamical model accounts for all the relevant harmonics of the terrestrial gravity field at the typical altitude of geostationary orbits, as well as solar radiation pressure and third-body perturbations caused by the Moon and the Sun. Orbit propagations are performed using two algorithms based on different equations of motion and numerical integration methods: (i) Gauss planetary equations for modified equinoctial elements with a Runge-Kutta numerical integration scheme based on 8–7th-order Dorman and Prince formulas; (ii) Cartesian state equations of motion in an Earth-fixed frame with a Runge-Kutta Fehlberg 7/8 integration scheme. The numerical results exhibit excellent agreement over integration times of decades. Some well-known phenomena emerge, such as the longitudinal drift due to the resonance between the orbital motion and Earth's rotation, attributable to the J_{22} term of the geopotential. In addition, the third-body perturbation due to Sun and Moon causes two major effects: (a) a precession of the orbital plane, and (b) complex longitudinal dynamics. This study proposes an analytical approach for the prediction of the precessional motion and show its agreement with the (more accurate) orbit evolution obtained numerically. Moreover, long-term orbit propagations show that the above mentioned complex longitudinal dynamics persists over time scales of several decades. Frequent and unpredictable migrations toward different longitude regions occur, in contrast with the known effects due only to the perturbative action of J_{22}.

1. Introduction

In the last decades, the space debris population has grown rapidly [1], creating a serious hazard for existing spacecraft and future space missions. As a result, multiple studies and proposals have appeared in the scientific literature that address the problem of space debris mitigation and removal [2–6]. Moreover, the International Academy Debris Committee has supplied several recommendations to space ventures and agencies to avoid the increasing deterioration of the space environment [7]. Specifically, two regions are critically crowded: (a) the spherical shell corresponding to low Earth orbits, particularly at altitudes between 400 and 800 km, and (b) the circular ring around the geostationary (GS) orbit [8]. The latter typically hosts telecommunications and remote sensing satellites. Nowadays, these spacecraft are equipped with the propellant needed to reach adequate graveyard orbits at the end of their operational life and thus avoid interfering with active satellites. However, in the sixties, seventies and early eighties GS spacecraft used to be abandoned in the synchronous orbit at the end of operations, which led to the accumulation of an enormous amount of debris in the region. Even today, if the propulsion system fails, the satellite drifts uncontrolled under the effects of orbital perturbations. Therefore, a thorough understanding of the long-term orbital dynamics and the ensuing hazard for current and future operations in the GS region is of paramount importance.

For GS satellites, the J_{22} term, related to the ellipticity of the terrestrial equator, has a dominant effect due to resonance between the orbital motion and Earth's rotation. This gives rise to four equilibrium positions, two stable and two unstable, at specific geographical

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Numerical simulation of the post-Newtonian equations of motion for the near Earth satellite with an application to the LARES satellite

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Abstract

We study the post-Newtonian perturbations in the orbit of a near-Earth satellite by integrating them with a high-fidelity orbit propagation software KASIOP. The perturbations of the orbital elements are evaluated for various cases from a low-Earth orbit to a geostationary one, and from an equatorial to a polar orbit. In particular, the numerical simulation is applied to the LARES-like satellite under a realistic orbital configuration. The relativistic perturbations include the Schwarzschild term, the effects of Lense-Thirring precession, and the post-Newtonian term due to the quadrupole moment of the Earth as well as the post-Newtonian gravitoelectric and gravitomagnetic forces, which are produced by the tidal potential of the solar system bodies, are also modeled. The latter three terms are usually ignored in most orbit-propagation software. The secular variations of the orbital elements are evaluated from the orbital positions propagated for a half year. For a medium altitude orbit like that of the LARES mission, the magnitude of the relativistic perturbations ranges from the order of 10^{-7} m/s^2 by the Schwarzschild effect to 10^{-15} m/s^2 by the relativistic tidal effects. The orbital integration shows that the secular variations in three orbital elements – the ascending node, the argument of perigee, and the mean anomaly at epoch – are larger than the systematic error as results of the relativistic perturbations. The magnitudes of the secular variation are investigated in terms of the orbital altitude, inclination, and the size of each perturbation force. The numerical simulation rendered in this study shows that the secular post-Newtonian perturbations with the magnitude lying beyond the Schwarzschild and the Lense-Thirring effects need to be taken into account in current and upcoming space geodesy missions.

