Data Reduction for Modelling Satellite Radar Cross Sections 1/2

Daniel Topa
daniel.topa@hii.com

Huntington Ingalls Industries
Mission Technologies

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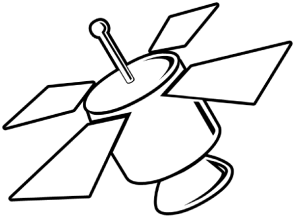
Models of Radar Cross Sections for Satellites



Spherical Harmonics Expansion

$$f(\theta,\phi) \approx a_{0,0}Y_0^0 + a_{1,-1}Y_1^{-1} + a_{1,0}Y_1^0 + a_{1,1}Y_1^1 + \dots$$

where

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}}P_n^m(\cos\theta)e^{im\phi}$$

Overview

Input and Final Output

Input: *.obj File

```
# Created with the Wolfram Language : www.wolfram.com

mtllib sp-006.mtl

# 6 vertex positions
v  0 0 -1
v  0 -1 0
v  -1 0 0
v  1 0 0
v  0 0 1
v  0 1 0
```

Output: Amplitude Vector

$$
\begin{aligned}
a_{0,0} &= 1.345 \pm 0.015 \\
a_{1,-1} &= 1.098 \pm 0.017 \\
a_{1,0} &= 1.210 \pm 0.017 \\
a_{1,1} &= 0.945 \pm 0.017 \\
a_{2,-2} &= 0.512 \pm 0.018 \\
a_{2,-1} &= 0.732 \pm 0.017 \\
a_{2,0} &= 1.110 \pm 0.017 \\
a_{2,1} &= 0.885 \pm 0.016 \\
a_{2,2} &= 0.658 \pm 0.017
\end{aligned}
$$

Beginning to End I

Data Creation and Analysis Steps

1. Start with CAD model: `*.stl`
2. Create `*.obj`(all facets, vertices)
3. Create `*.facet` (partitioned by materials)
   - Create `*.geo` (geometry)
   - Create `*.lib` (EM properties)
4. Run Mercury MoM
5. Generate `*.4112.txt`
6. Harvest θ, ϕ fields

Beginning to End II

7. Create `*.rcs`
8. Create amplitudes a

Big Picture: CAD to `*.geo`

Preparing to Run Mercury MoM

1. **Start with CAD model:** `*.stl`
2. **Finish with table** `*.geo`
3. **Partition CAD material by materials properties**

Biggest Challenge

Going from a CAD model to a model of different electromagnetic materials.

Software Components

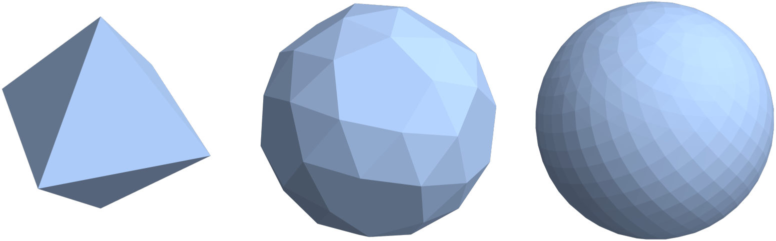
1. **converter:** `*.obj` ⇒ `*.facet`
2. **mesh analysis & repair:** `*.obj` ⇒ `*.facet`
3. **extractor: pull backscatter from** `*.4112.txt`
4. **converter: backscatter to** `*.rcs`
5. **calculator:** `*.rcs` **to spherical harmonic amplitudes**

CAD file (*.stl) to Mesh Structure File (*.obj)

Many Tools For Converting *.stl to *.obj

1. Blender
2. FreeCAD
3. OpenSCAD
4. SolidWorks
5. Tinkercad
6. MeshConvert.com
7. Online 3D Model Converter
8. others

Seeing the *.obj File



6 vertices **60 vertices** **600 vertices**

Decadal Improvement in Resolution:
Number of vertices increases $\times 10$

sp-006.obj

```
# Created with the Wolfram Language : www.wolfram.com

mtllib sp-006.mtl

# 6 vertex positions
v  0 0 -1
v  0 -1 0
v  -1 0 0
v  1 0 0
v  0 0 1
v  0 1 0

# 0 UV coordinates

# 0 vertex normals
```

```
# Mesh '' with 8 faces
usemtl Material_1
f  1/ 2/ 3/
f  2/ 1/ 4/
f  2/ 5/ 3/
f  5/ 2/ 4/
f  1/ 6/ 4/
f  6/ 1/ 3/
f  6/ 5/ 4/
f  5/ 6/ 3/
```

Components of the `*.obj`

1. **Headers and Comments (#):**
   - **Used for metadata or human-readable information.**
   - **Example:** `# Created with Wolfram Language`.
2. **Vertex Positions (`v`):**
   - **Specifies 3D coordinates for vertices.**
   - **Example:** `v 0 0 -1`.
3. **Faces (`f`):**
   - **Defines polygons by referencing vertex indices.**
   - **Example:** `f 1/2/3`.

