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DETERMINATION OF THE EARTH'S GEOID BY SATELLITE OBSERVATIONS

by

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ABSTRACT

Determinations of the geoid made by different authors have differed by more than forty meters in some geographic locations. The authors differed in the observations employed in the number of gravity coefficients they determined, and in a number of details in the method of solution. Experiments conducted with Doppler observations on satellites have shown moderate variations (rarely as much as 30 meters) in the geoid determined if the number of satellite orbital inclinations employed is reduced by one. Reduction of the number of gravity parameters used to represent the geoid also resulted in moderate variations in the principal geoid features, except under special circumstances which are described. Reducing the number of weeks of observations did not produce deviations greater than 25 meters. However, reducing the number of observing stations in addition resulted in distortions of the computed geoid which reached 100 meters. It appears that the most recent geoid heights determined from satellite observations are correct to about 20 meters at any location and that observational data being obtained and techniques of computation being utilized should improve the accuracy to 10 meters or better.

FOREWORD

The analyses described in this report were conducted under Bureau of Naval Weapons Task Assignment RT8801001/2101/S4390001 during the period September 1964 - January 1966 on the basis of satellite observations made during the period June 1961 - April 1964. These analyses were performed by many members of the Naval Weapons Laboratory under the direction of S. J. Smith and R. W. Hill.

This report was prepared to provide a basis for a presentation at the symposium on "The Mantles of the Earth and Terrestrial Planets" sponsored by the North Atlantic Treaty Organization and held at the University of Newcastle upon Tyne in April 1966.

APPROVED FOR RELEASE:

/s/ BERNARD SMITH
Technical Director

INTRODUCTION

Geoid heights determined by various scientists on the basis of careful analysis of satellite observations have produced results which differ by 40 meters or more in some geographic locations.(1) There are many differences in the methods used by the various authors which will be outlined in the next section. Finally, quantitative results obtained from systematic tests of some of these differences will be reported.

SOURCES OF DIFFERENCES IN SOLUTIONS FOR

GEOID HEIGHTS BASED ON SATELLITE OBSERVATIONS

Observations

Geodetic solutions reported to date have been based upon observations made by the Baker-Nunn camera network of the Smithsonian Astrophysical Observatory[2],[3] or the Doppler satellite tracking system of the U. S. Navy.[4][5] The camera observations are available for many satellites for time periods of several years. While daily observations have not been made by the complete Doppler system for such time spans because of failures which ultimately occur in the satellite power system or circuitry, only a small part of the data which has been obtained has been used in geodetic solutions made to date. The all-weather capability and the somewhat larger number of stations in the Doppler network has permitted the extraction of a large amount of information from short time periods of observation. The Baker-Nunn network is shown in figure 1. Since the observations must be referenced to a star background, the stations observe only on relatively clear nights. Since few satellites are actively illuminated, observation times are further limited to times near sunrise and sunset when the sun and satellite are in favorable positions to permit the camera to record a reflection of the sun off the satellite. Up to 1966, the Doppler equipment, consisting of 13 relatively fixed stations and five mobile vans has obtained data from the sites shown in figure 2 for time periods of six weeks to six years. The equipment has provided reliable data more than 90 percent of the time that a satellite is scheduled for observation. Thus data during four or more passes, depending on the satellite altitude, are obtained each day for each satellite with a stable oscillator unless another such satellite with a higher priority is above the radio horizon of the station during the pass. Other types of observations have not played a role in determining the complete specifications for the gravity field either due to lack of precision of the equipment or due to scarcity of observations. However, the Minitrack system of the National Aeronautics and Space Administration provided the first information on the latitude variation of the gravity field[6][7] and is still contributing to the refinement of this information[8],[9] through the determination of the direction to actively transmitting satellites. The direction is found by comparison of phase of the incoming signal on pairs of antenna systems. Another important contribution to verification of the geoid has been made by the analysis of observations[10] of synchronous satellites, which yields information on some of the gravity coefficients.

