Stitching Multiple Camera Frames into a Single Picture

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1 The Physical Problem

Consider a lenslet array with ξ' columns and η' rows. Each lenslet samples the incident wavefront $\psi(x, y)$ over a distinct region of space and produces a well-defined focal spot on the CCD array. The average of the x and y slopes of the wavefront over the lenslet is:

$$\theta_{\mu\nu} = \begin{bmatrix} \theta_{\mu\nu,x} \\ \theta_{\mu\nu,y} \end{bmatrix},$$

where:

$$\theta_{\mu\nu,x} = \int_{\mu-1}^{\mu} \int_{\nu-1}^{\nu} \frac{\partial \psi}{\partial x} \, dx \, dy, \quad \theta_{\mu\nu,y} = \int_{\mu-1}^{\mu} \int_{\nu-1}^{\nu} \frac{\partial \psi}{\partial y} \, dx \, dy.$$

A continuous wavefront is reduced to a discrete set of $N' = \xi'\eta'$ average slope measurements for each camera frame.

A simple case involving two frames is shown in Figure 1. The overlap region reveals the relative tilt difference between adjacent frames. The analysis assumes a perfect reference file.

2 The Mathematical Problem

The tilt of each frame is denoted as $c_{\mu\nu}$. For adjacent frames, the relative tilt difference is:

$$\Delta v_{\mu\nu} = v_{\mu\nu} - v_{\mu+1,\nu}, \quad \Delta h_{\mu\nu} = h_{\mu\nu} - h_{\mu,\nu+1}.$$

The least squares fit minimizes the sum of the squares of the differences:

$$\chi^2 = \sum_{\mu=1}^{\xi} \sum_{\nu=1}^{\eta} \left((\Delta v_{\mu\nu} - (c_{\mu\nu} - c_{\mu+1,\nu}))^2 + (\Delta h_{\mu\nu} - (c_{\mu\nu} - c_{\mu,\nu+1}))^2 \right).$$

The system of equations derived from minimizing χ^2 can be solved using Singular Value Decomposition (SVD).

Define the interaction matrix A and data matrix D such that:

$$A \cdot c = D.$$

The matrix A is singular and requires special handling. Using SVD, the inverse can be computed as:

$$A^{-1} = V \Sigma^{-1} U^T,$$

where Σ contains the singular values, U and V are orthogonal matrices, and $A = U \Sigma V^T$.