Components of the *.obj

4. **Material Library Reference (mtllib):**
   - **External *.mtl file that specifies visual materials for rendering (e.g., color, shading)**
   - **Example: sp-006.mtl.**
   - **Important Note: This *.mtl file is not related to the electromagnetic materials library in CAD models, which defines physical properties like permittivity, permeability, or conductivity.**

short.facet I

```
facimusFacet.f08  2020-06-25 11:34:36
    1
<partName>
0
     6
     36.180340       26.286556      -22.360680
    -44.721359        0.000000      -22.360680
     44.721359        0.000000       22.360680
     36.180340      -26.286556      -22.360680
      0.000000        0.000000       50.000000
      0.000000        0.000000      -50.000000
    1
<partName>
      3      20       0       0       0       0       0
      1       2       3       0
      4       5       6       0
      2       1       7       0
      4       3       8       0
```

short.facet II

```
         9         6        10         0
         3         4        11         0
         2         7        12         0
         3         2         8         0
         2        12         8         0
        12         5         8         0
```

short.facet I

1. *.facet is organized around **components**
2. Components have **distinct electromagnetic properties**
3. Consider *.facet as *.obj for each component

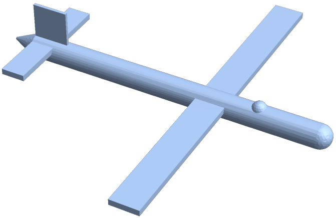
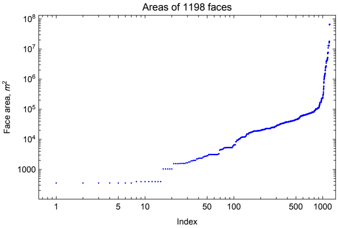
Components of the `*.facet`

1. **Aluminum substructures**
    1. **Header**
    2. **Vertex locations.**
    3. **Faces by vertex index**
2. **Titanium substructures**
    1. **Header**
    2. **Vertex locations.**
    3. **Faces by vertex index**
3. **...**

Achilles Heel: Minimum triangle size

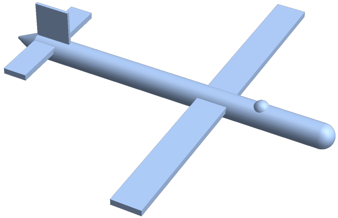
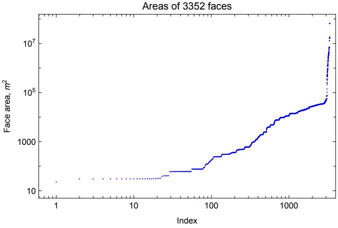
Mercury MoM is very sensitive to Spectral radius

Standard meshing, 0.05 m resolution

Mesh	Spectrum
	

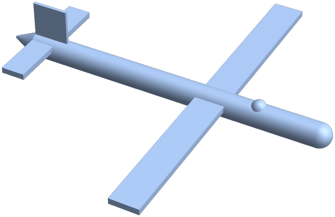
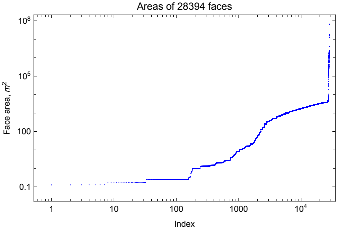
```
--------------------------------------------------------------------------------
                 ---| Mercury MOM Completed Sucessfully |---

--------------------------------------------------------------------------------
```

Standard meshing, 0.01 m resolution

Mesh	Spectrum
	

```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
 subroutine ACA_Sum_Update( A, S, Tol, RefNorm ) :  RHS: ACA did not converge
 =          0
```

Standard meshing, 0.001 m resolution

Mesh	Spectrum
	


```
--------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------FATAL ERROR-------
subroutine Geometry_TRI_Compute( Tris, tol ) :Have Triangles with effective zero area
nTris_With_Zero_Area =          60
```

1. `*.geo` describes model for Mercury MoM analysis
2. Points to `*.facet`
3. Points to Material Library
4. Configures Mercury MoM

sph-septum.geo I

```
Hemi Sphere, air diel

&MM_MOM
  bUseACA = .TRUE.,
  bSolve_ACA = .FALSE.,
  bOutOfCore = .FALSE.,
  bNormalizeToWaveLength = .TRUE.,
  bNormalize             = .FALSE.,
  dCloseLambda  = 0.100000,
  ACA_Factor_Tol = 0.000010,
  ACA_RHS_Tol = 0.000100,
  Point_Tolerance = 0.001000,
  Lop_Admissibility = WEAK,
  Kop_Admissibility = CLOSE
/

FREQUENCY
  ghz
```

sph-septum.geo II

```
  0.300000  0.000000  1

Excitation
  MONOSTATIC

Angle Cut
  1
  0.000000  360.000000  361
  AZIMUTH
  90.000000

Boundary Conditions
Mat.lib
2
R_Free_Space => Free_Space
R_PEC => PEC
3
1 BC_PEC R_PEC R_Free_Space
```

sph-septum.geo III

```
2 BC_PEC R_PEC R_Free_Space
3 BC_PEC R_PEC R_Free_Space

Geometry Type and Data
FACET
meters
sph_septum.facet
```

