Thermodynamics Scratch Sheet

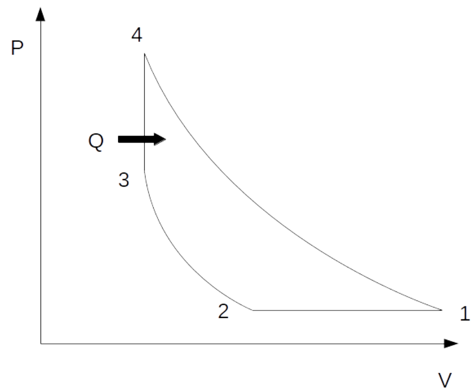

Figure 1: PV Diagram of Modern Atkinson Cycle

The engine has 6 cylinders and 3000 c.c. total swept volume. The fuel used will be standard octane gasoline. Intake is taken from the ambient atmosphere. A modern atkinson cycle is chosen in-lieu of the traditional Otto cycle for it's increased ideal efficiency and better mixing properties.
These parameters completely define our thermal cycle. Since the proportion of gasoline is small, and gasoline composed of nonpolar substances, an Ideal Gas model is assumed.

Initial Constraints

Our compression ratio is taken to be 10, a high compression ratio that prevents knocking.
Likewise, a standard air to fuel mixture ratio of 14.7 is chosen.

$$C_R = \frac{V_2}{V_3} \tag{1}$$

$$M_R = \frac{m_A}{m_G} = 14.7 \tag{2}$$

The volumes are constrained by the total c.c. of the engine. Likewise, it is noted process 3-4 is isochoric.

$$V_4 = V_3 \tag{3}$$

$$\frac{3000 \text{ c.c.}}{6} = V_1 - V_4 \tag{4}$$

Finally, point 1 and point 2 are at ambient pressure p_{amb} and temperature T_{amb}.

Process 2 to 3

Using the isentropic equations, p_3 is solved for.

$$\frac{p_3}{p_2} = C_R^{\gamma} \tag{5}$$

$$p_3 = p_{amb} C_R^{\gamma} \tag{6}$$

Process 3 to 4

The total volume V_2 determines the total energy input. By setting up volume mass relations and using the mix ratio, the total mass of the gasoline can be found. This leaves a V_2 in the unknowns.

$$V_G = m_g R_g \frac{T_2}{p_2} = m_g R_g \frac{T_{amb}}{p_{amb}} \tag{7}$$

$$V_A = m_a R_a \frac{T_{amb}}{p_{amb}} \tag{8}$$

$$V_2 = V_G + V_A = m_G \frac{T_{amb}}{p_{amb}} (R_G + M_R R_A) \tag{9}$$

$$m_G = \frac{V_2 p_{amb}}{T_{amb}(R_G + M_R R_A)} \tag{10}$$

$$Q = m_G H_G \tag{11}$$

Next, using energy conservation, an equation is set up to solve for p_4. Since this process is isochoric, W = 0.

$$\frac{3}{2} p_4 V_4 = \frac{3}{2} p_3 V_4 + Q \tag{12}$$

Process 4 to 1

Finally, isentropic relations are used to create one final equation.

$$\frac{p_4}{p_{amb}} = \left(\frac{V_1}{V_4}\right)^{\gamma} \tag{13}$$

Solving for Unknowns

Currently there are 5 unknowns: V_1 V_2 V_3 V_4 p_4
The 5 equations used to solve them are listed below:

$$\frac{3000 \text{ c.c.}}{6} = V_S = V_1 - V_4 \tag{14}$$

$$C_R = \frac{V_2}{V_3} \tag{15}$$

$$Q = m_G H_G = \frac{H_G p_{amb} V_2}{T_{amb}(R_G + M_R R_A)} \tag{16}$$

$$\frac{3}{2} p_4 V_4 = \frac{3}{2} p_3 V_4 + Q \tag{17}$$

$$\frac{p_4}{p_{amb}} = \left(\frac{V_1}{V_4}\right)^{\gamma} \tag{18}$$

Plugging in interim constants and replacing V_3 with V_4.

$$V_S = V_1 - V_4 \tag{19}$$
$$V_2 = C_R V_4 \tag{20}$$
$$Q = C_1 V_2 \tag{21}$$
$$\frac{3}{2} p_4 V_4 = \frac{3}{2} p_3 V_4 + C_1 V_2 \tag{22}$$
$$\frac{p_4}{p_{amb}} = \left(\frac{V_1}{V_4}\right)^\gamma \tag{23}$$

Substituting in $V_2 = C_R V_4$ and $p_3 = p_{amb} C_R^\gamma$ to get:

$$\frac{3}{2} p_4 V_4 = \frac{3}{2} p_{amb} C_R^\gamma V_4 + C_1 C_R V_4 \tag{24}$$
$$p_4 = p_{amb} C_R^\gamma + \frac{2}{3} C_1 C_R \tag{25}$$

Substituting in $V_1 = V_S + V_4$:

$$\left(\frac{p_4}{p_{amb}}\right)^{\frac{1}{\gamma}} = \frac{V_S + V_4}{V_4} \tag{26}$$
$$V_4 \left(\left(\frac{p_4}{p_{amb}}\right)^{\frac{1}{\gamma}} - 1 \right) = V_S \tag{27}$$
$$V_4 = \frac{V_S}{\left(\left(\frac{p_4}{p_{amb}}\right)^{\frac{1}{\gamma}} - 1 \right)} \tag{28}$$

Finally from V_4 V_2 and V_1 are derived:

$$V_2 = C_R V_4 \tag{29}$$
$$V_1 = V_S + V_4 \tag{30}$$

0.1 Other properties and power calculation

First the temperatures and molar amount is found. Then the power is derived from these:

$$n = \frac{p_{amb} V_2}{R T_{amb}} \tag{31}$$
$$T_4 = \frac{p_4 V_4}{nR} \tag{32}$$
$$T_3 = \frac{p_3 V_3}{nR} \tag{33}$$
$$W_{cyc} = \frac{3}{2} nR(T_4 - T_{amb}) - p_{amb}(V_1 - V_2) - \frac{3}{2} nR(T_3 - T_{amb}) \tag{34}$$
$$P = 6 W_{cyc} \frac{\omega}{2} \tag{35}$$