Electrodynamics: Homework 4

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1. We have
$$T^{i0} = F^i_\gamma F^{0\gamma} - \frac{1}{4}\eta^{i0} F_{\gamma\delta} F^{\gamma\delta}. \tag{1}$$
Given that $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$ and $i \in \{1, 2, 3\}$ we conclude
$$T^{i0} = F^i_\gamma F^{0\gamma} = \eta_{\rho\gamma} F^{i\rho} F^{0\gamma} \tag{2}$$
Remembering that $F^{\mu\nu}$ is antisymmetric and thus that $F^{00} = 0$, we notice that
$$T^{i0} = \eta_{\rho j} F^{i\rho} F^{0j} = -\sum_{j=1}^{3} F^{ij} F^{0j}. \tag{3}$$
Recalling that $F^{0j} = E_j$ and $F^{ij} = \sum_{k=1}^{3} \epsilon_{ijk} B_k$ we have
$$T^{i0} = -\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijk} B_k E_j = -(\mathbf{E} \times \mathbf{B})_i \tag{4}$$

2. (i) Indeed, $\mathcal{L}$ is real. This is because $\mathcal{L}_\phi$, $\mathcal{L}_A$ and $\mathcal{L}_{\text{int}}$ are.