

# Large-scale proof and libraries in Isabelle/HOL

Gerwin Klein



Australian Government

Department of Communications,  
Information Technology and the Arts  
Australian Research Council

NICTA Members



THE AUSTRALIAN NATIONAL UNIVERSITY



THE UNIVERSITY OF NEW SOUTH WALES



Department of State and  
Regional Development



NICTA Partners



The Place To Be



THE UNIVERSITY OF  
MELBOURNE



The University of Sydney



Queensland  
Government



Griffith  
UNIVERSITY



Queensland University of Technology



THE UNIVERSITY  
OF QUEENSLAND  
AUSTRALIA

**4** Verified



1 microkernel

8,700 lines of C

0 bugs\*

qed

\*conditions apply

**Windows**

An exception 06 has occurred at 0028:C11B3ADC in VxD DiskTSD(03) + 00001660. This was called from 0028:C11B40C8 in VxD voltrack(04) + 00000000. It may be possible to continue normally.

- \* Press any key to attempt to continue.
- \* Press CTRL+ALT+RESET to restart your computer. You will lose any unsaved information in all applications.

Press any key to continue

# Windows Vista®

Stunning. Breakthrough. Entertaining.

HP TouchSmart PC and Microsoft Windows Vista deliver you a PC experience designed to fit wherever life happens.

Innovative solutions brought to you by:

Microsoft



BEST BUY





for more info contact  
sales@sfy.com  
or visit  
www.sfy.com

for more info contact  
sales@sfy.com  
or visit  
www.sfy.com

the  
**Bay**

the  
**Bay**

# The Problem



# Small Kernels

## Small trustworthy foundation

- hypervisor, microkernel, nano-kernel, virtual machine, separation kernel, exokernel ...
- High assurance components in presence of other components

**seL4 API:**

- IPC
- Threads
- VM
- IRQ
- Capabilities

Untrusted



Trusted



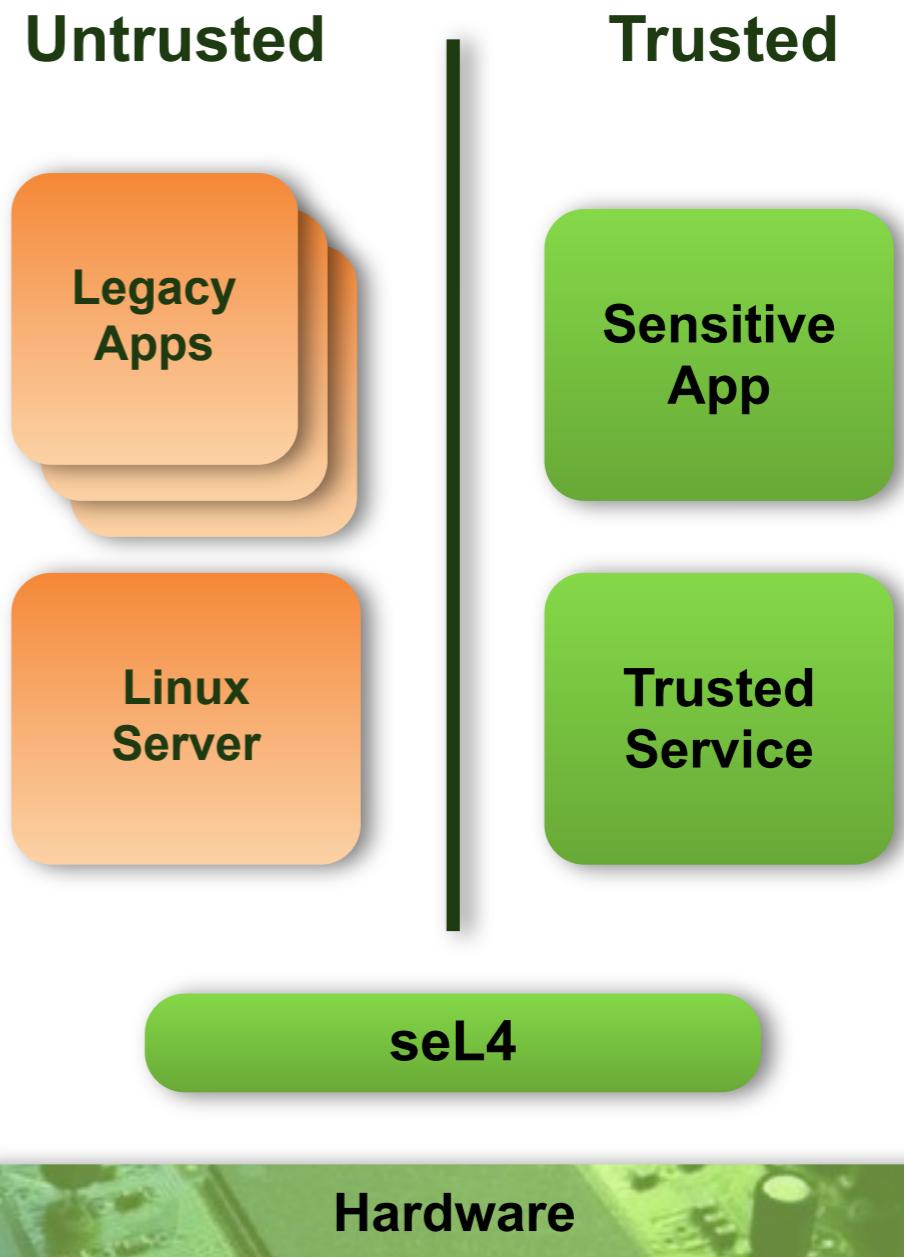
# Small Kernels

## Small trustworthy foundation

- hypervisor, microkernel, nano-kernel, virtual machine, separation kernel, exokernel ...
- High assurance components in presence of other components

