Principles of Formal Languages

Florian Rabe

Computer Science, University Erlangen-Nürnberg, Germany

2024

Administrative Information

Format

Room

- lectures and exercises in person — if we have a room
- interaction strongly encouraged — We don't want to lecture — we want to have a conversation during which you learn
- possibly make use of zoom

Recordings

- maybe prerecorded video lectures or recorded zoom meeting
- to be decided along the way

Background

Instructor

- PD Dr. Florian Rabe
- Prof. Dr. Michael Kohlhase
  Professor of Knowledge Representation and Processing

Course

- This course is given for the fourth time
- Still a bit experimental polishing and revising the materials this year
- Signature course of our research group same name!

Prerequisites

Required

- basic knowledge about formal languages, context-free grammars
  but we'll do a quick revision here

Helpful

- Algorithms and Data Structures mostly as a contrast to this lecture
- Basic logic we'll revise it slightly differently here
- all other courses as examples of how knowledge pervades all of CS

General

- Curiosity this course is a bit unusual
- Interest in big picture
  this course touches on lots of things from all over CS

Examination and Grading

Suggestion

- grade determined by single exam
- probably written, 90 minutes
- exercises indirectly graded through conversation during exam

unless we agree on something else by the second week

Exam-relevant

- anything mentioned in notes
- anything discussed in lectures
- anything done in exercises

none is a superset of another!

Materials and Exam-Relevance

Textbook

- does not exist
- normal for research-near specialization courses

Notes

- textbook-style but not as comprehensive
- developed along the way
- systematic write-up, not necessarily in lecture order

Slides

- not comprehensive
- used as visual aid, conversation starters

Communication

Open for questions

- open door policy in our offices
- always room for questions during lectures
- for personal questions, contact me during/after lecture
- matrix channel `https://matrix.to/#/#wuv:fau.de`
- studon forum not used

Materials

- notes and slides are at: `https://github.com/florian-rabe/Teaching/tree/master/WuV`
- currently last year's version, will change throughout the semester
  you can read ahead, but maybe don't print everything right away
- pull requests and issues welcome

Exercises

Learning Goals

- Get acquainted with state of the art of practice
- Try out real tools

Homeworks

- one major project as running example
- homeworks building on each other

build one large knowledge-based system

details on later slides

Overview and Essential Concepts

Representation and Processing

Common pairs of concepts:

Representation	Processing
Static	Dynamic
Situation	Change
Be	Become
Data Structures	Algorithms
Set	Function
State	Transition
Space	Time

Data and Knowledge

2×2 key concepts

Syntax	Data
Semantics	Knowledge

- Data: any object that can be stored in a computer
  Example: $((49.5739143, 11.0264941), "2020-04-21T16:15:00CEST")$
- Syntax: a system of rules that describes which data is **well-formed**
  Example: "a pair of (a pair of two IEEE double precision floating point numbers) and a string encoding of a time stamp"
- Semantics: system of rules that determines the meaning of well-formed data
- Knowledge: combination of some data with its syntax and semantics

Knowledge is Elusive

Representation of key concepts

- Data: using primitive objects
  implemented as bits, bytes, strings, records, arrays, ...
- Syntax: (context-free) grammars, (context-sensitive) type systems
  implemented as inductive data structures
- Semantics: functions for evaluation, interpretation, of well-formed data
  implemented as recursive algorithms on the syntax
- Knowledge: elusive
  emerges from applying and interacting with the semantics

Semantics as Translation

- Knowledge can be captured by a higher layer of syntax
- Then semantics is translation into syntax

Data syntax	Semantics function	Knowledge syntax
SPARQL query	evaluation	result set
SQL query	evaluation	result table
program	compiler	binary code
program expression	interpreter	result value
logical formula	interpretation in a model	mathematical object
HTML document	rendering	graphics context

Heterogeneity of Data and Knowledge

- Capturing knowledge is difficult
- Many different approaches to semantics
  - fundamental formal and methodological differences
  - often captured in different fields, conferences, courses, languages, tools
- Data formats equally heterogeneous
  - ontologies
  - programs
  - logical proofs
  - databases
  - documents

Challenges of Heterogeneity

Challenges

- collaboration across communities
- translation across languages
- conversion between data formats
- interoperability across tools

Sources of problems

- interoperability across formats/tools major source of
  - complexity
  - bugs
- friction in project team due to differing preferences, expertise
- difficult choice between languages/tools with competing advantages
  - reverting choices difficult, costly
  - maintaining legacy choices increases complexity

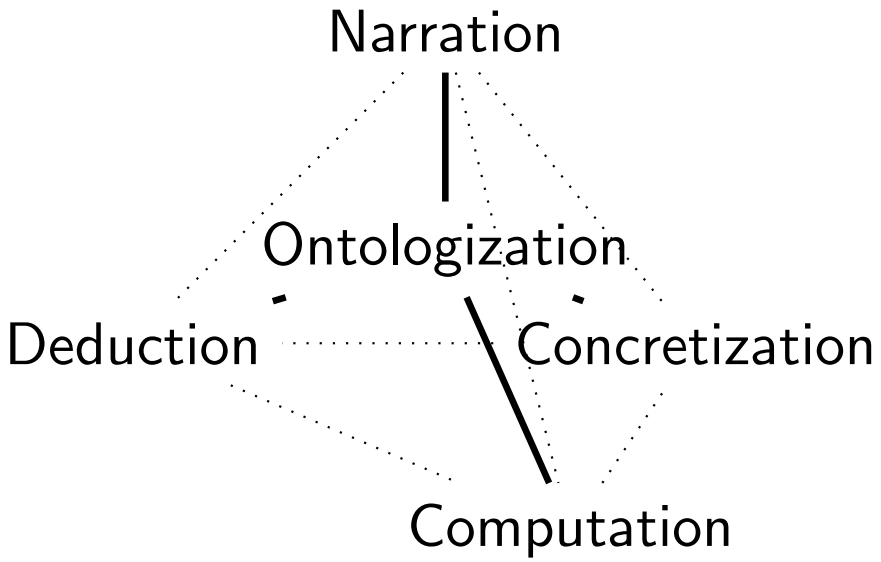
Aspects of Knowledge

- Tetrapod model of knowledge active research by our group
- classifies approaches to knowledge into five aspects

Aspect	KRLs (examples)
ontologization	ontology languages (OWL), description logics (ALC)
concretization	relational databases (SQL, JSON)
computation	programming languages (C)
deduction	logics (HOL)
narration	document languages (HTML, LaTeX)

Relations between the Aspects

Ontology is distinguished: capture the knowledge that the other four aspects share



Complementary Advantages of the Aspects

Aspect	objects	characteristic		
		advantage	joint advantage of the other aspects	application
ded.	formal proofs	correctness	ease of use	verification
comp.	programs	efficiency	well-definedness	execution
concr.	concrete objects	queriability	abstraction	storage/retrieval
narr.	texts	flexibility	formal semantics	human understanding

Aspect pair	characteristic advantage
ded./comp. narr./conc.	rich meta-theory simple languages
ded./narr. comp./conc.	theorems and proofs normalization
ded./conc. comp./narr.	decidable well-definedness Turing completeness

Structure of the Course

Aspect-independent parts

- shared characteristics
- general methods

Aspects-specific parts

- one part for each aspect
- high-level overview of state of the art
- focus on comparison/evaluation of the aspect-specific results

Structure of the Exercises

One major project

- representative for a project that a CS graduate might be put in charge of
- challenging heterogeneous data and knowledge
- requires integrating/combining different languages, tools

unique opportunity in this course because knowledge is everywhere

Concrete project

- develop a campo/studnon-style KRP system for a university
- lots of heterogeneous knowledge
  - course and program descriptions
  - legal texts
  - websites
  - grade tables
  - code to generate diplomas
- build a functional system applying the lessons of the course

we'll see how far we get — priority is learning

Language Layers

Layers of Language Design

Layer	Specified by	Implemented by
Syntax		
Context-Free	grammar	AST+parser+printer
Context-Sensitive	inference system	type checker
Semantics	inference system, interpretation, or translation	
Pragmatics	human preferences	human judgment

KRP = syntax + semantics

Layered Processing

Data is processed in phases

1. data representation format, e.g., string, JSON, XML, binary
2. parsed — well-formed context-free syntax tree
3. context-sensitive check by traversal of the syntax tree — well-typed syntax tree
4. computation by traversal of well-typed AST — semantics

Possible Errors

Layer	Error
CFS	not derivable from grammar
CSS	symbols not used as declared, other conditions
Sem.	ambiguous/undefined semantics
Pragmatics	not useful

Typical Errors by Layer

In a programming language:

Layer	Expression	Issue	Explanation
CFS	1/	syntax error	argument missing
CSS	1/"2"	typing error	wrong type
Sem.	1/0	run-time error	undefined semantics
Pragm.	1/1	code review	unnecessarily complex

Typical Errors by Layer

In a logic:

Layer	Expression	Issue	Explanation
CFS	$\forall x$	not well-formed	body missing
CSS	$\forall x.P(y)$	not well-typed	y not declared
Sem.	the $x \in \mathbb{N}$ with $x < 0$	not well-defined	no such x exists
Pragm.	$\exists x.x \neq x$	not useful	no model exists

Context-Free Grammars

The Chomsky Hierarchy

- CH-0, regular grammars:
  - equivalent to regular expressions and finite automata
  - not used much as grammars
- CH-1, context-free grammars (CFGs) our focus
- CH-2, context-sensitive grammars
  - important as languages, but awkward as grammars
  - instead: type system determines subset of context-free language
- CH-3, unrestricted grammars
  - Turing-complete, theoretically important
  - not used much as grammars

Definitions

- An alphabet is a set of symbols.
- A word is a list of symbols from the alphabet.
- A production is pair of words.
  - A production is written $lhs ::= rhs$.
  - Multiple productions for the same left-hand side are abbreviated $lhs ::= rhs_1 \mid \ldots \mid rhs_n$.
  - Right-hand side may also use regular expressions like * for repetition and [] for optional parts.
- A CFG is a set of productions where *lhs* is a single symbol.
  - If there is a production $N ::= rhs$, N is called non-terminal, otherwise terminal.
  - If a word contains non-terminal symbols, it is called non-terminal, otherwise terminal.
- A syntax tree is a tree whose nodes are labeled with productions $N ::= rhs$ where the non-terminals in rhs are exactly the lhs's of the children.
- The word produced by a syntax tree is read off by exhaustively replacing every lhs with the respective rhs.

Example: Syntax of Arithmetic Language

Numbers

$$
\begin{array}{lll}
N ::= 0 \mid 1 & & \text{literals} \\
\quad \mid \; N + N & & \text{sum} \\
\quad \mid \; N * N & & \text{product}
\end{array}
$$

Formulas

$$
\begin{array}{lll}
F ::= N \doteq N & & \text{equality} \\
\quad \mid \; N \leq N & & \text{ordering by size}
\end{array}
$$

Implementing CFGs via Inductive Data Types

Correspondence

CFG	IDT
non-terminal	type
production	constructor
non-terminal on left of production	return type of constructor
non-terminals on right of production	arguments types of constructor
terminals on right of production	notation of constructor
words derived from non-terminal N	expressions of type N

Classes of Languages

Functional languages:

- pure: ML, Haskell
- with OO: F#, Scala

inductive types are primitive

OO-languages:

- C#, Java, C++

inductive types simulated via classes

Untyped languages:

- Python, Javascript

inductive types simulated ad hoc

Implementing the Example

Done interactively. See the examples in the repository. See also the notes.

Exercise 3

Individually, using any programming language, implement the AST for the BOL language. Implement a printer for BOL by context-free traversal of the syntax tree.

Remarks:

- To simplify, you can drop most productions for concepts and relations.
- To simplify, you can drop properties altogether. But it would be nice to have them and allow for integers and strings as basic types.

Context-Sensitive Syntax

Vocabularies and Declarations

Generic structure of a context-sensitive language

- a vocabulary is a list of declarations
  - named: type/function/predicate symbol etc.
  - unnamed: axioms etc.
  - structural: namespaces/package, inclusion/import
- named declarations introduce atomic objects of various kinds
- for each kind, a non-terminal for complex expressions of that kind
- references to names introduced by declarations are base cases of expressions

Example: Typed Expressions

Vocabularies

$Voc ::= Decl^*$ — list of declarations

Declarations

$Decl ::= id : Type^* \to Type$ — typed function symbols

$\mid \; id : Type^* \to FORM$ — typed predicate symbols

Types

$Type ::= Nat \mid String$ — base types

Expressions

$Expr ::= 0 \mid 1 \mid Expr + Expr \mid Expr * Expr$ — as before

$\mid \; id(Expr^*)$ — application of a function symbol

Formulas

$Form ::= Expr \doteq Expr \mid Expr \leq Expr$ — as before

$\mid \; id(Expr^*)$ — application of a predicate symbol

Example: Vocabularies and Expressions

Example vocabulary V containing the following declarations:

- $fib : Nat \to Nat$
- $length : String \to Nat$
- $mod : Nat\ Nat \to Nat$
- $prime : Nat \to FORM$

Example expressions relative to V

- expressions: $fib(0)$, $mod(fib(fib(1)), 1 + 1)$
- formulas: $fib(0) = 0$, $prime(fib(1))$

Primitive vs. Declared

Primitive

- built into the language
- assumed to exist a priori — fundamentals of nature
- fixed semantics (usually interpreted by identity function)

	primitive	declared
introduced by	language designer	user
introduced in	grammar	vocabulary V
visible in	all vocabularies	V only
semantics given	explicitly	implicitly
. . . by	translation function	axioms

more expressive declarations $\rightarrow$ fewer primitives needed

paradoxical: more complex language may have simpler grammar

Quasi-Primitive = Declared in standard library

Standard library: a vocabulary *StdLib*

- present in every language — empty vocabulary by default
- one fixed vocabulary
  - implicitly included into every other vocabulary
  - implicitly fixed by any translation between vocabularies

  Combination of advantages
  - from the user's perspective: like a primitive
  - from the theory's/system's perspective: no special treatment

Examples

- sufficiently expressive languages
  - push many primitive objects to standard library — never all
  - simplifies language, especially when defining operations

    strings in C, BigInteger in Java, inductive type for $\mathbb{N}$
- inexpressive languages
  - many primitives — SQL, spreadsheet software
  - few (quasi)-primitives — few operations available in OWL

Example: Removing Built-in Operations

Grammar without built-in operations

Voc	$::=$	$Decl^*$	list of declarations
$Decl$	$::=$	$id : Type^* \to Type$	typed function symbols
	$\mid$	$id : Type^* \to FORM$	typed predicate symbols
$Type$	$::=$	$Nat \mid String$	base types
$Expr$	$::=$	$id(Expr^*)$	application of a function symbol
$Form$	$::=$	$id(Expr^*)$	application of a predicate symbol

Standard library:

- $0 : Nat$, $1 : Nat$, $sum : Nat\ Nat \to Nat$, $product : Nat\ Nat \to Nat$,
- $equals : Nat\ Nat \to FORM$, $lesseq : Nat\ Nat \to FORM$

Example: Removing more Primitive Operations

If we add type declarations, we can remove *Nat* as well

Voc	$::= Decl^*$	list of declarations
$Decl$	$::= id : Type^* \to Type$	typed function symbols
	$\mid id : Type^* \to FORM$	typed predicate symbols
	$\mid id : TYPE$	type symbols
$Type$	$::= id$	reference to a type symbol
$Expr$	$::= id(Expr^*)$	application of a function symbol
$Form$	$::= id(Expr^*)$	application of a predicate symbol

Note: *Type* and *Form* are non-terminals, *TYPE* and *FORM* are not

Add to default vocabulary: $Nat : TYPE$, $String : TYPE$

Context-Sensitivity

A reference to a declared name must respect the way in which it was declared in the vocabulary examples below relative to V above

- occur in a position where an expression of the right kind is expected
  example error: $prime(1) = 1$
- be applied to the right number of arguments
  example error: $prime(1, 1)$
- if a type system is used
  - arguments must have the right types
    $length(1)$
  - return type must match what is expected
    $fib(1)$ if a string is expected

Syntax Traversal

Context-free traversal — mutually recursive functions

- one function for each non-terminal/inductive type
- for each such function, one case for each production/constructor
- for each such case, one recursive call for each non-terminal on the rhs/constructor argument
- Examples
  - printer/serializer (return, e.g., string)
  - test if a feature is used (return boolean)
  - find set of used identifiers (return set of names)

Context-sensitive traversal: as above but

- functions take extra argument for vocabulary
- cases for identifier references look up declaration in vocabulary
- Examples: anything that looks up identifier declarations
  - substitution of identifiers with new expressions
  - typical syntax transformations, e.g., in compilers (return same expression kind)
  - semantics by translation

Type Checker

Syntax checker

- context-sensitive traversal where all functions return booleans
- case for vocabulary checks each declaration relative to the preceding vocabulary
- cases for identifier references check correct use of identifier

Type checker: as above but additionally

- functions for typed expression additionally take expected type as argument
- cases for identifiers references
  - check each argument against declared input type
  - compare output to expected type

Exercise 4

Implement a type-checker for BOL.

