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Comparison of iteration convergences of SIE and VSIE for solving electromagnetic scattering problems for coated objects

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[1] The surface integral equation approach and the hybrid surface-volume integral equation approach are compared for solving the problem of electromagnetic scattering by conducting objects with dielectric coating. The surface integral equation is formulated by the PMCHW method, and it is efficient for modeling bulk material scattering. The hybrid surface-volume integral equation approach is better conditioned for the volume scattering problems. For a special class of problems in which the dielectric coating is electrically thin (one to two grid sizes), the numbers of unknowns needed by the two approaches are of the same order, and the surface-volume integral equation approach converges significantly faster than the pure surface integral equation approach. Several numerical examples are given, and the results showed good agreements with the above statement. *Index Terms:* 0669 Electromagnetics: Scattering and diffraction; 0644 Electromagnetics: Numerical methods; 0619 Electromagnetics: Electromagnetic theory; *Keywords:* electromagnetic scattering, moment method, surface integral equation, volume integral equation

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1. Introduction

[2] With the aid of fast solvers, the integral equation approach has been widely used in solving problems of electromagnetic scattering from large and complex structures. It has been reported that a multilevel fast multipole algorithm has been applied to calculate the radar cross section of a model airplane in X-band (with 9.99 million unknowns) [*Song and Chew*, 2000]. Though there are some researches on fast matrix inversions, most fast algorithms developed so far are based on iterative solvers. It is known that for iterative solvers, the major computer resource usages are (1) time and memory needed to perform each matrix-vector multiplication, and (2) time needed for multiple iterations for a convergence solution. For solving an integral equation, the fast algorithms are mainly designed for the reduction of the computational complexity of a matrix-vector multiplication. For example, the multilevel fast multipole algorithm has been successfully applied to reduce the computational complexity from $O(N^2)$ to $O(N\log N)$ for a matrix-vector multiplication in solving electromagnetic scattering from 3-D complex conducting objects [*Song and Chew*, 1995, 2000; *Rokhlin*, 1990; *Coifman et al.*, 1993]. This leaves the number of iterations an important part for overall solution time when multiple right-hand sides are considered. For some structures, the matrix may be poorly conditioned and convergence can be very slow. The convergence rate depends on a number of factors, such as the iterative solvers used, the target characteristics, and the equations used to model the physical problem. For example, in the integral equation approach, the combined field integral equation (CFIE) convergences much faster than that of the electric field integral equation (EFIE) for conducting scatters [*Song and Chew*, 1995] as well as for homogeneous dielectric scatters [*Harrington*, 1989]. Various efforts have been made to increase the convergence rate in the iterative solution of a matrix equation. These include the use of CFIE, better conditioned formulations, and preconditioning. As a result, if a problem allows a choice of different operator equations, one criterion to make a selection is to consider the rates of convergence for the operators.

[3] To calculate the EM scattering from 3-D coated structures, both the surface integral equation (SIE) and the hybrid volume-surface integral equation (VSIE) approaches can be applied given that the coating material is piece-wise homogeneous (required by SIE approach). An intuitive comparison may conclude that the SIE may need less number of unknowns than that of the VSIE. This is true when the dielectric material region has a large fractional volume. However, for thin material coatings of one or two layers (one tenth of a wavelength per layer), we can show that the number of unknowns

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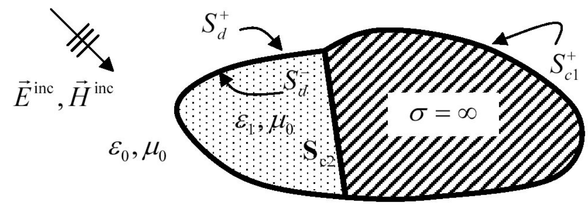


Figure 1. The configuration of the scattering problem. The scatter consists of a conducting part and a dielectric part (coating). The incident wave is a plane wave that is specified by $\vec{E}^{\mathrm{inc}}$, $\vec{H}^{\mathrm{inc}}$.

needed by SIE and VSIE are of the same order. It is known that for the modeling of large-scale problems, the multilevel fast multipole algorithm can be applied to both SIE and VSIE to reduce the computational complexity to the same order. Under this circumstance, a key factor to determine which algorithm to be used will depend on which one needs less iteration numbers to obtain a convergent solution. Because the integral operator in the VIE is less singular than that in the SIE, it is expected that the VSIE approach will need less number of iterations compared with the SIE approach for scattering problems with material coating. In this paper, we present some numerical results to compare the convergence rate of SIE versus VSIE for solving a scattering problem. In the following, we will first present a brief formulation of the two algorithms in frequency domain with time factor $\exp(-i\omega t)$, followed by some numerical examples to compare their performances in solving scattering problems. Based on the examples, we conclude that for thin material coating, the VSIE algorithm has significant advantage over the SIE approach if iterative solvers are used.

