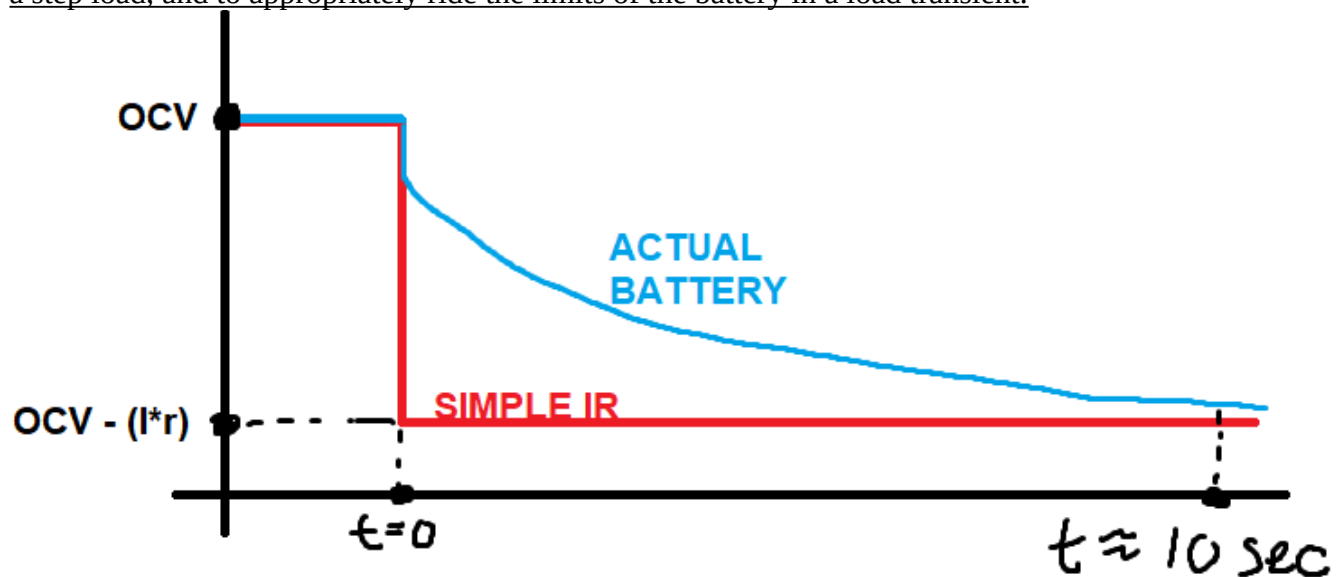


1) Purpose:

To have a battery model that can effectively capture the internal dynamics of a battery cell and give insights as to what transients will be present in response to a load change attached to the cell, we need a way to stimulate the battery cell to probe these internal parameters, which are otherwise unobservable when conducting static tests (constant current, constant power, etc.). To briefly reiterate, we care about these transients because the simple IR model of a cell (an ideal voltage source in series with a resistor representing the internal resistance) provides a lower bound for what the cell voltage will be when a step load is applied to the cell. For a given current demanded, a higher predicted voltage implies more power for the same amount of current. For a given power demanded, a higher predicted voltage implies less current. Being able to model these battery transients elevates us from the simple IR model, being able to accurately predict the battery voltage given any load condition, not just a step load, and to appropriately ride the limits of the battery in a load transient.



X axis: time

Y axis: voltage

Load step at $t=0$ seconds

OCV \rightarrow open circuit voltage of cell at rest

$I \rightarrow$ current drawn in step load

$r \rightarrow$ modeled internal battery resistance for simple IR model

In order to find these internal parameters, we need a way to excite the battery cell so that it is not in equilibrium. In the general case, the load a battery cell experiences can be equated to the instantaneous current out of the cell. A case can be made that instantaneous power is a better metric for the load demanded of the cell, but for modeling and simulation purposes thinking of the current drawn from the battery cell as the load will be easier to work with since current itself is a system state variable, while power is a product of system state variables ($V \times I$). Backing out what V and I would need to be given a cell output power can get very ugly (in the simple IR model, you would need to solve a quadratic equation. If your model is more complex... good luck).

Understanding the current in the battery cell as the demanded load of the cell, we would like a way to draw changing currents from the cell using an external circuit. What should this current waveform look like? The answer is a sinusoid. This is because driving an LTI (linear time invariant) system with an external force that is purely sinusoidal causes all system variables of that system to also be sinusoids, making analysis in the frequency domain equivalent to solving for state variables (V and I) of a resistor network. This is the basis of how we analyze circuit containing components that are linear time invariant, like resistors, capacitors, and inductors. We will make the assumption that the internals of a battery cell can be accurately modeled using resistors, capacitors, and inductors, as well as possibly ideal voltage or current sources, an assumption that may not be entirely valid. The internals of a battery cell do exhibit behaviors such as hysteresis, nonlinearities with respect to system state variables (current dependent RLC values for example), temperature dependence (an unmodeled system variable that we are not directly measuring), among other things. These effects will undermine our ability to model the internal dynamics of a cell using LTI components, but it has been shown in academic papers that the Randles model, a circuit containing only ideal voltage sources, resistors, and capacitors, is an adequate model for a battery cell. The effectiveness of an LTI system for our case in legged robots is still unknown, and is the topic of this research.

A problem with creating such a driver to push a sinusoid into the battery (applying a sinusoidal voltage or current across the battery terminals), is that one half of the sinusoid will have to apply positive power and the other half will have to apply negative power. This means the driver circuit will need to be able to both sink and source power. This difficulty is only exacerbated when going to lower frequencies as the amount of power sourced or sunk in each half cycle increases.