The type-checker must check that all identifier references are used according to their declaration:

- concept identifiers as concepts
- relation identifiers as relations
- individual identifiers as individuals
- properties identifiers of type Y as properties for values of type Yindividual identifiers as individuals

Kinds of Typing: Extrinsic and Intrinsic

Breakout Question

Is this an improvement over BOL?

Declarations

$D ::=$	**individual** $i : C$	typed atomic individual
$\mid$	**concept** c	atomic concept
$\mid$	**relation** $r \subseteq C \times C$	typed atomic relation
$\mid$	**property** $p \subseteq C \times T$	typed atomic property

rest as before

Actually, when is a language an improvement?

orthogonal, often mutually exclusive criteria

Trade-off for syntax design

- expressivity: easy to express knowledge
  e.g., big grammar, complex type system
- simplicity: easy to implement/interpret
  e.g., few, carefully chosen productions, types

Semantics

- specification, implementation, documentation

Intended users

- skill level
- prior experience with related languages
- amount of training needed
- innovation height, differential evaluation against existing languages

Actually, when is a language an improvement? (2)

Support software ecosystem

- optional tool support: IDEs, debuggers, heuristic checkers, alternative implementations, interpreter/REPL
- many/large well-crafted vocabularies and package managers to find them
- integrations with other languages: translations, common run-time platforms, foreign function interface

Long-term plans: re-answer the above question but now

- maintainability: syntax was changed, everything to be redone
- backwards compatibility: support for legacy input
- scalability: expressed knowledge content has reached huge sizes

General Idea

A **type system** for a syntax consists of

- some non-terminals $\mathcal{E}$, whose words are called $\mathcal{E}$-**expressions**,
  coarse, context-free, classification into disjoint sets
- for some symbols $\mathcal{E}$
  - set of types: $\mathcal{T}$-expressions for a non-terminal $\mathcal{T}$
  - typing relation $\Gamma \vdash^{L}_{V} e : T$ between $\mathcal{E}$-expressions e and $\mathcal{T}$-expressions T

  fine, context-sensitive, classification into disjoint or overlapping sets

Examples

- BOL: non-terminals $\mathcal{E}$ for expressions are C, I, R, P, F
  I-expressions typed by C-expressions
  overlapping, types undecidable/difficult to check
- SFOL: non-terminals $\mathcal{E}$ for expressions are Y, T, F
  T-expressions typed by Y-expressions disjoint, types easy to infer

Church vs. Curry Typing

	intrinsic	extrinsic
λ-calculus by	Church	Curry
type is	carried by object	given by environment
typing is a	function objects $\rightarrow$ types	relation objects $\times$ types
objects have	unique type	any number of types
types interpreted as	disjoint sets	unary predicates
type given by	part of declaration	additional axiom
example	**individual** "WuV":"course"	**individual** "Wuv", "WuV" `is-a` "course"
examples	SFOL, SQL	OWL, Scala, English
	most logics, functional PLs	ontology, OO, natural languages
	many type theories	set theories

Type Checking

	intrinsic	extrinsic
type is	carried by object	given by environment
typing is a	function objects $\rightarrow$ types	relation objects $\times$ types
objects have	unique type	any number of types
type given by example	part of declaration **individual** WuV: course	additional axiom **individual** Wuv, WuV `is-a` course
type inference for x	uniquely infer A from x	find minimal A with $x : A$
type checking	inferred=expected	prove $x : A$
subtyping $A <: B$	cast from A to B	$x : A$ implies $x : B$
typing decidable	yes unless too expressive	no unless restricted
typing errors	static (compile-time)	dynamic (run-time)
advantages	easy unique type inference	flexible allows subtyping

Examples: Curry-Typing in BOL

Semantics	objects	types	typing relation
absolute	individuals I	concepts C	i `is-a` C
SFOL	terms of type ι	predicates $C \subseteq \iota$	$c(i)$
English	proper nouns	common nouns	"i is a C"

Examples

System	typing	objects	types	typing relation
any	Church	expressions	non-terminals	derived from
BOL	Curry	individuals	concepts	`is-a`
SFOL	Church	terms	types	:
set theory	Curry	sets	sets	$\in$
OO	Curry	instances	classes	`isInstanceOf`

Subtyping

Subtyping works best with Curry Typing

- explicit subtyping as in $\mathbb{N} <: \mathbb{Z}$
- comprehension/refinement as in $\{x : \mathbb{N} | x \neq 0\}$
- operations like union and intersection on types
- inheritance between classes, in which case subclass $=$ subtype
- anonymous record types as in $\{x : \mathbb{N}, y : \mathbb{Z}\} <: \{x : \mathbb{N}\}$

Kinds of Types

Question

What kind of types are there?

Abstract vs. Concrete Types

Concrete type: values are

- given by their internal form,
- defined along with the type, typically built from already-known pieces.

product types, enumeration types, collection types

main example: concrete (inductive/algebraic) data types

Abstract type: values are

- given by their externally visible properties,
- defined in any environment that understands the type definition.

structures, records, classes, aggregation types

main example: abstract data types

Non-Recursive vs. Recursive Types

Non-Recursive types

- given by some expressions
- can be anonymous
- values given directly

integers, lists of strings, ...

Recursive types

- definition of the type must refer to the type itself
  so type must have a name
- type typically defines other named operations
- values obtained by fixed-point constructions

optional property of concrete and abstract data types

Atomic vs. Complex Types

Atomic type

- given by its name
- values are a set

integers, strings, booleans, . . .

Complex types

- arise by applying type symbol to arguments
- separate set of values for each tuple of arguments

two kinds of complex types (next slide)

Type operators vs. Dependent type Families

Both are complex: take arguments and return a type

Type operators take only type arguments, e.g.,

- type operator $\times$
- takes two types A, B
- returns type $A \times B$

Dependent types take also value arguments, e.g.,

- dependent type operator *vector*
- takes natural number n, type A
- returns type A^n of n-tuples over A

dependent types much more complicated, less uniformly used

harder to starndardize

Built-in vs. User-Defined Types

Built-in types

- syntax and semantics fixed by language designer
- part of grammar, implementation, etc.
- usually concrete, atomic, non-recursive

typical: integers, strings, lists

sometimes also called primitive or basic types

User-Defined types

- declared by users in vocabulary
- standard syntax prescribed by grammar, possibly customizable
- semantics given by operations and their axioms

anything the language can axiomatize

usually difficult to axiomatize recursive properties

Non-Recursive Data Types

Common Built-in Types

Typical (quasi-)primitive types

- natural numbers ($= \mathbb{N}$)
- arbitrary precision integers ($= \mathbb{Z}$)
- fixed precision integers (32 bit, 64 bit, ...)
- floating point (float, double, ...)
- Booleans
- characters (ASCII, Unicode)
- strings

Observation:

- essentially the same in every language
  including whatever language used for semantics
- semantics by translation trivial

Less Common Types

Problem

- quickly encounter primitive types not supported by common languages
- need to encode them using existing types
  typically as strings, ints, or prodcuts/lists thereof

Examples

- date, time, color, location on earth
- graph, function
- picture, audio, video
- physical quantities ($1m$, $1in$, etc.)
- gene, person

Specifying a Type

Components

- the type — eg: 'int'
- values of the type — eg: 0, 1, -1, ...
- string encoding — injective function from values to strings
- operations on type — eg: addition, multiplication, ...

Examples

- IEEE floating point numbers
- ISO 8601 for date/time

Type Operators

As for types, but taking n type argument.

Component

- the type — eg: 'list'
- arity n — eg: 1
- for argument types $A_1, \ldots, A_n$ with string encodings $E_1, \ldots, E_n$
  - values of the type — eg: $[a_1, \ldots, a_n]$ for a_i values of A_1
  - string encoding — eg: "[" $E(a_1)$ "," ... "," $E_1(a_n)$ "]"
  - operations on the type — eg: concatenation, element access, ...

No good standards, but part of most languages

Basic Atomic Types

typical in IT systems

- fixed precision integers (32 bit, 64 bit, . . .)
- IEEE float, double
- Booleans
- Unicode characters
- strings — could be list of characters but usually bad idea

typical in math

- natural numbers ($= \mathbb{N}$)
- arbitrary precision integers ($= \mathbb{Z}$)
- rational, real, complex numbers
- graphs, trees

clear: language must be modular, extensible

Advanced Atomic Types

general purpose

- date, time, color, location on earth
- picture, audio, video

domain-specific

- physical quantities ($1m$, $1in$, etc.)
- gene, person
- semester, course id, ...

clear: language must be modular, extensible

Collection Data Types

Homogeneous Collection Types

- sets
- multisets (= bags)
- lists — all unary type operators, e.g. *list A* is type of lists over *A*
- fixed-length lists (= Cartesian power, vector *n*-tuple)

dependent type operator

Heterogeneous Collection Types

- lists
- fixed-length lists (= Cartesian power, *n*-tuple)
- sets
- multisets (= bags)

all atomic types, e.g., *list* is type of lists over any objects

Aggregation Data Types

Products

- Cartesian product of some types $A \times B$
  values are pairs (x, y) numbered projections $_1$, $_2$ — order relevant
- labeled Cartesian product (= record) $\{a : A, b : B\}$
  values are records $\{a = x, b = y\}$
  named projections a, b — order irrelevant

Disjoint Unions

- disjoint union of some types $A \uplus B$
  values are $inj_1(x)$, $inj_2(y)$ numbered injections $_1$, $_2$ — order relevant
- labeled disjoint union $a(A)|b(B)$
  values are constructor applications $a(x)$, $b(y)$
  named injections a, b — order irrelevant

labeled disjoint unions uncommon

but recursive labeled disjoint union = inductive data type

A Basic Language for Data Types

Let BDL be given by

Types

$T ::= int \mid float \mid string \mid bool$		base types
$\mid$	$\texttt{list}\ T$	homogeneous lists
$\mid$	$(\texttt{ID} : T)^*$	record types
$\mid$	$\ldots$	additional types

Data

$D ::= (64\ bit\ integers)$		
$\mid$	$(IEEE\ double)$	
$\mid$	$"(Unicode\ strings)"$	
$\mid$	$\texttt{true} \mid \texttt{false}$	
$\mid$	D^*	lists
$\mid$	$(\texttt{ID} = D)^*$	records
$\mid$	$\ldots$	constructors for additional types

Recursive Concrete Data Types

Motivation

Idea

- describe infinite type in finite way
- describe words derived from grammars
- exploit inductive structure to catch all values

Name: usually called **inductive** data type, especially when recursive

Examples

Natural numbers Nat given by

- $zero$
- $succ(n)$ for every $n : Nat$

Lists $list\ A$ over type A given by

- empty list nil
- $cons(a, l)$ for every $a : A$ and $l : list\ A$

Arithmetic expressions E given by

- natural number $literal(n)$ for $n : Nat$
- sum $plus(e, f)$ for every $e, f : E$
- product $times(e, f)$ for every $e, f : E$

Rigorous Definition

Let T be the set of types that are known in the current context.

An **inductive data type** is given by

- a name n, called the **type**,
- a set of **constructors** each consisting of
  - a name
  - a list of elements of $T \cup \{n\}$, called the **argument** types

Notation:

```
inductive n = c(A,...) | ...

inductive Nat = zero | succ(Nat)
inductive E = Number(Nat) | sum(E,E) | times(E,E)
```

Induction Principle

The values of the inductive type are exactly the ones that can be built by the constructors.

- No junk: the constructors are jointly-surjective
  - no other values but union of their images
  - closed world
- No confusion: the constructors are jointly-injective in the following sense
  - each constructor is an injective function
  - images of the constructor are pairwise disjoint

Inductive definition: define function out of inductive type by giving one case per constructor pattern matching

- total because jointly-surjective
- well-defined because jointly-injective (no overlap between cases)

Special case: No recursion

A concrete data types without recursive constructor arguments are called a **labeled union**. They are isomorphic to the union of the products of the constructor arguments.

Example:

```
inductive Value = Number(Nat) | true | false
inductive Product(A,B) = Pair(A,B)
```

A concrete data types without any constructor arguments is called an **enumeration**. They have exactly one element per constructor.

Example:

```
inductive Boolean = true | false
inductive Color = red | blue | green
```

Generalization: Mutual Induction

Multiple inductive types whose definitions refer to each other.

Example:

```
inductive E = literal(Nat) | sum(E,E) | times(E,E)
inductive F = equal(E,E) | less(E,E)
```

BDL Extended with Named Inductive Types

V	$::=$	$Decl^*$	Vocabularies
$Decl$	$::=$	**inductive** $t\,\{(\texttt{ID} : T^* \to t)^*\}$	type definitions

Types

T	$::=$	$\ldots$	as before
	$\mid$	t	reference to a named type

Data

D	$::=$	$\ldots$	as before
	$\mid$	$\texttt{ID}(D^*)$	constructor application

Recursive Abstract Data Types

Breakout Question

What do the following have in common?

