Bauhaus-Universität Weimar

Photogrammetric Computer Vision

Exercise 2

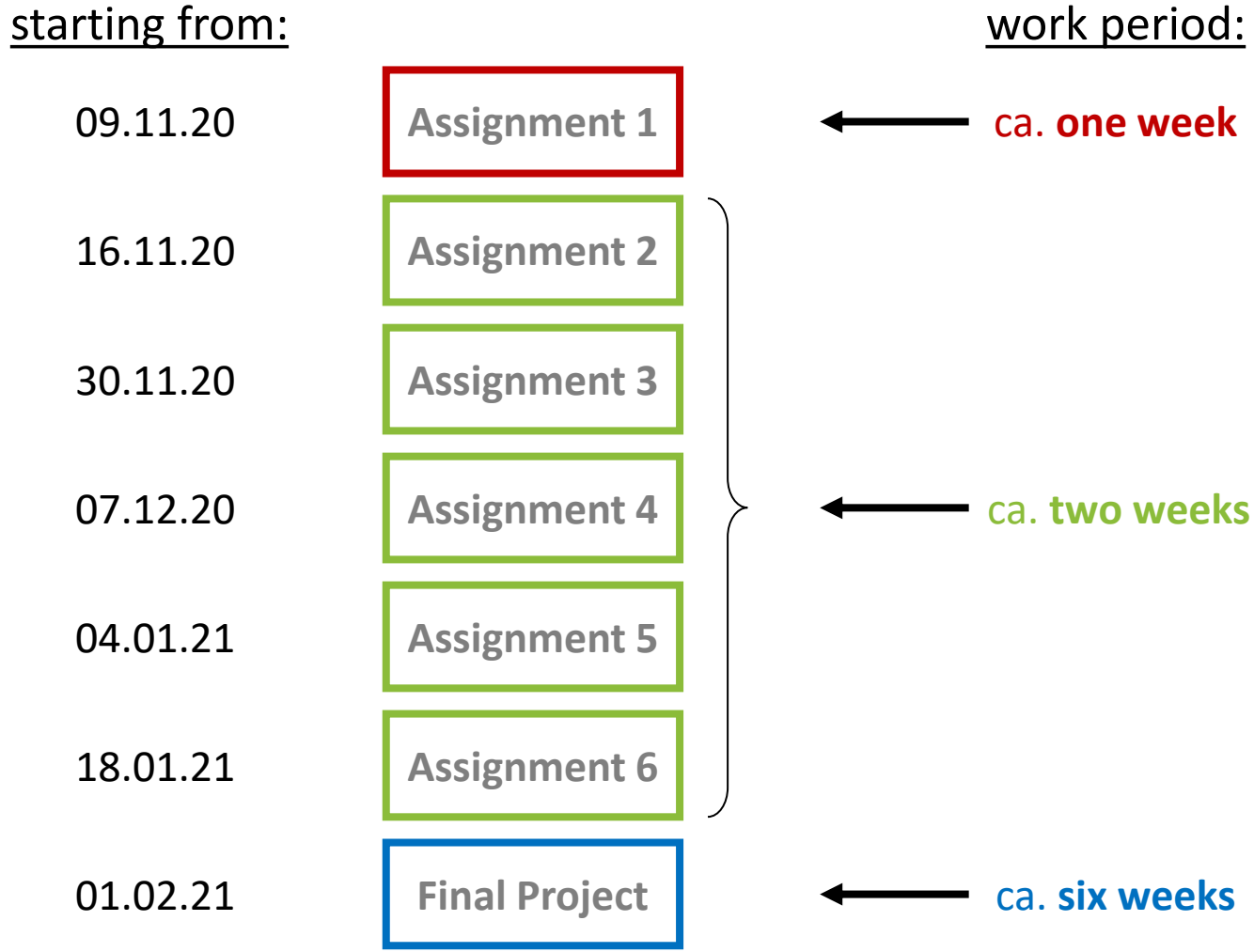
Winter semester 20/21

(Course materials for internal use only!)

