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Mathematical analysis II

Homework 2

To be handed in by Wednesday, 22.10.25, 23:59 h via OWL

Exercise 1 (Continuity and closedness). *(2+2+2=6 points)*

a) We define the following function $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ via

$$f(x,y) = \frac{x^2 y}{x^2 + y^2}.$$

Decide whether this function is continuous, and whether or not we can extend it to a function that is continuous on the whole of $\mathbb{R}^2$. *(Hint: You can use without proof that for any $x, y \neq 0$, it holds $|xy|/(x^2 + y^2) \leq 1$.)*

b) Let (X, d) be a metric space and $a \in X$. Show that the function

$$f : X \to [0, \infty), \qquad f(x) = d(x, a)$$

is continuous.

c) Let (X, d) be a metric space and $f : X \to \mathbb{R}$ be continuous. Show that the kernel

$$\mathrm{ker} f := f^{-1}(\{0\})$$

is closed in X. *(Reminder: for a set $A \subset \mathbb{R}$, the pre-image is defined as $f^{-1}(A) = \{x \in X : f(x) \in A\}$.)*

Exercise 2 (Total vs. partial derivatives). *(4 points)*

Show that the function defined via

$$f(x,y) = \begin{cases} (x^2 + y^2) \sin \frac{1}{x^2 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{else} \end{cases}$$

is totally differentiable, but its partial derivatives are not continuous.