Keywords: Post-Newtonian; Orbit propagation; General relativity; Space Geodesy; Earth Quadrupole Moment

1. Introduction

Nowadays, the Earth-orbiting satellites play a critical role for space geodesy. The precise orbit determination is a main tool to determine a number of important fundamental properties in astronomy and geodesy. Namely the whole process of estimating the orbital position is the key to understand the Earth system like the gravitational field, reference frame, and even climate change. As there are increasing demands for the more accurate products like the gravitational field or the reference frame than ever before, the more precise orbital dynamic models should be developed. The Post-Newtonian (PN) relativistic perturbations cannot be ignored any longer in satellite's equations of motion. The International Earth Rotation Service and Reference Systems Service (IERS) have provided the relativistic references models and guidelines for processing the Earth-related measurements and they started to include the PN perturbation to IERS Standards (1992) (McCarthy, 1992). However, at that time, only the Schwarzschild solution of general relativity – the largest

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A novel predictive algorithm for double difference observations of obstructed BeiDou geostationary earth orbit (GEO) satellites

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Abstract

Transmission link disturbances and device failure cause global navigation satellite system (GNSS) receivers to miss observations, leading to poor accuracy in real-time kinematic (RTK) positioning. Previously described solutions for this problem are influenced by the length of the prediction period, or are unable to account for changes in receiver state because they use information from previous epochs to make predictions. We propose an algorithm for predicting double difference (DD) observations of obstructed BeiDou navigation system (BDS) GEO satellites. Our approach adopts the first-degree polynomial function for predicting missing observations. We introduce a Douglas-Peucker algorithm to judge the state of the rover receiver to reduce the impact of predictive biases. Static and kinematic experiments were carried out on BDS observations to evaluate the proposed algorithm. The results of our navigation experiment demonstrate that RTK positioning accuracy is improved from meter to decimeter level with fixed ambiguity (horizontal < 2 cm, vertical < 18 cm). Horizontal accuracy is improved by over 50%, and the vertical accuracies of the results of the static and kinematic experiments are increased by 47% and 27% respectively, compared with the results produced by the classical approach. Though as the baseline becomes longer, the accuracy is weakened, our predictive algorithm is an improvement over existing approaches to overcome the issue of missing data.

Keywords: BDS GEO; Double difference observation prediction; RTK; Douglas-Peucker

1. Introduction

Since December 27, 2012, the Chinese BeiDou-2 regional navigation system has provided regional positioning, navigation, and timing (PNT) services across the Asia-Pacific region. The performance of BDS RTK has been assessed in many studies. Static relative positioning of the baseline reaches millimeter-level precision (He et al., 2013; Teunissen et al., 2014). These levels of precision are close to the levels achieved by Global Positioning System (GPS) (Shi et al., 2013; Yang et al., 2011; Jin, 2016; Montenbruck et al., 2015). At present, global navigation satellite system (GNSS) real–time kinematic (RTK) positioning technology can reach centimeter or even millimeter-level precision by successfully fixing carrier phase ambiguities in a good environment (Teunissen et al., 1997). In order to eliminate related systematic error, the classical synchronous RTK technique requires the reference station and rover station both to receive their signals from the same satellites simultaneously. However, owing to transmission link disturbances, urban canyon and device failure, GNSS receivers often lose information from obser-

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Choices for temporal gravity field modeling for precision orbit determination of CryoSat-2

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Abstract

In this paper we review the precision orbit determination (POD) performance of the CryoSat-2 mission where we used all tracking data between June-2010 and Jan-2023; with station and beacon coordinates provided in the ITRF2020 reference system, we use a mean gravity model, and we use spacecraft specific models for modeling drag and radiation pressure. To model time variable gravity (TVG) we distinguish between two components, there is a short term oceanic and atmospheric part for which we use the AOD1B model; for the longer term part we employ GRACE and GRACE-FO monthly potential coefficient solutions. Our experience is that adding TVG information is not necessarily successful during POD, and that attention must be paid to the proper processing of the GRACE and GRACE-FO data. To demonstrate this property we define four runs where we gradually implement TVG information. An evaluation criterion is the level of POD tracking residuals, the level of the empirical accelerations, and a comparison to precision orbit ephemeris provided by the Centre National d'Etudes Spatiales (CNES). Unexplained empirical accelerations found during POD are on the level of 3 nm/s^2 for the along-track component and 13 nm/s^2 for the cross-track component. The laser residuals converge at approximately 1.02 cm and the Doppler residuals are on the level of 0.406 mm/s, the radial orbit difference to the CNES POE-F (Precision Orbit Ephemeris version F) orbits narrows to 6.5 mm. Tracking residuals are not evenly distributed for DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite) beacons, the South Atlantic Anomaly effect is for instance clearly visible in the first empirical orthogonal function EOF mode of monthly binned DORIS residuals. After consideration of all possible TVG approaches our conclusion is that 3 hourly AOD1B model fields result in a small but visible improvement. The addition of TVG from GRACE and GRACE-FO is implemented in two different ways from which we can select a version that does lead to a reduction in the Doppler tracking residuals and which does reduce the level of solved for empirical accelerations.