BAKER-NUNN CAMERA SITES

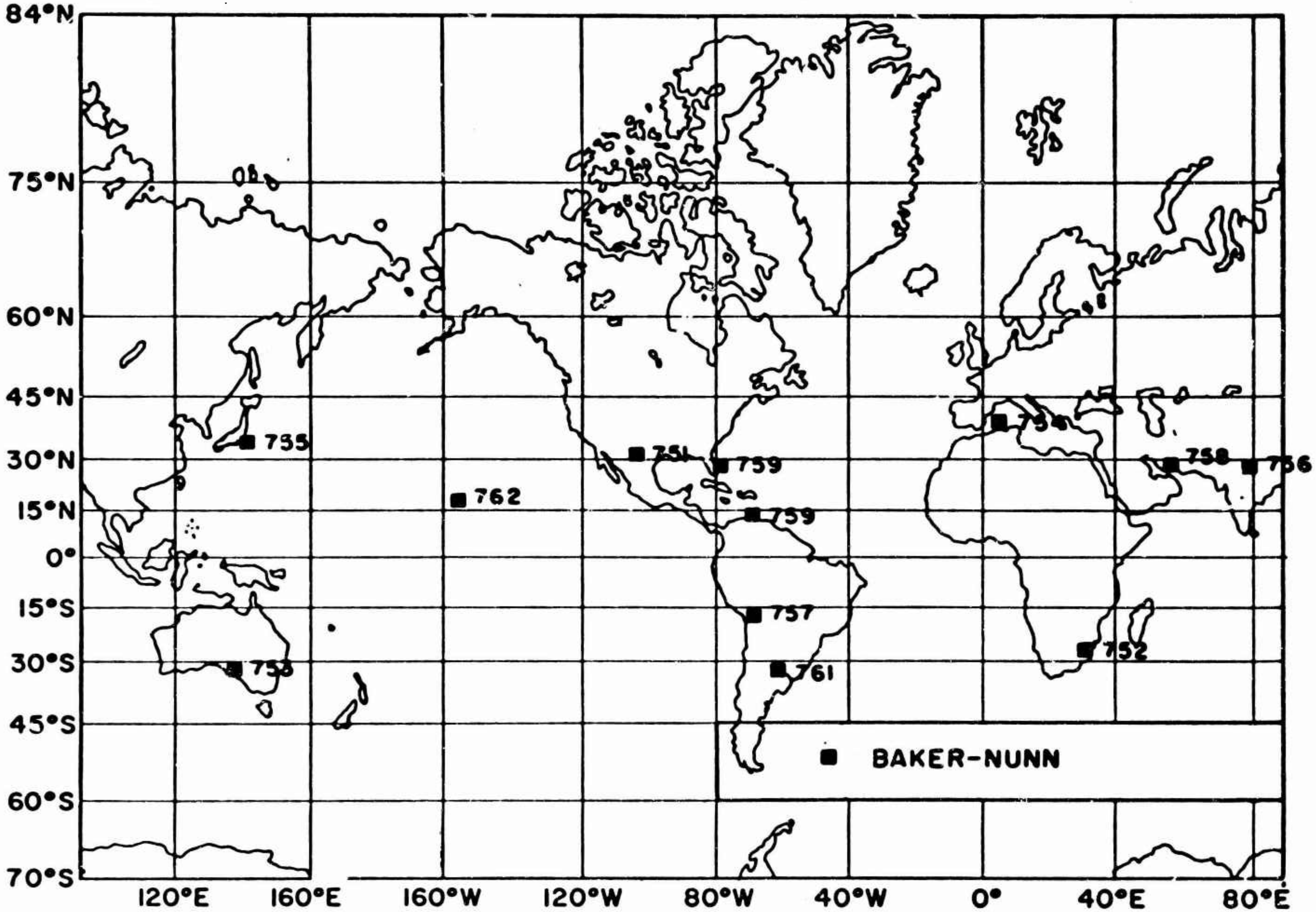

FIGURE 1

FIGURE 1

DOPPLER TRACKING STATION SITES

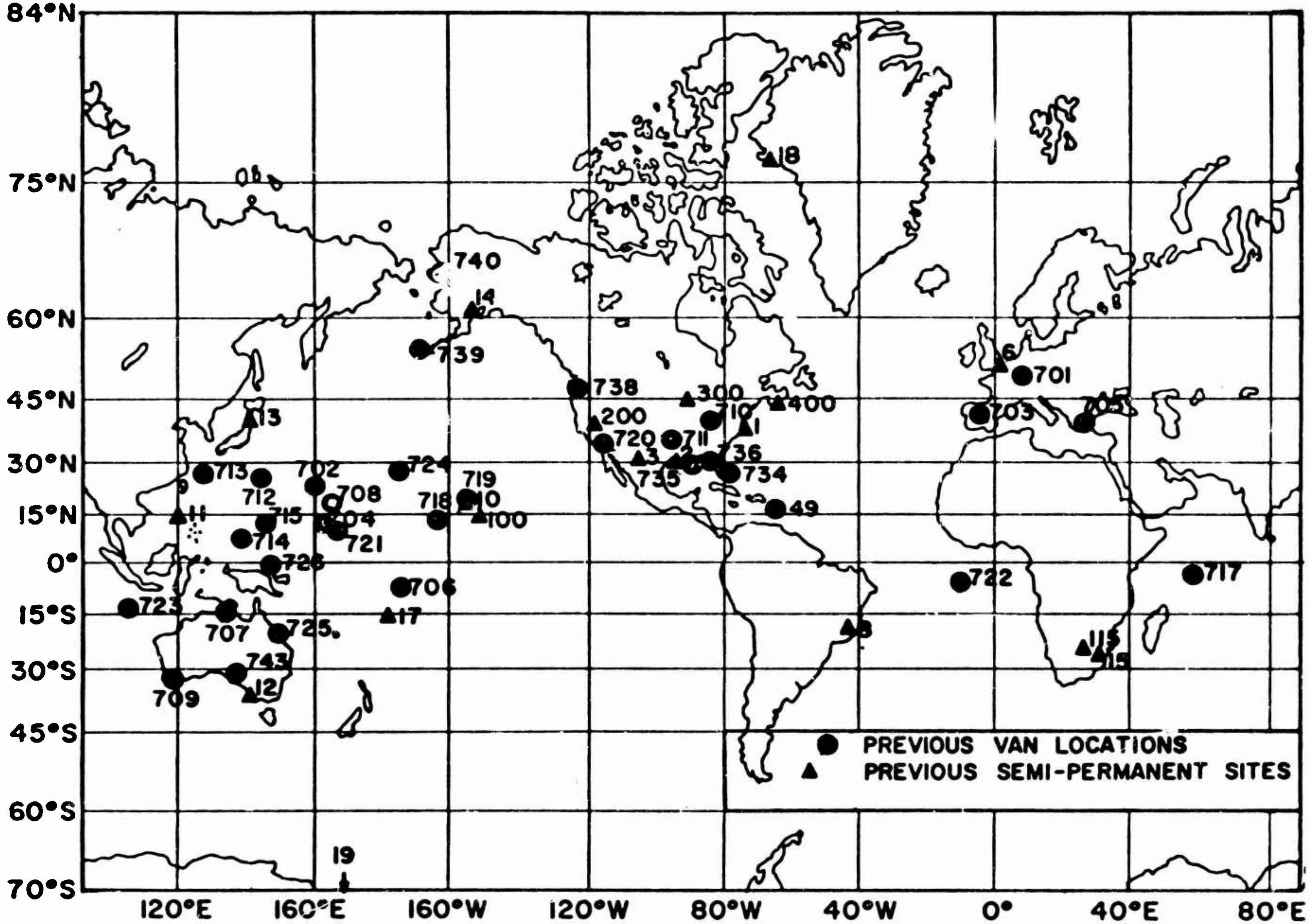

FIGURE 2

Parameters

The geoid is found by determining the coordinates for which the potential, defined by the following or some similar expression, is equal to a selected constant:

$$V = \frac{\mu}{r} \Sigma \left(\frac{R}{r}\right)^n P_n^m (\varphi) (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)$$

where φ, λ, r are the polar coordinates of a point on the geoid, $P_n^m(\varphi)$ are the associated Legendre polynomials of degree and order (n,m), R is a nomial earth's radius which scales the coefficients $C_{n,m}$, $S_{n,m}$ which are to be determined, and μ is the product of the earth's mass and the universal gravitational constant. The potential may be evaluated for a constant which minimizes the differences between the geoid and a reference ellipsoid, although a different choice may be made as in the last section of this paper as a computational expedient. Although attempts have been made to evaluate the coefficient, μ, from Doppler observations alone[1] and from optical observations together with a scale provided by survey[11], the most reliable estimate obtained to date has been obtained from the analysis of observations of lunar probes.[12][13] The geoids determined from satellite solutions have involved increasingly larger numbers of coefficients ranging from fourth to eighth degree and order. Coefficients of 13th and 14th order have also been obtained;[14][15] while these higher order coefficients do not directly influence the geoid by more than a meter, they do influence the satellite motion significantly and therefore could bias the determination of the lower coefficients if their effects are ignored. Other parameters which affect the observational data and must therefore be considered in the solution for the gravity coefficients include the coordinates of the observing stations, the orbital constants of the satellites, atmospheric drag and solar radiation parameters, and instrument biases. The strength of the solution for station coordinates may be improved in various ways: Constraints may be imposed on the solution such as fixing the longitude of one of the observing stations at an arbitrary value, or holding the relative coordinates of the stations within a datum to the positions found by survey. Rather than imposing survey constraints, the positions of the stations are sometimes introduced into the solution as additional observational data with weights corresponding to the estimated accuracy of the assumed positions. The six orbit constants for each span of data used in the solution are defined differently in accordance with the theory used, as discussed in the next section. The drag and radiation parameters modify physical models of varying levels of complexity. The most complex models include a parameter to scale external measures of time, latitude, longitude, and altitude variations in density and radiation. Simpler models include parameters to scale functions which vary only with altitude, while in still other models, extra parameters are introduced to account for the dominant effects of drag on satellite motion, without the use of a specific atmospheric model. Instrument biases are introduced only to take account of variations in the Doppler signals due to variations in the frequencies of the oscillator of the satellite or observing station. Either a frequency parameter or both frequency and frequency drift parameters are introduced for each pass of each satellite over each station.

Orbit Theory

The orbit constants are the six constants of integration of the satellite orbit which best fit the observations. The orbit is computed either by numerical integration of the equations of motion from these initial conditions or by general perturbation methods, wherein the quadratures are completed analytically after appropriate transformations and approximations are made. Since partial derivatives of the observations with respect to the orbit constants, gravity parameters and other constants in the equations of motion are required in the least squares solution, the partial derivatives of satellite position with respect to these parameters are also obtained either by numerical integration or by general perturbation methods. In some cases some or all of the partial derivatives are found by general perturbation methods while the satellite orbit is found by numerical integration. Although the methods differ in their accuracy, the differences are not sufficient to account for the differences in the solutions for the geoid.

Statistical Representation

Since the distribution of the observing stations on the earth is not uniform, some attempts have been made to compensate by introducing weights which tend to equalize the strength of the data from different geographic areas. Some experiments have also been performed in which the component of the optical sight line which is along the direction of motion of the satellite was given lower weight in order to compensate for variable atmospheric drag effects. The various methods of aggregating the 300 or so Doppler observations obtained on each satellite pass include a special form(16) of averaging groups of eight points, polynomial fitting to the pass, and transformation of the raw data to measurements of frequency, slant range and the equivalent of the time of closest approach for the pass. All representations of the data assume the observations are uncorrelated whether in the raw or in the transformed state.

Method of Solution

Each solution for the geoid involves the formation of the normal equations arising from imposing the condition that the values of the parameters shall minimize the sums of squares of the residuals of observation. These equations are sometimes solved simultaneously while in other cases subsets of the equations are solved for subsets of the parameters. It is expected that converged solutions obtained by either method would be equivalent, although statistical estimates of the accuracy of the solution are normally obtained only when the parameters are obtained from the simultaneous solution of the equations.

SENSITIVITY OF SOLUTIONS TO VARIATIONS IN OBSERVATIONS AND PARAMETERS USED

Solution NWL-5E

The most complete solution for the geoid obtained by the Naval Weapons Laboratory on the basis of Doppler observations is called NWL-5E. This parameter set was obtained as a simultaneous solution for the gravity coefficients through seventh degree and sixth order, the coordinates of the observing stations, the orbit parameters and a drag parameter for each span of data used, and a frequency and frequency drift parameter for each satellite pass over each station. The extent and distribution of the observational data upon which this solution is based is shown in table 1. The NWL-5E gravity parameters obtained in the solution are listed in appendix E, table 6, while the geoid contours obtained from these coefficients are shown in appendix A, figure 3. As a computational expedient, each geoid given in this report was defined to be the equipotential surface equivalent to the gravity coefficients which passes through a geocentric reference ellipsoid at zero degrees latitude and longitude. The next sections describe the sensitivity of this solution for the geoid to variations in the number of gravity parameters in the solution, the number and distribution of observations on satellites having different orbital inclinations, and to the number of observing stations. It is believed that these are the principal sources of variations in the solutions for the geoid obtained to date. The geoid contours obtained in the tests discussed in the next three paragraphs are shown in appendices B, C and D, respectively.

Effect of Reducing Number of Parameters

The NWL-5E observational data were used to conduct a series of tests to determine the influence of the number of gravity parameters on the solution for the geoid. First the solution was truncated from seventh degree to fourth degree by simply discarding the higher degree coefficients. Some of the features of the geoid were lost, and many of the other features were reduced in depth as may be seen by comparing the first two columns of table 2. The set of coefficients through fourth degree and order obtained in a solution which did not include higher order coefficients as parameters was termed the "best (4,4) solution." The features of this solution, shown in the third column table 2, are similar to the truncated (4,4) solution. Another method of reducing the number of gravity parameters in the solution involves a transformation to the space in which the gravity parameters are decoupled, and reduction of the number of gravity parameters in this "Q" space.[1] Solutions for the 40 and 50 most significant parameters in Q space, based on the same observational data used in the NWL-5E solution are given in the last two columns of table 2. It can be seen that the solution for the 40 most significant gravity parameters is inferior

TABLE 1

NUMBER OF SATELLITE PASSES USED IN SOLUTION NWL-5E

Station	Satellite 1961 $\alpha\eta 1$	1962 $\beta\mu 1$	1961 o1	Polar*	Total
Maryland	64	228	160	449	901
Texas	71	259	130	100	560
N. Mexico	87	314	195	446	1042
England	--	48	119	332	499
Brazil	84	193	--	312	589
Hawaii	78	--	--	354	432
Phillipines	66	203	--	353	622
Australia 12	44	164	--	197	405
Australia 709	--	--	--	145	145
Alaska	--	157	156	900	1213
So. Africa 15	76	160	--	--	236
So. Africa 115	--	--	--	331	331
Samoa	--	170	--	348	518
Greenland	--	--	--	707	707
Oahu	--	271	21	285	577
California 200	--	202	--	296	498
California 720	--	--	--	295	295
Minnesota	--	--	33	334	367
Maine	--	--	34	381	415
Marcus	--	116	--	--	116
Japan	75	214	--	419	708
Indiana	--	68	--	--	68
Oklahoma	--	77	--	--	77
Iwo Jima	--	50	--	--	50
Okinawa	--	96	--	112	208
Yap	--	--	--	49	49
Guam	--	--	--	35	35
Johnston	--	--	--	127	127
Kauii	--	--	--	212	212
Total Passes	645	2990	848	7519	12002
No. of Weeks of Data	5	7	10	15	37
Orbital Inclination	32°	50°	67°	90°	

