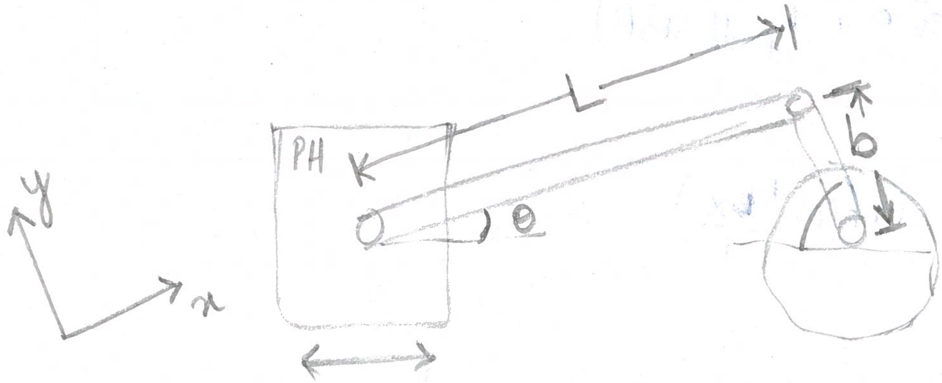
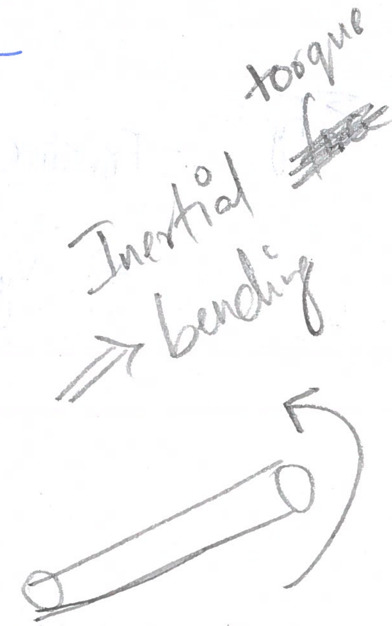
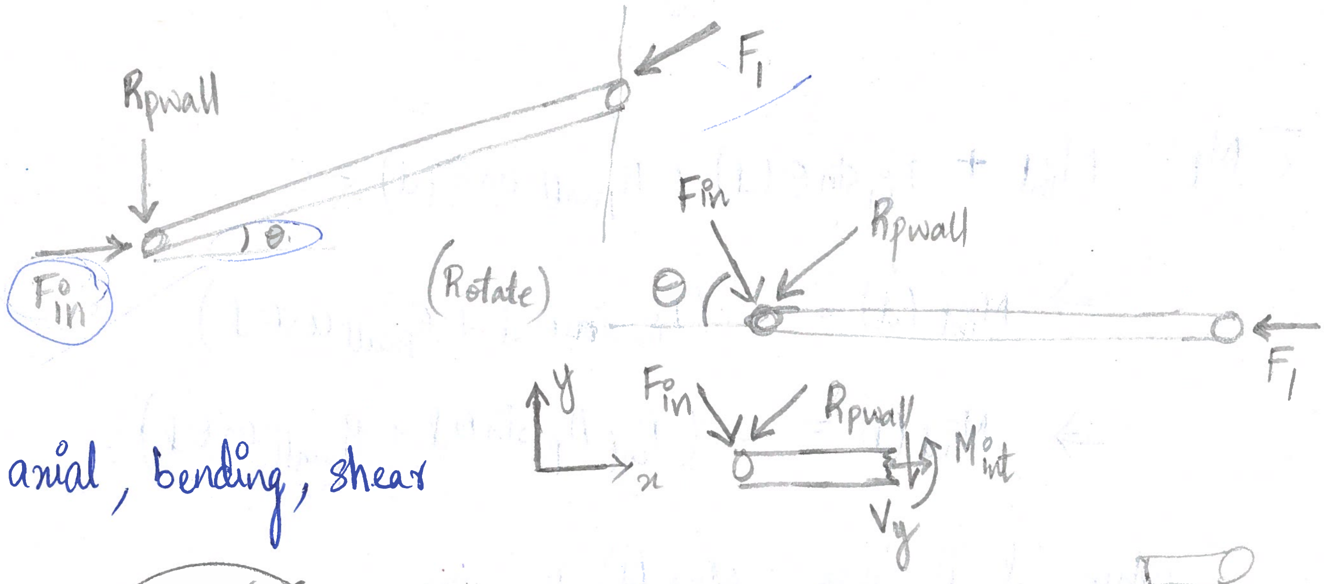
# MEEN 368 - GDP - Stress analysis - Connecting rod




