STAT 443: Assignment 3

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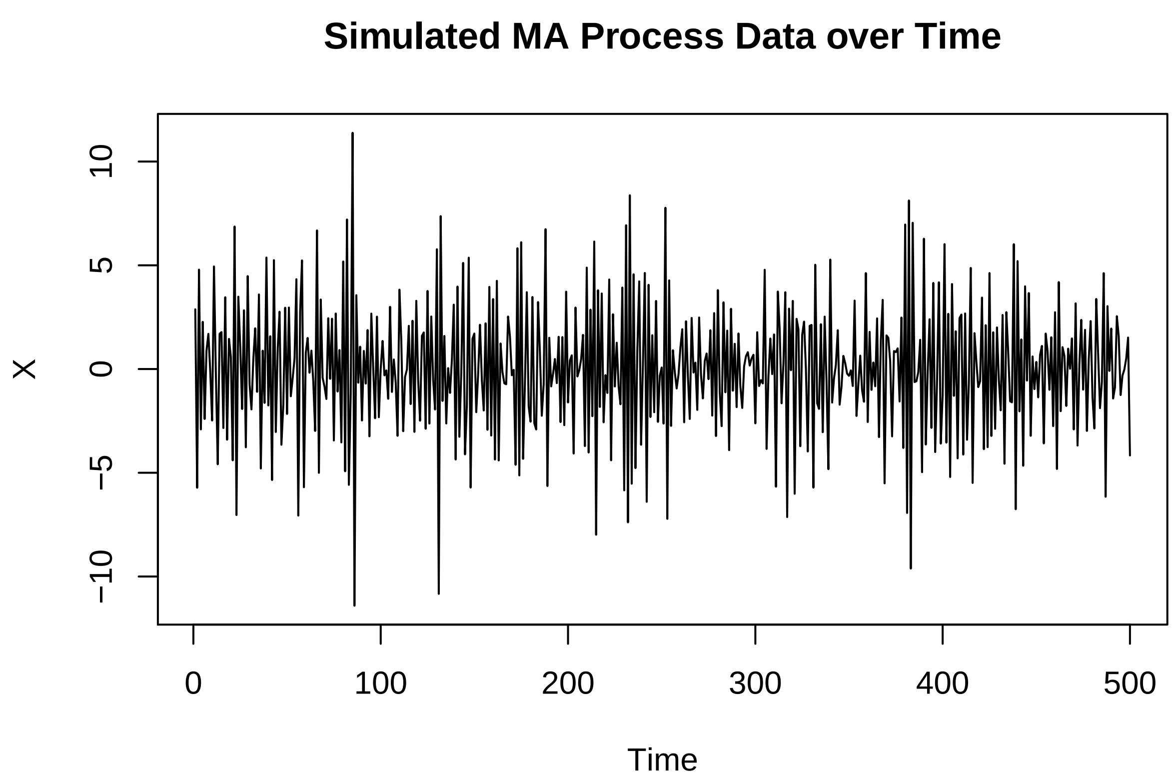
Question 1

a)

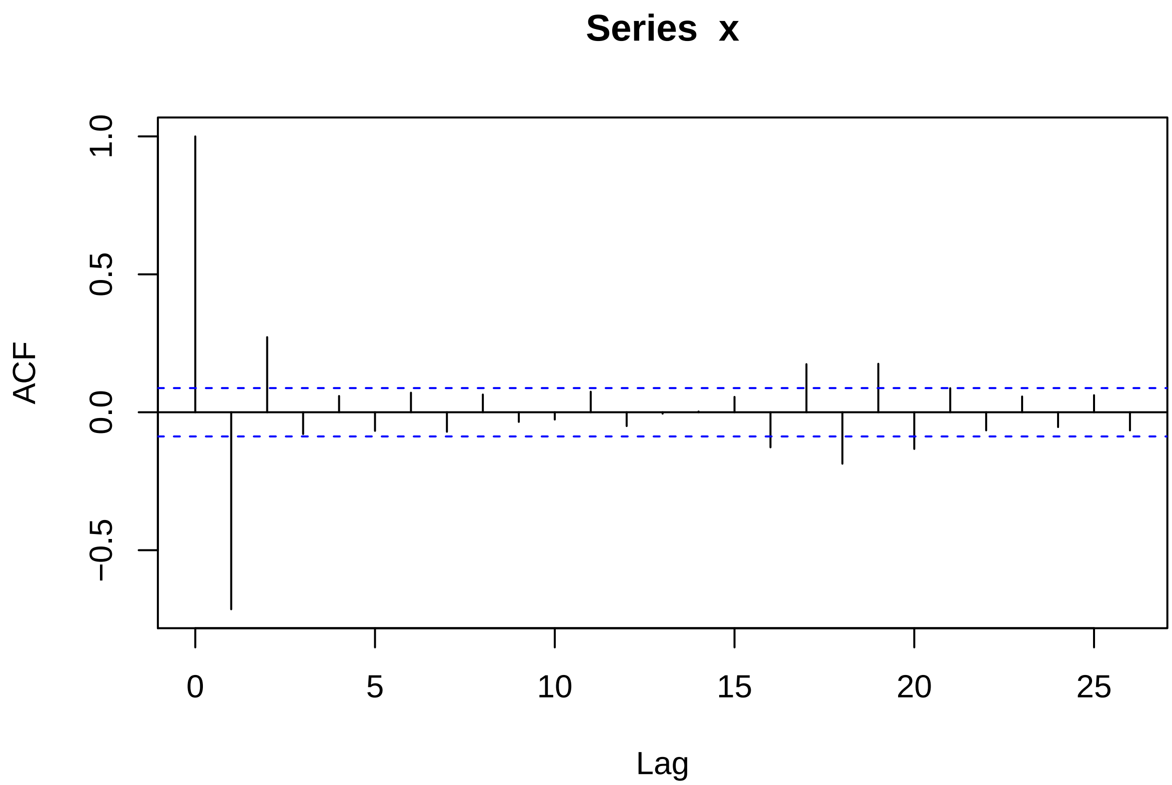
```
x = arima.sim(n=500, list(ma = c(-4.25, 5.75, -1.80)), sd=sqrt(0.2))
#create time series plot
plot(x, ylab = "X", main = "Simulated MA Process Data over Time")
```



We expect the ACF to decrease, and cut off at lag 3, ie, all autocorrelations for lags past 3 will be 0.

b)

```r
#plot sample acf
acf(x)
```



The acf looks like what we would expect, accounting for sampling error, which causes the sample acf to not match the theoretical pattern precisely. From the sample acf plot, we see that there are two statistically significant "spikes" at lags 1,2, followed by non-significant values for other lags.

