$$\begin{pmatrix}0.62797\\0.37203\end{pmatrix}\cdot\begin{pmatrix}0.8 & 0.2\\0.3 & 0.7\end{pmatrix}=\begin{pmatrix}0.62797\cdot0.8+0.37203\cdot0.2\\0.62797\cdot0.3+0.37203\cdot0.7\end{pmatrix}$$

$$=\begin{pmatrix}0.502376+0.074406\\0.188391+0.260421\end{pmatrix}=\begin{pmatrix}0.576782\\0.448812\end{pmatrix}$$

$$=(0.5768,\ 0.4489)$$ BY APPROXIMATION.

3. SMOOTHING

ORDER = 3

EQUAL WEIGHTS

TIME SERIES $\underbrace{4,5,2}_{3},5$

WINDOW = 3

OUTLIER.

4, 5, 2, 5

3.6667 4

3.67

$$\frac{4+5+2}{3}=\frac{11}{3}=3.6667$$

$$\frac{5+2+5}{3}=\frac{12}{3}=4$$

WE CAN NOTICE THAT 2 WAS OUR OUTLIER AND IT WAS REMOVED BY SMOOTHING.

$t=3$

$\underline{\alpha_3(1)} = (0.0492 \cdot 0.8 + 0.058 \cdot 0.3) \cdot 0.3 = (0.03936 + 0.0174) \cdot 0.3$
$= 0.017028$

$\underline{\alpha_3(2)} = (0.0492 \cdot 0.2 + 0.058 \cdot 0.7) \cdot 0.2 =$
$= 0.0406 \cdot 0.2 = 0.010088$

---

b) $Pr(\overset{\cancel{S_1, S_2}}{X_2, X_3, X_1}) = \alpha_3(1) + \alpha_3(2) = 0.017028 + 0.010088$
$= 0.027116$ => FULL PROBABILITY

CONDITIONAL.

$Pr(H_3 = S_1 / X_2, X_3, X_1) = \dfrac{\alpha_3(1)}{Pr(X_2, X_3, X_1)} = \dfrac{0.017028}{0.027116}$
$= 0.62797$

$Pr(H_3 = S_2 | X_2, X_3, X_1) = \dfrac{\alpha_3(2)}{Pr(X_2, X_3, X_1)} = \dfrac{0.010088}{0.027116} =$

$= 0.37203$

---

c) Prediction $H_4 | \overset{\cancel{S_1, S_2}}{X_2, X_3, X_1}$

$\cancel{(0.62797 \;\; 0.37203) * \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix}} = 0.8 \cdot 0.62797 + 0.3 \cdot 0.37203$

THIS NEEDS TO BE A COLUMN VECTOR MULTIPLIED BY TRANSITION MATRIX. PROBABILITY AT TIME 3 * TRANSITION MATRIX

2. TRANSITION MATRIX $M = \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix}$

EMISSION MATRIX

$$B = \begin{array}{c} \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \end{array} \begin{array}{l} S_1 \\ S_2 \end{array}$$

INITIAL PROBABILITIES

$$P = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

VISIBLE STATES $= \{x_2, x_3, x_1\}$

a) α_3?

$$\xrightarrow{\text{TIME}}$$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ S_1 & 0.16 & 0.0492 & 0.017028 \\ S_2 & 0.12 & 0.058 & 0.010088 \end{array} \text{ alphas}$$

$$\overbrace{\quad\quad\quad}^{\text{TIME}}$$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ S_1 & 0.16 & 0.0492 & 0.017028 \\ S_2 & 0.12 & 0.058 & 0.010088 \end{array}$$

alphas

$t = 1$

$\alpha_1(S_1) = 0.4 \cdot 0.4 = 0.16$

$\alpha_1(S_2) = 0.6 \cdot 0.2 = 0.12$

$t = 2$

$\cancel{\alpha_2(S_1) = 0.16 \cdot 0.3 \cdot 0.8 = 0.0384}$

$\cancel{\alpha_2(S_1) = 0.12 \cdot 0.3 \cdot 0.3}$

$\alpha_2(S_1) = (0.16 \cdot 0.8 + 0.12 \cdot 0.3) \cdot 0.3 = (0.128 + 0.036) \cdot 0.3 = 0.0492$