- Java class
- SQL schema for a table
- logical theory (e.g., Monoid)

Breakout Question

What do the following have in common?

- Java class
- SQL schema for a table
- logical theory (e.g., Monoid)

all are (essentially) abstract data types

Motivation

Recall subject-centered representation of assertion triples:

```
individual "FlorianRabe"
  is-a "instructor" "male"
  "teach" "WuV" "KRMT"
  "age" 40
  "office" "11.137"
```

Can we use types to force certain assertions to occur together?

- Every instructor should teach a list of courses.
- Every instructor should have an office.

Motivation

Inspires **subject-centered types**, e.g.,

```
concept instructor
  teach course*
  age: int
  office: string

individual "FlorianRabe": "instructor"
  is-a "male"
  teach "WuV" "KRMT"
  age 40
  office "11.137"
```

Incidental benefits:

- no need to declare relations/properties separately
- reuse relation/property names
  distinguish via qualified names: instructor .age

Motivation

Natural next step: inheritance

```
concept person
  age: int

concept male <: person

concept instructor <: person
  teach course*
  office: string

individual "FlorianRabe": "instructor" ⊓ "male"
  "teach" "WuV" "KRMT"
  "age" 40
  "office" "11.137"
```

our language quickly gets a very different flavor

Examples

Prevalence of abstract data types:

aspect	language	abstract data type
ontologization	UML	class
concretization	SQL	table schema
computation	Scala	class, interface
deduction	various	theory, specification, module, locale
		if recursive: coinductive type
narration	various	emergent feature

same idea, but may look very different across languages

Examples

aspect	type	values
computation	abstract class	instances of implementing classes
concretization	table schema	table rows
deduction	theory	models

Values depend on the environment in which the type is used:

- class defined in one specification language (e.g., UML), implementations in programing languages Java, Scala, etc.
  available values may depend on run-time state
- theory defined in logic,
  models defined in set theories, type theories, programming languages
  available values may depend on philosophical position

Definition

Given some type system, an **abstract data type** (ADT) is defined by

$$\textbf{class}\ a = \{c_1 : T_1[= t_1], \ldots, c_n : T_n[= t_n]\}$$

where

- c_i are distinct names
- T_i are types
- t_i are optional definitions; if given, $t_i : T_i$ required

Recursion

- general case: a may occur in the T_i
- if non-recursive: called a record type

BDL Extended with Named ADTs

V	$::=$	$Decl^*$	Vocabularies
$Decl$	$::=$	**class** $t\,\{\texttt{ID} : T^*\}$	ADT definitions

Types

T	$::=$	$\ldots$	as before
	$\mid$	t	reference to a named ADT

Data

D	$::=$	$\ldots$	as before
	$\mid$	$t\{(\texttt{ID} = D)^*\}$	ADT elements

Inheritance

Generalized ADT definition:

```
abstract class a extends a_1, ..., a_m {
  c_1 : T_1
  ⋮
  c_n : T_n
}
```

Terminology:

- a **inherits** from a_i
- a_i are **super**-X or **parent**-X of a where X is whatever the language calls its ADTs (e.g., X=class)

Flattening

Given ADT with inheritance as above, define **flattening** by

$$a^\flat = a_1^\flat \cup a_m^\flat \cup \{c_1 : T_1, \ldots, c_n : T_n\}$$

where duplicate field names are handled as follows

- same name, same type, same or omitted definition: merge
  details may be much more difficult
- otherwise: ill-formed

Subtleties

We gloss over several major issues:

- How exactly do we merge duplicate field names? Does it always work? implement abstract methods, override, overload
- Is recursion allowed, i.e., can I define an ADT $a = A$ where a occurs in A?
  common in OO-languages: use a in the types of its fields
- What about ADTs with type arguments?
  e.g., generics in Java, square-brackets in Scala
- Is mutual recursion between fields in a flat type allowed?
  common in OO-languages
- Is * commutative? What about dependencies between fields?

  no unique answers

  incarnations of ADTs subtly different across languages

Breakout question

When using typed concrete data,
how to fully realize abstract data types

- nesting: ADTs occurring as field types
- inheritance between ADTs
- mixins

ADTs in Databases

Nesting: field $a : A$ in ADT B

- field types must be base types, $a : A$ not allowed
- allow ID as additional base type
- use field $a : ID$ in table B
- store value of b in table A

Inheritance: B inherits from A

- add field $parent_A$ to table B
- store values of inherited fields of B in table A

general principle: all objects of type A stored in same table

Mixin: $A * B$

- essentially join of tables A and B on common fields
- some subtleties depending on ADT flattening

Subtyping

Subtyping

- relatively easy if all data types disjoint
- better with subtyping open problem how to do it nicely

Subtyping Atomic Types

- $\mathbb{N} <: \mathbb{Z}$
- ASCII $<:$ Unicode

Subtyping Complex Types

- covariance subtyping ($=$ vertical subtyping) same for disjoint unions

$$A <: A' \Rightarrow \mathit{list}\ A <: \mathit{list}\ A'$$

$$A_i <: A'_i \Rightarrow \{\ldots, a_i : A_i, \ldots\} <: \{\ldots, a_i : A'_i, \ldots\}$$

- structural subtyping ($=$ horizontal subtyping)

$$\{a : A, b : B\} :> \{a : A, b : B, c : C\}$$

$$a(A)|b(B) <: a(A)|b(B)|c(C)$$

Exercise 10

As a single group, write a joint document that specifies a choice of built-in datatypes for BOL and SFOL. They should be helpful in general and for the university ontology in particular.

For each new type, you have to give

- name
- values
- string encoding of each value (for printing, import/export)

Add the types and their values to your BOL and SFOL implementations and adjust your translation to translates values to themselves. (Note: Not all types will be supported by theorem provers; so your TPTP printer may have to be partial.)

A Basic Programming Language

See the running example of grammar and type system as well as the implementation of syntax, parser, printer, type-checker, and semantics by translation.

OWL-Style Ontology Languages

Components of an Ontology

8 main declarations

- **individual** — concrete objects that exist in the real world, e.g., "Florian Rabe" or "WuV"
- **concept** — abstract groups of individuals, e.g., "instructor" or "course"
- **relation** — binary relations between two individuals, e.g., "teaches"
- **properties** — binary relations between an individuals and a concrete value (a number, a date, etc.), e.g., "has-credits"
- axioms
  - **concept assertions** — the statement that a particular individual is an instance of a particular concept
  - **relation assertions** — the statement that a particular relation holds about two individuals
  - **property assertions** — the statement that a particular individual has a particular value for a particular property
  - **terminological axioms** — statements about relations between concepts, e.g., "instructor" $\sqsubseteq$ "person"

An Example Ontology Language: Structural Parts

The structural part declares/introduces names:

Vocabularies: Ontologies

$O \quad ::= D^*$

Declarations

D	$::=$	**individual** i	atomic individual
	$\mid$	**concept** c	atomic concept
	$\mid$	**relation** r	atomic relation
	$\mid$	**property** $p : T$	atomic property
	$\mid$	**axiom** F	axiom

Identifiers

$i, c, r, p ::=$ alphanumeric string

An Example Ontology Language: Expressions (1)

Expressions use the declared names to build complex entities

Individual expressions

I	$::= i$	atomic individuals

Concept expressions

C	$::= c$	atomic concepts
	$\mid \top$	universal concept
	$\mid \bot$	empty concept
	$\mid C \sqcup C$	union of concepts
	$\mid C \sqcap C$	intersection of concepts
	$\mid \forall R.C$	universal relativization
	$\mid \exists R.C$	existential relativization
	$\mid \mathrm{dom} R$	domain of a relation
	$\mid \mathrm{rng} R$	range of a relation
	$\mid \mathrm{dom} P$	domain of a property

An Example Ontology Language: Expressions (2)

Relation expressions

$R ::= r$		atomic relations
$\mid$	$R \cup R$	union of relations
$\mid$	$R \cap R$	intersection of relations
$\mid$	$R; R$	composition of relations
$\mid$	R^*	transitive closure of a relation
$\mid$	R^{-1}	dual relation
$\mid$	Δ_C	identity relation of a concept

Formulas

$F ::= C \equiv C$		concept equality
$\mid$	$C \sqsubseteq C$	concept subsumption
$\mid$	I `is-a` C	concept instance
$\mid$	$I\ R\ I$	relation instance
$\mid$	$I\ P\ V$	property instance

An Example Ontology Language: Expressions (3)

Built-in types and values are part of the expressions in order to use them with properties

Property expressions

$P ::= p$	atomic properties

Basic types and values

$T ::= \texttt{int} \mid \texttt{float} \mid \texttt{bool} \mid \texttt{string}$	types
$V ::=$ (omitted)	values

Typical: one kind of expression for every kind of declaration, plus some other expression kinds (here: formulas, types, and values)

Divisions of an Ontology

Abstract vs. concrete

- TBox: concepts, relations, properties, axioms
  everything that does not use individuals
- ABox: individuals and assertions

Named vs. unnamed

- Signature: individuals, concepts, relations, properties
  together called entities or resources
- Theory: assertions, axioms

Comparison of Terminology

Here	OWL	Description logics	ER model	UML	semantics via logics
individual	instance	individual	entity	object, instance	constant
concept	class	concept	entity-type	class	unary predicate
relation	object property	role	role	association	binary predicate
property	data property	(not common)	attribute	field of base type	binary predicate

domain	individual	concept
type theory, logic	constant, term	type
set theory	element	set
database	row	table
philosophy[1]	object	property
grammar	proper noun	common noun

[1] as in `https://plato.stanford.edu/entries/object/`

Ontologies as Sets of Triples

General idea:

- Turn everything into a relation/property assertion
- Represent ontologies as sets of subject-predicate-object triples
- Obtain efficient representation of ontologies using RDF and RDFS

Assertion	Triple		
	Subject	Predicate	Object
entities	recover from what's mentioned in assertions		
concept assertion	"Florian Rabe"	`is-a`	"instructor"
relation assertion	"Florian Rabe"	"teaches"	"WuV"
property assertion	"WuV"	"has credits"	7.5
axiom	only some special cases work, e.g.,		
subconcept axiom	"instructor"	`subClassOf`	"person"

Special Entities

RDF and RDFS define special entities for use in ontologies:

- "rdfs:Resource": concept of which all individuals are an instance and thus of which every concept is a subconcept
- "rdf:type": relates an entity to its type:
  - an individual to its concept (corresponding to `is-a` above)
  - other entities to their special type (see below)
- "rdfs:Class": special class for the type of classes
- "rdf:Property": special class for the type of properties
- "rdfs:subClassOf': a special relation that relates a subconcept to a superconcept
- "rdfs:domain": a special relation that relates a relation to the concepts of its subjects
- "rdfs:range": a special relation that relates a relation/property to the concept/type of its objects

Goal/effect: capture as many parts as possible as RDF triples.

Declarations as Triples using Special Entities

Assertion	Triple		
	Subject	Predicate	Object
individual	individual	"rdf:type"	"rdfs:Resource"
concept	concept	"rdf:type"	"rdf:Class"
relation	relation	"rdf:type"	"rdf:Property"
property	property	"rdf:type"	"rdf:Property"
concept assertion	individual	"rdf:type"	concept
relation assertion	individual	relation	individual
property assertion	individual	property	value
for special forms of axioms			
$c \sqsubseteq d$	c	"rdfs:subClassOf"	d
$\mathrm{dom}\, r \equiv c$	r	"rdfs:domain"	c
$\mathrm{rng}\, r \equiv c$	r	"rdfs:range"	c

A Real-Life Ontology Language

See online resources for OWL.

Some specialties:

- Slightly different names than in BOL
- No strict distinction between individuals, concepts, relations - just resources
- Some special axioms, e.g., to make relations transitive
- Multiple sublanguages with varying expressivity/implementability: Lite, DL, Full

BOL vs. OWL:

- BOL is simpler, more systematically structured
  good for teaching, prototypes
- OWL is the standard
  the one to use for better or worse

Exercise 1

As a team, build an ontology for a university.

Using git, OWL, and WebProtege are good ways to start.

(In WebProtege, set "suffix" to "user supplied name" in "New Entity Settings". Otherwise, it'll get messy when you share your ontology.)

Example: The Common Sense Ontology

Situation

- society uses one ontology for common sense knowledge
- changes over time

content relative to ontology: laws, regulations, etc.

Special aspects

- unwritten
- not actually fully agreed upon
- sometimes subject to political debate
- no formal ontology language good enough to capture practical nuances
- many society members not comfortable with formal languages

but still always exists implicitly

Idea: see political proposals as ontology evolution

OWL-Style vs. SQL-Style Ontology Languages

Recall: Ontologies

Main ideas

- Ontology abstractly describes concepts and relations
- Tool maintains concrete data set
- Focus on efficiently
  - identifying (i.e., assign names)
  - representing
  - processing
  - querying

  large sets of concrete data

Recall: TBox-ABox distinction

- TBox: general parts, abstract, fixed

  main challenge: correct modeling of domain
- ABox: concrete individuals and assertions about them, growing

  main challenge: aggregate them all

Concrete Data

Concrete is

- Base values: integers, strings, booleans, etc.
- Collections: sets, multisets, lists (always finite)
- Aggregations: tuples, records (always finite)
- User-defined concrete data: enumerations, inductive types
- Advanced objects: finite maps, graphs, etc.