We have acquired two large bi-polar power supplies capable of tracking an input signal with a bandwidth of around 5-10kHz with a peak current of +/- 20A each. You will need to write a script to inject sinusoidal currents into the cell with different bias currents. Basically, pick a fixed amplitude current sinusoid and vary both the frequency and the constant current added to vertically offset this sinusoid. Your fixed amplitude current sinusoid has to be big enough such that the corresponding voltage sinusoid is large enough to be measured. Remember that the voltage drop due to a current in a cell is related by the apparent resistance of the cell, and resistances of cells typically exist in the range of 5-20 milliOhms. A 1amp current sinusoid results in a 20mV voltage sinusoid. The voltage waveforms we are going to measure will be quite small while also being on top of the cell voltage, around 2.5V-4.2V, which is why I bought the 12 bit oscilloscope.

2) The Theory:

We need to apply voltage waveforms across the cell and measure the corresponding current waveforms produced (typically use Potentiostat), or we need to do the opposite and send a current waveform into the cell and measure the voltage waveform across the cell (typically use Galvanostat). In either case, whether we have a driver circuit that can control the voltage across the cell or the current into the cell, we will be able to measure the other, dependent waveform which is related to our input signal by the impedance of the cell.

$(Z = V\angle 0 / I\angle \Phi)$ if we are driving the voltage and measuring amplitude and phase of current
 $(Z = V\angle \Phi / I\angle 0)$ if we are driving the current and measuring amplitude and phase of voltage

Phasor Recap: Remember that we are representing the voltage and current values with phasors here, since we are operating in the frequency domain. Remember that a phasor only encodes two properties: the amplitude of the sine wave (just like you would quantify the voltage of a DC voltage), and the phase shift. The driving sinusoid which is typically assigned a phase shift of 0 radians, so you can think of the phase shift of the dependent signal to be a phase shift relative to the driving sinusoid (you could always take every signal present in a circuit and add on a constant phase shift and the solution would still be perfectly valid. You are just changing the zero crossing time of the voltages and currents, equivalent to a time domain time offset. Thus the convention is to assign your driving circuit a phase offset of 0 radians to keep it

simple). Phasors do not explicitly encode the frequency because it is redundant to do so. All waveforms in an LTI circuit, which are driven by a single frequency input sinusoid, will all have the same frequency as the driving sinusoid but potentially have a different amplitude and phase. **ALSO REMEMBER THAT ALL PHASOR MATH REPRESENTS FREQUENCY IN RAD/S AND NOT HERTZ! DO NOT ACCIDENTALLY DO MATH WITH THE FREQUENCY IN HERTZ!** $2\pi f = \omega$

Once we have the current and voltage phasors for a specific driving frequency, dividing them gives the impedance of the cell. This impedance is a complex number, and if we take multiple impedance measurements while sweeping the driving frequency we can make one of those Nyquist plots you read about in the EIS readings. The goal of EIS is to produce one of those Nyquist plots. Then you can fit a model to the data in the Nyquist plot in a least squares sense for a fairly good estimate for the frequency dependent internal impedance of the battery cell.

One subtlety here is recognizing how our assumption of the linearity of the system is going to affect how well our model will reflect the actual behavior of the battery cell which is an inherently nonlinear system. Our original protocol to divide the voltage and current phasors to obtain impedance needs to be inspected further. The impedance is given as the ratio of the voltage and current phasors: whatever their values may be does not matter, only their ratio.

Example ideal resistor with AC driving signals:

$$2V\angle 0 / 1A\angle 0 = 2\text{Ohms}$$

$$4V\angle 0 / 2A\angle 0 = 2\text{Ohms}$$

$$600V\angle 0 / 300A\angle 0 = 2\text{Ohms}$$

Despite the voltage and current waveforms across this imaginary ideal resistor being wildly different, the ideal resistor will always present itself as an ideal resistance, with zero phase shift. In reality, components present some sort of non linearity to input driving signals, especially that of the internal impedance of a battery cell. In the case of a battery cell, you might measure something like this:

Battery cell responding to AC voltage driving signal:

$$200\text{mV}\angle 0 / 1A\angle 0.13 = 200 \text{ mOhms}$$

$$400\text{mV}\angle 0 / 1.9A\angle 0.24 = 211 \text{ mOhms}$$

$$1V\angle 0 / 4.1A\angle 0.38 = 244 \text{ mOhms}$$

In this case, a doubling of our input voltage amplitude did not lead to a doubling of our measured current amplitude. There also might be phase shift that changes with input voltage amplitude, which should not be affected in an ideal LTI system. Batteries are nonlinear systems. To account for such effects, we will need to capture multiple Nyquist plots.

A Nyquist plot encodes the real and imaginary parts of the impedance of a system as you sweep through frequency. Varying the driving signal amplitude and bias current does not matter if the system is truly linear. In our case we will not make the assumption that the system is perfectly linear because we know battery cells exhibit nonlinear behavior. We will need multiple Nyquist plots to capture this effect, each Nyquist plot captured with a different input signal bias current (or vertical offset current).

In our case, we would like to conduct each frequency sweep with the input signal being a current waveform and measure the corresponding voltage waveform. We will conduct frequency sweeps for a list of bias currents, making a Nyquist plot that corresponds to a specific bias current. This will make a 2D lookup table to find the impedance of the battery cell, indexed by both the driving frequency and the bias current. Your objective to generate this table, so that I can use it to fit a frequency and bias current dependent equivalent circuit model for use to inform constraints on battery voltage in the robot trajectory optimizer.