1

a) $\ddot{x}(t) - t x(t) = e^t$

linear ODE, 2nd order, non homogenous

because $\ddot{x}(t) - t x(t) \neq 0$

b) PDE because its multiple variables

c) linear ODE, $\frac{d}{dt}\left(f^{(4)}(t)\right) = f^{(5)}(t)$

so it is 5th order

$\frac{d}{dt}(t^2) = 2t$ is not constant • f

So its non homogenous

d) ODE, non-linear because $\sin\theta$

2nd order

2

$$A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \qquad B = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

a)

$$AB = \begin{vmatrix} 0\cdot 1 + 1\cdot 2 & 1\cdot 1 + 2\cdot 0 \\ 3\cdot 0 + 4\cdot 1 & 3\cdot 1 + 4\cdot 0 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$$

b)

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{vmatrix} 3 \\ 5 \end{vmatrix} \qquad \begin{aligned} x + 2y &= 3 \\ 3x + 4y &= 5 \end{aligned}$$

$$x = 3 - 2y \qquad 3x = 9 - 6y$$

$$9 - 6y + 4y = 5$$

$$\boxed{\begin{aligned} y &= 2 \\ x &= -1 \end{aligned}}$$

c)

$$v \cdot w = 1\cdot 0 + 1\cdot 1 + 0\cdot 1 = \boxed{1}$$

$$\cos\theta = \frac{v\cdot w}{|v||w|} \qquad |v| = |w| = \sqrt{1^2 + 1^2 + 0^2}$$

$$\cos\theta = \frac{1}{2} \qquad \boxed{\theta = 60^\circ}$$

2)

$v^t w = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 = \boxed{1}$

$v w^t = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}}$

3 a) $\dfrac{1+i}{1-i} \dfrac{(1+i)}{(1+i)} = \dfrac{\cancel{1} + 2i - \cancel{1}}{1+1} = i$

$\mathrm{Re}|i| = 0$ $\qquad\qquad$ $\mathrm{Im}|i| = 1$

b) $e^{17i + \ln 2}$ $\qquad$ $|e^{17i} e^{\ln 2}| = 2$

rotation $\qquad$ mag

$\mathrm{Re}|2| = 2$ $\qquad\qquad$ $\mathrm{Im}|2| = 0$

c) $f(t) = e^{2t + \pi i t}$

$f(t) = e^{\pi i t} \cdot e^{2t} \leftarrow$ $\boxed{\text{spiral outward}}$

rotation

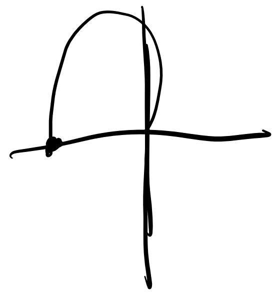
d)

$$f(t) = e^{\pi i t} \cdot e^{2t}$$

rotation

$t > 0$,

$$\text{Im}\left[e^{\pi i t}\right] = 0$$



$$e^{\pi i} = -1$$

$$f(1) = -1 \cdot e^{2}$$

$$\boxed{t = 1,\ f(1) = -e^{2}}$$

(4) a)

$$e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

$$\cos(x+y) = \text{Re}\left[e^{i(x+y)}\right]$$

$$\sin(x+y) = \text{Im}\left[e^{i(x+y)}\right]$$

$$e^{i(x+y)} = e^{ix}e^{iy} = (\cos x + i\sin x)(\cos y + i\sin y)$$

$$= \cos x\cos y + i\sin x\cos y + i\sin y\cos x - \sin x\sin y$$

$$\boxed{\begin{aligned}\cos(x+y) &= \cos x\cos y - \sin x\sin y\\ \sin(x+y) &= \sin x\cos y + \sin y\cos x\end{aligned}}$$

b) $A\cos(wx - \phi) = A\cos(wx)\cos(-\phi) - A\sin(wx)\sin(-\phi)$

$\cos(x) + \sin(x)$ $\boxed{w = 1}$, same freq

$\cos(-\phi) = -\sin(-\phi)$ $\cos\phi = \sin(\phi)$ $\boxed{\phi = \pi/4}$

$\cos\phi = \sin\phi = \frac{\sqrt{2}}{2}$

$A\frac{\sqrt{2}}{2}\left(\cos(x) + \sin(x)\right) = \cos x + \sin x$

$\boxed{A = \sqrt{2}}$ $\cos x + \sin x = \sqrt{2}\cos\left(1 \cdot x - \pi/4\right)$

5 a) $(r-2)^4$ $r = 2$

$$\boxed{y(t) = \left(c_1 + c_2 t + c_3 t^2 + c_4 t^3\right) e^{2t}}$$

b) $\boxed{y(t) = \left(c_1 + c_2 t + c_3 t^2 + c_4 t^3\right) e^{2t} + \frac{1}{16}}$

(6)

a) $r^2 + 2r + 2 = 0$ $\quad \dfrac{-2 \pm \sqrt{4 - 4(1)(2)}}{2}$

$\dfrac{-2 \pm 2i}{2}$ $\quad r = -1 \pm i$ $\quad \boxed{r = -1 + i \quad r = -1 - i}$

b) $f(t) = c_1 e^{-t + it} + c_2 e^{-t - it}$

$$\boxed{f(t) = e^{-t}\left(c_1 e^{it} + c_2 e^{-it}\right)}$$

c) $e^{it} = \cos(t) + i\sin(t)$

$y(t) = e^{-t}\left(c_1 \cos(t) + c_1 i \sin(t) + c_2 \cos(t) - c_2 i \sin(t)\right)$

$$y(t) = e^{-t}\left((c_1 + c_2)\cos(t) + i(c_1 - c_2)\sin(t)\right)$$

$c_1 = a_1 + b_1 i$ $\quad a_1 + a_2 = A$ $\quad i(b_1 i - b_2 i) = B$

$c_2 = a_2 + b_2 i$ $\quad b_1 + b_2 = 0$ $\quad b_2 - b_1 = B$

$c_1 + c_2 = A$ $\quad a_1 - a_2 = 0$

$i(c_1 - c_2) = B$

$$\boxed{y(t) = e^{-t}\left(A\cos(t) + B\sin(t)\right)}$$

d)

$y(0) = 1 \cdot A \qquad A = 1$

$y'(t) = -e^{-t}\left(\cancel{-A\sin(t)} + B\cos(t)\right)$

$y'(0) = 1 \cdot B \qquad B = 1$

$$\boxed{y(t) = e^{-t}(\cos(t) + \sin(t)}$$