

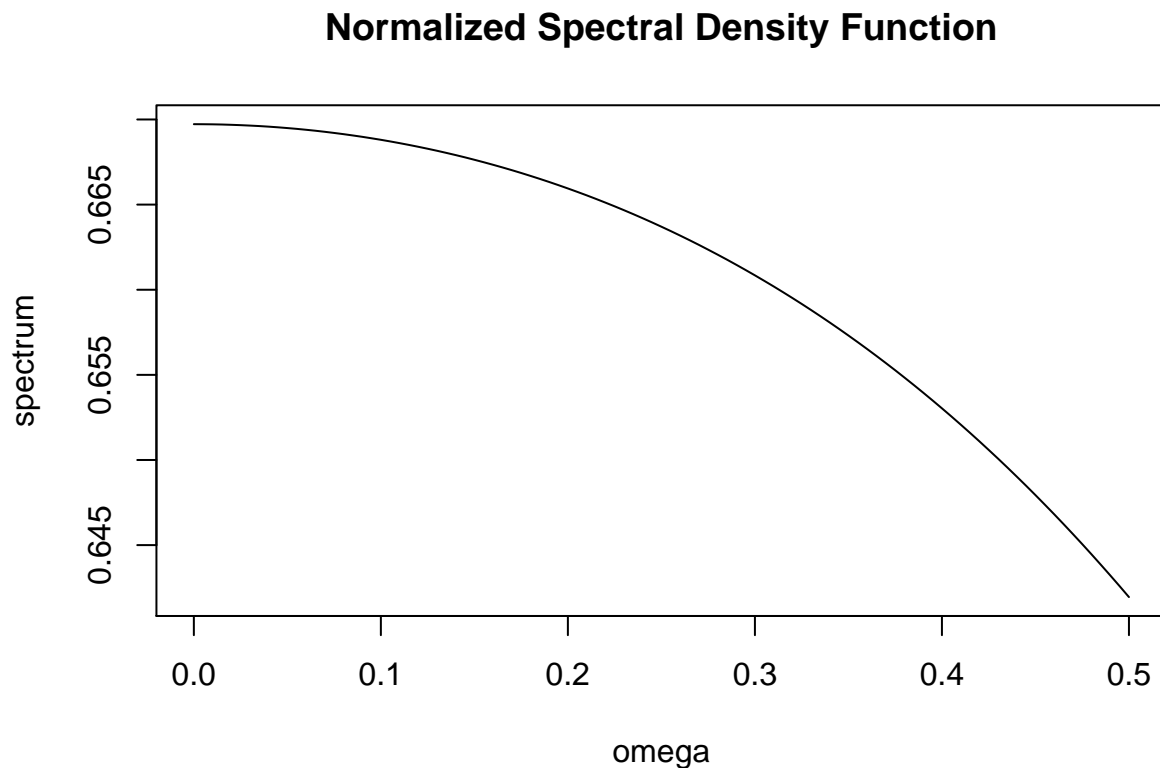
STAT 443: assignment 4

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Q1c) Plot the normalized spectral density and comment on its behaviour.

```
q1c = function(w) ((1+1.065*cos(w)+0.169*cos(2*w)-0.13*cos(3*w))/pi)
plot(q1c, xlab="omega", ylab="spectrum", main="Normalized Spectral Density Function", xlim=c(0,0.5))
```



