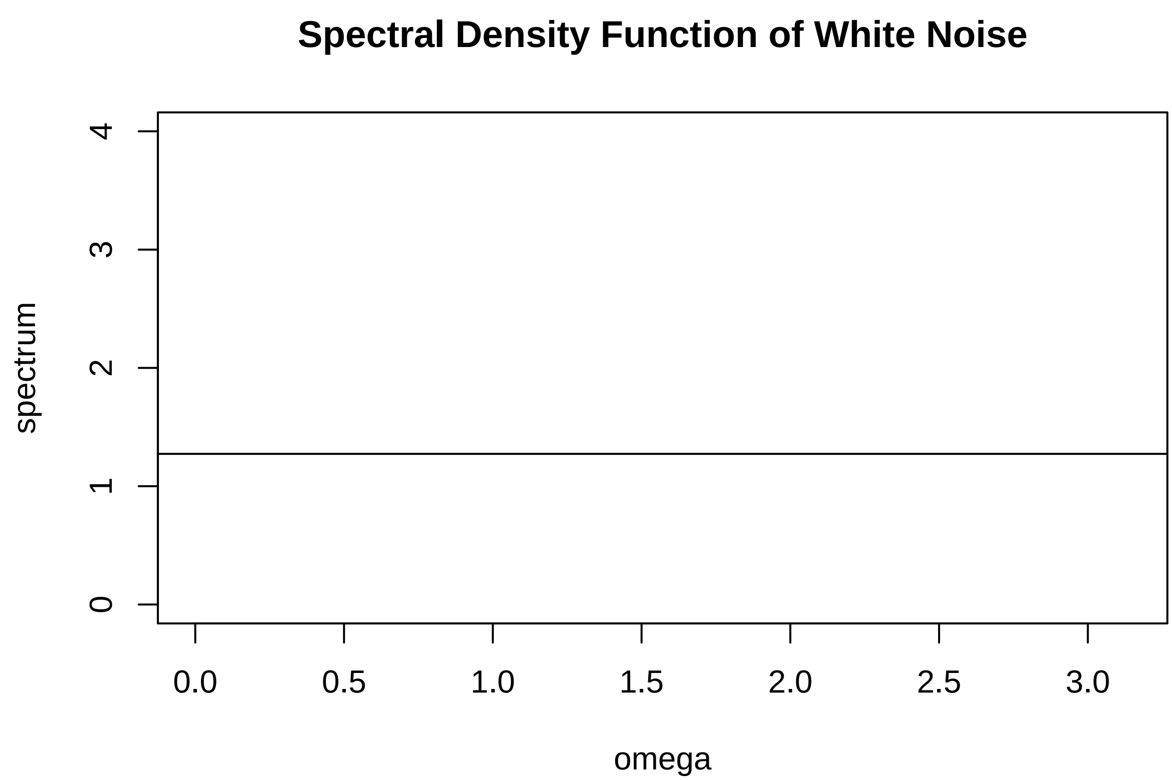
STAT 443: Lab 10

Flora Zhang 52135365

28 Mar, 2022

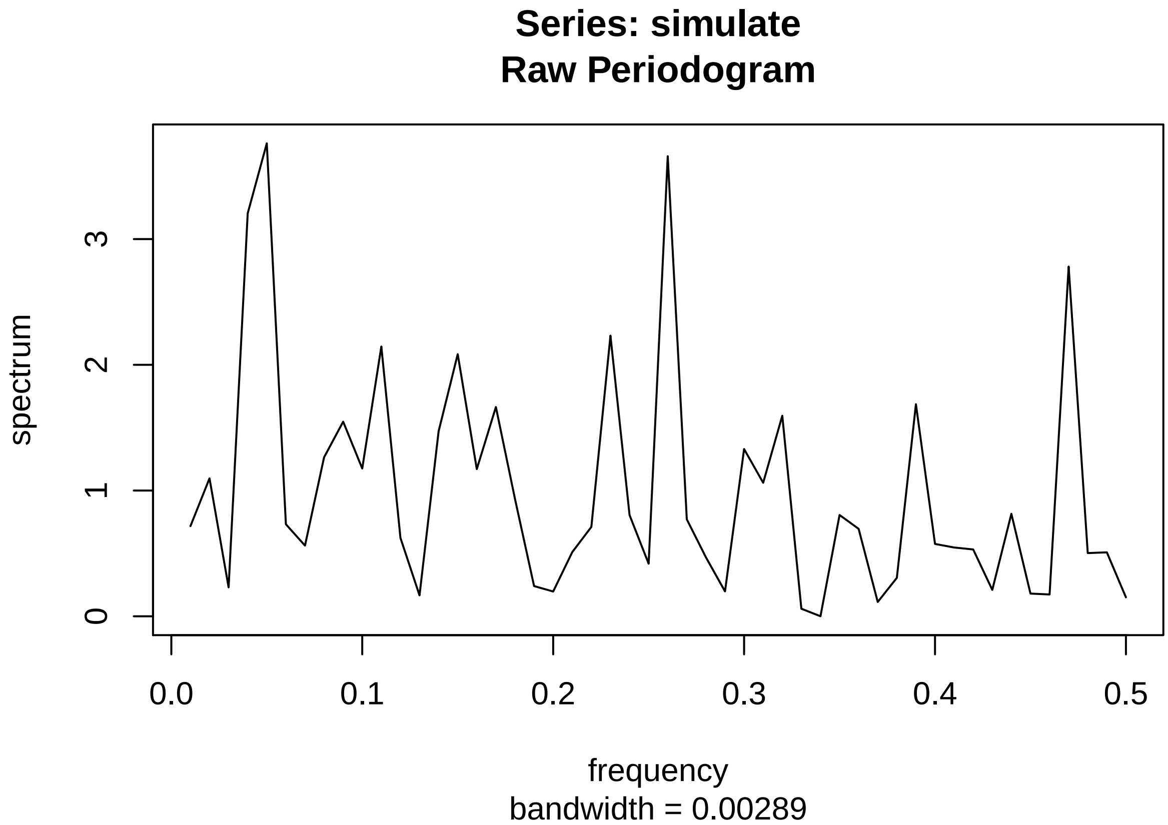
1a) WN(0, 4)

```
plot(x = NA, y = NA, xlim = c(0,pi), ylim = c(0,4), xlab="omega", ylab="spectrum", main="Spectral Densi
abline(h = 4/pi)
```



b)

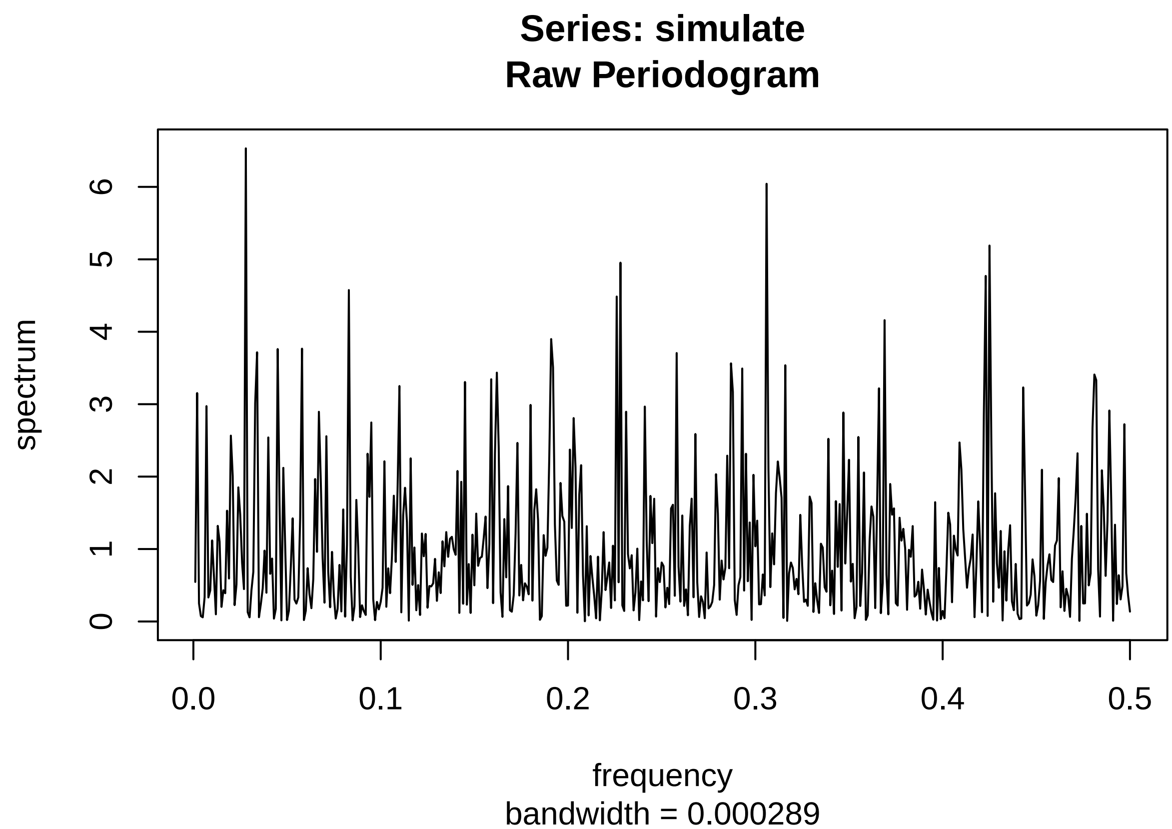
```
N = 100
simulate = arima.sim(n=N, model=list(0,0,0))
spec.pgram(simulate, log="no")
```