Computer Vision in Engineering – Prof. Dr. Rodehorst
M.Sc. Mariya Kaisheva
mariya.kaisheva@uni-weimar.de

Agenda

7 exercise classes *
distributed within 12 weeks
(between 09.11.20 and 01.02.21)



* **Disclaimer:** In case of unexpected circumstances, the planned schedule for the exercise classes might change in the course of the semester.

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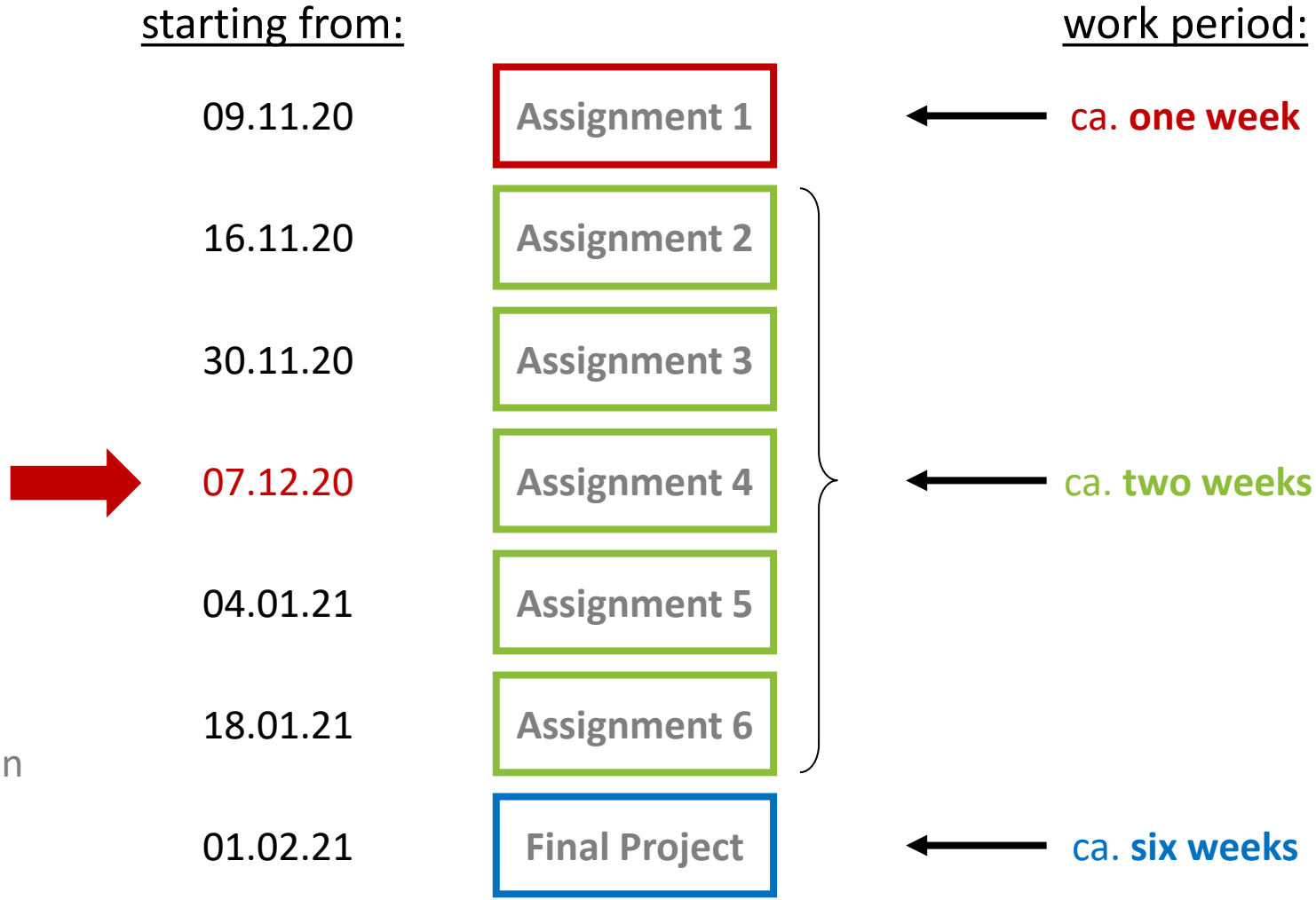