1. `*.lib` **contains electromagnetic properties**
2. **Makes model more realistic**
3. **Designed to be a library**

Materials.lib I

```
Material Library

Mag
  DIELECTRIC_PROP
  1
  GHz
  0.330000, (1.000000, 0.000000), (8.000000,
      -8.000000)
```

Running Mercury MoM

```
./MMoM_4.1.12 example.geo
```

Sprawling Toolset: Languages

1. **Fortran**
2. **Python**
3. **Mathematica**
4. **Shell scripts**

Sprawling Toolset: Purposes

1. **Automation**
2. **Conversions**
3. **Data Analysis**
4. **Diagnostics**

Sprawling Toolset: Design

1. Object oriented
2. Emphasis on error tracking
3. Some crude
4. Some refined

Major Fortran Tools I

1. `aeneas.f08`
2. `createFacetFile.f08`
3. `esjufjoll.08`
4. `facimusFacet.f08`
5. `facet-maker.f08`
6. `geo-writer.f08`
7. `harvestRCSfromMoM.f08`
8. `json-writer.f08`

Major Fortran Tools II

9. `gather.f08`
10. `revised-reader.f08`
11. `shaeffer.f08`
12. `sigma.f08`

esjufjollf.f08 Execution I

Listing 1: Excerpt from esjufjoll.f08

```
! dantopa:hot/eriksjokull % ./eriksjokull
! List of 10 input files in ../elevations/list-of-files.txt:
!  1. PTW-elev-0p045.4112.txt.
!  2. PTW-elev-0p050.4112.txt.
!  3. PTW-elev-0p055.4112.txt.
!  4. PTW-elev-0p060.4112.txt.
!  5. PTW-elev-0p065.4112.txt.
!  6. PTW-elev-0p070.4112.txt.
!  7. PTW-elev-0p075.4112.txt.
!  8. PTW-elev-0p080.4112.txt.
!  9. PTW-elev-0p085.4112.txt.
! 10. PTW-elev-0p090.4112.txt.
!
! * Properties of azimuth   mesh:
! * minimum value = -180.000000, maximum value = 179.000000, length = 359.000000
! * number of samples = 360, interval size = 1.00000000
!
! #  #  Dimensions for RCS data containers  #  #
!
! # Expected dimensions:
! # Number of radar frequencies scanned by MoM:   28
```

esjufjollf.f08 Execution II

```
! # Number of azimuth   angles scanned by MoM:  360
! # Number of elevation angles scanned manually: 10
!
! # Container for each MoM 4112.txt file: rcs_table_rank_2
! # Free angle dimension = 360 indices run from 1 to 360
! # Frequency dimension  = 28 indices run from 1 to 28
!
! # Container for all MoM 4112.txt files: rcs_table_rank_3
! # Free angle dimension  = 360 indices run from 1 to 360
! # Frequency dimension   = 28 indices run from 1 to 28
! # Fixed angle dimension = 10 indices run from 1 to 10
!
! Analyzing file 001/010: 'PTW-elev-0p045.4112.txt', elevation = 45.
! Analyzing file 002/010: 'PTW-elev-0p050.4112.txt', elevation = 40.
! Analyzing file 003/010: 'PTW-elev-0p055.4112.txt', elevation = 35.
! Analyzing file 004/010: 'PTW-elev-0p060.4112.txt', elevation = 30.
! Analyzing file 005/010: 'PTW-elev-0p065.4112.txt', elevation = 25.
! Analyzing file 006/010: 'PTW-elev-0p070.4112.txt', elevation = 20.
! Analyzing file 007/010: 'PTW-elev-0p075.4112.txt', elevation = 15.
! Analyzing file 008/010: 'PTW-elev-0p080.4112.txt', elevation = 10.
! Analyzing file 009/010: 'PTW-elev-0p085.4112.txt', elevation = 5.
! Analyzing file 010/010: 'PTW-elev-0p090.4112.txt', elevation = 0.
```

facet-maker.f08 Execution I

Listing 2: Excerpt from facet-maker.f08

```
! dantopa:rcs/facet % ./facet-maker B20-standard-1m
     (master)fortran-alpha
!
! target directory: ./data/
! input  file: ./data/B20-standard-1m.obj
! output file: ./data/B20-standard-1m.facet
!
! Opening ./data/B20-standard-1m.obj to read data lists.
!
! Opening ./data/B20-standard-1m.facet for writing.
!
! completed at 2020-04-08 16:05:04
```

gather.f08 Execution I

Listing 3: Excerpt from gather.f08

```
! dantopa:3d/xylorimba % ./gather
    (master)fortran-alpha
! 1. file = PTW-elev-0n179.4112.txt, -179, elevation angle = 0.
! 2. file = PTW-elev-0n178.4112.txt, -178, elevation angle = 0.
! 3. file = PTW-elev-0n177.4112.txt, -177, elevation angle = 0.

! 358. file = PTW-elev-0p178.4112.txt, 178, elevation angle = 0.
! 359. file = PTW-elev-0p179.4112.txt, 179, elevation angle = 0.
! 360. file = PTW-elev-0p180.4112.txt, 180, elevation angle = 0.
!
! completed at 2020-05-12 21:49:23
```

revised-reader.f08 Excerpts I

Listing 4: Excerpt from revised-reader.f08 (lines 39-45)

```
! dantopa:readers/cleat % ./revised-reader

    (master)fortran-alpha
! B-20A.4112.txt has 14843 lines
! B-20A.4112.txt has 28 frequencies.
! 3.00000000
! '...Finished' found at line 854
! 4.00000000
! '...Finished' found at line 1368
```