2. Discretization of SIE and VSIE

[4] The general problem of electromagnetic scattering by a coated conducting scatterer is shown in Figure 1, in which, the coating can be of full or of partial. For the sake of simplicity, we assume the coating material is of non-magnetic, (i.e., $\mu = \mu_0$), the dielectric permittivity (ε_1) is homogeneous, and the background is free-space. The surface integral equation for a coated scattering problem has been presented in a number of references. Here we use the PMCHW formulation [*Wu and Tsai*, 1977] that was developed by Poggio, Miller, Chang, Harrington, and Wu.

[5] To establish the equations, we assume that the equivalent electric current and magnetic current on the dielectric surface S_d are $\vec{J}_d$, and $\vec{M}_d$, and the induced electric current on S_{c1} and S_{c2} are $\vec{J}_{c1}$and $\vec{J}_{c2}$, respectively. The integral equations are established using the boundary conditions on the dielectric surface and the conducting surfaces. For this purpose, we define two operators $\overleftrightarrow{L}$ and $\overleftrightarrow{K}$ that map the electric current to electric field, and magnetic current into magnetic field,

$$\overleftrightarrow{L}_{\alpha}^{\pm}\cdot\vec{J} = \int_S (\overleftrightarrow{I} + k_{\alpha}^{-2}\nabla\nabla)G(k_{\alpha},\vec{r},\vec{r}\,')\cdot\vec{J}(\vec{r}\,')dS', \vec{r}\in S^{\pm}, \tag{1}$$

$$\overleftrightarrow{K}_{\alpha}^{\pm}\cdot\vec{M} = \nabla\times\int_S G(k_{\alpha},\vec{r},\vec{r}\,')\cdot\vec{M}(\vec{r}\,')dS', \vec{r}\in S^{\pm}, \tag{2}$$

where the superscript "$\pm$" indicates that field is evaluated on the positive side or negative side of the surface, $G(k_\alpha,\vec{r},\vec{r}\,') = \exp(ik_\alpha|\vec{r}-\vec{r}\,'|)/(4\pi|\vec{r}-\vec{r}\,'|)$ is the 3-D scalar Green's function, and the subscript $\alpha = 0, 1$ indicates the wavenumber that is used in the Green's function. On the two sides of the dielectric surface, the tangent components of electric field and magnetic field are continuous. This leads to the following equations [*Wu and Tsai*, 1977; *Luo*, 2001]:

$$\begin{aligned}(ik_0\eta_0\overleftrightarrow{L}_0^{+} + ik_1\eta_1\overleftrightarrow{L}_1^{-})\cdot\vec{J}_d - (\overleftrightarrow{K}_0^{+} + \overleftrightarrow{K}_1^{-})\cdot\vec{M}_d \\ + ik_0\eta_0\overleftrightarrow{L}_0\cdot\vec{J}_{c1} - ik_1\eta_1\overleftrightarrow{L}_1^{-}\cdot\vec{J}_{c2} = -\vec{E}^{\mathrm{inc}},\end{aligned} \tag{3}$$

$$\begin{aligned}(\overleftrightarrow{K}_0^{+} + \overleftrightarrow{K}_1^{-})\cdot\vec{J}_d + (ik_0\eta_0^{-1}\overleftrightarrow{L}_0^{+} + ik_1\eta_1^{-1}\overleftrightarrow{L}_1^{-})\cdot\vec{M}_d \\ + \overleftrightarrow{K}_0\cdot\vec{J}_{c1} - \overleftrightarrow{K}_1^{-}\cdot\vec{J}_{c2} = -\vec{H}^{\mathrm{inc}}.\end{aligned} \tag{4}$$

Here, $\eta_\alpha = \sqrt{\mu_\alpha/\varepsilon_\alpha}$ is the wave impedance in the media with ε_α and μ_α. On the two conducting surfaces, the tangent component of the electric field vanishes, leading to

$$\begin{aligned}ik_0\eta_0\overleftrightarrow{L}_0^{+}\cdot\vec{J}_d - \overleftrightarrow{K}_0^{+}\cdot\vec{M}_d + ik_0\eta_0\overleftrightarrow{L}_0^{+}\cdot\vec{J}_{c1} \\ = -\vec{E}^{\mathrm{inc}}, \vec{r}\in S_{c1},\end{aligned} \tag{5}$$

$$-ik_1\eta_1\overleftrightarrow{L}_1\cdot\vec{J}_d + \overleftrightarrow{K}_1\cdot\vec{M}_d + ik_1\eta_1\overleftrightarrow{L}_1\cdot\vec{J}_{c2} = 0, \vec{r}\in S_{c2}. \tag{6}$$

It should be pointed out that the tangent components are used and implied in equations (3) to (6).

