

Generating Equivalent Rhythmic Notations based on Rhythm Tree Languages

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TENOR'17

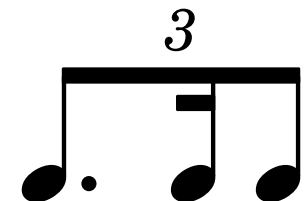
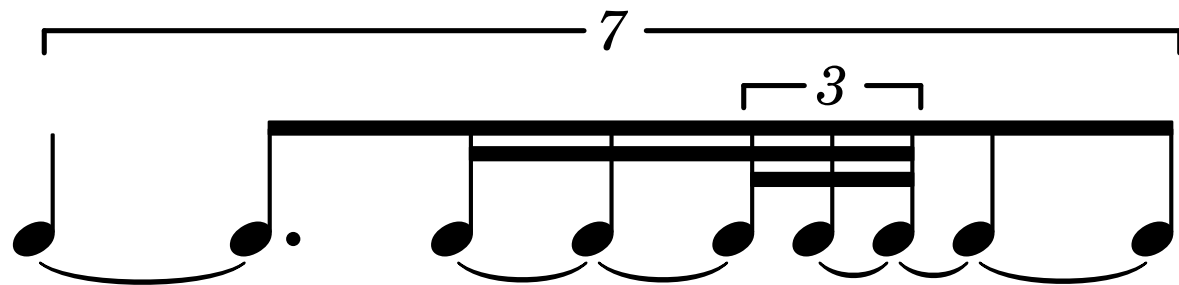
May 26, 2017

A Coruña

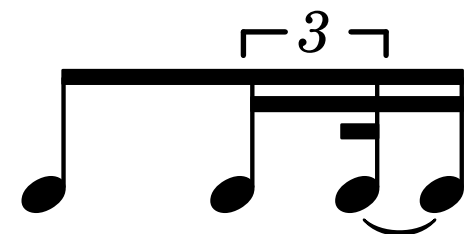
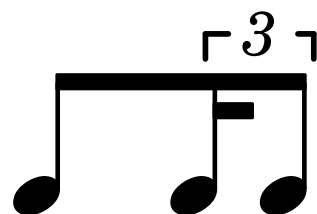
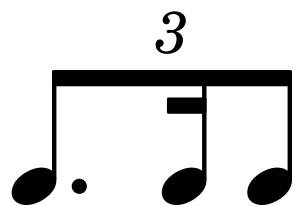
problem

enumerate **all** rhythms notations
equivalent to a given rhythm

simplify,... or complexify



emphasize meter



problem

enumerate **all** rhythms notations
equivalent to a given rhythm

equivalent: defining the same sequences of durations (IOI)

enumerate:

- from simplest to more complicated... or opposite
- lazily: don't compute all rhythms first and then sort

all: really?

definition of a rhythm notation domain = language:

what kind of tuples? how many nested divisions?...

plan

enumerate **all** rhythms notations
equivalent to a given rhythm

1. rhythm tree representation
rhythm (formal) languages
2. efficient lazy enumeration algorithm
3. conclusion & applications

rhythm tree representation

hierarchical representation of sequences of events with durations

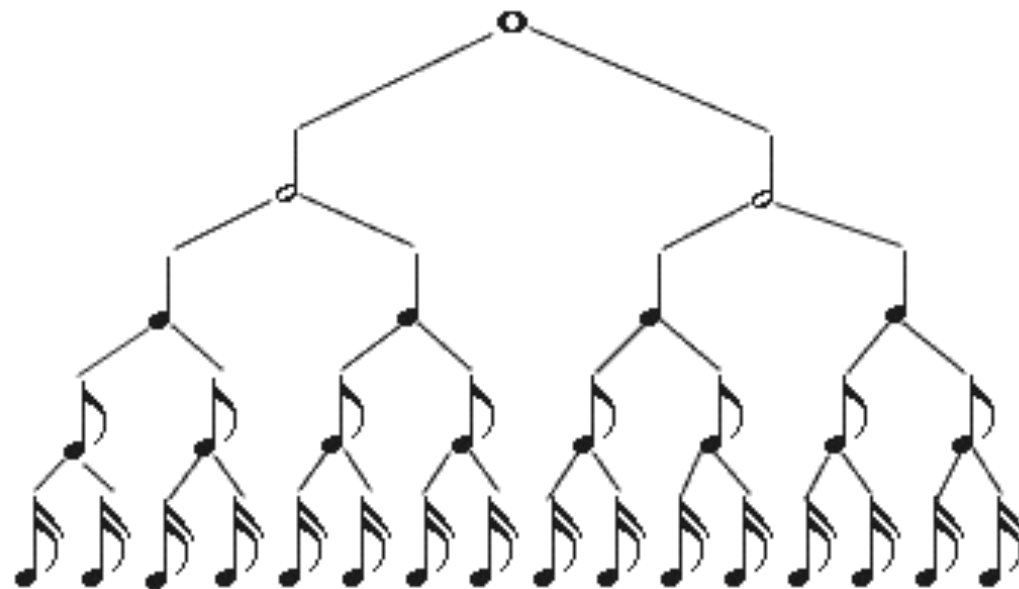
used in composition assistance environments since years

Laurson

Patchwork: A Visual Programming Language
Helsinki: Sibelius Academy, 1996

Agon, Haddad, Assayag

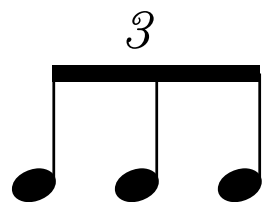
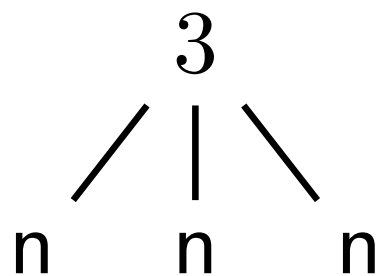
Rhythmic structures representation and rendering
ICMC, 2002



rhythm trees

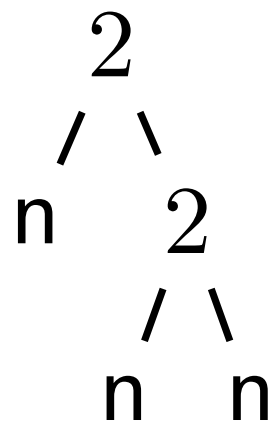
principle: “*the data is in the structure*”