Keywords: Orbit determination; Temporal gravity modelling; Performance analysis

1. Introduction

The CryoSat-2 mission described in (Wingham et al., 2006) was primarily developed for studying ocean ice thickness whereby use is made of an advanced radar instrument called SIRAL that has synthetic aperture processing capabilities. Other applications of this mission are to study ice sheet topography cf (Khan et al., 2022), also it should be mentioned that the CryoSat-2 mission contributed to observing ocean topography so that the mission contributes to the RADS project, cf (Naeije et al., 2000). The orbit of CryoSat-2 is close to polar and the altitude is approximately 725 km at an inclination of 88 degree. The mission was launched in April 2010, orbit determination is realized with the help of 10s integrated Doppler observations from the international DORIS service cf. (IDS, 2023a) (Willis et al., 2016) and satellite laser ranging obser-

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ORBIT DYNAMICS AND KINEMATICS WITH FULL QUATERNIONS

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Abstract: Full quaternions constitute a compact notation for describing the motion of a body in the space. An important result about full quaternions is that they can be partitioned into a unit quaternion (which describes the orientation with respect to a suitable reference), and a modulus (which represents the translational motion along the direction indicated by the unit quaternion). Since vectors and scalars are also full quaternions, the equations of body motion can be rewritten in quaternion form. In this paper the orbit dynamics and kinematics of a point mass moving in the space are transformed in quaternion form. Simple application examples are presented. *Copyright © 2004 IFAC*

Keywords: Aerospace Trajectories, System Models, Quaternions, Satellite Control Applications.

1. INTRODUCTION

When dealing with satellite attitude and orbit control, one of the first design issue is the formulation of spacecraft dynamics. According to classical approach, rigid body motion can be decomposed into:

1. *orbital motion*, depending on position and velocity of the satellite Centre of Mass (COM) ;
2. *attitude kinematics and dynamics*, described by Euler parameters (i.e.: unit quaternions) or Euler angles.

This methodology is very well known, has been widely treated in literature (Wertz, 1978; Kaplan, 1976), and is commonly used in applications: for example it has been employed in the design of a *drag-free* controller for the European satellite GOCE (Canuto *et al*, 2002). In this case, satellite attitude corresponds to the orientation of a body-fixed reference frame w.r.t. a *local orbital* frame, univocally defined by orbit position and velocity. Assuming that the orientation of the body frame w.r.t. an inertial frame is known, it becomes necessary to parameterize the orientation of the orbital frame w.r.t. the inertial reference. The problem, apparently straightforward, which suggested the present work, is transforming the inertial coordinates of the three unit vectors constituting the orbital frame into a set of four Euler parameters. Two alternative solutions have been considered:

1. to build the rotation matrix and then exploit the well known conversion rules allowing to pass to quaternion parameterization;
2. to associate a *full quaternion* notation (i.e.: non-unitary quaternion) to orbital frame.

The former solution has been employed in attitude determination of the GOCE satellite. The latter one, which is credited to be original, has been developed with the aim of finding a direct way to express the motion of the local orbital frame entrained by the COM motion.

A full quaternion can describe the modulus and the orientation of a vector w.r.t. a given reference frame. This implies, considering the satellite orbit, that *position and velocity can be alternatively denoted with a vector or with the associated full quaternion.* Since that, orbital dynamics and kinematics can be rewritten substituting vector notation with full quaternions. This results in harmonization of motion equations: both orbital dynamics/kinematics and attitude dynamics/kinematics can be rewritten in

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Advances in Geosciences

Kinematic and reduced-dynamic precise orbit determination of low earth orbiters

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Abstract. Various methods for kinematic and reduced-dynamic precise orbit determination (POD) of Low Earth Orbiters (LEO) were developed based on zero- and double-differencing of GPS carrier-phase measurements with and without ambiguity resolution. In this paper we present the following approaches in LEO precise orbit determination:

– zero-difference kinematic POD,

– zero-difference dynamic POD,

– double-difference kinematic POD with and without ambiguity resolution,

– double-difference dynamic POD with and without ambiguity resolution,

– combined GPS/SLR reduced-dynamic POD.

All developed POD approaches except the combination of GPS/SLR were tested using real CHAMP data (May 20-30, 2001) and independently validated with Satellite Laser Ranging (SLR) data over the same 11 days.

With SLR measurements, additional combinations are possible and in that case one can speak of combined kinematic or combined reduced-dynamic POD. First results of such a combined GPS/SLR POD will be presented, too.

This paper shows what LEO orbit accuracy may be achieved with GPS using different strategies including zero-difference and double-difference approaches. Kinematic versus dynamic orbit determination is presently an interesting issue that will also be discussed in this article.

Key words. POD, kinematic orbit, dynamic orbit, LEO, CHAMP, ambiguity resolution, GPS, SLR

1 Introduction

Today more and more low Earth orbiting satellites (LEOs) of new scientific missions are equipped with a GPS receiver for precise orbit determination (POD). Although POD with GPS has been tested using data from various satellites (e.g. TOPEX/Poseidon, GPS/MET, etc.), there are still many open issues concerning the optimum way to determine LEO satellite orbits with GPS: on one hand the quality of spaceborne GPS receivers has considerably improved, and on the other hand much progress was achieved in the modeling aspects of POD.

Over the last year we developed various methods to compute LEO satellite orbits based on techniques ranging from reduced-dynamic to purely kinematic precise orbit determination. These algorithms allow to process GPS code and phase observations on the zero- or double-difference level. They have been thoroughly tested using simulated data and various analyses of real CHAMP data have been performed.