We see that the sample periodogram of the simulated series roughly follows the same shape as the actual, but there are differences as there are many peaks and fluctuations due to noise.

c)

```
N = 1000
simulate = arima.sim(n=N, model=list(0,0,0))
spec.pgram(simulate, log="no")
```

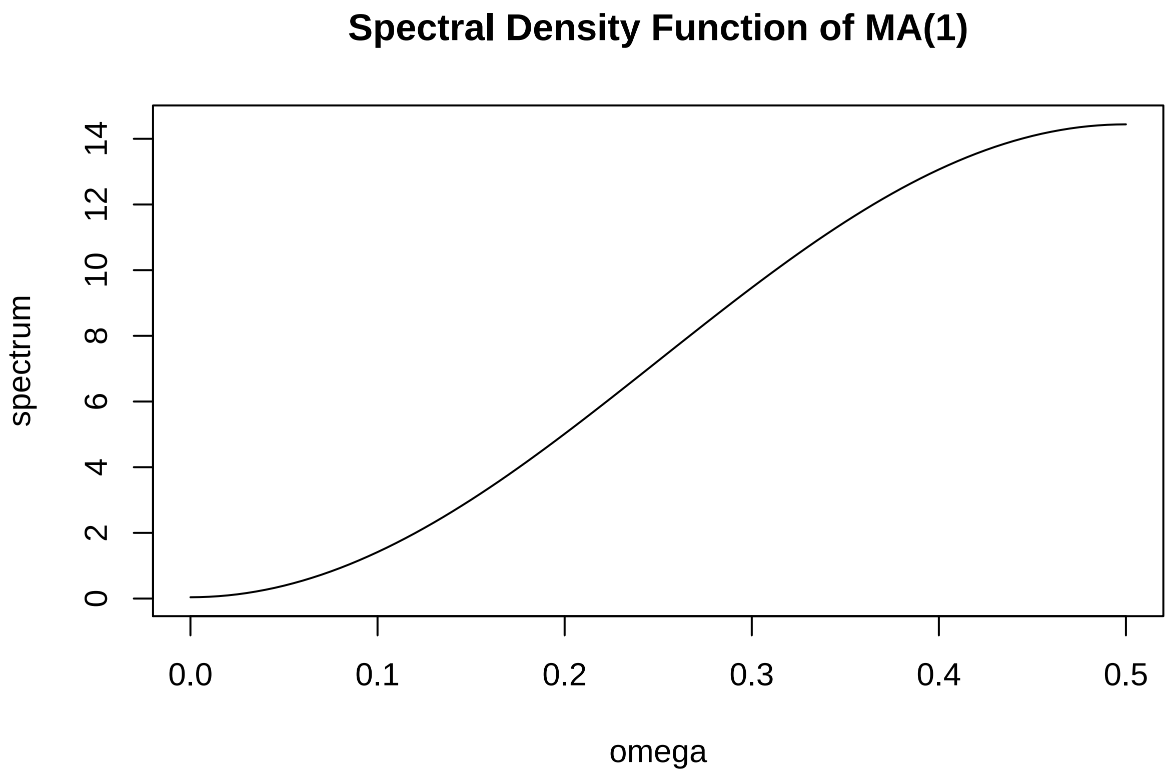


Similarly, we see that the sample periodogram roughly follows the same shape, but there are still differences as there are many peaks and fluctuations due to noise.

d) By repeating parts (b) and (c) several times, we see that the sample periodograms roughly follow