[6] The volume-surface integral equations are also formulated using the equivalent principles and the boundary conditions [*Lu and Chew*, 1999, 2000]. In this formulation, the dielectric material is removed and replaced with induced volume electric current $\vec{J}_V$, and the conducting object is removed and replaced with induced surface electric current $\vec{J}_S$. The volume current and the surface current radiate in free space. Two integral equations are needed to determine the two unknown functions. One is established with the volume equivalent theorem which states that the total electric

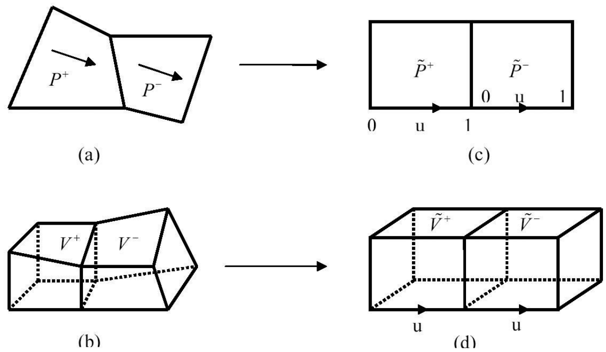


Figure 2. (a) Two adjacent patches in surface mesh. (b) Two adjacent volume cells in volume mesh. (c) The two adjacent surface patches in u-v plane as two unit squares. (d) The two adjacent volume cells in u-v-w space as two unit cubes.

field inside the dielectric material is the superposition of the incident and the scattered field, i.e.,

$$\vec{E} = \vec{E}^{\,\mathrm{inc}} + ik_0\eta_0\vec{L}_0 \cdot \vec{J}_V + ik_0\eta_0\vec{L}_0 \cdot \vec{J}_S, \vec{r} \in V. \tag{7}$$

It should be noted that by Ampere's law, the volume electric current $\vec{J}_V$ is related to the total electric field intensity by $\vec{J}_V = i\omega(\varepsilon_0 - \varepsilon_1)\vec{E}$. Hence there are only two unknown functions, $\vec{J}_V$ and $\vec{J}_S$ in equation (7). The other integral equation is formulated with the aid of the boundary condition on the conducting surface, and is given by

$$ik_0\eta_0\vec{L}_0 \cdot \vec{J}_V + ik_0\eta_0\vec{L}_0 \cdot \vec{J}_S = -\vec{E}^{\,\mathrm{inc}}, \vec{r} \in S_c. \tag{8}$$

Again the tangent components are used and implied in equation (8). The two integral equations, (7) and (8), will be solved simultaneously to determine the unknown currents. If the VSIE and SIE are used to model the same scatterer, then the volume V in equation (7) is the dielectric region, and the surface S_c in equation (8) is the union of S_{c1} and S_{c2} that appeared in equations (5) and (6).

[7] To solve the integral equations by the method of moments (MoM), the structures are subdivided into meshes that consists of small surface patches (for surface), and small volume cells (for dielectric region). In our simulation, the quadrilateral patches of first order and hexahedron volumes of first order are used to discretize the surface and volume regions, respectively. In both SIE and VSIE approaches, the surface currents are approximated by roof-top basis function which is defined on two adjacent cells $\Omega^\pm$ (a cell is either a surface patch in surface mesh or a volume element in volume mesh). Typical mesh cells are shown in Figure 2. The basis functions for both surface mesh and volume mesh are defined as

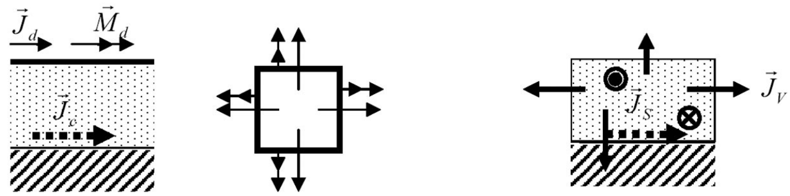
$$\vec{f}_n^{\,\Omega}(r) = \frac{\pm D_n}{\sqrt{g}} T_n(u)\frac{\partial\vec{r}}{\partial u}, \vec{r} \in \Omega^\pm, \Omega = S \text{ or } V, \tag{9}$$

where $T_n(u) = u$ if $\vec{r} \in \Omega^+$, and $T_n(u) = 1 - u$ if $\vec{r} \in \Omega^-$, D_n is the basis function scaling factor which is equal to the common edge length for surface basis function, and is equal to the area of the common face for volume basis function. In the basis function definition, $\sqrt{g}$ is the Jacobian of the transformation that maps an irregular cell in one coordinate system into a regular cell in another coordinate system. As shown in Figure 2, an arbitrary quadrilateral patch is mapped into $u-v$ plane as a unit square, and an arbitrary hexahedron volume cell is mapped into $u-v-w$ space as a unit cubic.

