BTC, HYPE, and the Black-Scholes Equation

MTH 693b Project Presentation

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MTH 693b : Computational Partial Differential Equations

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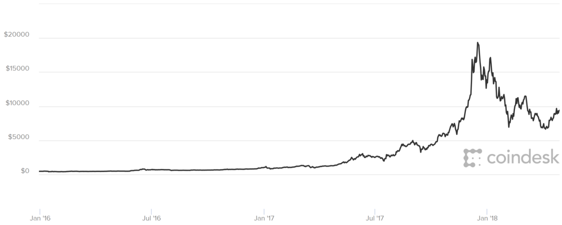
Roadmap

- Motivation and Background
- Models:
  - Price Modeling
  - Volatility Modeling
  - Monte Carlo Simulation
- Black-Scholes Equation
  - Stability
  - Discretization / Scheme
  - Boundary Conditions
- Results and Conclusions

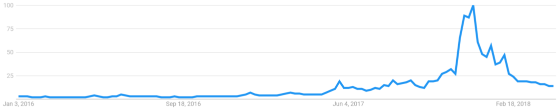
Money Good

Motivation and Background

- Explosion of Cryptocurrencies in 12/17
- Big Data, Behavioral Economics and the Internet
- Accurate and Fast Algorithms for High Frequency Trading
- Maximize Profits with Mathematical Theory on our Side!
- Roll the dice on the Market and the Computers



BTC/USD (Jan 2016 - Present)

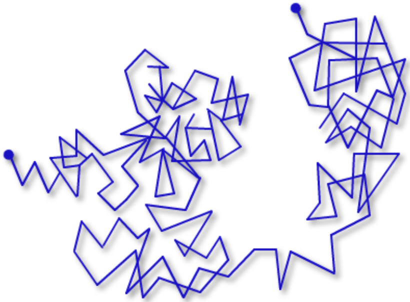


Google Search Trends for "Bitcoin"

Modeling The Price of Bitcoin

Everyone's favorite geometric Brownian motion!

$$S_t = S_{t-1} e^{[(\mu - \frac{\sigma^2}{2})dt + (\sigma\epsilon)\sqrt{dt}]}$$



Now with a Twist!

$$\mu' = \mu(1 + H_{ype} * g(t))$$

Modeling Volatility

TGARCH

Historical Volatility

$$\sigma_{HV} = STD(log(\frac{R_t}{R_{t-1}})) \quad (1)$$

Threshold Generalized Autoregressive Conditional Heteroskedasticity

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \gamma_1 D_{t-1} a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2)$$

$$D_t = \begin{cases} 0 & \text{if } a_t < 0 \\ 1 & \text{if } a_t \geq 0 \end{cases}$$

Algorithm

- Fit Models and TGARCH parameters to data with Least Squares
- Monte Carlo Loop: (Many Iterations)
  - S_t,B_t,RF_t to T with σ_{HV}
  - Price-Volatility Feedback Loop: (Few Iterations)
    - Compute σ_t from S_t , B_t , RF_t
    - Use σ_t to update S_t
- Compute Averages and Variation of S_t and σ_t
- Backwards time iteration of Black-Scholes.

HYPE

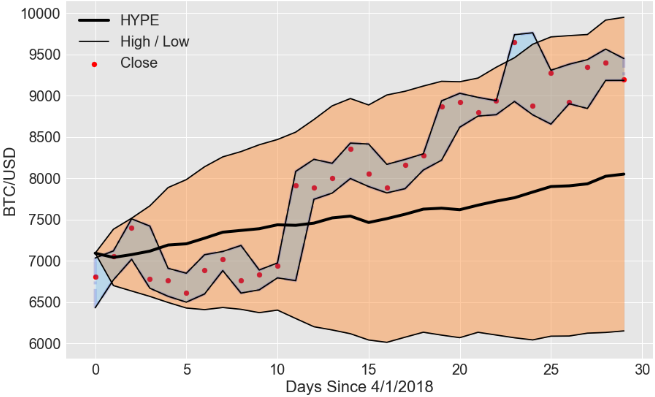


Figure: Hype Model evaluation for April 2018, N=100

Black-Scholes Equation

A European Call Price Model

The Black-Scholes Equation

$$\frac{\partial C}{\partial t}+\frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0$$

- C(S,t) = Price of a Call
- $\sigma(t)$ = volatility of the security
- r = Risk Free rate of return

Stability

A simple Change of Variables...