By making use of dynamical models (e.g. Earth's gravity field, tides, air-drag, solar radiation pressure) satellite orbits can be determined using different types of measurements, e.g. pseudo-range, carrier-phase (GPS), range (SLR) and doppler type of measurements (DORIS). In this case the quality of the dynamical models are crucial for the orbit obtained. On the other hand, the GPS technique, by tracking many satellites every epoch, allows a purely kinematic approach without making use of any dynamical model. Kinematic orbit determination is independent of the gravity field and of all the non-conservative forces acting on the satellites.

In the first part of the paper the observation equations and the treatment of the parameters in all methods will be presented and in the following, results for the CHAMP satellite, including the validation with SLR, will be given.

2 LEO GPS observation equation

The observation equation for LEO zero-difference POD using carrier-phase measurements for the frequency i between LEO receiver and GPS satellite s can be written as follows (in units of length):

$$L^s_{LEO,i} = \rho^s_{\mathrm{LEO}} + c\,(\delta t_{\mathrm{LEO}} + \delta t_{sys,i}) - c\,(\delta t^s + \delta t^{sys,i}) +$$

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Dynamics of a Geostationary Satellite

Clément Gazzino

Observations of Resonance Effects on Satellite Orbits Arising from the Thirteenth- and Fourteenth-Order Tesseral Gravitational Coefficients

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Abstract. Orbit parameters for several satellites, obtained on the basis of Doppler observations with gravity parameters through $(n, m) = (7, 6)$, yielded residuals of fit which showed a periodic error with an amplitude of 100 to 150 meters and a period of 2.5 to 5.5 days. Analysis of the residuals, which agree in period with the beat period between the orbit period and $1/m$ times the earth's rotational period relative to the plane of the orbit, yielded values for the gravity coefficients corresponding to $(n, m) = (15, 13)$, $(13, 13)$, and $(15, 14)$.

Periodic variation in the prediction errors for the polar satellite 1963 49*B* were noted by R. Newton (Johns Hopkins University, private communication), who attributed the errors to a resonance phenomenon existing between the nodal period of the satellite and the sidereal period corresponding to the gravitational harmonics ($C_{13,13}$, $S_{13,13}$). Calculations described below were then performed by S. J. Smith, R. W. Hill, and F. Rowell of the Naval Weapons Laboratory, Dahlgren, Virginia, which confirmed this hypothesis and yielded values for $C_{13,13}$, $S_{13,13}$, $C_{15,13}$, $S_{15,13}$, $C_{15,14}$, and $S_{15,14}$ on the basis of Doppler observations made on satellites 1963 49*B*, $1961o_1$, and $1962\beta\mu_1$, which have orbital inclinations of 90, 67, and 50 degrees, respectively.

To evaluate the magnitude of the effects we transformed the frequency observations for each pass of the satellite over each station to an 'along-track error,' which represents the distance the station would have to be moved parallel to the velocity vector of the satellite (at the time of closest approach of the satellite to

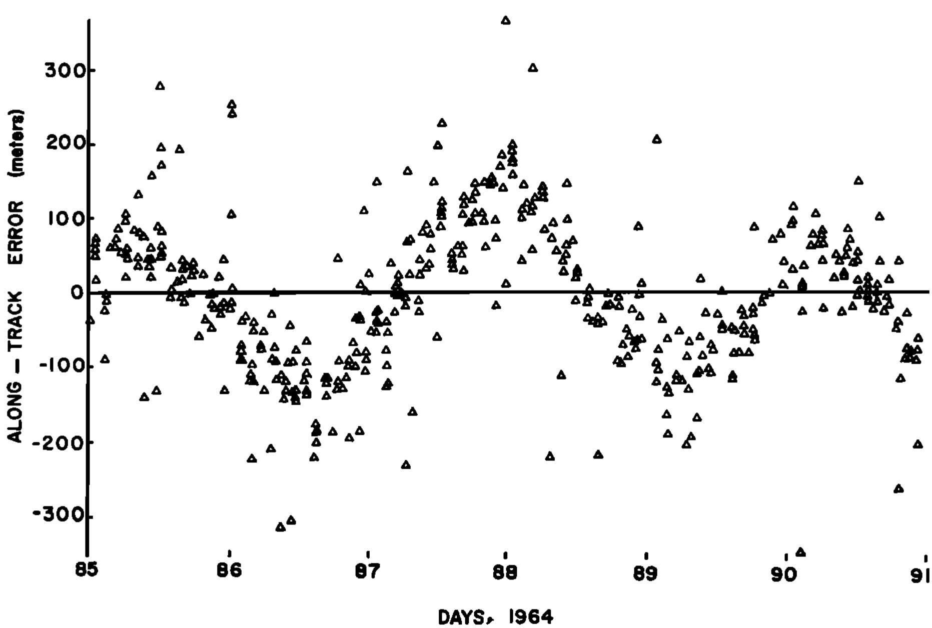

Fig. 1. Along-track errors for satellite 1963 49*B* with NWL-5A geodetic parameters.

