6.2210 PSet1 — Miguel Flores-Acton

Problem 1

A)

$$A_r = \pi R_3^2 - \pi R_2^2$$

$$A_r = \pi(R_3^2 - R_2^2)$$

$$\frac{\partial \rho}{\partial z} = A_r \rho_V$$

$$\frac{\partial \rho}{\partial z} = \pi(R_3^2 - R_2^2)\rho_V$$

$$L = 2\pi R_1$$

$$\frac{\partial \rho}{\partial z} = L\rho_S$$

$$\frac{\partial \rho}{\partial z} = 2\pi R_1 \rho_S$$

Because the net charge has to be zero we can set the derivatives $\frac{\partial \rho}{\partial z}$ equal to the negative of the other

$$2\pi R_1 \rho_S = -\pi(R_3^2 - R_2^2)\rho_V$$

$$\rho_S = -\frac{(R_3^2 - R_2^2)\rho_V}{2R_1}$$

B)

Since $\nabla \times E = 0$, and because of radial symmetry, we can say that the field must also be purely radial, therefore the only non-zero component of E is in the $\hat{r}$ direction.

C)

$$\int \vec{E} \cdot d\vec{a} = \frac{q_e}{\varepsilon_0}$$

Choosing a cylinder that encloses all the charges:
Since the cylinder is uniform to infinity on the z axis, we know that the none of E points in the $\hat{z}$ direction.

Given that L is the length of the cylinder and r is the radius:

$$2\pi r L \mathrm{E}_r \varepsilon_0 = 2\pi R_1 - \frac{(R_3^2 - R_2^2)\rho_V}{2R_1} L + \pi(R_3^2 - R_2^2) L \rho_V$$

$$2rE$$

Defining a cylinder around