Input Slides 2024-12: My Two Slides

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December 15, 2024

Overview

Sealing the Mesh

Boo

Fourier and Extensions to 2- and 3-D

$e^{in\theta}$ **From periodic functions in 1D to radial and angular decompositions in 2D and 3D.**

Why We Love Fourier for Smooth Functions

Smooth Functions, Beautiful Representations

- **Weierstrass Approximation Theorem:** Any continuous function on $[a, b]$ can be uniformly approximated by polynomials. Fourier provides a similar approximation, using trigonometric bases instead of polynomials.
- **Riesz-Fischer Theorem:** Fourier coefficients (a_n, b_n) belong to l^2 space, guaranteeing convergence in the L^2 sense. This bridges the gap between smoothness and square-integrability.
- **Uniform Convergence for Smooth Periodic Functions:** For sufficiently smooth functions (C^∞ or C^k), Fourier series converge uniformly, ensuring no oscillatory artifacts (Gibbs phenomenon disappears).

Fourier and Extensions to 2- and 3D

1D:

$$f(\theta) = \sum_{n=-\infty}^{\infty} a_m e^{in\theta}$$

2D:

$$f(r,\theta) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n,2} a_n^m R_n^m(r) e^{in\theta}$$

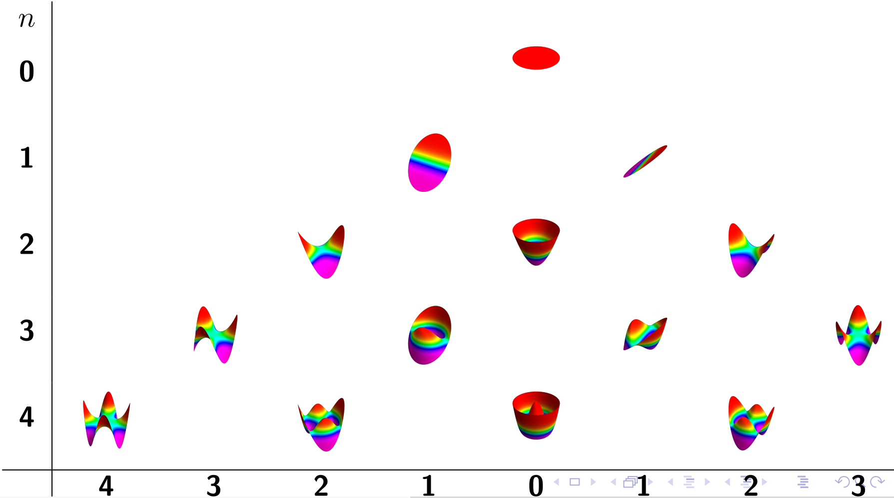
3D:

$$f(r,\theta,\phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} a_n^m \sqrt{\frac{(2m+1)(m-n)!}{4\pi(m+n)!}} P_l^m(\cos\theta) e^{in\theta}$$

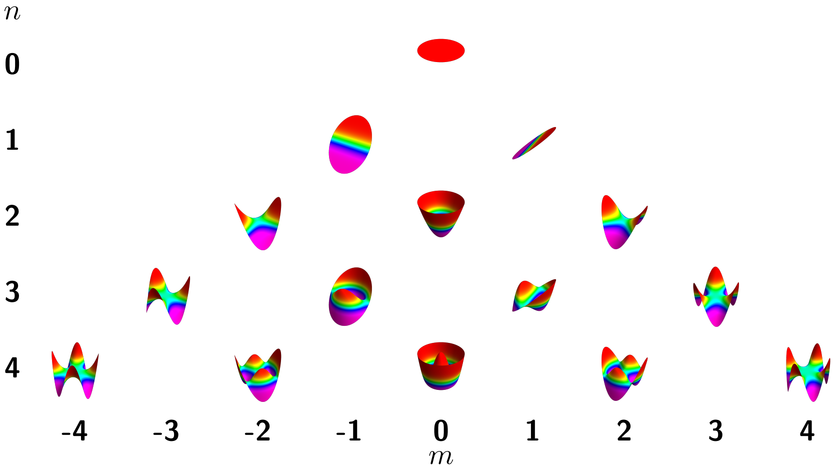
Fourier and Extensions to 2- and 3D

- **1D Fourier Series:** Decomposes a periodic function $f(\theta)$ into a sum of complex exponentials with coefficients a_n capturing the amplitudes of each frequency component.
- **2D Fourier-Bessel:** Extends Fourier analysis to two dimensions using radial functions $R_n^m(r)$, often employed in circular domains or optical applications.
- **3D Spherical Harmonics:** Represents functions on a sphere using harmonics $Y_l^m(\theta, \phi)$ and radial components $R_l(r)$, crucial in fields like quantum mechanics and gravitational modeling.

