Example of well log linear inversion

The method presented here was proposed by (Savre, 1963)

Savre, W.C., 1963, Determination of a more accurate porosity and mineral composition in complex lithologies with the use of the sonic, neutron, and density surveys: Journal of Petroleum Technology, v. 15, no. 9, pp. 945–959, doi: 10.2118/617-PA

Neutron log

true porosity (percentage of porosity)

$$\phi_n^o \approx \phi + (1-\phi)\,0.49\,G \qquad (1)$$

percentage of gypsum in matrix

observed apparent neutron porosity

Density log

fluid density

average grain density of rock matrix

$$\rho_b^o \approx \phi\rho_f + (1-\phi)\tilde{\rho} \qquad (2)$$

observed bulk density of the formation

percentage of porosity

fractions of anhydrite, dolomite and gypsum in rock matrix

$$(3) \qquad \tilde{\rho} = A\rho_A + D\rho_D + G\rho_G$$

density of gypsum (2.35 g/cm³)

density of dolomite (2.82 g/cm³)

density of anhydrite (2.98 g/cm³)

$$A + D + G = 1 \qquad (4)$$

Sonic log

observed average sonic transit time measured from the log (in micro-sec/ft)

$$\mu s/ft = \frac{1}{0.3048}\,\mu s/m$$

$$\Delta t^o \approx \phi\Delta t_f + (1-\phi)\widetilde{\Delta t} \qquad (5)$$

average matrix transit time from the Wyllie time-average equation

sonic transit time through the interstitial fluid (in micro-sec/ft)

$$(6) \qquad \widetilde{\Delta t} = A\Delta t_A + D\Delta t_D + G\Delta t_G$$

sonic transit time through anhydrite (50.0 micro-sec/ft)

sonic transit time through dolomite (40.0 micro-sec/ft)

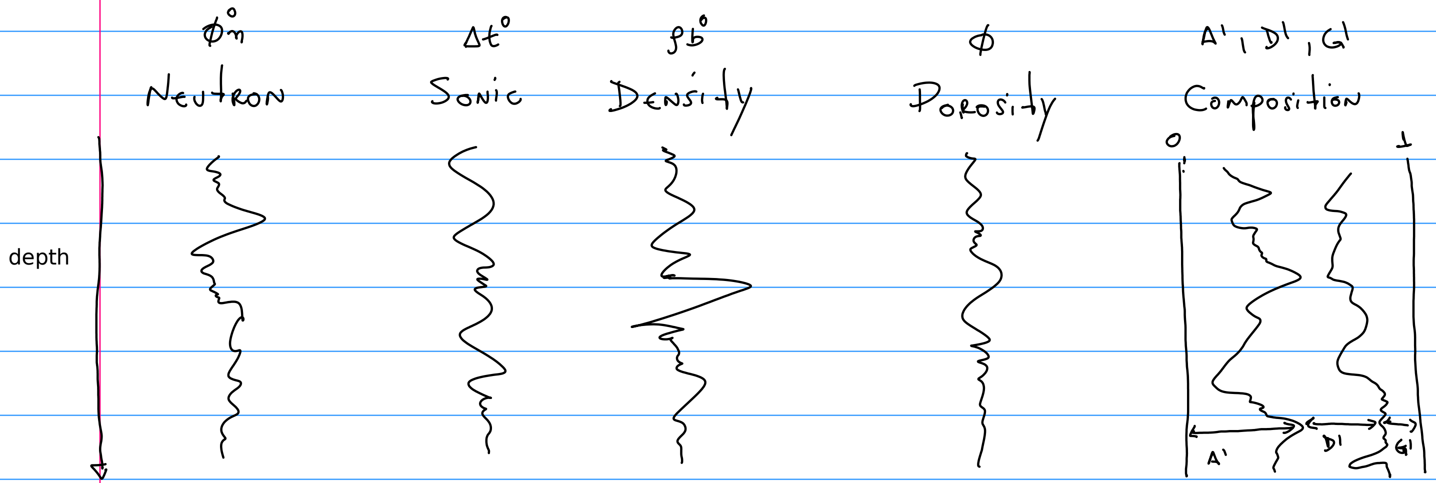
sonic transit time through gypsum (52.6 micro-sec/ft) percent of rock matrix

percent of bulk volume

$$(7)\quad\begin{cases}\phi_n^o \approx \phi + 0.49\,G' \\ \Delta t^o \approx \phi\,\Delta t_f + A'\Delta t_A + D'\Delta t_D + G'\Delta t_G \\ \rho_b^o \approx \phi\,\rho_f + A'\rho_A + D'\rho_D + G'\rho_G \\ 1 = \phi + A' + D' + G' \end{cases}$$

constraint

$$\left.\begin{aligned} A' &= A \times (1-\phi) \\ D' &= D \times (1-\phi) \\ G' &= G \times (1-\phi) \end{aligned}\right\}\quad(8)$$



Solution for a single point

(Consider that all data are measured at the same points)

$$(9)\quad \mathbf{d}^o = \begin{bmatrix} \phi_n^o \\ \Delta t^o \\ \rho_b^o \\ 1 \end{bmatrix} \qquad (10)\quad \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0.49 \\ \Delta t_f & \Delta t_A & \Delta t_D & \Delta t_G \\ \rho_f & \rho_A & \rho_D & \rho_G \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad (11)\quad \mathbf{p} = \begin{bmatrix} \phi \\ A' \\ D' \\ G' \end{bmatrix}$$

$$\min \Gamma(\mathbf{p}) \quad (12)$$

$$\Gamma(\mathbf{p}) = (\mathbf{d}^o - \mathbf{A}\mathbf{p})^T(\mathbf{d}^o - \mathbf{A}\mathbf{p}) \quad (13)$$

goal function

$$\nabla\Gamma(\mathbf{p}^*) = \mathbf{0} \quad (14)$$

particular parameter vector minimizing the goal function

arbitrary parameter vector

$$\nabla\Gamma(\mathbf{p}) = -2\mathbf{A}^T(\mathbf{d}^o - \mathbf{A}\mathbf{p}) \quad (15)$$

$$\nabla\Gamma(\mathbf{p}^*) = -2\mathbf{A}^T\mathbf{d}^o + 2\mathbf{A}^T\mathbf{A}\mathbf{p}^* = \mathbf{0}$$

$$(\mathbf{A}^T\mathbf{A})\,\mathbf{p}^* = \mathbf{A}^T\mathbf{d}^o \quad (16)$$