

MEEN 368 DESIGN REPORT: PISTON-CRANKSHAFT ASSEMBLY

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MEEN 368

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Thermodynamics: Standard Air Analysis

The engine has 6 cylinders and 3000 c.c. total swept volume. The fuel used will be standard octane gasoline. Intake is taken from the ambient atmosphere. A Modern Atkinson cycle is chosen in-lieu of the traditional Otto cycle. The Modern Atkinson Cycle is more thermodynamically efficient than the Otto Cycle - reducing fuel and pollution.

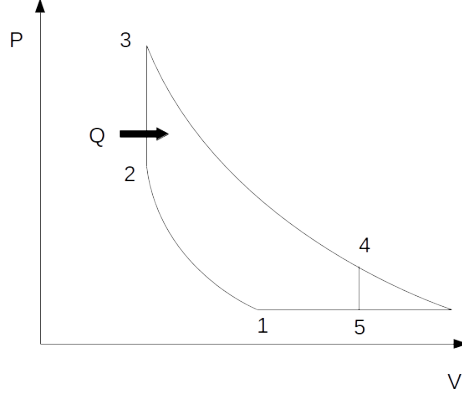


Figure 1: PV Diagram of Modern Atkinson Cycle

Our compression ratio is taken to be 10, a high compression ratio that prevents knocking. A lean air to fuel mixture ratio of 29.4 is chosen to ensure complete combustion and limit the pressures. These parameters completely define our thermal cycle.

Since the proportion of gasoline is small, and gasoline composed of nonpolar substances, an air standard model is used. The air is not assumed cold - the cold assumption results in far too significant of errors during the combustion process. Gas Tables are used to get air properties.

Initial Constraints and Constants

The compression ratio and air-to-fuel ratio are listed below.

$$C_R = \frac{V_1}{V_2} = 10 \quad (1)$$

$$M_R = \frac{m_A}{m_G} = 29.4 \quad (2)$$

The total volume swept by the piston is constrained by the total c.c. of the engine.

$$\frac{3000 \text{ c.c.}}{6} = V_4 - V_2 \quad (3)$$

Point 1 and 5 are at ambient pressure p_{amb} , temperature T_{amb} , and density ρ_{amb} .

$$p_{amb} = 101325 \text{ Pa} \quad (4)$$

$$T_{amb} = 288 \text{ K} \quad (5)$$

$$\rho_{amb} = 1.225 \text{ kg m}^{-3} \quad (6)$$

The lower value of the heat of combustion of gasoline referenced from the U.S. EIA.

$$H_G = 46.7 \text{ MJ kg}^{-1} \quad (7)$$

State 1

This is the start of the compression stroke right after the intake stroke. The air is at ambient conditions, noted in the previous section. The specific volume is calculated and the internal energy taken from an air table.

$$u_1 = 205.71 \text{ kJ kg}^{-1} \quad (8)$$

$$v_1 = 0.8163 \text{ m}^3 \text{ kg}^{-1} \quad (9)$$

State 1-2

The process from state 1 to state 2 is adiabatic compression. The adiabatic compression is assumed isentropic. The compression ratio dictates the final specific volume.

$$C_R = \frac{v_1}{v_2} \quad (10)$$

$$v_2 = \frac{v_1}{C_R} \quad (11)$$

$$v_2 = 0.08163 \text{ m}^3 \text{ kg}^{-1} \quad (12)$$

$$\rho_2 = 12.25 \text{ kg m}^{-3} \quad (13)$$

Since it's an isentropic process, the final state can be found from the air table.

$$T_2 = 703 \text{ K} \quad (14)$$

$$u_2 = 514.95 \text{ kJ kg}^{-1} \quad (15)$$

State 2-3

The process from state 2 to state 3 is an isochoric process with heat added from combustion. The specific heat added is determined from the air to fuel ratio.

$$m_t = m_G + m_A = m_G + M_R m_G \quad (16)$$

$$Q = m_G H_G \quad (17)$$

$$q = \frac{Q}{m_t} = \frac{H_G}{1 + M_R} \quad (18)$$

$$q = 1.5362 \text{ MJ kg}^{-1} \quad (19)$$

With no work done on the gas, the final internal energy is given by adding the heat and initial internal energy.

$$u_3 = q + u_2 \quad (20)$$

$$u_3 = 2051.1 \text{ kJ kg}^{-1} \quad (21)$$

The density is identical, since it's an isochoric process. Additional properties are now derived from the air table.

