Connecting Rod

Theory

The connecting rod can be modelled as having a rectangular cross section and as a two-force member carrying axial loads only. The internal axial reaction force N can be found using Equation 1.

<Insert eqn 1>

Axial stress due to N can be found as in Equation 2.

<Insert eqn 2>

The next consideration is stress caused due to the maximum inertial torque possessed by the rod. That can be determined using the mass moment of inertia about the pivot point and maximum angular acceleration of the rod. Maximum angular acceleration occurs when the piston is at top dead center at the start of the expansion stroke. It is given by Equation 3 and the inertial torque can be found using Equation 4.

<Insert eqns 3 and 4>

An assumption that was made was the minimum angular acceleration is zero. Therefore, the minimum inertial torque would be zero as well. The axial stress caused by the inertial torque can be found using Equation 5.

<Insert eqn 5>

The two axial stress can then be combined by simply superimposing them. Following the sign convention, both the forces will cause compressive axial stresses and add up to give a total compressive stress. Since no other forces are taken into consideration, this total stress also becomes the von Mises stress acting in the rod as described by Equation 6.

<Insert eqn 6>

Finally, the yielding factor of safety can be determined using Equation 7.

<Insert eqn 7>

In parallel to failure by yielding, buckling failure must also be considered. An end condition value of C equal to 1 was determined to be most appropriate for this scenario. The length condition can be evaluated as given in Equation 8.

<Insert eqn 8>

Based on the length condition, the critical buckling force for either case can be found using Equation 9 and 10.

<Insert eqns 9 and 10>

The buckling factor of safety can then be calculated as the ratio of the critical force and force F1 as given in Equation 11.

<Insert eqn 11>

Yet another consideration for the rod design is failure due to fatigue. Two approaches are used – linear elastic fracture mechanics (LEFM) and the stress life method. The former yields how many cycles the part can take before an initial crack reached critical crack size while the latter tells the factor safety associated with infinite fatigue life for the part.

As for LEFM, the maximum crack size can be found using Equation 12 where the stress intensity modification factor *Beta* is chosen to be 1 as a conservative estimate. Furthermore, the part can be modelled as a flat plate under axial loading with a surface crack.

<Insert eqn 12>

An initial crack size of 1 mm is used. The number of cycles to failure can be found using Paris law as given in Equation 13.

<Insert eqn 13>

As for the stress-life method, the Soderberg criteria was used to remain conservative. No stress concentrations are taken into account as the rod is a straight uniform part. Therefore, the theoretical and fatigue stress concentration factors will be set to be unity. The axial stresses due to force F1 and inertial torque in the rod cycle between zero and their maximum values as defined in Equations 2 and 5 respectively. Therefore, the fatigue stresses can be calculated as given in Equations 14 and 15.

<Insert eqns 14 and 15>

Equations 16 through 18 define the surface modification factors and fully corrected endurance limit.

<Insert eqns 16, 17 and 18>

Finally, the fatigue factor of safety can be calculated using Equation 19.

<Insert eqn 19>

Results

An Excel sheet was coded for simultaneously calculating the yielding, buckling and fatigue factors of safety as well predicting LEFM fatigue life given the connecting rod's dimensions and material properties. The resulting sheet can be seen in Figure 1 depicting values for the chosen material and dimensions.

<Insert Excel sheet page>

A summary of the chosen material, rod dimensions and various factors of safety is given in Table 1.

Material	Cross sectional Area	Yielding FoS	Buckling FoS	LEFM Fatigue Life	Infinite Fatigue Life FoS
AISI 4130 Steel	40 mm x 40 mm	5.59	5.52	3×10^{15} cycles	5.24

This material was chosen over other options such as T6 6061 Aluminium because of its ability to provide infinite fatigue life and while being cheaper than titanium.