the same shape as the spectral density function, but the peaks or fluctuations change and vary. Additionally, the height of the spectrums and location of fluctuations change as you change the simulations. As a result, the periodogram is not a consistent estimator of the spectral density function, as the variance does not approach zero as the N, sample size, increases.

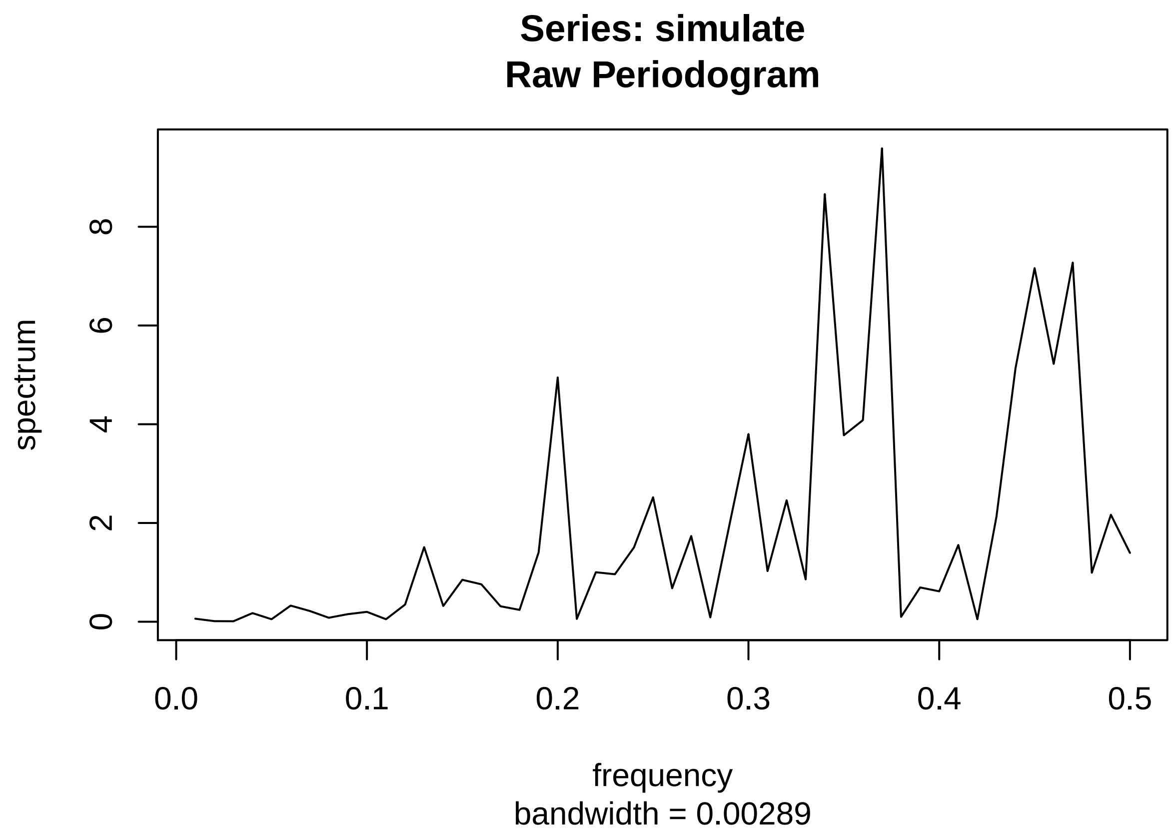
2a) alpha = -0.9

```
fcn = function(w) (4*(1+(-0.9)^2 +2*(-0.9)*cos(2*pi*w)))
plot(fcn, xlab="omega", ylab="spectrum", main="Spectral Density Function of MA(1)", xlim=c(0,0.5))
```