**seL4 API:**

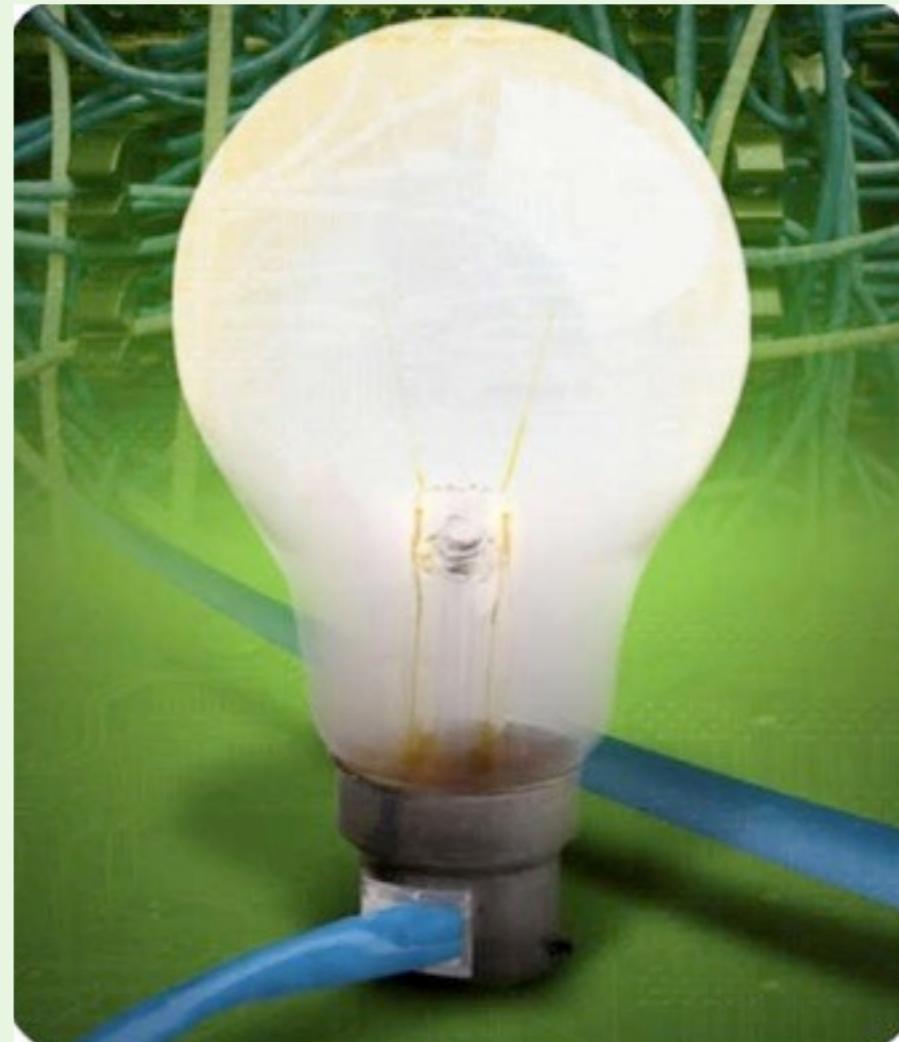
- IPC
- Threads
- VM
- IRQ
- Capabilities



# The Proof



# The Proof



# Functional Correctness



**Proof**

Specification



Code

# Functional Correctness

What

Specification

Proof



Code

## definition

```
schedule :: unit s_monad where
  schedule ≡ do
    threads ← allActiveTCBs;
    thread ← select threads;
    switch_to_thread thread
  od
  OR switch_to_idle_thread
```

# Functional Correctness



## What

Specification

### definition

```
schedule :: unit s_monad where
  schedule ≡ do
    threads ← allActiveTCBs;
    thread ← select threads;
    switch_to_thread thread
  od
  OR switch_to_idle_thread
```