*1963 38B, 1963 38C or 1963 49B

TABLE 2

EFFECT OF NUMBER OF GRAVITY COEFFICIENTS ON GEOID

Location		NWL-5E	(4,4) Truncation	(4,4) Best Fit	"Top 40" Solution	"Top 50" Solution
England	Latitude	55°N	45°N	40°N	50°N	55°N
	Longitude	340°E	0°	355°E	335°E	345°E
	Height	61 m	57 m	59 m	64 m	62 m
So. Africa		50°S	50°S	50°S	50°S	45°S
		20°E	35°E	40°E	60°E	15°E
		33 m	48 m	37 m	58 m	35 m
India		5°N	10°N	20°N	10°N	5°N
		75°E	75°E	75°E	70°E	75°E
		- 110 m	- 84 m	- 91 m	- 55 m	- 100 m
Japan		0°	0°	10°N	5°N	5°N
		145°E	145°E	150°E	145°E	150°E
		71 m	68 m	57 m	61 m	66 m
No. Pacific		35°N	--	--	35°N	35°N
		185°E	--	--	185°E	185°E
		- 36 m	(-13m)[1]	(-5m)[1]	- 37 m	- 42 m
E. Pacific		20°N	30°N	30°N	30°N	15°N
		245°E	265°E	265°E	275°E	245°E
		- 72 m	- 45 m	- 60 m	- 24 m	- 57 m
W. Atlantic		15°N	--	--	--	20°N
		305°E	--	--	--	305°E
		- 56 m	(-19m)[1]	(-23m)[1]	(-9m)[1]	- 46 m
So. America		25°S	25°S	30°S	15°S	20°S
		295°E	285°E	280°E	280°E	295°E
		11 m	16 m	9 m	67 m	14 m
So. Pacific		75°S	70°S	70°S	75°S	75°S
		180°E	195°E	185°E	180°E	185°E
		- 77 m	- 52 m	- 52 m	- 50 m	- 68 m

[1] Geoid height at location given under NWL-5E solution

to the (4,4) solution, although the latter involves a smaller number of parameters. However, this does not indicate that solutions in Q space are without application: The transformation was designed to obtain a solution in cases where the full parameter set is indeterminate, which was not the case in this example.

Effect of Satellite Orbital Inclination

The NWL-5E solution was based upon observations of satellites having four different orbital inclinations. Solutions were also obtained omitting data from each of the four inclinations in turn. A summary of the geoid features for each of these solutions is given in table 3. Omission of the data observed on the satellite with an orbital inclination of 32 degrees resulted in the largest disturbance of the solution. However, the geoid heights generally agree to 15 meters.

Effect of Number of Observations and Number of Stations

In order to test the influence of the number of observations and the number of observing stations on the solution, solutions were made using data obtained during one week for each of three satellites. In the first of three solutions summarized in table 4, data from all observing stations were used to determine gravity coefficients through the seventh degree and sixth order. A second test, which limited the number of observing stations to eight, resulted in gross distortions of the computed geoid. However, adding three pair of thirteenth and fourteenth order gravity coefficients as parameters of the solution resulted in a computed geoid close to that obtained with more extensive observations. The number of passes used in these last two solutions, which was only 1/40 of the number used in the NWL-5E solution, were distributed as shown in table 5.

TABLE 3

EFFECT OF SATELLITE INCLINATION ON SOLUTION FOR GEOID

Location		NWL 5E Solution	Solution Omitting Orbital Inclination of: 32°	50°	67°	90°
England	Latitude	55°N	60°N	60°N	55°N	50°N
	Longitude	340°E	340°E	345°E	345°E	345°E
	Height	61 m	73 m	89 m	63 m	54 m
So. Africa		50°S	35°S	30°S	55°S	45°S
		20°E	15°E	15°E	50°E	50°E
		33 m	49 m	46 m	36 m	22 m
India		5°N	5°N	5°N	5°N	5°N
		75°E	70°E	75°E	75°E	75°E
		- 110 m	- 95 m	- 90 m	- 110 m	- 125 m
Japan		0°	0°	10°N	0	10°S
		145°E	145°E	145°E	145°E	160°F
		71 m	106 m	79 m	73 m	68 m
No. Pacific		35°N	30°N	35°N	35°N	35°N
		185°E	180°E	185°E	185°E	200°E
		- 36 m	- 34 m	- 39 m	- 39 m	- 63 m
E. Pacific		20°N	20°N	20°N	20°N	20°N
		245°E	240°E	240°E	245°E	250°E
		- 72 m	- 56 m	- 63 m	- 73 m	- 46 m
W. Atlantic		15°N	10°N	15°N	20°N	20°N
		305°E	305°E	305°E	305°E	305°E
		- 56 m	- 38 m	- 57 m	- 54 m	- 74 m
So. America		25°S	25°S	25°S	30°S	10°S
		295°E	295°E	295°E	300°E	285°E
		11 m	32 m	36 m	11 m	3 m
So. Pacific		75°S	75°S	75°S	75°S	70°S
		180°E	180°E	185°E	185°E	195°E
		- 77 m	- 67 m	- 88 m	- 85 m	- 85 m

TABLE 4

EFFECT OF NUMBER OF OBSERVATIONS AND STATIONS ON SOLUTION FOR GEOID

Location		NWL-5E Solution	Solutions Based on Three Weeks of Data: All Stations Without Resonant Parameters	8 Stations Without Resonant Parameters	8 Stations With Resonant Parameters
England	Latitude	55°N	40°N	30°N	50°N
	Longitude	340°E	340°E	335°E	345°E
	Height	61 m	77 m	273 m	65 m
So. Africa		50°S	40°S	30°S	50°S
		20°E	15°E	345°E	15°E
		33 m	63 m	220 m	18 m
India		5°N	10°N	30°N	10°N
		75°E	75°E	90°E	75°E
		- 110 m	- 101 m	- 37 m	- 129 m
Japan		0°	0°	0°	30°N
		145°E	145°E	140°E	150°E
		71 m	81 m	237 m	46 m
No. Pacific		35°N	30°N	--	45°N
		185°E	180°E	--	190°E
		- 36 m	- 16 m	--	- 59 m
E. Pacific		20°N	10°N	20°N	50°N
		245°E	245°E	230°E	280°E
		- 72 m	- 65 m	- 26 m	- 88 m
W. Atlantic		15°N	5°N	0°	10°N
		305°E	315°E	310°E	300°E
		- 56 m	- 37 m	- 21 m	- 88 m
So. America		25°S	40°S	30°S	30°S
		29[illegible]°E	260°E	270°E	295°E
		11 m	33 m	174 m	- 7 m
So. Pacific		75°S	70°S	75°S	65°S
		180°E	190°E	165°E	175°E
		- 77 m	- 59 m	2 m	- 91 m

TABLE 5

NUMBER OF SATELLITE PASSES USED IN 3 ARC SOLUTION

Station	Satellite 1962 βμ1	1961 o1	Polar	Total
Maryland	19	22	--	41
New Mexico	30	20	28	78
England	6	26	16	48
Brazil	8	--	26	34
Australia	22	--	11	33
So. Africa	14	--	--	14
Samoa	15	--	25	40
Hawaii	29	--	--	29
Total	143	68	106	317

SUMMARY

While differences in various published solutions for the geoid based on satellite data were not tested under controlled conditions, the differences do not appear to be unreasonable in view of the effects of variations in the number of parameters on the solution (table 2) and of the effects of biases under conditions where the data density is limited (table 5). The latter tests show that the principal geoid features can be obtained on the basis of data obtained from a small number of stations during a short time period provided that all significant parameters are considered in the solution. The sensitivity of the solution to the satellite inclinations considered (table 3) tends to indicate that the recent solutions based on the Doppler system, which yields the highest data density, provides geoid undulations to an accuracy of about 20 meters. Considering that future solutions will include three times the number of gravity coefficients and three times the number of satellite inclinations,[15] it seems reasonable to expect that 10 meter accuracy will be obtained in the geoid features in the future.