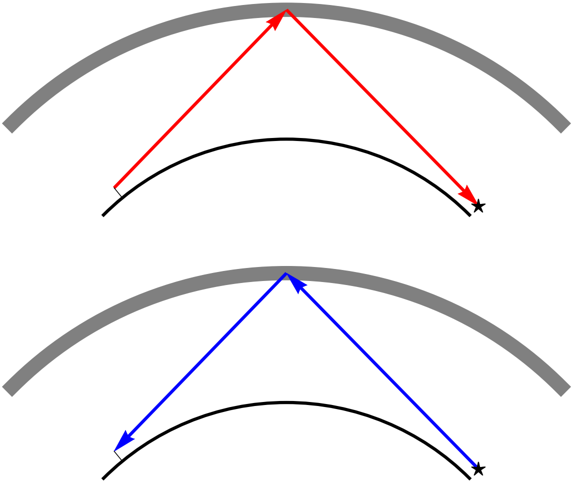
The first few Zernike disk polynomials



Lowest Zernike disk polynomials



Energy Out, Energy In



Energy Out, Energy In

1. radar (left) irradiates target (star)
2. backscatter travels from target to radar

Radar Cross Section: Definition

$$\sigma_\star = 4\pi \lim_{r\to\infty} r^2 \left| \frac{E_{\textsf{incident}}}{E_{\textsf{scattered}}} \right|^2 \tag{2.1}$$

Skolnik 1962, (2.36)

Radar Cross Section: Discussion

- Radar cross section is a far field phenomenon
- Assumes single polarization to and from target
- Target is completely metallic:
  E field results from surface currents
- Shape is quasi-dimensional
  - Dimensions in two known directions
  - Dish antennae, solar panels, booms
- Resonant scattering:
  Ratio of typical dimension to wavelength ≈ 1
- Kolosov 1987, §4.6

Radar Cross Section: Conceptual Overview

- **Radar Basics:**
  - Transmit energy Skolnik 1962, p. 21.
  - Receive scattered signal.
  - Direction, strength → object properties Knott, Schaeffer, and Tulley 2004, p. 45.
- **What is RCS?**
  - Measures object "visibility" to radar Lab 2002, Section 2.
  - Depends on:
    - Material
    - Geometry
    - Orientation Peebles 2007, pp. 3-4.
- **Key Question:** Power reflected vs. power transmitted.

Factors Influencing Radar Cross Section

- **Shape:**
  - Smooth → directional reflection Knott, Schaeffer, and Tulley 2004, p. 47.
  - Complex → scattered energy.
- **Material:**
  - Metal → strong reflection.
  - Absorbers → reduced RCS Knott, Schaeffer, and Tulley 2004, Section 3.2.
- **Size vs. Wavelength:**
  - Large → high scattering.
  - Small → "invisible" (Rayleigh scattering) Kolosov 1987, p. 188.
- **Orientation:**
  - Aligned → max RCS Knott, Schaeffer, and Tulley 2004,

Input files

1. `B-20.geo`
    1. Points to facet file
    2. Configure linear algebra solver
    3. Radar frequency range
    4. Angular sampling ranges
    5. Boundary conditions
    6. Mono– or Bistatic
    7. Surface or Volume integral elements
    8. Length units
2. `B-20.facet`
    1. Vertex list

Linear algebra (don't alter)

```
&MM_MOM
 bUseACA = .TRUE.,
 bSolve_ACA = .TRUE.,
 bOutOfCore = .TRUE.,
 bNormalizeToWaveLength = .FALSE.,
 bNormalize = .FALSE.,
 dCloseLambda = 0.100000,
 ACA_Factor_Tol = 0.000010,
 ACA_RHS_Tol = 0.000100,
 Point_Tolerance = 0.001000,
 nLargestBlockSize = -1,
 MemorySize_GB = -1.000000,
 stackSize_GB = -1.000000,
 nFillThreads = -1,
 nFillMKLThreads = 1,
 nLUThreads = -1,
 nLUMKLThreads = 1,
 nRHSThreads = 1,
 nRHSMKLThreads = 1,
 bOutputACAGrouping = .FALSE.,
 bOutputRankFraction = .FALSE.,
 bLimitLUColumns = .FALSE.,
 Lop_Admissibility = WEAK,
 Kop_Admissibility = CLOSE
```

Memory management (don't alter)

```
&Scratch_Memory
 Scratch_RankFraction_Z = 0.300000,
 Scratch_RankFraction_LU = 0.600000,
 Scratch_RankFraction_RHS = 2.000000,
 Scratch_RankFraction_Solve = 1.000000,
 MemoryFraction_Z = 0.950000,
 MemoryFraction_Scratch_LU = 0.500000,
 MemoryFraction_LU = 1.000000,
 MemoryFraction_RHS = 0.500000,
 MemoryFraction_Solve = 0.900000,
```

Quadrature (don't alter)

```
&QUADRATURE
 NTRISELF = 7,
 NTRINEAR = 3,
 NTRIFAR = 3,
 NTETSELF = 11,
 NTETNEAR = 4,
 NTETFAR = 4,
 NQGAUSS = 4
```

