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Mathematical analysis II

Homework 10

To be handed in by Wednesday, 17.12.25, 23:59 h via OWL

Exercise 1 (Fubini's theorem). *(5 points)*

Let $a, b > 0$. Calculate the area of the ellipse

$$E = \left\{ x, y \in \mathbb{R} : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}.$$

Exercise 2 (A counterexample). *(5 points)*

Let for $x + y \neq 2$

$$f(x, y) = \frac{y - x}{(2 - x - y)^3}.$$

Show (without the help of AI!) that

$$\int_0^1 \int_0^1 f(x, y)\, dx\, dy \neq \int_0^1 \int_0^1 f(x, y)\, dy\, dx.$$

Why this is no contradiction to Fubini's theorem?