**Loading:** axial, bending, shear

$$\sigma_{axial,1} = F_{net,ax} \cdot \cancel{A_c}$$

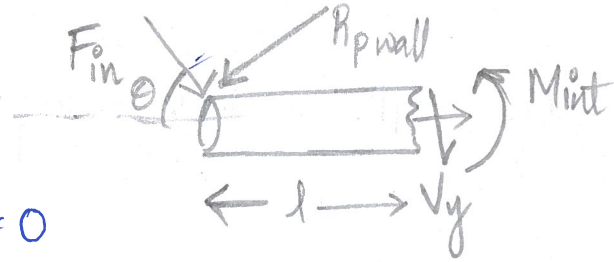
$$\boxed{\text{Incorrect.}}$$

$$= \left(-F_1 + F_{in}\cos\theta - R_{pwall}\sin\theta\right) A_c$$

$$\frac{F_1 \delta}{A}$$

$$= \left[\frac{-F_{in}}{\frac{b}{a}\sin\left(\cos^{-1}\left(\frac{x}{b}\right)\right)} + F_{in}\cos\theta - R_{pwall}\sin\theta\right] A_c$$

$$\Rightarrow \sigma_{axial,1} = \left[P_{cyl} A_p \left(\cos\theta - \frac{1}{\frac{b}{a}\sin\left(\cos^{-1}\frac{x}{b}\right)}\right) - R_{pwall}\sin\theta\right] A_c$$




$\Sigma F_y$ : $$-F_{in}\sin\theta - R_{pwall}\cos\theta - V_y = 0$$

$$\Rightarrow V_y = -(F_{in}\sin\theta + R_{pwall}\cos\theta)$$

$$\cancel{=} \quad \cancel{\frac{b}{a}\sin\left(\cos^{-1}\frac{bx}{b}\right)}$$

$$\Rightarrow V_y = -(P_{cyl}A_p\sin\theta + R_{pwall}\cos\theta)$$

$\Sigma M_l$ : $$M_{int} + F_{in}\sin\theta(l) + R_{pwall}\cos\theta(l) = 0$$

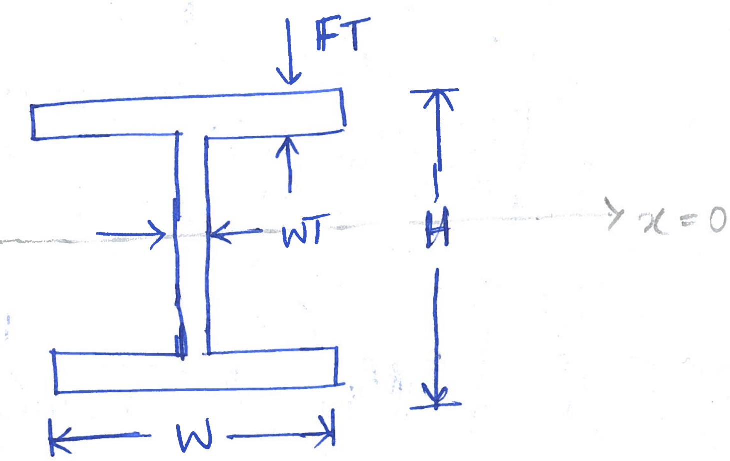
$$\Rightarrow M_{int}(l) = -(F_{in}\sin\theta\, l + R_{pwall}\cos\theta\, l)$$

$$\Rightarrow M_{int}(l) = -(P_{cyl}A_p\sin\theta\, l + R_{pwall}\cos\theta\, l)$$

when $l$ is max, $M_{int}(l)$ is max.

$$M_{int,max} = M_{int}(L) = -L(P_{cyl}A_p\sin\theta + R_{pwall}\cos\theta)$$

Connecting rod $\Rightarrow$ I-beam

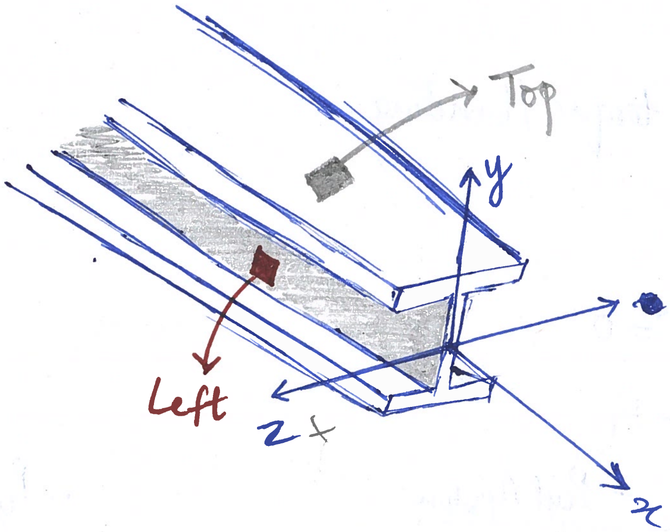



Area moment of inertia :

$$I = \frac{1}{12}\left[WH^3 - W(H-2FT)^3 + WT(H-2FT)^3\right]$$

$$A_c = 2(W \times FT) + WT(H-2FT)$$

Ex 3-6




Top :

$$\sigma_x = \sigma_{axial,1} - \frac{M_{int}(H/2)}{I}$$

$$\Rightarrow \sigma_x = \left[ P_{cyl} A_p \left( \cos\theta - \frac{1}{\frac{b}{a}\sin\left(\cos^{-1}\frac{x}{b}\right)} \right) - R_{pwall}\sin\theta \right] \Big/ A_c$$