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starting from:		work period:
09.11.20	Assignment 1	← ca. **one week**
16.11.20	Assignment 2	← ca. **two weeks**
30.11.20	Assignment 3	
14.12.20	Assignment 4	← ca. **one week** followed by 2 weeks vacation
04.01.21	Assignment 5	← ca. **two weeks**
18.01.21	Assignment 6	
01.02.21	Final Project	← ca. **six weeks**

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Agenda

Topics:

Assignment 1.	Points and lines in the plane, first steps in MATLAB / Octave
Assignment 2.	**Projective transformation (Homography)**
Assignment 3.	Camera calibration using direct linear transformation (DLT)
Assignment 4.	Orientation of an image pair
Assignment 5.	Projective and direct Euclidean reconstruction
Assignment 6.	Stereo image matching
Final Project. *	TBA

*required only for Digital Engineering and some HCI students

Agenda

	Beginning:	Submission deadline:
Assignment 1.	~~09.11. 2020~~	~~15.11. 2020~~
Assignment 2.	**16.11. 2020**	**29.11. 2020**
Assignment 3.	30.11. 2020	13.12. 2020
Assignment 4.	14.12. 2020	06.01. 2021
Assignment 5.	04.01. 2021	17.01. 2021
Assignment 6.	18.01. 2021	31.01. 2021
Final Project. *	01.02. 2021	14.03. 2021

Update

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Assignment 1 – sample solution

Part 1

1. You would like to compute the connecting line between two 2D points. What happens, if the two points are identical?

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ w \end{pmatrix}, \quad \mathbf{l} = \mathbf{x} \times \mathbf{x} = \begin{pmatrix} yw - wy \\ wx - xw \\ xy - yx \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad |\mathbf{l}| = 0$$

Not defined!

2. Where does the general line $x \cos \varphi + y \sin \varphi = d$ intersect the line $(0, 0, 1)^T$ given in homogeneous coordinates? How can this point be interpreted?

$$\mathbf{l}_1 = \begin{pmatrix} \cos\phi \\ \sin\phi \\ -d \end{pmatrix}, \quad \mathbf{l}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 = \begin{pmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{pmatrix}$$

Section at infinity in the direction l_1

3. Show that the horizon is a straight line by showing that three points on the horizon are always collinear.

$$\det \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 0 & 0 & 0 \end{pmatrix} = x_1 \cdot y_2 \cdot 0 - x_3 \cdot y_2 \cdot 0 + x_2 \cdot y_3 \cdot 0 - x_1 \cdot y_3 \cdot 0 + x_3 \cdot y_1 \cdot 0 - x_2 \cdot y_1 \cdot 0 = 0$$

Part 2

All objects in MATLAB are matrices. A matrix is created with `[1,2;3,4]`, where semicolons separate the rows. For a matrix multiplication use * and for a matrix `A` is `A'` the transpose. For the solution of this exercise the commands `cross`, `sin`, `cos`, `pi` and `inv` can be helpful.

1. The two points $\mathbf{x} = (2, 3)^T$ and $\mathbf{y} = (-4, 5)^T$ are given.
   a. Determine the connecting line $\mathbf{l}$ between the two points.
   b. *Move* $\mathbf{x}$ and $\mathbf{y}$ in the direction $\mathbf{t} = (6, -7)^T$,
      rotate afterwards using the angle $\varphi = 15°$ and finally
      scale with factor $\lambda = 8$.
   c. Accomplish the same operations with the line $\mathbf{l}$.
2. Examine whether the transformed points $\mathbf{x}'$ and $\mathbf{y}'$ are on the transformed line $\mathbf{l}'$.