Concrete is not

- Uninterpreted symbols
- Variables (free or bound) — λ-abstraction, quantification
- Symbolic expressions — formulas, algorithms

Two Approaches to Representing Concrete Data

Curry-typed ontology languages (e.g., BOL, OWL)

- Representation based on knowledge graph
- Ontology written in BOL-like language
- Data maintained as set of triples — tool = triple store
- Typical language/tool design
  - ontology and query language separate — e.g., OWL, SPARQL
  - triple store and query engine integrated — e.g., Virtuoso tool

Church-typed languages (e.g., SQL, UML)

- Representation based on abstract data types
- Ontology written as set of related ADTs — SQL database schema
- Data maintained as tables — tool = (relational) database
- Typical language/tool design
  - ontology and query language integrated — e.g., SQL
  - table store and query engine integrated — e.g., SQLite tool

Evolution of Approaches

Our usage is non-standard

- Common
  - ontologies = untyped approach, OWL, triples, SPARQL
  - relational databases = typed approach, tables, SQL
- Our understanding: two approaches evolved from same idea
  - ontolgoies = Curry-typed ontology + data store
  - relational database = Church-typed ontology + data store

Evolution

- Typed-untyped distinction minor technical difference
- Optimization of respective advantages causes speciation
- Today segregation into different
  - jargons
  - languages, tools
  - communities, conferences
  - courses

Data structures for Curry-typed concrete data

Central data structure = knowledge graph

- nodes = individuals i
  - identifier
  - sets of concepts of i
  - key-value sets of properties of i
- edges = relation assertions
  - from subject to object
  - labeled with name of relation

Processing strengths

- store: as triple set
- edit: Protege-style or graph-based
- visualize: as graph — different colors for concepts, relations
- query: match, traverse graph structure
- untyped data simplifies integration, migration

Data structures for Church-typed concrete data

Central data structure = relational database

- tables = abstract data type
- rows = objects of that type
- columns = fields of ADT
- cells = values of fields

Processing strengths

- store: as CSV text files, or similar
- edit: SQL commands or table editors
- visualize: as table view
- query: relational algebra
- typed data simplifies selecting, sorting, aggregating

Identifiers

Curry-Typed Knowledge graph

- concept, relation, property names given in TBox
- individual names attached to nodes

Church-Typed Database

- table, column names given in schema
- row identified by distinguished column (= key)
  options
  - preexistent characteristic column
  - added upon insertion
    - UUID string
    - incremental integers
    - concatenation of characteristic list of columns
- column/row identifiers formed by qualifying with table name

Axioms

Curry-Typed Knowledge Graph

- traditionally very expressive axioms
- yields inferred assertions
- triple store must do consequence closure to return correct query results
- not all axioms supported by every triple store

Church-Typed Database

- typically no axioms
- instead consistency constraints, triggers
- allows limited support for axioms without calling it that way
- stronger need for users to program the consequence closure manually

Open/Closed World

- Question: is the data complete?
  - closed world: yes
  - open world: not necessarily
- Dimensions of openness
  - existence of individual objects
  - assertions about them
- Sources of openness
  - more exists but has not yet been added
  - more could be created later
- Orthogonal to typed/untyped distinction, but in practice
  - knowledge graphs use open world
  - databases use closed world

Open world is natural state, closing adds knowledge

Closing the World

Derivable consequences

- induction: prove universal property by proving for each object
- negation by failure: atomic property false if not provable
- term-generation constraint: only nameable objects exist

Enabled operations

- universal set: all objects
- complement of concept/type
- defaults: assume default value for property if not otherwise asserted

Monotonicity problem

- monotone operation: bigger world = more results
- examples: union, intersection, $\exists R.C$, join, IN conditions
- counter-examples: complement, $\forall R.C$, NOT IN conditions

technically, non-monotone operations in open world dubious

Summary

	semantic web	relational databases
ontology aspect	TBox of ontology	SQL schema
conceptual model	knowledge graph	set of tables
concrete data aspect	ABox of ontology	SQL database
concrete data storage	set of triples	set of rows of the tables
concrete data formats	RDF	CSV
concrete data tool	triple store	database implementation
typing	soft/Curry	hard/Church
query language	SPARQL	SQL SELECT query
openness of world	tends to be open	tends to be closed

Exercise 9

Absolute semantics:

- Via concrete data: Export your ontology to a triple store like Virtuoso and run a concrete query in SPARQL.
- Via deduction: Export your ontology in OWL format to a reasoner like FaCT++ and prove a theorem. Potentially, do this by installing a plugin for a reasoner in your ontology IDE.

Relative semantics:

- Via deduction: finish exercise 8 and prove a theorem via an SFOL theorem prover.

Data Languages vs. Ontology Languages

Data as Part of Ontology

Terminology vs. Data

- schema describes general shape of the data
  - small
  - rarely changed
  - carefully designed to model the world
- data must conform to the schema
  - often large
  - easy to change, extend
  - requires expensive curation

Examples

- OWL-style: TBox for terminology, ABox for data
- SQL-style: schema for terminology, tables for data

Standalone Data Languages

Ontology-backing common but not technically necessary

- OWL-style data with optional terminology: RDF
- SQL-style data with optional schema: CSV

Ontology-backing possible but not as common or standardized

- variety of structurally similar languages JSON, XML, YAML
- various schema and typing standards
- expression-only, no vocabularies
  adding vocabularies would create integrated ontology+data language

major challenge: data interoperability

CSV

RDF

Standalone Data Languages

Overview

General Properties

- general purpose or domain-specific
- typed or untyped
  typical: Church-typed but no type operators, quasi untyped
- text or binary serialization
- libraries for many programming languages
  - data structures
  - serialization (data structure to string)
  - parsing (string to data structure, partial)

Candidates

- XML: standard on the web, notoriously verbose
- JSON: JavaScript objects, more human-friendly text syntax
  older than XML, probably better choice than XML in retrospect
- YAML: line/indentation-based

Breakout Question

What is the difference between JSON, YAML, XML?

Typical Data Representation Languages

XML, JSON, YAML essentially the same

except for concrete syntax

Atomic Types

- integer, float, boolean, string
- need to read fine-print on precision

(Not Very) Complex Types

- heterogeneous lists — a single type for all lists
- records — a single type for all records

Example: JSON

JSON:

```
{
    "individual" : "FlorianRabe",
    "age" : 40,
    "concepts" : ["instructor","male"],
    "teach" : [
        {"name" : "Wuv", credits : 7.5},
        {"name" : "KRMT", credits : 5}
    ]
}
```

Weirdnesses:

- atomic/list/record = basic/array/object
- record field names are arbitrary strings, must be quoted
- records use : instead of =

Example: YAML

inline syntax: same as JSON but without quoted field names

alternative: indentation-sensitive syntax

individual : "*FlorianRabe*"
age : 40
concepts :
  – "*instructor*"
  – "*male*"
teach :
  – *name* : "*WuV*"
    credits : 7.5
  – *name* : "*KRMT*"
    credits : 5

Weirdnesses:

- atomic/list/record = scalar/collection/structure
- records use : instead of =

Example: XML

Weird structure but very similar

- elements both record (= attributes) and list (= children)
- elements carry name of type (= tag)

```
<Person individual="Florian Rabe" age="40">
 <concepts>
   <Concept>instructor</Concept/>
   <Concept>male</Concept/>
 </concepts>
 <teach>
   <Course name="WuV" credits="7.5"/>
   <Course name="KRMT" credits="5"/>
 </teach>
</Person>
```

- Good: Person, Course, Concept give type of object
  easier to decode
- Bad: value of record field must be string
  concepts cannot be given in attribute
  integers, Booleans, whitespace-separated lists coded as strings

Structure Sharing

Problem

- Large objects are often redundant specially when machine-produced
- Same string, URL, mathematical objects occurs in multiple places
- Handled in memory via pointers
- Size of serialization can explode

Solution 1: in language

- Add definitions to language common part of most languages anyway
- Users should introduce name whenever object used twice
- Problem: only works if
  - duplication anticipated
  - users introduced definition
  - duplication within same context

structure-sharing most powerful if across contexts

Structure Sharing (2)

Solution 2: in tool

- Use factory methods instead of constructors
- Keep huge hash set of all objects
- Reuse existing object if already in hash set
- Advantages
  - allows optimization
  - transparent to users
- Problem: only works if
  - for immutable data structures
  - if no occurrence-specific metadata e.g., source reference

In data representation language

- Allow any subobject to carry identifier
- Allow identifier references as subobjects

allows preserving structure-sharing in serialization

supported by XML, YAML

Syntax Trees as Data

Two kinds of data in syntax

- Basic data that occurs literally in the source
  - in BOL, SFOL: strings for names, basic types/values
  - typically leaves of the syntax tree
  - often needs a codec
- The syntax tree structure
  - names of the productions/constructors
  - names of the children/constructor arguments
  - nesting of nodes

Representing a Syntax Tree in XML

Context-free traverser of the AST (printer)

- For basic data, use the type as the tag and give the string encoding in an attribute:

$$< \text{identifier value}=\text{"NAME"} / >$$

$$< \text{int value}=\text{"ENCODED\_VALUE"} / >$$

- For node $N = c(e_1, \ldots, e_n)$ where
  - c is the constructor/production name
  - e_i are the subtrees

$$XML(N) =< c > XML(e_1) \ldots XML(e_n) < /c >$$

optionally: wrap every $XML(e_i)$ in another tag that gives the name of the constructor argument, e.g., for $C \sqcup D$

$$<union><left>XML(C)</left><right>XML(D)</right></union>$$

$$<union>XML(C)XML(D)</union>$$

Exercise 11

As a group, define an XML schema for BOL using the RelaxNG compact syntax. See `https://relaxng.org/compact-tutorial-20030326.html`.

Use names for all productions and constructor arguments, to obtain the tag names for the tree structure.

Use the string encodings for your agreed-upon primitive types to encode values of primitive types.

Individually, extend your implementations of BOL with XML importers and exporters relative to that schema.

The exporter is a context-free traverser. For the importer, use an XML library for your chosen programming language to parse the input. Then write a context-free traverser over the XML data structures that translates the XML into your BOL data structures.

Exchange your vocabularies with each other. Type-check each other's vocabularies after importing.

Data Interoperability through Codecs

Data Interoperability

Situation

- languages focus on different aspects frequent need to exchange data
- generally, lots of aspect/language-specific objects
  proofs, programs, tables, sentences
- but same/similar primitive data types used across systems
  should be easy to exchange

Problem

- crossing system barriers usually requires interchange language
  serialize as string and reparse
- interchange languages typically untyped
  RDF, CSV, XML, JSON, YAML, ...

Solution

- standardize basic data types
- standardize encoding in interchange languages

Failures of Encodings

Y2K bug

- date encoded as tuple of integers, using 2 digits for year
- needed fixing in year 2000
- estimated $300 billion spent to change software
- possible repeat: in 2038, number of seconds since 1970-01-01 (used by Unix to encode time as integer) overflows 32-bit integers

Genes in Excel

- 2016 study found errors in 20% of spreadsheets accompanying genomics journal papers
- gene names encoded as strings but auto-converted to other types by Excel
  - "SEPT2" (Septin 2) converted to September 02
  - REKIN identifiers, e.g., "2310009E13", converted to float $2.31E+1$

https://genomebiology.biomedcentral.com/articles/10.1186/s13

Failures of Encodings (2)

Mars Climate Orbiter

- two components exchanged physical quantity
- specification required encoding as number using unit Newton seconds
- one component used wrong encoding (with pound seconds as unit)
- led to false trajectory and loss of $300 million device

Shellshock

- bash allowed gaining root access from 1998 to 2014
- function definitions were encoded as source code
- not decoded at all; instead, code simply run (as root)
- allowed appending "; ..." to function definitions

SQL injection similar: complex data encoded as string, no decoding

General Definition

Throughout this section, we fix a data representation language L.

L-words called codes

Given a data type T, a codec for T consists

- coding function: $c : T \to L$
- partial decoding function: $d : L \to^{?} T$
- such that

$$d(c(x)) = x$$

Codec Operators

Given a data type operator T taking n type arguments, a codec operator C for T

- takes n codecs C_i for T_i
- returns a codec $C(C_1, \ldots, C_n)$ for $T(T_1, \ldots, T_n)$

Codecs for Base Types

We define codecs for the base types using strings as the data representation language L.

Easy cases:

- StandardFloat: as specified in IEEE floating point standard
- StandardString: as themselves, quoted
- StandardBool: as *true* or *false*
- StandardInt (64-bit): decimal digit-sequences as usual

Breakout Question

How to encode unlimited precision integers?

Codecs for Unlimited Precision Integers

Encode $z \in \mathbb{Z}$

- L is strings: decimal digit sequence as usual
- L is JSON:
  - IntAsInt: decimal digit sequence as usual
    JSON does not specify precision
    but target systems may get in trouble
  - IntAsString: string containing decimal digit sequence
    safe but awkward
  - IntAsDecList: list of decimal digits safe but awkward
  - IntAsList1: as list of digits for base 2^{64}
    OK, but we can do better
  - IntAsList2: as list of
    - integer for the number of digits, sign indicate sign of z
    - list of digits of $|z|$ for base 2^{64}

    Question: Why is this smart?

Codecs for Unlimited Precision Integers

Encode $z \in \mathbb{Z}$

- L is strings: decimal digit sequence as usual
- L is JSON:
  - IntAsInt: decimal digit sequence as usual
    JSON does not specify precision
    but target systems may get in trouble
  - IntAsString: string containing decimal digit sequence
    safe but awkward
  - IntAsDecList: list of decimal digits safe but awkward
  - IntAsList1: as list of digits for base 2^{64}
    OK, but we can do better
  - IntAsList2: as list of
    - integer for the number of digits, sign indicate sign of z
    - list of digits of $|z|$ for base 2^{64}

    Question: Why is this smart?
    Can use lexicographic ordering for size comparison

Codecs for Lists

Encode list x of elements of type T

- L is strings: e.g., comma-separated list of T-encoded elements of x
- L is JSON:
  - ListAsString: like for strings above
  - ListAsArray: lists JSON array of T-encoded elements of x

Additional Types

Examples: semester

Extend BDL:

Types	
$T ::= Sem$	semester
Data	
$D ::= sem(int, bool)$	i.e., year + summer?

Define standard codec:

$$sem(y, true) \rightsquigarrow "SSY"$$

$$sem(y, false) \rightsquigarrow "WSY"$$

where Y is encoding of y

Additional Types (2)

Examples: timestamps

Extend BDL:

Types

T ::= *timestamp*

Data

D ::= (productions for dates, times, etc.)