Listing 5: Excerpt from revised-reader.f08 (lines 123-127)

```
! 26. nu = 28.0000000, start = 13344, stop = 13703, terms = 360
! 27. nu = 29.0000000, start = 13858, stop = 14217, terms = 360
! 28. nu = 30.0000000, start = 14372, stop = 14731, terms = 360
!
! completed at 2020-05-09 15:30:20
```

sigma.f08 Overview I

Listing 6: Excerpt from sigma.f08

```
! nb: /Users/dantopa/Mathematica_files/nb/ert/mercury/snake/fortran-01.nb
program rcs

! Read the Mercury Methods of Moments processed into a table of mean total RCS
    values
! Use the method of least squares to find
!   RCS ( yaw angle )            radar frequency fixed
!   RCS ( radar frequency )      yaw angle fixed
! Daniel Topa, ERT Corp

! Class structure
!   RCStable: table of mean total RCS ( nu, alpha )
!   LinearSystem: Sytem Matrix A, data vector b
!        flavors: Fourier, monomial
!        tied to RCStable
!   LeastSquaresResults:
!        amplitudes
!        errors
!        residual error vector
!        tied to linear system
```

Mathematica Commands I

```
(* Generate a mesh for a convex hull of points on a sphere *)
mesh = ConvexHullMesh[SpherePoints[60]] // Region;

(* Extract vertex coordinates from the mesh *)
vlist = MeshCoordinates[mesh];

(* Label each point in the mesh *)
labeledPoints = MapIndexed[
    Text[Style[ToString[#2[[1]]], Bold, Black], #1] &, vlist
];
```

Python Tool for *.obj to *.facet I

```
from datetime import datetime
from Facet import Facet
from Vertex import Vertex

import io
import os
import sys

DEFAULT_ELEMENT_DESCRIPTION    = '3␣{}␣0␣0␣0␣0␣0'
DEFAULT_FILE_EXTENSION_OUTPUT = '.facet'
DEFAULT_PART_COUNT            = '1'
DEFAULT_PART_MIRROR           = '0'
DEFAULT_PART_NAME             = '<PTW␣MeshModel>'
DEFAULT_SUBPART_COUNT         = '1'
DEFAULT_SUBPART_NAME          = '<PTW␣MeshSheet>'

argumentCount = len(sys.argv)

# output argument-wise
if argumentCount == 2:
    objectFileName = sys.argv[1]
    outputFileName = os.path.splitext(objectFileName)[0] +
         DEFAULT_FILE_EXTENSION_OUTPUT
elif argumentCount == 3:
```

Python Tool for *.obj to *.facet II

```
    objectFileName = sys.argv[1]
    outputFileName = sys.argv[2]
else:
    sys.stderr.write('Usage: python Obj2Facet.py <input-obj-file-name> 
        [<output-facet-file-name>]\n')
    sys.exit()

facetCount  = 0
facetLines  = ""
vertexCount = 0
vertexLines = ""
with io.open(objectFileName, 'r', encoding='utf-8') as objectFile:
    line = objectFile.readline()
    lineNumber = 1
    while line:
        tokens = line.strip().split(' ')
        if len(tokens) == 4:
            type = tokens[0]

            if type.lower() == 'f':
                facetLines += ' '.join(tokens[1:4])
                facetLines += ' 0'
                facetLines += '\n'
                facetCount += 1
```

Python Tool for *.obj to *.facet III

```
            elif type.lower() == 'v':
                vertexLines += ' '.join(tokens[1:4])
                vertexLines += '\n'
                vertexCount += 1

        line        = objectFile.readline()
        lineNumber += 1

    objectFile.close()

with io.open(outputFileName, 'w', encoding='utf-8') as outputFile:
    outputFile.write('FACET FILE V3.4 ')
    outputFile.write(datetime.today().strftime('%d-%b-%Y %H:%M:%S'))
    outputFile.write('\n')

    outputFile.write(DEFAULT_PART_COUNT)
    outputFile.write('\n')
    outputFile.write(DEFAULT_PART_NAME)
    outputFile.write('\n')
    outputFile.write(DEFAULT_PART_MIRROR)
    outputFile.write('\n')

    outputFile.write(str(vertexCount))
```