## Proof

How

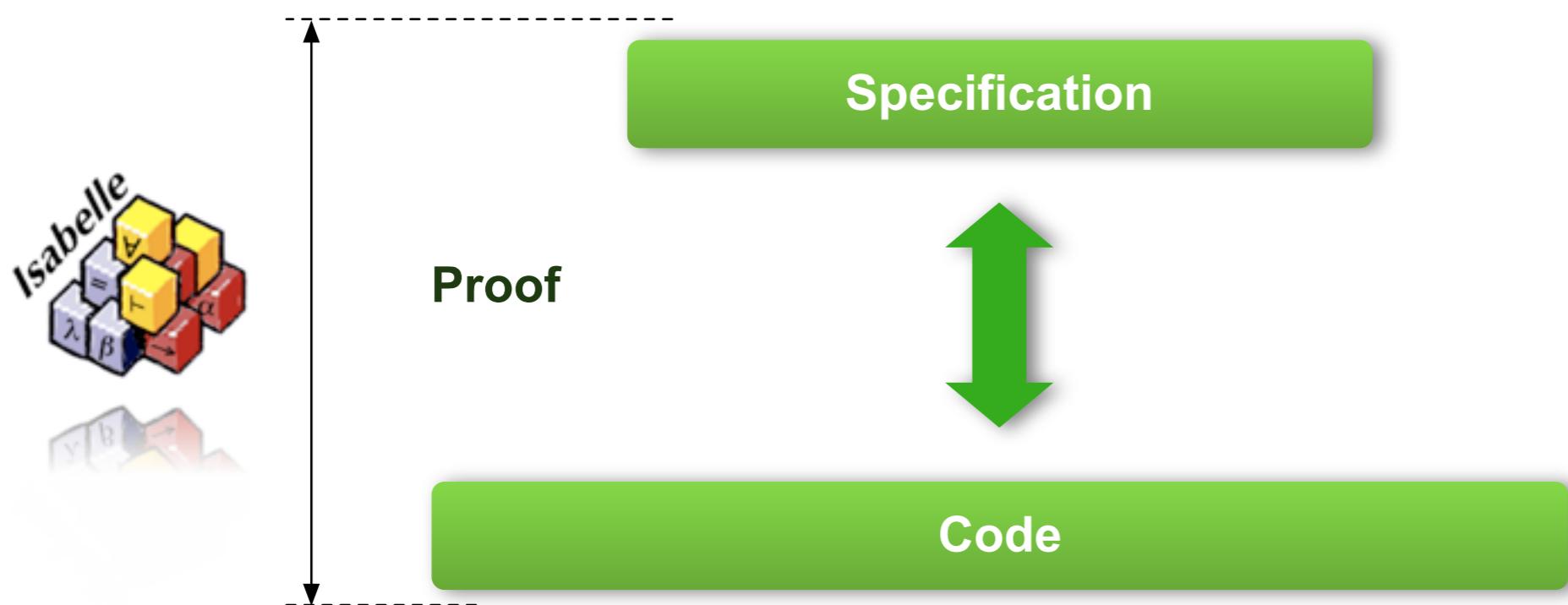
```
void
schedule(void) {
  switch ((word_t)ksSchedulerAction) {
    case (word_t)SchedulerAction_ResumeCurrentThread:
      break;

    case (word_t)SchedulerAction_ChoseNewThread:
      chooseThread();
      ksSchedulerAction = SchedulerAction_ResumeCurrentThread;
      break;