(c)

```r
theoretical_acf = ARMAacf(ma=c(-4.25, 5.75, -1.80))
theoretical_acf
```

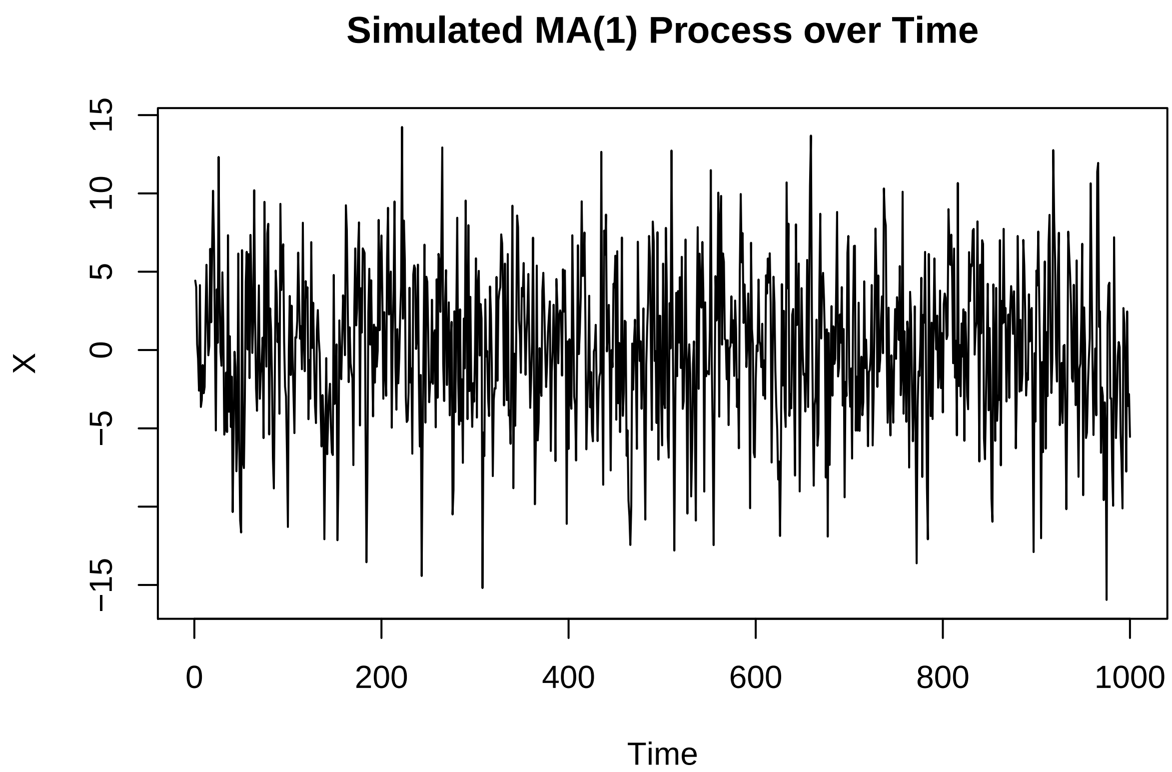
```
##          0           1           2           3           4
##  1.00000000 -0.70509347  0.24203016 -0.03251151  0.00000000
```

We see that: p(h) = 1 for h = 0 p(h) = -0.71 for h = 1 p(h) = 0.24 for h = 2 p(h) = -0.033 for h = 3 p(h) = 0.00 for h = 4 (onwards)

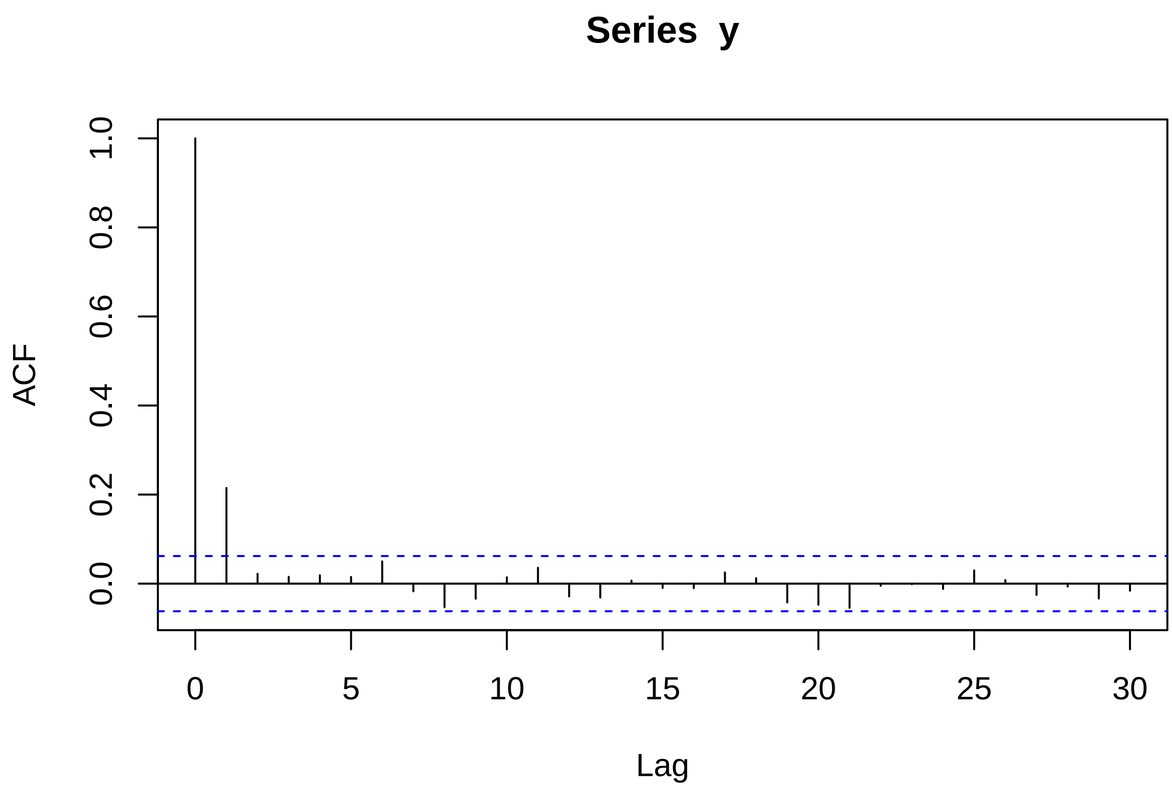
(d) The sample acf does not behave exactly like the theoretical acf due to sample errors. We can see that for the theoretical acf, the all autocorrelations for lags past 3 are 0. Similarly, the sample acf decreases as lag increases, with significant spikes at lags 1 and 2, which follows by non-significant values for other lags.

