Crank Shaft Calculations

- l_c is the length of each crank.
- l_s is the length of each shaft section between each crank.
- t_b is the thickness of shaft crank connectors.
- l_a is the length of each crank & shaft subsection. $l_a = l_s + 2t + l_c$
- l_b is the length of the shaft between each bearing location and closest crank connector.
- l_s is the length of the shaft between the two bearings.

The crankshaft consists of six separate pistons acting on the shaft. Since each piston undergoes 4 strokes, a total cycle of 4π is needed. Thus each piston stroke is offset by 120 degrees. Piston n is the nth piston from the bearing next to the flywheel. Each force on the crank shaft is a vector equation given below.

$$\vec{F}_n = f(\theta + 120°(n-1)) \tag{1}$$

The direction of each crank shaft is offset by 120 degrees too, so the piston cycles are offset by 120 degrees. The radius vector of each crank is given below.

$$\vec{r}_n = r_c\Big\langle \cos(\theta + 120°(n-1))\ \hat{\imath},\ \sin(\theta + 120°(n-1))\ \hat{\jmath}\Big\rangle \tag{2}$$

For the shaft coordinates, the $\hat{k}$ vector is parallel to the axis of the shaft, from the first bearing to the second. The torque equation along the shaft is simply written out using cross products.

$$\vec{T}(x) = \sum_{n=1}^{6} \langle x - l_a(n-1) - l_b - \frac{l_c}{2} - t\rangle^0\ \vec{F}_n \times \vec{r}_n \tag{3}$$

There are two reaction forces at each of the bearings, $\vec{R}_1$ and $\vec{R}_2$. The force and moment equations are written out to solve for these.

$$\sum \vec{F} = \vec{R}_1 + \vec{R}_2 + \sum_{n=1}^{6} \vec{F}_n = 0 \tag{4}$$

$$\sum \vec{M} = \vec{R}_2 \times l_s\hat{\jmath} + \sum_{n=1}^{6} \vec{F}_n \times (x - l_a(n-1) - l_b - \frac{l_c}{2} - t)\ \hat{\jmath} = 0 \tag{5}$$

$$\vec{R}_2 = -\sum_{n=1}^{6} \frac{\vec{F}_n(x - l_a(n-1) - l_b - \frac{l_c}{2} - t)}{l_s} \tag{6}$$

$$\vec{R}_1 = \sum_{n=1}^{6} \frac{\vec{F}_n(x - l_a(n-1) - l_b - \frac{l_c}{2} - t)}{l_s} - \sum_{n=1}^{6} \vec{F}_n \tag{7}$$

The shear and moment equations are derived below. The shear and moment is not affected by the crank offset, since there are no axial forces.

$$\vec{V}(x) = \vec{R}_1\langle x\rangle^0 + \sum_{n=1}^{6} \vec{F}_n \ \langle x - l_a(n-1) - l_b - \frac{l_c}{2} - t\rangle^0 \tag{8}$$

$$\vec{M}(x) = \vec{R}_1\langle x\rangle + \sum_{n=1}^{6} \vec{F}_n \ \langle x - l_a(n-1) - l_b - \frac{l_c}{2} - t\rangle \tag{9}$$

$$\tag{10}$$

The maximum moment and shear stresses are given. The shear stress equation only applies for the on axis shaft.

$$\sigma_M(x) = \pm\frac{|\vec{M}(x)|d(x)}{2I(x)} \tag{11}$$

$$\tau(x) = \frac{|T(x)|d(x)}{2J(x)} \tag{12}$$