Wildfire Spread as a System of Partial Differential Equations*

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Wildfires have devastated wildlife and Our structures time and time again. The exponential effect of climate change puts a very rapid timer on a very serious threat. Especially in California. Climate change has nearly doubled the destruction of wildfires in western California every 15 years for the past half century. We present two models, with the hope of simulating wildfire spread for various wildlife distributions. We have wrapped our problem into a discrete grid as a system of partial differential equations. However, our model does not play nicely within our simple leap-frog scheme. This has motivated the creation of a simplified model, We show that the simulations are often tied to the sensitive parameters that we have implemented further yielding instability within our scheme. Nonetheless we provide insight to analyzing and discussing future adaptations and possible improvements to similar models and schemes.

I. INTRODUCTION

A. Motivation

Wildfires throughout the centuries have ravaged wildlife, whether caused by man or nature. The devastation and destruction ripple through the coming decades in the aftermath. Especially in California, where each year we see the fiery blaze cascade through the country side. Unfortunately, wildfires will not go away in the coming decade. Wildfires are only exacerbated by climate change. In the last thirty years Abatzoglou et al found that climate change has nearly doubled the area of wildlife burned in Western N. American forests [2]. In order to preserve the wildlife and prevent the desolation of our structures we must first understand our enemy. Once we can visualize and anticipate the spread of fire in wildfires we can inform our response teams to efficiently mitigate key choke points in wildfires. This motivates our work in this paper, the core belief that modeling the spread of wildfires will provide key insight to the fire response teams and civilians in danger.

B. Background

Wildfire modeling and prevention have been studied for decades. Scott found the key parameters that determined the spread of wildfire was based off of various metrics of the forest. [1]

We see from Fig 1. that the dominant metrics for fire spread are Landscape, Fuel type, Moisture, and wind. We implement the these metrics into a single term that groups all of these effects into spread and intensity.

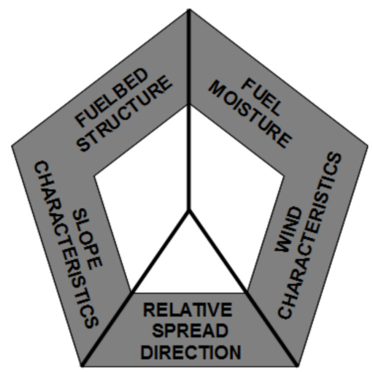

FIG. 1. The five major components to Scott's[1] fire spreading model. We toss these terms into a single component of our Fire spreading term.

Where we have the spread term act as a physical heat transfer term, convection to the surroundings. Where our source term is simply an initial starting point of heat. We evolve the system around three key concentrations; Fire, Fuel/Wildlife and Response

C. Fire, Life and Response

Our model breaks down wildfires into three key components. Fire, denoted as F. Which represents the relative concentration of fire at that point in space and time. Life represents the relative concentration of wildlife, this is our representation of the fuel load at a specific point. Response is a term that represents the relative affect of fire fighters or other response teams that limit the spread of fire at a point in space and time. These will be the key terms in our model, as we evolve our system we sill the dynamic relationship between each component.

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II. METHODS

A. Creating the Model

We set out to represent each term in our model from our intuition and literature of similar nature. We have developed our own representation of wildfires, with this creation comes its own set of problems that we will see later on. For now let us discuss our two models. One comprised of linear and non-linear terms, the other comprised of only linear terms.

1. Non-linear Model

Here we lay the framework for our interesting but implementation is difficult within a linear scheme, such as the one we will lay out. Intialiy we placed our spreading term as dependent on the relative concentration of fire at a point, with the spread direction dependent on the divergence of the fire. A simple reduction due the response and an increase from our sourcing term L(t,x,y).

$$\dot{F}(t,x,y) = r_F F(\nabla \cdot F) - \gamma R + \phi L \tag{1}$$

Our response evolves with some constant growth, then increases as the total amount of area on Fire grows. Then Focuses in on points where the fire is spreading rapidly.

$$\dot{R}(t,x,y) = R_0 + r_R(A_F)(\nabla \cdot F) \tag{2}$$

where $r_R(A_F)$, the area on fire is given by,

$$A_F = \iint_D F(t,x,y)dxdy \tag{3}$$

and r_R(x) the response growth parameter is given by.

$$r_R(x) = \sum_{i=1}^{N} k_i(x)^i \tag{4}$$

i is related to the degree of response, N represents the total degrees of response to a wildfire and k_i is our scaling coefficient. All of our non-linearity packed into $\dot{R}(t,x,y)$. A very explosive term that gave us quite the trouble. Which motivated a more simplistic case that can be readily solved with finite differences.

$$\dot{L}(t,x,y) = -r_L\dot{F} \tag{5}$$

A simple representation of the fuel decreasing relative to the fire currently going at a point in space and time. This model while fun and intriguing does not provide much insight to us, as we cannot readily visualize and evolve the system due to the non-linear terms not playing nicely. Thus we suggest the following simplified version.