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Q1 Q2 # Radiation Belt Response to Fast Reverse Shock at Geosynchronous Orbit

Q3 Q4 Q5 Ankush Bhaskar[1,2,3], David Sibeck[1], Shrikanth G. Kanekal[1], Howard J. Singer[4], Geoffrey Reeves[5], Denny M. Oliveira[1,6], Suk-Bin Kang[1], and Colin Komar[1]

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Abstract

Fast reverse shocks (FRSs) cause the magnetosphere to expand, by contrast to the well-known compressions caused by the impact of fast forward shocks (FFS). Usually, FFSs are more geoeffective than FRSs, and consequently the inner magnetosphere dynamic responses to both shock types can be quite different. In this study, we investigate for the first time the radiation belt response to an FRS impact using multi-satellite observations and numerical simulations. Spacecraft on the dayside observed decreases in magnetic field strength and energetic (∼40–475 keV) particle fluxes. Timing analysis shows that the magnetic field signature propagated from the dayside to the nightside magnetosphere. Particles with different energies vary simultaneously at each spacecraft, implying a non-dispersive particle response to the shock. Spacecraft located at lower L-shells did not record any significant signatures. The observations indicate a local time dependence of the response associated with the shock inclination, with the clearest signatures being observed in the dusk–midnight sector. Simulations underestimate the amplitude of the magnetic field variations observed on the nightside. The observed decreases in the electron intensities result from a combination of radial gradient and adiabatic effects. The radial gradients in the spectral index appear to be the dominant contributor to the observed variations of electrons seen on the dayside (near noon and dusk) and on the nightside (near midnight). This study shows that even an FRS can affect the radiation belts significantly and provides an opportunity to understand their dynamic response to a sudden expansion of the magnetosphere.

Unified Astronomy Thesaurus concepts: Fast solar wind (1872); Van Allen radiation belt (1758); Planetary magnetosphere (997); Interplanetary particle acceleration (826); Solar wind (1534); Interplanetary magnetic fields (824); Interplanetary discontinuities (820); Solar-planetary interactions (1472); Magnetohydrodynamical simulations (1966); Solar activity (1475); Solar-terrestrial interactions (1473);

1. Introduction

Interplanetary (IP) shocks are a frequent feature of the solar wind (Burlaga 1971; Richter et al. 1985). Fast mode IP shocks occur when the relative speed between the ambient solar wind and the shock speed is larger than the local magnetosonic speed (Landau & Lifshitz 1960; Richter et al. 1985; Tsurutani et al. 2011; Oliveira 2017). Fast IP shocks that are moving away from the Sun are classified as fast forward shocks (FFSs), while fast shocks that propagate toward the Sun are classified as fast reverse shocks (FRSs) (Burlaga 1995; Tsurutani et al. 2011; Oliveira 2017). Similarly to FFSs, FRSs are carried antisunward by the continuous solar wind flow as seen from a reference frame defined as a spacecraft in the IP space or the Earth itself (Richter et al. 1985; Burlaga 1995; Tsurutani et al. 2011; Oliveira 2017; Oliveira & Samsonov 2018). FFSs are more numerous during solar maxima, while FRS occurrence rates have no clear correlation with solar activity (Echer et al. 2003; Kilpua et al. 2015; Cavus et al. 2019). Kilpua et al. (2015) showed that the occurrence rates of FFSs are higher than the occurrence rates of FRSs during all solar phases, except during solar minima.

The steepening conditions across the fronts of FFSs and FRSs are different. In the case of FFSs, all plasma parameters (particle number density, thermal temperature, and velocity), along with the interplanetary magnetic field (IMF) increase. Conversely, in the case of FRSs, all solar wind and IMF parameters decrease, except the solar wind plasma velocity (Landau & Lifshitz 1960; Burlaga 1971; Richter et al. 1985; Burlaga 1995; Burguess 1995; Tsurutani et al. 2011). See Figure 2 of Oliveira (2017) for comparisons between schematic profiles of FFSs and FRSs. These distinct shock conditions are responsible for different magnetospheric responses to the impacts of FFSs and FRSs. For CIRs at 1 au, there are often reverse shocks without forward shocks. However, at large distances from the Sun all CIRs typically have both forward and reverse shocks (Smith & Wolfe 1976). Moreover, FRSs are found to be more efficient in accelerating the particles in interplanetary space as compared to FFSs (Tsurutani et al. 1982). It is therefore important to study the impact of FRSs on the magnetosphere and radiation belt.