b)

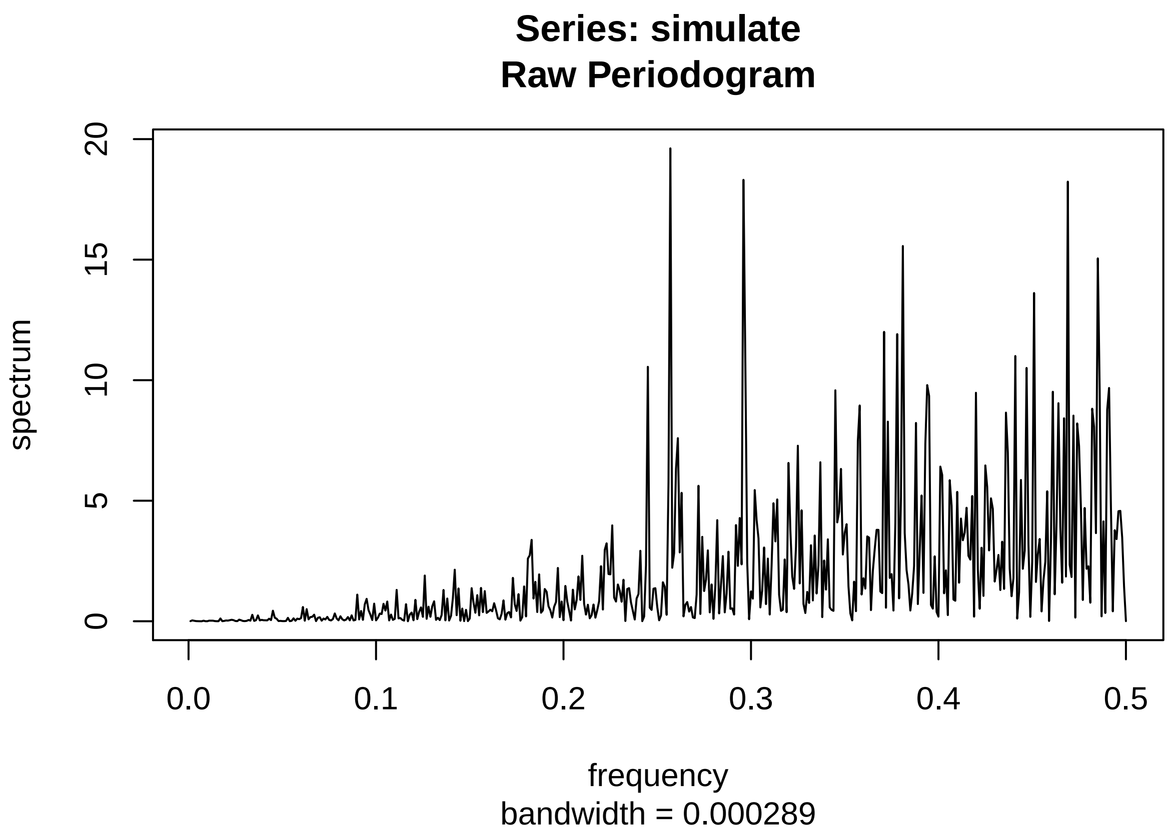
```
N = 100
alpha = -0.9
simulate = arima.sim(n=N, model=list(ma=alpha))
spec.pgram(simulate, log="no")
```



