Elliptic Integrals in Orbital Mechanics

Daniel Topa
daniel.topa@hii.com

Huntington Ingalls Industries
Mission Technologies

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Why Elliptic Integrals

1. Trajectory corrections and transfer orbits
2. Orbital corrections (J2)
3. Long-term simulations of orbits

Overview

Orbital Period T

$$T = \int_0^{2\pi} \sqrt{\frac{a^3}{\mu\left(1 - e\cos E\right)^2}} dE \tag{1.1}$$

Incomplete Elliptic Integrals

$$K(k) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \qquad \textbf{1st kind} \qquad (2.1)$$

$$E(k) = \int_0^{\phi} \sqrt{1 - k^2 \sin^2 \theta}\, d\theta \qquad \textbf{2nd kind} \qquad (2.2)$$

$$\Pi(n; k, \phi) = \int_0^{\phi} \frac{1}{1 - n^2 \sin \theta} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \qquad \textbf{3rd kind} \qquad (2.3)$$

Incomplete Elliptic Integral of the First Kind

$$K(k) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \qquad \textbf{1st kind} \qquad (2.1)$$

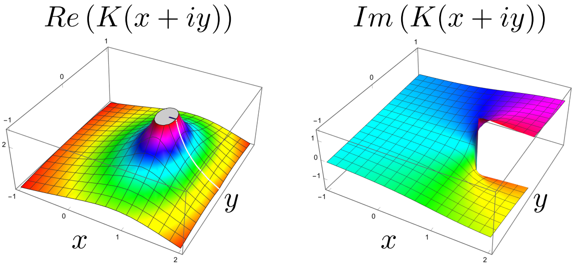

Table: $K(x+iy)$ **is analytic in the complex plane excluding** $[1,\infty)$

Control Factors

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Professional Societies: Computational Mechanics

Bibliography I

[1] Amparo Gil, Javier Segura, and Nico M. Temme. Numerical Methods for Special Functions. Society for Industrial and Applied Mathematics, 2007.

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