FFSs are known to be the most geoeffective class of IP shocks (Echer et al. 2004; Alves et al. 2011; Tsurutani et al. 2011; Oliveira & Raeder 2015; Oliveira & Samsonov 2018). The first and most dramatic magnetospheric response to the impact of FFSs are characterized by positive sudden impulses (SI^+) in the horizontal component of the geomagnetic field measured by magnetometers located in the space (Patel & Coleman 1970; Huttunen et al. 2005; Wang et al. 2010; Su et al. 2015; Rudd et al. 2019) and on the ground (Siscoe et al. 1968; Smith et al. 1986; Takeuchi et al. 2002b; Rudd et al. 2019), as a consequence of a sudden increase of the solar wind

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ORIGINAL ARTICLE

Reduced dynamic and kinematic precise orbit determination for the Swarm mission from 4 years of GPS tracking

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Abstract
Precise science orbits for the first 4 years of the Swarm mission have been generated from onboard GPS measurements in a systematic reprocessing using refined models and processing techniques. Key enhancements relate to the introduction of macro-models for a more elaborate non-gravitational force modeling (solar radiation pressure, atmospheric drag and lift, earth albedo), as well as carrier phase ambiguity fixing. Validation using satellite laser ranging demonstrates a 30% improvement in the precision of the reduced dynamic orbits with resulting errors at the 0.5–1 cm level (1D RMS). A notable performance improvement is likewise achieved for the kinematic orbits, which benefit most from the ambiguity fixing and show a 50% error reduction in terms of SLR residuals while differences with respect to reduced dynamic ephemerides amount to only 1.7 cm (median of daily 3D RMS). Compared to the past kinematic science orbits based on float-ambiguity estimates, the new kinematic position solutions exhibit a factor of reduction of two to three in Allan deviation at time scales of 1000s and higher, and promise an improved recovery of low-degree and -order gravity field coefficients in Swarm gravity field analyses.

Keywords POD · GPS · SLR · Ambiguity fixing · Non-gravitational forces

Introduction

Swarm is a small-satellite "Earth Explorer" mission of the European Space Agency (ESA) dedicated to the exploration of the earth's magnetic field (Friis-Christensen et al. 2008; Olsen et al. 2016). Further science objectives include investigations of the earth's atmosphere (Siemes et al. 2016) and gravity field (Jäggi et al. 2016; da Encarnação et al. 2016). The Swarm constellation is made up of three identical spacecraft. These satellites orbit the earth in polar orbits with an 87° inclination and initial altitudes of about 470 km (Swarm-A, -C) and 520 km (Swarm-B). While the Swarm-A/C satellites remain close to each other with mutual separations of about 50–200 km, Swarm-B exhibits a different orbital period and its orbital plane drifts relative to that of the Swarm-A/C pair by about 25° per year (Sieg and Diekmann 2016).

Key payloads of each Swarm satellite include an absolute scalar magnetometer and a vector field magnetometer, the Langmuir probe and the thermal ion imager for measuring the electric field, a set of accelerometers, and star cameras. The spacecraft are, furthermore, equipped with a dual frequency GPS receiver (Zangerl et al. 2014) for precise orbit determination and onboard navigation. GPS observations and the derived orbit determination support the geocoding of other instrument data, but likewise contribute to ionospheric research, thermospheric density determination, and gravity field recovery.

Kinematic and reduced dynamic precise science orbits (PSOs) are generated by TU Delft for the ESA on a routine basis (van den IJssel et al. 2015) using the GNSS High-precision Orbit determination Software Tools (GHOST; Wermuth et al. 2010). Over the 4 years of operations conducted so far, Swarm GPS tracking—and, in consequence, the orbit

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Dynamic and Reduced-Dynamic Precise Orbit Determination of Satellites in Low Earth Orbits

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Article

Experimental Study on the Precise Orbit Determination of the BeiDou Navigation Satellite System

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Abstract: The regional service of the Chinese BeiDou satellite navigation system is now in operation with a constellation including five Geostationary Earth Orbit satellites (GEO), five Inclined Geosynchronous Orbit (IGSO) satellites and four Medium Earth Orbit (MEO) satellites. Besides the standard positioning service with positioning accuracy of about 10 m, both precise relative positioning and precise point positioning are already demonstrated. As is well known, precise orbit and clock determination is essential in enhancing precise positioning services. To improve the satellite orbits of the BeiDou regional system, we concentrate on the impact of the tracking geometry and the involvement of MEOs, and on the effect of integer ambiguity resolution as well. About seven weeks of data collected at the BeiDou Experimental Test Service (BETS) network is employed in this experimental study. Several tracking scenarios are defined, various processing schemata are designed and carried out; and then, the estimates are compared and analyzed in detail. The results show that GEO orbits, especially the along-track component, can be significantly improved by extending the tracking network in China along longitude direction, whereas IGSOs gain more improvement if the tracking network extends in latitude. The involvement of MEOs and ambiguity-fixing also make the orbits better.