Comparing this sample periodogram to the spectral density function, we can see that they have similar shapes as they both gradually exponentially slope upwards, then peak near frequency = 0.5. However,the periodogram shows fluctuations and looks more jagged, which is a result of noise. The peak of the periodogram also does not seem to perfectly match that of the spectral density function.

c)

```
N = 1000
beta = -0.9
simulate = arima.sim(n=N, model=list(ma=beta))
spec.pgram(simulate, log="no")
```

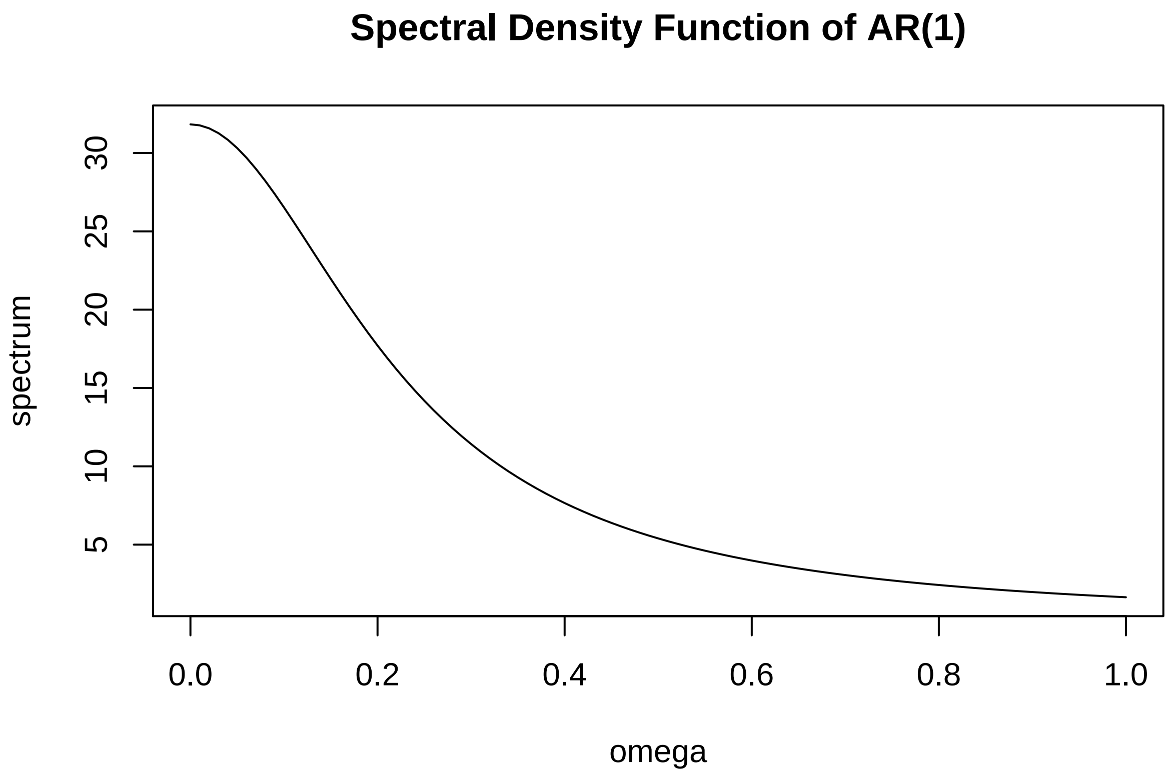


Comparing this sample periodogram to the spectral density function, we can see that they have similar shapes as they both gradually exponentially slope upwards, then peak near frequency = 0.5. However,the periodogram shows fluctuations and looks more jagged, which is a result of noise. The peak of the periodogram also does not seem to perfectly match that of the spectral density function. The periodogram here has a lot more fluctuations compared to that in (b).

d) Comparing both periodograms from (b) and (c) to the true spectral density function, we see that they all have similar shapes as they gradually exponentially slope upwards, then peak near frequency = 0.5. The sample periodograms both have fluctuations and appear jagged as a result of noise, with the periodogram from (c) having more fluctuations compared to (b). Additionally, the height of the spectrums and location of fluctuations change as you change the simulations. The peaks of the periodograms seem slightly off from the true values as well. As a result, the periodogram is not a consistent estimator of the spectral density function, as the variance does not approach zero as the N, sample size, increases.