- leaves contain events
- branching defines durations
by uniform division of a time interval

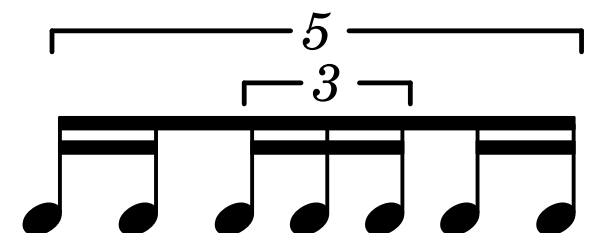
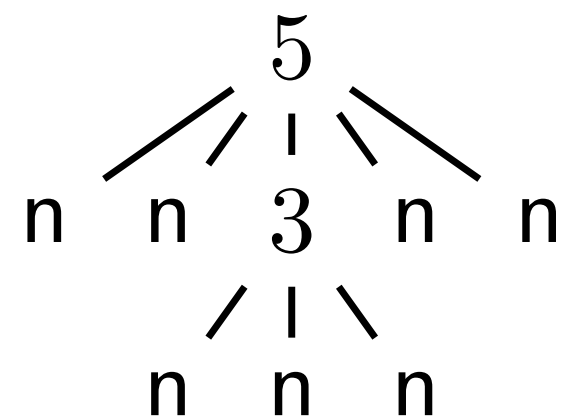


duration
sequence

$$\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$$



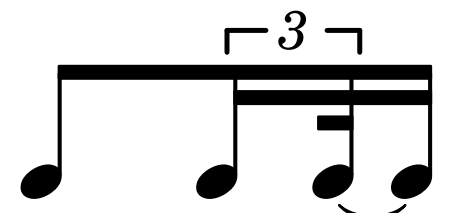
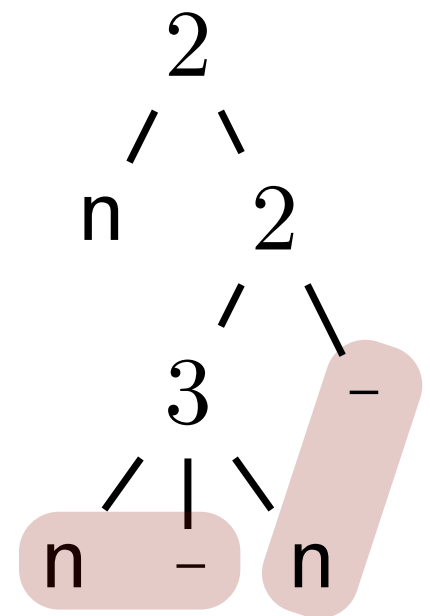
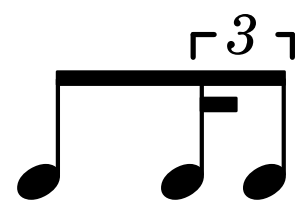
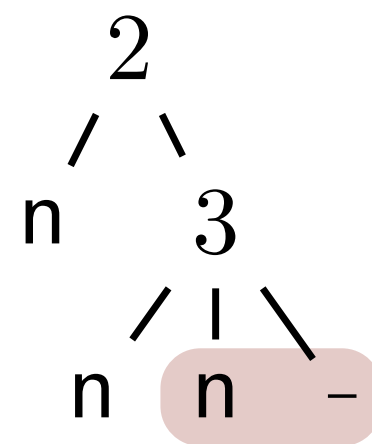
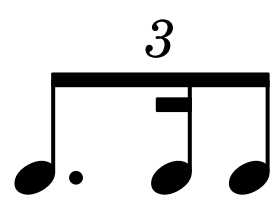
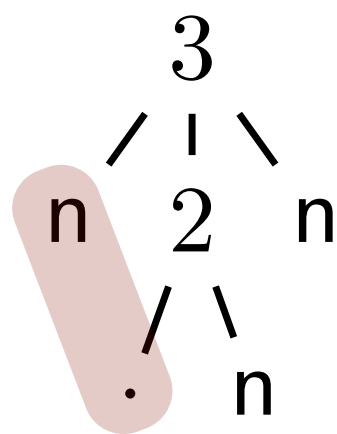
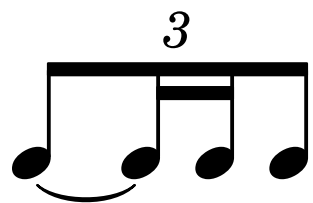
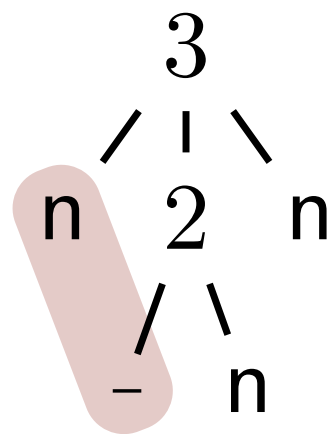
$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4}$$