[8] In VSIE, we have used a vector $i\omega\varepsilon\bar{E}$ as the unknown function and it is approximately represented by a set of volume rooftop basis functions which is also defined in equation (9). It can be seen that the unknown function is proportional to the induced volume current $\bar{J}_V$. Galerkin's testing scheme is used in both SIE and VSIE to convert the integral equations into matrix equations.

[9] Now let us compare the number of unknowns used in the SIE and VSIE algorithms. For a single closed surface that is modeled by N_c^S patches, the number of unknowns is $2N_c^S$ if the surface is PEC (because each quad patch has four edges and each pair of patches share one basis function). For full coating, if the coating is one layer and the external surface is modeled also by N_c^S patches, then the total number of unknowns for the SIE approach is $N_S = 6N_c^S$ ($2N_c^S$ is used for electric current on the PEC core surface, $2N_c^S$ is used for the electric current on the external dielectric surface, and $2N_c^S$ is used for the magnetic current of the dielectric surface). The assignment of basis function for each material patch is shown in Figure 3a, in which the top view shows 8 basis functions for each patch. Since each pair of patch share one basis, there are 4 unknowns for each patch on the average. Similarly, the total number of unknowns for a two layer coating case is estimated as $N_S = 10N_c^S$ assuming the two layers have different dielectric constants.

[10] For VSIE modeling of the same structure with the same mesh size, the number of PEC current is the same as that in SIE, i.e., $2N_c^S$. Here, N_c^S is the number of volume cells (assuming that each surface patch corresponds to one volume cell for one layer coating). The number of volume unknowns (for electric dielectric material only) is $4N_c^S$ (the factor 4 in this case comes from the fact that each volume, on the average, needs 2 full basis functions and 2 half basis functions. One half basis vector points to the external background media, and one half basis function points to the PEC core surface, refer to [*Lu and Chew*, 1999, 2000] for the definition of a half basis function). The basis function assignment for a single volume cell is shown in Figure 3b, in which 6 basis functions, one for each of the six



(a) SIE mesh cross view (left) and top view (right).

(b) VSIE mesh cross view.

Figure 3. (a) The basis function assignment for the SIE mesh. (b) The basis function assignment for VSIE mesh. The dash-tailed arrow stands for the basis on conducting surface, single arrow stands for basis for $\vec{J}_d$ or $\vec{J}_V$, and double arrow stands for the basis for $\vec{M}_d$.

faces of the cube, are assigned to a cell. Because each pair of cells will share one basis function, there are 4 independent basis functions for each cell on the average. Hence the number of unknowns for one layer full coating for VSIE is $N_V = 6N_c^S$. For two layer full coating with different material permittivity in each layer, the VIE would need $9N_c^S$ (the two half bases in between the layers are combined into one full basis). Obviously, the numbers of the unknowns needed for one layer full coating case by the VSIE and SIE are about the same if the same mesh sizes are used. The comparisons for the number of unknowns are summarized in Table 1.

[11] In a fast algorithm such as the MLFMA [*Song and Chew*, 2000], the memory mainly consists of two parts, one is for the storage of the near-neighbor interaction matrix elements, and one is for the FMM translation, aggregation, and disaggregation matrices. The memory for the FMM part depends only on the structure (shape and size), hence it will be the same for both the SIE and VSIE. The number of the near-neighbor matrix elements is proportional to the number of unknowns. As a result, the memory needed for the fast algorithm would be about the same scale when applied to SIE and VSIE. Similar conclusion applies to the total CPU time for one matrix-vector multiplication. One major difference in CPU time is for matrix filling, for which the VSIE may need more time than SIE since volume integration is needed. However the matrix-filling time for both SIE and VSIE are of the same order in FMM, i.e., they differ only by a constant factor. Ignoring the matrix-filling time, we expect that the MLFMA applications to SIE and VSIE require similar computer resources for solving a scattering problem involving material coating. In the following section, we provide some numerical examples to compare the performances for the two algorithms. Conclusions of the comparison will be useful in determining which integral equation approaches to be used for solving a coated object scattering problems by an iterative solver.

