

# Mathematical analysis II

## Homework 5

To be handed in by Wednesday, 12.11.25, 23:59 h via OWL

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*Explicitly implicit*

**Exercise 1.**

(3+2=5 points)

Let the function  $F : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $F(x, y) = ye^x - x \ln(y) - 1$  and the point  $P = (0, 1)$  be given.

- a) Show that there exists a neighborhood  $U$  of  $P$  and a function  $g : U \rightarrow \mathbb{R}$  such that we can write  $y = g(x)$  in  $U$ . Show also that the function  $g$  is continuously differentiable.
- b) Calculate  $g'(x)$  and  $g'(0)$ .

**Exercise 2.**

(2+2+1=5 points)

Let  $F : \mathbb{R}^3 \rightarrow \mathbb{R}$  be given by

$$F(x, y, z) = x^3 - y^3 + z^3 + 2z^2 - 3xyz.$$

- a) Show that there is a neighborhood of the point  $P = (x_0, y_0) = (1, -1)$  and a function  $g : \mathbb{R}^2 \rightarrow \mathbb{R}$  with  $g(1, -1) = -1$  such that  $F(x, y, g(x, y)) = 0$  in this neighborhood.
- b) Show that  $g$  has a stationary point in  $(1, -1)$ . (Hint: partial derivatives wrt.  $x$  and  $y$  and chain rule.)
- c) Calculate the tangent plane of  $g$  in the point  $(1, -1)$ . Give a *geometric* explanation why is your result not surprising (“gradient is zero” does not count as geometric).