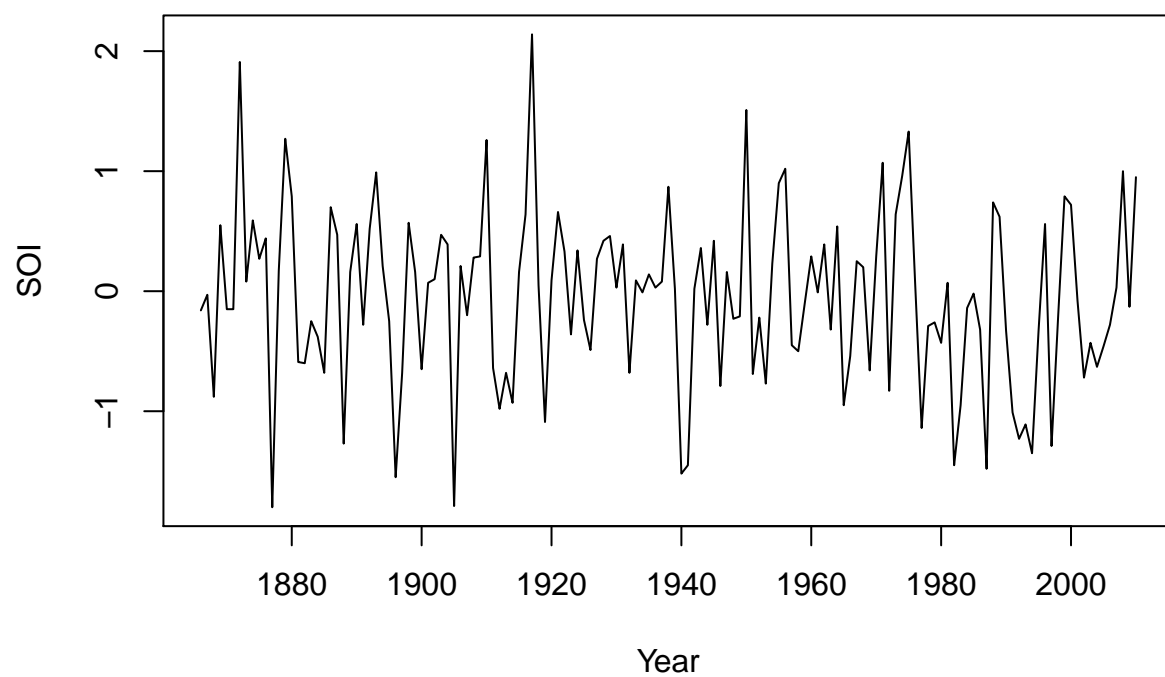
The normalized spectral density function is seen to decrease exponentially as omega increases.

3)

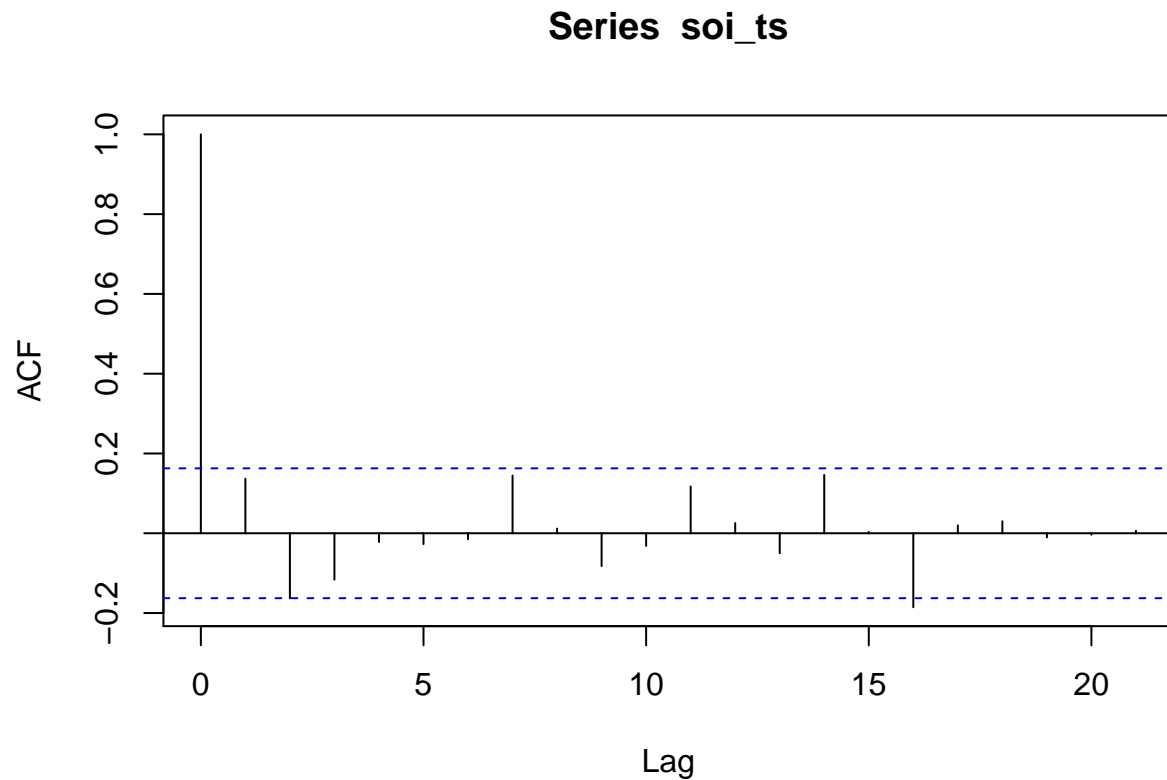
a)

```
soi = read.table("soi.txt", header=TRUE)
soi_ts = ts(data=soi$annual, start=1866, end=2010)
plot(soi_ts, xlab= "Year", ylab="SOI", main="Plot of Annual SOI")
```