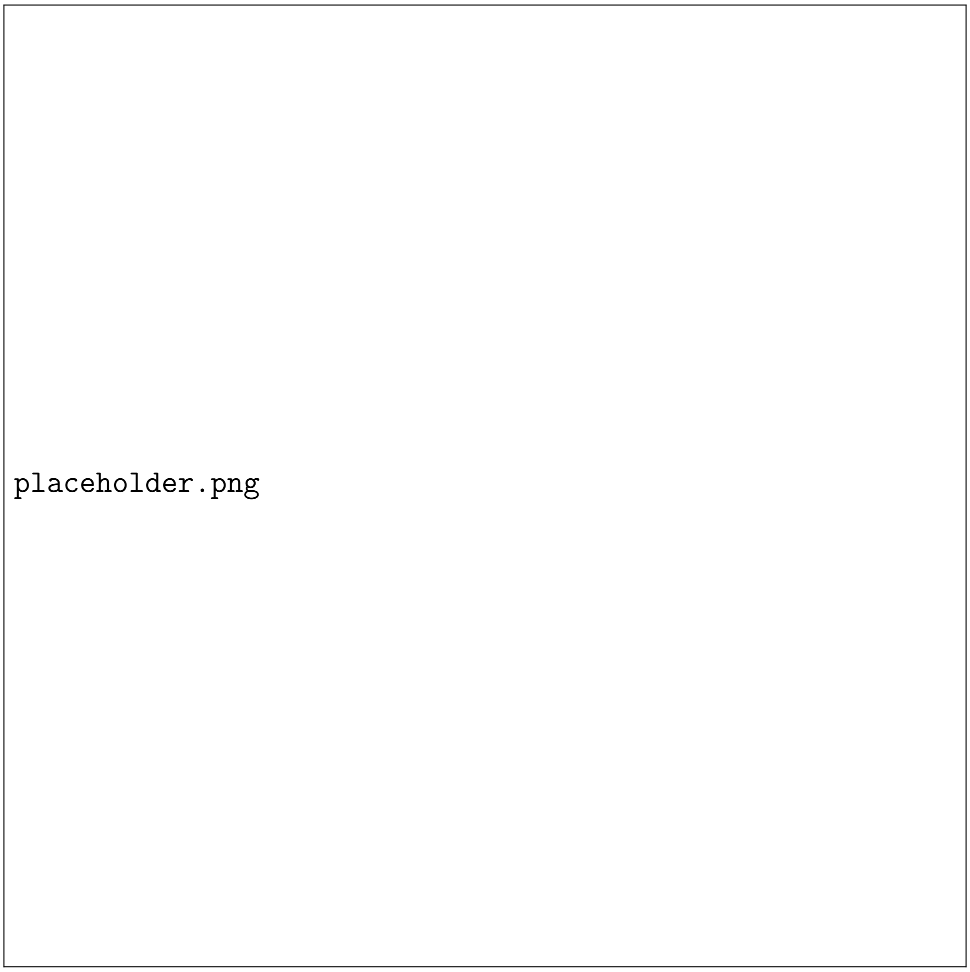

Figure 1: Overlapping frames. The red and blue outlines represent consecutive frames with overlap regions shaded in gray.

3 Example

Consider a non-rectangular quilt (Figure 2). The existence function $\epsilon_{\mu\nu}$ determines if data is present in a given panel:

$$\epsilon_{\mu\nu} = \begin{cases} 1 & \text{if data exists in panel } (\mu, \nu), \\ 0 & \text{otherwise.} \end{cases}$$

In a practical implementation, the solution vector c must adjust for missing data. This ensures a robust solution to the stitching problem. For example, if a frame is missing data, the existence function accounts for it, and A is adjusted accordingly.

4 References

1. R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 1990.

2. G.H. Golub and C.F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 1996.

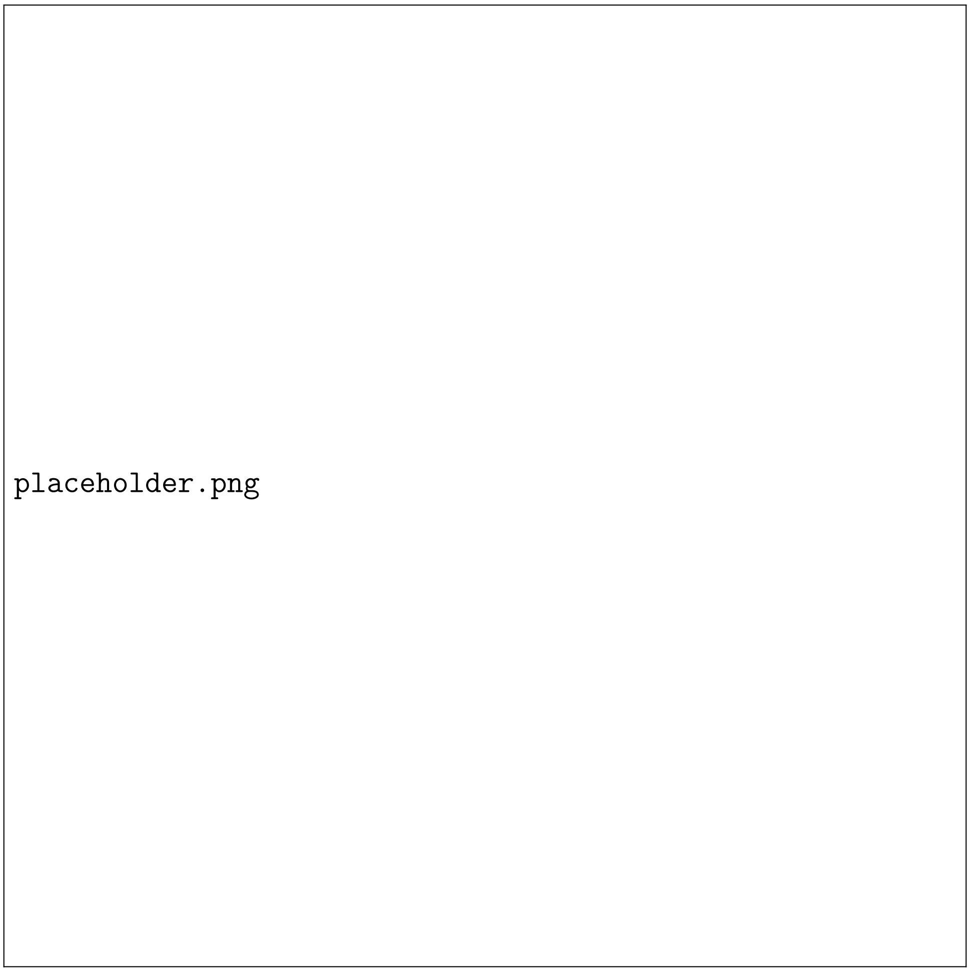

Figure 2: Non-rectangular quilt example. The existence function $\epsilon_{\mu\nu}$ ensures valid data regions are identified.

3. W.H. Press et al., *Numerical Recipes in FORTRAN*, Cambridge University Press, 1992.

4. R.A. Horn and C.R. Johnson, *Topics in Matrix Analysis*, Cambridge University Press, 1991.