Python Tool for *.obj to *.facet IV

```
    outputFile.write('\n')
    outputFile.write(vertexLines)

    outputFile.write(DEFAULT_SUBPART_COUNT)
    outputFile.write('\n')
    outputFile.write(DEFAULT_SUBPART_NAME)
    outputFile.write('\n')

    outputFile.write(DEFAULT_ELEMENT_DESCRIPTION.format(facetCount))
    outputFile.write('\n')
    outputFile.write(facetLines)

    outputFile.close()
```

obj_model.py for Obj2Facet_Python3.py I

```python
from face   import Face
from vertex import Vertex

class ObjModel(object):

#region Constructors

    def __init__(self                         ,
                 faces   : [Face   ] = None,
                 vertices: [Vertex] = None) -> None:

        self.__faces   : [Face   ] = [] if faces    is None else faces
        self.__vertices: [Vertex] = [] if vertices is None else vertices

#endregion

#region Properties

    @property
    def faces(self) -> [Face]:
        return self.__faces

    @faces.setter
    def faces(self, value: [Face]) -> None:
```

obj_model.py for Obj2Facet_Python3.py II

```python
            self.__faces = value

    @property
    def vertices(self) -> [Vertex]:
        return self.__vertices

    @vertices.setter
    def vertices(self, value: [Vertex]) -> None:
        self.__vertices = value

#endregion

#region Overridden/Implemented Methods

    def __str__(self) -> str:
        return (
            "faces=["                                                    +
            " ".join(f"({face!s})" for face in self.__faces)            +
            "],vertices=["                                               +
            " ".join(f"({vertex!s})" for vertex in self.__vertices)     +
            "]"
        )

#endregion
```

`obj_model.py` for `Obj2Facet_Python3.py` III

OBJ to FACET Conversion Tools I

```
|-- face.py
|-- facet_converter.py
|-- facet_file_reader.py
|-- facet_file_writer.py
|-- facet_model.py
|-- file_read_exception.py
|-- file_reader.py
|-- obj_converter.py
|-- obj_facet_conversion.py
|-- obj_file_reader.py
|-- obj_file_writer.py
|-- obj_model.py
|-- part.py
|-- subpart.py
|-- test/
```

OBJ to FACET Conversion Tools II

```
|   |-- test_facet_file_reader.py
|   `-- test_obj_file_reader.py
`-- vertex.py
```

Tools for *.4112.txt

```
|-- MercuryMoMProcessor.pyproj
|-- MercuryMoMProcessor.sln
|-- README.md
|-- file_read_exception.py
|-- mercury_mom_output_file_reader.py
|-- mercury_mom_processor.py
|-- output
|-- result_data.py
|-- result_set.py
|-- sample
|    |-- sphereCourse.4112.txt
|    |-- test_mercury_mom_output_file_reader.py
```

Shell Scripts

1. Initialize environment for Mercury MoM
2. Run special cases
3. Data wrangling

Initialize Runtime Environment I

```
# #!/bin/zsh
# https://stackoverflow.com/questions/9901210/bash-source0-equivalent-in-zsh
#printf '%s\n' "$(date) $(tput bold)${(%):-%N}$(tput sgr0)"

#!/bin/bash
printf "%s\n" "$(date),␣$(tput␣bold)${BASH_SOURCE[0]}$(tput␣sgr0)"

echo "ulimit␣-s␣unlimited"
      ulimit -s unlimited

echo "export␣OMP_NUM_THREADS=10"
      export OMP_NUM_THREADS=10

echo 'export␣LD_LIBRARY_PATH="/Users/dantopa/Dropbox/2nd-generation/RCS-project/Mercury␣MOM/MM_Distribution_4.1.12/Linux64/redistributables/":${LD_LIBRARY_PATH}'
      export LD_LIBRARY_PATH="/Users/dantopa/Dropbox/2nd-generation/RCS-project/Mercury␣MOM/MM_Distribution_4.1.12/Linux64/redistributables/":${LD_LIBRARY_PATH}


echo "\${OMP_NUM_THREADS}␣=␣${OMP_NUM_THREADS}"
echo "\${LD_LIBRARY_PATH}␣=␣${LD_LIBRARY_PATH}"
```

Resolution Sweep for Calibration I

```
#!/bin/bash
printf "%s\n" "$(date),␣$(tput␣bold)${BASH_SOURCE[0]}$(tput␣sgr0)"

# counts steps in batch process
export counter=0
function new_step(){
    counter=$((counter+1))
    echo ""
    echo "Step␣${counter}:␣${1}"
}

new_step "Set␣environment␣variables"

export OMP_NUM_THREADS=10
export LD_LIBRARY_PATH="/usr/lib64":"$(pwd)":"${LD_LIBRARY_PATH}"

echo "ulimit␣=␣$(ulimit)"
echo "\${OMP_NUM_THREADS}␣=␣${OMP_NUM_THREADS}"
echo "\${LD_LIBRARY_PATH}␣=␣${LD_LIBRARY_PATH}"

facets="Sphere-S000001␣Sphere-S00001␣Sphere-S0001␣Sphere-S001␣Sphere-S01"

new_step "cd␣/Dropbox/2nd-generation/RCS-project/4.1.12/Linux64/bin"
          cd /Dropbox/2nd-generation/RCS-project/4.1.12/Linux64/bin
```