Standard codec: encode as string as defined in ISO 8601

Ontology-Backed Data Interchange

Research Goal for Aspect-Independent Data in Tetrapod

Standardization of Common Data Types

- Ontology language optimized for declaring types, values, operations
  semantics must exist but can be extra-linguistic
- Vocabulary declaring such objects
  should be standardized, modular, extensible

Standardization of Codecs

- Fixed small set of primitive objects
  should be (quasi-)primitive in every language
  not too expressive, possibly untyped
- Standard codecs for translating common types to interchange languages

Codec for type A and int. lang. L

- coding function A-values $\to$ L-objects
- partial decoding function L-objects $\to$ A-values
- inverse to each other
  in some sense

Design

1. Specify types
   - types
   - constructors
   - operations

   can be done in appropriate type theory
2. Pick data representation language L
3. Specify codecs for type system and L
   - at least one codec per base type
   - at least one codec operator per type operator

   on paper
4. Every system implements
   - type system (as they like) typically aspect-specific constraints
   - codecs as specified
   - function mapping types to codecs
5. Systems can exchange data by encoding-decoding

   type-safe because codecs chosen by type

BDL-Mediated Interoperability

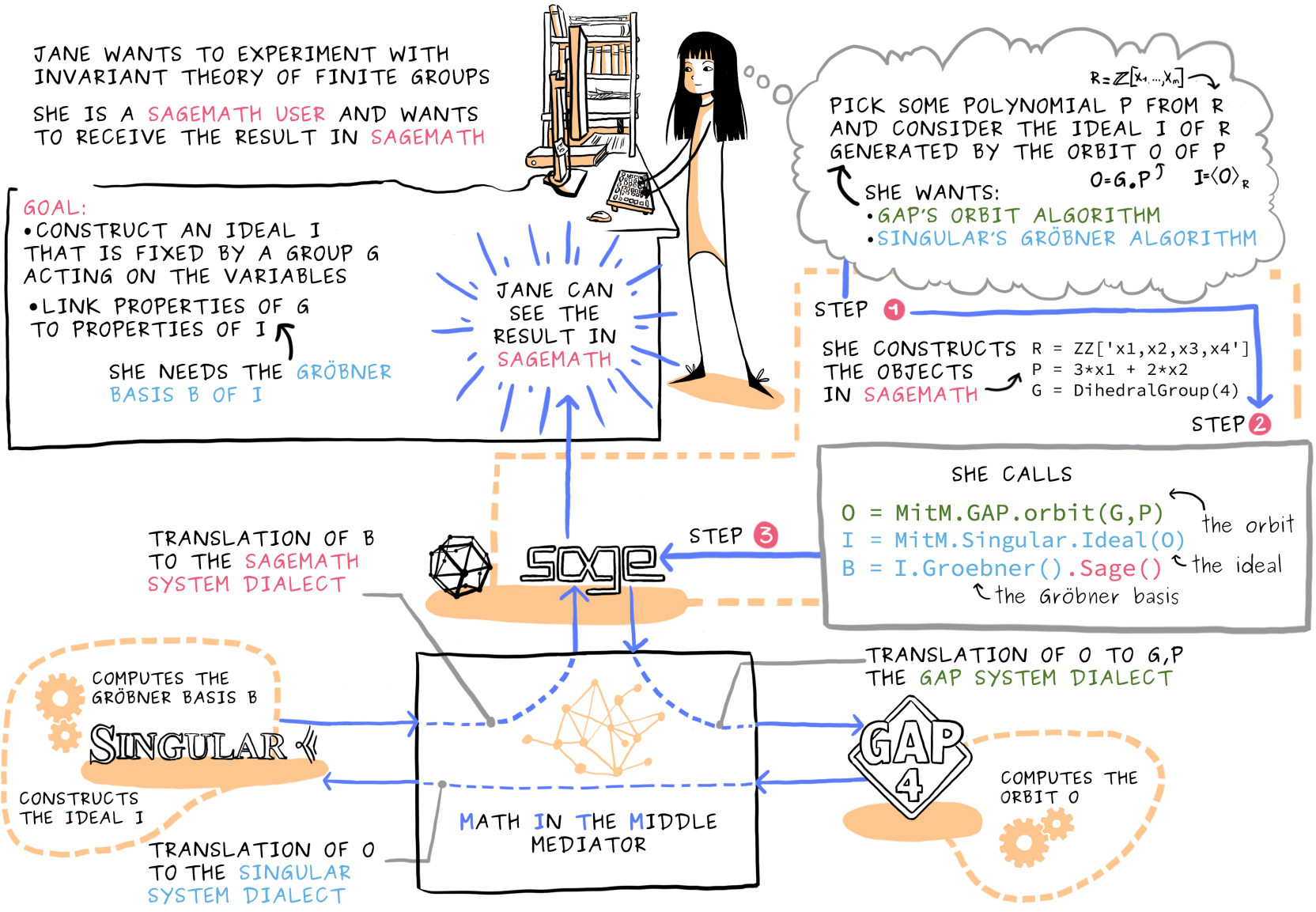
Idea

- define data types in BDL or similar typed ontology language
- use ADTs
- generate corresponding
  - class definitions for programming languages PL
    one class per ADT
  - table definitions in SQL one table per ADT
- use codecs to convert automatically when interchanging data between PL and SQL

Open research problem

no shiny solution yet that can be presented in lectures

Example Application: OpenDreamKit research project



Codecs in ADT Definitions

SQL table schema = list of fields where field is

- name
- type — only types of database supported

BDL semantic table schema = list of fields where field is

- name
- type T of type system — independent of database
- codec for T using primitive objects of database as codes
  see research paper `https://kwarc.info/people/frabe/Research/WKR_`

Codec could be chosen automatically, but we want to allow multiple users a choice of codecs for the same type.

Example

Ontology based on BDL-ADTs with additional codec information:

```
schema Instructor
  name:     string       codec StandardString
  age:      int          codec StandardInt
  courses:  list Course  codec CommaSeparatedList CourseAsName
schema Course
  name:      string      codec StandardString
  credits:   float       codec StandardFloat
  semester:  Semester    codec SemesterAsString
```

Generated SQL tables:

```
CREATE TABLE Instructor
  (name string, age int, courses string)
CREATE TABLE Course
  (name string, credits float, semester string)
```

Open Problem: Non-Compositionality

Sometimes optimal translation is non-compositional

- example translate *list*-type in ADT to comma-separated string in DB
- better break up *list B* fields in type *A* into separate table with columns for *A* and *B*

Similar problems

- a pair type in an ADT could be translated to two separate columns
- an option type in an ADT could translated to a normal column using SQL's NULL value

Open Problem: Querying

- General setup
  - write SQL-style queries using at the BDL level
  - automatically encode values when writing to database from PL
  - automatically decode query results when reading from DB
- But queries using semantic operations cannot always be translated to DB
  - operation $\mathit{IsSummer} : \mathit{Semester} \to \mathit{bool}$ in BDL
  - query SELECT $*$ FROM course WHERE$\mathit{IsSummer}$(semester)
  - how to map *IsSummer* to SQL?
- Ontology operations need commuting operations on codes
  - given $f : A \to B$ in BDL, codecs C, D for A and B
  - SQL function f' commutes with f iff

$$B.\mathit{decode}(f'(C.\mathit{encode}\, a)) = f(a)$$

for all $a : A$

Typed First-Order Logic

Syntax

We give an overview of first-order logic as the main example of the deductive corner of the Tetrapod. The grammar is given in Section 6.3 of the notes.

Type Systems

Overview

General Goal

- subdivide expressions into groups (called sorts, types, kinds, etc.)
- written $\vdash_V E : T$ for expression E of type T and vocabulary V

Basic Type System

- expressions are always subdivided by their non-terminals
- the types are the non-terminals
- type of expression immediately clear from expression
- context only needed to check well-formedness of expression, not to infer its type

Refined Type System

- for some non-terminals, the expressions are additionally subdivided
- the types are other expressions

  E and T can be from same or different non-terminals
- relation between E and T entirely up to the language

Local Variables

Type Systems with Variables

- $\Gamma \vdash E : T$ for expression E of type T
- $\Gamma = x_1 : Y_1, \ldots, x_n : Y_n$ declares free variables that may occur in E and T
- confusingly: Γ usually also called context

Choice of Γ

- for BOL: nothing (BOL has no variable binding)
- for SFOL: variables introduced by $\forall$ and $\exists$

Algorithms for Type Systems

Judgment $\Gamma \vdash_V E : T$

Type Checking

- input: V, Γ, E, T
- output: boolean

Type Inference

- input: Γ, V, E
- output: T

in practice: also return error messages, e.g., as exceptions

Advanced Variants

- as above but additionally return E' and T'
- E and T have gaps that are filled by the algorithms resulting in E' and T'

Implementing a Type-Checker

Structure of Syntax

- structural level: vocabularies (and morphisms), declarations
- expressions: some non-terminals are designated as expressions
  - usually at least one per declaration kind
  - usually includes (or can be extended to include) formulas

Structure of Type-Checker

- function check-N for each non-terminal N
  - takes context (V and Γ) and N-word
  - returns error message(s)
- whenever N-words are typed by Y-words, instead
  - function check-N takes V, Γ, N-word, and Y-word (expected type)
  - function infer-N takes V, Γ, N-word; returns Y-word (inferred type)

Exercise 6

Implement the syntax (with printer but not necessarily with parser) and type-checking of SFOL.

Theory Morphisms

Syntax

Syntax like for BOL: morphism $m : V \to W$ maps

- type symbols to type expressions

$$y := Y$$

- function symbols to function expressions

$$f := [x_1, \ldots, x_n]T$$

n-ary function expression = term with n free variables

- predicate symbols to predicate expressions

$$p := [x_1, \ldots, x_n]F$$

n-ary predicate expression = formula with n free variables

Example

V:

```
type person
type int
fun age: person → int
pred sibling ⊆ person person
```

W:

```
type human
type int
fun minus: int int → int
fun birthYear: person → int
fun currentYear: → int
pred parent ⊆ person person
```

One possible morphism $m : V \to W$:

```
person := human
int    := int
age    := [p] minus(currentYear, birthYear(p))
sibling:= [x,y] ∀ p:human. parent(p,x) ↔ parent(p,y)
```

Type-Checking (1)

The easy cases — like for BOL

- for V-type symbol $y := Y$ mapped by $y := Y$ well-typed if

$$\vdash_W Y : \texttt{type}$$

  i.e., if Y is a type-expression
- V-axiom asserting F: mapping of named symbols must be such that $m(F)$ is a theorem

Type-Checking (2)

Conditions for function/predicate symbols slightly technical

- V-function symbol $f : Y_1 \dots Y_n \to Y$ mapped by $f := [x_1, \dots, x_n]T$

$$x_1 : m(Y_1), \dots, x_n : m(Y_n) \vdash_W f' : m(Y)$$

  i.e., well-typed if T is a well-typed term with
  - free variables whose types correspond to the arguments of f
  - output corresponding to the output type of f
- V-predicate symbol $p \subseteq Y_1 \dots Y_n$ mapped by $p := [x_1, \dots, x_n]F$

$$x_1 : m(Y_1), \dots, x_n : m(Y_n) \vdash_W F : \texttt{form}$$

  i.e., well-typed if F is a well-typed formula with free variables whose types correspond to the arguments of p

  like for function symbols but no output type

Homomorphic Extension

Given morphism $m : V \to W$

- In principle like for BOL: map V-expression E to W-expression $m(E)$ by replacing every symbol reference with the expression provided by m
- But one subtlety for function/predicate application
  - if m contains $f := [x_1, \ldots, x_n]T$:

$$m(f(t_1, \ldots, t_n)) = T[x_1/m(t_1), \ldots, x_n/m(t_n)]$$

    (term T with each variable x_i substituted with t_i)
  - if m contains $p := [x_1, \ldots, x_n]F$:

$$m(p(t_1, \ldots, t_n)) = F[x_1/m(t_1), \ldots, x_n/m(t_n)]$$

    (formula F with each variable x_i substituted with t_i)

Theory Morphisms Preserve Theorems

Given

- well-typed theories V, W
- well-typed theory morphism $m : V \to W$
- well-typed V-formula F that is a theorem

i.e., consequence of the V-axioms

Then:

$$m(F) \quad \text{is a } W\text{-theorem}$$

Enables Big Picture Applications

- reuse theorems across theories
- show equivalence of theories
- find structural connection between seemingly large theories
- build module system using inheritance and functors
- build large theories from small systematically

Semantics

Theorems

Semantics for an SFOL theory V

- theorem: a formula that is implied (must be true) by the axioms
- contradiction: a formula that must be false due to the axioms

  if there is negation: F is contradiction iff $\neg F$ is theorem
- V is inconsistent iff all formulas are theorems
  (equivalently: iff all formulas are contradictions)

more details on semantics later

Kinds of Formulas

- ill-formed: cannot be represented in AST, parse error
- ill-typed: can be represented in AST, but rejected by type-checker
- well-typed: accepted by type-checker and one of
  - theorem
  - contradiction
  - other

    usually, most formulas are neither theorem not contradiction

Implementing Semantics

Automated Theorem Prover (ATP)

- input:
  - an SFOL-theory V
  - a well-typed V-formula F (called the conjecture)
- output: F is theorem/F is contradiction/timeout

Theorem status undecidable for most logics

- theorem provers try proving/refuting until interrupted
- typical: proof/refutation in a few seconds/minutes or never

many ATPs for SFOL: Vampire, E, Spass, ...

TPTP: A Standard Concrete Syntax

SFOL Syntax Standardization

- basically only one choice for abstract syntax of SFOL
- but lots of options for concrete syntax

  recall: concrete syntax = grammar with all the terminal symbols

TPTP Concrete Syntax

- designed by Sutcliffe for CASC competition

  annual competition of SFOL ATPs
- gradually became de facto standard syntax for SFOL
- today: every SFOL ATP supports TPTP as input syntax
- various extensions of TPTP for other logics

Online interface for SFOL ATPs at `http://www.tptp.org`

TPTP Syntax: Declarations

Theory

- text file containing one line per declaration/axiom/conjecture

Declarations

- named: line `tff(decl_N, decl, N: TYPE).` where
  - `N` is the name
  - `TYPE` is
    - `$tType` for a type declaration
    - `Y1*...*Yn>Y` for a function declaration
    - `Y1*...*Yn>$o` for a predicate declaration
- axiom: line `tff(N, axiom, F).` where
  - `N` is some name
  - `F` is the formula

TPTP Syntax: Expressions

Expressions

- ![X:TYPE]:F and ?[X:TYPE]:F for quantifiers
- F & G, F | G, F => G, ~F for connectives
- s(T1 ,..., Tn) for function/predicate applications

Identifiers

- variables must start with upper-case letter
- type/function/predicate symbols must start with a lower-case letter
- remaining characters alphanumeric

Conjecture (the formula to prove)

- like an axiom
- but with conjecture instead of axiom

Exercise 7

Extend your implementation of SFOL as follows:

- Add a printer for printing SFOL theories and expressions that produces strings in TPTP syntax.
- Add a function that takes a theory and a conjecture and produces the corresponding TPTP input. Use an ATP to (try to) prove the conjecture.
- Optionally:
  - Add theory morphisms to the AST.
  - Implement the homomorphic extension — a function that takes a morphism $m : V \to W$ and a V-expression E and returns the W-expression $m(E)$.

    actually one function each for terms, types, formulas
  - Add a check method for morphisms that calls the homomorphic extension, your type checker, and a theorem prover as needed.
  - Check an example theory morphism.

Natural Language

Languages

We give an overview of narrative languages as one corner of the Tetrapod.

Formal Languages for Natural Language

Formal Languages

- Controlled natural languages: GF, . . . ,
- Word processors: Microsoft Word, Libre Office Write, . . .
- Web-oriented languages: Markdown, HTML, . . .

Features

- Unrestricted or barely restricted natural language
- Usually: visual presentation, e.g., fonts, colors
- Possibly: formal structure, e.g.,
  - sections
  - lists, enumerations
  - cross-references
- Rarely: explicit representation of ontological knowledge, e.g., definitions, references

Semantics by Translation

Relative Semantics

- define semantics of a language l relative to a language L whose semantics is already fixed
- by translation of l into L

Compositional Translation

- translation is given as context-free traversal of checked syntax tree
- one function per non-terminal N
- one case per production $N ::= R$
- one recursive call per non-terminal in R

A Narrative Semantics of BOL

We give a semantics of BOL by translation into English according to Section 6.6 of the notes.