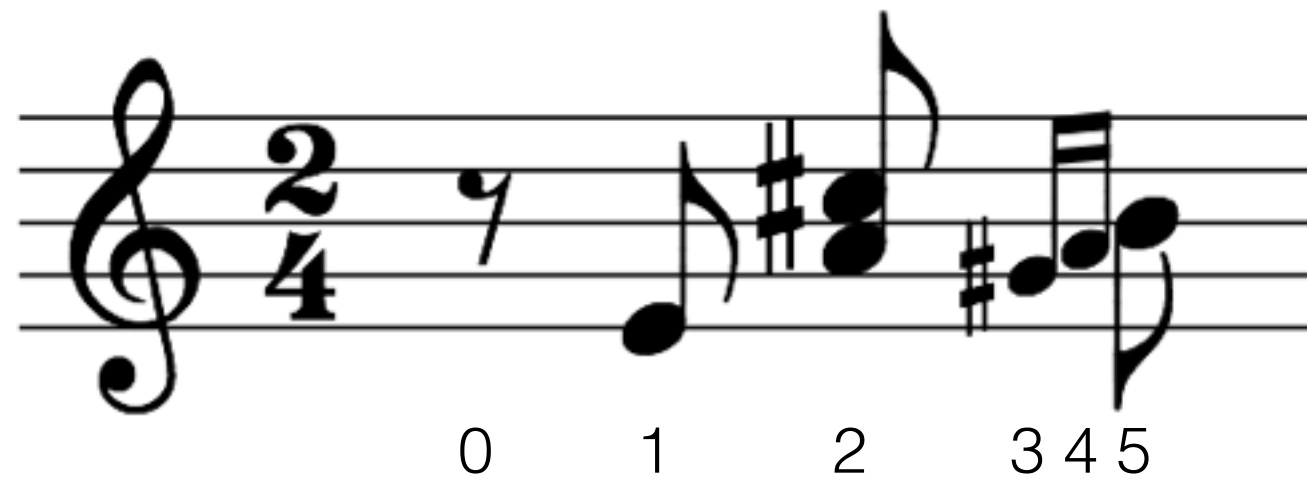
$$\frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{15} \quad \frac{1}{15} \quad \frac{1}{15} \quad \frac{1}{5} \quad \frac{1}{5}$$

rhythm trees: ties and dots

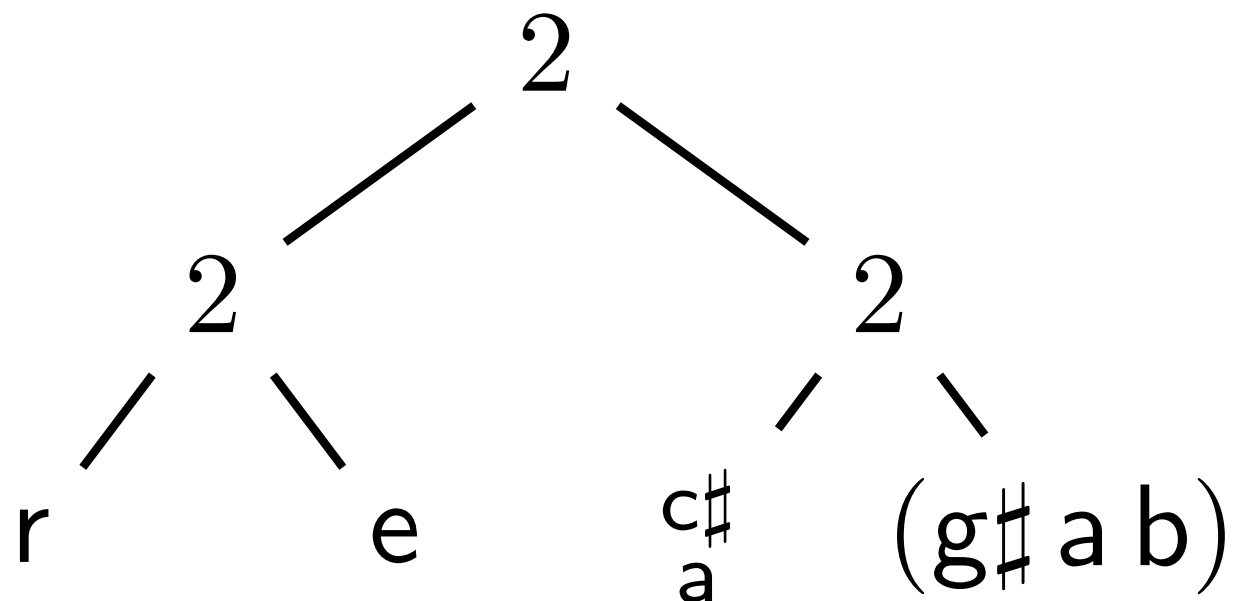
duration
sequence $\frac{1}{2} \frac{1}{6} \frac{1}{3}$



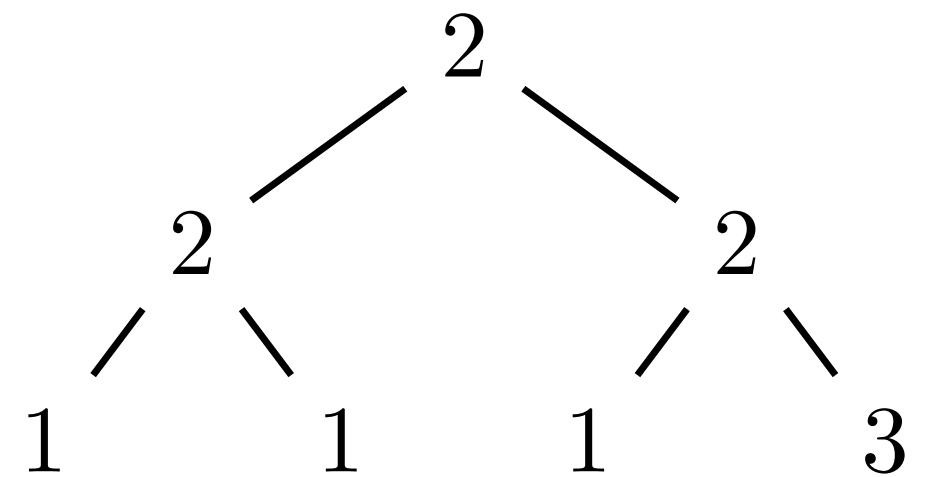
pitch, rests, grace notes...