Part 2

$$l^T x = 0$$

$$l^T H^{-1} H x = 0$$

$$l^T H^{-1} = l'^T \qquad Hx = x'$$

$$H^{-T} l = l'$$

$$l'^T x' = 0$$

```
function Exercise1                          % Planar similarity transformation
%        =========
x = [ 2; 3; 1];                  % Points x and y in homogeneous coordinates
y = [-4; 5; 1];
l = cross(x, y)                          % Joining line l using cross product

H = Scale(8) * Rot(15) * Trans(6,-7);        % Transformation concatenation
                              % 1. Translation, 2. Rotation, 3. Global scaling
x2 = H * x                               % Apply transformation to points x and y
y2 = H * y
l2 = inv(H') * l;                           % Apply transformation to line l

x2' * l2                                 % Incidence test: scalar product 0?
y2' * l2

function T = Trans(x,y)
%        ==============
T = [ 1   0   x ;                                        % Translation matrix
      0   1   y ;
      0   0   1 ];

function R = Rot(a)
%        ==========
phi = a * pi / 180;                                       % Degree -> radiant
R = [ cos(phi) -sin(phi)   0 ;                              % Rotation matrix
      sin(phi)  cos(phi)   0 ;
         0         0       1 ];

function S = Scale(s)
%        ============
S = [ s   0   0 ;                                    % Global scaling matrix
      0   s   0 ;
      0   0   1 ];
```

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Assignment 2

Assignment 2: *Projective Transformation*

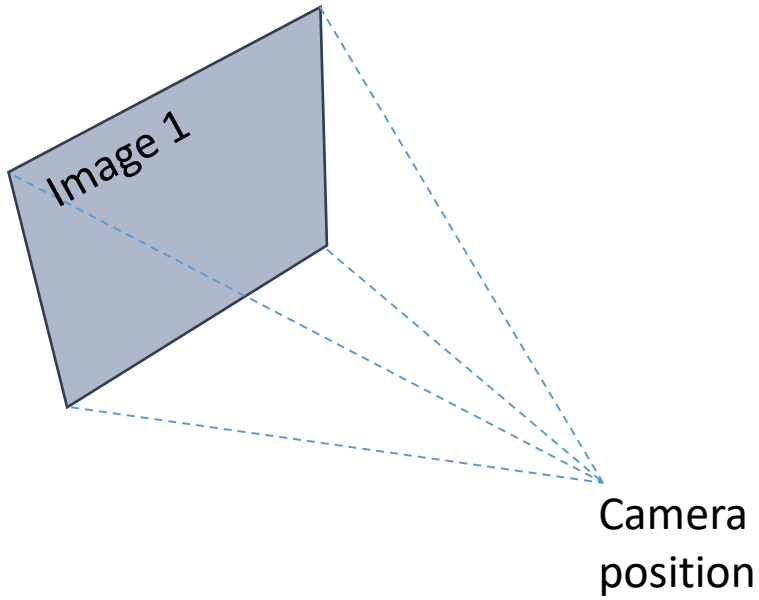
1) Image acquisition
   - 3 images
   - at least **30%** overlap

2) Correspondence analysis
   - **interactive** point selection

3) Homography computation
   - $\mathbf{H_{12}}$ (first to second image)
   - $\mathbf{H_{32}}$ (third image to intermediate mosaic)

4) Projective rectification
   - auxiliary program ***geokor***

5) Visualization



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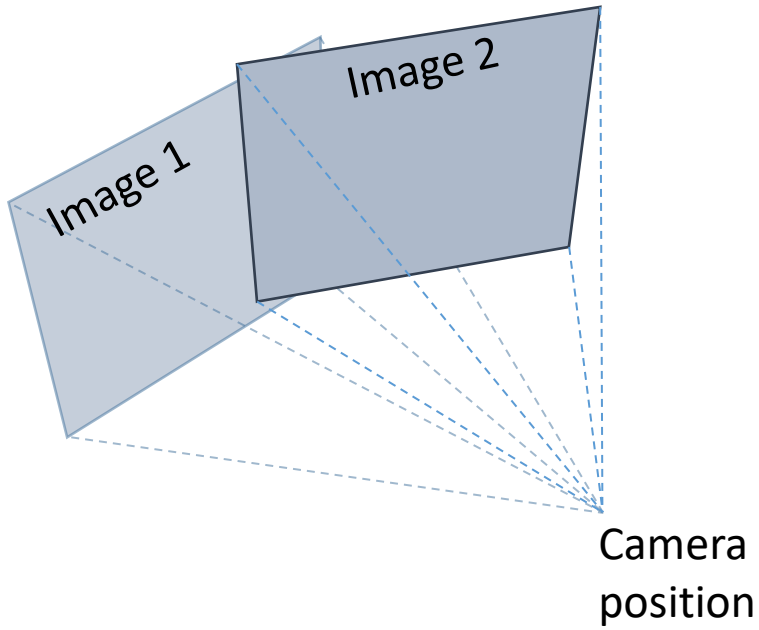