LaTeX

The sTeX Dialect of LaTeX

LaTeX is a language for natural language documents.

sTeX enriches LaTeX to allow describing the ontology in addition to the narrative document. In particular: definitions and references to them:

sTeX primitives

- \begin{smodule}{<name>} starts a module (groups definitions)
- \symdecl*{<sym>} delcares/reserves a symbol (name)
- \begin{ sdefinition } starts a definition
- \definiendum{<sym>}{<print>} makes a definiendum for <sym> in a definition
- \sr{<sym>}{<print>} references <sym> (e.g. as a link on <print>

Exercise 5

Implement a semantics by translation for BOL that translates BOL to sTeX.

Concretely, your translation should take an ontology and a name N. And it should return a file N.tex containing

- for every named declaration: one English sentence with an sTeX declaration
- for every axiom: one English sentence in which all identifier references carry a hyperlink to the corresponding sTeX declaration

Run this on your university ontology to produce university.tex and university.pdf. Write a second sTeX document containing a mission statement for the university in such a way that every mention of a concept carries a hyperlink to the respective declaration in university.pdf.

Kinds of Semantics

Recall

Recall:

Syntax	Data
Semantics	Knowledge

Representing

- syntax = formal language
  - grammar — context-free part
  - type system — context-sensitive well-formedness
- data = words in the syntax
  - set of vocabularies
  - set of typed expressions for each vocabulary
- semantics = ???
- knowledge = emergent property of having well-formed words with semantics

Semantics as Querying

General Idea

- Semantics answers questions about the syntax
- Based on the intended meaning of the syntax
- Specifies the meaning by giving the answers

Special Cases

- Deductive semantics: answers the question whether a formula is a theorem
- Computational semantics: answers the question what the result of a program is
- Narrative semantics: allows answering any question if we understand natural language
- Concrete data semantics: answers the question what objects with certain properties exist

Semantics as Imperfect Modeling

- Actual meaning of real-world difficult to model
  - practical argument: any practically interesting system has too many rules
    cf. physics, e.g., three-body problem already chaotic
  - theoretical argument: no language can fully model itself
    cf. Gödel's incompleteness theorems
- Practical semantics is approximation of ideal semantics
- Use practical purpose as guide for defining semantics
  - a set of questions that can be asked e.g., judgments or queries
  - a definition of what the answers are

restrict questions according to practical needs

Aspects of Semantics

- Documentation: answers given as text
  - pro: easy to read for humans, critical to build intuitions
  - con: often ambiguous, contradictory, or incomplete
- Specification: correct answers defined by rule system

  also called calculus or inference system
  - pro: good stepping stone between the other two levels
  - con: accomplishes the pros of neither of them
- Implementation: answers computed by algorithm
  - pro: easy to automate, critical for efficiency and scale
  - con: essentially impossible to understand or analyze
- Unit testing: set of query/answer pairs
  - pro: easy to write, automate
  - con: does not cover the whole semantics

Rule system is sweet spot to connect human- and machine-friendly definitions.

Relative Semantics by Translation

Components:

- Two syntaxes
  - object-language l e.g., BOL
  - meta-language L e.g., SFOL, Scala, SQL, English
- Semantics of L assumed fixed captures what we already know
- Semantics of l by translation into L

semantics of l relative to to existing semantics of L

Problem: just kicking the can?

Discussion of Semantics by Translation

Advantages

- a few meta-languages yield semantics for many languages
- easy to develop new languages
- good connection between syntax and semantics via compositionality, substitution theorem

Disadvantages

- does not solve the problem once and for all
- impractical without implementation of semantics of meta-language
- meta-languages typically much more expressive than needed for object-languages
- translations can be difficult, error-prone

Also needed: absolute semantics

Absolute vs. Relative Semantics

Absolute = self-contained, no use of meta-language L

Get off the ground

- semantics for a few important meta-languages
  e.g., FOL, assembly language, set theory
- relative semantics for all other languages, e.g.,
  - model theory: logic $\rightarrow$ set theory
  - compilation: Scala $\rightarrow$ JVM $\rightarrow$ assembly

Redundant semantics

- common to give
  - relative and absolute semantics for same syntax
  - multiple relative semantics translations to different aspects
  - sometimes even maybe multiple absolute ones
- Allows understanding syntax from multiple perspectives
- Allows cross-checking show equivalence of two semantics

Example: Recall Syntax of Arithmetic Language

Syntax: represented as formal grammar

Numbers

$$
\begin{array}{lll}
N ::= 0 \mid 1 & & \text{literals} \\
\quad \mid \; N + N & & \text{sum} \\
\quad \mid \; N * N & & \text{product}
\end{array}
$$

Formulas

$$
\begin{array}{lll}
F ::= N \doteq N & & \text{equality} \\
\quad \mid \; N \leq N & & \text{ordering by size}
\end{array}
$$

Implementation as inductive data type

Example: Absolute Semantics

Represented as judgments defined by sets of rules

- unclear what judgments to use
- here: computation $\vdash N \rightsquigarrow N$ and truth $\vdash F$

For numbers n: Rules to normalize numbers into values

$$\frac{}{\vdash N + 0 \rightsquigarrow N} \qquad \frac{}{\vdash N * 0 \rightsquigarrow 0} \qquad \frac{}{\vdash N * 1 \rightsquigarrow N}$$

$$\frac{}{\vdash N * (R + S) \rightsquigarrow N * R + N * S}$$

and their commutative variants as well as

$$\frac{}{\vdash L + (M + N) \rightsquigarrow (L + M) + N}$$

For formulas f: rules to determine true formulas

$$\frac{}{\vdash N \doteq N} \qquad \frac{}{\vdash 0 \leq N} \qquad \frac{\vdash L \leq M}{\vdash L + N \leq M + N}$$

Example: Absolute Semantics (2)

Checking if an absolute semantics works as intended is hard.

Here: number rules allow

1. eliminating all cases where arguments of $*$ are 0, 1, or $+$; thus, no more $*$
2. eliminating all cases where arguments of $+$ are 0
3. shift brackets of nested $+$ to the left
4. left: 0 or $(\ldots(1+1)\ldots+1)$ — isomorphic to natural numbers

formula rules allow

1. concluding equality if identical normal forms
2. reducing $M+1 \leq N+1$ to $M \leq N$, repeat until $0 \leq N$

Example: Relative Semantics

Semantics: represented as translation into known language

Problem: Need to choose a known language first
Here: unary numbers represented as strings

Built-in data (strings and booleans):

S ::=	""	empty
|	(Unicode)	character sequence
B ::=	`true`	truth
|	`false`	falsity

Built-in operations to work on the data:

- concatenation of strings $S ::= \texttt{conc}(S, S)$
- replacing all occurrences of c in S_1 with S_2
  $S ::= \texttt{replace}(S_1, c, S_2)$
- equality test: $B ::= S_1 == S_2$
- prefix test: $B ::= \texttt{startsWith}(S_1, S_2)$

Example: Relative Semantics

Represented as function from syntax to semantics

- mutually recursive, inductive functions for each non-terminal symbol
- compositional: recursive call on immediate subterms of argument

For numbers n: semantics $[\![n]\!]$ is a string

- $[\![0]\!] = ""$
- $[\![1]\!] = "|"$
- $[\![m + n]\!] = \texttt{conc}([\![m]\!], [\![n]\!])$
- $[\![m * n]\!] = \texttt{replace}([\![m]\!], "|", [\![n]\!])$

For formulas f: semantics $[\![f]\!]$ is a boolean

- $[\![m \doteq n]\!] = [\![m]\!] == [\![n]\!]$
- $[\![m \leq n]\!] = \texttt{startsWith}([\![n]\!], [\![m]\!])$

Example: Equivalence of Semantics

For formulas

- if $\vdash F$, then $[\![F]\!] = \mathit{true}$ — usually called soundness
- if $[\![F]\!] = \mathit{true}$, then $\vdash F$ — usually called completeness

For numbers

- $\vdash N \rightsquigarrow 0$ iff $[\![N]\!] = ""$
- $\vdash N \rightsquigarrow (\ldots(1+1)\ldots+1)$ iff $[\![N]\!] = "|\ldots|"$

Relative Semantics for BOL

Semantics of BOL

Aspect	kind of semantic language	semantic language
deduction	logic	SFOL
concretization	database language	SQL
computation	programming language	Scala
narration	natural language	English

Narrative Semantics of BOL in English

We discussed earlier

- the rough design of how natural languages can be seen as a formal systems
  - grammar book = syntax
  - sentences = formulas
  - dictionary + common sense statements = standard library
  - domain-specific dictionary + sentences = vocabulary

  , ,
- the translation from BOL to it
- the non-compositional aspects of natural language

see details in the lecture notes

Deductive Semantics of BOL in SFOL

We discuss

- the grammar of SFOL
- context-sensitive languages with variable binding (of which SFOL is an example)
- an implementation of SFOL in Scala
- the translation from BOL to SFOL
- compositionality of the translation
- the issue of
  - non-compositionality
  - the need for a semantic prefix

see details in the lecture notes

Relative Semantics by Translation

General Definition

A semantics by translation consists of

- syntax: a formal system I
- semantic language: a formal system L
  different or same aspect as I
- semantic prefix: a vocabulary P in L
  formalizes fundamentals that are needed to represent I-objects
- interpretation: translates every I-vocabulary T to an L-vocabulary P, $[\![T]\!]$

Common Principles

Properties shared by all semantics by translation

not part of formal definition, but best practices

- *l*-declaration translated to *L*-declaration for the same name
- vocabularies translated declaration-wise
- one inductive function for every kind of complex *l*-expression
  - individuals, concepts, relations, properties, formulas
  - maps *l*-expressions to *L*-expressions
- atomic cases (base cases): *l*-identifier translated to *L*-identifier of the same name or something very similar
- complex cases (step cases): compositional

Compositionality

Case for operator $*$ in translation function compositional iff interpretation of $*(e_1, \ldots, e_n)$ only depends on on the interpretation of the e_i

$$[\![*(e_1, \ldots, e_n)]\!] = [\![*]\!]([\![e_1]\!], \ldots, [\![e_n]\!])$$

for some function $[\![*]\!]$

Example: ;-operator of BOL in translation to FOL

- translation: $[\![R_1; R_2]\!] = \exists m : \iota.[\![R_1]\!](x, m) \wedge [\![R_2]\!](m, y)$
- special case of the above via
  - $* = ;$
  - $n = 2$
  - $[\![;]\!] = (p_1, p_2) \mapsto \exists m : \iota.p_1(x, m) \wedge p_2(m, y)$
- Indeed, we have $[\![R_1; R_2]\!] = [\![;]\!]([\![R_1]\!], [\![R_2]\!])$

Compositionality (2)

Translation compositional iff

- one translation function for each non-terminal all written $[\![-]\!]$
- each defined by one induction on syntax
  i.e., one case for production
  mutually recursive
- all cases compositional

Substitution theorem: a compositional translation satisfies

$$[\![E(e_1, \ldots, e_n)]\!] = [\![E]\!]([\![e_1]\!], \ldots, [\![e_n]\!])$$

for

- every expression $E(N_1, \ldots, N_n)$ with non-terminals N_i
- some function $[\![E]\!]$ that only depends on E

Compositionality (3)

$$[\![E(e_1, \ldots, e_n)]\!] = [\![E]\!]([\![e_1]\!], \ldots, [\![e_n]\!])$$

for every expression $E(N_1, \ldots, N_n)$ with non-terminals N_i

Now think of

- variable x_i of type N_i instead of non-terminal N_i
- $E(x_1, \ldots, x_n)$ as expression with free variables x_i of type N_i
- expressions e derived from N as expressions of type N
- $E(e_1, \ldots, e_n)$ as result of substituting e_i for x_i
- $[\![E]\!](x_1, \ldots, x_n)$ as (semantic) expression with free variables x_i

Then both sides of equations act on $E(x_1, \ldots, x_n)$:

- left side yields $[\![E(e_1, \ldots, e_n)]\!]$ by
  - first substitution e_i for x_i
  - then semantics $[\![-]\!]$ of the whole
- right side yields $[\![E]\!]([\![e_1]\!], \ldots, [\![e_n]\!])$ by
  - first semantics $[\![-]\!]$ of all parts
  - then substitution $[\![e_i]\!]$ for x_i

semantics commutes with substitution

Non-Compositionality

Examples

- deduction: cut elimination, translation from natural deduction to Hilbert calculus
- computation: optimizing compiler, e.g., loop unrolling
- concretization: query optimization, e.g., turning a WHERE of a join into a join of WHEREs,
- narration: ambiguous words are translated based on context

Typical sources

- subcases in a case of translation function
  - based on inspecting the arguments, e.g., subinduction
  - based on context
- custom-built semantic prefix

Translation vs. Embedding

Translation

- as above, I and L are at the same level
- I-declarations represented as L-declarations

also called shallow embedding

Embedding

- L is used as meta-language to represent I

e.g., L is programming language to implement I

- I-declarations represented as L-objects using an inductive type

also called deep embedding

Exercise 8

Implement the translation from BOL to SFOL. Translate your university ontology.

Formulate a BOL-conjecture and prove it by sending the translated version to an SFOL theorem prover.

Denotational Semantics

Relative Semantics: Denotation vs. Translation

Translation semantics

- vocabularies mapped to vocabularies, expressions to expressions
- symbols in input vocabulary yield symbols in output vocabulary
- examples:
  - BOL to SFOL
  - compiling a programming language to another language

Denotational semantics

- symbols in vocabulary given concrete value
  value/meaning/denotation/interpretation
- model: maps every symbols to its concrete value
- expressions interpreted relative to fixed situation
- examples
  - interpreting a program
    situation = input+run-time environment
  - interpreting logical formulas in a model situation = model

Interpreted vs. Uninterpreted Symbols

Interpreted = symbols with fixed semantics

- base types and their operations — integer, etc.
- concrete data types — enumerations, inductive types

semantics fixed by language

Uninterpreted = semantics open to interpretation

- e.g, $a : \texttt{type}$, $f : a \to a$, etc.
- axioms/definitions constrain/specify possible interpretations

semantics constrained by vocabulary, fixed by situation

Relative to situation

- semantics of all symbols fixed
- semantics of every expression can be determined

often but not necessarily computable

Example

Vocabulary: rules of the world

- type a
- operation $f : a \to a$
- relations $r : a \to \texttt{prop}$
- axioms F about f, r

Situation: one concrete world S

- specific set a^S
- specific function f^S from a^S to a^S
- specific subset r^S of a^S
- proof that F holds about f^S, r^S

Aspect	Abstract Vocabulary	Situation
ontology	TBox	initial situation through ABox
data	schema	database
deduction	theory	model

Terminology

"vocabulary" and "situation" are not standard names. They are introduced here to unify the different kinds of languages.