Table 1. Comparison of the Number of Unknowns for SIE (PMCHW Formulation) and VSIE for a PEC Target With Full Material Coating

	One Layer		Two Layers	
	SIE	VSIE	SIE	VSIE
Number of PEC patches	N	N	N	N
Number of unknowns for PEC patch	2N	2N	2N	2N
$\mu_r = 1$				
Number of unknowns for dielectric	4N	4N	8N	7N
Total number of unknowns	6N	6N	10N	9N
$\mu_r \neq 1$				
Number of unknowns for dielectric	4N	8N	8N	14N
Total number of unknowns	6N	10N	10N	16N

3. Numerical Examples

[12] The first example is a fully coated conducting box. The PEC box (the core) dimension is 0.8m×0.4m× 0.1m, and the coating thickness is 0.02 m on all sides (to be more specific, the external material surface is also a box with dimension of 0.84m×0.44m×0.14m). The relative permittivity of the coating material is $\varepsilon_r = 2.56$. Three meshes are considered with the average edge length being 0.0627m (coarse), 0.0424m (middle), and 0.0324m (fine), respectively. The wavelength of the incident plane wave is 1.0m. The RCSs calculated by the VSIE algorithm are almost the same (the rms difference between the three mesh cases is less than 0.23dB in VSIE), but those from the SIE approach are different. The RCS results are shown in Figure 4. This figure shows that with similar mesh size, the two algorithms produced similar results. However the numbers of iterations needed to reduce the residue error in CG solvers from 1.0 to 1E-3 are quite different, as shown in Table 2. The numbers of iterations for every angle in the observation plane ($\theta = 90°$) are shown in Figure 5, from which we can see that the numbers of iterations for VSIE are much smaller than that for SIE for all the angles of incidence. Since the solution from the previous incident angle is used as

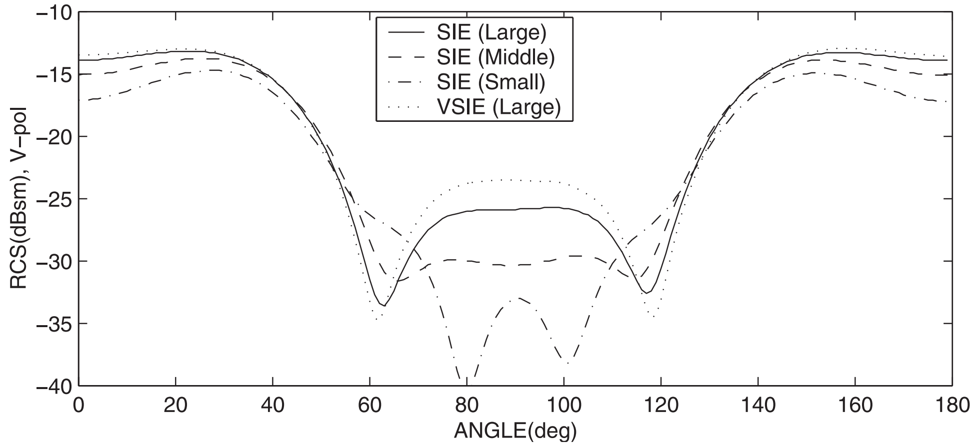


Figure 4. The RCSs of the coated box for three different meshes sizes (coarse, middle, and fine). The results of VSIE for the three meshes are almost the same and hence only one curve for VSIE (fine mesh) is displayed in this figure.

initial guess, the iteration numbers for incident angle 2 to 181 are smaller than that of the first incident angle for which the initial guess is set to zero. It should also be pointed out that the same CG solver is used for both VSIE and SIE.

[13] The next example shows the results for a circular cylinder of finite length [*Medgyesi-Mitschang and Putnam*, 1984]. The cylinder is made up of two sections, one section is PEC, and the other is the dielectric, as shown in Figure 6. The dimensions in the figure are a=0.0508 m, b=0.1016 m, and d=0.0762 m. The permittivity of the material is $\varepsilon_r = 2.6$. The cylinder is discretized into 1546 patches with 4368 unknowns. The average mesh size is $0.058\lambda_0$ at 3.0 GHz. The monostatic radar cross section calculated by the SIE and VSIE algorithms are shown in Figure 7, and the average numbers of CG iterations are shown in Table 2.