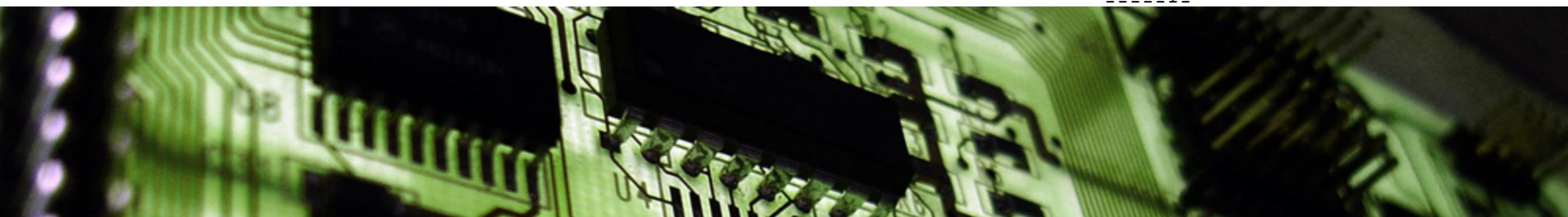
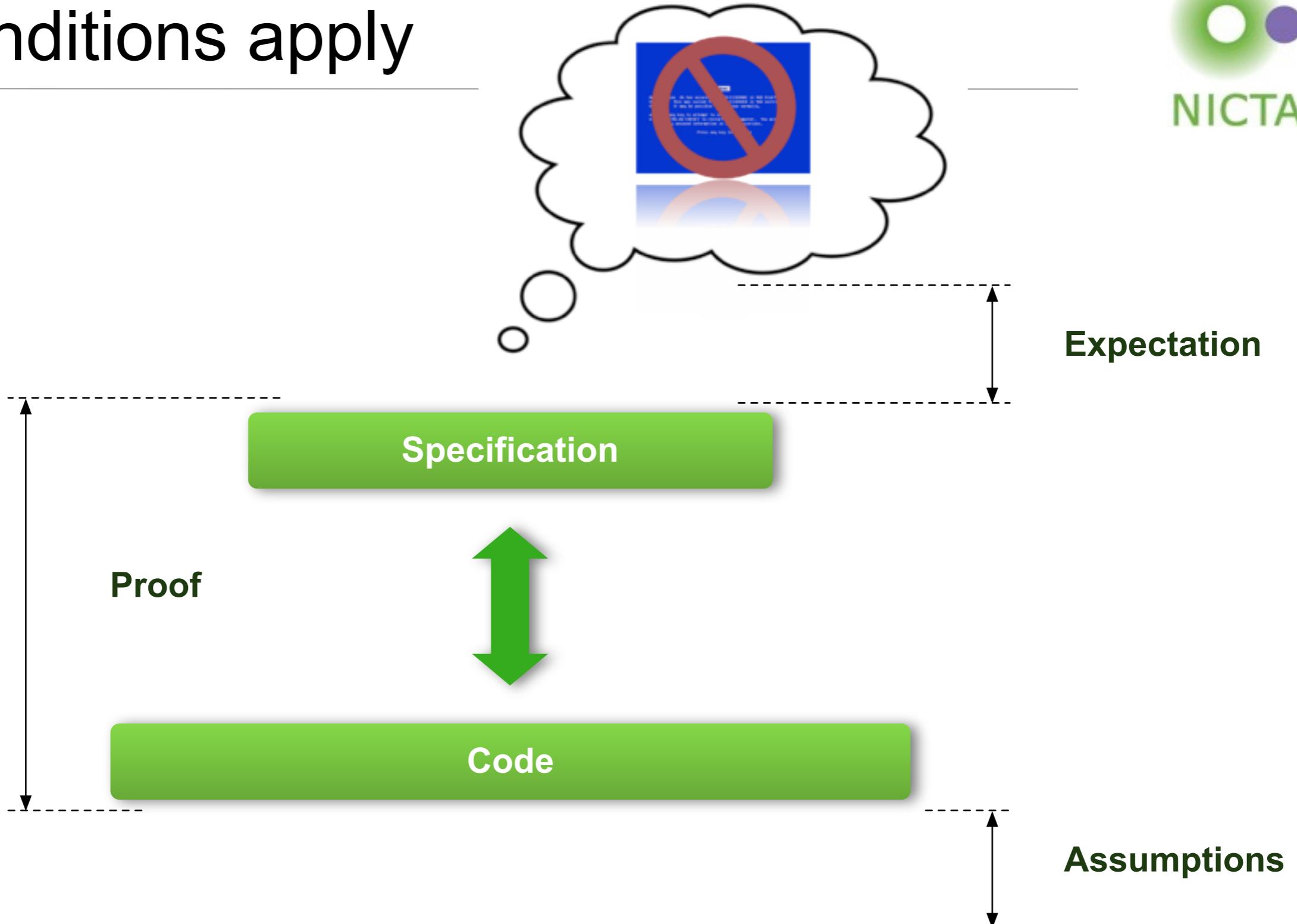
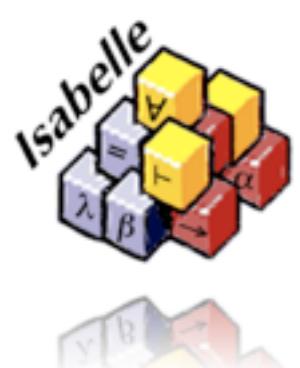
    default: /* SwitchToThread */
      switchToThread(ksSchedulerAction);
      ksSchedulerAction = SchedulerAction_ResumeCurrentThread;
      break;
  }
}

void
chooseThread(void) {
  prio_t prio;
  tcb_t *thread, *next;
```

