



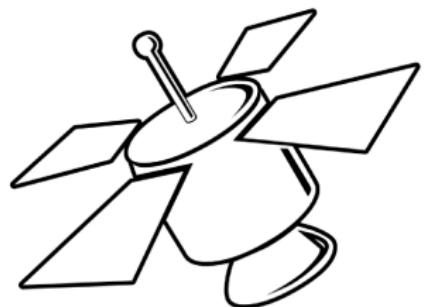
# Input Slides 2024-12: My Two Slides

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**Huntington Ingalls Industries**  
**Mission Technologies**

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# Models of Radar Cross Sections for Satellites



## Spherical Harmonics Expansion

$$f(r, \theta, \phi) \approx a_{0,0} Y_0^0 + a_{1,-1} Y_1^{-1} \\ + a_{1,0} Y_1^0 + a_{1,1} Y_1^1 + \dots$$

where

$$Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^m(\cos \theta) e^{im\phi}$$



# Overview

1 Writ Large

2 Radiation

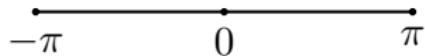
3 Results in 2D



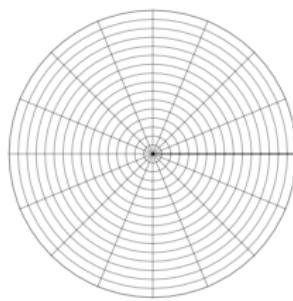
# Goal: RCS Models in 3D

**RCS models have been built in 2D.**  
**Extend to 3D using the same MoM code.**

# Fourier Domains in 1-, 2-, and 3D

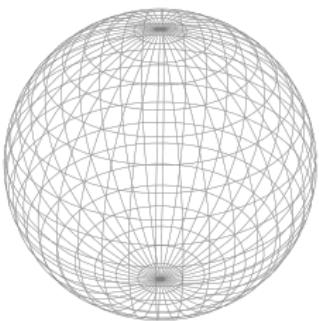


$$\theta \in [-\pi, \pi]$$



$$\theta \in [-\pi, \pi]$$

$$r \in [0, 1]$$



$$\theta \in [-\pi, \pi]$$

$$r \in [0, 1]$$

$$\phi \in [0, \pi]$$



# Fourier and Extensions to 2- and 3-D

$e^{in\theta}$  From periodic functions in 1D to radial and angular  
decompositions in 2D and 3D.

# Why We Love Fourier for Smooth Functions

## Smooth Functions, Beautiful Representations

- **Weierstrass Approximation Theorem:** Any continuous function on  $[a, b]$  can be uniformly approximated by polynomials. Fourier provides a similar approximation, using trigonometric bases instead of polynomials.
- **Riesz-Fischer Theorem:** Hunting license - Fourier coefficients  $(a_n, b_n) \in l^2$ , guaranteeing convergence in the  $L^2$  sense.
- **Uniform Convergence for Smooth Periodic Functions:** Smooth ( $C^\infty$  or  $C^k$ ) functions, Fourier series converge uniformly, ensuring no oscillatory artifacts (Gibbs phenomenon disappears).
- **Orthogonality of Basis Sines and cosines form an**



# Fourier and Extensions to 2- and 3D

1D:

$$f(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta}$$

2D:

$$f(r, \theta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n,2} a_n^m R_n^m(r) e^{in\theta}$$

3D:

$$f(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^n a_n^m \sqrt{\frac{(2m+1)(m-n)!}{4\pi(m+n)!}} P_l^m(\cos\theta) e^{in\theta}$$