Resolution Sweep for Calibration II

```
for f in ${facets}; do
    new_step "Run ${f}"
    cp sphere/sphereTemplate.geo ${f}.geo
    cp sphere/${f}.facet .
    sed -i 's/FILE_A/'${f}'/' ${f}.geo
    sed -i 's/FILE_B/'${f}'.facet/' ${f}.geo
    echo "./MMoM_4.1.12 ${f}.geo > sphere/${f}-run.out"
          ./MMoM_4.1.12 ${f}.geo > sphere/${f}-run.out
    mv ${f}.geo      sphere/.
    mv ${f}.4112.txt sphere/.
    rm ${f}.facet
done

new_step "Exit"
echo "time used = ${SECONDS} s"
date
```

Sweeping Polar Angle I

```
#! /bin/bash
printf "%s\n" "$(tput␣bold)$(date)␣${BASH_SOURCE[0]}$(tput␣sgr0)"

# generated by geo_writer.f08.
#  2020-05-09 23:05:02

# sequence of operations
# 1. bring in geometry file for specific elevation
# 2. run Mercury MoM - pipe into outputs folder
# 3. move Mercury MoM results file to outputs folder
# 4. remove geometry file

# sequence of operations
# 1. bring in geometry file for specific elevation
# 2. run Mercury MoM - pipe into outputs folder
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# 4. remove geometry file

# directory structure:

# ./
#   -- bin
#        MMoM_4.1.12
#        elevation-sweep.sh
```

Sweeping Polar Angle II

```
#    -- inputs
#         facet file
#         geo files
#         materials library (empty file)
#    -- outputs
#         MoM results *.4112.txt
#         MoM run log *.out

# counts steps in batch process
export counter=0
export SECONDS=0
function new_step(){
    export counter=$((counter+1))
    export subcounter=0
    echo ""; echo ""
    echo "Step␣${counter}:␣${1}"
}
function sub_step(){
    subcounter=$((subcounter+1))
    echo ""
    echo "␣␣Substep␣${counter}.${subcounter}:␣${1}"
}

# # # shared files
```

Sweeping Polar Angle III

```
echo 'cp␣../inputs/PTW-Materials.lib␣.'
      cp ../inputs/PTW-Materials.lib .
echo 'cp␣../inputs/PTW.facet␣.'
      cp ../inputs/PTW.facet .

export ticks=${SECONDS}
new_step 'elevation␣angle␣=␣-89␣deg␣from␣North␣Pole'

sub_step 'cp␣../inputs/PTW-elev-0n089.geo␣.'
          cp ../inputs/PTW-elev-0n089.geo .

sub_step './MMoM_4.1.12␣PTW-elev-0n089.geo␣>␣../outputs/PTW-elev-0n089.out'
          ./MMoM_4.1.12 PTW-elev-0n089.geo > ../outputs/PTW-elev-0n089.out

sub_step 'mv␣*.4112.*␣../outputs/.'
          mv *.4112.* ../outputs/.

sub_step 'rm␣PTW-elev-0n089.geo'
          rm PTW-elev-0n089.geo

echo ""
echo "time␣used␣to␣run␣PTW-elev-0n089.geo␣=␣$((SECONDS-ticks))␣s"


```

Sweeping Polar Angle IV

```
new_step 'clear␣shared␣files'
sub_step 'rm␣PTW-Materials.lib'
          rm PTW-Materials.lib
sub_step 'rm␣PTW.facet'
          rm PTW.facet
sub_step 'rm␣-r␣tempDir/'
          rm -r tempDir/

echo ""
echo "time␣used␣=␣${SECONDS}␣s"
date
```

Running Code, Sorting Output I

```
#!/bin/bash
printf "%s\n" "$(date), $(tput bold)${BASH_SOURCE[0]}$(tput sgr0)"

# counts steps in batch process
export counter=0
export SECONDS=0
function new_step(){
    counter=$((counter+1))
    echo ""
    echo "Step ${counter}: ${1}"
}

new_step "Set environment variables"
export OMP_NUM_THREADS=10
export LD_LIBRARY_PATH="/usr/lib64":"$(pwd)":"${LD_LIBRARY_PATH}"
export mom=$(dirname $PWD)

# MoM Users Manual, p. 3
ulimit -s unlimited

echo "ulimit = $(ulimit)"
echo "\${OMP_NUM_THREADS} = ${OMP_NUM_THREADS}"
echo "\${LD_LIBRARY_PATH} = ${LD_LIBRARY_PATH}"

```

Running Code, Sorting Output II

```
new_step "Run␣Sciacca␣example:␣PTW"

new_step "Identify␣source␣files"
export stem="B-20A"
facets="${stem}-S-1000m␣${stem}-S-0100m␣${stem}-S-0050m␣${stem}-S-0010m"
echo "\${facets}␣=␣${facets}"

new_step "cd␣${mom}"
          cd ${mom}

for f in ${facets}; do
    new_step "Run␣${f}"
    cp ${stem}/template-${stem}.geo ${f}.geo
    cp ${stem}/${f}.facet .
    sed -i 's/FILE_A/'${f}'/'       ${f}.geo
    sed -i 's/FILE_B/'${f}'.facet/' ${f}.geo
    echo "./MMoM_4.1.12␣${f}.geo␣>␣${stem}/${f}-run.out"
          ./MMoM_4.1.12 ${f}.geo > ${stem}/${f}-run.out
    mv ${f}.geo      ${stem}/.
    mv ${f}.4112.txt ${stem}/.
    rm ${f}.facet
done

new_step "Exit"
```

