Two Ways

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1.

Superposition

When only the voltage source is connected and the current source is set to 0A, we can think of it as the current source is disconnected. This makes just 3 resistors in series which their equivalent resistance is $R_{eq} = R_1 + R_2 + R_3$.
Therefore the current through the resistors is $I = V_s/(R_1 + R_2 + R_3)$
Multiplying the current times the resistances, we get that
$e_1 = (V_s \times (R_2 + R_3))/(R_1 + R_2 + R_3)$, and that $e_2 = (V_s \times R_3)/(R_1 + R_2 + R_3)$
Now calculating the circuit if $V_s = 0$ the equivalent resistance is $R_{eq} = R_1||(R_2 + R_3)$ which expanded makes $1/(1/R_1 + 1/(R_2 + R_3))$.
Therefore $e_1 = I_s/(1/R_1 + 1/(R_2 + R_3))$, simplifying to
$e_1 = (I_s \times R_1 \times R_2 \times R_3)/((R_2 \times R_3) + R_1)$
Since $e_2 = (e_1 \times R_3)/(R_2 + R_3)$ we can substitute e_1 to get the equation $e_2 = (I_s \times R_1 \times R_2 \times {R_3}^2 \times (R_2 + R_3))/((R_2 \times R_3) + R_1)$.
Combining both of these, we get that
$e_1 = (V_s \times (R_2 + R_3))/(R_1 + R_2 + R_3) + (I_s \times R_1 \times R_2 \times R_3)/((R_2 \times R_3) + R_1)$
$e_2 = (V_s \times R_3)/(R_1 + R_2 + R_3) + (I_s \times R_1 \times R_2 \times {R_3}^2 \times (R_2 + R_3))/((R_2 \times R_3) + R_1)$

Nodal

With nodal analysis, we can come up with the two equations because we have two unknowns. $(V_s - e_1)/R_1 + I_s + (e_2 - e_1)/R_2 = 0$ and $e_2/R_3 = (e_1 - e_2)/R_2$

Solving these two equations with some algebra gets us the solutions:
$e_1 = (R_2 \times V_s + I_s \times R_1 \times R_2)/((R_1 + R_2) - (R_1 \times R_2)/(R_2 + R_3))$
$e_2 = (R_2 \times R_3 \times (V_s + I_s \times R_1))/((R_2 + R_3) \times ((R_2 + R_1) - (R_1 \times R_3)/(R_2 + R_3)))$

Equivalence

Rearanging the equations from the node and superposition analysis, they are the same.

2.

Superposition

Calculating when only I_1 is connected, we first find the equivalent resistance for the circuit, which is $R_{eq} = 75\Omega || 150\Omega = 50\Omega$

That means that $e_1 = I_1 \times 50$ and then we can treat each pair of resistors as voltage dividers to calculate e_2 and e_3

Therefore we get
$e_2 = (I_1 \times 50 \times 45)/75 = I_1 \times 30$ and $e_3 = (I_1 \times 50)/2 = I_1 \times 25$

Calculating when only I_2 is connected, we can first calculate the current divider

Current through the bottom will be $I_2 \times (105/225) = I_2 \times (7/15)$
Current through the top will be $I_2 \times (120/225) = I_2 \times (8/15)$

This means $e_3 = 75 \times I_2 \times (7/15) = I_2 \times 35$ and
$e_2 = -45 \times I_2 \times (7/15) = -I_2 \times 21$

Refrencing e_1 from e_3 we can calculate
$e_1 = I_2 \times 35 - 75 \times I_2 \times (8/15)$ and simplifying
$e_1 = -I_2 \times 5$

Adding up the superpositions of I_1 and I_2 we get:
$e_1 = 50I_1 - 5I_2$
$e_2 = 30I_1 - 21I_2$
$e_3 = 25I_1 + 35I_2$

Nodal

Calculating with nodal analysis:
$(e_1 - e_2)/30 = I_2 + e_2/45$ and $(e_1 - e_3)/75 + I_2 = e_3/75$ and
$I_1 = (e_1 - e_2)/30 + (e_1 - e_3)/75$

After solving with algebra:
$e_1 = 50I_1 - 5I_2$
$e_2 = 30I_1 - 21I_2$
$e_3 = 25I_1 + 35I_2$

Equivalence

These are the same