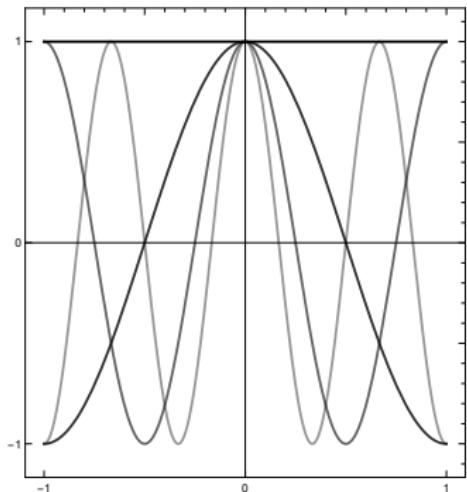
# Fourier and Extensions to 2- and 3D

- **1D Fourier Series:** Decomposes a periodic function  $f(\theta)$  into a sum of complex exponentials with coefficients  $a_n$  capturing the amplitudes of each frequency component.
- **2D Fourier-Bessel:** Extends Fourier analysis to two dimensions using radial functions  $R_n^m(r)$ , often employed in circular domains or optical applications.
- **3D Spherical Harmonics:** Represents functions on a sphere using harmonics  $Y_l^m(\theta, \phi)$  and radial components  $R_l(r)$ , crucial in fields like quantum mechanics and gravitational modeling.