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APPENDIX A

GEOID HEIGHTS FOR NWL 5E SOLUTION

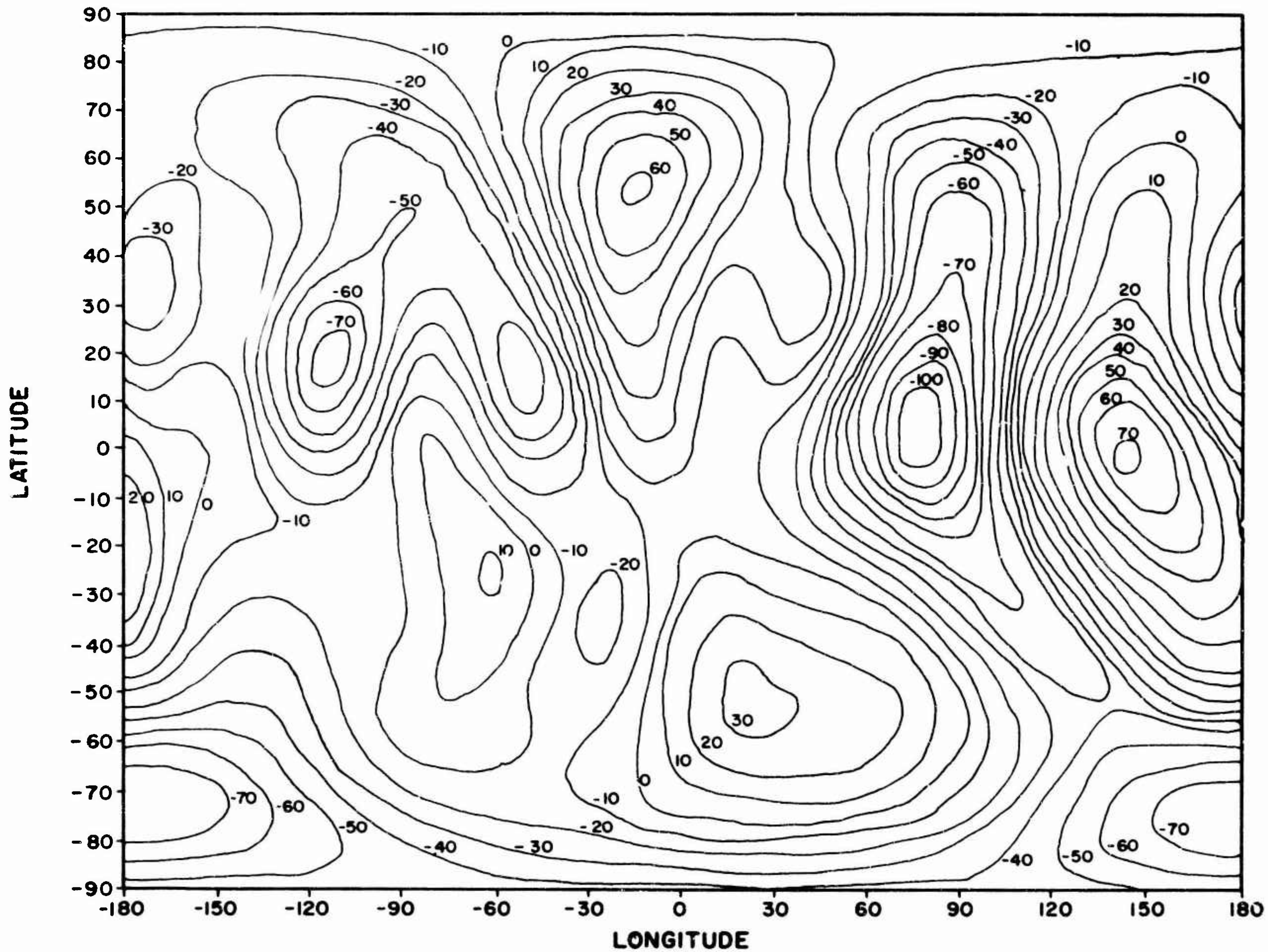

FIGURE 3

APPENDIX B

GEOID HEIGHTS FOR (4, 4) TRUNCATION OF NWL 5E SOLUTION

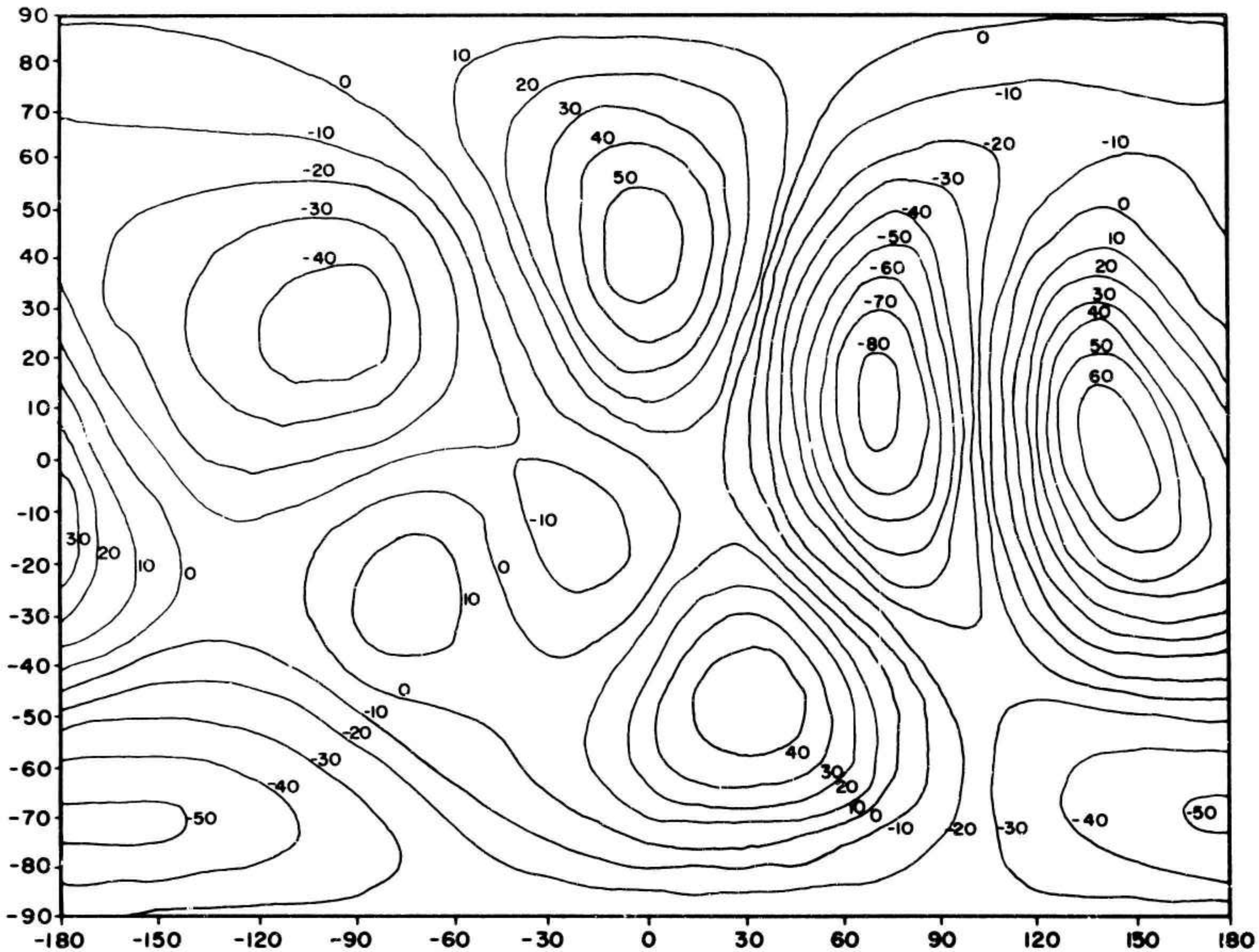

FIGURE 4

GEOID HEIGHTS FOR BEST (4, 4) SOLUTION

