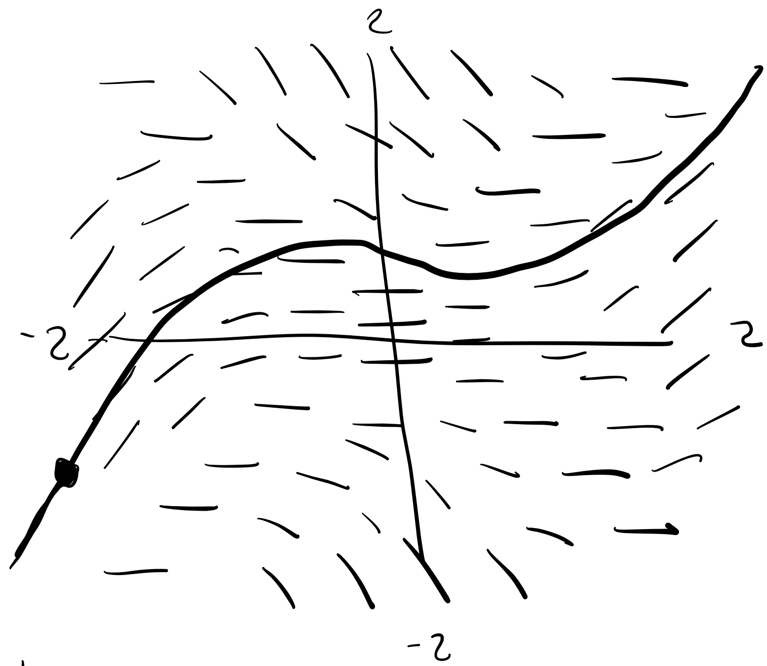
1) $y' = x^2 - y^2 = C \qquad y^2 = x^2 - C$

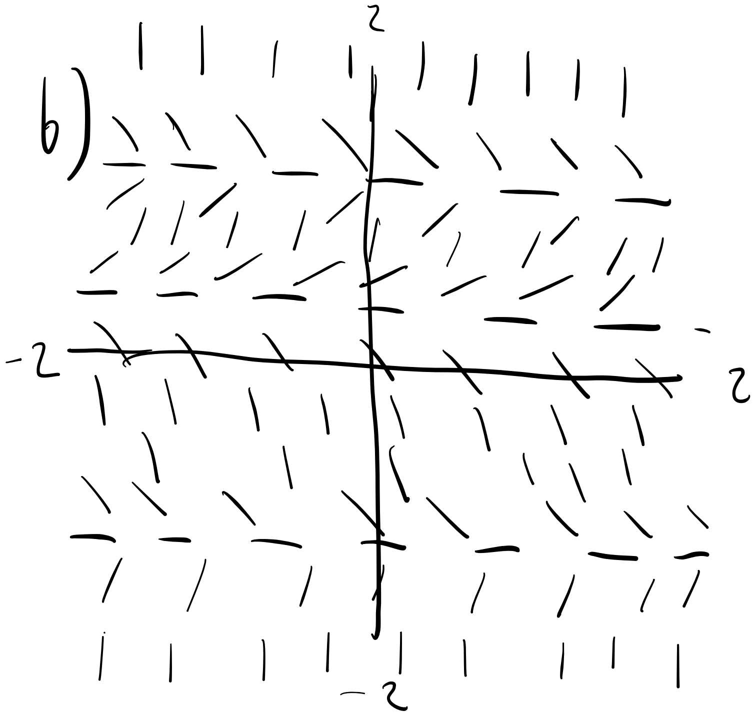
$y = \sqrt{x^2 - C}$

isoclines of y' are in the form $y = \sqrt{x^2 - C}$



1 local min
1 local max

b)



$y'(t) = 0$

$0 = 3y - y^5 - 1$

$y = 0.33473$
$y = -1.38879$
$y = 1.21465$

constant solutions

c) $y' = \frac{-1}{3^5} + 1 - 1 < 0 \quad 3y >> y^5$

when $x \approx \frac{1}{3}$

$3y$ gets smaller & smaller → negative rate increases

falls to $\lim_{x \to \infty} y(x) = -1.38879$

$\lim_{x \to -\infty} y(x)$ is opposite so $= 0.33473$

2 a) $\frac{dP}{dt} = r\,P(t)\left(1 - \frac{P(t)}{k}\right)$

$$\frac{dP}{r\,P(t)\left(1 - \frac{P(t)}{k}\right)} = dt \qquad t = \frac{k}{r}\int \frac{dP}{P(k-P)}$$

$$\frac{tr}{k} = \int \frac{dP}{P} - \int \frac{dP}{kP - P^2}$$

$t =$

$$\frac{tr}{k} = \frac{1}{k}\ln|P| - \ ?$$

③ a) $\frac{dV}{dy} = -3x^3 + 2x^2 y + 4y^3$ $\frac{dV}{dx} = 4x^3 - 9x^2 y + 2xy^2$

$V(x,y) = -3x^3 y + x^2 y^2 + y^4 + \phi(x)$

$V(x,y) = x^4 - 3x^2 y + x^2 y^2 + \alpha(y)$

$V(x,y) = x^4 - 3x^3 y + x^2 y^2 + y^4 + C$

b) $V(x,y) = C$

$$\frac{d}{dt} V(x,y) = \frac{dV}{dx} \cdot \frac{dx}{dt} + \frac{dV}{dy} \frac{dy}{dt}$$

$$= \frac{dV}{dx} f(x,y) + \frac{dV}{dy} g(x,y)$$

$$= \frac{dV}{dx}\left(\frac{dV}{dy}\right) + \frac{dV}{dy}\left(-\frac{dV}{dx}\right) = 0$$

$\frac{d}{dt} V(x,y) = 0$ means that V does not change along solution paths of the system (aka. follow level curves)