ACT 2012
Math Practice I

Daniel Topa

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1 True/False

1. Which of the following statements is true?

- ○ A Every real number is rational.
- ○ B Some rational numbers are integers.
- ○ C All integers are whole numbers.
- ○ D All non-negative real numbers are positive.
- ○ E The number 3.1416 is irrational.

1.1 A. False

Counterexample: a real number which is irrational: $\sqrt{2}$

1.2 B. True

Every integer n is the rational number $\frac{n}{1}$

1.3 C. False

Counterexample: an integer which in not a whole number: -1.

1.4 D. False

The number 0 is non-negative yet not positive.

1.5 E. False

The number π is irrational. But the problem poses a finite approximation of π which is rational:

$$3.1416 = \frac{31416}{10000}$$

2 Rational numbers

2. Which of the following is not a rational number?

- ○ F -3
- ○ G $-\sqrt{3}$
- ○ H $\sqrt{9}$
- ○ J 17%
- ○ K $\frac{8}{10}$

2.1 F. Rational

$$-3 = \frac{-3}{1}$$

2.2 G. Irrational

There are no integers p and q such that

$$-\sqrt{3} = \frac{p}{q}$$

.

2.3 H. Rational

$$\sqrt{9} = 3 = \frac{3}{1}$$

2.4 J. Rational

The symbol % is Latin shorthand for *per centum*:

$$17\% = \frac{17}{100}$$

.

2.5 K. Rational

$$p = 8,\ q = 10$$

Caveat: stricter definitions require that p and q be in simplest form. In this case the rational number is $\frac{4}{1}$.

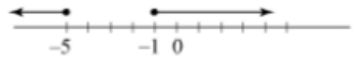
3 Precendence

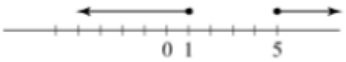
3.1 D. -22

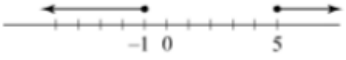
$$\begin{aligned} 2^3 - 3\left[5 - \left(4 - 3^2\right)\right] &= 2^3 - 3\left[5 - (4 - 9)\right] \\ &= 2^3 - 3\left[5 - (-5)\right] \\ &= 2^3 - 3\left[10\right] \\ &= 2^3 - 30 \\ &= 8 - 30 \\ &= -22 \end{aligned} \tag{1}$$

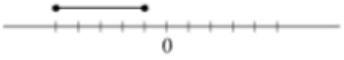
4 Regions

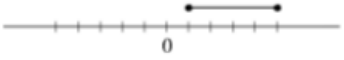
4. Which of the following graphs corresponds to the inequality: $|x+3| \geq 2$?

F 

G 

H 

J 

K 

The region represents everything outside of the ball of radius 2 centered at $x = -3$. This is the complement of the region $-5 \leq x \leq -1$.

4.1 F.

5 Remainders

5. Ramona bought 10 yards of fabric to make curtains. Each pair of curtains requires 2 feet 3 inches of fabric. After making the maximum number of pairs of curtains, how much fabric will remain unused?

- A 3 inches
- B 7 inches
- C 9 inches
- D 1 foot 3 inches
- E All of the fabric will be used.

5.1 C. 9 inches

Convert to feet: 10 yards = 30 feet of fabric. Each unit needs 2 ft 3in = $2\frac{1}{4}$ feet of fabric.

Estimate: $12 \times 2\frac{1}{4} = 24+3 = 27$ ft. We can make one more item, for a total of $27+2\frac{1}{4} = 29\frac{1}{4}$ ft. Leftover $30 - 29\frac{1}{4} = \frac{3}{4}$ ft $= 9$ in.

6 Domain

6. What is the domain of the function $f(x)=\frac{x-2}{x^2-x-2}$?

- ○ F All real numbers
- ○ G All real numbers except -1
- ○ H All real numbers except 2 and -1
- ○ J All real numbers except 2
- ○ K All real numbers except -2 and 1

6.1 G. $\mathbb{R}\setminus\{-1\}$

The function is not defined when the denominator is 0. Reduce function to simplest form and qualify denominator. Factor denominator:

$$x^2 - x - 2 = (x+1)(x-2)$$

Simplify input fynction:

$$\frac{x-2}{x^2-x-2} = \frac{\cancel{x-2}}{(x+1)\cancel{(x-2)}} = \frac{1}{x+1}.$$

Therefore, the input function is not defined when $x = -1$.