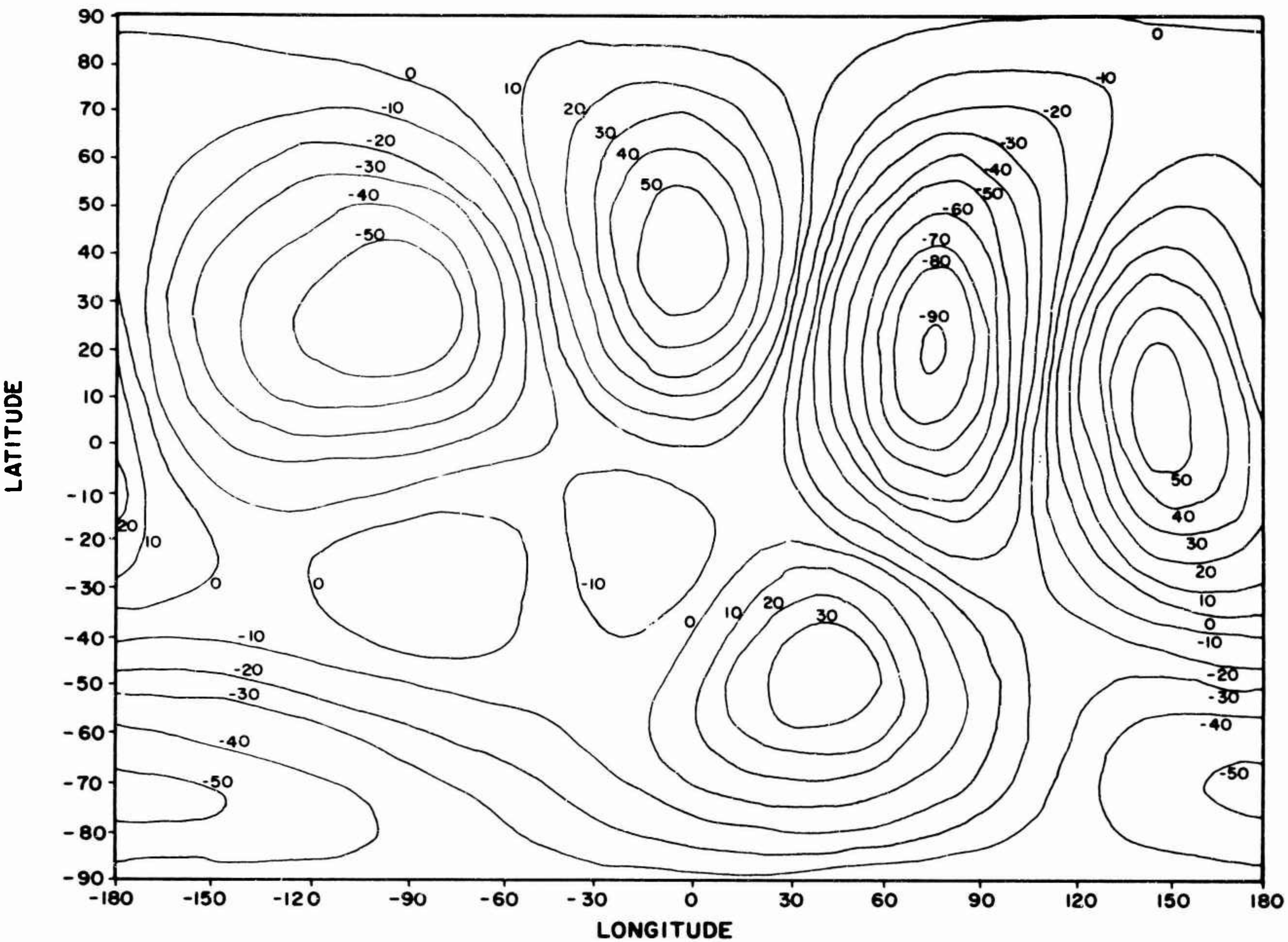

FIGURE 5

LONGITUDE

FIGURE 5

GEOID HEIGHTS FOR MOST SIGNIFICANT 40 PARAMETERS

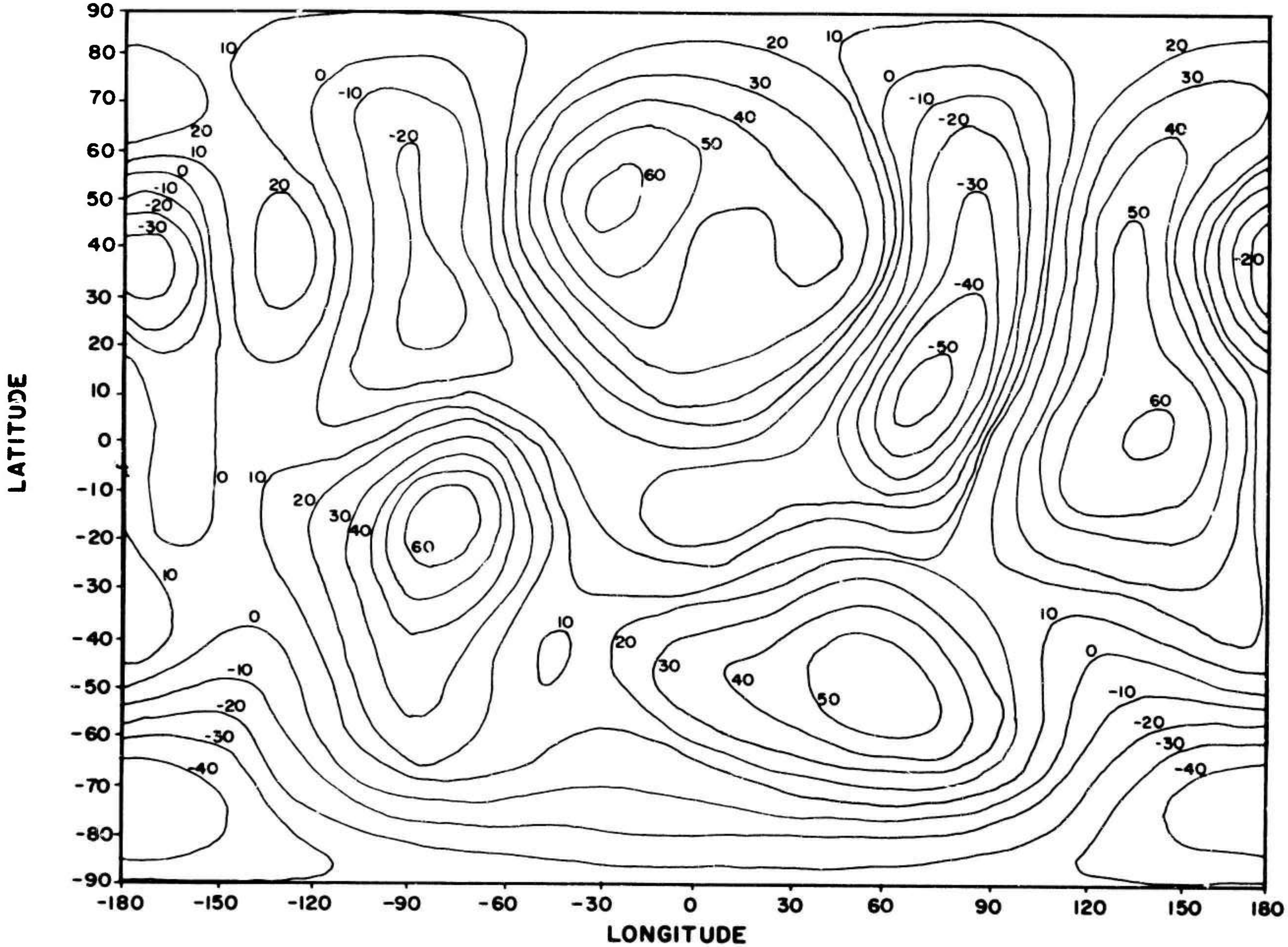

FIGURE 6

GEOID HEIGHTS FOR MOST SIGNIFICANT 50 PARAMETERS

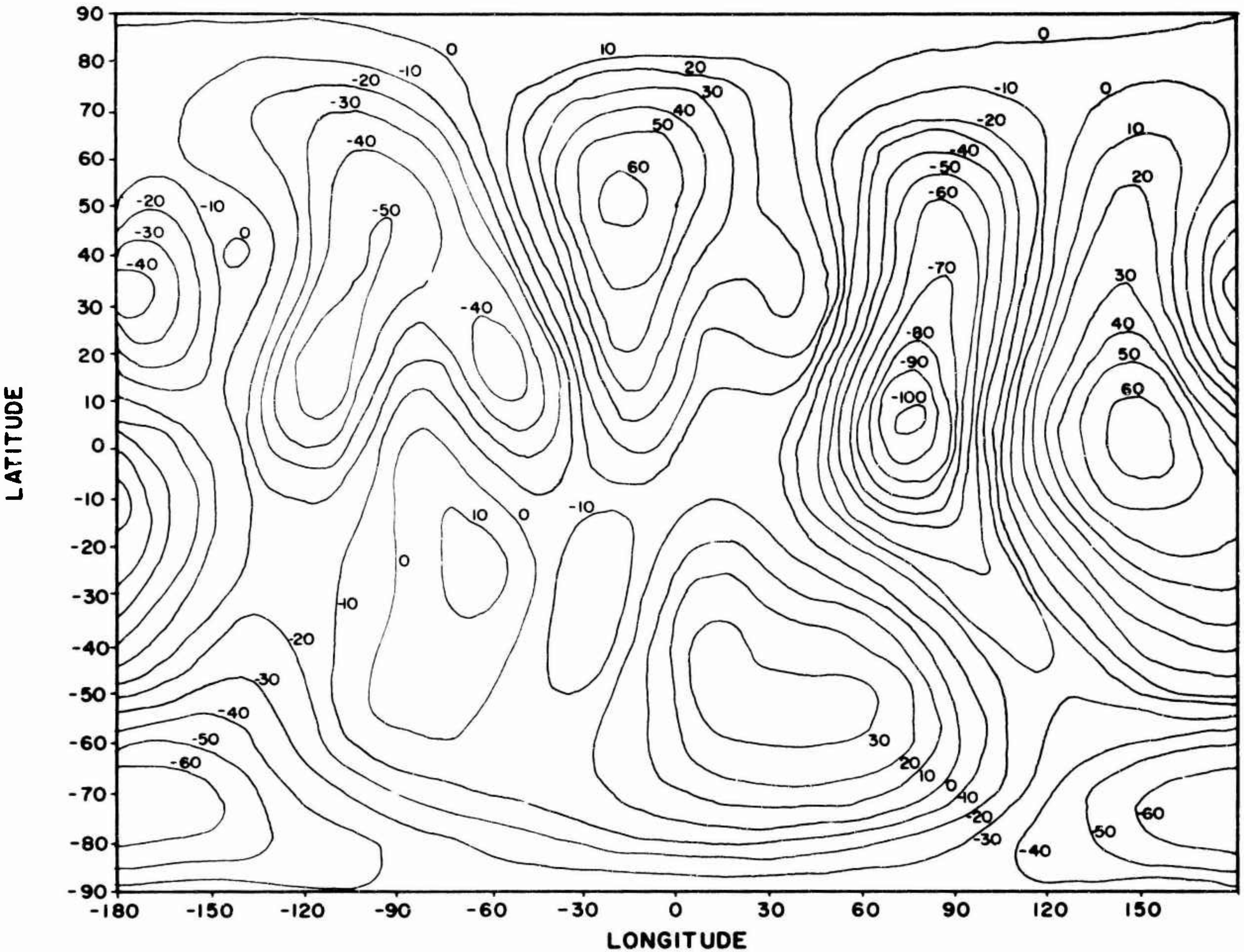

FIGURE 7

APPENDIX C

