Problem Set 4

Due Friday, October 4th by 5pm

(10 points per question. Please scan and upload to Canvas as a PDF)

Question 0: If you worked with up to two classmates, please list their names.

1. A finite string whose elements belong to $\{a, b\}$ is an A-PALINDROME just in case it is a palindrome that has 'a' as a middle letter.

   Task 1: Provide a recursive definition of the set of a-palindromes without appealing to the property of being a palindrome as in the rough definition above.

   Proof

   1. Base case 'a' is an a-palindrome
   2.
   3. palindrome = letter + palindrome + letter where letter is in {a,b}
   4. And any a-palindrome can be built this way

   Task 2: Prove by induction that every a-palindrome has an even number of 'b's.

   Proof

   1. Base case: "a" is a palindrome, number of b is even
   2. Recursive Step: for any a-palindrome, a...a does not change the number of b's so they are still even. If b...b, then number of B's increases by 2 and an even + 2 is still even
   3. Conclusion: an a-palindrome has an even number of b's

2. No wfs of $\mathcal{L}^{\text{PL}}$ ever contains consecutive atomic formulas (e.g., '$(PP \wedge Q)$'). Between every atomic formula, there needs to be an operator.

3. Let $\mathcal{I}^{+}(\varphi) = 1$ for every sentence letter φ in $\mathcal{L}^{\text{PL}}$.

   Task 1: Show that $\mathcal{V}_{\mathcal{I}^{+}}(\varphi) = 1$ for every wfs φ of $\mathcal{L}^{\text{PL}}$ that does not include negation.

   Proof

   Repeat here and below...

   Task 2: Show that every contradiction contains negation.

Proof

Repeat here and below...

4. Complete the proof of **Rule 4 ($\neg$E)** from Chapter 4.
5. Complete the proof of **Rule 6 ($\wedge$E)** from Chapter 4.