7 Prime numbers

Memorize the primes less than 100:

$$\{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97\}$$

7. Which of the following sets of numbers contains at least one number that is not prime?

- ○ A {3,5,7,73}
- ○ B {2,23,31,131}
- ○ C {3,5,7,11,71}
- ○ D {11,13,17,19,29}
- ○ E {1,3,5,53}

7.1 **A. All are prime**

7.2 **B. 2 is not prime**

7.3 **C. All are prime**

7.4 **D. All are prime**

7.5 **E. 1 is not prime, by formal definition.**

8 Precedence

8. If $x = -4$ and $y = 3$, what is the value of $x - y(x + y)$?

- F 7
- G -7
- H -1
- J 1
- K -49

8.1 **F. -7**

$$\begin{aligned} x - y(x + y) &= (-4) - (-3)\,((-4) - (-3)) \\ &= (-4) - (-3)\,(-4 + 3) \\ &= (-4) - (-3)\,(-1) \\ &= (-4) - 3 \\ &= -7 \end{aligned} \tag{2}$$

9 Real numbers

9. Which of the following is not a real number?

- A 143%
- B $\frac{5}{5}$
- C $\frac{8}{0}$
- D π
- E $\sqrt{10}$

9.1 **A. Rational** $\frac{143}{100}$

9.2 **B. Integer** 1

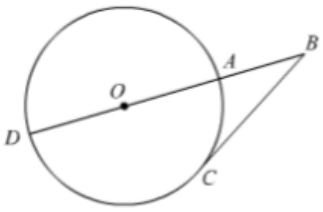
9.3 **C. Not defined - division by** 0

9.4 **D. Irrational**

9.5 **E. Irrational**

10 Geometry

10.



$\overline{BC}$ is tangent to circle O at C and $\text{m}\overset{\frown}{AC} = 73°$. Find m$\angle OBC$.

- F 73°
- G $53\frac{1}{2}°$
- H 34°
- J 17°
- K 23°

Notation unknown.

11 Distance

11. At noon, Ashley starts traveling north at an average speed of 40 miles per hour. From the same place one hour later, Ricky starts traveling south at an average speed of 45 miles per hour. Which equation could be used to determine at what time they will be 210 miles apart?

- A $40x + 45(x - 1) = 210$
- B $40x - 45(x - 1) = 210$
- C $40x = 45(x - 1)$
- D $40x + 45(x + 1) = 210$
- E $40(x - 1) + 45x = 210$

11.1 B. $210 = 40x - 45(x-1)$

Foundation: distance = velocity times time

$$d = vt$$

.
Ashley's distance from origin $d_1 = v_1 t = 40t$
Ricky's distance from origin $d_2 = v_2(t-1) = 45(t-1)$ for $t > 1$

Distance between Ashley and Ricky: $D = d_1 - d2$. When does $D = -210$? When

$$-210 = 40t - 45(t-1)$$

$t = 33$ hours.

12 Algebra

12. What is the solution set for $5(x-3)+4 = 5-2(x+1)$?

- F $\left\{\frac{8}{3}\right\}$
- G $\{-2\}$
- H $\{2\}$
- J $\{1\}$
- K $\varnothing$

12.1 H. $\{2\}$

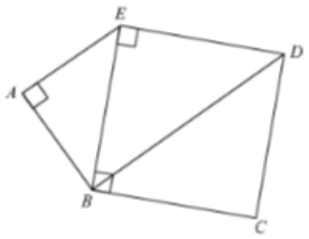
$$\begin{aligned} 5(x-3)+4 &= 5-2(x+1) \\ 5x-15+4 &= 5-2x-2 \\ 5x-11 &= 3-2x \\ 7x &= 14 \\ x &= 2 \end{aligned} \tag{3}$$

12.2 Check

If $x = 2$, $x-3 = -11$, and $x+1 = 3$. Therefore $5(x-3)+4 = -5+4 = -1$ and $5-2(x+1) = 5-2\cdot 3 = -1$.

