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Zimní semestr 2025/26

Mathematical analysis II

Homework 1

To be handed in by Wednesday, 15.10.25, 23:59 h via OWL

Exercise 1 (Metrics). *(3+1=4)*

a) Let $X = [1, \infty)$. Show that the function defined via

$$d(x, y) = \left| \frac{1}{x} - \frac{1}{y} \right|$$

is a metric.

b) Let $X = (0, \infty)$. Is the function defined by

$$d(x, y) = |x|^{|y|}$$

a metric? Show or disprove!

Exercise 2 (Open and closed sets). *(3+3=6)*

In here, we assume $X = \mathbb{R}$.

a) Let $d : X \times X \to [0, \infty)$ be a metric, and define

$$\delta(x, y) = \min\{d(x, y),\ 1\}.$$

Show that δ is really a metric, and that d and δ define the same open and closed sets.

b) Let

$$d(x, y) = \begin{cases} 1 & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases}$$

Show that under this metric, every set is both open and closed.