a) $\cos^2 x = \frac{1+\cos 2x}{2}$ $\quad \cos^5 = \left(\frac{1+\cos 2x}{2}\right)^2 \cos x$

$\left(\frac{1+2\cos 2x + \frac{1+\cos 4x}{2}}{4}\right)\cos x$ $\quad \left(\frac{1+2\cos 2x + \cos^2 2x}{4}\right)\cos x$

$\frac{3+4\cos 2x + \cos 4x}{8}\cos x$ $\quad \frac{3\cos x + 4\cos 2x\cos x + \cos 4x \cos x}{8}$

$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

$\frac{6\cos x + 4\cos(3x) + 4\cos(x) + \cos 5x + \cos 3x}{16}$

$\cos^5 x = \frac{1}{16}(10\cos x + 5\cos 3x + \cos 5x)$

$\boxed{c_0 = 0,\ c_1 = \frac{10}{16},\ c_3 = \frac{5}{16},\ c_5 = \frac{1}{16}}$

b) $\cos^5 x = \left(\frac{e^{ix} + e^{-ix}}{2}\right)^5 = \frac{(e^{ix} + e^{-ix})^5}{32}$ binomial theorem

$e^{5ix} + 5e^{4ix}e^{-ix} + 10e^{3ix}e^{-2ix} + 10e^{2ix}e^{-3ix} + 5e^{ix}e^{-4ix} + e^{-5ix}$

$= \left(e^{5ix} + e^{-5ix} + 5e^{3ix} - 5e^{3ix} + 10e^{ix} + 10e^{-ix}\right)\frac{1}{32}$

$\boxed{c_0 = 0,\ c_1 = \frac{10}{16},\ c_3 = \frac{5}{16},\ c_5 = \frac{1}{16}}$

c) $\sin^2 x = \frac{1-\cos 2x}{2}$ $\quad \sin^4 x = \frac{1-2\cos 2x + \cos^2 2x}{4}$

$\sin^4 x = \frac{2-4\cos 2x + 1 + \cos 4x}{8}$ $\quad \sin^5 x = \frac{3\sin x - 4\sin x\cos 2x + \sin x\cos 4x}{8}$

$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$\frac{1}{8}(3\sin x - 2(\sin 3x - \sin x)) + \frac{1}{2}\sin 5x - \frac{1}{2}\sin 3x$

$\sin^5 x = \frac{1}{16}(10\sin x - 5\sin 3x + \sin 5x)$ $\quad \boxed{c_0 = 0,\ d_1 = \frac{10}{16},\ d_3 = \frac{-5}{16},\ d_5 = \frac{1}{16}}$

d) $\sin\theta = \frac{e^{jx} - e^{-jx}}{2j}$ $\quad \sin^5 = \frac{1}{(2j)^5}(e^{jx} - e^{-jx})^5$

$e^{5jx} - 5e^{4jx}e^{-jx} + 10e^{3jx}e^{-2jx} - 10e^{2jx}e^{-3jx} + 5e^{jx}e^{-4jx} - e^{-5jx}$

$\frac{1}{32j}\left(e^{5jx} - e^{-5jx} - 5(e^{3jx} - e^{-3jx}) + 10(e^{jx} - e^{jx})\right)$

$\boxed{c_0 = 0,\ d_1 = \frac{10}{16},\ d_3 = -\frac{5}{16},\ d_5 = \frac{1}{16}}$

e) $\cos x = \frac{e^{jx} + e^{-jx}}{2}$ $\quad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$

Euler is easier to do operations, while trig is easier to interpret in final form