# Lowest Order Fourier Basic Functions

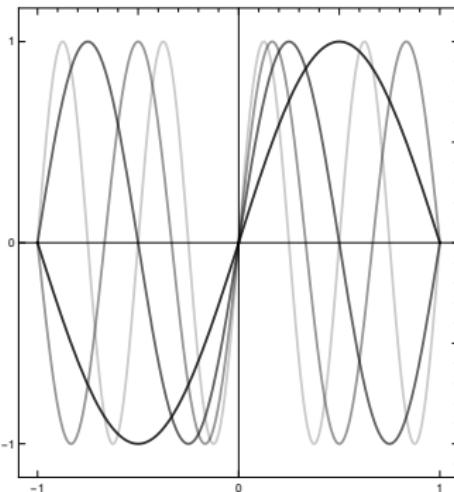
$$f(-\theta) = f(\theta)$$

**Even Parity**

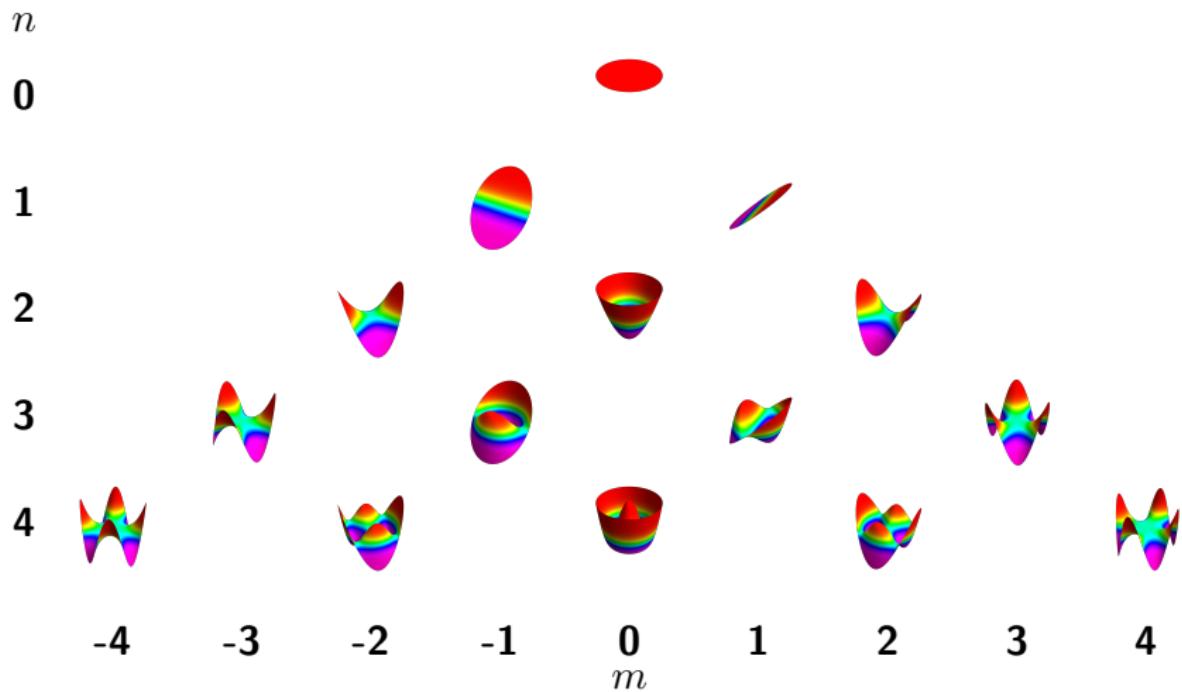


$$f(-\theta) = -f(\theta)$$

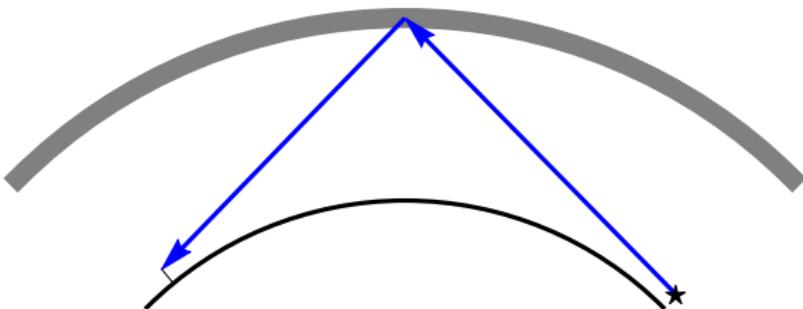
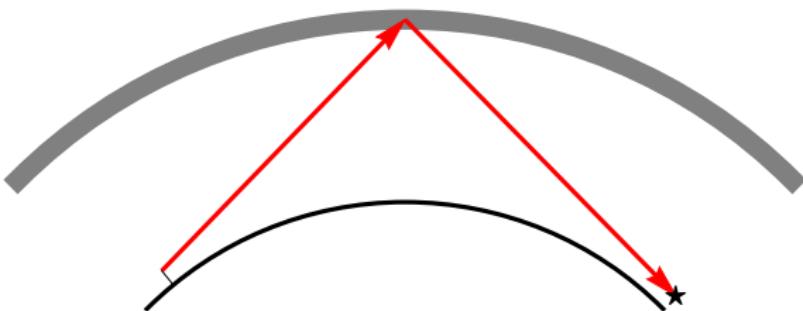
**Odd Parity**



# Lowest Order Zernike Disk Polynomials



## Energy Out, Energy In





# Energy Out, Energy In

- ① radar (left) **irradiates** target (star)
- ② **backscatter** travels from target to radar



# Radar Cross Section: Definition

$$\sigma_* = 4\pi \lim_{r \rightarrow \infty} r^2 \left| \frac{E_{\text{incident}}}{E_{\text{scattered}}} \right|^2 \quad (2.1)$$

Skolnik 1962, (2.36)

## Radar Cross Section: Discussion

- Radar cross section is a **far field** phenomenon
- Assumes **single polarization** to and from target
- Target is **completely metallic**:  
 $E$  field results from surface currents
- Shape is **quasi-dimensional**
  - Dimensions in two known directions
  - Dish antennae, solar panels, booms
- **Resonant scattering**:  
Ratio of typical dimension to wavelength  $\approx 1$
- **Kolosov 1987, §4.6**

# Radar Cross Section: Conceptual Overview

- **Radar Basics:**
  - Transmit energy (Skolnik 1962, p. 21).
  - Receive scattered signal.
  - Direction, strength → object properties (Knott, Schaeffer, and Tulley 2004, p. 45).
- **What is RCS?**
  - Measures object “visibility” to radar Lab 2002, Section 2.
  - Depends on:
    - Material
    - Geometry
    - Orientation (Peebles 2007, pp. 3-4).
- **Key Question:** Power reflected vs. power transmitted.

# Factors Influencing Radar Cross Section

- **Shape:**

- Smooth → directional reflection (Knott, Schaeffer, and Tulley 2004, p. 47).
- Complex → scattered energy.

- **Material:**

- Metal → strong reflection.
- Absorbers → reduced RCS (Knott, Schaeffer, and Tulley 2004, Section 3.2).