[14] The last example is a triangle-shaped conducting plate with dielectric coating on its three sides. The structure mesh and dimension are shown in Figure 8. The coating material has relative dielectric permittivity $\varepsilon_r = 4.5 + 9.0i$. The plate is on x-y plane and the average

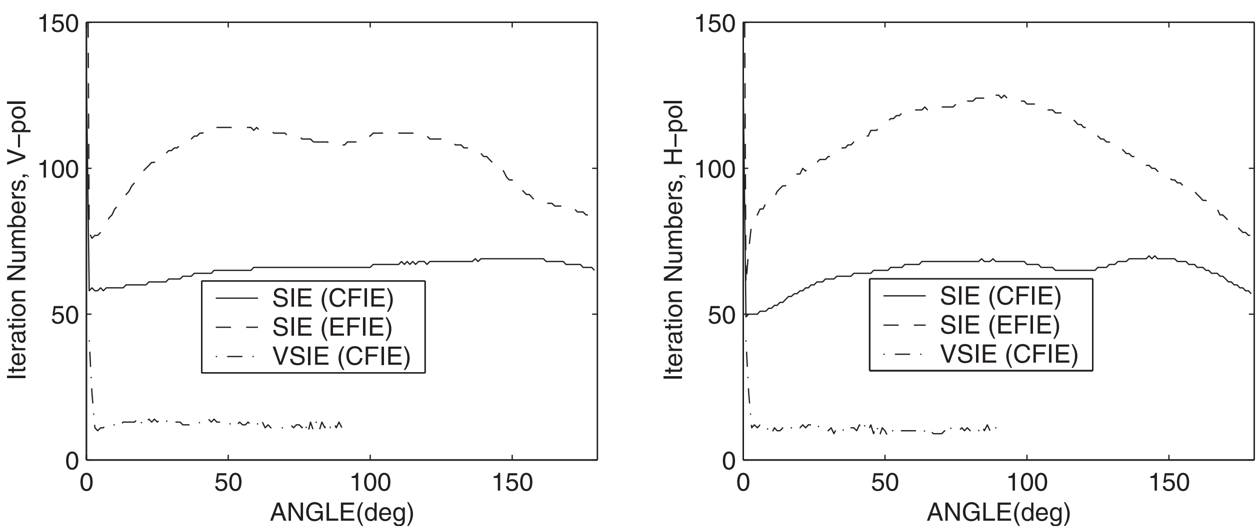


Figure 5. The numbers of CG iterations needed by SIE and VSIE to reduce the relative residue error from 1 to 1E-3 in solving the matrix equation associated with the coated box geometry. The CFIE is used for both integral equations and the standard CG solver is used. The incident angles are $\theta = 90°$, and $\phi = 0°$ to $\phi = 180°$ with 1 degree increment.

Table 2. Comparison of the Run-Time Parameters for the SIE and VSIE Programs Using the Same CG Solver[a]

Examples	Residue Error	Number of Unknowns		Average Iteration Number		RCS Error (rms), dB	
		SIE	VSIE	SIE(V/H)	VSIE(V/H)	V-Pol	H-Pol
Cylinder	0.001	4368	7920	65/66	22/25	0.42	0.29
Box (mesh 1)	0.005	1512	1884	324/117	22/21	3.05[b]	4.26[b]
Box (mesh 2)	0.005	3240	3772	146/89	10/10	0.91[b]	2.36[b]
Box (mesh 3)	0.005	5964	6672	112/77	7/7	0.25	0.88
Sphere	0.001	5184	5184	161/160	94/97	0.48	0.66
Triangle plate	0.002	11840	10610	128/83	9/10	2.75[c]	1.75[c]

[a]The number of iterations are the average for all monostatic incident angles except for the bistatic sphere case for which the iteration number is for the first incident angle.

[b]The mesh sizes for SIE and VIE are different (the mesh for SIE is too coarse).

[c]The large error in this case is caused by the rapid oscillation in the RCS because the target size is large.

mesh size for the conducting surfaces and the dielectric material regions at 1.0 GHz are 0.018 m and 0.0232 m, respectively. The thickness of the plate is 0.048 m and the dielectric coating thickness (on the three sides) is also t=0.048 m, as indicated in Figure 8. The monostatic RCS is calculated for $\theta = 90°$, and $\phi = 0$ to 360° to with angle increment of 1°, and is shown in Figure 9. Other run time parameters (numbers of unknowns and numbers of iterations, etc.) are listed in Table 2. The three peak values of RCS in Figure 9 corresponds to the three edge reflections at $\phi = 63.5°$, 180°, and 270°, respectively.

[15] From Table 2 we can make the following observations:

1. It can be seen that for the problems of scattering by coated objects, the SIE and VSIE require about the same number of unknowns to describe the problem when the coating in thin (one to two layers). A special case is the cylinder for which the coating material is thick (there are 10 layers in that case). Since the permittivity of the coating material is homogeneous, the SIE basis function is distributed on the external surface of the material only. We would expect that if the 10 layers have different material permittivities, the number of the unknowns for SIE would be about the same as that used in VSIE.