with additional symbols for leaves



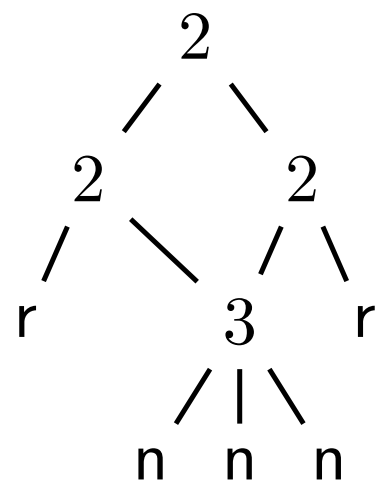
with event counters



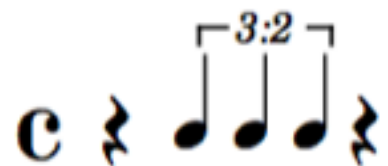
rhythm dags

extension of RT to *join then split* = *ratio* notations (p in the time of q)
 no symbol $_$: sum of durations represented by node sharing (*data in structure*)

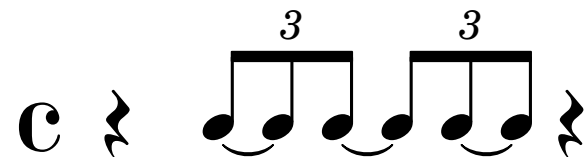
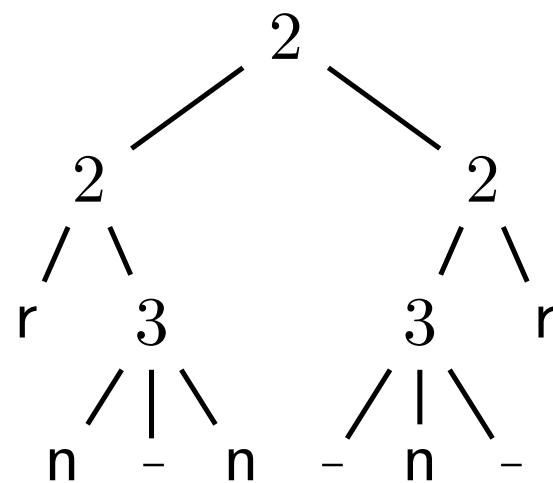
dag



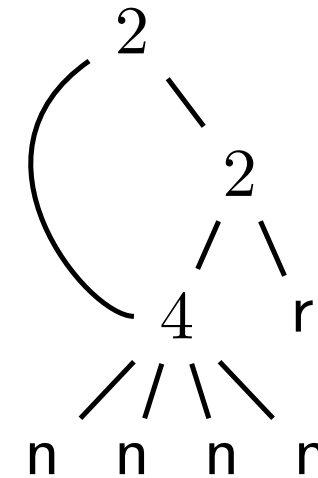
$$\frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4}$$



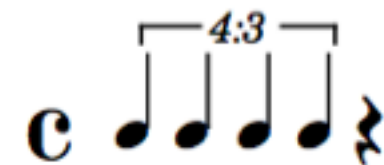
\equiv tree



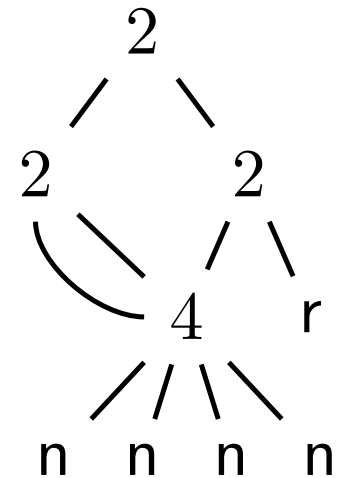
dag



$$\frac{3}{16} \quad \frac{3}{16} \quad \frac{3}{16} \quad \frac{3}{16} \quad \frac{1}{4}$$



\equiv dag



rhythm languages

fix the set of rhythm notations that we want to support
using formal language tools

Lee

The rhythmic interpretation
of simple musical sequences
Musical Structure and Cognition, 1985

Andrew McLoed, Mark Steedman

Meter Detection in Symbolic Music
Using a Lexicalized PCFG

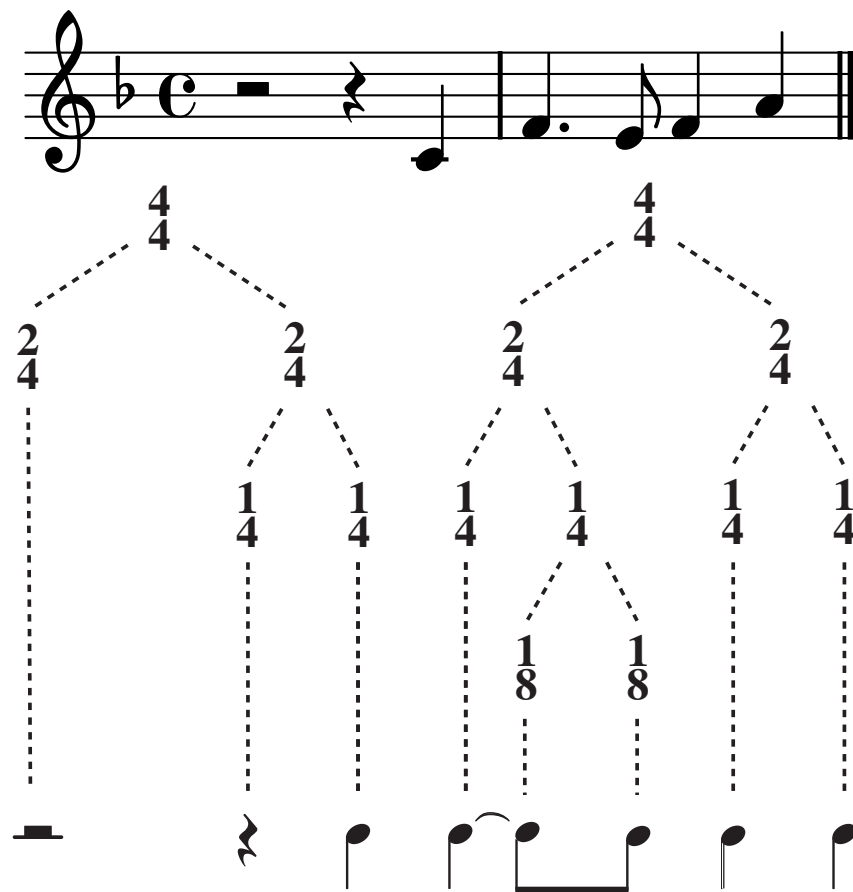
SMC 2017

$$\mathbf{C} \rightarrow \mathbf{O} \mid \mathbf{-} \mid \frac{2}{4} + \frac{2}{4}$$

