Miguel Flores-Acton September 16, 2024

6.1200 Problem Set 2

Problem 1 *(Collaborators: None)*

Case : d > 20 If $d \leq 20$, then that means that $a, b, c \leq 20$. Adding together a, b, c we get that $a + b + c + d \leq 80$. Plugging in the values to the equation $100 - (\leq 80) = e$. We get that $e \geq 20$. $\square$

Problem 2 *(Collaborators: None)*

If $\log_2 3$ is rational, then it can be represented by $\frac{a}{b}$ where a and b are coprime natural numbers. We can rearange the inequality to $2^{\frac{p}{q}} = 3$ and further to $2^a = 3^b$. This means that 2 is only 2^a is only divisible by 2 and 3^b is only divisible by 3. Since both 2 and 3 are prime numbers, this will never be true except for zero, and we know that $\log_2 3$ is a non zero number. $\square$

Problem 3 *(Collaborators: None)*

Proof :

The proof is by induction, using the inductive hypothesis $F(n)F(n+1) = F(0)^2 + F(1)^2 + \ldots + F(n)^2$

BaseCase :

$F(0)^2 = F(0) \times F(1)$: $0 = 0$ is true

InductiveStep :

If we're trying to evaluate $F(n) \times F(n+1)$ that means by induction $F(n-1) \times F(n)$ is assumed to be true, therefore:

Assume $n \geq 1$ $F(n-1) \times F(n) + F(n)^2 = F(n) \times F(n+1)$

$F(n) \times (F(n-1) + F(n)) = F(n) \times F(n+1)$

And we know that because of the fibbonacci sequence $F(n-1) + F(n) = F(n+1)$, so:

$F(n) \times F(n+1) = F(n) \times F(n+1)$

This means that $\forall n \in \mathbb{N}. F(n)F(n+1) = F(0)^2 + F(1)^2 + \ldots + F(n)^2$ $\square$

Problem 4 *(Collaborators: None)*

Part 4(a)

Part 4(b)

Shivam never mentions x_1¿0, however just brings it out of thin air in the end of the inductive step.

Part 4(c)

Zach's problem is that when saying the statment $x_i = 4x_{i-1}$ for all $i \geq 0$ he is wrong. Also, in the $x_{i+1} = 6x_i - 8x_{i-1}$, when $i = 1$ $8x_{i-1}$ relys on x_0 which is not defined.

Part 4(d)

Proof :

Using regular induction on the hypothesis $R(i) :=$ "$x_i \geq 3x_{i-1} AND x_i > 0$".

Problem 5 *(Collaborators: None)*

Part 5(a)

$P(k) :=$ "for all pairs $m, n > 0$ with $m \times n = k$, we have $(k-1)$ is the splits needed to break up the chocolate bar"

Part 5(b)

Proof :

The proof is by induction, using $P(k)$ as our inductive hypothesis

BaseCase :

If $k = 1$ then $m, n = 1$ and there are no splits we have to do. And $k - 1 = 0$

InductiveStep :