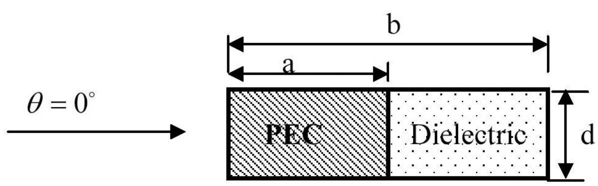


Figure 6. Side view of a circular cylinder [*Medgyesi-Mitschang and Putnam*, 1984] that is made up by a conducting section and a dielectric section (the first incident angle for RCS calculation is from the PEC end).

2. The rms differences of the solutions from the SIE and VSIE are within 1 dB with two exceptions. The first exception is for the full-coated box (coarse and middle size mesh), in which case, the mesh sizes for SIE are too coarse. The second exception is for the partially coated triangular plate in which case, the RCS values are rapidly oscillating as function of angles due to large electrical size, and any small angular misalignment in the RCS will results in large errors (it is suggested that the moving average error should be used in this case instead of the point-wise rms error). From this point of view, we can characterize that the two algorithms will produce solutions with the same order of accuracy when mesh sizes are comparable and the same order of basis functions are used.

3. The iteration numbers needed to reduce the residue error to a specified value for VSIE are much smaller than that for SIE, using the same conjugate gradient solver. The numbers of iteration for the first incident angle (for which the initial guesses for both programs are set to zero) are also significantly different, with the VSIE convergence faster than the SIE. One reason for this fact is that the integral operator for the VIE part in the VSIE has lower order of singularity compared with the one in SIE. Another cause for the low convergence rate of SIE is related to the testing function [*Sheng et al.*, 1998]. It has been shown that for the surface integral equation with material surfaces, the best testing scheme is the TENH option [*Sheng et al.*, 1998]. In this paper, we have used the TETH formulation for the SIE program.

[16] In addition to the above examples, we also compared the number of iterations for the same scatterer at different frequencies. In this experiment, we consider a conducting sphere of radius 0.4 m at five frequencies uniformly sampled from 0.5 to 0.7 GHz. The sphere is coated with a 0.04m thick dielectric material of $2.56+0.5i$. The number of unknowns is kept at 5184 for

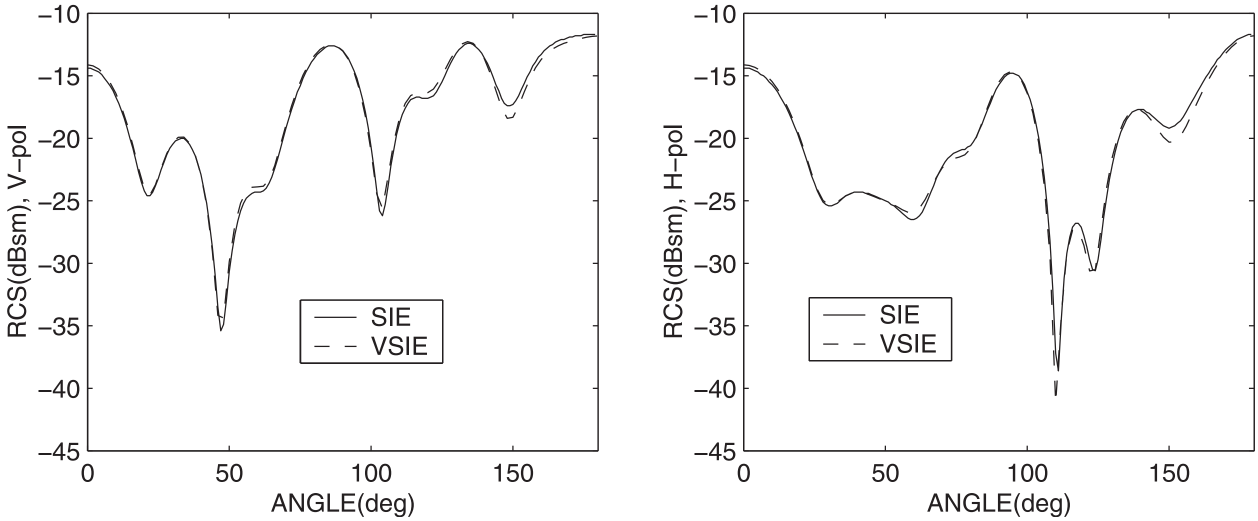


Figure 7. The monostatic RCSs calculated by SIE and VSIE for the circular cylinder in Figure 6. In both algorithms, the combined field integral equation is used (the combination constant is 0.6). The incident angle increament is 1 degree.

both SIE and VSIE formulations. It is found from calculation that the average iteration numbers are 25 for VSIE and 84.6 for SIE when the CG residue error is set to 0.005 for both solvers.

