

$$\begin{pmatrix} 0.62797 \\ 0.37203 \end{pmatrix} \cdot \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix} = \begin{pmatrix} 0.62797 \cdot 0.8 + 0.37203 \cdot 0.2 \\ 0.62797 \cdot 0.3 + 0.37203 \cdot 0.7 \end{pmatrix}$$

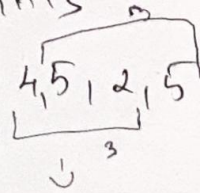
$$= \begin{pmatrix} 0.502376 + 0.074406 \\ 0.188391 + 0.260421 \end{pmatrix} = \begin{pmatrix} 0.576782 \\ 0.448812 \end{pmatrix}$$

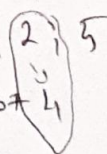
$$= (0.5768, 0.4488) \text{ BY APPROXIMATION.}$$

3. SMOOTHING

ORDER = 3

EQUAL WEIGHTS

TIME SERIES  $4, 5, 2, 5$   
  
 WINDOW = 3

$4, 5, 2, 5$   
  
 3.6667  
 3.67

$$\frac{4+5+2}{3} = \frac{11}{3} = 3.6667$$

$$\frac{5+2+5}{3} = \frac{12}{3} = 4$$

WE CAN NOTICE THAT 2 WAS OUR OUTLIER AND IT WAS REMOVED BY SMOOTHING.



$$t=3$$

$$\alpha_3(1) = (0.0492 \cdot 0.8 + 0.058 \cdot 0.3) \cdot 0.3 = (0.03936 + 0.0174) \cdot 0.3 = 0.017028$$

$$\alpha_3(2) = (0.0492 \cdot 0.2 + 0.058 \cdot 0.7) \cdot 0.2 = 0.0406 \cdot 0.2 = 0.010088$$

$$b) P_2(X_2, X_3, X_1) = \alpha_3(1) + \alpha_3(2) = 0.017028 + 0.010088 = 0.027116 \Rightarrow \text{FULL PROBABILITY}$$

CONDITIONAL.

$$P_2(H_3 = S_1 | X_2, X_3, X_1) = \frac{\alpha_3(1)}{P_2(X_2, X_3, X_1)} = \frac{0.017028}{0.027116}$$

$$P_2(H_3 = S_2 | X_2, X_3, X_1) = \frac{\alpha_3(2)}{P_2(X_2, X_3, X_1)} = \frac{0.010088}{0.027116}$$

$$= 0.37203$$

c) Prediction  $H_4 | X_2, X_3, X_1$

$$(0.62797 \ 0.37203) \cdot \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix} = 0.8 \cdot 0.62797 + 0.3 \cdot 0.37203$$

THIS NEEDS TO BE A COLUMN VECTOR MULTIPLIED BY TRANSITION MATRIX. PROBABILITY AT TIME 3 \* TRANSITION MATRIX



2. TRANSITION MATRIX  $M = \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix}$

EMISSION MATRIX

$$B = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \begin{matrix} S_1 \\ S_2 \end{matrix}$$

INITIAL PROBABILITIES

$$P = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

VISIBLE STATES =  $\{ \underline{x_2}, \underline{x_3}, x_1 \}$

a)  $x_3 \rightarrow$  TIME  $\begin{matrix} 1 & 2 & 3 \end{matrix}$

$S_1$	$\begin{pmatrix} 0.16 \rightarrow 0.0492 & 0.017028 \end{pmatrix}$	$\xrightarrow{\text{alphas}}$	$S_1$	$\begin{pmatrix} 0.16 & 0.0492 & 0.017028 \end{pmatrix}$
$S_2$	$\begin{pmatrix} 0.12 \rightarrow 0.058 & 0.010088 \end{pmatrix}$		$S_2$	$\begin{pmatrix} 0.12 & 0.058 & 0.010088 \end{pmatrix}$
				alphas