$$p_3 = 8.187 \text{ MPa} \quad (22)$$

$$T_3 = 2385 \text{ K} \quad (23)$$

$$\rho_3 = 12.25 \text{ kg m}^{-3} \quad (24)$$

$$p_{3r} = 4443.2 \quad (25)$$

State 3-4

This is the expansion stroke - modeled as an adiabatic process. The pressure it expands to is chosen so enough power is produced.

$$p_4 = 2.25p_o = 227.98 \text{ kPa} \quad (26)$$

$$p_{4r} = 123.73 \quad (27)$$

Modeling the adiabatic process as isentropic, the relative pressure can be used to find the final properties.

$$T_4 = 1022 \text{ K} \quad (28)$$

$$u_4 = 778 \text{ kJ kg}^{-1} \quad (29)$$

$$\rho_4 = 0.777 \text{ kg m}^{-3} \quad (30)$$

Complete Cycle Properties

The isentropic processes have no heat transfer - $W = \Delta U$.

The isobaric process has a simple work relation - $W = P\Delta V$.

The specific work is derived from this:

$$w_{cyc} = (u_3 - u_4) - (u_2 - u_1) - p_{amb}(v_4 - v_1) \quad (31)$$

$$w_{cyc} = 1038.3 \text{ kJ kg}^{-1} \quad (32)$$

The total air mass per cylinder is now solved for:

$$\frac{3000 \text{ c.c.}}{6} = m_t(v_4 - v_2) \quad (33)$$

$$m_t = 414.81E - 6 \text{ kg} \quad (34)$$

One cycle is completed per piston for every 2 rotations. The power is given by the mass, specific work, and rotational frequency. The power is evaluated at 8000 R.P.M.

$$P = 6m_tw_{cyc}\frac{f}{2} \quad (35)$$

$$P = 172.28 \text{ kW} = 231.0 \text{ hp} \quad (36)$$

$$\eta = \frac{w_{cyc}}{q} = 67.6\% \quad (37)$$

Real world efficiencies are expected to be far lower.

System Dynamics

To derive the piston forces, first the linear kinematics of the piston head needs to be derived with respect to $\alpha = \omega t$, where $\theta = 0$ is TDC and x is the distance below TDC.

The crankshaft radius is r and the connecting rod length is l_{CR} . The height from the crank center is given by $r \cos(\alpha) = r \cos(\omega t)$. The height from the connecting rod is derived from noting the triangles formed by the crankshaft and connecting rod share an edge. $l_{CR} \cos(\beta)$ is solved for in terms of α .

$$r \sin(\alpha) = l_{CR} \sin(\beta) \quad (38)$$

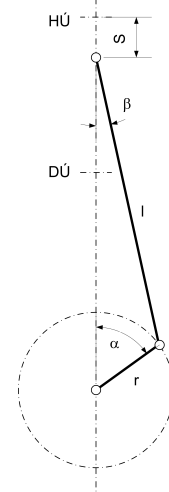
$$\frac{r}{l_{CR}} \sin(\alpha) = \sin(\beta) \quad (39)$$

$$\cos(\beta) = \sqrt{1 - \left(\frac{r}{l_{CR}} \sin(\alpha)\right)^2} \quad (40)$$

$$l_{CR} \cos(\beta) = \sqrt{l_{CR}^2 - (r \sin(\alpha))^2} \quad (41)$$

The position is given by $r \cos(\omega t) + l_{CR} \cos(\beta)$. The equation is adjusted such that $x(0) = 0$ and downwards is positive. The derivatives are derived with a symbolic algebra program. These dynamic equations are used to later derive the forces on the piston and crankshaft.

Crank Linkage



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$$x(t) = (r + l_{CR}) - \left(r \cos(\omega t) + \sqrt{l_{CR}^2 - r^2 \sin^2(\omega t)} \right) \quad (42)$$

$$v(t) = \frac{\omega r^2 \sin(\omega t) \cos(\omega t)}{\sqrt{l_{CR}^2 - r^2 \sin^2(\omega t)}} + \omega r \sin(\omega t) \quad (43)$$

$$a(t) = \frac{\omega^2 r^4 \sin^2(\omega t) \cos^2(\omega t)}{(l_{CR}^2 - r^2 \sin^2(\omega t))^{\frac{3}{2}}} - \frac{\omega^2 r^2 \sin^2(\omega t)}{\sqrt{l_{CR}^2 - r^2 \sin^2(\omega t)}} + \frac{\omega^2 r^2 \cos^2(\omega t)}{\sqrt{l_{CR}^2 - r^2 \sin^2(\omega t)}} + \omega^2 r \cos(\omega t) \quad (44)$$

Forces

To get the force of the piston, pressures equations are needed. The pressure of the expansion and compression strokes relative to piston position are approximated with the adiabatic relation $PV^\gamma = c$. The pressure of the intake and compression strokes are derived using mass flow rates and Bernoulli's equation. At the TDC, the height of the air column is given by $z = \frac{V_{\text{comp}}}{A_C}$, where A_C is the cross sectional area of the chamber and V_{comp} is the air volume at compression. Thus air volume at any point of piston movement is $V(t) = A_C(x(t) + z)$.