$$\frac{2}{4} \rightarrow \mathbf{D} \mid \mathbf{-} \mid \frac{1}{4} + \frac{1}{4}$$

$$\frac{1}{4} \rightarrow \mathbf{Q} \mid \mathbf{Z} \mid \frac{1}{8} + \frac{1}{8}$$

$$\frac{1}{8} \rightarrow \mathbf{Q} \mid \mathbf{7} \mid \dots$$



parse tree \cong rhythm tree

rhythm grammars

definition of allowed nested divisions
by rules of an acyclic Context-Free grammar

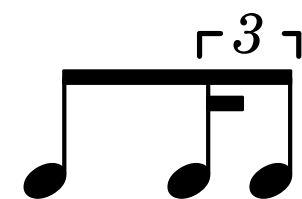
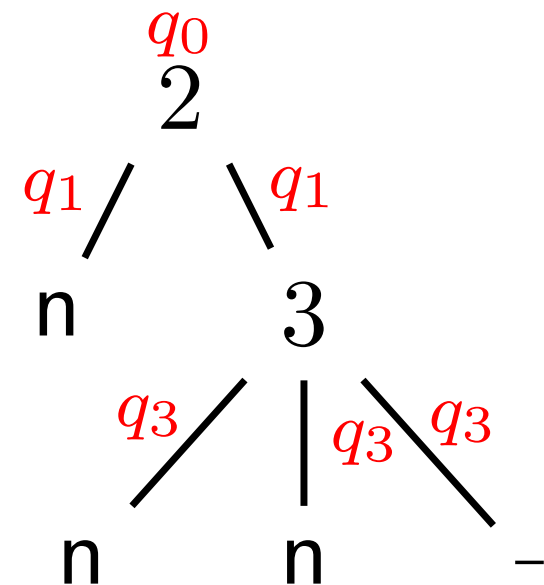
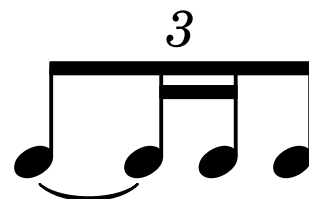
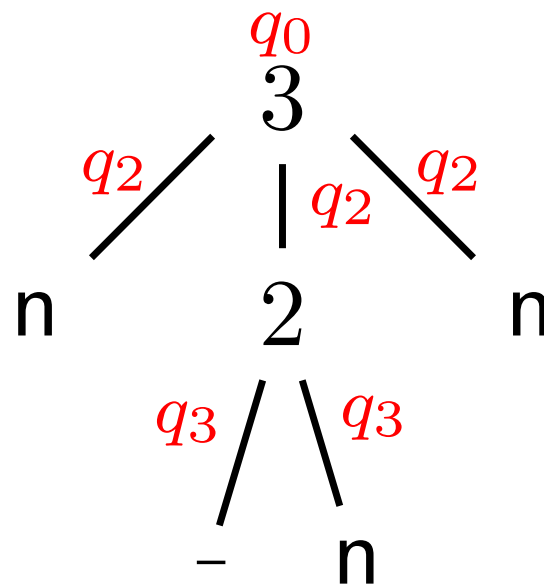
divide by 2 then by 2 or 3, or divide by 3 then by 2

$$\begin{array}{llll}
 G = & q_0 \rightarrow q_1, q_1 & q_1 \rightarrow q_3, q_3 & q_2 \rightarrow q_3, q_3 & q_3 \rightarrow n \\
 & q_0 \rightarrow q_2, q_2, q_2 & q_1 \rightarrow q_3, q_3, q_3 & q_2 \rightarrow n & q_3 \rightarrow - \\
 & q_0 \rightarrow n & q_1 \rightarrow n & q_2 \rightarrow - &
 \end{array}$$

language $L(G)$
of generated
rhythm trees

=

derivation trees
of the grammar



weighted rhythm grammars

rhythm tree ranking: assign a weight value to every tree of $L(G)$

by extension of grammar with weight values in production rules

$q_0 \xrightarrow{0.25} q_1, q_1$	$q_1 \xrightarrow{0.2} q_3, q_3$	$q_2 \xrightarrow{0.5} q_3, q_3$	$q_3 \xrightarrow{0.15} n$
$q_0 \xrightarrow{0.45} q_2, q_2, q_2$	$q_1 \xrightarrow{0.7} q_3, q_3, q_3$	$q_2 \xrightarrow{0.1} n$	$q_3 \xrightarrow{0.35} _$
$q_0 \xrightarrow{0.1} n$	$q_1 \xrightarrow{0.1} n$	$q_2 \xrightarrow{0.25} _$	
	$q_1 \xrightarrow{0.25} _$		



weight domains

	sum	product
penalties	min	+
probabilities	max	.