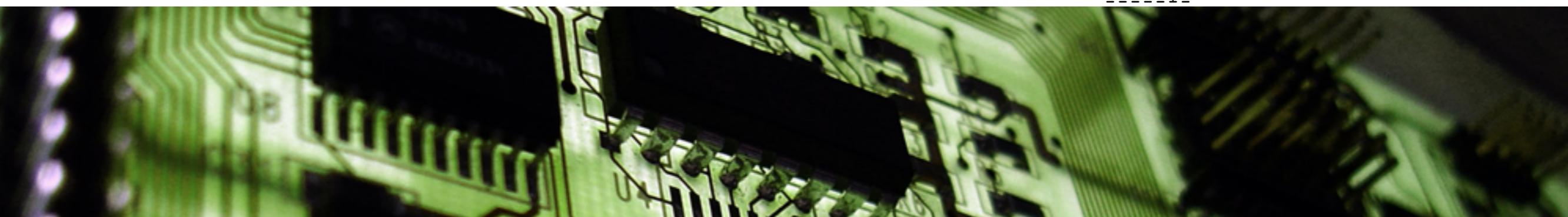
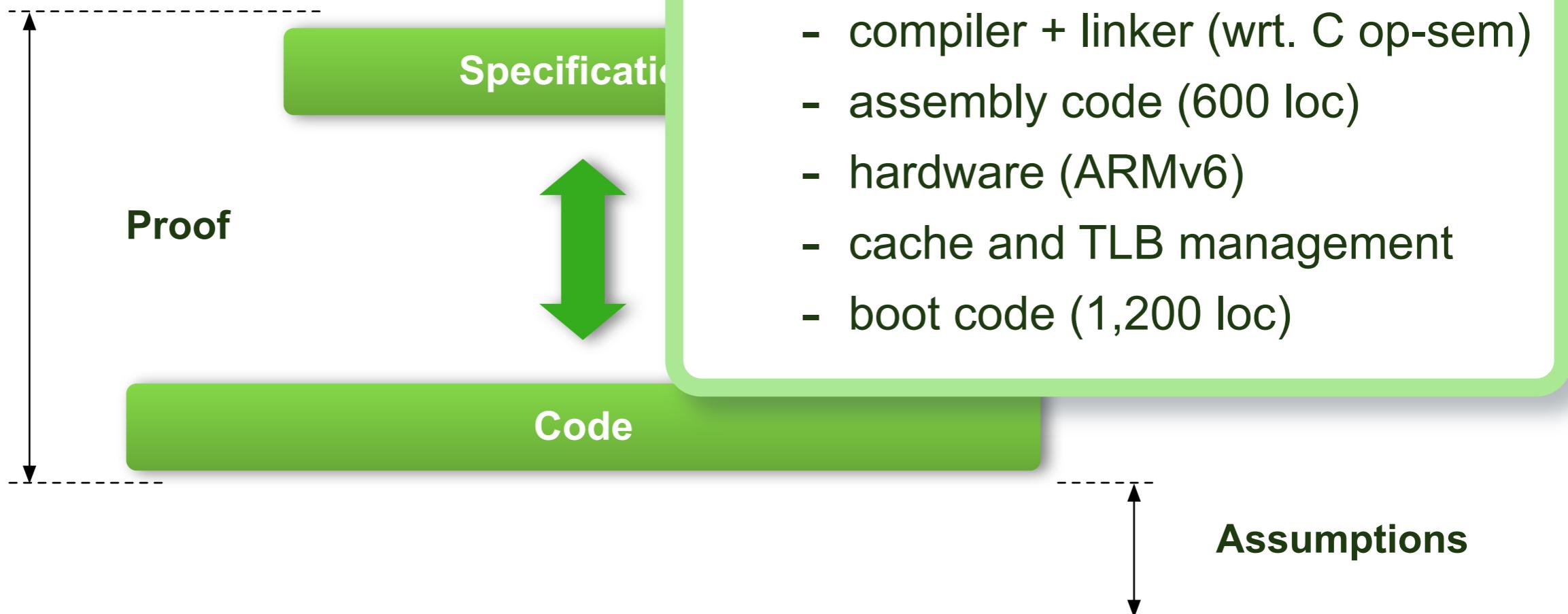
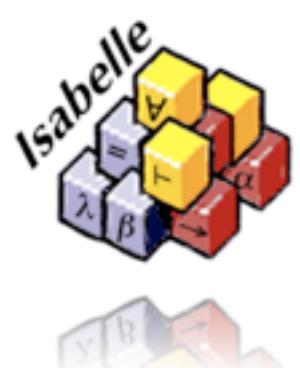
\*conditions apply