Question 2: (a)

```
y = arima.sim(n=1000, list(ma = c(5)), sd=sqrt(0.9))
plot(y, xlab = "Time", ylab="X" , main = "Simulated MA(1) Process over Time", )
```



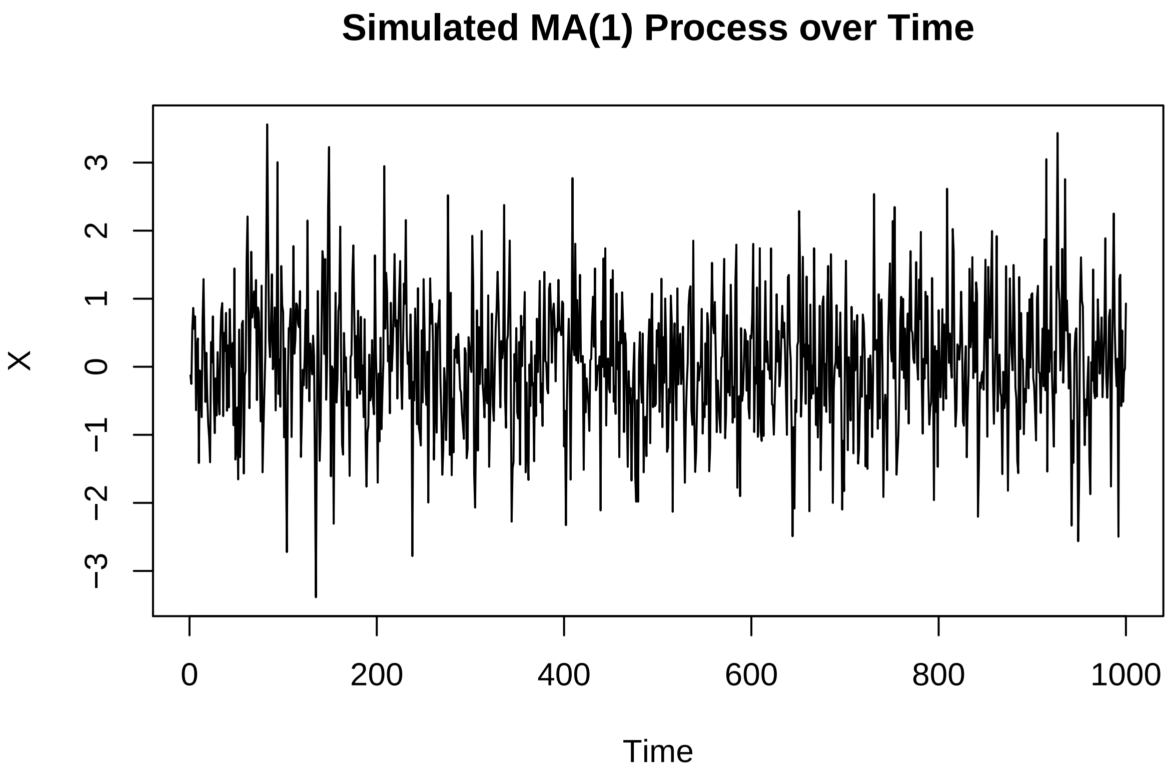

```
#plot sample acf
acf(y)
```



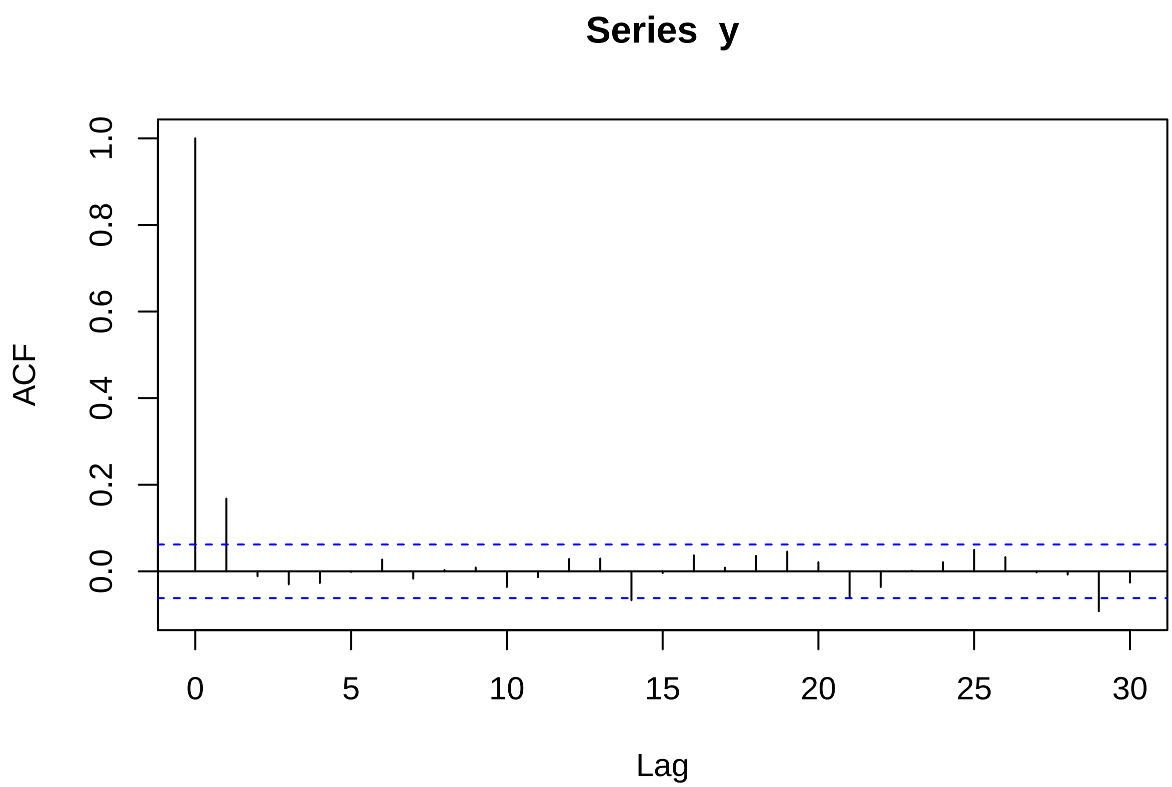
From the sample acf plot, we can see that there is a significant positive "spike" at lag 1, followed by generally non-significant values for other lags.

