Problem Set 3

Due Friday, September 27th by 5pm

(10 points per question. Please scan and upload to Canvas as a PDF)

Question 0: If you worked with up to two classmates, please list their names!

1. For each of the following examples, complete the following tasks:

Task 1: Regiment the following argument in $\mathcal{L}^{\text{PL}}$.

Task 2: Then, evaluate the resulting $\mathcal{L}^{\text{PL}}$ argument by either writing a semantic proof that it is valid, or else specifying truth values for the sentence letters and proving that the argument is invalid.

(a) *If the lawyer did it, then the doctor did not. Therefore, if the doctor did it, then the lawyer did not.* (B = the lawyer did it; G = the doctor did it.)

Proof

$$B \to \neg G \models G \to \neg B$$

1. $B \to \neg G$:PR
2. $\quad G$:AS
3. $\quad \neg B$:1,2$\to$E
4. $G \to \neg B$:2-3 $\to$I

Argument is valid

(b) *Naïve realism is false. This is because if naïve realism were true, then naïve realism would be false.* (R = naïve realism is true.)

Proof

$$R \to \neg R \models \neg R$$

1. $R \to \neg R$:PR
2. $\quad R$:AS
3. $\quad \neg R$:1,2 $\to$E
4. $\neg R$:2-3 $\neg$I

Argument is valid

2. Write a semantic proof that the following is a tautology.

   (a) $\big((P \vee Q) \wedge (P \vee R)\big) \rightarrow \big(P \vee (Q \wedge R)\big)$

Proof

P	Q	R	$(P \vee Q) \wedge (P \vee R) \rightarrow P \vee (Q \wedge R)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

It is a tautology because it is true for all values

3. Appeal to the definitions in order to prove the following lemmas from the book.

   (a) **Lemma 2.1** If $\Gamma \vDash \varphi$, then $\Gamma \cup \Sigma \vDash \varphi$.

Proof

$\Gamma \vDash \varphi$ means that Γ entails φ meaning that if all the premisis in Γ are true, then the premis φ must be true. For every premisis in Σ that is added, it only has the possibility to reduce the set of combinations of the sentence letters which make φ true. Therefore φ must still be entailed in the union set.

   (b) **Lemma 2.3** $\Gamma \vDash \varphi$ just in case $\Gamma \cup \{\neg\varphi\}$ is unsatisfiable.

Proof

If $\Gamma \vDash \varphi$, then there is never a case when Γ is true and φ is false. However if we add $\neg\varphi$ to Γ, there will now be a case when Γ is true in which φ is asserted to False in the premisis and φ will be true on the proposition. This means that $\Gamma \vDash \varphi$ in the case $\Gamma \cup \{\neg\varphi\}$ would be unsatisfiable.

4. Recall the definition of the interpretation set $|\varphi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi) = 1\}$ from the book, proving each of the following claims for arbitrary wfss φ and ψ of $\mathcal{L}^{\text{PL}}$:

   (a) $|\varphi \wedge \psi| = |\varphi| \cap |\psi|$.[1]

[1] $|\varphi| \cap |\psi| := \{\mathcal{I} : \mathcal{I} \in |\varphi| \text{ and } \mathcal{I} \in |\psi|\}$ is the intersection of the interpretation sets $|\varphi|$ and $|\psi|$.

Proof

We can define $|\varphi \wedge \psi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi \wedge \psi) = 1\}$
and since we know that $\mathcal{V}_{\mathcal{I}}(\varphi \wedge \psi) = 1$ is true if $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ and $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ is true, we can use the definistion of $|\varphi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi) = 1\}$
for both φ and ψ to rise to the conclusion that it means that $|\varphi| \cap |\psi|$.

(b) $|\varphi \vee \psi| = |\varphi| \cup |\psi|$.[2]

Proof

Just like before, we can define $|\varphi \vee \psi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi \vee \psi) = 1\}$
and since we know that $\mathcal{V}_{\mathcal{I}}(\varphi \vee \psi) = 1$ is true if $\mathcal{V}_{\mathcal{I}}(\varphi) = 1$ or $\mathcal{V}_{\mathcal{I}}(\psi) = 1$ is true, we can use the definistion of $|\varphi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi) = 1\}$
for both φ and ψ to rise to the conclusion that it means that $|\varphi| \cup |\psi|$.

(c) $|\neg\varphi| = |\varphi|^c$.[3]

Proof

We know that $|\neg\varphi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\neg\varphi) = 1\}$
and that means $|\neg\varphi| := \{\mathcal{I} : \mathcal{V}_{\mathcal{I}}(\varphi) = 0\}$
and that is the definition of $|\varphi|^c$

5. Prove that $\varphi, \psi \vDash \chi$ just in case $\vDash (\varphi \wedge \psi) \to \chi$.

Proof

$\vDash (\varphi \wedge \psi) \to \chi$ means that for every case in which φ and ψ are both true χ must be true as well. $\varphi, \psi \vDash \chi$ also means the same thing, where now φ and ψ are premisis but also both be true for χ to be true.

[2] $|\varphi| \cup |\psi| := \{\mathcal{I} : \mathcal{I} \in |\varphi| \text{ or } \mathcal{I} \in |\psi|\}$ is the union of the interpretation sets $|\varphi|$ and $|\psi|$.
[3] $|\varphi|^c := \{\mathcal{I} : \mathcal{I} \notin |\varphi|\}$ is the complement within the set of all $\mathcal{L}^{\text{PL}}$ interpretations.