Pset3

4.1)

a. You need 2 points. This is because each point contains two numbers, and the total degrees of freedom in 2D is 3. Therefore two points, containing a total of 4 numbers, is enough to constrain all 3 DOF because 4 > 3.

b. You need three points for similar reasons. Each point in 3D carries 3 numbers, and their are 7 DOF in 3D space. Therefore you need at least three points, or nine numbers, to constrain all 7 DOF.

4.4)

a. The decision variables would be the translational component of a 2D transform since we know the rotation to be already correct. This means we are searching for an x and y that when added to the scene points make the two shapes overlap.

b. The decision variables do not show up in the constraints since the translation part of a transformation could be infinite in either x or y. This is unlike the rotation matrix which could also be a part of the transformation we are solving for which must be constrained to be a valid rotation matrix.

c. The objective function is the sum of the square of the L2 norms of the relative position between corresponding points.

For P_O = [0 0], sum = 327

For P_O = [3 10], sum = 0

For P_O = [6 12], sum = 39

d. It would be quadratic with respect to the x and y of P_O. This means the problem is convex and can be solved quickly and a global optimum will be reached.

```python
import numpy as np

def obj(p_s, p_m):
    s = 0
    for i in range(len(p_s)):
        s += np.linalg.norm(p_s[i] - p_m[i]) ** 2
    return s

p_s = np.array([[1,5],
                [3,10],
                [5,10]])

p_om = np.array([[-2,-5],
                [0,0],
                [2,0]])

p_o = np.array([[0,0],
                [3,10],
                [6,12]])

for p in p_o:
    print(p)
    print(obj(p_s, p_om + p))
```

4.5) Done on paper
4.8) Paper

Survey: Meshcat Mustard

(4.5) a) $\min\limits_{p_x\, p_y\, a\, b} \sum\limits_i \left\| \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} a & -b \\ b & a \end{pmatrix} p^{mc_i} - p^{s_i} \right\|^2$

$a^2+b^2=1$

$$\left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} p^{mc_i} - p^{s_i} \right\|^2$$

$$\left\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|^2 + \left\| \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\|^2$$

$$0$$

Sum can't be smaller than 0

Will converge correctly if $p^{mc_1}{}_{,x} < 0$
and if $p^{mc_2}{}_{,x} > 0$

$+ J^T F$

b) $\min_{p_x p_y a b} \sum_i \left\| \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} a & -b \\ b & a \end{pmatrix} p^{mc_i} - p^{s_i} \right\|^2$

$\downarrow$

$\left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} p^{mc_i} - p^{s_i} \right\|^2$

$\downarrow$

$\left\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|^2 + \left\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|^2$

$\downarrow$

Zero is smallest possible sum $\rightarrow 0$

Will converge if $p^{mc_1}_x > 0$
and if $p^{mc_2}_x < 0$

c) $W = \sum_{i=1}^{N_s} (p^{s_i} - p^{\bar{s}})({}^{\theta}p^{m_i} - {}^{\theta}p^{\bar{m}})^T$

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${}^{\theta}p^{\bar{m}} = \frac{1}{N_s} \sum_{i=1}^{N_s} {}^{\theta}p^{mc_i}$

$p^{\bar{s}} = \frac{1}{N_s} \sum_{i=1}^{N_s} p^{s_i}$

$W = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$N_s = 2$

${}^{\theta}p^{\bar{m}} = \frac{1}{2}($

4.8 a) 1)

Points	$d^2=$
(b_1, r_1)	2
(b_2, r_2)	16
(b_2, r_3)	2
(b_3, r_4)	2
(b_3, r_5)	17
(b_4, r_6)	2

error = 41

2) r_2, r_5 outliers, largest errors

3) error_new = error − (16 + 17)

error_new = 8

b) 1) b_4, $d = 4$

2) r_1, $d = \frac{1}{\sqrt{2}}$

3) Scheme 2