- **Size vs. Wavelength:**

- Large → high scattering.
- Small → "invisible" (Rayleigh scattering) (Kolosov 1987, p. 188).

- **Orientation:**

- Aligned → max RCS (Knott, Schaeffer, and Tulley 2004,



# Input files

## ① B-20.geo

- ① Points to facet file
- ② Configure linear algebra solver
- ③ Radar frequency range
- ④ Angular sampling ranges
- ⑤ Boundary conditions
- ⑥ Mono- or Bistatic
- ⑦ Surface or Volume integral elements
- ⑧ Length units

## ② B-20.facet

- ① Vertex list



# Linear algebra (don't alter)

```
&MM_MOM
bUseACA = .TRUE.,
bSolve_ACA = .TRUE.,
bOutOfCore = .TRUE.,
bNormalizeToWaveLength = .FALSE.,
bNormalize = .FALSE.,
dCloseLambda = 0.100000,
ACA_Factor_Tol = 0.000010,
ACA_RHS_Tol = 0.000100,
Point_Tolerance = 0.001000,
nLargestBlockSize = -1,
MemorySize_GB = -1.000000,
stackSize_GB = -1.000000,
nFillThreads = -1,
nFillMKLThreads = 1,
nLUThreads = -1,
nLUMKLThreads = 1,
nRHSThreads = 1,
nRHSMKLThreads = 1,
bOutputACAGrouping = .FALSE.,
bOutputRankFraction = .FALSE.,
bLimitLUColumns = .FALSE.,
Lop_Admissibility = WEAK,
Kop_Admissibility = CLOSE
```



# Memory management (don't alter)

```
&Scratch_Memory
  Scratch_RankFraction_Z = 0.300000,
  Scratch_RankFraction_LU = 0.600000,
  Scratch_RankFraction_RHS = 2.000000,
  Scratch_RankFraction_Solve = 1.000000,
  MemoryFraction_Z = 0.950000,
  MemoryFraction_Scratch_LU = 0.500000,
  MemoryFraction_LU = 1.000000,
  MemoryFraction_RHS = 0.500000,
  MemoryFraction_Solve = 0.900000,
```



# Quadrature (don't alter)

```
&QUADRATURE
NTRISELF = 7,
NTRINEAR = 3,
NTRIFAR = 3,
NTETSELF = 11,
NTETNEAR = 4,
NTETFAR = 4,
NQGAUSS = 4
```



# Radar frequencies

FREQUENCY  
ghz

```
0.003000 0.030000 28 !Freq Start, Freq Stop, Num Frequencies
```



# Sampling

Angle Cut

1

0.000000 359.000000 360

AZIMUTH

90.000000



# Monostatic or bistatic

Excitation  
MONOSTATIC



# Boundary Conditions

```
Boundary Conditions
B-20-Materials.lib
4
V_FREE_SPACE => Free_Space
V_PEC => PEC
V_PMC => PMC
V_NULL => NULL
1
0 BC_PEC V_FREE_SPACE
```



## Final settings

SIE	surface integral elements
B-20A.facet	CAD description
m	meters



# Mercury MoM is Single Precision

**Example: 8 MHz**  
**Despite exact binary representation**

$$8_{10} = 1000_2$$

Start Frequency = 7.9999994E-03GHz



## Run sequence - launch

```
$./MMoM.4.1.12 b20.geo
-----
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
----- Reporting output in MB from Linux command: vmstat -s -S M -----
53113 M free memory
```



## Run sequence - sample output

```
Freq    = 30.00E+00 MHz
Lambda = 9.99E+00 m
k       = 628.75E-03 m-1
subroutine Solve_SetUp( Surface, bk, pSys, pD, Nodes ) : ...Finished
-----
---| Time : Time total for RHS solve
---| Twall = 0.0004168 ; Tcpu = 0.0002319 ; Ratio = 1.80
-----
---| Out Of Core Times: Diagonal Blocks
---|
---| nWrites.....: 2.
---| GigaBytes Write.....: 0.
---| Write Time (Hr).....: 0.00
---| Average Write Rate (MBytes/sec)..: 19.
---| nReads.....: 5.
---| GigaBytes Read.....: 0.
---| Read Time (Hr).....: 0.0002
---| Average Read Rate (MBytes/sec)..: 48.
---|
-----
Z Column Summary IO 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
```