$$t = a(t_1, \dots, t_n)$$

$$weight(t, q) = \sum_{q \xrightarrow{v} q_1, \dots, q_n} v \cdot \prod_{i=1}^n weight(t_i, q_i)$$

grammars construction

- incremental construction
by composition of elementary languages
using composition operators
(union, intersection, *etc*)
- learning weighted grammar from score corpus
→ compact representation of rhythm notations in corpus

enumeration: k -best parsing

$best(k, q_i)$ is the k^{th} best weighted tree generated by q_i

defined recursively thanks to the monotonicity of weight evaluation

$$\begin{array}{llll}
 q_0 \xrightarrow{0.25} q_1, q_1 & q_1 \xrightarrow{0.2} q_3, q_3 & q_2 \xrightarrow{0.5} q_3, q_3 & q_3 \xrightarrow{0.15} \mathbf{n} \\
 q_0 \xrightarrow{0.45} q_2, q_2, q_2 & q_1 \xrightarrow{0.7} q_3, q_3, q_3 & q_2 \xrightarrow{0.1} \mathbf{n} & q_3 \xrightarrow{0.35} _ \\
 q_0 \xrightarrow{0.1} \mathbf{n} & q_1 \xrightarrow{0.1} \mathbf{n} & q_2 \xrightarrow{0.25} _ & \\
 & q_1 \xrightarrow{0.25} _ & &
 \end{array}$$

$$best(1, q_0) = \min \left\{ \begin{array}{l} 2(best(1, q_1), best(1, q_1)), \\ 3(best(1, q_2), best(1, q_2), best(1, q_2)), \\ \mathbf{n} \end{array} \right\}$$

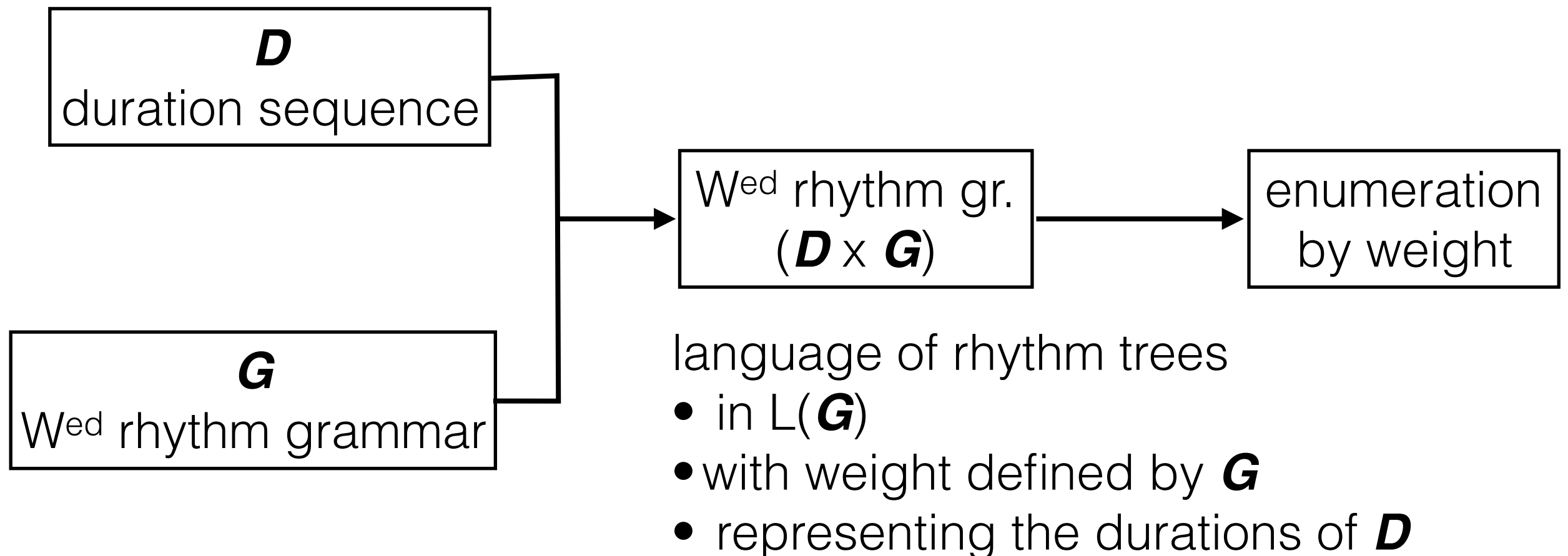
assuming it is the first term, then

$$best(2, q_0) = \min \left\{ \begin{array}{l} 2(best(1, q_1), best(2, q_1)), \\ 2(best(2, q_1), best(1, q_1)), \\ 3(best(1, q_2), best(1, q_2), best(1, q_2)), \\ \mathbf{n} \end{array} \right\} \text{ etc}$$

enumeration of rhythms

problem reformulation

given a grammar \mathbf{G} and a rhythm of duration sequence \mathbf{D}
enumerate **all** rhythm in $L(\mathbf{G})$ and of duration sequence \mathbf{D} ,
according to their weight in \mathbf{G}



product grammar $\mathbf{D} \times \mathbf{G}$

$\langle D', q_i \rangle$: trees generated from q_i and of duration sequence D'

$$D = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 6 & 3 \end{bmatrix}$$

from

we obtain

$$q_0 \xrightarrow{0.25} q_1, q_1$$

$$q_0 \xrightarrow{0.45} q_2, q_2, q_2$$

$$\langle D, q_0 \rangle \xrightarrow{0.25} \langle [\tfrac{1}{2}], q_1 \rangle, \langle [\tfrac{1}{6}, \tfrac{1}{3}], q_1 \rangle$$

$$\langle D, q_0 \rangle \xrightarrow{0.45} \langle [\tfrac{1}{3}], q_2 \rangle, \langle -[\tfrac{1}{6}, \tfrac{1}{6}], q_2 \rangle, \langle [\tfrac{1}{3}], q_2 \rangle$$

$$q_1 \xrightarrow{0.2} q_3, q_3$$

$$q_1 \xrightarrow{0.7} q_3, q_3, q_3$$

$$\langle [\tfrac{1}{6}, \tfrac{1}{3}], q_1 \rangle \xrightarrow{0.2} \langle [\tfrac{1}{6}, \tfrac{1}{12}], q_3 \rangle, \langle -[\tfrac{1}{4}], q_3 \rangle$$

$$\langle [\tfrac{1}{6}, \tfrac{1}{3}], q_1 \rangle \xrightarrow{0.7} \langle [\tfrac{1}{6}], q_3 \rangle, \langle [\tfrac{1}{6}], q_3 \rangle, \langle -[\tfrac{1}{6}], q_3 \rangle$$