13 Geometry

13.



If $m\angle EAB = m\angle BED = m\angle EBC = 90°$, and $AB = AE = 2$, and $DE = 3$, what is the length of $\overline{BD}$?

- ○ A 4
- ○ B 3.5
- ○ C 5
- ○ D $\sqrt{17}$
- ○ E $\sqrt{13}$

13.1 D. $\sqrt{17}$

$$BD = \sqrt{BE^2 + DE^2} \tag{4}$$

Given $DE = 3$, we need to find BE. Use

$$BE = \sqrt{AB^2 + AE^2} = \sqrt{2^2 + 2^2} = \sqrt{8} \tag{5}$$

By (4),

$$BD = \sqrt{3^2 + \sqrt{8}^2} = \sqrt{9 + 8} = \sqrt{17} \tag{6}$$

14 Number Line

14. Which of the following represents the distance between −8 and 5 on the number line?

- ○ F $|5 - (-8)|$
- ○ G $-|-8 - 5|$
- ○ H $|5 + (-8)|$
- ○ J $|-8 + 5|$
- ○ K $-|-8 + 5|$

14.1 **F.** $5 - (-8)$

15 Geometry

15. Which of the following is not equal to the others?

- ○ A 1.125
- ● B $\frac{9}{8}$
- ○ C $1\frac{1}{8}$
- ○ D 1125%
- ○ E $\frac{3}{4}+\frac{3}{8}$

15.1 **D.** $1125\% = \frac{1125}{100} = 11.25$

16 Factor quadratics

16. What is the complete factorization of $6x^2 + 6x - 36$?

- ○ F $3(x-2)(2x+6)$
- ○ G $6(x-2)(x+3)$
- ○ H $(2x-4)(3x+9)$
- ○ J $6(x+2)(x-3)$
- ● K $2(3x^2+3x-18)$

16.1 **G.** $6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x - 2)(x + 3)$

17 Binomial Theorem

17. How many different slates of officers—president, vice-president, secretary, and treasurer—can be selected from an organization of 20 members?

- ○ A 4845
- ● B 5
- ○ C About $1.0137(10^{17})$
- ○ D 116,280
- ○ E 80

17.1 **A.** 4845

Number of unique combinations of $k = 4$ distinct items from a pool of $n = 20$ items is given by "binomial n k":

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{7}$$

For this problem

$$\binom{n}{k} = \frac{20!}{4!(20-4)!} = \frac{20!}{4!16!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16!}}{4!\cancel{16!}} \tag{8}$$

Further simplification

$$\frac{20 \cdot 19 \cdot 18 \cdot 17}{4!} = 5 \cdot 19 \cdot 3 \cdot 17 = 4845 \tag{9}$$

18 Shapes

18. What is another name for a quadrilateral with congruent angles and congruent sides?

- ○ F A rhombus
- ○ G A rectangle
- ○ H A square
- ○ J A pentagon
- ● K A parallelogram

18.1 K. parallelogram

19 Algebra

19. What is the completely simplified form of $2x - 3[x - 2(3x - 2)]$?

- ○ A $17x + 12$
- ○ B $17x - 12$
- ○ C $-19x - 12$
- ○ D $-17x + 4$
- ○ E $-19x + 4$

19.1 A. $17x + 12$

$$\begin{aligned} 2x - 3\left[x - 2(3x - 2)\right] &= 2x - 3\left[x - 6x + 4\right] \\ &= 2x - 3x + 18x + 12 \\ &= 2x - 3x + 18x + 12 \\ &= 17x + 12 \end{aligned} \tag{10}$$