(b)

```
y = arima.sim(n=1000, list(ma = c(0.2)), sd=sqrt(0.9))
plot(y, xlab = "Time", ylab="X" , main = "Simulated MA(1) Process over Time")
```




```
#plot sample acf
acf(y)
```

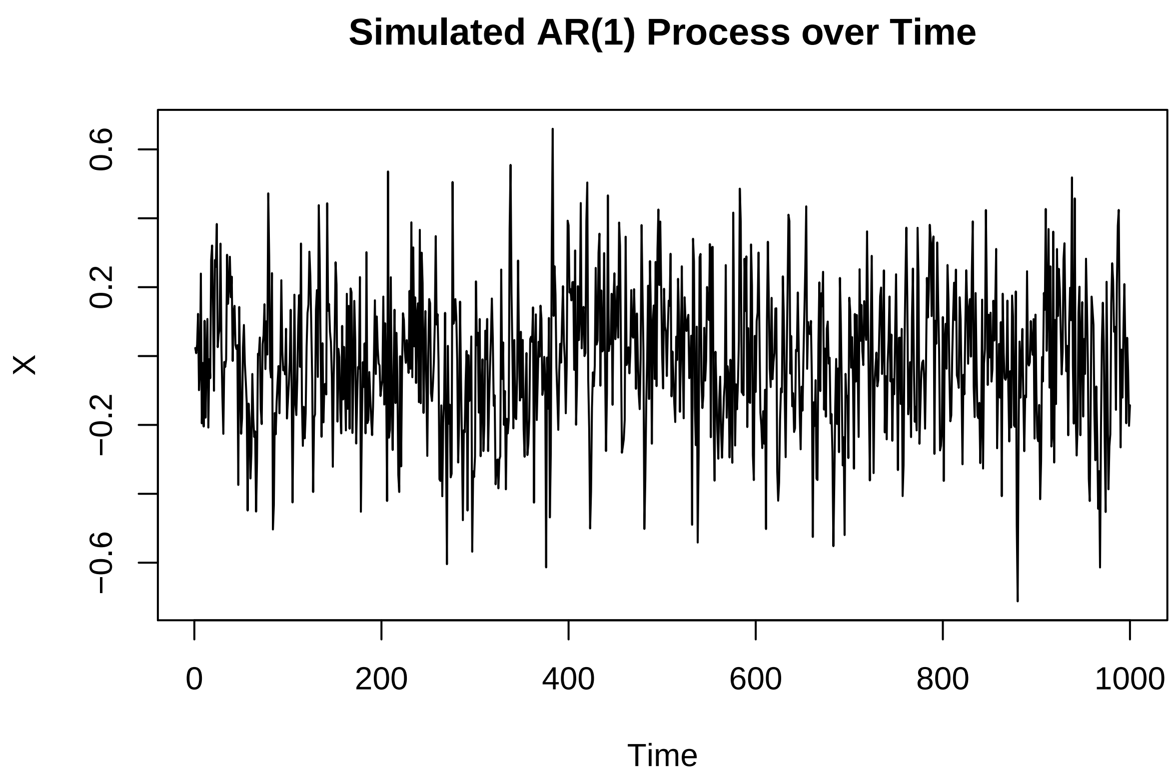


Similar to 2a), from the sample acf plot, we can see that there is a significant positive “spike” at lag 1, followed by generally non-significant values for other lags.

