**Supplementary figures and explanations for the manuscript**

Parallel cross-validation: a scalable fitting method for Gaussian process models

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SFig. 1: Sensitivity analysis of the simulation study presented in Section 3.1

To investigate the dependence of the results of the simulation study on the parameter grid for θ and λ we repeat the study with 29 x 29 parameter (841 parameter combinations) and the larger parameter ranges [θ/4, 4θ] and [λ/4,4λ]. The results in the following figure are similar to the one in Fig. 2 and indicate that they do not depend much on the choice of the parameter grid.
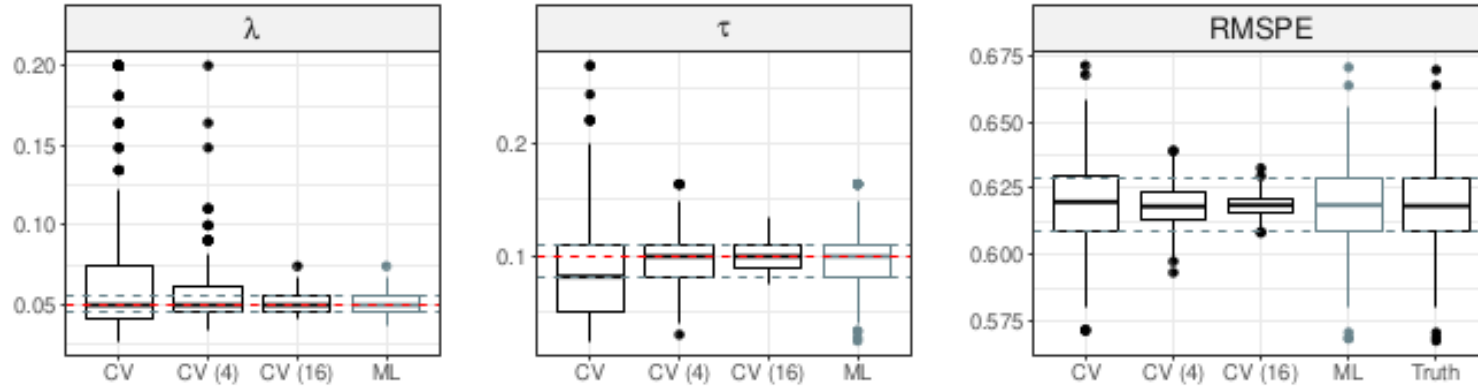
SFig. 2: Bivariate plots of the results from the simulation study given in Section 3.1

The figure shows the number of times a parameter configuration is chosen to be best fitting one. The total number of simulations is 400 (i.e., each panel summarized 400 estimations). The true parameter values used to simulate the data are indicated by the red dashed lines.

