a) $1 + 3r + 2r^2 = 0 \quad (2r+1)(r+1) = 0 \quad r = -1, -\frac{1}{2}$

$y(t) = Ae^{-\frac{1}{2}t} + Be^{-t} + C \qquad 1 + 3\cdot 2 + 2y'' = 1$

$1 = A + B + C \qquad \frac{3}{3} - \frac{16}{3} - B = C \qquad y''(0) = -3$

$2 = -\frac{1}{2}A - B + C \qquad \frac{-13}{3} - B = C \qquad 2 = -\frac{1}{2}\left(\frac{16}{3}\right) - B - \frac{13}{3} - B$

$-3 = \frac{1}{4}A + B + C$

$-3 = \frac{1}{4}A + 1 - A \qquad 2B = -\frac{6}{3} - \frac{8}{3} - \frac{13}{3}$

$-4 = -\frac{3}{4}A \quad A = \frac{16}{3} \qquad B = \frac{-27}{6} = \frac{-9}{2}$

$1 = \frac{16}{3} - \frac{9}{2} + C \quad \frac{33}{6} - \frac{32}{6} = C = \frac{1}{6}$

$$\boxed{y(t) = \frac{16}{3}e^{-\frac{1}{2}t} - \frac{9}{2}e^{-t} + \frac{1}{6}}$$

b) $n = 1 : \; 8y[1] - 6\cdot 1 + 2 = 1 \qquad 5 = 8y[1] \qquad y[1] = \frac{5}{8}$

$8z^2 - 6z + 1 = 0 \qquad z = \frac{6 \pm \sqrt{36 - 32}}{16} \qquad z = \frac{6 \pm 2}{16} = \frac{1}{2}, \frac{1}{4}$

$y[n] = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{4}\right)^n + C$

$1 = A + B + C \qquad \frac{5}{2} = 2A + B + 4C \qquad \frac{9}{2} = 4A + 5B + 5C$

$2 = 2A + 4B + C \qquad B + C = \frac{1}{2} \qquad \frac{9}{2} - 5 = -A$

$\frac{5}{8} = \frac{A}{2} + \frac{B}{4} + C \qquad 4B + C = 1 \qquad A = \frac{1}{2}$

$3B = \frac{1}{2} \qquad B = \frac{1}{6} \qquad C = \frac{3}{6} - \frac{1}{6} = \frac{1}{3}$

$$\boxed{y[n] = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^n + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)^n + \frac{1}{3}}$$

c) $y'(t) = -\frac{1}{24}e^{-\frac{3}{4}t} - \frac{1}{4}e^{-\frac{1}{2}t} \qquad 0 = \frac{1}{6} + \frac{-A}{24} + \frac{B}{96} \qquad A = 6$

$y''(t) = \frac{1}{96}e^{-\frac{3}{4}t} + \frac{1}{8}e^{-\frac{1}{2}t} \qquad 0 = \frac{1}{2} - \frac{A}{4} + \frac{B}{8} \qquad B = 8$

$y(t) = \frac{1}{6}e^{-\frac{3}{4}t} + \frac{1}{2}e^{-\frac{1}{2}t} + \frac{1}{3} \qquad C = \frac{1}{3}$

$$\boxed{y(t) + 6y'(t) + 8y''(t) = \frac{1}{3}x(t)}$$