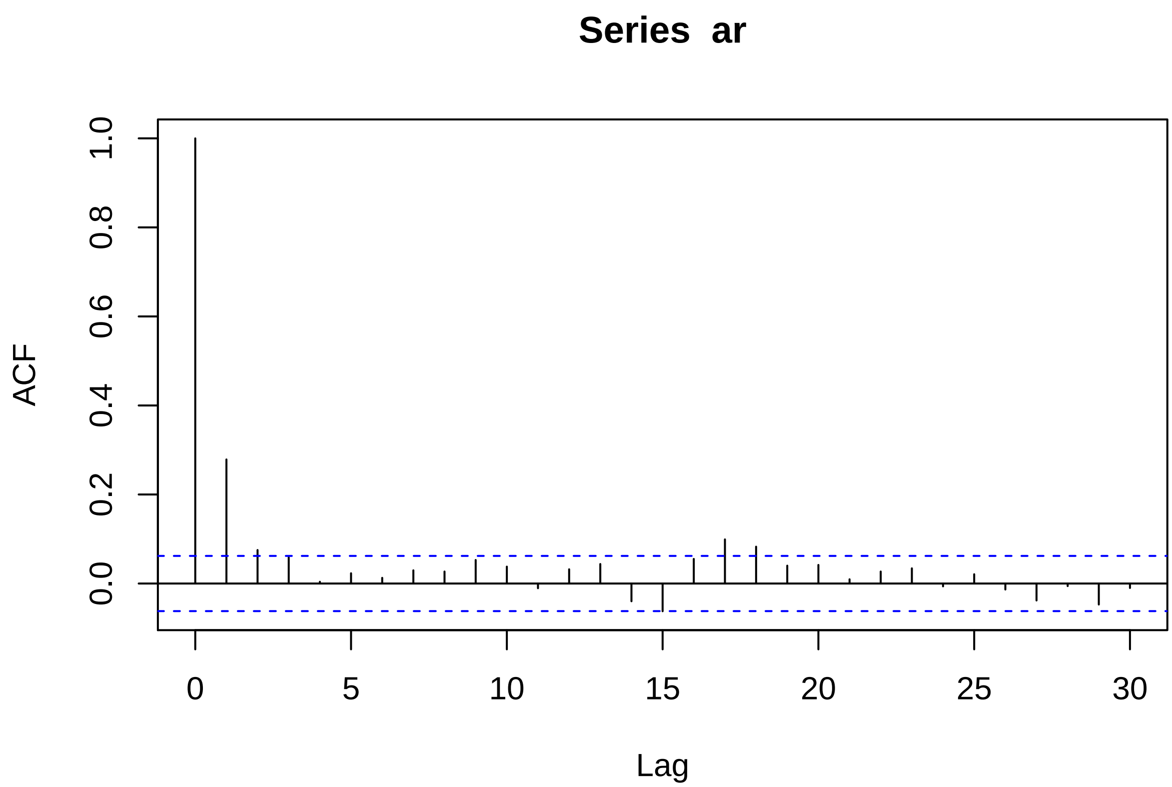
(c) After repeating parts (a) and (b), we see that the acf plots for both are the same. This is because of the invertibility of moving average processes, ie, beta (5) and 1/beta (0.2) show the same acf.

Question 3: (a)

```
alpha = 0.3
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
plot(ar, xlab = "Time", ylab="X", main="Simulated AR(1) Process over Time")
```


Simulated AR(1) Process over Time
X
0.6
0.2
−0.2
−0.6
0
200
400
600
800
1000
Time