$t = 1$

$$x_1(S_1) = 0.4 \cdot 0.4 = 0.16$$

$$x_1(S_2) = 0.6 \cdot 0.2 = 0.12$$

$t = 2$

~~$$x_2(S_1) = 0.16 \cdot 0.3 \cdot 0.8 = 0.0384$$~~

~~$$x_2(S_2) = 0.12 \cdot 0.3 \cdot 0.3$$~~

$$x_2(S_1) = (0.16 \cdot 0.8 + 0.12 \cdot 0.3) \cdot 0.3 = (0.128 + 0.036) \cdot 0.3 = 0.32 \cdot 0.0492$$

$$x_2(S_2) = (0.16 \cdot 0.2 + 0.12 \cdot 0.7) \cdot 0.5 = (0.032 + 0.084) \cdot 0.5 = 0.058$$



$$MLE = \frac{\# \text{TRANSITIONS FROM } s_i \text{ TO } s_j}{\# \text{ALL TRANSITIONS FROM } s_i}$$

WE GET THE FOLLOWING MATRICES:

TRANSITION COUNTS: SUNNY CLOUDY RAIN

	SUNNY	CLOUDY	RAIN	SUM
SUNNY	2	2	0	4
CLOUDY	0	0	3	3
RAIN	1	1	2	4

ESTIMATE  $M^*$  = NORMALIZED IT

$$= \begin{pmatrix} 2/4 & 2/4 & 0/4 \\ 0/3 & 0/3 & 3/3 \\ 1/4 & 1/4 & 2/4 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$$

~~$M_1 =$  6 TO 10~~

$$MLE = \frac{6 \text{ TRANSITIONS}}{11 \text{ OCCASIONS}} = 0.5454$$

$$\text{SUNNY} = \frac{2 \text{ 110 occasions}}{2 \times 2} = 0.5$$

2+2  $\rightarrow$  <sup>4</sup> NO. OF TRANSITIONS FROM SUNNY

$$P_{AIN} = \frac{2}{4} = \frac{2}{4} = 0.5$$

$$2+1+1 \rightarrow \text{NO. OF TRANSITIONS FROM } n=3$$

$$\text{cloudy} = \frac{2}{2} = \frac{2}{3} = \frac{0.3333}{3} = 0$$

THERE IS NO TRANSITION FROM CLOUDY TO CLOUDY

IF THE TOTAL NUMBER OF TRANSITIONS WOULD BE:

$2+2+2+1+1+3 = 11$ , i.e. JUST CHANGE THE DENOMINATOR ABOVE.



## 2. THEORETICAL PROBLEMS

### 1. 12 CONS. DAYS

SUNNY  $\xrightarrow{1}$  SUNNY  $\xrightarrow{1}$  CLOUDY  $\xrightarrow{1}$  RAIN  $\xrightarrow{1}$  RAIN  $\xrightarrow{1}$  SUNNY  $\xrightarrow{2}$  SUNNY  $\xrightarrow{2}$  CLOUDY  $\xrightarrow{2}$  RAIN  
 RAIN  $\xrightarrow{2}$  CLOUDY  $\xrightarrow{2}$  RAIN

SUNNY TO SUNNY = 2 OCCASIONS

SUNNY TO CLOUDY = 2 OCCASIONS

RAIN TO RAIN = 2 OCCASIONS

RAIN TO SUNNY = 1 OCCASION

RAIN TO CLOUDY = 1 OCCASION

CLOUDY TO RAIN = 3 OCCASIONS

3 STATES = SUNNY, CLOUDY, RAIN

$n_{ij}$ : THE NUMBER OF TRANSITIONS FROM  $S_i$  TO  $S_j$  WE HAVE OBSERVED

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=1}^N n_{ij}}$$

$$\sum_{j=1}^N \hat{p}_{ij} = 1$$

MAXIMUM LIKELIHOOD ESTIMATE

TRANSITION PROBABILITIES  $\prod_{i,j=1}^N \hat{p}_{ij}^{n_{ij}} \rightarrow \max \hat{p}_{ij} \geq 0$

$$\ln \prod_{i,j=1}^N \hat{p}_{ij}^{n_{ij}} = \sum_{i,j} n_{ij} \ln \hat{p}_{ij}$$