Running Code, Sorting Output III

```
echo "time used = ${SECONDS} s"
date
```

Facet (Face)

- Discretized as small triangular or quadrilateral elements.
- Supports surface currents ($\vec{J}$) induced by incident fields.
- Enforces boundary conditions derived from Maxwell's equations:
  - PEC: $\vec{E}_t = \vec{0}$
  - Dielectric: $\vec{E}_t^{(1)} = \vec{E}_t^{(2)}, \quad \vec{H}_t^{(1)} - \vec{H}_t^{(2)} = \vec{K}$
- Surface currents are discretized using basis functions (e.g., RWG).
- Integral equations relate $\vec{J}$ to scattered fields via Green's functions.

Edges

- **Shared boundaries between adjacent facets.**
- **Enforces physical continuity of surface current, $\vec{J}$.**
- **Charge conservation at the edge:**

$$\nabla_s \cdot \vec{J} = -j\omega\rho \tag{4.1}$$

where ρ is the surface charge density.

- **Used in testing (e.g., Galerkin's method) to evaluate interaction integrals.**

Boundary Conditions

- **Maxwell's boundary conditions on facets:**

$$\textbf{PEC:} \quad \vec{E}_t = \vec{0}$$

$$\textbf{Dielectric:} \quad \vec{E}_t^{(1)} = \vec{E}_t^{(2)}$$

$$\vec{H}_t^{(1)} - \vec{H}_t^{(2)} = \vec{K}$$

- **Continuity enforced on edges:**

$$\vec{J}_{\textbf{facet 1}} \cdot \hat{n}_{\textbf{edge}} = \vec{J}_{\textbf{facet 2}} \cdot \hat{n}_{\textbf{edge}}$$

- **Ensures no spurious currents or charge accumulation.**

Interplay Between Face and Edge

- **Facet**: Supports surface currents $\vec{J}$ and tangential electric field $\vec{E}_t$.
- **Edge**: Ensures:
  - Continuity of $\vec{J}$ across facets.
  - Charge conservation, (4.1):
- Maxwell's equations are satisfied numerically:

$$\nabla \times \vec{H} = \vec{J} + j\omega\epsilon\vec{E}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

RWG Basis Functions – Overview

- Used to represent surface currents ($\vec{J}$) in MoM simulations.
- Defined on pairs of adjacent triangular elements sharing an edge.
- Ensures:
  - Continuity of surface current across shared edges.
  - Sparse and efficient numerical representation.
- Piecewise linear variation within triangles.

RWG Basis Function Definition

- **For two adjacent triangles T^+ and T^- sharing edge l_n:**
- **RWG function $\vec{f}_n(\vec{r})$:**

$$\vec{f}_n(\vec{r}) = \begin{cases} \frac{l_n}{2A^+}(\vec{r} - \vec{r}_+), & \vec{r} \in T^+ \\ \frac{l_n}{2A^-}(\vec{r}_- - \vec{r}), & \vec{r} \in T^- \\ 0, & \textbf{otherwise} \end{cases} \tag{1}$$

- **Parameters:**
  - l_n: **Length of the shared edge.**
  - A^+, A^-: **Areas of triangles T^+ and T^-.**
  - $\vec{r}_+$, $\vec{r}_-$: **Opposite vertices in T^+, T^- relative to l_n.**

Surface Current Representation

- Total surface current density $\vec{J}(\vec{r})$:

$$\vec{J}(\vec{r}) = \sum_{n} I_n \vec{f}_n(\vec{r}) \tag{4.2}$$

- I_n: Coefficients representing the current magnitude for basis function n.
- RWG basis functions provide local support, simplifying matrix assembly.

Matrix Assembly in MoM

- **Integral form of Maxwell's equations discretized using RWG functions.**
- **Resulting system of equations:**

$$\mathbf{ZI} = \mathbf{V} \tag{4.3}$$

- **Terms:**
  - $\mathbf{Z}$: **Impedance matrix from basis function interactions.**
  - $\mathbf{I}$: **Vector of current coefficients (I_n).**
  - $\mathbf{V}$: **Excitation vector from incident fields.**

Key Properties of RWG

- **Continuity**:

$$\vec{J}_{\text{facet 1}} \cdot \hat{n}_{\text{edge}} = \vec{J}_{\text{facet 2}} \cdot \hat{n}_{\text{edge}} \tag{4}$$

  Ensures smooth current flow across edges.

- **Sparse Representation**:
  - Non-zero support only on two triangles sharing an edge.
- **Accuracy**:
  - Captures linear current variations.
  - Suitable for arbitrary geometries.