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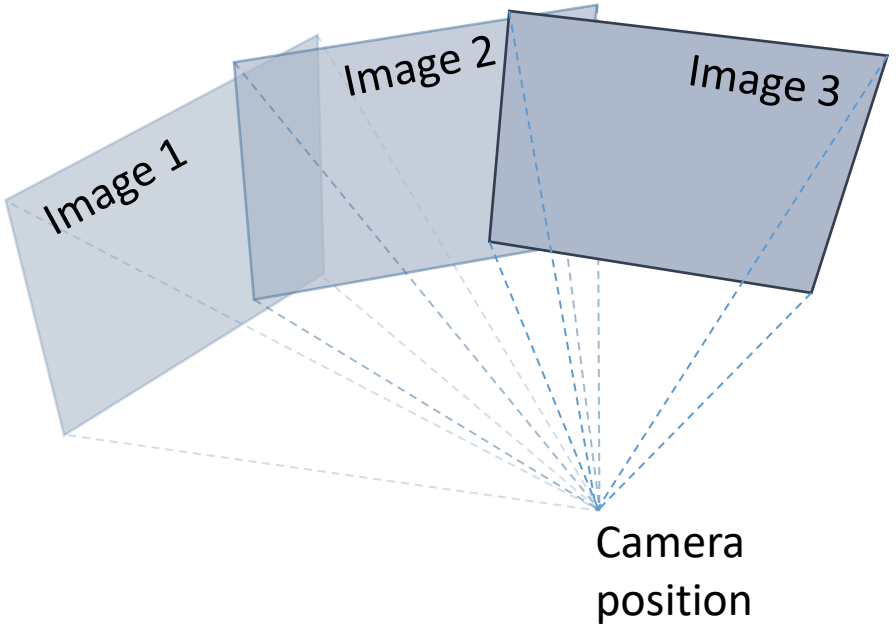