Plot of Annual SOI



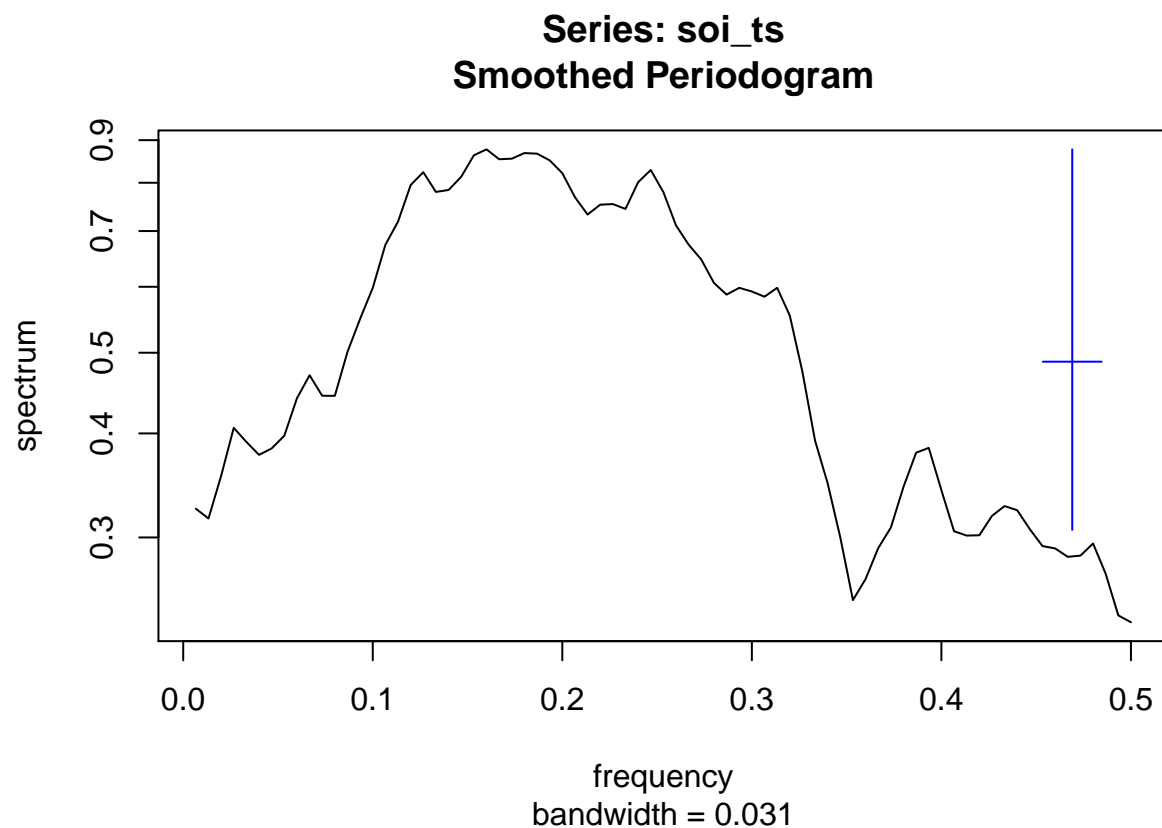
```
acf(soi_ts)
```



From the time series plot, we see that there are fluctuations in the data. There appears to be no trend in the data. The ACF plot appears to cutoff after lag 0, with no significant values for higher lags, except at around lag 16.

- b) Comment on what you observe and estimate the wavelength and angular frequency for the dominating frequency.

```
periodogram_3b = spec.pgram(soi_ts, log="yes", spans = sqrt(2 * length(soi_ts)))
```



```
#periodogram$freq[10]
```

From the smoothed periodogram, we see that it is still slightly spiky and fluctuates. However, it clearly peaks around a frequency of 0.15.

Wavelength and frequency:

```
periodogram_3b$freq[which.max(periodogram_3b$spec)] #calculating the frequency where the periodogram ac
```

```
## [1] 0.16
```

```
which.max(periodogram_3b$spec)
```

```
## [1] 24
```

So, $p=24$, and the angular frequency, $w_p = 2 * \pi * p / N = 2 * \pi * 24 / 145 = 1.039975$. So, the wavelength = $2 * \pi / w_p = 2 * \pi / 1.039975 = 6.04$ years.

c)

```
#fourier frequency w_p
fourier_frequency = function(p, N) {
  if(p <= N/2){
```

```

    omega_p = (2*pi*p)/N
    return(omega_p)
  }
}

fourier_frequency(10, 145)

```

```
## [1] 0.4333231
```

Input: - p - N: number of terms in the list (positive integer)

d)

```

for(p in 1:72){
  N = 145
  t = c(1:145)
  alpha = cos(fourier_frequency(p,N)*t)
  beta = sin(fourier_frequency(p,N)*t)
  linear_model = lm(soi$annual ~alpha + beta)
  summary_model = summary(linear_model)
  if(pf(summary_model$fstatistic[1], 2, 142, lower.tail = F)<=0.05){
    print(summary_model)
  }
}

```

```

##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.61615 -0.44079 -0.00697  0.48828  1.98342