2. Linear Model

The concept behind the linear model was to implement the heat transfer term. We see the fruition of this term with the Laplcian in Eq 1. However, our model implements this term, scaled by the amount of fire in that area. The meaning of this term is to implement the physical nature of the fire spreading to its neighbors, as any heat source would. The next term in the rate of change of fire is the response term, culling the spread of fire. The last term is our sourcing term, where we provide fuel to the fire at the initial outbreak of the fire.

$$\dot{F}(t,x,y) = r_F\nabla^2 F - \gamma R + \phi\delta_{x0,y0}L \tag{6}$$

Our rate of change of the response has some base response, meaning our fire fighting teams efforts grow the longer the fire rages on. The divergence term simply states that response teams will surge to the areas that are spreading fire rapidly.

$$\dot{R}(t,x,y) = R_0 + r_R\nabla \cdot F(t,x,y) \tag{7}$$

For simplicity we assumed a linear form for the initial sourcing term. Each step decreasing, yielding equation blah.

$$\dot{L}(t) = -L_0 \tag{8}$$

In our simplest case this is our model. Where r_F , r_R , R_0 , L_0 , γ , ϕ are parameters to be fit with data. Assuming some size of our grid we can simulations for different fuel load distributions, then compare the relative are burned between our simulation and the data. The parameters can also be used scale different effects in different simulations. Say we increase γ by a order of magnitude, we can reasonably see the effect between simulations.

3. Initial Conditions and Boundary Values

We choose our boundary conditions to reflect an isolated area and a single source point. r_i is related to the intensity of the initial outbreak.

$$F(0,x,y) = r_i\delta_{x0,y0} \tag{9}$$

At all points on our boundaries we set the Fire to zero, as we expect the grid to be isolated and nothing comes in or out.

$$F(t,B) = 0 \tag{10}$$

The initial starting rate of change is simply related to how much fuel is at that point.

$$\dot{F}(0,x,y) = L_0 \tag{11}$$

Lastly to prevent any strange interactions we limit F,R, and L to be greater than or equal to zero.

$$F(t,x,y) \geq 0, R(t,x,y) \geq 0, L(t,x,y) \geq 0 \tag{12}$$

B. Discretization

1. *Leap-Frog*

Making use of a simple leap frog scheme we are able to discretize our model within an appropriate degree of accuracy and speed. However, mimetics could readily be implemented. Our discretization of the linear and non-linear models are as follows;

a. ***Non-Linear***

$$F_{i,j}^{n+1} = 2\Delta t[r_F F_{i,j}^n D - \gamma R_{i,j}^n + \phi L_{i,j}^n] - F_{i,j}^{n-1} \tag{13}$$

where D is given by,

$$D = \frac{1}{2\Delta s}[F_{i+1,j}^n - F_{i-1,j}^n + F_{i,j+1}^n - F_{i,j-1}^n] \tag{14}$$

where $\Delta s = \Delta x = \Delta y$

$$R_{i,j}^{n+1} = 2\Delta t[R_0 + r_R(A_F)D] - R_{i,j}^{n+1} \tag{15}$$

To evaluate the integral in this term we made use of a simple 2D trapezoid integration, a quick and dirty solution to yield fast results.

$$L_{i,j}^{n+1} = -r_L 2\Delta t[F_{i,j}^{n+1} - F_{i,j}^{n-1}] - L_{i,j}^{n-1} \tag{16}$$

b. ***Linear***

$$F_{i,j}^{n+1} = 2\Delta t[r_F L - \gamma R_{i,j}^n + \delta_{x_0,y_0}\phi L_{i,j}^n] - F_{i,j}^{n-1} \tag{17}$$

where the Laplacian L is given by,

$$L = \frac{1}{(\Delta s)^2}[F_{i+1,j}^n + F_{i-1,j}^n + F_{i,j+1}^n + F_{i,j-1}^n - 4F_{i,j}^n] \tag{18}$$

$$R_{i,j}^{n+1} = 2\Delta t[R_0 + r_R D] - R_{i,j}^{n+1} \tag{19}$$

$$L_{i,j}^{n+1} = -2\Delta t L_0 - L_{i,j}^{n-1} \tag{20}$$

Where i,j are spatial points discreetly representing x and y. n is the discretization of time.

Each color in Fig 2. represents a different value of L, in this case we have distributed five different values randomly across this grid. This yields a randomized forest with unpredictable wildlife and various spread parameters for each type of wildlife in a discrete point.