Keywords: BeiDou; tracking network; precise orbit determination; ambiguity-fixing

AIMING AT A 1-CM ORBIT FOR LOW EARTH ORBITERS: REDUCED-DYNAMIC AND KINEMATIC PRECISE ORBIT DETERMINATION

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Abstract. The computation of high-accuracy orbits is a prerequisite for the success of Low Earth Orbiter (LEO) missions such as CHAMP, GRACE and GOCE. The mission objectives of these satellites cannot be reached without computing orbits with an accuracy at the few cm level. Such a level of accuracy might be achieved with the techniques of reduced-dynamic and kinematic precise orbit determination (POD) assuming continuous Satellite-to-Satellite Tracking (SST) by the Global Positioning System (GPS). Both techniques have reached a high level of maturity and have been successfully applied to missions in the past, for example to TOPEX/POSEIDON (T/P), leading to (sub-)decimeter orbit accuracy. New LEO gravity missions are (to be) equipped with advanced GPS receivers promising to provide very high quality SST observations thereby opening the possibility for computing cm-level accuracy orbits. The computation of orbits at this accuracy level does not only require high-quality GPS receivers, but also advanced and demanding observation preprocessing and correction algorithms. Moreover, sophisticated parameter estimation schemes need to be adapted and extended to allow the computation of such orbits. Finally, reliable methods need to be employed for assessing the orbit quality and providing feedback to the different processing steps in the orbit computation process.

Keywords: precise orbit determination, reduced-dynamic, kinematic, GPS, LEO

1. Introduction

The launch of CHAMP in July 2000 has triggered significant efforts by many scientific institutes in the field of precise orbit determination (POD). Without very high precision orbit determination, one of the most important mission objectives of CHAMP cannot be reached, namely a significant improvement in global Earth gravity field modeling (Reigber et al., 1999). High-precision orbit determination becomes even more of a challenge for the upcoming GRACE mission (launch in March 2002) and the future GOCE mission (expected launch in early 2006). These missions are much more demanding in terms of gravity field modeling performance than CHAMP and even more stringent orbit accuracy requirements are imposed. In order to get the most out of these missions, an orbit accuracy at the cm level is aimed at (NRC, 1997; ESA, 1999). All previously mentioned missions are Low Earth Orbiters (LEOs) flying at very low altitudes, in the 240–450 km height range

remote sensing

Article

Real-Time Precise Orbit Determination of Low Earth Orbit Satellites Based on GPS and BDS-3 PPP B2b Service

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Abstract: This study investigates and verifies the feasibility of the precise point positioning (PPP)-B2b enhanced real-time (RT) precise orbit determination (POD) of low Earth orbit (LEO) satellites. The principles and characteristics of matching various PPP-B2b corrections are introduced and analyzed. The performance and accuracy of broadcast ephemeris and PPP-B2b signals are compared and evaluated by referring to the precise ephemeris. The root mean square (RMS) errors in the Global Positioning System (GPS) and BeiDou Navigation Satellite System (BDS)-3 broadcast ephemeris orbits in the along direction are larger than those in the other two (radial and cross) directions, and correspondingly, the along component PPP-B2b corrections are greatest. The continuity and smoothness of the GPS and BDS-3 broadcast ephemeris orbits and clock offsets are improved with the PPP-B2b corrections. The availability of PPP-B2b corrections is comprehensively analyzed for the TJU-01 satellite. Several comparative schemes are adopted for the RT POD of the TJU-01 satellite using the broadcast ephemeris and PPP-B2b corrections. The RT POD performance is improved considerably with the broadcast ephemeris corrected by the PPP-B2b signals. The RMS of the RT orbital errors in the radial, along, and cross directions is 0.10, 0.13, and 0.09 m, respectively, using BDS-3 and GPS PPP-B2b corrections, with reference to the solutions calculated with the precise ephemeris. The accuracy is improved by 5.1%, 43.9%, and 28.7% in the three directions, respectively, relative to that achieved with the broadcast ephemeris. It is concluded that a greater proportion of received PPP-B2b satellite signals corresponds to a greater improvement in the accuracy of the RT POD of the LEO satellite.

Keywords: GPS; BDS-3; broadcast ephemeris; real time; PPP-B2b; precise orbit determination

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1. Introduction

For the last few years, with an increasing frequency of space missions, there has been a growing need for the precise and real-time (RT) orbit determination of low Earth orbit (LEO) satellites, which has been widely studied [1–3]. According to the actual Global Positioning System (GPS) broadcast ephemeris with the Jet Propulsion Laboratory's global differential GPS corrections, the root mean square (RMS) error in the three-dimensional (3D) direction of the RT POD of the Challenging Mini Satellite Payload (CHAMP) spacecraft is 30 cm [4]. The RT POD of Meteorological Operational Satellite-A (MetOp-A) has been performed using observations recorded by the global navigation satellite system (GNSS) receiver of an onboard atmospheric sounding instrument and the results indicate that the accuracy can reach 0.5 m in the three axial directions of the GPS broadcast ephemeris [5]. Through dynamic model compensation, the standard deviation (STD) of the RT POD of the Fengyun-3C using BDS/GPS pseudo-range measurements can reach the meter level [6].

