# Problem Set 8

## Due Friday, October 11th by 5pm

(20 points per question. Please scan and upload to Canvas as a PDF)

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**Note:** Complete the following steps for two of the following three problems given below:

(I) Regiment the argument in $\mathcal{L}^{\text{FOL}}$. (5pts)

(II) State whether the argument is valid or invalid. (5pts)

(III) If the argument is invalid, provide a countermodel along with a semantic argument that proves that the argument is invalid. If the argument is valid, provide a semantic argument that proves that it is valid. (10pts)

1. Laura is in love with the sun. Who loves the sun doesn't have a broken heart. It follows that Laura doesn't have a broken heart.

Proof

(I)
$Lx$: $x$ is in love with the sun
$Bx$: $x$ has a broken heart
laura $= l$

$Ll$
$(\forall x)(Lx \rightarrow \neg Bx)$
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$\neg Bl$

(II) Argument is valid

(III) Proof:

1. Assume a $\mathcal{L}^{\text{FOL}}$ model $M = \{\mathbb{D}, I\}$ where $V_I(Ll) = 1$.
2. It follows that $\{l\} \in I(L)$ and $\{l\} \in \mathbb{D}^1$ and $l \in \mathbb{D}$
3. Considering some v.a. $\hat{a}$ where $\hat{a}(x) = l$ then $Ll \rightarrow \neg Bl$ and therefore $V_I(Ll) \rightarrow V_I(\neg Bl)$.
4. Knowing that $V_I(Ll) = 1$ in our model, we know that $1 \rightarrow V_I(\neg Bl)$
5. Based on the semantics of $\rightarrow$ we then know that the previous expression equates to $V_I(\neg Bl)$
6. Therefore we can conlude that $\neg Bl$ is proven by the premises $\square$

2. All who exalted, was converted by a believer. Thus some believer converted someone who exalted.

Proof

(I)
$Ex$: $x$ extalted
$Cxy$: $x$ converted $y$
$Bx$: $x$ is a believer

$(\forall x)(Ex \rightarrow (\exists y)(By \wedge Cyx))$

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$(\exists x)((\exists y)(By \wedge Ex \wedge Cyx))$

(II) Argument is invalid in case when no one exalted

(III) Proof:

1. Assume a $\mathcal{L}^{\text{FOL}}$ model $M = \{\mathbb{D}, I\}$ such that $V_I((\forall x)(\neg Ex)) = 1$.
2. It follows that $V_I^{\hat{a}}((\forall x)(\neg Ex)) = 1$ for all v.a. including $\hat{a}$ defined over $\mathbb{D}$.
3. Accordingly, $V_I^{\hat{a}}(\neg Ea) = 1$ and from the semantics, $V_I^{\hat{a}}(Ea) = 0$.
4. Using the semantics for $\rightarrow$ we can conclude that $0 \rightarrow ... = 1$
5. Therefore $V_I^{\hat{a}}(Ea \rightarrow (\exists y)(By \wedge Cya)) = 1$
6. Assuming $\hat{c}$ where $V_I^{\hat{a}}(Bc \wedge Ea \wedge Cca) = 1$.
7. Using the semantics of $\wedge$, $V_I^{\hat{a}}(Ea) = 1$
8. However, previously, $V_I^{\hat{a}}(Ea) = 0$.
9. Therefore argument fails this model and is invalid $\square$

3. Hesperus is rising. Hesperus is Phosphorus. Therefore Phosphorus is rising.