Radar frequencies

```
FREQUENCY
 ghz
 0.003000 0.030000 28 !Freq Start, Freq Stop, Num Frequencies
```

Sampling

```
Angle Cut
 1
 0.000000 359.000000 360
 AZIMUTH
 90.000000
```

Monostatic or bistatic

```
Excitation
 MONOSTATIC
```

Boundary Conditions

```
Boundary Conditions
B-20-Materials.lib
4
V_FREE_SPACE => Free_Space
V_PEC => PEC
V_PMC => PMC
V_NULL => NULL
1
0 BC_PEC V_FREE_SPACE
```

Final settings

`SIE`	surface integral elements
`B-20A.facet`	CAD description
`m`	meters

Mercury MoM is Single Precision

Example: 8 MHz
Despite exact binary representation

$$8_{10} = 1000_2$$

```
Start Frequency = 7.9999994E-03GHz
```

Run sequence - launch

```
$./MMoM_4.1.12 b20.geo
--------------------------------------------------------------------------------
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
------------ Reporting output in MB from Linux command:  vmstat -s -S M ------------
 53113 M free memory
```

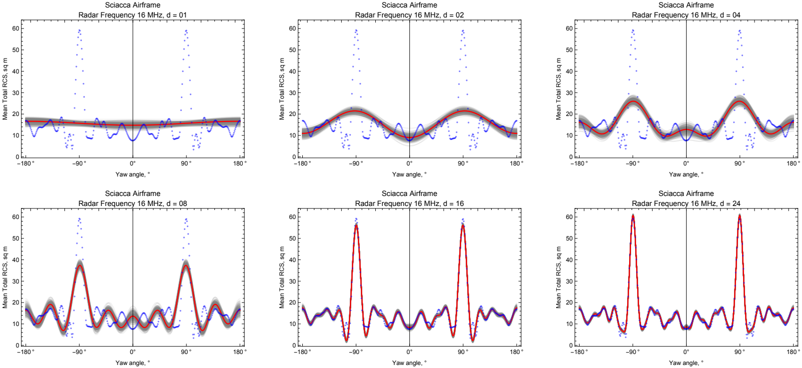
Run sequence - sample output

```
 Freq   = 30.00E+00 MHz
 Lambda = 9.99E+00 m
 k      = 628.75E-03 m-1
subroutine Solve_SetUp( Surface, bk, pSys, pD, Nodes ) :  ...Finished
--------------------------------------------------------------------------------
---| Time :  Time total for RHS solve
---| Twall = 0.0004168 ; Tcpu = 0.0002319 ; Ratio = 1.80
--------------------------------------------------------------------------------
---| Out Of Core Times:  Diagonal Blocks
---|
---| nWrites.........................:  2.
---| GigaBytes Write.................:  0.
---| Write Time (Hr).................:  0.00
---| Average Write Rate (MBytes/sec).:  19.
---| nReads..........................:  5.
---| GigaBytes Read..................:  0.
---| Read Time (Hr)..................:  0.0002
---| Average Read Rate (MBytes/sec)..:  48.
---|
--------------------------------------------------------------------------------
Z Column Summary IO 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
```

Run sequence - completion

```
$./MMoM_4.1.12 b20.geo
--------------------------------------------------------------------------------
HOSTNAME = 3dd5a4b0d3c8
HOSTTYPE =
CPU =
OSTYPE =
MACHTYPE =
NUMBER_OF_PROCESSORS =
OMP_NUM_THREADS =
PROCESSOR_ARCHITECTURE =
PROCESSOR_IDENTIFIER =
------------ Reporting output in MB from Linux command:  vmstat -s -S M ------------
 53113 M free memory
```

Fourier Transform Visualizations at 16 MHz



Note

Blue: Data **Red:** Approximation **Gray:** Error

Fourier Transform Visualizations at 16 MHz

- **Approximation Order (d):** The number of terms in the Fourier approximation increases as $d = 1, 2, 4, 8, 16, 24$.
- **Low-Order Approximations:** Approximations with smaller d (e.g., $d = 1, 2, 4$) fail to capture fine structures, leading to significant residual error.
- **Higher-Order Approximations:** As d grows, the approximation better resolves finer features, and the error (gray) shrinks significantly, especially at smooth regions.
- **Error Behavior:** The error decreases non-uniformly—large errors persist near abrupt changes or peaks due to Gibbs phenomena, but smooth regions converge faster.
- **Key Insight:** Fourier approximations demonstrate trade-offs: computational complexity increases with d, but fidelity improves.
- **General Notes:**

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Fourier Transform Visualizations at 16 MHz

- **Key Insight:** Fourier approximations demonstrate trade-offs: fidelity improves with d, as does computational cost.
- **General Notes:**
  - Fourier series resolve functions as sums of sines and cosines.
  - Low-frequency terms: broad trends; high-frequency:e fine details.
  - Convergence is faster for smooth functions but slower for discontinuities or sharp changes.
  - More terms improves fidelity, but can introduce numerical artifacts.

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