GEOID HEIGHTS WITHOUT 30 DEGREE SATELLITE INCLINATION

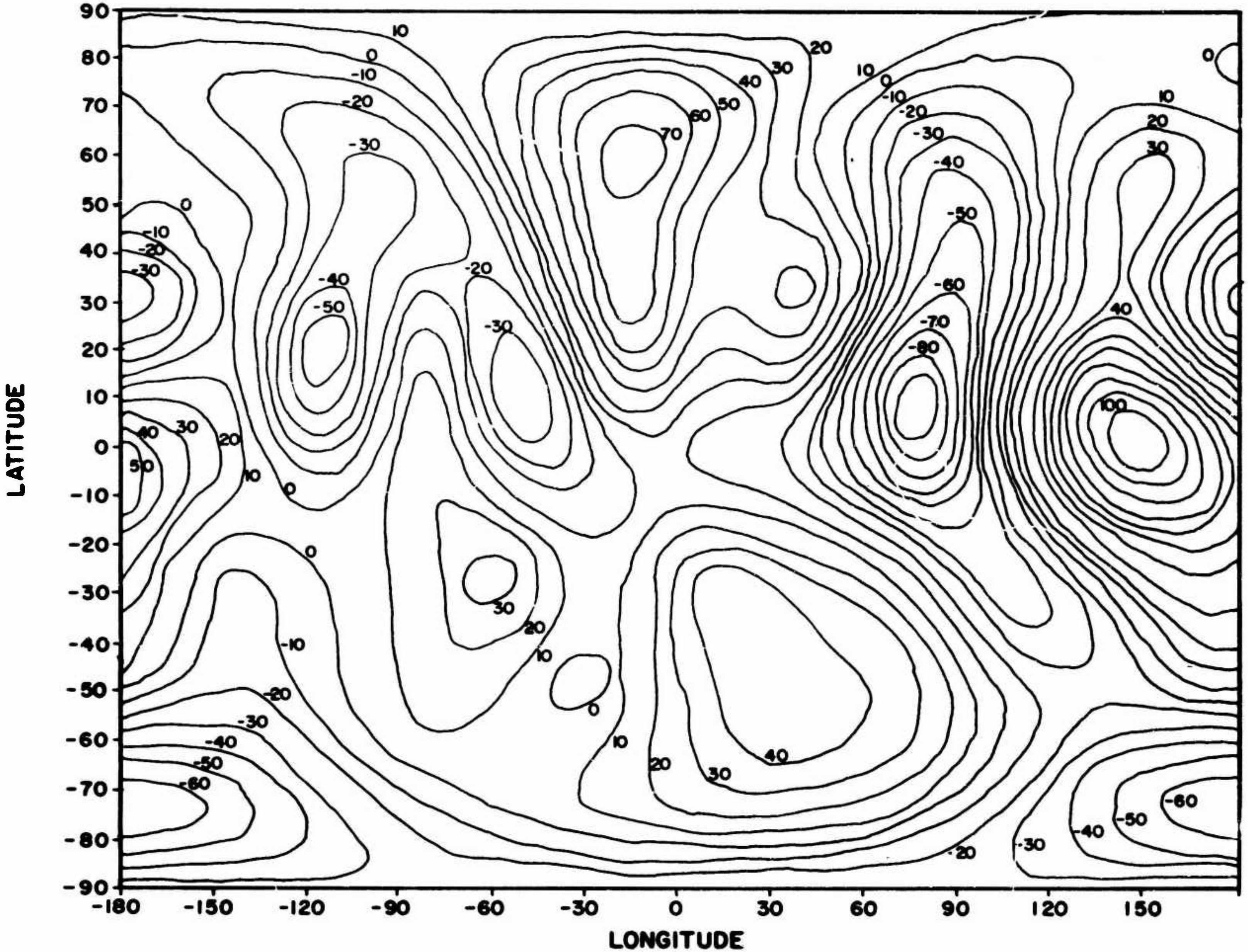

FIGURE 8

GEOID HEIGHTS WITHOUT 50 DEGREE SATELLITE INCLINATION

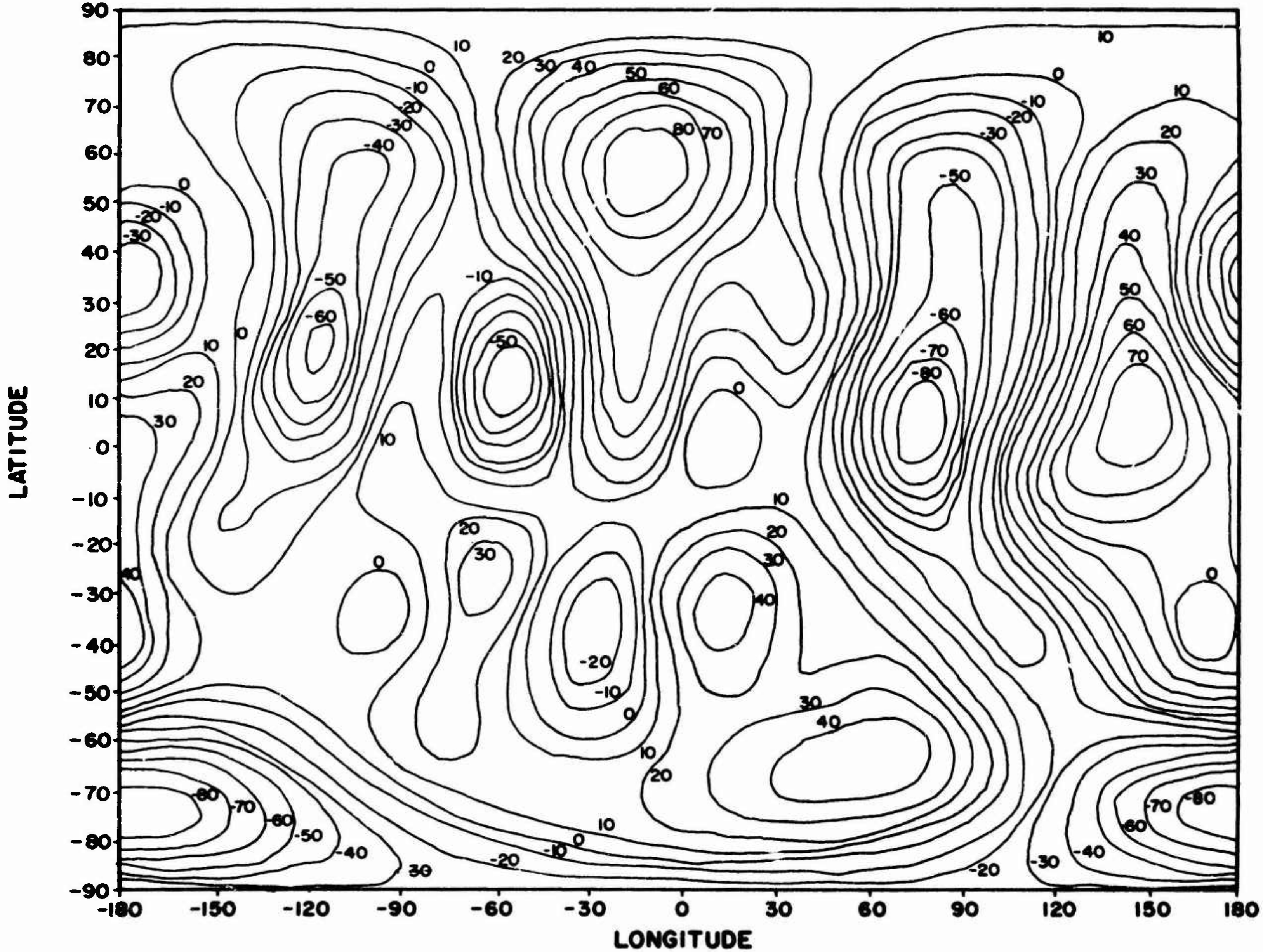

FIGURE 9

GEOID HEIGHTS WITHOUT 67 DEGREE SATELLITE INCLINATION

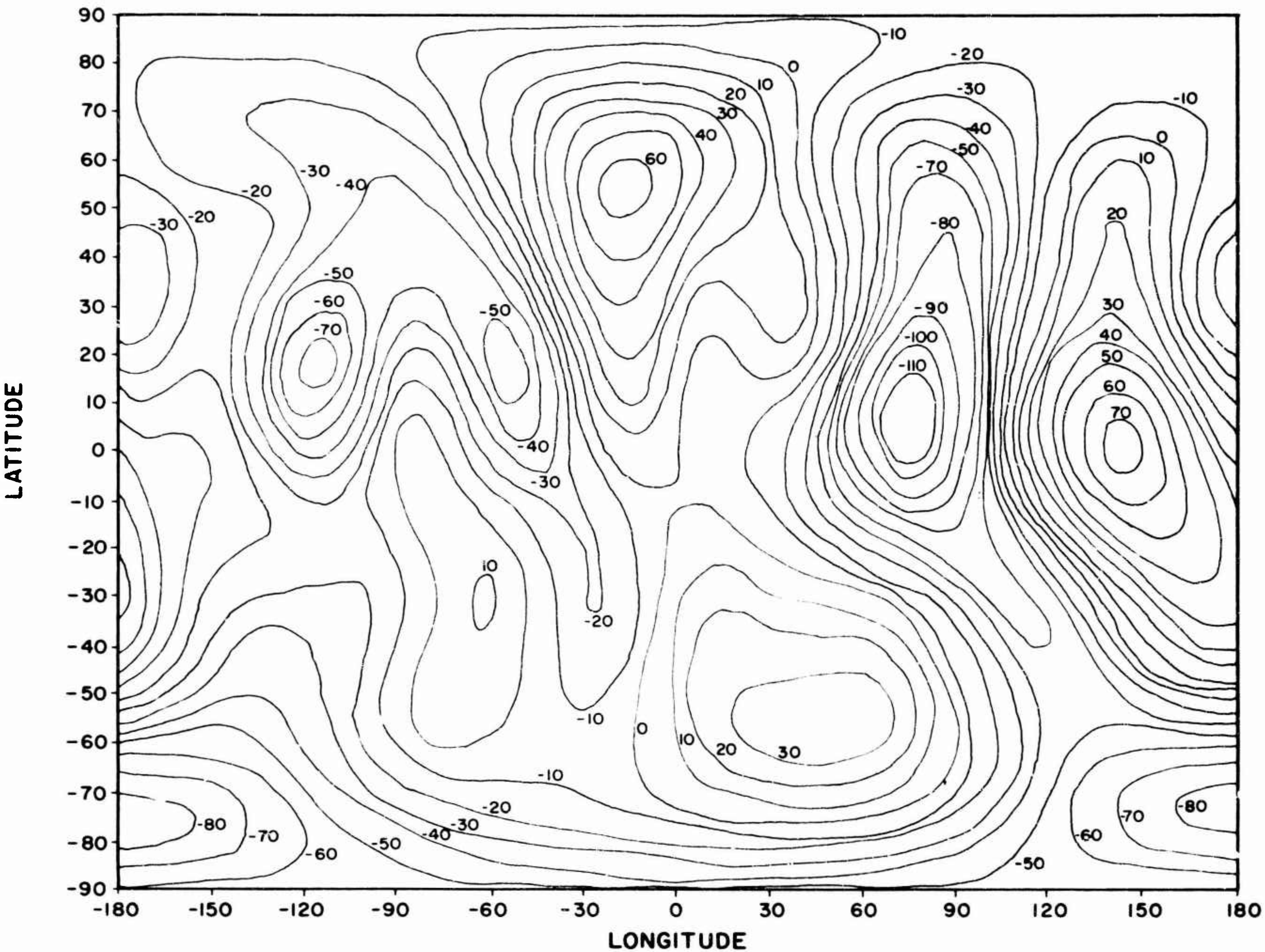

FIGURE 10