20 Exponents

20. What is the value of -4^{-2}?

- ○ F 16
- ○ G -16
- ○ H $\frac{1}{16}$
- ○ J $\frac{-1}{16}$
- ○ K 8

20.1 H. $\frac{1}{16}$

$$-4^{-2} = \frac{1}{-4^2} = \frac{1}{(-4)(-4)} = \frac{1}{16} \tag{11}$$

21 Complex numbers

21. What is the product of the complex numbers $-1 + 3i$ and $2 + 4i$?

- ○ A $-14 + 2i$
- ○ B $10 + 2i$
- ○ C $-14 - 2i$
- ○ D $-14 - 10i$
- ○ E $-10 - 10i$

21.1 B. $10 + 2i$

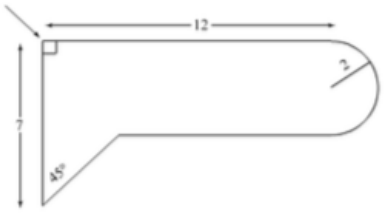
Product of complex numbers:

$$(x_1 + iy_1)(x_2 + iy_2) = x_1x_2 + y_1y_2 + i(x_1y_2 + x_2y_1)$$

$$(-1 + i3)(2 + i4) = -2 - (i^2)12 + i(-4 + 6) = -2 + 12 + i2 = 10 + i2 \tag{12}$$

22 Geometry

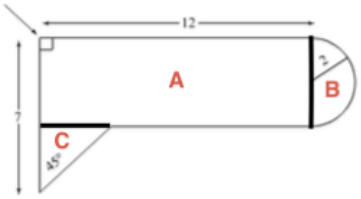
22.



In the diagram, the long sides of the figure are parallel, the right end is capped with a semicircle, and the left end has a triangular extension as shown. What is the area of this figure?

- ○ F $98 + \frac{9}{2} + 2\pi$
- ○ G $84 + 2\pi$
- ○ H $57 + 4\pi$
- ○ J $57 + 2\pi$
- ○ K $\frac{1}{2}(105 + 4\pi)$

22.1 **K.** $\frac{1}{2}(105 + 4\pi) = 52\frac{1}{2} + 2\pi$



Use the partition

Area A:

$$A = 12 \times 4 = 48$$

Area B:

$$B = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi 4 = 2\pi$$

Area C:

$$C = \frac{1}{2}3 \times 3 = 4\frac{1}{2}$$

Total area:

$$A + B + C = 52\frac{1}{2} + 2\pi$$

23 Factoring

23. What is the simplest form of $\frac{12\sqrt{20}}{\sqrt{6}}$?

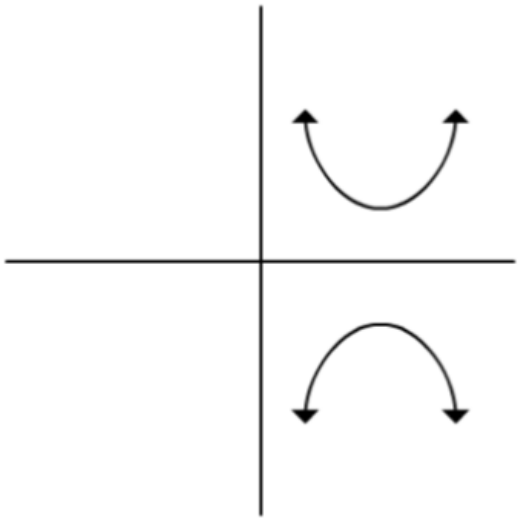
- A $4\sqrt{30}$
- B $2\sqrt{120}$
- C $\frac{12\sqrt{10}}{\sqrt{3}}$
- D $\frac{12\sqrt{30}}{3}$
- E None of these

23.1 **C.** $\frac{12\sqrt{10}}{\sqrt{3}}$

$$\frac{12\sqrt{20}}{\sqrt{6}} = 12\sqrt{\frac{20}{6}} = 12\sqrt{\frac{10}{3}} = \frac{12\sqrt{10}}{\sqrt{3}} \tag{13}$$

24 Symmetry

24.