- $u(y,t) = C(e^y, t)$
- $\mu^{cv} = \mu - \frac{\sigma^2}{2}$
- $\tau = T - t$

$$\frac{\partial u}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial y^2} - \mu^{cv}\frac{\partial u}{\partial y} + ru = 0$$

One more time for fun!

- $\phi = ue^{-\alpha\tau}$
- $r^{cv} = r + \alpha$

$$\frac{\partial \phi}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2 \phi}{\partial y^2} - \mu^{cv}\frac{\partial \phi}{\partial y} + r^{cv}\phi = 0$$

Discretization

Reverse Time Central Space

- Explicit
- Forward Time with $k = -k$
- $S \in [K, S_{max}]$
- $t \in [0, T]$
- $O(k) + O(h^2)$

The Scheme

- $Upper = \lambda(\mu^{cv} - \frac{\sigma^2}{2h})$
- $Diagonal = 1 + kr^{cv} + \frac{\lambda\sigma^2}{h}$
- $Lower = -\lambda(\mu^{cv} + \frac{\sigma^2}{2h})$

Boundary Conditions

- C(S,T) = Max(0,S-K)
- C(K,t) = 0

RESULTS

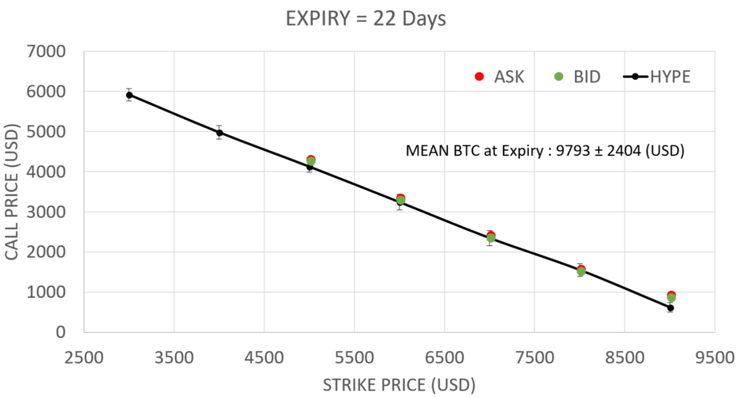


Figure: Hype Model evaluation for 30 day BTC Calls, N=15

RESULTS

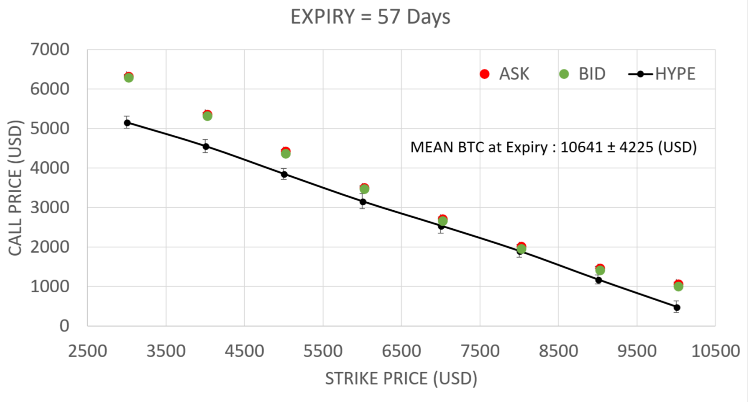


Figure: Hype Model evaluation for 60 day BTC Calls, N=15

Conclusion

Closing Remarks

- Volatility Model provides slight enhancement
- Hype Drift line can account for large traffic
- Black-Scholes equation is extremely susceptible to exponential growth
- Finite Differences works well, possibly try other methods.

Future work:

- Implicit Scheme (CN)
- Machine Learning for parameters fits
- Other Indicators besides Google Trends

Thanks for listening!

Questions or Comments?