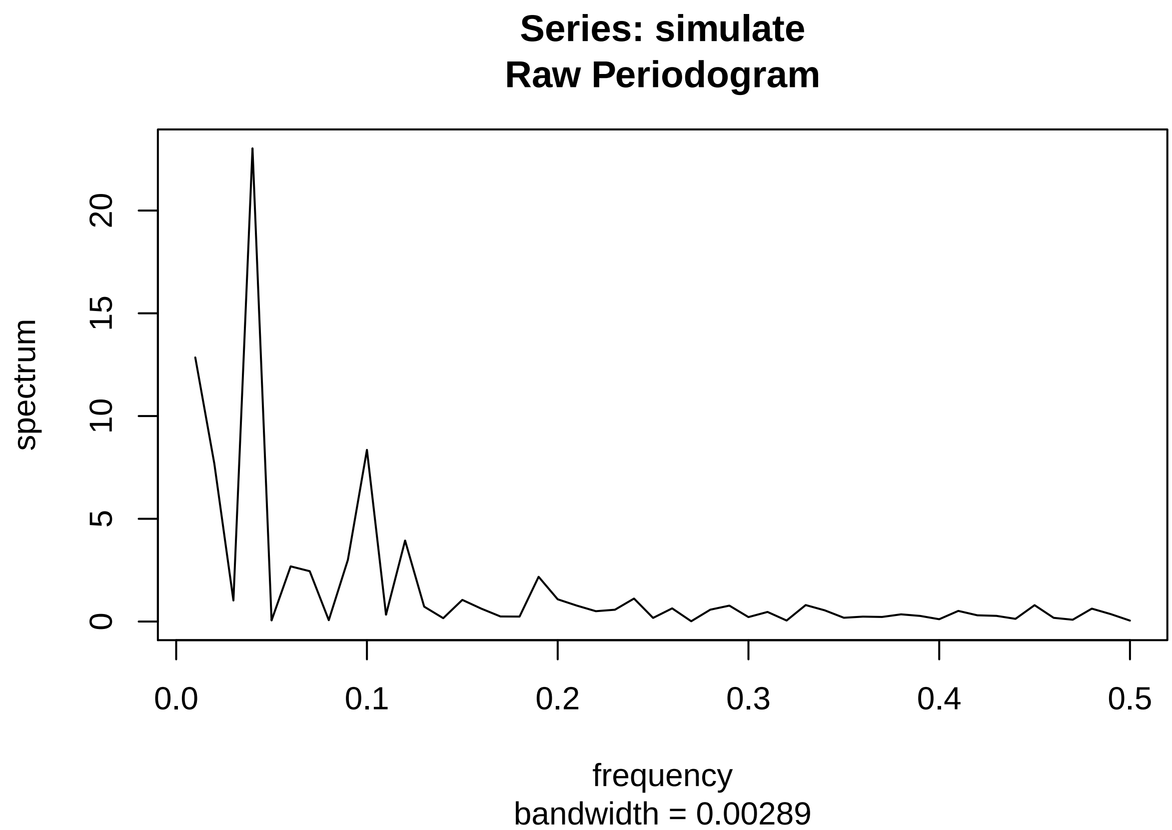
3a)

```
fcn = function(w) (4/(pi*(1-1.6*cos(w) + 0.8^2)))
plot(fcn, xlab="omega", ylab="spectrum", main="Spectral Density Function of AR(1)")
```



b)