$\alpha_2(S_2) = (0.16 \cdot 0.2 + 0.12 \cdot 0.7) \cdot 0.5 = (0.032 + 0.084) \cdot 0.5 = 0.058$

$$MLE = \frac{\#\text{TRANSITIONS FROM } S_i \text{ TO } S_j}{\#\text{ALL TRANSITIONS FROM } S_i}$$

WE GET THE FOLLOWING MATRICES:

TRANSITION COUNTS:

$$\begin{array}{c|ccc|c} & \text{SUNNY} & \text{CLOUDY} & \text{RAIN} & \text{SUM} \\ \hline \text{SUNNY} & 2 & 2 & 0 & 4 \\ \text{CLOUDY} & 0 & 0 & 3 & 3 \\ \text{RAIN} & 1 & 1 & 2 & 4 \end{array}$$

ESTIMATE M^* = NORMALIZE IT

$$M^* = \begin{pmatrix} 2/4 & 2/4 & 0/4 \\ 0/3 & 0/3 & 3/3 \\ 1/4 & 1/4 & 2/4 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$$

$$\cancel{MLE = \frac{6 \text{ TRANSITIONS}}{11 \text{ OCCASSIONS}} = 0.5454}$$

$$\left.\begin{array}{l} \text{SUNNY} = \dfrac{2}{2+2} = \dfrac{2}{4} = 0.5 \\ \quad 2+2 \rightarrow \text{NO. OF TRANSITIONS FROM SUNNY} \\ \text{RAIN} = \dfrac{2}{2+1+1} = \dfrac{2}{4} = 0.5 \\ \quad 2+1+1 \rightarrow \text{NO. OF TRANSITIONS FROM RAIN} \\ \text{CLOUDY} = \cancel{\tfrac{3}{3}} = \cancel{\dfrac{1}{3}} = \cancel{0.3333}\ \dfrac{0}{3} = 0 \end{array}\right\} \text{NOT SURE ABOUT THE NUMBER OF TRANSITIONS}$$

THERE IS NO TRANSITION FROM CLOUDY TO CLOUDY

IF THE TOTAL NUMBER OF TRANSITIONS WOULD BE:

$2+2+2+1+1+3 = 11$, WE JUST CHANGE THE DENOMINATOR ABOVE.

2. THEORETICAL PROBLEMS

1. 12 CONS. DAYS

SUNNY $\xrightarrow{1}$ SUNNY $\xrightarrow{1}$ CLOUDY $\xrightarrow{1}$ RAIN $\xrightarrow{1}$ RAIN $\xrightarrow{1}$ SUNNY $\xrightarrow{2}$ SUNNY $\xrightarrow{2}$ CLOUDY $\xrightarrow{2}$ RAIN $\xrightarrow{2}$
RAIN $\xrightarrow{1}$ CLOUDY $\rightarrow$ RAIN

SUNNY TO SUNNY = 2 OCCASIONS

SUNNY TO CLOUDY = 2 OCCASIONS

RAIN TO RAIN = 2 OCCASIONS

RAIN TO SUNNY = 1 OCCASSION

RAIN TO CLOUDY = 1 OCCASSION

CLOUDY TO RAIN = 3 OCCASSIONS

3 STATES = SUNNY, CLOUDY, RAIN

n_{ij}: THE NUMBER OF TRANSITIONS FROM S_i TO S_j WE HAVE OBSERVED

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=1}^{n} n_{ij}} \qquad \sum_{j=1}^{n} \hat{p}_{ij} = 1$$

MAXIMUM LIKELIHOOD ESTIMATE

TRANSITION PROBABILITIES $\prod_{ij=1}^{n} p_{ij}^{n_{ij}} \to \max \quad p_{ij} \geq 0$

$$\ln \prod_{ij} p_{ij}^{n_{ij}} = \sum_{ij} \ln p_{ij}^{n_{ij}} = \sum_{ij} n_{ij} \ln p_{ij}$$