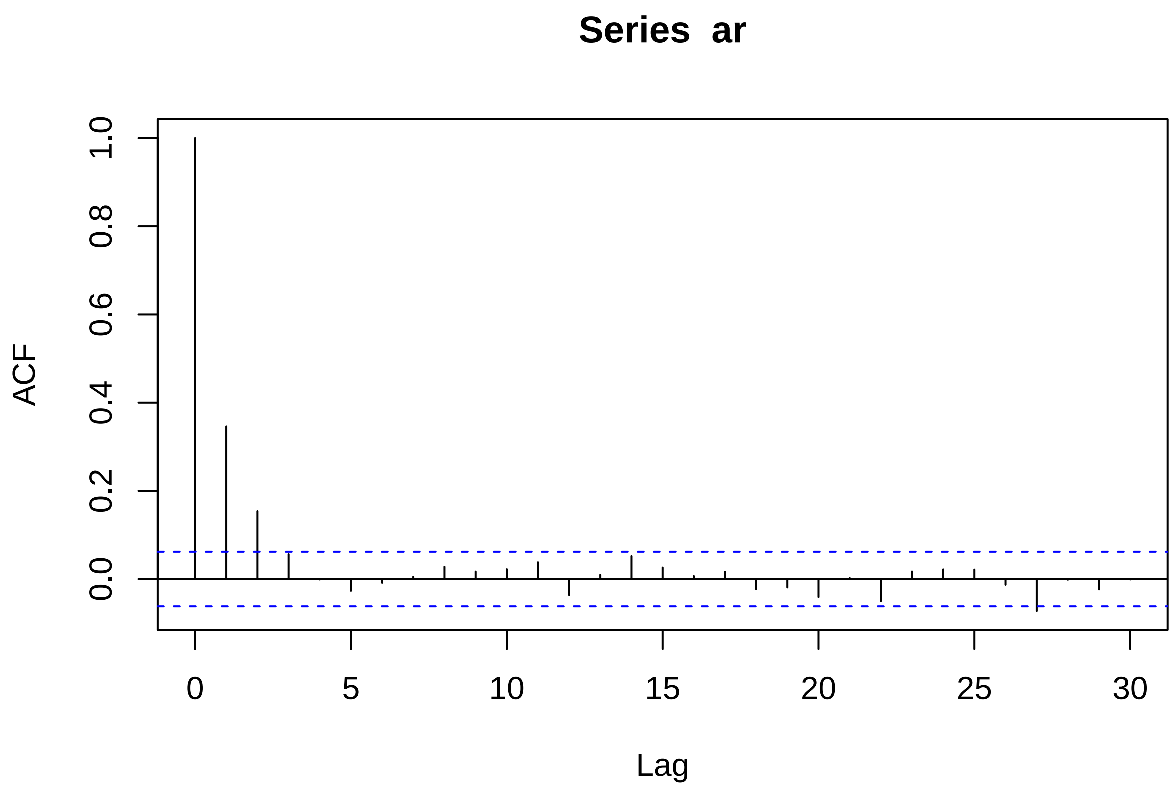
```
acf(ar)
```



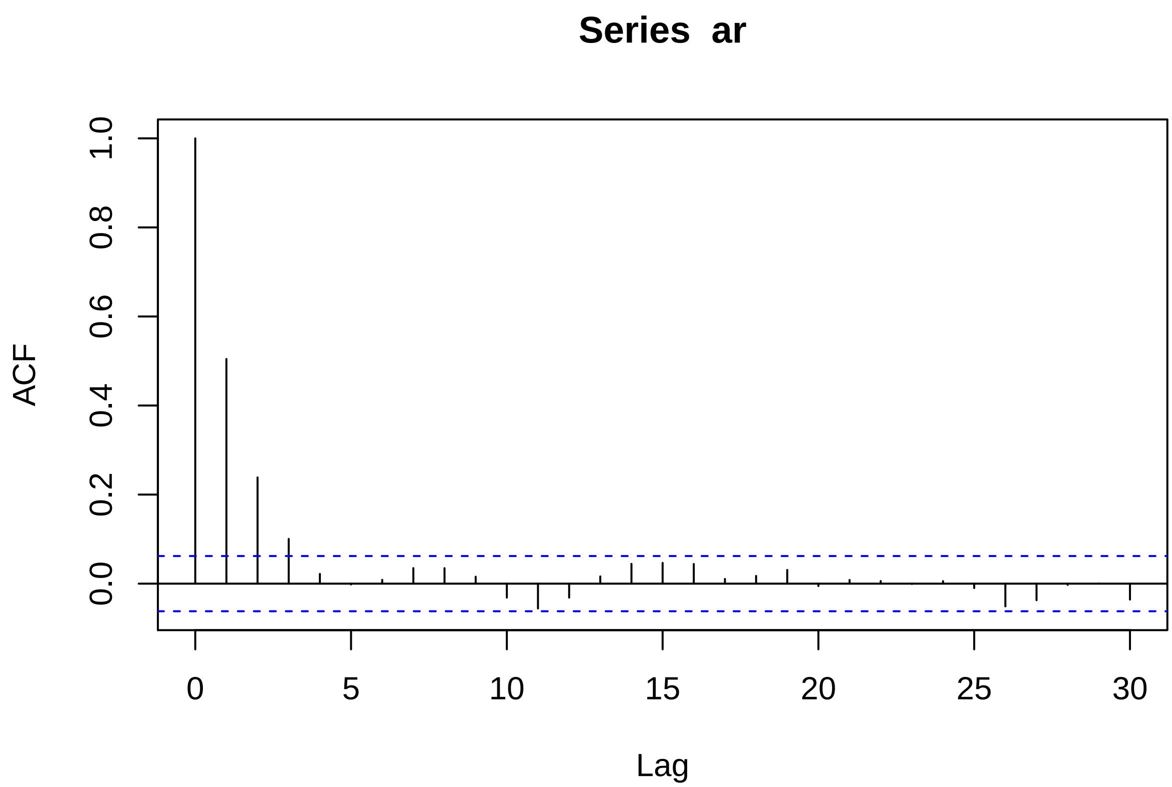
From the sample acf, we can see that there is a significant positive "spike" at lag 1, followed by generally non-significant values for the other lags. The ACF exponentially decreases to 0 as the lag increases.

(b)

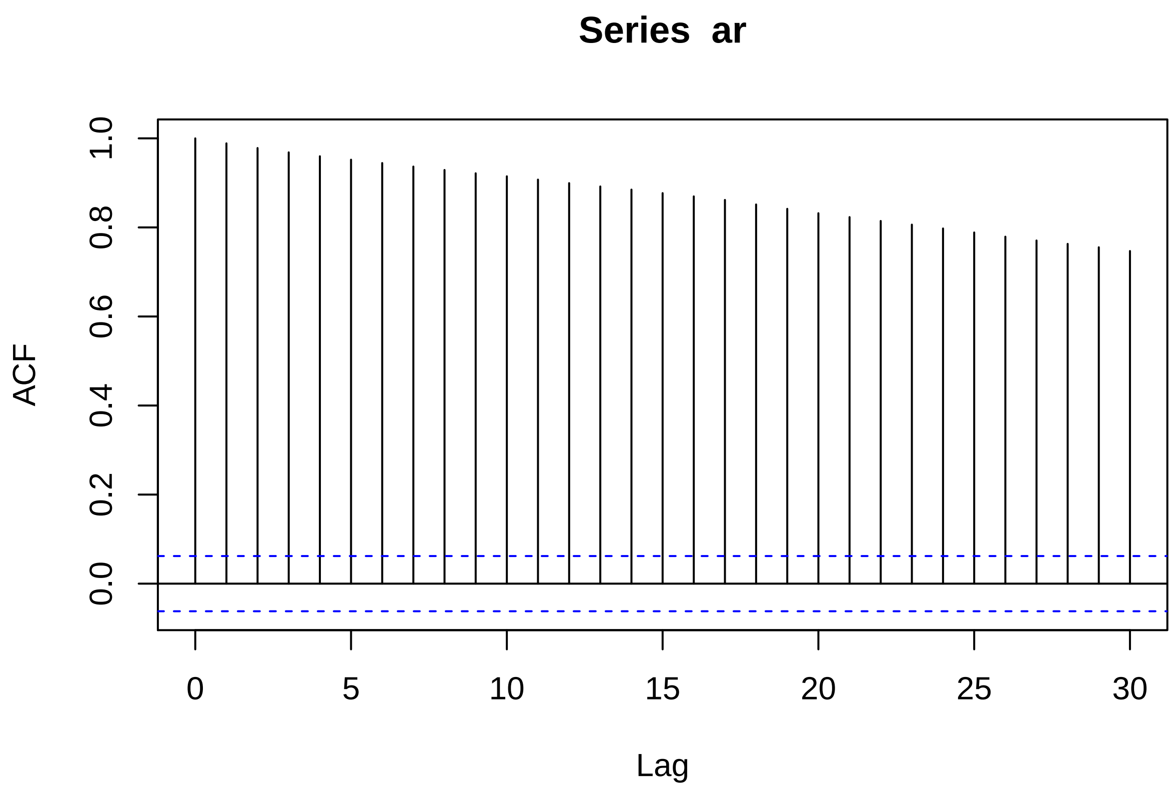
```
alpha = 0.3
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
acf(ar)
```




```
alpha = 0.5
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
acf(ar)
```