The standard names vary by knowledge aspect:

Aspect	Vocabulary	Situation	Vocabulary+Situation
ontology	TBox	ABox	ontology
data	schema	database	SQL dump
deduction	theory	model	concrete theory
computation	program	environment	execution
narration	dictionary	technical jargon	

Approaches to Uninterpreted Symbols

Usually only one extreme

- Computation: typically mostly interpreted symbols, situation only provides input/environment — programs can be run directly
- Deduction: typically only uninterpreted symbols — focus on studying the possible models

Some attempts at combining

- reasoning about programs — e.g., functions with pre-/postconditions
- logics with built-in base types — e.g., SMT solving

Initial Semantics

Some situations can be captured in vocabulary

- ABox part of ontology
- schema and table entries part of SQL syntax
- trickier when vocabulary symbols represent abstract sets/functions

  usually meta-language needed to define concrete semantics

Any vocabulary induces default situation

- inhabitants of a type are exactly the terms of that type
- functions map exactly as given by axioms
- every situation must be an extension
- called initial situation, or initial semantics
- examples:
  - ABox, database tables if part of vocabulary
  - Herbrand model of logical theory

Absolute Semantics for BOL

Judgments

Goal: Answer the question whether a formula is a theorem.

Use deduction judgment:

$$\Gamma \vdash_{V}^{BOL} F$$

for formula F

Notation:

- We drop the superscript ^{BOL} whenever clear.
- We drop the subscript $_{V}$ whenever clear.
- We drop the context Γ if it is empty.

Lookup Rules

The main rules that need to access the vocabulary:

$$\frac{f \text{ in } V}{\vdash_V f}$$

for assertions or axioms f

Assumptions in the context are looked up accordingly:

$$\frac{x : f \text{ in } \Gamma}{\Gamma \vdash f}$$

Rules for Subsumption and Equality

Subsumption is an order with respect to equality:

$$\frac{}{\vdash c \sqsubseteq c}$$

$$\frac{\vdash c \sqsubseteq d \qquad \vdash d \sqsubseteq e}{\vdash c \sqsubseteq e}$$

$$\frac{\vdash c \sqsubseteq d \qquad \vdash d \sqsubseteq c}{\vdash c \equiv d}$$

Equal concepts can be substituted for each other:

$$\frac{\vdash c \equiv d \qquad x : C \vdash f(x) : \mathtt{prop} \qquad \vdash f(c)}{\vdash f(d)}$$

This completely defines equality.

Rules relating Instancehood and Subsumption

$$\frac{\vdash i \texttt{ is-a } c \qquad \vdash c \sqsubseteq d}{\vdash i \texttt{ is-a } d}$$

Read:

- if
  - $i \texttt{ is-a } c$
  - $c \sqsubseteq d$
- then $i \texttt{ is-a } d$

$$\frac{x : I, x \texttt{ is-a } c \vdash x \texttt{ is-a } d}{\vdash c \sqsubseteq d}$$

Read:

- if
  - assuming an individual x and $x \texttt{ is-a } c$, then $x \texttt{ is-a } d$
- then $c \sqsubseteq d$

Induction

Consider from before

$$\frac{x : I,\, x \,\texttt{is-a}\, c \vdash x \,\texttt{is-a}\, d}{\vdash c \sqsubseteq d}$$

Question: Do we allow proving the hypothesis by checking for each individual x? induction

Induction

Consider from before

$$\frac{x : I,\ x\ \texttt{is-a}\ c \vdash x\ \texttt{is-a}\ d}{\vdash c \sqsubseteq d}$$

Question: Do we allow proving the hypothesis by checking for each individual x? induction

- Open world: no

Induction

Consider from before

$$\frac{x : I,\ x\ \texttt{is-a}\ c \vdash x\ \texttt{is-a}\ d}{\vdash c \sqsubseteq d}$$

Question: Do we allow proving the hypothesis by checking for each individual x? induction

- Open world: no
- Closed world: yes

$$\frac{\Gamma[x = i] \vdash f[x = i] \ \text{ for every individual } i}{\Gamma, x : I \vdash f(x)}$$

effectively applicable if only finitely many individuals

Rules for Union and Intersection of Concepts

Union as the least upper bound:

$$\frac{}{\vdash c \sqsubseteq c \sqcup d} \qquad \frac{}{\vdash d \sqsubseteq c \sqcup d}$$

$$\frac{\vdash c \sqsubseteq h \qquad \vdash d \sqsubseteq h}{\vdash c \sqcup d \sqsubseteq h}$$

Dually, intersection as the greatest lower bound:

$$\frac{}{\vdash c \sqcap d \sqsubseteq c} \qquad \frac{}{\vdash c \sqcap d \sqsubseteq d}$$

$$\frac{\vdash h \sqsubseteq c \qquad \vdash h \sqsubseteq d}{\vdash h \sqsubseteq c \sqcap d}$$

Rules for Existential and Universal

Easy rules:

- Existential

$$\frac{\vdash i\ r\ j \qquad \vdash j\ \texttt{is-a}\ c}{\vdash i\ \texttt{is-a}\ \exists r.c}$$

- Universal

$$\frac{\vdash i\ \texttt{is-a}\ \forall r.c \qquad \vdash i\ r\ j}{\vdash j\ \texttt{is-a}\ c}$$

Other directions are trickier:

- Existential

$$\frac{\vdash i\ \texttt{is-a}\ \exists r.c \qquad j : I,\ i\ r\ j,\ j\ \texttt{is-a}\ c \vdash f}{\vdash f}$$

- Universal

$$\frac{j : I,\ i\ r\ j \vdash j\ \texttt{is-a}\ c}{\vdash i\ \texttt{is-a}\ \forall r.c}$$

Selected Rules for Relations

Inverse:

$$\frac{\vdash i\ r\ j}{\vdash j\ r^{-1}\ i}$$

Composition:

$$\frac{\vdash i\ r\ j \qquad \vdash j\ s\ k}{\vdash i\ (r;s)\ k}$$

Transitive closure:

$$\frac{}{\vdash i\ r^*\ i} \qquad \frac{\vdash i\ r\ j \qquad \vdash j\ r^*\ k}{\vdash i\ r^*\ k}$$

Identity at concept c:

$$\frac{\vdash i\ \texttt{is-a}\ c}{\vdash i\ \Delta_c\ i}$$

Overview

General Ideas

- Recall
  - syntax = context-free grammar
  - semantics = translation to another language
- Example: BOL translated to SQL, SFOL, Scala, English
- Querying = use semantics to answer questions about syntax

Note:

- Not the standard definition of querying
- Design of a new Tetrapod-level notion of querying

ongoing research

- Subsumes concepts of different names from the various aspects

Propositions

syntax with propositions =
designated non-terminals for propositions

Examples:

aspect	basic propositions
ontology language	assertions, concept equality/subsumption
programming language	equality for some types
database language	equality for base types
logic	equality for all types
natural language	sentences

Aspects vary critically in how propositions can be formed

- any program in computation
- quantifiers in deductions — undecidable
- *IN* in databases

Propositions as Queries

Propositions allow defining queries

	Query	Result
deduction	proposition	yes/no
concretization	proposition with free variables	true ground instances
computation	term	value
narration	question	answer

Semantics of Propositions

syntax with propositions =
designated non-terminals for propositions

needed to ask queries

semantics with theorems =
designates some propositions as theorems or contradictions

needed to answer queries

Note:

- A propositions may be neither theorem nor contradiction.
- We say that language has negation if:
  F theorem iff $\neg F$ contradiction and vice versa.

We write $\vdash F$ if F is theorem.

Deductive Queries

Definition

We assume

- a semantics $[\![-]\!]$ from I to L
- I has propositions
- there is an operation `True` that maps translations of I-propositions to L-propositions — called truth lifting
- L has semantics with propositions

We define

- a deductive query is an I-proposition p
- the result is
  - yes if `True`$[\![p]\!]$ is a theorem of L
  - no if `True`$[\![p]\!]$ is a contradiction in L

The Truth-Lift operator

Problem with type-preserving translation $[\![-]\!]$

- must translate $F : \mathtt{prop}^I$ to $[\![F]\!] : [\![\mathtt{prop}^I]\!]$
- but need not satisfy $[\![\mathtt{prop}^I]\!] = \mathtt{prop}^L$

  I-propositions maybe not translated to L-propositions
- If not, $\vdash^L [\![F]\!]$ cannot be used to answer query $\vdash^I F$

Solution: L-operator $\mathtt{True} : [\![\mathtt{prop}^I]\!] \to \mathtt{prop}^L$

- maps translations of I-propositions to L-propositions
- then use $\vdash^L \mathtt{True}[\![F]\!]$ to answer $\vdash^I F$

Example: probabilistic semantics with cutoff

- $\mathtt{prop}^I$: propositions; $\mathtt{prop}^L$: truth values
- probabilistic semantics: $[\![\mathtt{prop}^I]\!] = [0; 1]$
- $\mathtt{True}$ maps $p \in [0; 1]$ to truth value, e.g., via cutoff:
  $\mathtt{True}p = p \geq 0.75$

Situational Semantics as Translation

Example: standard semantics of SFOL in set theory

- set theory propositions: truth values $\{0, 1\}$
- situations (= models): provide interpretations for all vocabulary symbols
- semantics is model-specific interpretation function: given vocabulary V, V-model M, and V-expression E, write $[\![E]\!]_M$ for semantics of E in M

Reformulate as semantics by translation

- vocabulary V translated to set-theoretical vocabulary containing required components of model
- expression E translated to mapping $[\![E]\!] : M \mapsto [\![E]\!]_M$

Truth-Lift operator defines theorems

- $[\![\texttt{prop}']\!]$ is set of mappings μ from models to truth values
  standard notation for $[\![F]\!](M) = 1$: $[\![F]\!]_M = 1$ or $M \models F$
- $\texttt{True}\mu = 1$ iff $\mu(M) = 1$ for all models M (theorem = true in all models)

Breakout question

What can go wrong?

Problem: Inconsistency

In general, (in)consistency of semantics

- Some propositions may be both a theorem and a contradiction.
- In that case, queries do not have a result.

In practice, however:

- If this holds for some propositions, it typically holds for all of them.
- In that, we call L inconsistent.
- We usually assume L to be consistent.

Problem: Incompleteness

In general, (in)completeness of semantics

- We cannot in general assume that every proposition in L is either a theorem or a contradiction.
- In fact, most propositions are neither.
- So, queries do not necessarily have a result.
- We speak of incompleteness.

  Note: not the same as the usual (in)completeness of logic

In practice, however:

- It may be that L is complete for all propositions in the image of $\texttt{True}[\![-]\!]$.
- This is the case if I is simple enough

  typical for ontology languages

Problem: Undecidability

In general, (un)decidability of semantics:

- We cannot in general assume that it is decidable whether a proposition in L is a theorem or a contradiction.
- In fact, it usually isn't.
- So, we cannot necessarily compute the result of a query.
- However: If we have completeness, decidability is likely.

  run provers for F and $\neg F$ in parallel

In practice, however:

- It may be that L is decidable for all propositions in the image of $\texttt{True}[\![-]\!]$.
- This is the case if I is simple enough

  typical for ontology languages

Problem: Inefficiency

In general, (in)efficiency of semantics:

- Answering deductive queries is very slow.
- Even if we are complete and decidable.

In practice, however:

- Decision procedures for the image of $\texttt{True}[\![-]\!]$ may be quite efficient.
- Dedicated implementations for specific fragments.
- This is the case if I is simple enough

typical for ontology languages

Contexts and Free Variables

Concepts

Recall the analogy between grammars and typing:

grammars	typing
non-terminal	type
production	constructor
non-terminal on left of production	return type of constructor
non-terminals on right of production	arguments types of constructor
terminals on right of production	notation of constructor
words derived from non-terminal N	expressions of type N

We will now add contexts and substitutions.

Contexts

Independent of whether I already has contexts/variables, we can define:

- A context Γ is of the form $x_1 : N_1, \ldots, x_n : N_n$ where the
  - x_i are names
  - N_i are non-terminals

  We write this as $\vdash_I \Gamma$.
- A substitution for Γ is of the form $x_1 := w_1, \ldots, x_n := w_n$ where the
  - x_i are as in Γ
  - w_i derived from the corresponding N_i

  We write this as $\vdash_I \gamma : \Gamma$.
- An expression in context Γ of type N is a word w derived from N using additionally the productions $N_i ::= x_i$.
  We write this as $\Gamma \vdash_I w : N$.
- Given $\Gamma \vdash w : N$ and $\vdash \gamma : \Gamma$ as above, the substitution of γ in w is obtained by replacing every x_i in w with w_i. We write this as $w[\gamma]$.

Contexts under Compositional Translation

Consider a compositional semantics $[\![-]\!]$ from I to L between context-free languages.

- Every $\vdash_I w : N$ is translated to some $\vdash_L [\![w]\!] : N'$ for some N'.
- Compositionality ensures that N' is the same for all w derived from N.
- We write $[\![N]\!]$ for that N'.
- Then we have

$$\vdash_I w : N \quad \text{implies} \quad \vdash_L [\![w]\!] : [\![N]\!]$$

Now we translate contexts, substitutions, and variables as well:

$$[\![x_1 : N_1, \ldots, x_n : N_n]\!] := x_1 : [\![N_1]\!], \ldots, x_n : [\![N_n]\!]$$

$$[\![x_1 := w_1, \ldots, x_n := w_n]\!] := x_1 := [\![w_1]\!], \ldots, x_n := [\![w_n]\!]$$

$$[\![x]\!] := x$$

Then we have

$$\Gamma \vdash_I w : N \quad \text{implies} \quad [\![\Gamma]\!] \vdash_L [\![w]\!] : [\![N]\!]$$

Substitution under Compositional Translation

From previous slide:

$$[\![x_1 : N_1, \ldots, x_n : N_n]\!] := x_1 : [\![N_1]\!], \ldots, x_n : [\![N_n]\!]$$

$$[\![x_1 := w_1, \ldots, x_n := w_n]\!] := x_1 := [\![w_1]\!], \ldots, x_n := [\![w_n]\!]$$

$$[\![x]\!] := x$$

$$\Gamma \vdash_I w : N \qquad \text{implies} \qquad [\![\Gamma]\!] \vdash_L [\![w]\!] : [\![N]\!]$$

We can now restate the substitution theorem as follows:

$$[\![E[\gamma]]\!] = [\![E]\!][[\![\gamma]\!]]$$

Concretized Queries

Definition

We assume

- as for deductive queries
- semantics must be compositional

We define

- a concretized query is an l-proposition p in context Γ
- a single result is a
  - a substitution $\vdash_l \gamma : \Gamma$
  - such that $\vdash_L \texttt{True}[\![p[\gamma]]\!]$
- the result set is the set of all results

Example

1. BOL ontology:

   $$\textit{concept male},\ \textit{concept person},\ \textit{axiom male} \sqsubseteq \textit{person},$$
   $$\textit{individual FlorianRabe},\ \textit{assertion FlorianRabe isa male}$$

2. Query $x : \textit{individual} \vdash_{BOL} x$ isa *person*
3. Translation to SFOL: $x : \iota \vdash_{SFOL} \textit{person}(x)$
4. SFOL calculus yields theorem $\vdash_{SFOL} \textit{person}(\textit{FlorianRabe})$
5. Query result $[\![\gamma]\!] = x := \textit{FlorianRabe}$
6. Back-translating the result to BOL: $\gamma = x := \textit{FlorianRabe}$

   back translation is deceptively simple:
   translates SFOL-constant to BOL-individual of same name

Yes/No vs. Wh-Questions

Queries about $\vdash F$ are yes/no questions

- specialty of deductive semantics
- but maybe only because everything else is ever harder to do deductively

Queries about ground instances of $\Gamma \vdash F$ are Wh questions

- specialty of concrete databases
- for the special case of retrieving finite results sets from a fixed concrete store
- only situation where Wh questions are easy

Breakout question

What can go wrong?