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05874  -0.968  0.33493
## alpha        0.12266    0.08306   1.477  0.14196
## beta        -0.22334    0.08306  -2.689  0.00803 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7073 on 142 degrees of freedom
## Multiple R-squared:  0.06215,    Adjusted R-squared:  0.04894
## F-statistic: 4.705 on 2 and 142 DF,  p-value: 0.01051
##
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.65443 -0.48333  0.02351  0.49596  2.13006

```

```

##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05926  -0.959  0.3392
## alpha       0.21402    0.08381   2.554  0.0117 *
## beta        -0.03790    0.08381  -0.452  0.6518
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7136 on 142 degrees of freedom
## Multiple R-squared:  0.04522,    Adjusted R-squared:  0.03177
## F-statistic: 3.363 on 2 and 142 DF,  p-value: 0.03742
##
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.94455 -0.46721  0.03778  0.44087  1.99156
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05925  -0.959  0.3391
## alpha       -0.07272    0.08379  -0.868  0.3869
## beta         0.20607    0.08379   2.459  0.0151 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7134 on 142 degrees of freedom
## Multiple R-squared:  0.04571,    Adjusted R-squared:  0.03227
## F-statistic: 3.401 on 2 and 142 DF,  p-value: 0.03608
##
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.7122 -0.4693  0.0678  0.4179  2.3094
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05918  -0.960  0.3386
## alpha       -0.15147    0.08369  -1.810  0.0724 .
## beta        -0.16460    0.08369  -1.967  0.0512 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7126 on 142 degrees of freedom
## Multiple R-squared:  0.04789,    Adjusted R-squared:  0.03448
## F-statistic: 3.571 on 2 and 142 DF,  p-value: 0.03067
##
##