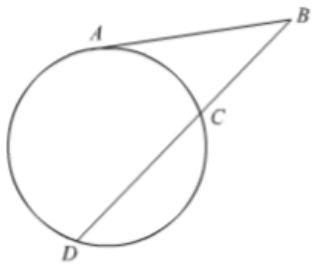
What symmetry is shown in the accompanying graph?

- ○ F Symmetry to the x-axis.
- ○ G Symmetry to the y-axis.
- ○ H Symmetry to the origin.
- ○ J Symmetry to the line $y = x$.
- ○ K No symmetry is shown in this graph.

24.1 F. Symmetry with respect to the $x-$axis

25 Geometry

25.



In the diagram, $BC = 4$ and $CD = 5$. What is the length of the tangent $\overline{AB}$?

- A $\sqrt{20}$
- B 3
- C $\sqrt{41}$
- D 6
- E 5

26 Symmetry

26. A number is randomly selected from $\{1,2,3,\cdots,20\}$. What is the probability that it is prime?

- F $\frac{9}{20}$
- G $\frac{2}{5}$
- H $\frac{7}{20}$
- J $\frac{3}{10}$
- K $\frac{1}{4}$

26.1 G. $\frac{2}{5}$

$$\{1, 2^\star, 3^\star, 4, 5^\star, 6, 7^\star, 8, 9, 10, 11^\star, 12, 13^\star, 14, 15, 16, 17^\star, 18, 19^\star, 20\}$$

Of the 20 numbers, 8 are prime. As a fraction, $\frac{8}{20} = \frac{2}{5}$ are prime.

27 Powers

27. If $a > 0$ and $b > 0$, express in simplest radical form: $\left(6\sqrt{48a}\right)\left(4\sqrt{30ab^3}\right)$.

- ○ A $24\sqrt{1440a^2b^3}$
- ○ B $288ab\sqrt{10b}$
- ○ C $288a\sqrt{10b^3}$
- ○ D $144a\sqrt{40b^3}$
- ○ E $12a\sqrt{40b^3}$

27.1 **C.** $288a\sqrt{10b^3}$

$$\left(6\sqrt{48a}\right)\left(4\sqrt{30ab^3}\right) = 24\sqrt{1440a^2b^3} = 24 \cdot 12a\sqrt{10b^3} = 288a\sqrt{10b^3}$$

28 Algebra

28. If $a \neq b$, what is equal to $\frac{a-b}{b-a}$?

- ○ F 1
- ○ G $\frac{a+b}{b+a}$
- ● H $\frac{a-b}{a-b}$
- ○ J 0
- ○ K -1

28.1 K. -1

$$\frac{a-b}{b-a} = \frac{\cancel{a-b}}{(-1)\cancel{(a-b)}} = -1$$

29 Logarithms

29. If $\log_2 3 = a$, $\log_2 5 = b$ and $\log_2 7 = c$, then what is $\log_2 35\sqrt{6}$?

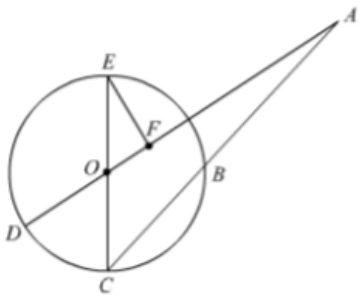
- ○ A $b+c+\frac{1}{2}(1+a)$
- ○ B $bc+\frac{1}{2}a$
- ○ C $\frac{1}{2}a(bc)$
- ○ D $a+b+c+\frac{1}{2}$
- ○ E There is not enough information.

29.1 A. $a+b+\frac{1}{2}(1+a)$

$$\log_2 35\sqrt{6} = \log_2\left(5\cdot 7\cdot 6^{\frac{1}{2}}\right) = \log_2\left(5\cdot 7\cdot 2^{\frac{1}{2}}3^{\frac{1}{2}}\right) = a+b+\frac{1}{2}(1+a)$$

30 Geometry

30.



In the diagram, which of the following is an inscribed angle?

- ○ F $\angle DAC$
- ○ G $\angle DOC$
- ○ H $\angle ECB$
- ○ J $\angle DOE$
- ○ K $\angle EFO$

30.1 H. $\angle ECB$