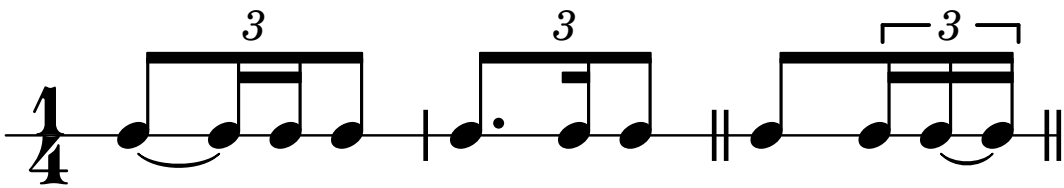
enumeration

for the product of

$$\mathbf{D} = \frac{1}{2} \frac{1}{6} \frac{1}{3} \quad \text{and}$$

$$\mathbf{G} = \begin{array}{l} q_0 \xrightarrow{0.1} q_1, q_1 \\ q_0 \xrightarrow{0.3} q_2, q_2, q_2 \\ q_0 \xrightarrow{0.05} \mathbf{n} \end{array} \quad \begin{array}{l} q_1 \xrightarrow{0.2} q_3, q_3 \\ q_1 \xrightarrow{0.7} q_3, q_3, q_3 \\ q_1 \xrightarrow{0.1} \mathbf{n} \\ q_1 \xrightarrow{0.25} _ \end{array} \quad \begin{array}{l} q_2 \xrightarrow{0.5} q_3, q_3 \\ q_2 \xrightarrow{0.1} \mathbf{n} \\ q_2 \xrightarrow{0.25} _ \end{array} \quad \begin{array}{l} q_3 \xrightarrow{0.15} \mathbf{n} \\ q_3 \xrightarrow{0.35} _ \end{array}$$

we obtain the following rhythms



enumeration (bis)

D = $\frac{1}{2} \frac{1}{6} \frac{1}{3}$

G =

$q_0 \xrightarrow{0.1} n$	$q_1 \xrightarrow{0.1} n$	$q_2 \xrightarrow{0.1} n$	$q_4 \xrightarrow{0.1} n$
$q_0 \xrightarrow{0.25} q_1, q_1$	$q_1 \xrightarrow{0.25} -$	$q_2 \xrightarrow{0.25} -$	$q_4 \xrightarrow{0.25} -$
$q_0 \xrightarrow{0.45} q_1, q_1, q_1$	$q_1 \xrightarrow{0.2} q_2, q_2$	$q_2 \xrightarrow{0.5} q_3, q_3$	$q_4 \xrightarrow{0.5} q_5, q_5$
$q_0 \xrightarrow{0.45} q_4, q_4, q_4, q_4, q_4$	$q_1 \xrightarrow{0.7} q_2, q_2, q_2$	$q_2 \xrightarrow{0.5} q_3, q_3, q_3$	$q_4 \xrightarrow{0.5} q_5, q_5, q_5$
$q_0 \xrightarrow{0.45} q_4, q_4, q_4, q_4, q_4, q_4, q_4$		$q_3 \xrightarrow{0.15} n$	$q_5 \xrightarrow{0.1} n$
		$q_3 \xrightarrow{0.35} -$	$q_5 \xrightarrow{0.25} -$
		$q_3 \xrightarrow{0.5} q_5, q_5$	

The musical notation consists of two staves. The first staff is in 4/4 time and contains six measures of music. The second staff is in 7/4 time and contains five measures of music. Both staves feature complex rhythmic patterns with triplets, quintuplets, and septuplets, as well as various rests and accidentals.

rhythm notations of value [1/2 1/6 1/3] (schema-06.txt)

The image displays a musical score for 58 staves, organized into two columns. Each staff contains a pair of rhythmic patterns, typically separated by a double bar line. The patterns are composed of eighth and sixteenth notes, often grouped into triplets and quintuplets. The score is divided into two main sections, with the first section ending at staff 29 and the second starting at staff 31. The patterns are labeled with numbers 1 through 58, indicating their sequence. The notation includes various rests, beams, and slurs to indicate the timing and grouping of the notes.

Staff 1: Pattern 1 (4/4 time, 8 measures).
Staff 2: Pattern 2 (4/4 time, 8 measures).
Staff 3: Pattern 3 (4/4 time, 8 measures).
Staff 4: Pattern 4 (4/4 time, 8 measures).
Staff 5: Pattern 5 (4/4 time, 8 measures).
Staff 6: Pattern 6 (4/4 time, 8 measures).
Staff 7: Pattern 7 (4/4 time, 8 measures).
Staff 8: Pattern 8 (4/4 time, 8 measures).
Staff 9: Pattern 9 (4/4 time, 8 measures).
Staff 10: Pattern 10 (4/4 time, 8 measures).
Staff 11: Pattern 11 (4/4 time, 8 measures).
Staff 12: Pattern 12 (4/4 time, 8 measures).
Staff 13: Pattern 13 (4/4 time, 8 measures).
Staff 14: Pattern 14 (4/4 time, 8 measures).
Staff 15: Pattern 15 (4/4 time, 8 measures).
Staff 16: Pattern 16 (4/4 time, 8 measures).
Staff 17: Pattern 17 (4/4 time, 8 measures).
Staff 18: Pattern 18 (4/4 time, 8 measures).
Staff 19: Pattern 19 (4/4 time, 8 measures).
Staff 20: Pattern 20 (4/4 time, 8 measures).
Staff 21: Pattern 21 (4/4 time, 8 measures).
Staff 22: Pattern 22 (4/4 time, 8 measures).
Staff 23: Pattern 23 (4/4 time, 8 measures).
Staff 24: Pattern 24 (4/4 time, 8 measures).
Staff 25: Pattern 25 (4/4 time, 8 measures).
Staff 26: Pattern 26 (4/4 time, 8 measures).
Staff 27: Pattern 27 (4/4 time, 8 measures).
Staff 28: Pattern 28 (4/4 time, 8 measures).
Staff 29: Pattern 29 (4/4 time, 8 measures).
Staff 30: Pattern 30 (4/4 time, 8 measures).
Staff 31: Pattern 31 (4/4 time, 8 measures).
Staff 32: Pattern 32 (4/4 time, 8 measures).
Staff 33: Pattern 33 (4/4 time, 8 measures).
Staff 34: Pattern 34 (4/4 time, 8 measures).
Staff 35: Pattern 35 (4/4 time, 8 measures).
Staff 36: Pattern 36 (4/4 time, 8 measures).
Staff 37: Pattern 37 (4/4 time, 8 measures).
Staff 38: Pattern 38 (4/4 time, 8 measures).
Staff 39: Pattern 39 (4/4 time, 8 measures).
Staff 40: Pattern 40 (4/4 time, 8 measures).
Staff 41: Pattern 41 (4/4 time, 8 measures).
Staff 42: Pattern 42 (4/4 time, 8 measures).
Staff 43: Pattern 43 (4/4 time, 8 measures).
Staff 44: Pattern 44 (4/4 time, 8 measures).
Staff 45: Pattern 45 (4/4 time, 8 measures).
Staff 46: Pattern 46 (4/4 time, 8 measures).
Staff 47: Pattern 47 (4/4 time, 8 measures).
Staff 48: Pattern 48 (4/4 time, 8 measures).
Staff 49: Pattern 49 (4/4 time, 8 measures).
Staff 50: Pattern 50 (4/4 time, 8 measures).
Staff 51: Pattern 51 (4/4 time, 8 measures).
Staff 52: Pattern 52 (4/4 time, 8 measures).
Staff 53: Pattern 53 (4/4 time, 8 measures).
Staff 54: Pattern 54 (4/4 time, 8 measures).
Staff 55: Pattern 55 (4/4 time, 8 measures).
Staff 56: Pattern 56 (4/4 time, 8 measures).
Staff 57: Pattern 57 (4/4 time, 8 measures).
Staff 58: Pattern 58 (4/4 time, 8 measures).