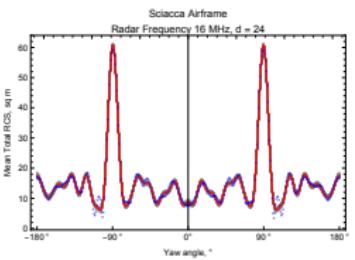
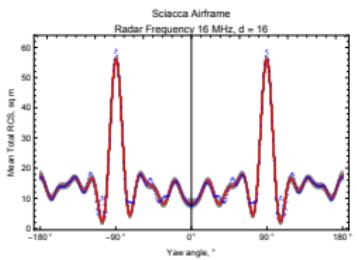
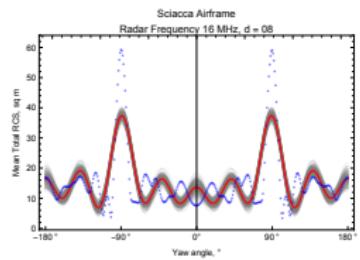
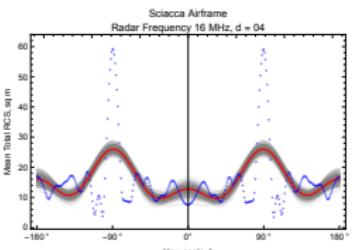
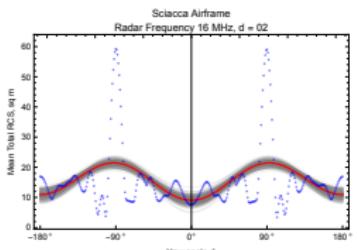
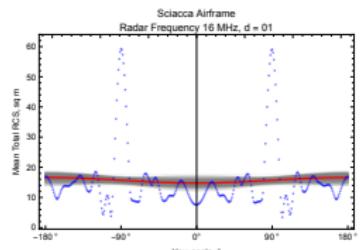


## Run sequence - completion

```
$./MMoM.4.1.12 b20.geo
-----
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
----- Reporting output in MB from Linux command: vmstat -s -S M -----
53113 M free memory
```



# Fourier Transform Visualizations at 16 MHz



## Note

Blue: Data

Red: Approximation

Gray: Error

# Fourier Transform Visualizations at 16 MHz

- **Approximation Order ( $d$ ):** The number of terms in the Fourier approximation increases as  $d = 1, 2, 4, 8, 16, 24$ .
- **Low-Order Approximations:** Approximations with smaller  $d$  (e.g.,  $d = 1, 2, 4$ ) fail to capture fine structures, leading to significant residual error.
- **Higher-Order Approximations:** As  $d$  grows, the approximation better resolves finer features, and the error (gray) shrinks significantly, especially at smooth regions.
- **Error Behavior:** The error decreases non-uniformly—large errors persist near abrupt changes or peaks due to Gibbs phenomena, but smooth regions converge faster.
- **Key Insight:** Fourier approximations demonstrate trade-offs: computational complexity increases with  $d$ , but fidelity improves.

# Fourier Transform Visualizations at 16 MHz

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- **Error Behavior:** The error decreases non-uniformly—large errors persist near abrupt changes or peaks due to Gibbs phenomena, but smooth regions converge faster.

# Fourier Transform Visualizations at 16 MHz

- **Key Insight:** Fourier approximations demonstrate trade-offs: fidelity improves with  $d$ , as does computational cost.
- **General Notes:**
  - Fourier series resolve functions as sums of sines and cosines.
  - Low-frequency terms: broad trends; high-frequency: fine details.
  - Convergence is faster for smooth functions but slower for discontinuities or sharp changes.
  - More terms improves fidelity, but can introduce numerical artifacts.

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