Piston Calculations

Gasket Forces

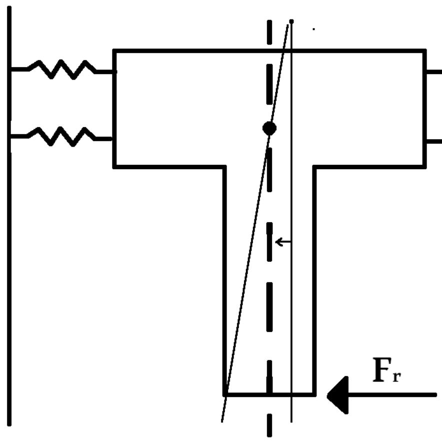

Figure 1: Ideal Gasket Forces

The gaskets exhibit no vertical forces on the wall. Only the horizontal force from the connected rod is considered. The gaskets are simply modeled as springs connecting the piston to either side of the wall. Since the gaskets must maintain connection to the piston chamber wall, small deflections and angles are assumed. Thus small angle approximations are made.

- θ is the angle the piston is rotated with respect to the vertical.
- x is the displacement of the piston.
- z_{G_1} is the vertical distance from the top of the piston head to the center of the top gasket.
- z_{G_2} is the vertical distance from the top of the piston head to the center of the bottom gasket.
- z_{CG} is the vertical distance from the top of the piston head to the center of mass.
- h_h is the height of the piston head.
- d_h is the diameter of the piston head.
- h_t is the height of the piston tail.

- d_t is the diameter of the piston tail.
- d_{go} and d_{gi} is the inner and outer diameters of the gaskets.
- h_g is the gasket height.
- Δt is the thickness of the gaskets, $\frac{d_{go}-d_{gi}}{2}$

First, the center of mass of the piston is calculated

$$z_{CG} = \frac{\frac{h_h}{2} d_h^2 h_h + (h_h + \frac{h_t}{2}) d_t^2 h_t}{d_h^2 h_h + d_t^2 h_t} \tag{1}$$

The spring equation for the forces are written out.

$$\begin{aligned} F_{G_{11}} &= k(x - (z_{CG} - z_{G_1})\theta) && (2)\\ F_{G_{12}} &= k(-x + (z_{CG} - z_{G_1})\theta) && (3)\\ F_{G_{21}} &= k(x - (z_{CG} - z_{G_2})\theta) && (4)\\ F_{G_{22}} &= k(-x + (z_{CG} - z_{G_2})\theta) && (5) \end{aligned}$$

The moment and force equations are written out.

$$\begin{aligned} F_R &= -F_{G_{11}} + F_{G_{12}} - F_{G_{21}} + F_{G_{22}} && (6)\\ 0 &= F_R(h_h + h_t - z_{CG}) + (F_{G_{12}} - F_{G_{11}})(z_{G_1} - z_{CG}) + (F_{G_{22}} - F_{G_{21}})(z_{G_2} - z_{CG}) && (7) \end{aligned}$$

Finally, x and theta are solved for.

$$\begin{aligned} \alpha &= 2h_h z_{CG} - h_h z_{G_1} - h_h z_{G_2} + 2h_t z_{CG} - h_t z_{G_1} - h_t z_{G_2} && (8)\\ \beta &= -4z_{CG}^2 + 3z_{CG} z_{G_1} + 3z_{CG} z_{G_2} - z_{G_1}^2 - z_{G_2}^2 && (9)\\ x &= \frac{F_R(\alpha + \beta)}{2k\left(z_{G_1}^2 - 2z_{G_1} z_{G_2} + z_{G_2}^2\right)} && (10)\\ \theta &= \frac{F_R(2h_h + 2h_t - 4z_{CG} + z_{G_1} + z_{G_2})}{2k\left(z_{G_1}^2 - 2z_{G_1} z_{G_2} + z_{G_2}^2\right)} && (11) \end{aligned}$$

The spring constant of the gaskets is derived in terms of material and geometric properties. To simplify calculation, the gasket will be treated as a flat area cross section.

$$\begin{aligned} \epsilon &= \frac{\sigma}{E} && (12)\\ \Delta t &= t\epsilon = \frac{Pt}{A_C E} && (13)\\ A_C &= d_{go} h_g && (14)\\ k &= \frac{P}{\Delta t} = \frac{d_{go} h_g E}{\Delta t} && (15) \end{aligned}$$

Gasket Stresses and Pressures

The average pressure difference from the reaction forces is given below:

$$\Delta p_{G_X \text{avg}} = \frac{F_{G_X}}{d_g o h_g} \tag{16}$$

The minimum pressure which the gaskets must be fitted to is determined by the maximum pressure of the air p_{AM} and the minimum change on pressure from the piston forces.