Compression Stroke

The compression stroke is derived using adiabatic relations $PV^\gamma = c$.

$$P_{\text{amb}} V_1^\gamma = P(t) V(t)^\gamma \quad (45)$$

$$P_{\text{Compression}}(t) = P_{\text{amb}} \left(\frac{V_1}{A_C(x(t) + z)} \right)^\gamma \quad (46)$$

Expansion Stroke

The expansion stroke is derived the same way.

$$P_4 V_{\text{comp}}^\gamma = P(t) V(t)^\gamma \quad (47)$$

$$P_{\text{Expansion}}(t) = P_4 \left(\frac{V_{\text{comp}}}{A_C(x(t) + z)} \right)^\gamma \quad (48)$$

Intake/Exhaust Strokes

The pressure difference in the intake/exhaust strokes is derived using bernoulli equations. A 0 air velocity in the piston chamber is assumed. For simplicity, the density is assumed constant. The pressure difference needed to balance the air flow rates is found.

Let A_I be the cross sectional area of the intake valve.

$$Q = A_I v_a \rho_{\text{amb}} = A_C v(t) \rho_{\text{amb}} \quad (49)$$

$$v = \frac{A_C v(t)}{A_I} \quad (50)$$

$$\Delta P = \frac{1}{2} \rho_{\text{amb}} v^2 \quad (51)$$

$$\Delta P = \frac{1}{2} \rho_{\text{amb}} \left(\frac{A_C v(t)}{A_I} \right)^2 \quad (52)$$

Gas Forces

The force on the piston from the gasses is a function of pressure and chamber area. Compressive forces are taken to be positive.

Simply,

$$F_{\text{Gasses}} = A_C (P(t) - P_{\text{amb}})$$

The total gas force function is given by a piecewise function. Let θ_c is the angle such that $x(t) = 9z$ and $v(t) < 0$. Then F_{Gas} is

$$F_{\text{Gas}}(t) = \begin{cases} A_C \left(P_4 \left(\frac{V_4}{A_C(x(t)+z)} \right)^\gamma - P_{\text{amb}} \right) - (m_P + m_{CR})a(t) & 0^\circ \leq \omega t \mod 720^\circ \leq 180^\circ \\ A_C \frac{1}{2} \rho_{\text{amb}} \left(\frac{A_C v(t)}{A_I} \right)^2 - (m_P + m_{CR})a(t) & 180^\circ < \omega t \mod 720^\circ \leq 360^\circ \\ -A_C \frac{1}{2} \rho_{\text{amb}} \left(\frac{A_C v(t)}{A_I} \right)^2 - (m_P + m_{CR})a(t) & 360^\circ < \omega t \mod 720^\circ \leq 540^\circ \\ A_C \frac{1}{2} \rho_{\text{amb}} \left(\frac{A_C v(t)}{A_I} \right)^2 - (m_P + m_{CR})a(t) & 540^\circ < \omega t \mod 720^\circ \leq 360^\circ + \theta_c \\ A_C \left(P_{\text{amb}} \left(\frac{V_4}{A_C(x(t)+z)} \right)^\gamma - P_{\text{amb}} \right) - (m_P + m_{CR})a(t) & 360^\circ + \theta_c < \omega t \mod 720^\circ < 720^\circ \end{cases} \quad (53)$$

Acceleration Forces

Additionally there exists a body force present due to the acceleration of the piston head. Since compressive forces are positive, the acceleration term is inverted. Assuming $l_{CR} \gg r$, the acceleration of the connecting rod is assumed equivalent to piston acceleration. For the forces on the piston, only the piston mass matters. For the forces on the crankshaft, both the piston mass m_p and connecting rod mass m_{CR} are needed.

$$F_{\text{accel}}(t) = -m_p a(t)$$

The total force on the piston is from both the acceleration terms and gas force terms.

$$\vec{F}(t) = \left\langle -F(t) \hat{i}, F(t) \frac{r \sin(\omega t)}{\sqrt{l^2 - r^2 \sin^2(\omega t)}} \hat{j} \right\rangle \quad (54)$$

$$\vec{F}_n(t) = \vec{F}(t + 120^\circ(n-1)\omega^{-1}) \quad (55)$$