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Quarkonium and Bertrand's Theorem

Daniel Topa
dantopa@gmail.com

University of New Mexico
Department of Mathematics and Statistics

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Overview

1. Background

Bertrand's Theorem

Bertrand's theorem

16 languages

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In classical mechanics, **Bertrand's theorem** states that among central-force potentials with bound orbits, there are only two types of central-force (radial) scalar potentials with the property that all bound orbits are also closed orbits.[1][2]

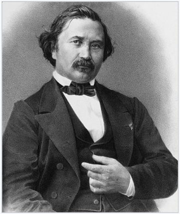
The first such potential is an inverse-square central force such as the gravitational or electrostatic potential:

$$V(r) = -\frac{k}{r} \text{ with force } f(r) = -\frac{dV}{dr} = -\frac{k}{r^2}.$$

The second is the radial harmonic oscillator potential:

$$V(r) = \frac{1}{2}kr^2 \text{ with force } f(r) = -\frac{dV}{dr} = -kr.$$

The theorem is named after its discoverer, Joseph Bertrand.



Joseph Bertrand

Wikipedia contributors 2023

No Bertrand Theorem

PHYSICAL REVIEW D **76,** 094005 (2007)

Large degeneracy of excited hadrons and quark models

P. Bicudo

Departamento de Física and CFTP, Instituto Superior Técnico, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal

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The pattern of a large approximate degeneracy of the excited hadron spectra (larger than the chiral restoration degeneracy) is present in the recent experimental report of Bugg. Here we try to model this degeneracy with state of the art quark models. We review how the Coulomb Gauge chiral invariant and confining Bethe-Salpeter equation simplifies in the case of very excited quark-antiquark mesons, including angular or radial excitations, to a Salpeter equation with an ultrarelativistic kinetic energy with the spin-independent part of the potential. The resulting meson spectrum is solved, and the excited chiral restoration is recovered, for all mesons with $J > 0$. Applying the ultrarelativistic simplification to a linear equal-time potential, linear Regge trajectories are obtained, for both angular and radial excitations. The spectrum is also compared with the semiclassical Bohr-Sommerfeld quantization relation. However, the excited angular and radial spectra do not coincide exactly. We then search, with the classical Bertrand theorem, for central potentials producing always classical closed orbits with the ultrarelativistic kinetic energy. We find that no such potential exists, and this implies that no exact larger degeneracy can be obtained in our equal-time framework, with a single principal quantum number comparable to the nonrelativistic Coulomb or harmonic oscillator potentials. Nevertheless we find it plausible that the large experimental approximate degeneracy will be modeled in the future by quark models beyond the present state of the art.

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Bicudo 2007

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[2] Wikipedia contributors. Bertrand’s theorem — Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/w/index.php?title=Bertrand%27s_theorem&oldid=1188516830. [Online; accessed 13-September-2024]. 2023.

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