Problem: Open World

In general, semantics uses open world:

- open world: result contains all known results
  same query might yield more results later
- closed world: result set contains all results
  always relative to concrete database for L

In practice, however,

- system explicitly assumes closed world typical for databases
- users aware of open world and able to process results correctly

Problem: Infinity of Results

In general, there may be infinitely many results:

- e.g., query for all x such that $\vdash x$,

In practice, however,

- systems pull results from finite database e.g., SQL, SPARQL
- systems enumerate results, require user to explicitly ask for more e.g., Prolog

Problem: Back-Translation of Results

In general, $[\![-]\!]$ may be non-trivial to invert

- easy to obtain $[\![p]\!]$ in context $[\![\Gamma]\!]$ — just apply semantics
- possible to find substitutions

$$\vdash_L \delta : [\![\Gamma]\!] \quad \text{where} \quad [\![\Gamma]\!] \vdash_L \texttt{True}[\![p]\!][\delta]$$

easiest case: just look them up in database

- but how to translate δ to l-substitutions γ with

$$\vdash_l \gamma : \Gamma \quad \text{where} \quad [\![\Gamma]\!] \vdash_L \texttt{True}[\![p[\gamma]]\!]$$

substitution theorem: pick such that $[\![\gamma]\!] = \delta$

the more $[\![-]\!]$ does, the harder to invert

In practice, however:

- often only interested in concrete substitutions
- translation of concrete data usually identity

But: practice restricted to what works even if more is needed

Computational Queries

Definition

We assume

- the same as for deductive queries
- semantics has equality/equivalence $\doteq$

We define

- a computational query is an I-expression e
- the result is an I-expression e' so that $\vdash_L [\![e]\!] \doteq [\![e']\!]$

intuition: e' is the result of evaluating e

If semantics is compositional, e may contain free variables

evaluate to themselves

Problem: Back-Translation of Results

In general, $[\![-]\!]$ may be non-trivial to invert

- easy to obtain $E := [\![e]\!]$
- possible to find E' with $\vdash_L E' \doteq E$ by working in the semantics
- non-obvious how to obtain e' such that $[\![e']\!] = E'$

In practice, however:

- evaluation meant to simplify, i.e., only useful if E' very simple
- simple E' usually in the image of $[\![-]\!]$
- typical case: E' is concrete data and $e' = E'$ called a value

Problem: Non-Termination

In general, computation of E' from E might not terminate

- while-loops
- recursion
- $(\lambda x.x\,x)\,(\lambda x.x\,x)$ with β-rule
- simplification rule $x \cdot y \rightsquigarrow y \cdot x$

  similar: distributivity, associativity

In practice, however:

- image of $[\![-]\!]$ part of terminating fragment

But: if I is Turing-complete or undecidable, general termination not possible

Problem: Lack of Confluence

In general, there may be multiple E' that are simpler than E

- there may be multiple rules that apply to E
- e.g., $f(g(x))$
  - call-by-value: first simplify $g(x) \rightsquigarrow y$, then $f(y) \rightsquigarrow z$
  - call-by-name: first plug $g(x)$ into definition of f, then simplify
- Normal vs. canonical form
  - normal: $\vdash_L E \doteq E'$
  - canonical: normal and $\vdash_L E_1 \doteq E_2$ iff $E_1' = E_2'$

equivalent expressions have identical evaluation
allows deciding equality

In practice, however:

- image of $[\![-]\!]$ part of confluent fragment
- typical: evaluation to a value is canonical form

works for BDL-types but not for, e.g., function types

Narrative Queries

Definition

We assume

- semantics into natural language

We define

- a narrative query is an L-question about some l-expressions
- the result is the answer to the question

Problem: Unimplementable

very expressive = very difficult to implement

- Natural language understanding
  - no implementable syntax of natural language
    needs restriction to controlled natural language
  - specifying semantics hard even when controlled
- Knowledge base for question answering needed
  - very large — must include all common sense
  - might be inconsistent — common sense often is
  - finding answers still very hard

In practice, however:

- accept unreliability — attach probability measures to answers
- implement special cases
  e.g., lookup in databases like Wikidata
- search knowledge base for related statements — Google, Watson

Syntactic Querying

Search

- "search" not systematically separated from "querying"
- often interchangeable
- querying tends to imply formal languages for queries with well-specified semantics e.g., SQL
- search tends to imply less targeted process e.g., Google

we will not distinguish between the two

Syntactic vs. Semantic Querying

Semantic querying

- Query results specified by vocabulary V but (usually) not contained in it
- Query answered using semantics of language
- Challenge: apply semantics to find results
  - deductive query $\vdash f : \texttt{prop}$ requires theorem prover
  - computation query $\vdash e : E$ requires evaluator
  - concrete query $\Gamma \vdash f : \texttt{prop}$ requires enumerating all substitutions, running theorem prover/evaluator on all of them

what we've looked at so far

Syntactic querying

- Query is an expression e
- Result is set of occurrences of e in V
- Independent of semantics
- Much easier to realize

Challenges for Syntactic Search

Easier to realize → scale until new challenges arise

- large vocabularies
  - narrative: all text documents in a domain
    e.g., all websites, all math papers
  - deductive: large repositories of formalization in proof assistants
    10^5 theorems
  - computational: package managers for all programming languages
  - concrete: all databases in a domain TBs routine
- incremental indexing: reindex only new/changed parts
- incremental search to handle large result sets pagination
- sophisticated techniques for
  - indexing: to allow for fast retrieval
  - similarity: to select likely results
  - quality: to rank selected results
- integration of some semantic parts

Overview

- Deduction
  - semantic: theorem proving called search
  - syntactic: text search
- Concretization
  - semantic: complex query languages (nestable queries) SQL, SPARQL
  - syntactic: search by identifier (linked data)
- Computation
  - semantic: interpreters called execution
  - mixed: IDEs search for occurrences, dependencies
  - syntactic: search in IDE, package manager
- Narration:
  - semantic: very difficult
  - syntactic: bag of words search

Abstract Definition: Document

Document =

- file or similar resource that contains vocabularies
- often with comments, metadata
- different names per aspect
  - deduction: formalization, theory, article
  - computation: source files
  - concretization: database, ontology ABox
  - narrative: document, web site

Library =

- collection of documents
- usually structured into folders, files or similar
- often grouped by user access e.g., git repository
- vocabularies interrelated within and across libraries

Abstract Definition: Document Fragment

Fragment = subdivision of documents into nested semantic units

Examples

- deductive: theory, section, theorem, definition, proof step, etc.
- computational: class, function, command, etc.
- concrete: table, row, cell
- narrative: section, paragraph, etc.

Assign unique **fragment URI**, e.g., LIB/DOC?FRAG where

- LIB: base URI of library e.g., repository URL
- DOC: path to document within library
  e.g., folder structure, file name
- FRAG: id of fragment within document
  e.g., class name/method name

Abstract Definition: Index(er)

Indexer consists of

- data structure O for indexable objects
  specific to aspect, index design
  e.g., words, syntax trees
- function that maps library to index
  the indexing

Index entry consists of

- object that occurred in the library
- URI of the containing fragment
- information on where in the fragment it was found

Index = set of index entries

Abstract Definition: Query and Result

Given

- indexer I with data structure O
- set of libraries
- union of their indexes — computed once, queried often

Query = object $\Gamma \vdash^I q : O$

Result consists of

- index entry with object o
- substitution for Γ such that q matches o

  definition of "match" index-specific, e.g., $q[\gamma] = o$

Result set = set of all results in the index

Bag of Words Search

Definition:

- Index data structure = sequences of words (n-grams) up to a certain length
- Query = bag of words — bag = multiset
- Match: (most) words in query occur in same n-gram or n-grams near each other

Example implementations

- internet search engines for websites
- Elasticsearch: open source engine for custom vocabularies

Mostly used for narrative documents

- can treat concrete values as words — e.g., numbers
- could treat other expressions as words — works badly

Symbolic Search

Definition:

- Index data structure = syntax tree (of any grammar) of expressions o with free/bound variables
- Query = expression q with free (meta-)variables
- Match: $q[\gamma] =_\alpha o$, i.e., up to variable renaming

Example implementation

- MathWebSearch

  see separate slides on MathWebSearch in the repository

Mostly used for formal documents

- deductive
- computational

Knowledge Graph Search

Definition:

- Index data structure = assertion forming node/edge in a knowledge graph
- Index = big knowledge graph G
- Query = knowledge graph g with free variables
- Match: $g[\gamma]$ is part of G

Example implementations

- SPARQL engines without consequence closure

  i.e., the most common case in practice
- graph databases

Mainly used for ABoxes of untyped ontologies

Value Search

Definition:

- Index data structure = BDL values v
- Query = BDL expression q with free variables
- Match: $q[\gamma] = v$

Example implementations

- no systematic implementation yet
- special cases part of most database systems

Could be used for values occurring in any document

- all aspects
- may need to decode/encode before putting in index

Cross-Aspect Occurrences

Observation

- libraries are written in one primary aspect
- indexer focuses on one aspect and kind of object
- but documents may contain indexable objects of any index

Cross-Aspect Occurrences: Examples

- Any library can contain
  - metadata on fragments
    - relation assertions induce knowledge graph structure between fragments
    - property assertions contain values narrative, symbolic objects, or values
  - cross-references to fragments of any other library
  - narrative comments
- Narrative text may contain symbolic expressions

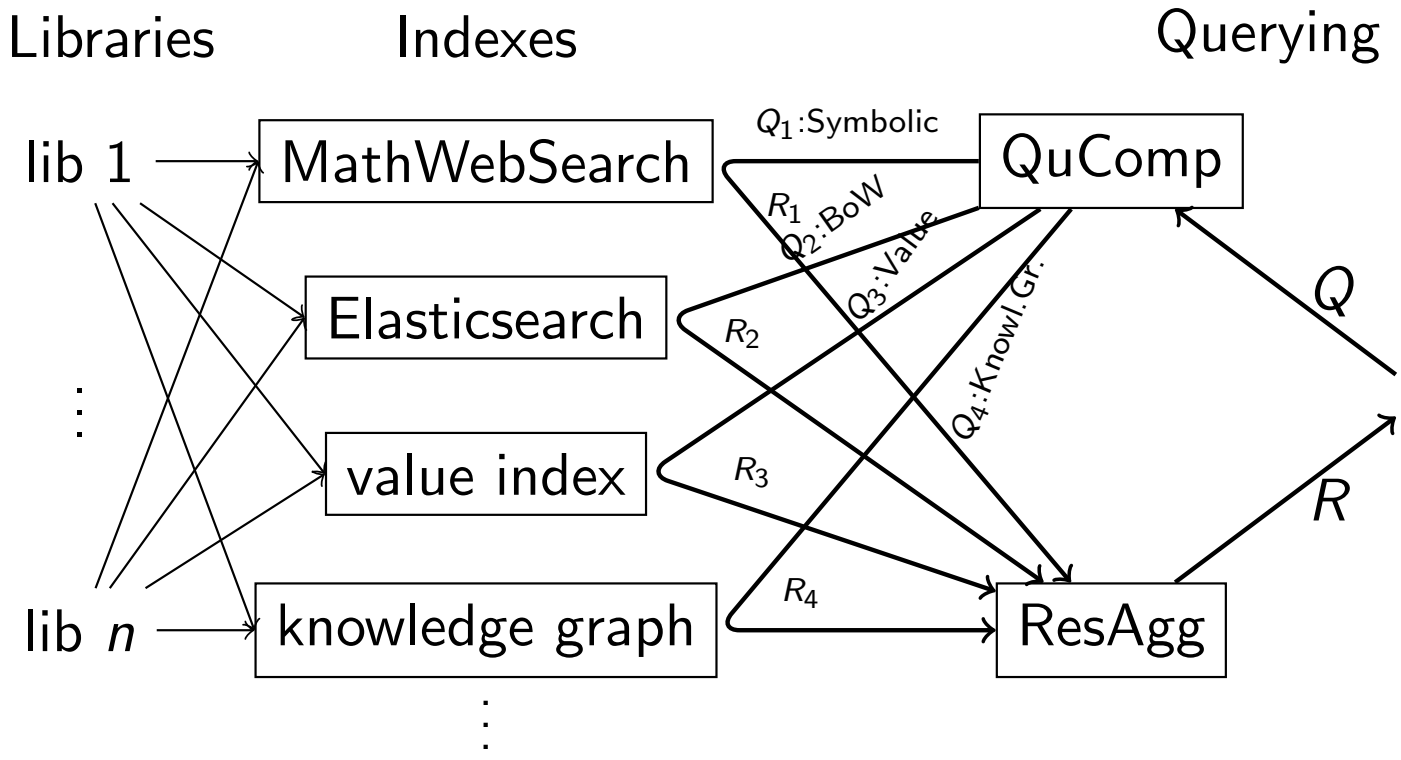
  STEM documents
- Database table may have columns containing
  - text
  - encoded BDL values
  - symbolic expression (often as strings)
- Symbolic fragments may contain database references

  e.g., when using database for persistent memoization

A New Indexing Design

recent paper `https://kwarc.info/people/frabe/Research/BKR_mdql_20.pdf`

with K. Bercic



A New Indexing Design (2)

Tricky question: What is the query language that allows combining queries for each index?

Easy:

- query = conjunction of atomic queries
- each atom queries one index
- QuComp splits into atoms
- ResAgg take intersection of results

Better: allow variables to be shared across atoms

open research question

A New Indexing Design: Example

Consider

- table of graphs with human-recognizable names and arc-transitivity property
  indexed into
  - value index for the graph `sparse6` codec
  - Boolean computed property for the arc-transitivity in knowledge graph
  - text index for name
- papers from arXiv in narrative index
  indexed into
  - narrative index for text
  - MathWebSearch for formulas
  - knowledge graph for metadata

Query goal: find arc-transitive graphs mentioned by name in articles with h-index greater than 50

Integrating Semantic Querying

Word search

- find multi-meaning words for only one meaning
  "normal" in math
- special treatment of certain queries e.g., "weather" in Google

Symbolic search

- match query $e \doteq e'$ against occurrence $e' \doteq e$
- similarly: associativity, commutativity, etc.
- slippery slope to deductive queries

Value search

- match query 1.5 against interval 1.4 ± 0.2
- match query $5 \cdot x$ against 25
- slippery slope to computational queries

frontiers of research — in our group: for STEM documents