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Zimní semestr 2025/26

Mathematical analysis II

Homework 5

To be handed in by Wednesday, 12.11.25, 23:59 h via OWL

Explicitly implicit

Exercise 1. *(3+2=5 points)*

Let the function $F : \mathbb{R}^2 \to \mathbb{R}$, $F(x, y) = ye^x - x\ln(y) - 1$ and the point $P = (0, 1)$ be given.

a) Show that there exists a neighborhood U of P and a function $g : U \to \mathbb{R}$ such that we can write $y = g(x)$ in U. Show also that the function g is continuously differentiable.

b) Calculate $g'(x)$ and $g'(0)$.

Exercise 2. *(2+2+1=5 points)*

Let $F : \mathbb{R}^3 \to \mathbb{R}$ be given by

$$F(x, y, z) = x^3 - y^3 + z^3 + 2z^2 - 3xyz.$$

a) Show that there is a neighborhood of the point $P = (x_0, y_0) = (1, -1)$ and a function $g = \mathbb{R}^2 \to \mathbb{R}$ with $g(1, -1) = -1$ such that $F(x, y, g(x, y)) = 0$ in this neighborhood.

b) Show that g has a stationary point in $(1, -1)$. (Hint: partial derivatives wrt. x and y and chain rule.)

c) Calculate the tangent plane of g in the point $(1, -1)$. Give a *geometric* explanation why your result is not surprising ("gradient is zero" does not count as geometric).