$$p_{\text{fit}} \geq p_{AM} - \frac{\min\{F_{G_X}, 0\}}{d_{go}h_g} \tag{17}$$

The radial gasket stress is simply the pressure on the gasket. Since the gasket is thin walled, a constant radial stress is assumed. Since the gaskets are supported by the piston head, the tangential stress is derived using a thick walled pressure vessel equation, with a solid pressure vessel. Since the inner radius is 0, the resulting stresses are identical.

$$\sigma_{gr} = -p \tag{18}$$

$$\sigma_{gt} = -p \tag{19}$$

The pressures also attempts to push out the gasket. The push out stress is derived below. The shear force is given pressure acting on the area of the gasket directly exposed to the chamber.

$$V = \frac{\pi}{4}(d_{go}^2 - d_h^2)p \tag{20}$$

$$\tau_{\text{max}} = \frac{3V}{2A} \tag{21}$$

$$\tau_{gp} = \frac{3V}{2h_g\pi\frac{d_h}{2}} = \frac{3\pi(d_{go}^2 - d_h^2)p}{4h_g\pi d_h} \tag{22}$$

Finally, friction creates a shear force on the gasket. Since the gasket reactions are equal and opposite, $p_{\text{avg}} = p_{\text{fit}}$.

$$V = \mu p_{\text{avg}}A_o \tag{23}$$

$$\tau_{\text{max}} = \frac{3V}{2A_h} \tag{24}$$

$$\tau_f = \frac{3\mu p_{\text{fit}}d_{go}^2}{d_h^2} \tag{25}$$

The minimum and maximum of each gasket stress is tabulated below for fatigue analysis.

$$\sigma_{gr\ \text{min}} = \sigma_{gt\ \text{min}} = p_{\text{fit}} + \frac{\min\{F_{G_X}, 0\}}{d_{go}h_g} \tag{26}$$

$$\sigma_{gr\ \text{max}} = \sigma_{gt\ \text{max}} = p_{\text{fit}} + \frac{\max\{F_{G_X}\}}{d_{go}h_g} \tag{27}$$

$$\tau_{g\ \text{max}} = \frac{3\pi(d_{go}^2 - d_h^2)p_{AM}}{4h_g\pi d_h} - \frac{3\mu p_{\text{fit}}d_{go}^2}{d_h^2} \tag{28}$$

$$\tau_{g\ \text{min}} = -\frac{3\mu p_{\text{fit}}d_{go}^2}{d_h^2} \tag{29}$$

Axial Piston Stress Calculations

The axial force on the head is determined by the pressure. The resultant stress from the from the pressure in the piston major diameter is given below.

$$\sigma_{z\ \text{Head}} = -p \tag{30}$$

In the gasket channels, a greater compression stress results from the smaller diameter and push out forces on the gasket. Stress concentration factors K_1 and K_2 are added due to the geometry.

$$\sigma_{z\ \text{Pressure}} = -K_1 p \frac{d_h^2}{d_{gi}^2} \tag{31}$$

$$\sigma_{z\ \text{Push Out}} = -K_2 p \frac{d_{go}^2 - d_h^2}{d_h^2 - d_{gi}^2} \tag{32}$$

$$\sigma_{z\ \text{Channel}} = -K_1 p \frac{d_h^2}{d_{gi}^2} - K_2 p \frac{d_{go}^2 - d_h^2}{d_h^2 - d_{gi}^2} \tag{33}$$

The tail of the piston also has a concentration factor due to the diameter change, represented by K_3. The tail also experiences a stress concentration at the joint, which has a hole of diameter d_j and concentration factor K_4.

$$\sigma_{z\ \text{Tail}} = -K_3 p \frac{d_h^2}{d_t^2} \tag{34}$$

$$\sigma_{z\ \text{Joint}} = -K_4 p \frac{d_h^2}{d_j^2} \tag{35}$$

Finally, a push out shear is present due to the joint. An upper limit is given by distributing the shear force among the smallest cross section.

$$V = \frac{F_R}{2} \tag{36}$$

$$\tau \leq \frac{3F_R}{4A_c} \tag{37}$$

The vertical deflection is estimated using $\delta = L\frac{\sigma}{E}$.

$$\delta_{\text{Head}} = -\frac{l_h p}{E} \tag{38}$$

$$\delta_{\text{Tail}} = -\frac{l_t p \frac{d_h^2}{d_t^2}}{E} \tag{39}$$

$$\delta_{\text{Total}} = \frac{l_h p - l_t p \frac{d_h^2}{d_t^2}}{E} \tag{40}$$