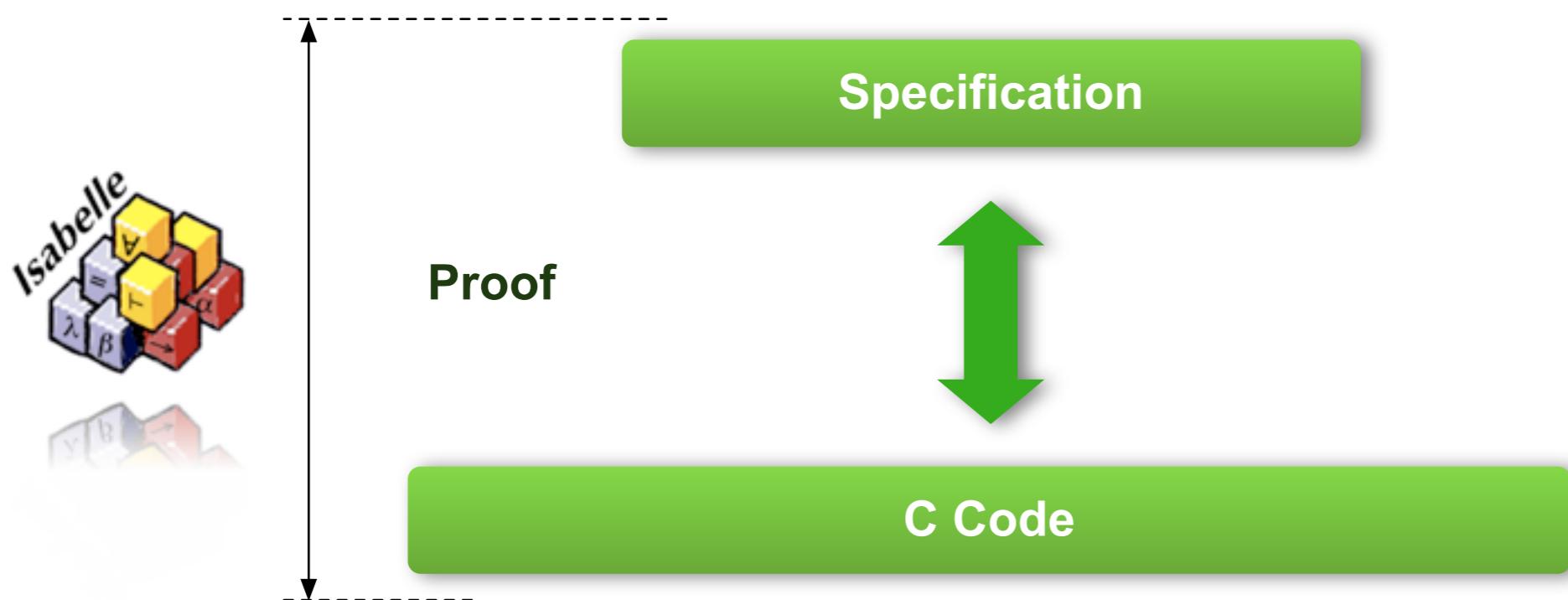
\*conditions apply



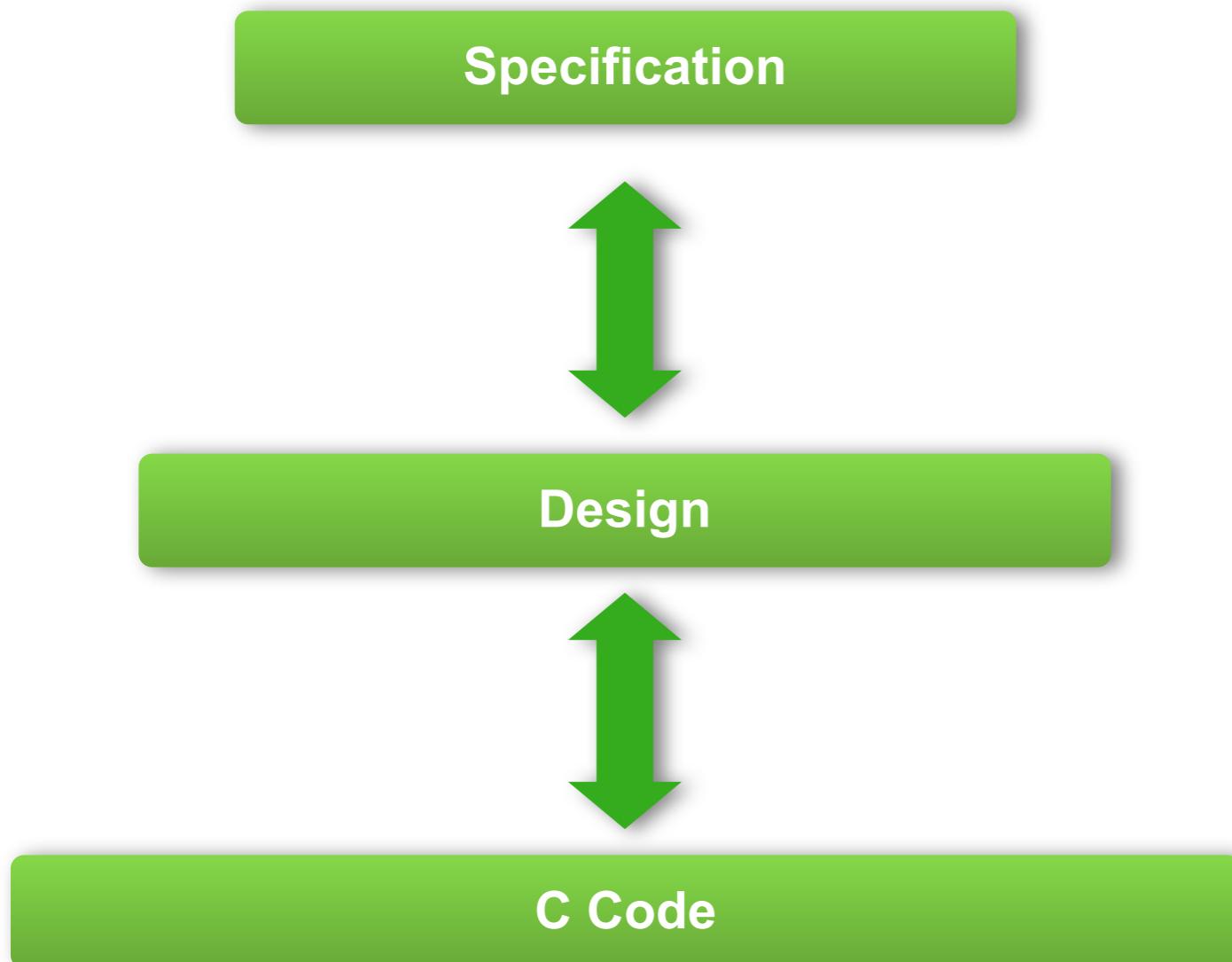
