Representing transformation constraints

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1 Implicit representation

Let $T_1 = T_{(R_1,\mathbf{t}_1)}$ and $T_2 = T_{(R_2,\mathbf{t}_2)}$ be two rigid-body transformations. Let $T_{2/1} = T_{(R_{2/1},\mathbf{t}_{2/1})}$ be the relative transformation $T_1^{-1} \circ T_2$. Then, we recall that

$$\begin{aligned} R_{2/1} &= R_1^T R_2 \\ \mathbf{t}_{2/1} &= R_1^T(\mathbf{t}_2 - \mathbf{t}_1) \end{aligned}$$

We define the following mapping from $SE(3)$ to $\mathbf{R}^6$

$$\widetilde{\log}(T_{(R,\mathbf{t})}) = \begin{pmatrix} \mathbf{t} \\ \log R \end{pmatrix},$$

and the implicit constraint

$$h(\mathbf{q}) = \widetilde{\log}(T_{2/1}) = \begin{pmatrix} \mathbf{t}_{2/1} \\ \log R_{2/1} \end{pmatrix} \tag{1}$$

This constraint has several interesting geometrical properties.

1.1 Pre-grasp

When manipulating objects, it is convenient to define pre-grasp position. A pre-grasp position is a position where the gripper is aligned with the x-axis of the object handle. This position is defined by

$$h(\mathbf{q}) = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T \tag{2}$$

where $\lambda > 0$ is the pre-grasp distance.

- T_1 is the position of the gripper,
- T_2 is the position of the handle.

1.2 Symmetric handle

When the handle is symmetric around axis z, the last component of the constraint can be dropped. The grasp constraint is then

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} h(\mathbf{q}) \quad = \quad 0 \tag{3}$$

The corresponding pre-grasp constraint is then

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} h(\mathbf{q}) \quad = \quad \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 \end{pmatrix}^T \tag{4}$$

2 Explicit representation

When the handle is not symmetric and attached to a freeflying object, the position of the freeflying object can be expressed explicitely with respect to the robt configuration. In manipulation planning, this corresponds to an explicit constraint where some configuration variables (of the object) depend on others.

$$\mathbf{q}_{out} = f(\mathbf{q}_{in}) \tag{5}$$

where $\mathbf{q}_{out}$ is a member of $SE(3)$ represented by a vector of dimension 7 (translation, unit quaternion), and $\mathbf{q}_{in}$ are the configuration variables of the kinematic chain holding the gripper.

More precisely, using additive notation for Lie group operations in $\mathbf{R}^n$ and $SE(3)$,

$$\mathbf{q}_{out} + \mathbf{q}_h = f_g(\mathbf{q}_{in}) \tag{6}$$

where

- $\mathbf{q}_h \in SE(3)$ is the position of the handle in the object frame,
- f_g is the mapping from the robot configuration to $SE(3)$ that maps to $\mathbf{q}_{in}$ the position of the gripper in the world frame when the kinematic chain holding the gripper is in configuration $\mathbf{q}_{in}$.

2.1 Implicit representation of an explicit constraint

In the general case, explicit constraint (5) can be represented implicitly as

$$h(\mathbf{q}) = \log(\mathbf{q}_{out} - f(\mathbf{q}_{in})).$$

In the case of explicit constraint (6), this writes

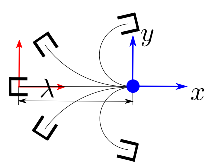
$$h(\mathbf{q}) = \log\left((\mathbf{q}_{out} + \mathbf{q}_h) - f_g(\mathbf{q}_{in})\right), \tag{7}$$

while (1) writes

$$h(\mathbf{q}) = \widetilde{\log}\left((\mathbf{q}_{out} + \mathbf{q}_h) - f_g(\mathbf{q}_{in})\right).$$

Symmetric handle (3) with expression (7) of h still represents the grasp constraint of a symmetric handle around z-axis, but unfortunately, (4) does not represent a correct pregrasp position.

3 Summary

	$h(\mathbf{q}) = \log(\mathbf{q}_{out} - f(\mathbf{q}_{in}))$	$h(\mathbf{q}) = \widetilde{\log}\left((\mathbf{q}_{out} + \mathbf{q}_h) - f_g(\mathbf{q}_{in})\right)$
non symmetric grasp	yes	yes
non symmetric pre-grasp	yes	yes
symmetric grasp	yes	yes
symmetric pre-grasp		yes