GEOID HEIGHTS WITHOUT 90 DEGREE SATELLITE INCLINATION

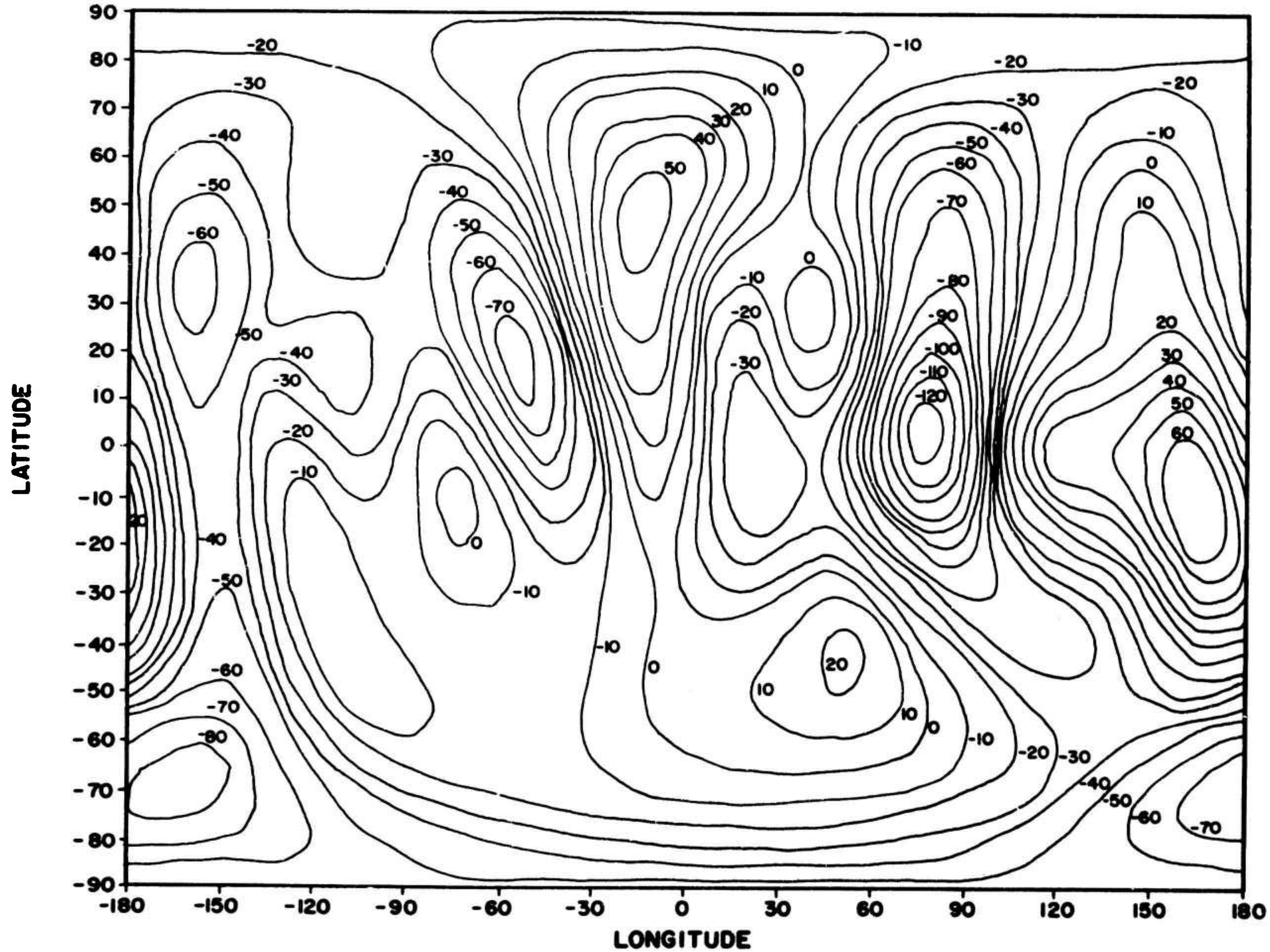

FIGURE 11

APPENDIX D

GEOID HEIGHTS FOR 3 ARC SOLUTION WITHOUT RESONANT PARAMETERS (ALL STATIONS)

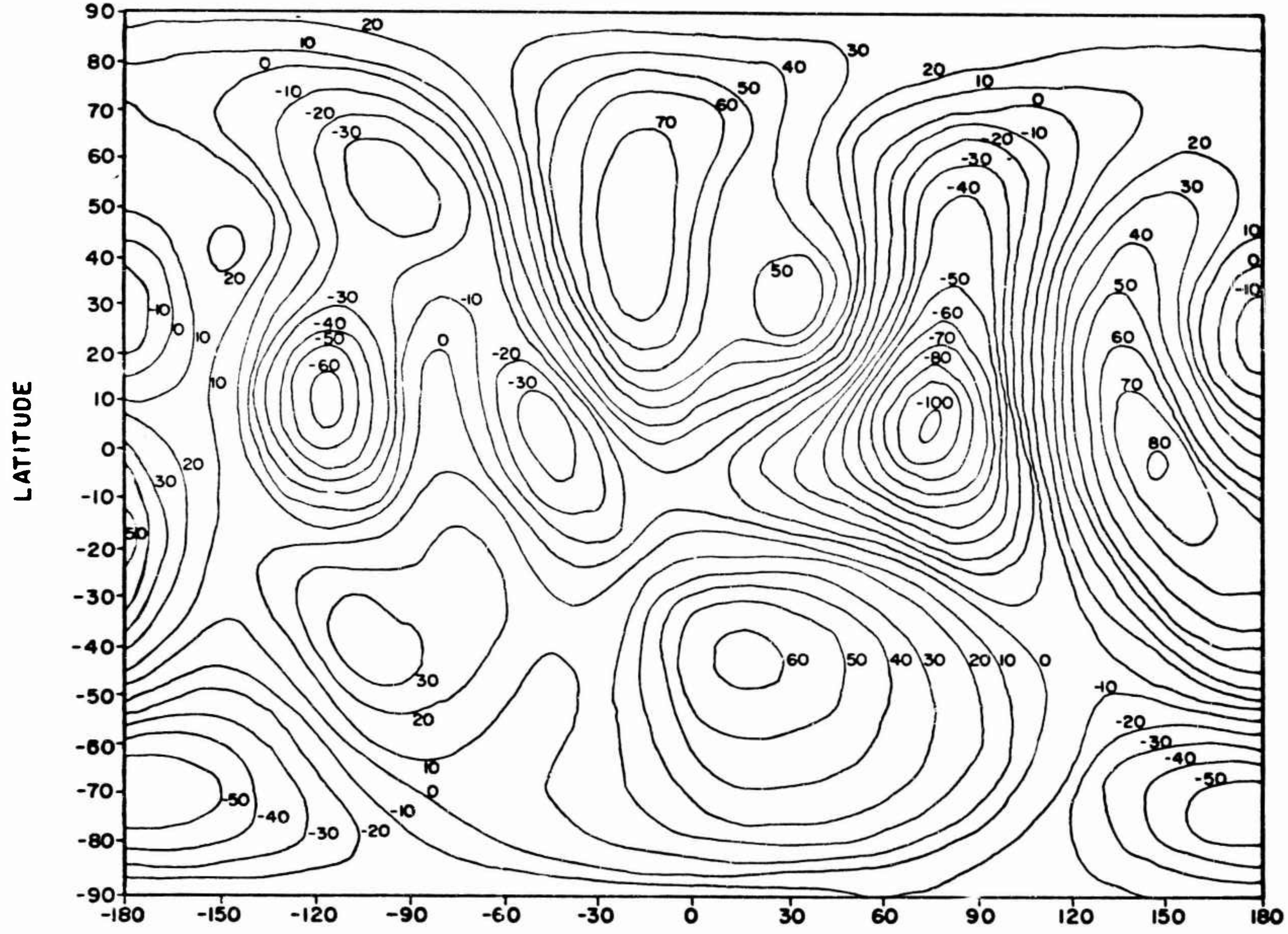

FIGURE 12

GEOID HEIGHTS FOR 3 ARC SOLUTION WITHOUT RESONANT PARAMETERS (8 STATIONS)