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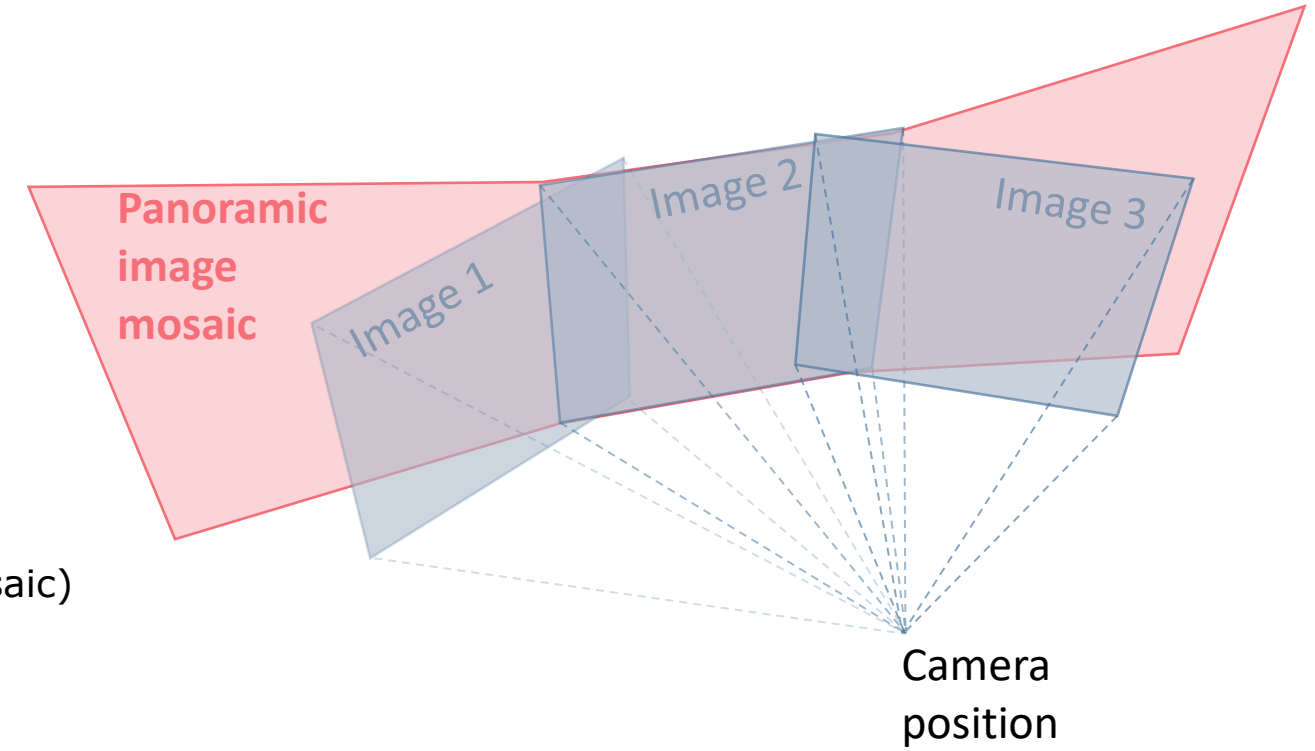

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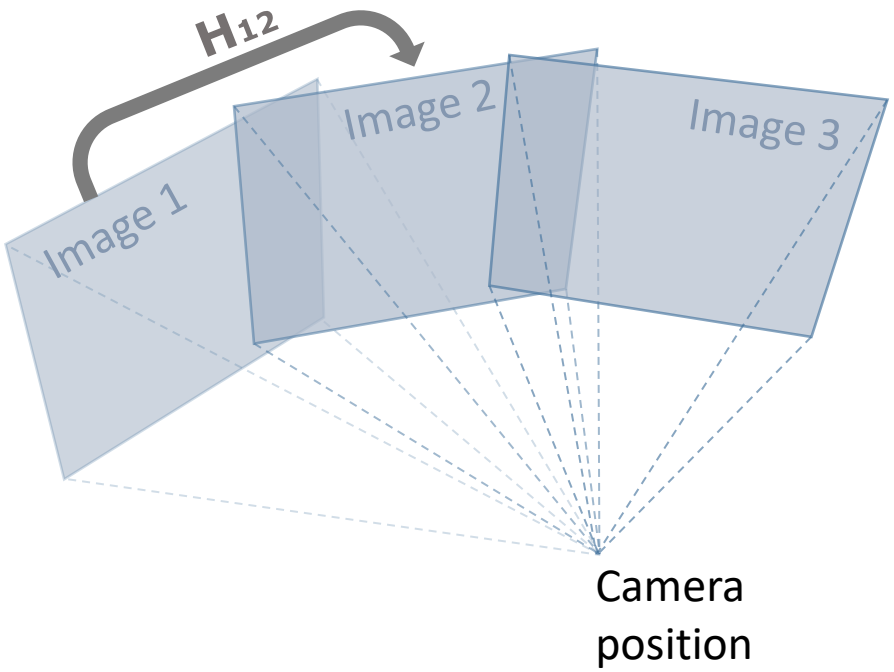

