a) $S_N = 1 + a + a^2 + \dots a^{N-1}$

$aS_N = a + a^2 + \dots a^N$

$S_N - aS_N = 1 - a^N$

$S_N(1-a) = 1 - a^N$

$\boxed{S_N = \frac{1-a^N}{1-a}}$

fails when $\boxed{a = 1 \rightarrow S_N = N}$

b) $\frac{1-a^N}{1-a}$ $a \rightarrow \infty$, if $|a| < 1$ $a^N \rightarrow 0$

$\boxed{S_\infty = \frac{1}{1-a} \text{ for } |a| < 1}$

c) $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ $\sum_{n=1}^{\infty} n a^{n-1} = \frac{1}{(1-a)^2}$

$\sum_{n=1}^{\infty} n a^n = \frac{a}{(1-a)^2}$ $S_d = \sum_{n=0}^{\infty} (n+1) a^n = \sum_{n=1}^{\infty} n a^n + \sum_{n=0}^{\infty} a^n$

$S_d = \frac{1}{1-a} + \frac{a}{(1-a)^2} = \boxed{\frac{1}{(1-a)^2} \text{ for } |a| < 1}$