$A_c \rightarrow 2(W \cdot FT) + WT(H - 2 \cdot FT)$

$$+ \frac{L\left(P_{cyl} A_p \sin\theta + R_{pwall}\cos\theta\right)(H/2)}{1/12\left[WH^3 - W(H - 2FT)^3 + WT(H - 2FT)^3\right]}$$

$\tau_{yx} = 0$

$\sigma' = \sigma_x$

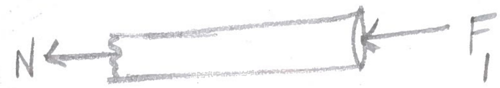
Bottom :

$$\sigma_x = \sigma_{axial,1} + \frac{M_{int}(H/2)}{I} = \sigma'$$

③

Loading: axial and torque/bending.

Internal normal force:




$$\Sigma F_x: \ -N - F_1 = 0$$

$$\Rightarrow N = -F_1$$

$$\Rightarrow \boxed{N = \frac{-P_{cyl} A_{piston}}{\frac{b}{L} \sin\left(\cos^{-1} \frac{x}{b}\right)}}$$

$P_{cyl_{max}} \rightarrow N_{max}, \sigma_{x_{max}}$

$P_{cyl_{min}} \rightarrow N_{min}, \sigma_{x_{min}}$

Axial stress due to N:

$$\boxed{\sigma_{x_N} = \frac{N}{A_c} = \frac{-P_{cyl} A_{piston}}{A_c \left(\frac{b}{L}\right) \sin\left(\cos^{-1} \frac{x}{b}\right)}}$$

Torque/bending due to inertial forces:

**Assumption:** Ignoring rotational friction at the pivot point.

$$I_{\underset{(mass)}{pivot}} \ddot{\theta} = M_{bending}$$

$$\Rightarrow M_{bending} = \left(\frac{1}{3} m L^2\right) \ddot{\theta}$$

Need θ wrt time

$$\theta = \sin^{-1}\left[\frac{b}{L} \sin\left(\cos^{-1} \frac{x}{b}\right)\right]$$

$\ddot{\theta} =$

$\ddot{\theta}$ is max when $x = 0$ (TDC) at the start of the expansion stroke

$$\ddot{\theta}(x=0) = \frac{\cancel{bL}\,(\cancel{b^2}L^2 - b^{\cancel{4}\,2})}{\cancel{Lb}\,\cancel{(L^2-b^2)}\,\cancel{(b^2)}\,b \cdot \sqrt{L^2-b^2}}$$

$$= \frac{1}{b\sqrt{L^2-b^2}}$$

$$\Rightarrow M_{bending,max} = I_{mass} * \ddot{\theta}_{max} = I_{mass} \times \frac{1}{b\sqrt{L^2-b^2}}$$

$\ddot{\theta}$ minimum $= 0$ (ASSUMPTION)

$$\Rightarrow M_{bending,min} = 0$$

Axial stress due to bending moment :

$$\sigma_{x\,bend} = \pm \frac{M_{bend}\, y}{I_{area}} = \pm \frac{M_{bend} (H/2)}{I_{area}}$$

$\sigma_{x\,bend}$

Combining stresses :

$$\sigma_{x} = \sigma_{xN} \pm \sigma_{x\,bend}$$

$$\sigma_{max} = \sigma_{xN_{max}} - \sigma_{x\,bend,max}$$

$$\sigma_{x\,min} = \sigma_{xN_{min}}$$

$$= \frac{-P_{cyl} A_{piston}}{A_c (b/L) \sin(\cos^{-1} x/b)} \pm \frac{M_{bend} (H/2)}{I_{area}} = \sigma'$$

Factor of safety : $n = \dfrac{S_y}{\sigma'}$

Alternative failure mechanism: buckling

End condition, $c = 1$ (Fig. 4-18)