Summary of RWG Functions

- Represent surface currents in MoM using triangular mesh discretization.
- Defined on pairs of adjacent triangles sharing a common edge.
- Ensure:
  - Continuity of surface currents across edges.
  - Sparse, efficient representation of $\vec{J}$.
- Efficient matrix assembly in MoM simulations.

Impedance Matrix

- **Each element Z_{mn} evaluates interaction between basis functions:**

$$Z_{mn} = \iint \vec{f}_m(\vec{r}) \cdot \vec{G}(\vec{r}, \vec{r}') \cdot \vec{f}_n(\vec{r}')\, \mathbf{d}S\, \mathbf{d}S' \qquad (2)$$

- **Terms:**
  - $\vec{f}_m(\vec{r})$: **RWG basis functions.**
  - $\vec{G}(\vec{r}, \vec{r}')$: **Green's function coupling source and observation points.**
- **Dense matrix, costly to compute and store.**

Impedance Matrix

- Each element Z_{mn} evaluates interaction between basis functions:

$$Z_{mn} = \iint \vec{f}_m(\vec{r}) \cdot \vec{G}(\vec{r}, \vec{r}') \cdot \vec{f}_n(\vec{r}')\, \mathbf{d}S\, \mathbf{d}S' \tag{2}$$

- Terms:
  - $\vec{f}_m(\vec{r})$: RWG basis functions.
  - $\vec{G}(\vec{r}, \vec{r}')$: Green's function coupling source and observation points.
- Dense matrix, costly to compute and store.

Excitation Vector

- **Represents contribution of incident fields:**

$$V_m = \iint \vec{f}_m(\vec{r}) \cdot \vec{E}_{\textbf{inc}}(\vec{r})\, \mathbf{d}S \tag{3}$$

- **Terms:**
  - $\vec{E}_{\textbf{inc}}(\vec{r})$: **Incident electric field.**
  - $\vec{f}_m(\vec{r})$: **RWG basis function.**

Physical and Numerical Behavior

- **Surfaces Reflect**:
  - Represent scattering and reflection of electromagnetic waves.
  - Surface currents ($\vec{J}$) induced by incident fields.
- **Edges Ring**:
  - Enforce continuity of surface currents across facets.
  - Numerical challenges can cause spurious oscillations.
  - Proper charge conservation ensures stable edge behavior.

Challenges in Solving the System

- $\mathbf{Z}$ is dense:
  - High memory requirement ($O(N^2)$).
  - Computationally expensive for direct solvers ($O(N^3)$).
- Ill-conditioning may require preconditioning.

Solution Techniques

- **Direct Solvers**:
  - **Gaussian elimination or LU decomposition.**
  - **Cost:** $O(N^3)$.
- **Iterative Solvers**:
  - **Conjugate Gradient (CG), GMRES.**
  - **Cost per iteration:** $O(N^2)$.
  - **Requires preconditioning for convergence.**
- **Fast Multipole Method (FMM)**:
  - **Reduces complexity to** $O(N \log N)$.
  - **Approximates far-field interactions.**

Summary of Linear System and Solutions

- **Linear system:**

$$\mathbf{ZI} = \mathbf{V} \tag{1}$$

- **Key challenges:**
  - **Dense, large-scale matrix $\mathbf{Z}$.**
  - **Computational cost of direct solvers.**
- **Efficient techniques:**
  - **Iterative solvers for large systems.**
  - **FMM for reducing complexity.**

Literature Survey I

- **Electromagnetic Scattering and MoM:**
  - Harrington (1967, 1987): Foundational work on the Method of Moments for electromagnetic problems Harrington 1967; Harrington 1987.
  - Rao (1980): Triangular patch modeling for arbitrarily shaped surfaces Rao 1980.
  - Mosig (2024): Historical insights into MoM and its applications in electrodynamics Mosig 2024.
- **Radar Cross Section (RCS):**
  - Gordon (1975): Far-field approximations for scattered fields Gordon 1975.

Literature Survey II

- Knott et al. (2004): Comprehensive guide on RCS prediction and measurement Knott, Schaeffer, and Tulley 2004.
- Crocker (2020): Dynamic RCS data handling and analysis Crocker 2020.

- **Müntz-Szász Theorem and Approximation:**
  - Siegel (1972), Sedletskii (2008): Extensions of approximation theorems in weighted spaces Siegel 1972; Sedletskii 2008.
  - Szasz (1916): Approximation by aggregates of powers Szász 1916.
- **Numerical Integration and Harmonics:**

Literature Survey III

- Colombo (1981): Harmonic analysis on spheres for numerical applications Colombo 1981.
- Bellet et al. (2022): Quadrature techniques on the cubed sphere Bellet, Brachet, and Croisille 2022.

- Computational Methods and Advancements:
  - Newman (1991): Introduction to MoM for computational physics Newman and Kingsley 1991.
  - Taddei et al. (2014): Fast MoM algorithms for phased arrays Taddei et al. 2014.

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Data Reduction for Modelling Satellite Radar Cross Sections 1/2

Daniel Topa
daniel.topa@hii.com

Huntington Ingalls Industries
Mission Technologies

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