2. *Evolving the System*

In order to evolve the system we simply initialized the grid, run our Leap-Frog scheme, each step checking if the fire at each point was greater than zero. If all the points are zero then we exit the simulation. If all the points are greater than zero we assume the system has entered a burned state. Where the entire grid has been burned to crisp.

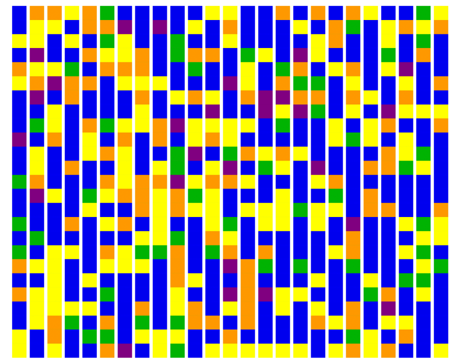

FIG. 2. 25x25 Grid of our forest, rigid representation of what our model looks like through a magnifying glass.

III. RESULTS

A. Stability

1. *Non-Linear Model*

As you have probably figured, the non-linear model simulation does not end very well. Clearly we see in FiG

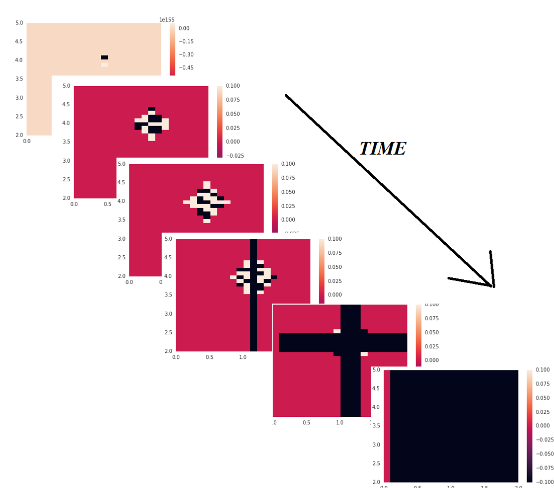

FIG. 3. The instability of our non-linear model is what motivated the creation of the linear model. As we clearly see the explosiveness of R(t,x,y) is this simulation within just a few time steps.

3. that our non-linear terms are immensely explosive and push the system to one end of infinity. We see that initially the model worked well, where black is low response and white is high fire. However due to the rapid growth of the response function r_F the response term quickly dominates the system.

2. Linear Model

The stability of our linear model is completely dependent on our constants/parameters. This motivates a need for a parameter sensitive analysis, with a bifurcation diagram of a linearized model.

B. Evolution of the Linear Model

We are currently implementing our scheme and linear model. We predict that our linear model will have large regions of stability that will be easy to dial into. However, we must first provide some insightful starting points for the parameters.

IV. DISCUSSION

Implementation of scheme are clearly hindering the dynamic relationship between each term. As we see that the non-linear terms dominate the system. Motivating a need for a new scheme and revaluation of the model to be compatible within a numerical context. The linear model shows promise but until our parameters have found a stable home we cannot fully endorse one model or the other.

A. Parameter Sensitivity

Our models are extremely sensitive to many parameters. This means that our results should be taken with a grain of salt. We found that adjusting a single parameter could shift the stability of our scheme to a unstable region. Yielding an unfortunate result, these instabilities are riddled throughout the problem and pop up one after another. The key to figuring out the

B. Stability Analysis

Linearization of our model to translate the system to a system of ODEs is currently being implemented. This will allow us to determine regions of instability and equilibrium points. This would provide insight to where the model will break down. Thus we can analyze any bifurcations that arise.

V. CONCLUSION

Wild fires currently storm across California and Mexico during the writing of this paper. Speaking volumes of the urgency that we will need to respond to the escalation of wildfires due to climate change. If we do not act swiftly and efficiently we will see a loss of wildlife on a scale unseen before in our species lifetime. Motivating our work in this paper we have created two similar and complex models within a second order accuracy scheme with mixed results. We provide an analysis of the stability within a linear scheme but for any meaningful conclusions we would need to adapt our model and scheme to be compatible. Nonetheless the models could also be improved upon, implementation of a delay differential equation would further complicate the system but would be an accurate representation of the delayed response of the fire fighters to the active fire zones. Implementing R(t) as R(t-k) provides the desiered result. However the complication of this term shifts our model a new system of differential equations that This may not be an easy problem to solve but the problems that need to solved rarely are.

[1] Scott, Joe H. *Introduction to Wildfire Behavior Modeling.* National Interagency Fuels, Fire, Vegetation & Technology Transfer, Wild Fire Managment RD&A (2012)

[2] Abatzoglou, John T., and A. Park Williams. *Impact of anthropogenic climate change on wildfire across western US forests.* Proceedings of the National Academy of Sciences 113, no. 42 (2016)