```

```
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66045 -0.42827  0.06489  0.42080  2.11943
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05860  -0.970  0.33384
## alpha        0.21868    0.08288   2.639  0.00925 **
## beta        -0.14669    0.08288  -1.770  0.07889 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7057 on 142 degrees of freedom
## Multiple R-squared:  0.06637,    Adjusted R-squared:  0.05322
## F-statistic: 5.047 on 2 and 142 DF,  p-value: 0.007627
```

The p values that give significant Fourier frequencies at the 5% level are: $p = 0.01051, 0.03742, 0.03608, 0.03067, 0.007627$.

e) The significant fourier frequencies are:

```
for(p in 1:72){
  N = 145
  t = c(1:145)
  alpha = cos(fourier_frequency(p,N)*t)
  beta = sin(fourier_frequency(p,N)*t)
  linear_model = lm(soi$annual ~alpha + beta)
  summary_model = summary(linear_model)
  if(pf(summary_model$fstatistic[1], 2, 142, lower.tail = F)<=0.05){
    print(summary_model)
    print(p)
  }
}
```

```
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.61615 -0.44079 -0.00697  0.48828  1.98342
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05874  -0.968  0.33493
## alpha        0.12266    0.08306   1.477  0.14196
## beta        -0.22334    0.08306  -2.689  0.00803 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```

## Residual standard error: 0.7073 on 142 degrees of freedom
## Multiple R-squared:  0.06215,    Adjusted R-squared:  0.04894
## F-statistic: 4.705 on 2 and 142 DF,  p-value: 0.01051
##
## [1] 16
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.65443 -0.48333  0.02351  0.49596  2.13006
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05926  -0.959   0.3392
## alpha        0.21402    0.08381   2.554   0.0117 *
## beta        -0.03790    0.08381  -0.452   0.6518
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7136 on 142 degrees of freedom
## Multiple R-squared:  0.04522,    Adjusted R-squared:  0.03177
## F-statistic: 3.363 on 2 and 142 DF,  p-value: 0.03742
##
## [1] 20
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.94455 -0.46721  0.03778  0.44087  1.99156
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05925  -0.959   0.3391
## alpha       -0.07272    0.08379  -0.868   0.3869
## beta         0.20607    0.08379   2.459   0.0151 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7134 on 142 degrees of freedom
## Multiple R-squared:  0.04571,    Adjusted R-squared:  0.03227
## F-statistic: 3.401 on 2 and 142 DF,  p-value: 0.03608
##
## [1] 23
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.7122 -0.4693  0.0678  0.4179  2.3094

```



```
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05918  -0.960  0.3386
## alpha      -0.15147    0.08369  -1.810  0.0724 .
## beta       -0.16460    0.08369  -1.967  0.0512 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7126 on 142 degrees of freedom
## Multiple R-squared:  0.04789,    Adjusted R-squared:  0.03448
## F-statistic: 3.571 on 2 and 142 DF,  p-value: 0.03067
##
## [1] 25
##
## Call:
## lm(formula = soi$annual ~ alpha + beta)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66045 -0.42827  0.06489  0.42080  2.11943
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05683    0.05860  -0.970  0.33384
## alpha        0.21868    0.08288   2.639  0.00925 **
## beta       -0.14669    0.08288  -1.770  0.07889 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7057 on 142 degrees of freedom
## Multiple R-squared:  0.06637,    Adjusted R-squared:  0.05322
## F-statistic: 5.047 on 2 and 142 DF,  p-value: 0.007627
##
## [1] 41
```

```
a0 = -0.05683
```

```
a16 = 0.12266
```

```
a20 = 0.21402
```

```
a23 = -0.07272
```

```
a25 = -0.15147
```

```
a41 = 0.21868
```

```
b16 = -0.22334
```

```
b20 = -0.032790
```

```
b23 = 0.20607
```

```
b25 = -0.1646
```

```
b41 = -0.14669
```

```
N = 145
```

```
Y_t = a0 + a16*cos(fourier_frequency(16,N)*t) + a20*cos(fourier_frequency(20,N)*t) + 23*cos(fourier_fr
```