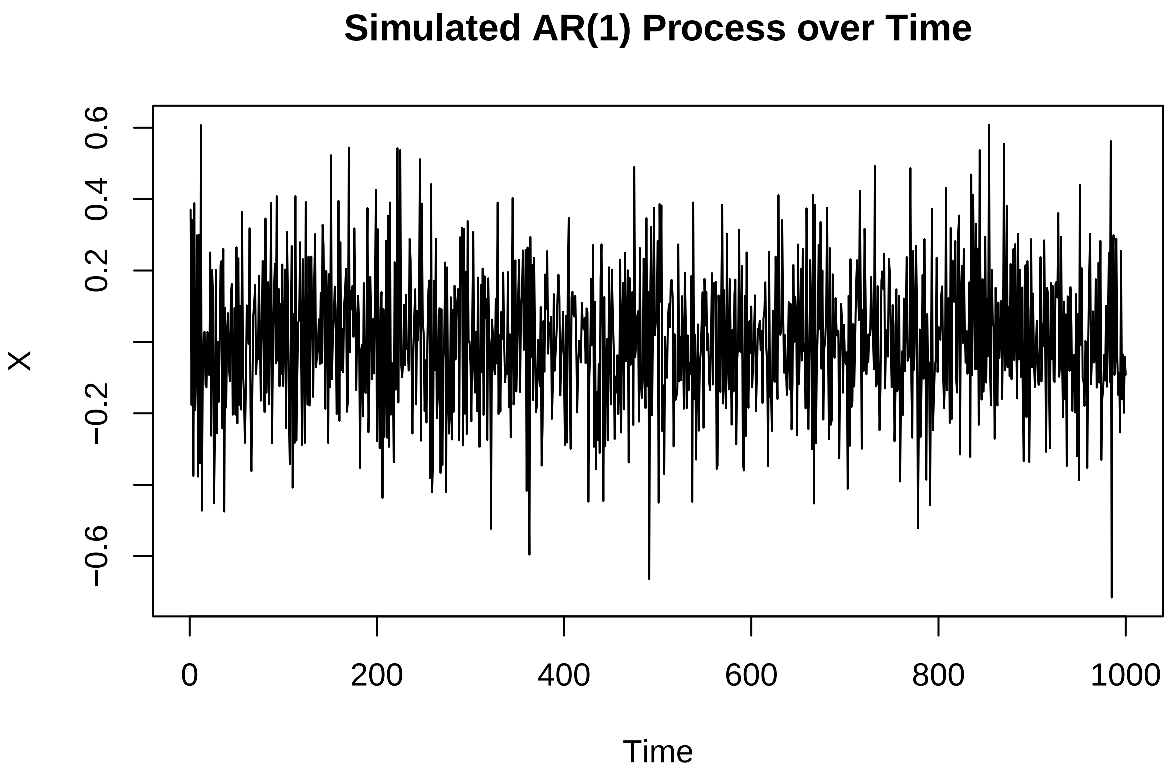

```
alpha = 0.99
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
acf(ar)
```



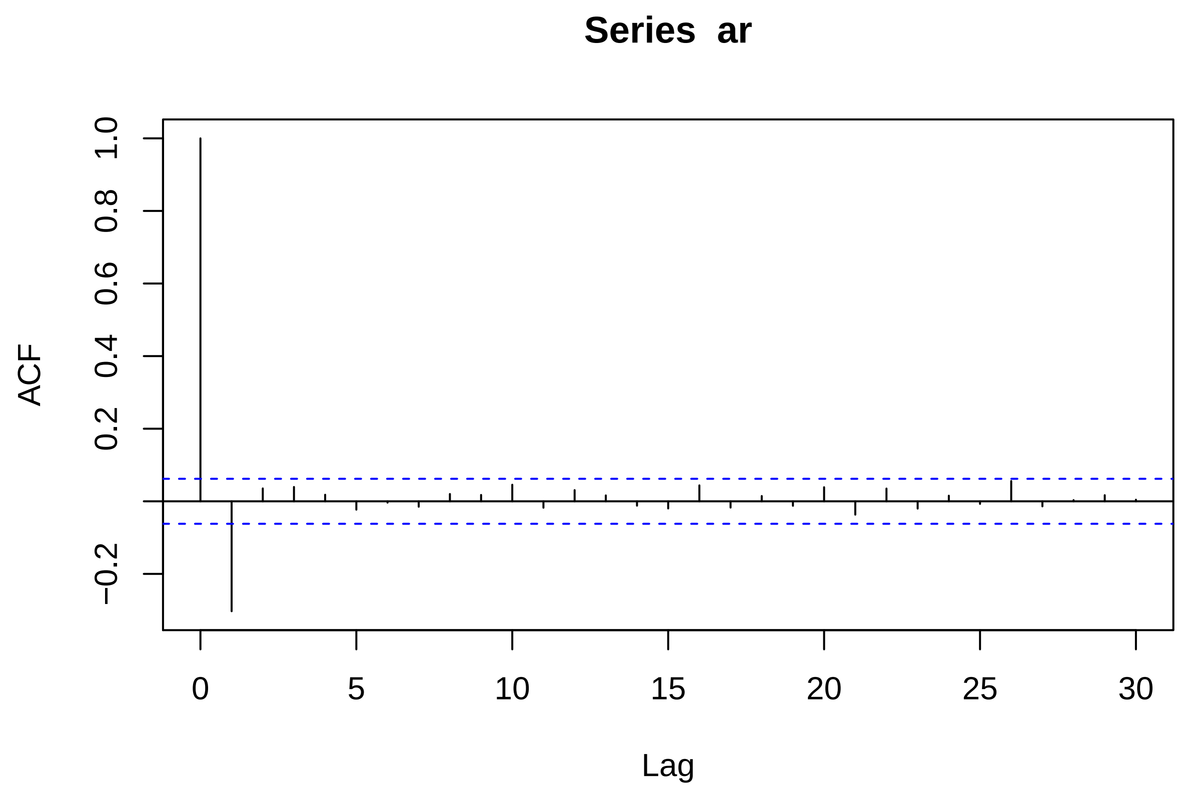
As a -> 1, we see that the acf function has larger, and more significant spikes as lag increases. Additionally, when a = 1, the model becomes not stationary

(c)