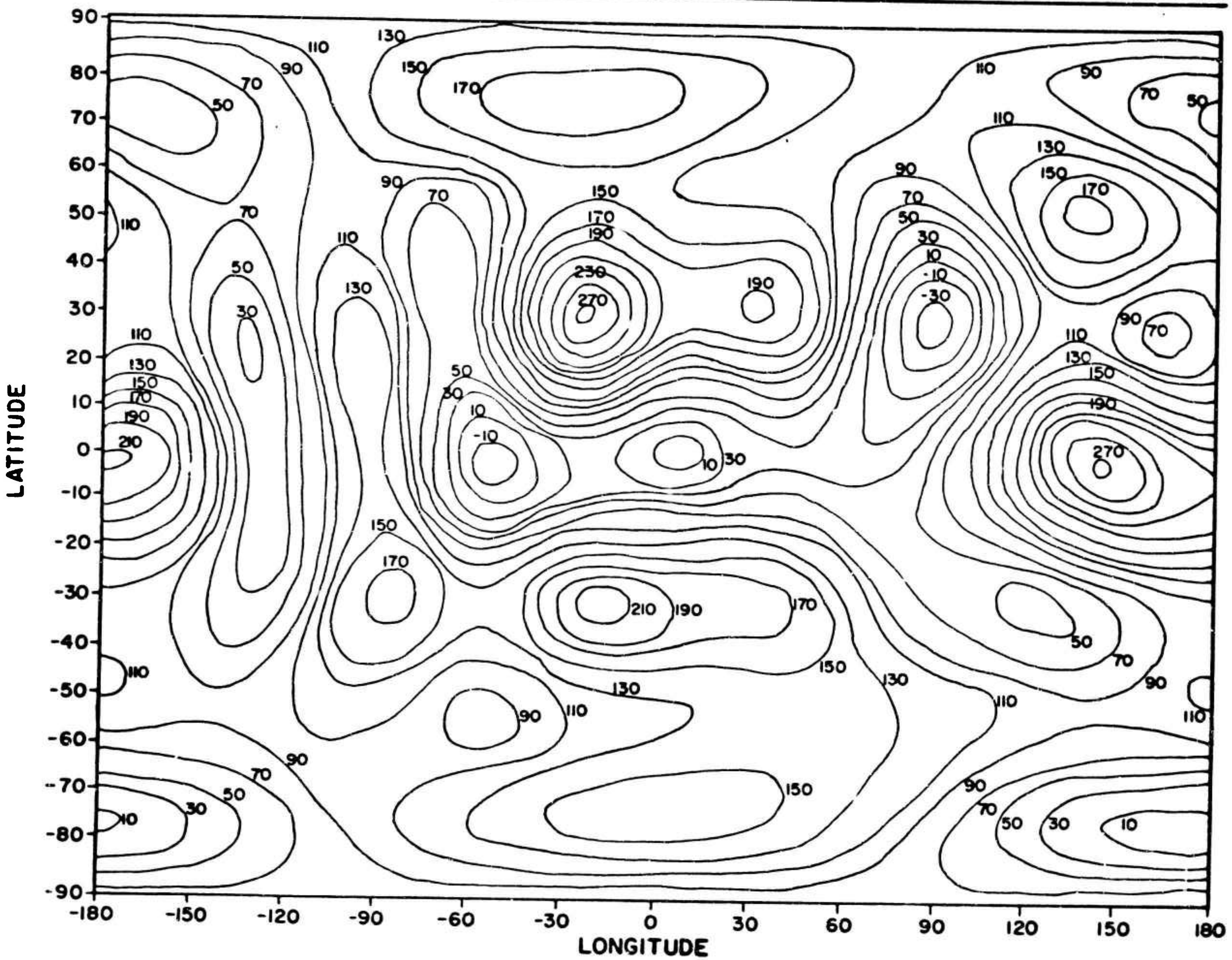

FIGURE 13

GEOID HEIGHTS FOR 3 ARC SOLUTION WITH RESONANT PARAMETERS (8 STATIONS)

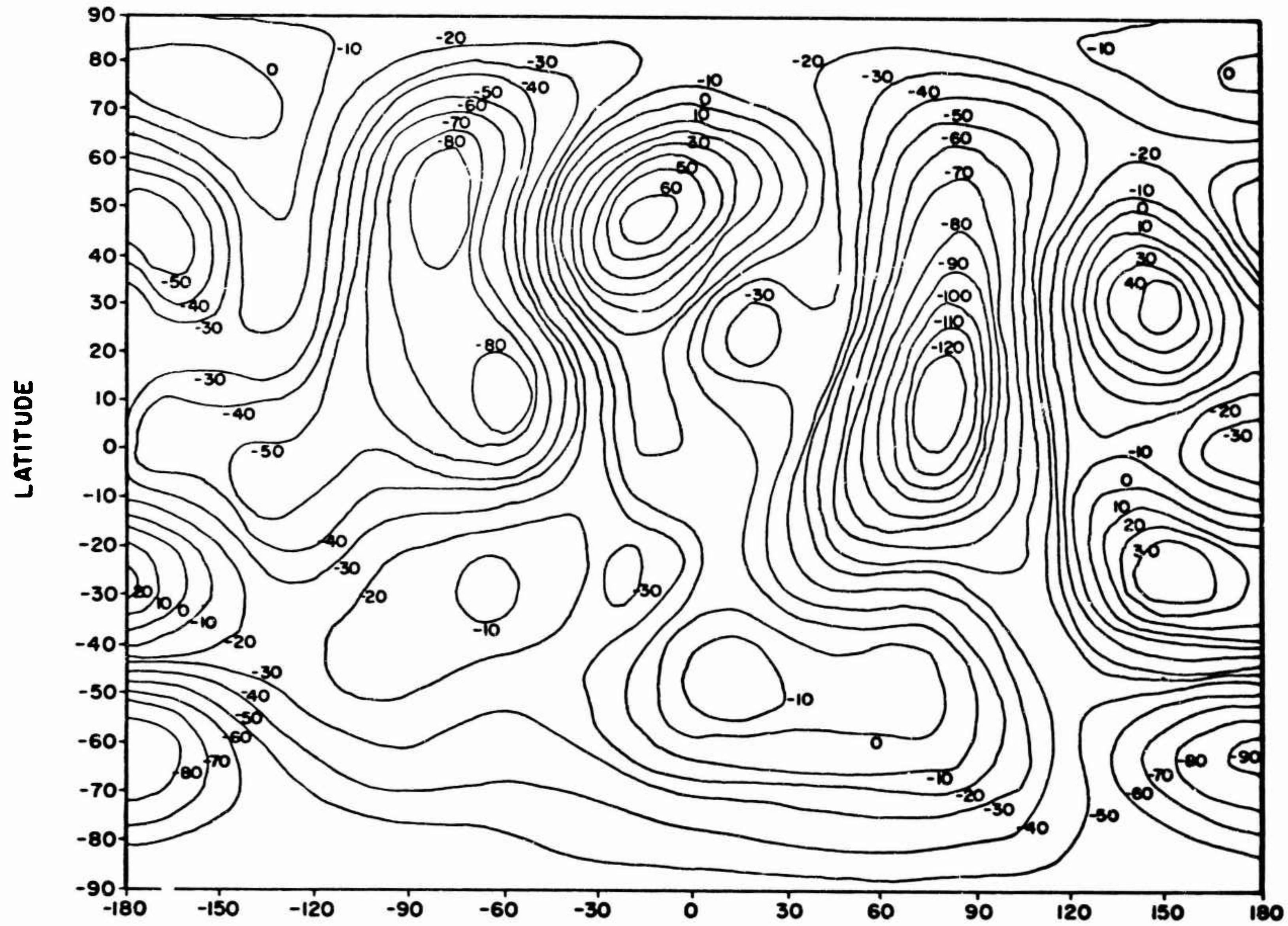

FIGURE 14

APPENDIX E

TABLE 6

NWL-5E NORMALIZED GRAVITY COEFFICIENTS[1]

n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$	n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$
2	0	-484.194		6	1	-.085	.192
3	0	.984		6	2	.129	-.457
4	0	.507		6	3	-.020	-.134
5	0	.045		6	4	-.193	-.316
6	0	-.219		6	5	-.093	-.786
7	0	.105		6	6	-.324	-.360
2	1	.016	.062	7	1	.331	.083
2	2	2.446	-1.519	7	2	.350	-.195
3	1	2.148	.274	7	3	.323	.045
3	2	.978	-.906	7	4	-.467	-.244
3	3	.585	1.625	7	5	.055	.021
4	1	-.495	-.575	7	6	-.477	-.244
4	2	.274	.671				
4	3	1.030	-.247				
4	4	-.413	.336				
5	1	.032	-.119				
5	2	.637	-.328				
5	3	-.389	-.124				
5	4	-.549	.148				
5	5	.215	-.594				

$$V = \mu \Sigma \left[R^n C_{nm} \frac{P_n^m \left(\frac{z}{r}\right)}{r^{n+1}} \cos m\lambda + R^n S_{nm} \frac{P_n^m \left(\frac{z}{r}\right)}{r^{n+1}} \sin m\lambda \right]$$

$$\bar{C}_{n,m} = \left[(n-m)!\ (2n+1)K/(n+m)!\right]^{-\frac{1}{2}}\ C_{nm}, \text{ where } K = 1 \text{ when } m = 0,$$

$$K = 2 \text{ when } m \neq 0.$$

where

- P_n^m is the associated Legendre polynomial
- R^n is the earth's radius
- μ is the earth's gravity constant
- λ is longitude with respect to Greenwich
- z and r are distances above the equatorial plane and from the center of earth, respectively

[1] Multiply all coefficients by 10^{-6}. $\mu = 398605.42\ km^3/sec^2$.

APPENDIX F

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13. ABSTRACT

Determinations of the geoid made by different authors have differed by more than forty meters in some geographic locations. The authors differed in the observations employed in the number of gravity coefficients they determined, and in a number of details in the method of solution. Experiments conducted with Doppler observations on satellites have shown moderate variations (rarely as much as 30 meters) in the geoid determined if the number of satellite orbital inclinations employed is reduced by one. Reduction of the number of gravity parameters used to represent the geoid also resulted in moderate variations in the principal geoid features, except under special circumstances which are described. Reducing the number of weeks of observations did not produce deviations greater than 25 meters. However, reducing the number of observing stations in addition resulted in distortions of the computed geoid which reached 100 meters. It appears that the most recent geoid heights determined from satellite observations are correct to about 20 meters at any location and that observational data being obtained and techniques of computation being utilized should improve the accuracy to 10 meters or better.

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