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Engineering Notes

Numerical Evaluation of Post-Newtonian Perturbations on the Global Navigation Satellite System

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I. Introduction

PRECISE orbital knowledge is one of the most stringent requirements for a global navigation satellite system (GNSS) because the positions of the GNSS satellites serve as reference points. If the reference points are provided with high precision, better solutions can be achieved in most applications, such as positioning and time synchronization. One of the most important issues for precise orbit determination is that the orbital dynamic model should have an accuracy equivalent to that of the measurements. The measurement technology of the GNSS, based on the time and frequency transfer through atomic clocks, has achieved an accuracy on the order of 1×10^{-16}, which is several orders of magnitude lower than that of the relativistic effect. The continuing progress in atomic clocks is expected to improve the accuracy at the rate of one order of magnitude per decade. There have been several extensive studies about the relativistic effects on the signal and time transfer in the GNSS [1]. The major relativistic effect of gravitational blue shift has already been taken into account in the Global Positioning System (GPS). The clocks on GPS satellites are adjusted so that the clocks appear to a ground station to have its chosen frequency. The details and additional relativistic effects on the atomic clocks in the GNSS satellites will not be discussed here because this Note focuses on the relativistic effects on the equations of motion of the GNSS satellites. Readers interested in the relativistic effects on the atomic clocks in the GNSS satellites can refer to the paper by Ashby [2].

According to the advances in measurement systems, the dynamic model (i.e., the equations of motion of satellites) also needs to take into account the relativistic effects. Therefore, the International Earth Rotation and Reference System Service (IERS) has first recommended that the largest relativistic effect must be included (i.e., the Schwarzschild terms), and this recommendation appeared in the IERS standards of 1992 [3]. The latest IERS conventions of 2010 [4] suggested that two additional relativistic perturbations (i.e., the Lense–Thirring effect and de Sitter precession) should be taken into account for precise orbit determination. However, these two additional relativistic terms are still often ignored in GPS data processing [5].

These relativistic perturbations are derived from the post-Newtonian (PN) approximation of a metric tensor defined with respect to the Earth's center, namely, the geocentric metric tensor. These approximated equations of motion can be described as a sum of the Newtonian term and the PN corrections. The aforementioned three PN corrections, or perturbations, are calculated by assuming the Earth has a spherical-symmetric gravitational field and a constant rotation rate. However, there are other PN perturbations caused by the nonsymmetrical shape of our planet and the PN tidal gravitational potential owing to the external bodies in our solar system. These additional PN corrections have not been considered yet because they are estimated to be minute as compared with other perturbations, such as due to the radiation pressures due to the sun and the Earth [6]. Advanced perturbation models and measurement technology (i.e., atomic clocks) prompt the investigation of the impact of previously ignored PN corrections. Even small errors in attitude or rigid-body models of spacecraft can affect the satellite's orbit determination results [7]. Therefore, it is important to evaluate the impact of PN corrections by conducting comprehensive numerical simulations; the results are likely to be very useful for defining the accuracy of the orbital dynamic models of the GNSS orbit for precise orbit determination.

Our main motivation in this work was to investigate the effects of the full set of first-order PN perturbations on the GNSS orbits, specifically on the GPS and the Russia's global navigation satellite system (GLONASS) orbits. There were also several other satellite navigation systems, such as BeiDou (People's Republic of China), Galileo (European Union), and the Quasi-Zenith Satellite System (Japan). However, these systems were not studied here because they are not yet fully operational. The full set of first-order PN corrections was first implemented by Roh et al. [8] in the high-fidelity orbit propagation software KASIOP (which stands for Korea Astronomy and Space Science Institute Orbit Propagator), and their effects on orbital elements were numerically evaluated for the laser geodynamics satellite (LAGEOS) and laser relativity satellite (LARES) orbits. The details about KASIOP can be found in [8]. The LAGEOS-1 and -2 missions provided the first experimental results for the Lense–Thirring effect, which is also known as the frame-dragging effect. The LARES satellite was launched in 2012 to measure the frame-dragging effect with an accuracy of ~1% [9]. However, these satellites have a small cannonball-shaped spacecraft with a low area-to-mass ratio to reduce any nonconservative perturbations such as atmospheric drag and radiation pressure from the sun and the Earth. The properties of the GPS and the GLONASS spacecraft are quite different from those of the LAGEOS and the LARES systems in many aspects, such as their box-wing shape, large solar panels, and semimajor axes of ~26,000 km, which are nearly twice that of the LAGEOS system. In this Note, the full first-order PN corrections were numerically simulated and investigated for the GPS and the GLONASS cases using the software that was previously developed by Roh et al. [8].

The PN equations of motion studied in this work are briefly introduced in Sec. II, and the details of the numerical simulations and the results are presented in Sec. III. Lastly, the summary and concluding remarks are presented in Sec. IV.

II. Background and Simulation Scope

A. Post-Newtonian Equations of Motion

The general theory of relativity (GR) deals with gravity. The space–time around a body with a certain mass is curved, and gravity can be thought of as a curvature of space–time in the GR. Namely, the geometry of space–time, described mathematically using a metric tensor, defines the motion of a particle in the space–time continuum. Resolution B1.3 of 2000 [10] adopted by the International Astronomical Union (IAU) states the relativistic reference frames based on the metric tensors defined for the geocentric and barycentric

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