\*conditions apply



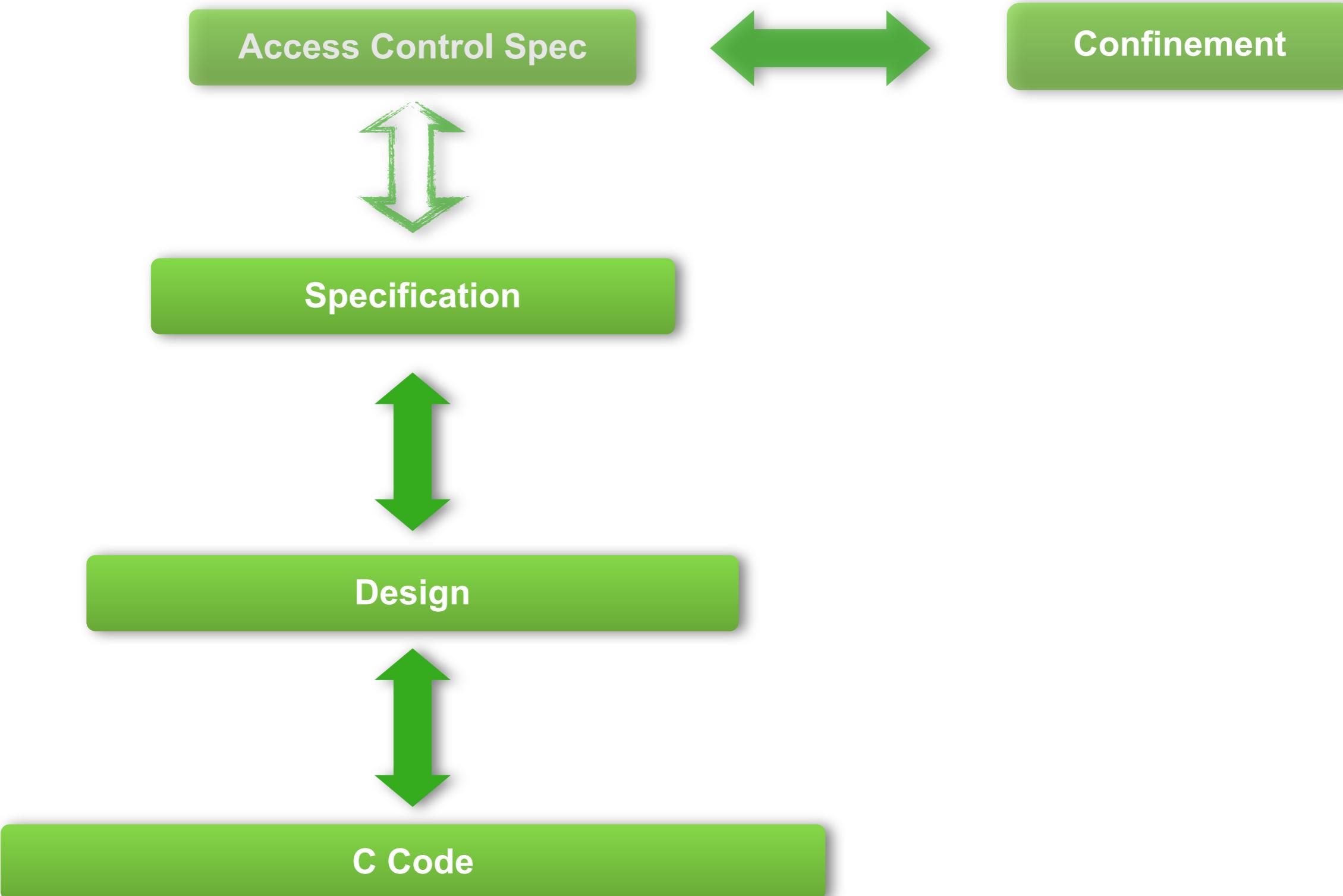
# Proof Architecture



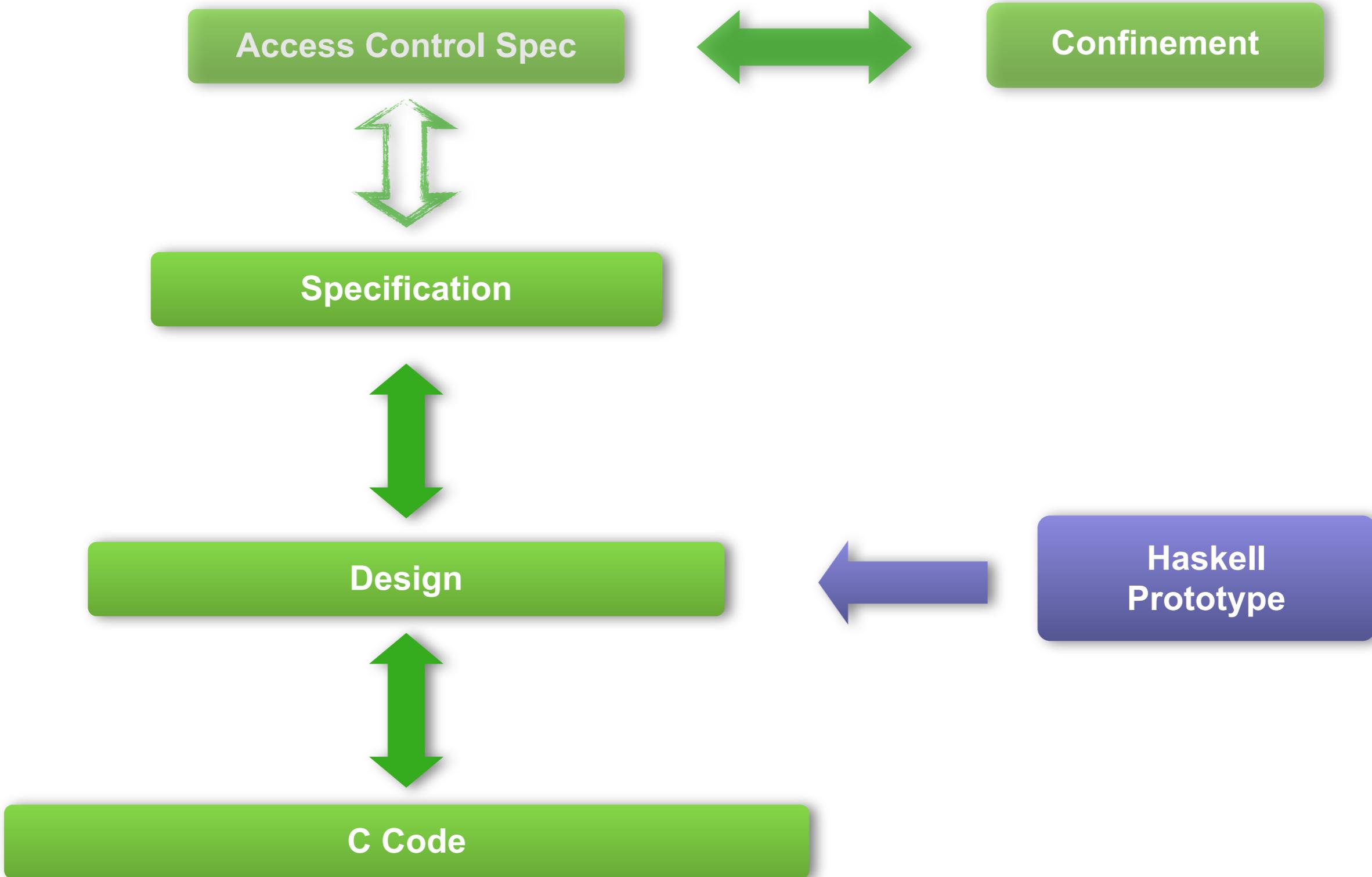