merged polyrhythms



merging both rhythms gives duration sequence $[1/5 \ 2/15 \ 1/15 \ 1/5 \ 1/15 \ 2/15 \ 1/5]$ and 6 alt. notations (with a complex enough grammar):

conclusion

- formal grammars defining languages of weighted rhythm notations
- ranked enumeration of equivalent rhythms in a given language

applications

transcription

backend procedure (once a rhythm quantization is found)

score editors / composition assistants

propose several rhythm notations

rendering of text based notation languages

logical layout (beaming *etc*):

rhythm grammar to specify (structured) layout preferences?

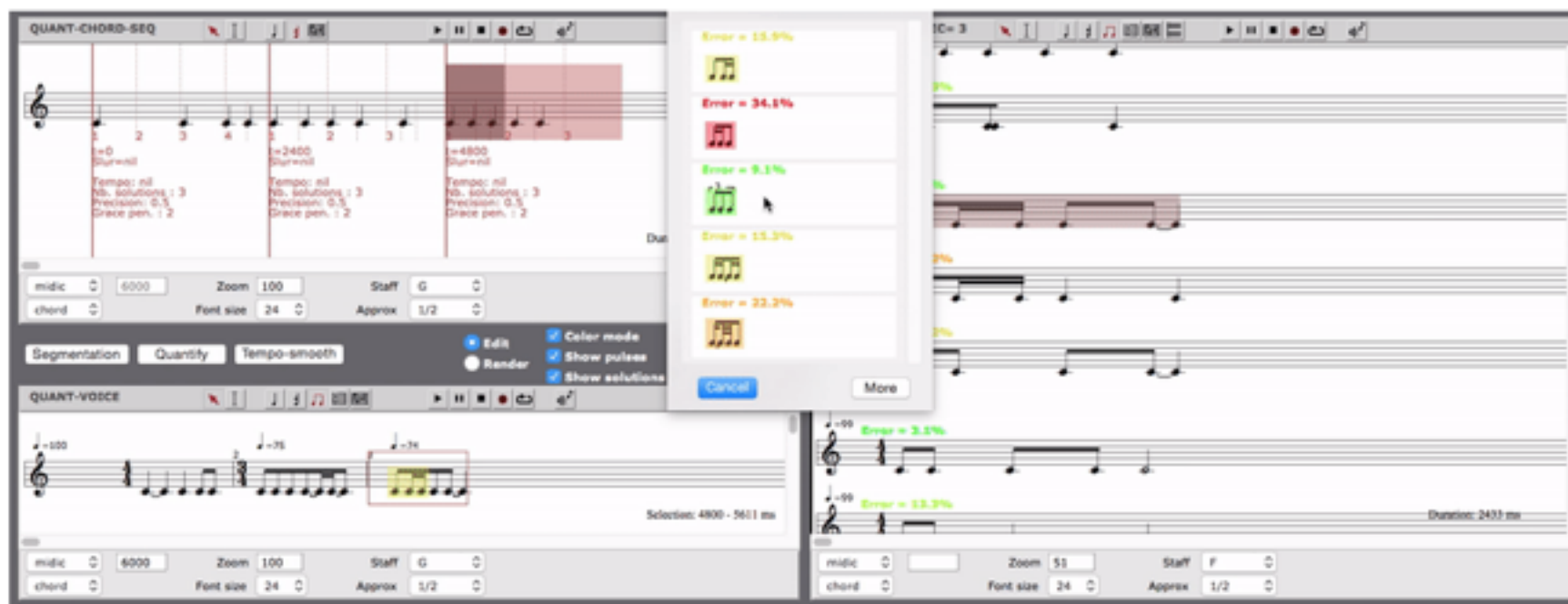
transcription

1. enumerate rhythms in language $L(\mathbf{G})$ **exactly** of duration sequence \mathbf{D}
2. enumerate rhythms in language $L(\mathbf{G})$ **close** to duration sequence \mathbf{D}
(\mathbf{D} unquantized in case 2.)

→ base of an interactive rhythm quantization procedure

implemented as an Open Music library

<http://repmus.ircam.fr/cao/rq>



and a C++ library

<https://gforge.inria.fr/projects/qparse>