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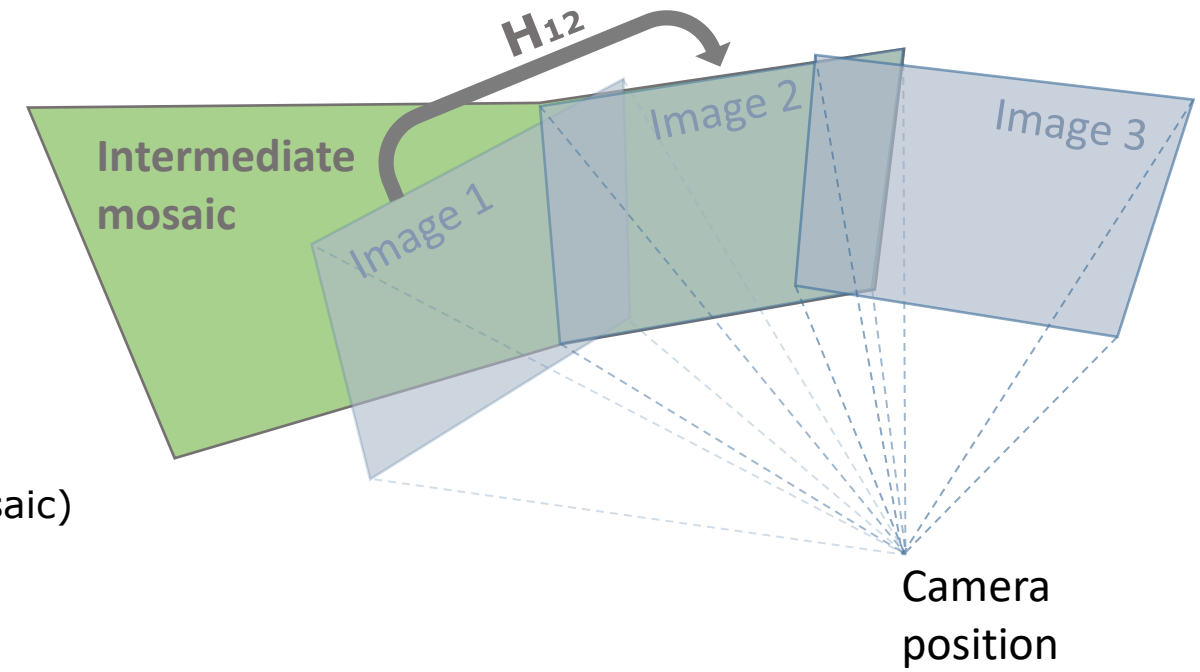

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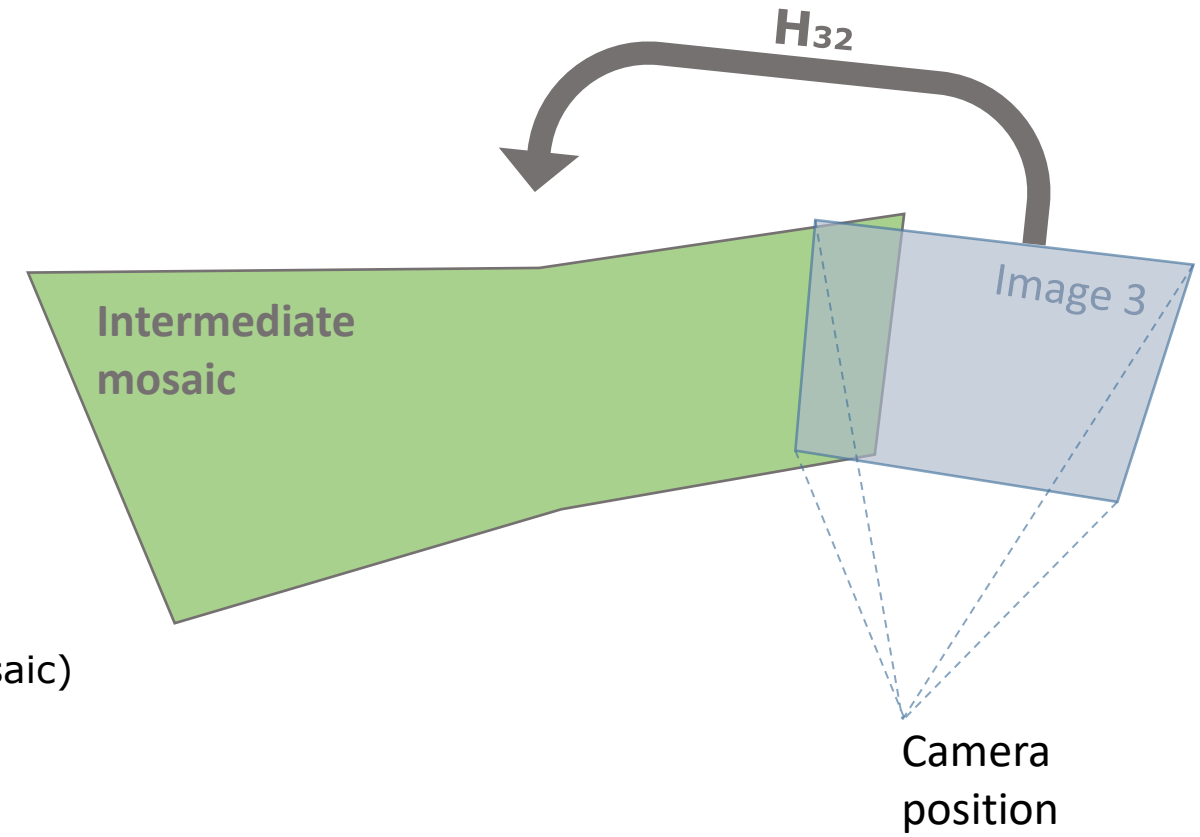