f)

```
plot(soi_ts, y_lab = "SOI", main="SOI over Time")
```

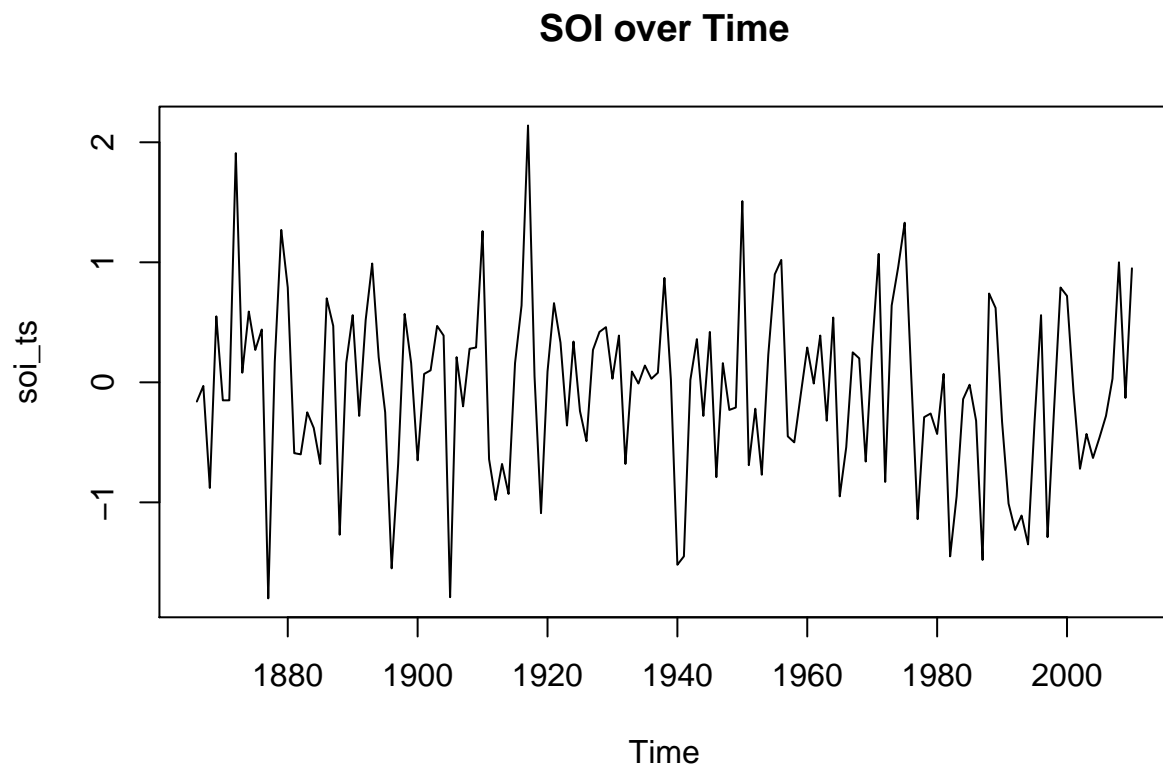
```
## Warning in plot.window(xlim, ylim, log, ...): "y_lab" is not a graphical  
## parameter
```

```
## Warning in title(main = main, xlab = xlab, ylab = ylab, ...): "y_lab" is not a  
## graphical parameter
```

```
## Warning in axis(1, ...): "y_lab" is not a graphical parameter
```

```
## Warning in axis(2, ...): "y_lab" is not a graphical parameter
```

```
## Warning in box(...): "y_lab" is not a graphical parameter
```



```
plot(Y_t, type="l")
```