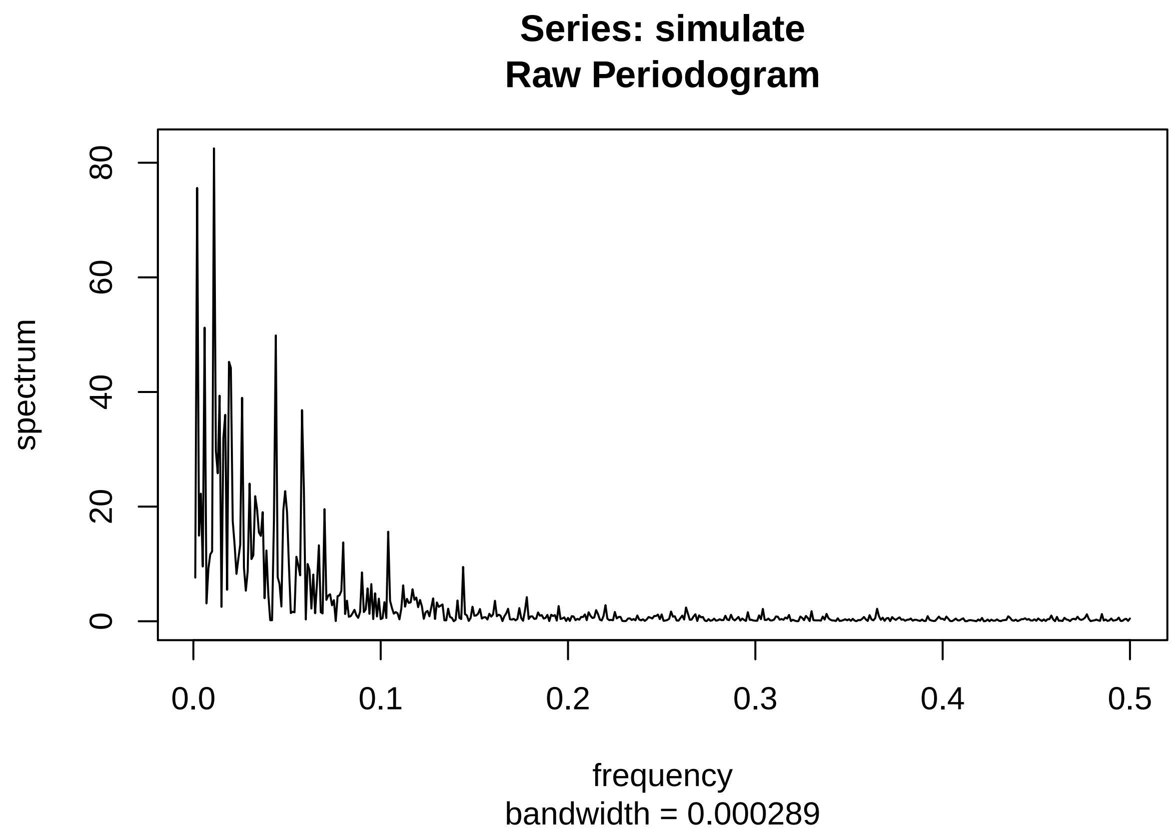
```
N = 100
alpha = 0.8
simulate = arima.sim(n=N, model=list(ar=alpha))
spec.pgram(simulate, log="no")
```



Comparing the sample periodogram to the spectral density function, we can see that they have similar shapes as they both peak near zero, then exponentially slope down. However,the periodogram shows fluctuations and looks more jagged, which is a result of noise.

c)

```
N = 1000
alpha = 0.8
simulate = arima.sim(n=N, model=list(ar=alpha))
spec.pgram(simulate, log="no")
```



Comparing this sample periodogram to the spectral density function, we can see that they have similar shapes as they both peak near zero, then exponentially slopes down. However,the periodogram shows fluctuations and looks more jagged, which is a result of noise. This periodogram has more fluctuations compared to the previous one for (b).

d) Comparing both periodograms from (b) and (c) to the true spectral density function, we see that they all have similar shapes as they reach a peak near zero, then begins to exponentially slope down. The sample periodograms both have fluctuations and appear jagged as a result of noise, with the periodogram from (c) having more fluctuations compared to (b). Additionally, the height of the spectrums and location of fluctuations change as you change the simulations. The peaks of the periodograms seem slightly off from the true values as well. As a result, the periodogram is not a consistent estimator of the spectral density function, as the variance does not approach zero as the N, sample size, increases.