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KARLOVA UNIVERZITA
Zimní semestr 2025/26

Mathematical analysis II

Homework 7

To be handed in by Wednesday, 26.11.25, 23:59 h via OWL

Exercise 1 (Extrema with constraints). *(6 points)*

Calculate the minimal distance of the point $x_0 = (1, -1, 1)$ to the set $M := \{(x, y, z) \in \mathbb{R}^3 : z^2 = 2xy + 1\}$. (Hint for easier calculations: how to say "náměstí vzdálenosti" in English? Explain why you are allowed to do that/why you don't lose anything.)

Exercise 2 (Directional derivative). *(2+2=4 points)*

a) Let $v \in \mathbb{R}^n$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be totally differentiable in the point $x_0 \in \mathbb{R}^n$. Show that $D_v f(x_0) = \nabla f(x_0) \cdot v$.

b) Calculate $D_v f(x_0)$ once using the definition and once with the help of the identity from part a):

$$f(x, y) = x^2 + y^2, \qquad x_0 = (1, 1), \qquad v = (1, 1).$$