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Zimní semestr 2025/26

Mathematical analysis II

Homework 3

To be handed in by Wednesday, 29.10.25, 23:59 h via OWL

Exercise 1 (Chain rule and extrema). *(2+2+2=6 points)*

a) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Show that for $z = f(\frac{xy}{x^2+y^2})$ it holds

$$x\partial_x z + y\partial_y z = 0.$$

b) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable and $a \in \mathbb{R}^n$ and $b \in \mathbb{R}$ be constant. Calculate the gradient of the function $g : \mathbb{R}^n \to \mathbb{R}$, $g(x) = f(\langle a, x\rangle + b)$. Here, the notation $\langle a, x\rangle$ is the scalar product of these vectors. Remember that the gradient is defined as $\nabla g = (\partial_{x_1} g, ..., \partial_{x_n} g)$.

c) Let

$$f(x, y) = (x - y - 1)^2.$$

Find and classify all extrema (minimum/maximum/saddle point).

Exercise 2 (Derivatives of higher order). *(2+2=4 points)*

a) Calculate for the following function the derivatives up to second order:

$$f(x, y) = (x + y + 3)^2 - e^{2x+y^2}.$$

b) For the vectorial function $v = (v_1, v_2, v_3) : \mathbb{R}^3 \to \mathbb{R}^3$ we define the divergence and curl via

$$\operatorname{div} v = \nabla \cdot v = \partial_1 v_1 + \partial_2 v_2 + \partial_3 v_3,$$
$$\operatorname{curl} v = \nabla \times v = \begin{pmatrix} \partial_2 v_3 - \partial_3 v_2 \\ \partial_3 v_1 - \partial_1 v_3 \\ \partial_1 v_2 - \partial_2 v_1 \end{pmatrix}.$$

(Here $\partial_i v_j$ means $\partial v_j / \partial x_i$ for any $i, j \in \{1, 2, 3\}$.) Show that $\operatorname{div} \operatorname{curl} v = 0$.

Exercise 3 (Not for handing in). Let

$$f(t) = (1 + t, t^2, 1 - t), \qquad g(x, y, z) = 1 + x + xyz.$$

Calculate once with and once without the help of the chain rule $D(g \circ f)(0)$.