Case 1: $L \geq \sqrt{\dfrac{2\pi^2 EI}{S_y A_c}}$

$$P_{cr_1} = \frac{\pi^2 EI}{L^2}$$

$$n = \frac{P_{cr_1}}{F_1}$$

Case 2: $L \leq \sqrt{\dfrac{2\pi^2 EI}{S_y A_c}}$

$$P_{cr_2} = S_y A_c - \left(\frac{S_y L}{2\pi}\right)^2 \frac{A_c^2}{EI}$$

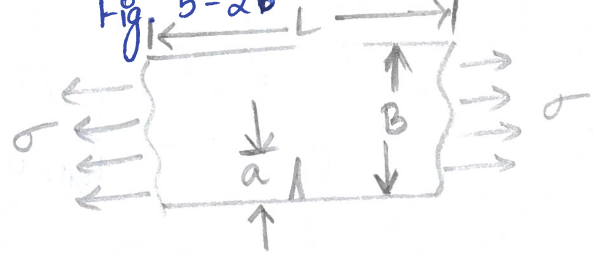
$$n = \frac{P_{cr_2}}{F_1}$$

$$\Rightarrow n = P_{cr} \times \frac{b/2 \sin\left(\cos^{-1} x/b\right)}{P_{cyl} A_{piston}}$$

## Fatigue analysis

### 1. LEFM

Fig. 5-26




Max crack size: $a_f = \dfrac{1}{\pi}\left(\dfrac{K_{IC}}{\beta \sigma}\right)^2$

$$\sigma = \sigma_{x_{max}} = \sigma'_{max}$$

$$\beta\left(\frac{a}{B}, \frac{L/2}{B}\right)$$

No. of cycles to failure with given crack $a_0$:
(Paris law)

$$N_f = \frac{1}{C(\Delta\sigma \beta \sqrt{\pi})^m} \left[ \frac{a^{1-m/2}}{1-m/2} \right]_{a_0}^{a_f}$$

$$\Delta\sigma = \sigma_{x_{max}} - \sigma_{x_{min}}$$

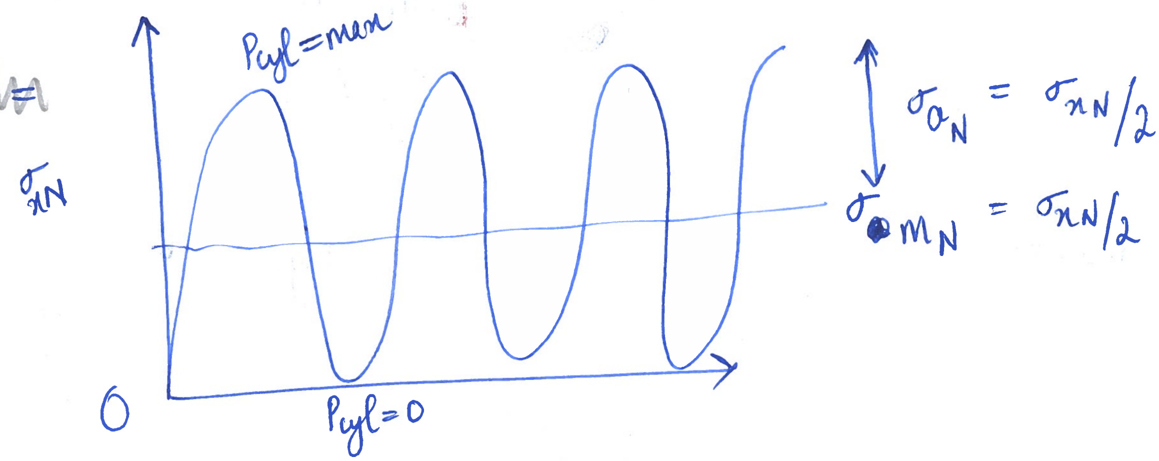
## 2. Stress - Life method

No stress concentrations along the beam.

$$k_t = k_f = 1.$$

$\cancel{\sigma_m \neq 0}$, $\cancel{\sigma_a = \sigma_{x_{max}}}$

$\cancel{\sigma_a =}$



$$\sigma_a' = \sqrt{\left(\sigma_{a_{bend}} + \frac{\sigma_{aN}}{0.85}\right)^2} = \sigma_{a_{bend}} + \frac{\sigma_{aN}}{0.85}$$

$$\sigma_m' = \sigma_{m_{bend}} + \sigma_{mN}$$

$$k_a = \boxed{\phantom{0}}\, S_{ut}^{\boxed{\phantom{0}}} \quad \text{(surface finish)}$$

$$k_b = \min\left(0.88\left(\underset{0.808\sqrt{BH}}{\cancel{0.370d}}\right)^{-0.107},\ 0.91\left(\underset{0.808\sqrt{BH}}{\cancel{0.370d}}\right)^{-0.157}\right)$$

$$S_e = k_a k_b S_e'$$

$$= k_a k_b \min\left(\frac{S_{ut}}{2}\text{ kpsi},\ 100\text{ kpsi}\right)$$

Soderberg: $n = \left(\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_y}\right)^{-1}$

$n > 1$

~~If $n < 1$~~, Design for infinite life.