3
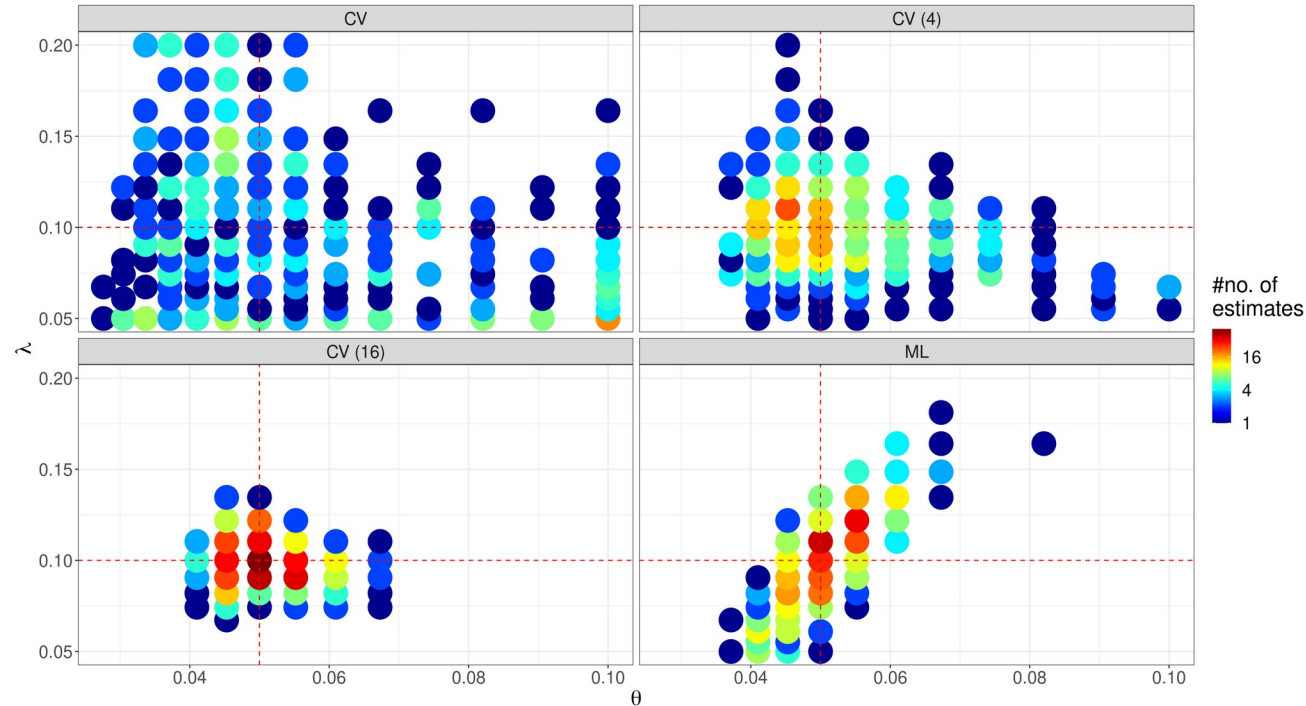
SFig. 3: Simulation study of Section 3.2 under misspecification

The figure shows the results of a simulation study similar to the one described in Section 3.2. The only difference is that here the spatial predictions are based on $\lambda_{prediction} = \lambda_{simulation} + 0.1$ and $\theta_{prediction} = \theta_{simulation} + 0.1$, i.e., the parameters of the model used for the prediction are different from the parameters used for the generation of the spatial fields. The figure shows a similar patter as Fig. 3 in the manuscript.
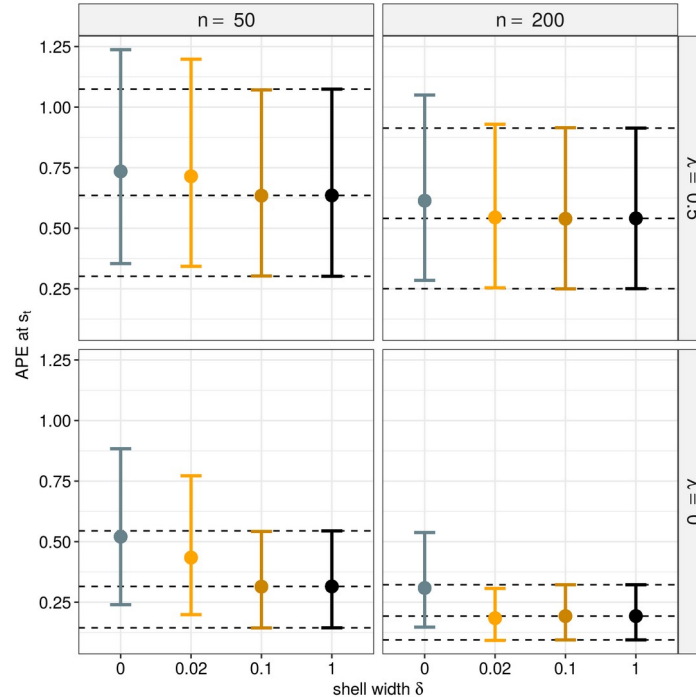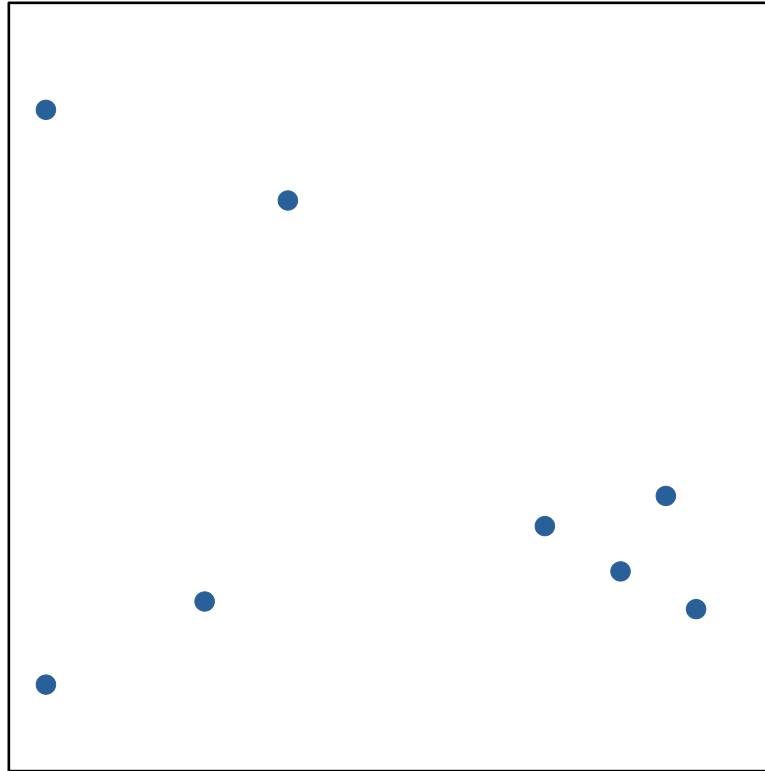
Illustration of the division of the data into subsets given in Section 2.4