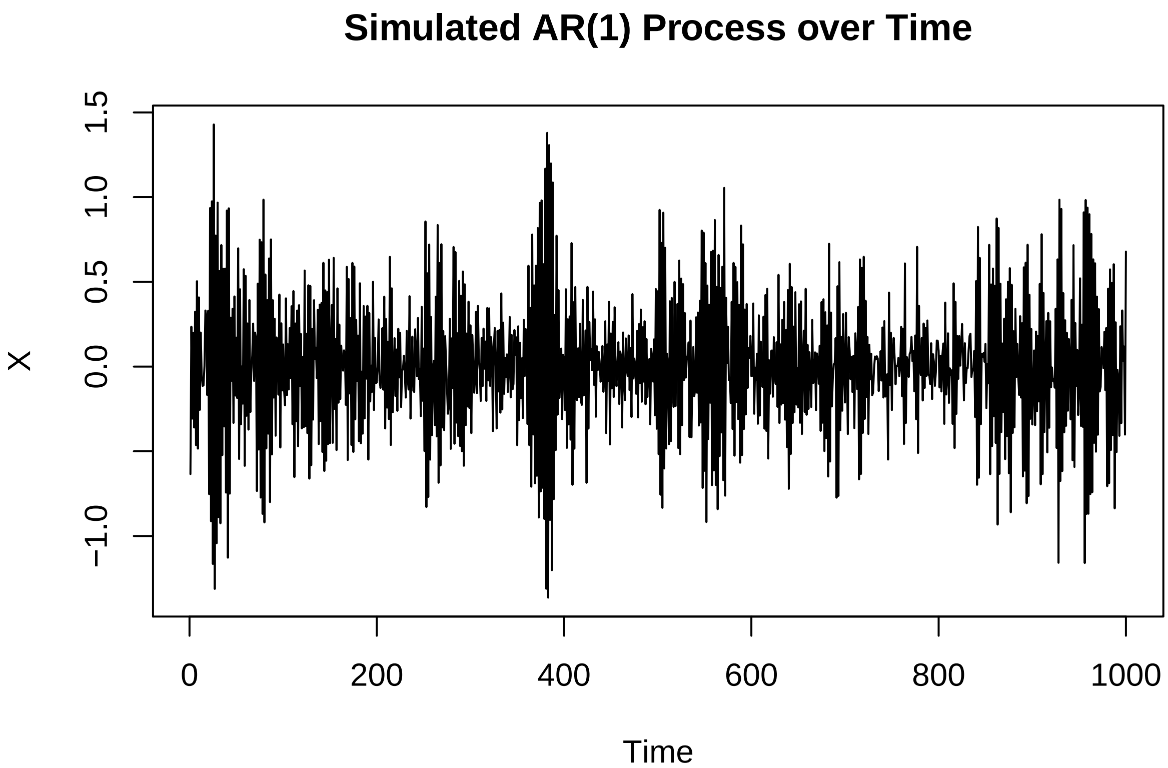
```
alpha = -0.3
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
plot(ar, xlab = "Time", ylab="X", main="Simulated AR(1) Process over Time")
```


Simulated AR(1) Process over Time
X
0.6
0.4
0.2
−0.2
−0.6
0
200
400
600
800
1000
Time

```
acf(ar)
```

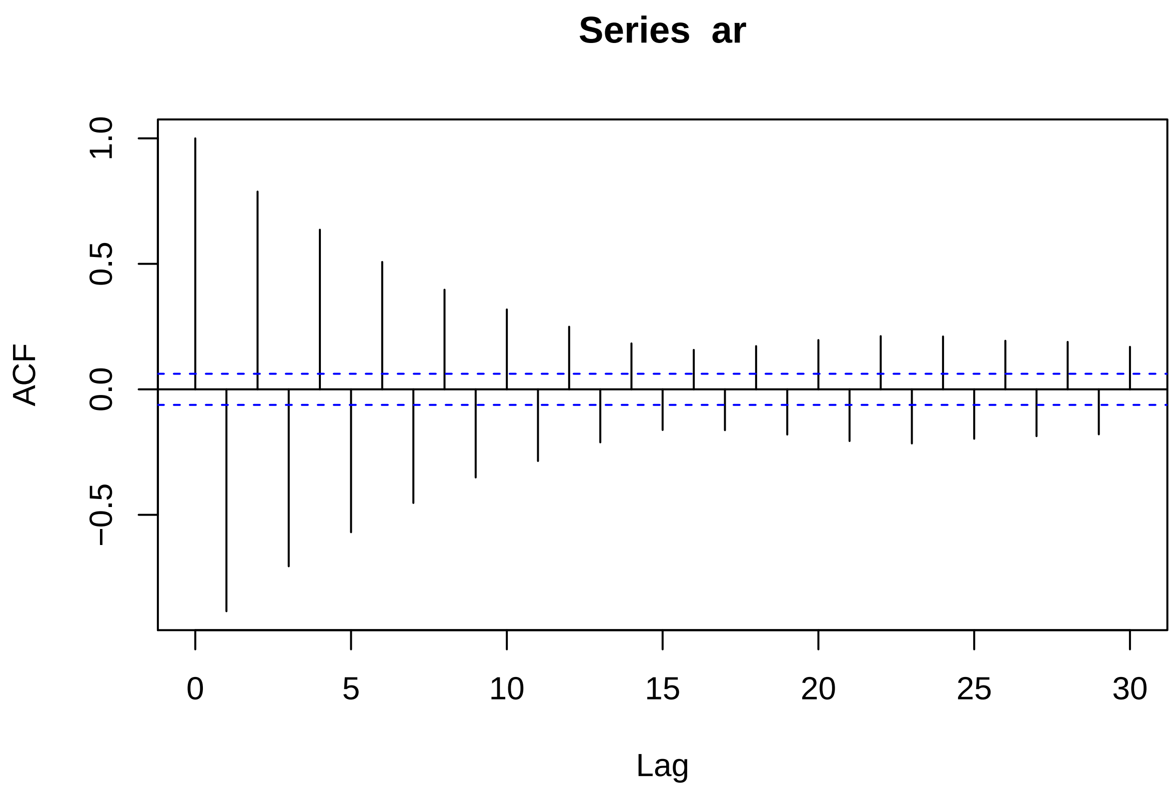



```r
alpha = -0.9
ar = arima.sim(n=1000, list(ar=alpha), sd=0.2)
plot(ar, xlab = "Time", ylab="X", main="Simulated AR(1) Process over Time")
```



Simulated AR(1) Process over Time
X
1.5
1.0
0.5
0.0
−1.0
0
200
400
600
800
1000
Time


```
acf(ar)
```



We can see that for negative a, the ACFs still exponentially decays to 0 as lag increases. However, the signs for the autocorrelations alternative between positive and negative. Similarly, as the |a| approaches 1, we observe larger, and more significant spikes as lag increases.