[17] The comparison shows that the VSIE has numerical advantage over SIE algorithm in solving problems of electromagnetic scattering from coated conducting scatterers. The comparisons, and hence the conclusions, are made with some restrictions: (1) the basis functions are of the first order, (2) the mesh model is of first order, (3) only the standard conjugate gradient solver (without any preconditioning) is tested, and (4) the implementation of fast solvers to the two approaches is not considered (in fact, it is much easier to implement the MLFMA for VSIE than for SIE). It is expected that the conclusion may be extended into higher order basis functions and higher order mesh models. However, the conclusions (especially for the convergence rate) for other CG solvers or CG solvers with preconditioning need to be investigated separately.

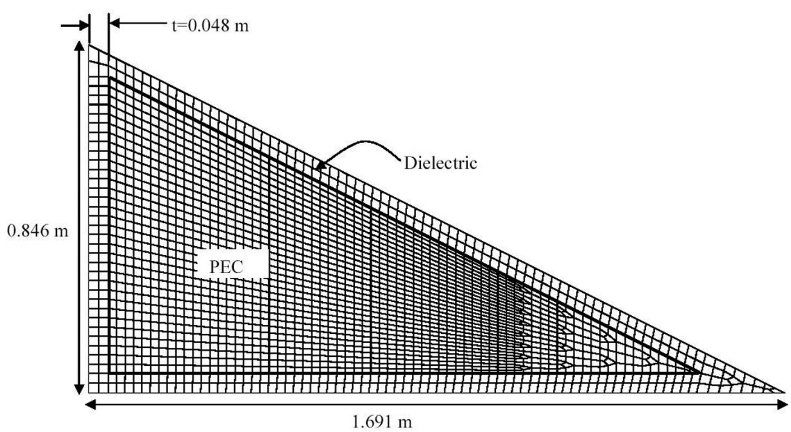


Figure 8. The top view of the mesh and the dimension of the partially coated triangular plate of finite thickness. The thicknesses of the PEC part and the material part are the same, and both are equal to 0.048 m. The coating on all three sides is uniform with thickness of 0.048 m. The thick line in the sketch indicates the interface of the PEC and the dielectric material.

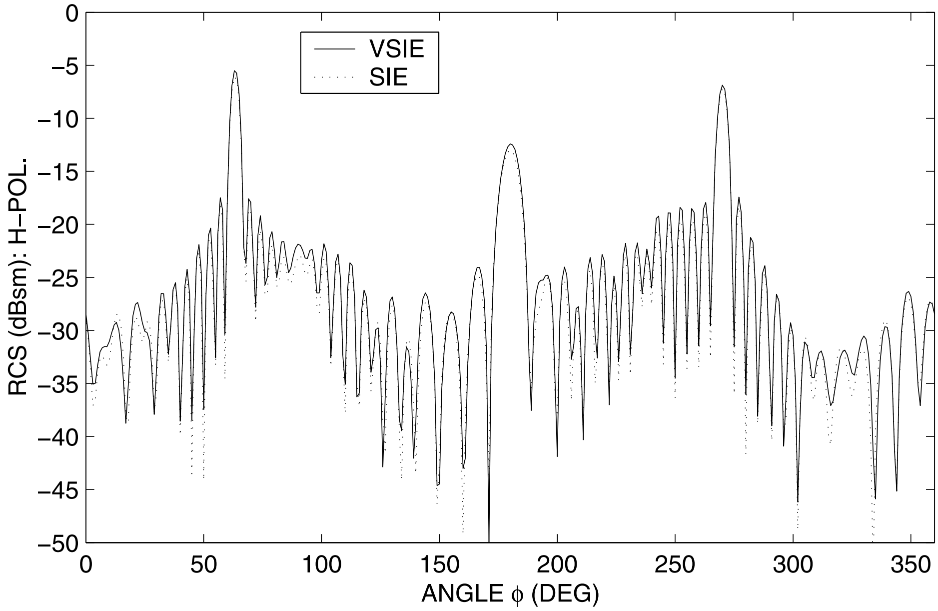


Figure 9. The monostatic RCS of a triangle PEC plate of finite thickness that is coated by dielectric material on its three sides. CFIE with combination constant of 0.7 is used in both SIE and VSIE programs.

4. Conclusions

[18] This paper presented comparisons of the two integral equation approaches, the SIE approach (based on PMCHW formulation), and the VSIE approach, for solving electromagnetic scattering problems. For thin dielectric coating problems (with one or two coating layers), the numbers of unknowns needed by both approaches are of the same order, and hence CPU time and memory requirement for the two approaches are of the same order assuming the same solver is used. Numerical examples showed that the VSIE has faster iteration convergence than that of SIE. The solution accuracies are of the same order. This faster iteration convergence of VSIE algorithm has significant impact on fast iterative solvers for which the computational complexities are reduced to lower orders, and the number of iterations directly affects the total solution time.

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