With the following configuration the division into four subsets leads to unbalanced number of points per subset:
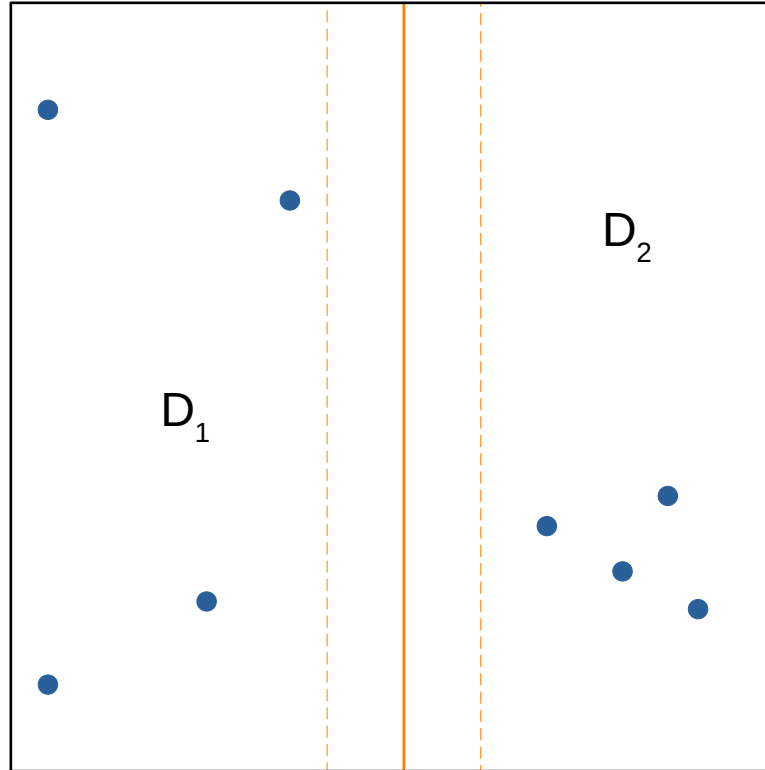- 8 points in the domain D = [0,1] x [0,1]
- Boundary width = 0.1

Recursion 1:

Split the domain D into 2 subsets such that $D_1 \cup D_1^{shell}$ and $D_2 \cup D_2^{shell}$ contain the same number of points.

- $D_1 \cup D_1^{shell}$ and $D_2 \cup D_2^{shell}$ both contain 4 points
- No points are in the boundary regions

Recursion 2:

Split the 2 subsets from the previous recursion into 2 subsets each.

- D'_1 U $D'_1{}^{shell}$ and D'_2 U $D'_2{}^{shell}$ contain 2 points. No points are in the boundary regions
- D'_3 U $D'_3{}^{shell}$ and D'_4 U $D'_4{}^{shell}$ contain 4 points. 2 points are in each boundary region
→ The subsets have an unbalanced number of points