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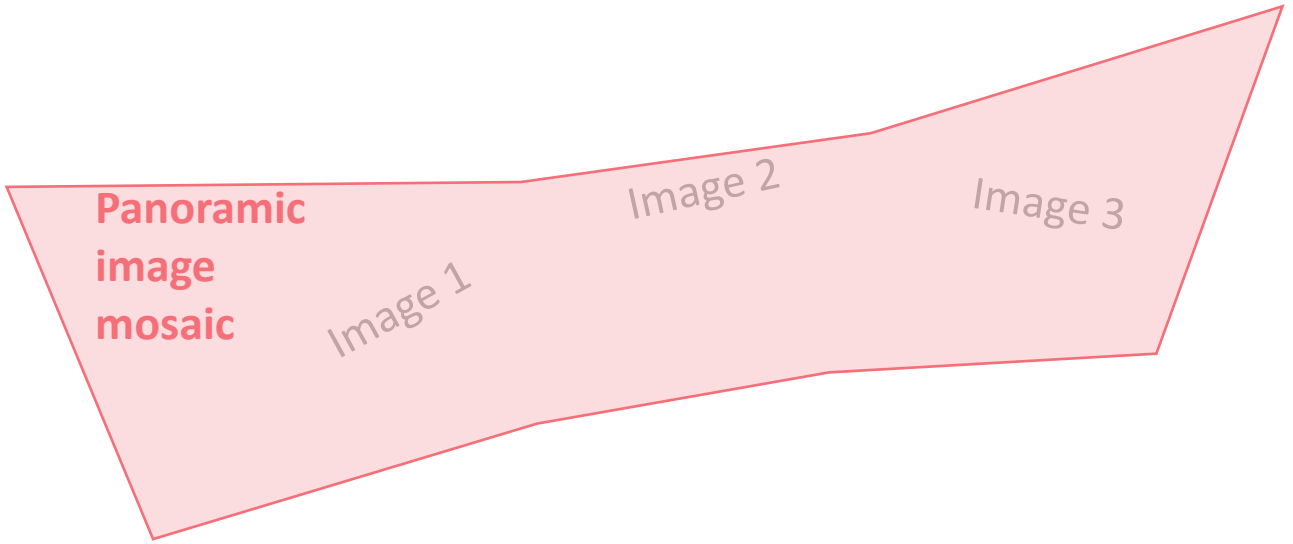

Assignment 2: *Example Result*

Image Source: *student submission; WS 19/20*

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Question from Last Week

Geometry	Coordinates
Euclidean	Cartesian
Projective	Homogeneous