

Gasket Forces

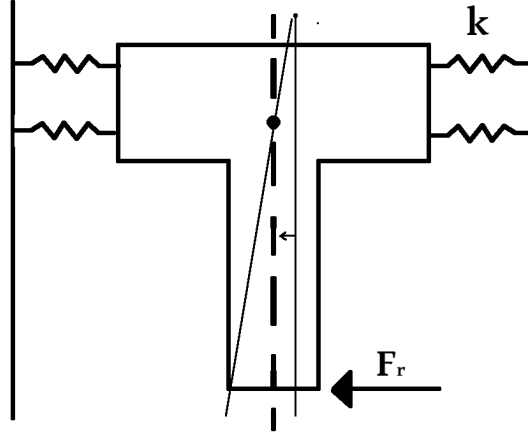


Figure 1: Ideal Gasket Forces

The gaskets exhibit no vertical forces on the wall. Only the horizontal force from the connected rod is considered. The gaskets are simply modeled as springs connecting the piston to either side of the wall. Since the gaskets must maintain connection to the piston chamber wall, small deflections and angles are assumed. Thus small angle approximations are made.

- θ is the angle the piston is rotated with respect to the vertical.
- x is the displacement of the piston.
- z_{G_1} is the vertical distance from the top of the piston head to the center of the top gasket.
- z_{G_2} is the vertical distance from the top of the piston head to the center of the bottom gasket.
- z_{CG} is the vertical distance from the top of the piston head to the center of mass.
- h_h is the height of the piston head.
- d_h is the diameter of the piston head.
- h_t is the height of the piston tail.
- d_t is the diameter of the piston tail.

- d_{go} and d_{gi} is the inner and outer diameters of the gaskets.
- h_g is the gasket height.
- t is the thickness of the gaskets, $\frac{d_{go}-d_{gi}}{2}$

First, the center of mass of the piston is calculated

$$z_{CG} = \frac{\frac{h_h}{2}d_h^2h_h + (h_h + \frac{h_t}{2})d_t^2h_t}{d_h^2h_h + d_t^2h_t} \quad (1)$$

The spring equation for the forces are written out.

$$F_{G_{11}} = k(x - (z_{CG} - z_{G_1})\theta) \quad (2)$$

$$F_{G_{12}} = k(-x + (z_{CG} - z_{G_1})\theta) \quad (3)$$

$$F_{G_{21}} = k(x - (z_{CG} - z_{G_2})\theta) \quad (4)$$

$$F_{G_{22}} = k(-x + (z_{CG} - z_{G_2})\theta) \quad (5)$$

The moment and force equations are written out.

$$F_R = -F_{G_{11}} + F_{G_{12}} - F_{G_{21}} + F_{G_{22}} \quad (6)$$

$$0 = F_R(h_h + h_t - z_{CG}) + (F_{G_{12}} - F_{G_{11}})(z_{G_1} - z_{CG}) + (F_{G_{22}} - F_{G_{21}})(z_{G_2} - z_{CG}) \quad (7)$$

Finally, x and θ are solved for.

$$\alpha = 2h_h z_{CG} - h_h z_{G_1} - h_h z_{G_2} + 2h_t z_{CG} - h_t z_{G_1} - h_t z_{G_2} \quad (8)$$

$$\beta = -4z_{CG}^2 + 3z_{CG}z_{G_1} + 3z_{CG}z_{G_2} - z_{G_1}^2 - z_{G_2}^2 \quad (9)$$

$$x = \frac{F_R(\alpha + \beta)}{2k(z_{G_1}^2 - 2z_{G_1}z_{G_2} + z_{G_2}^2)} \quad (10)$$

$$\theta = \frac{F_R(2h_h + 2h_t - 4z_{CG} + z_{G_1} + z_{G_2})}{2k(z_{G_1}^2 - 2z_{G_1}z_{G_2} + z_{G_2}^2)} \quad (11)$$

The spring constant of the gaskets is derived in terms of material and geometric properties. To simplify calculation, the gasket will be treated as a flat area cross section.

$$\epsilon = \frac{\sigma}{E} \quad (12)$$

$$\Delta t = t\epsilon = \frac{Pt}{A_C E} \quad (13)$$

$$A_C = d_{go}h_g \quad (14)$$

$$k = \frac{P}{\Delta t} = \frac{d_{og}h_g E}{t} \quad (15)$$