# Proof Architecture



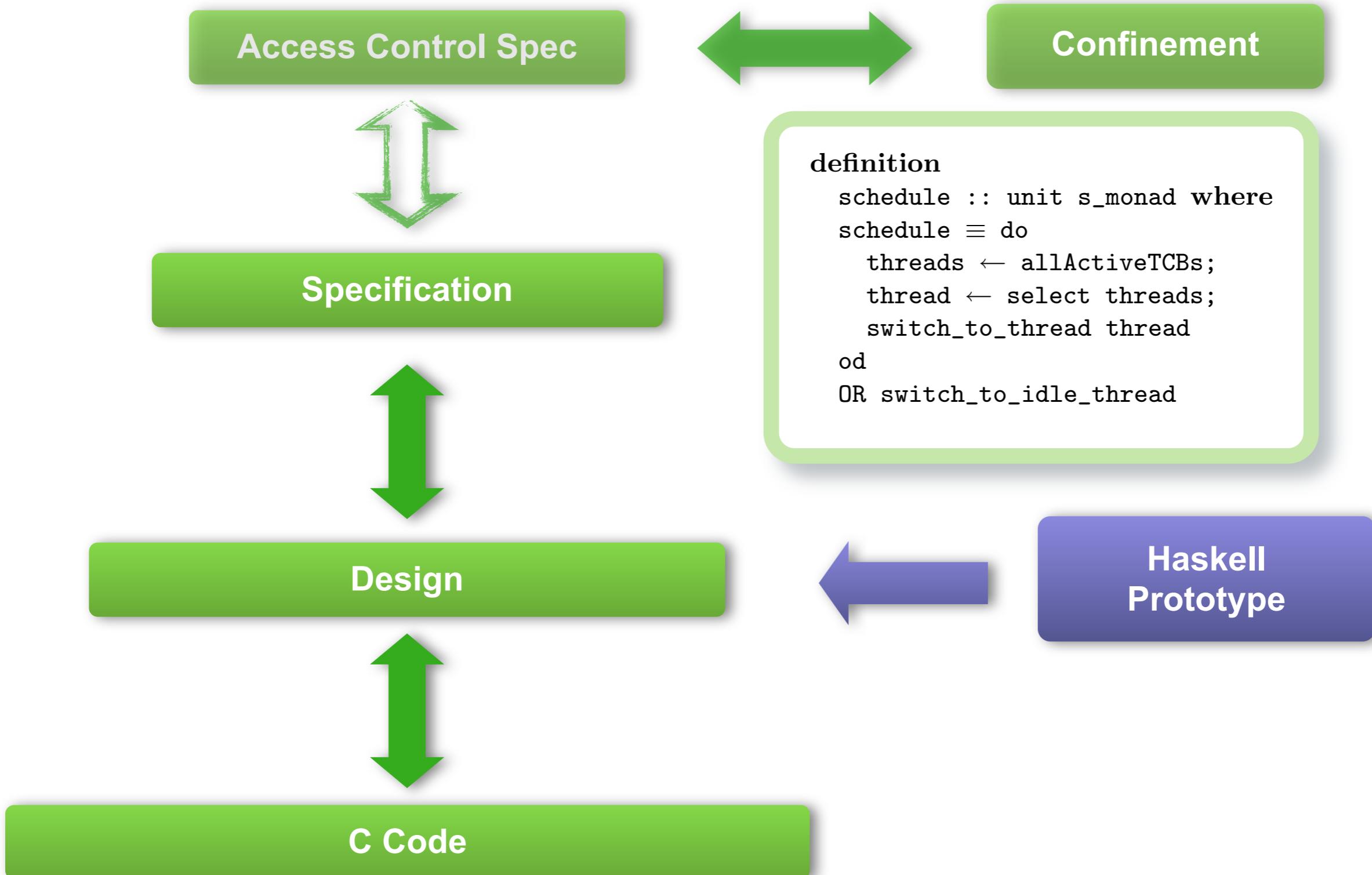
# Proof Architecture



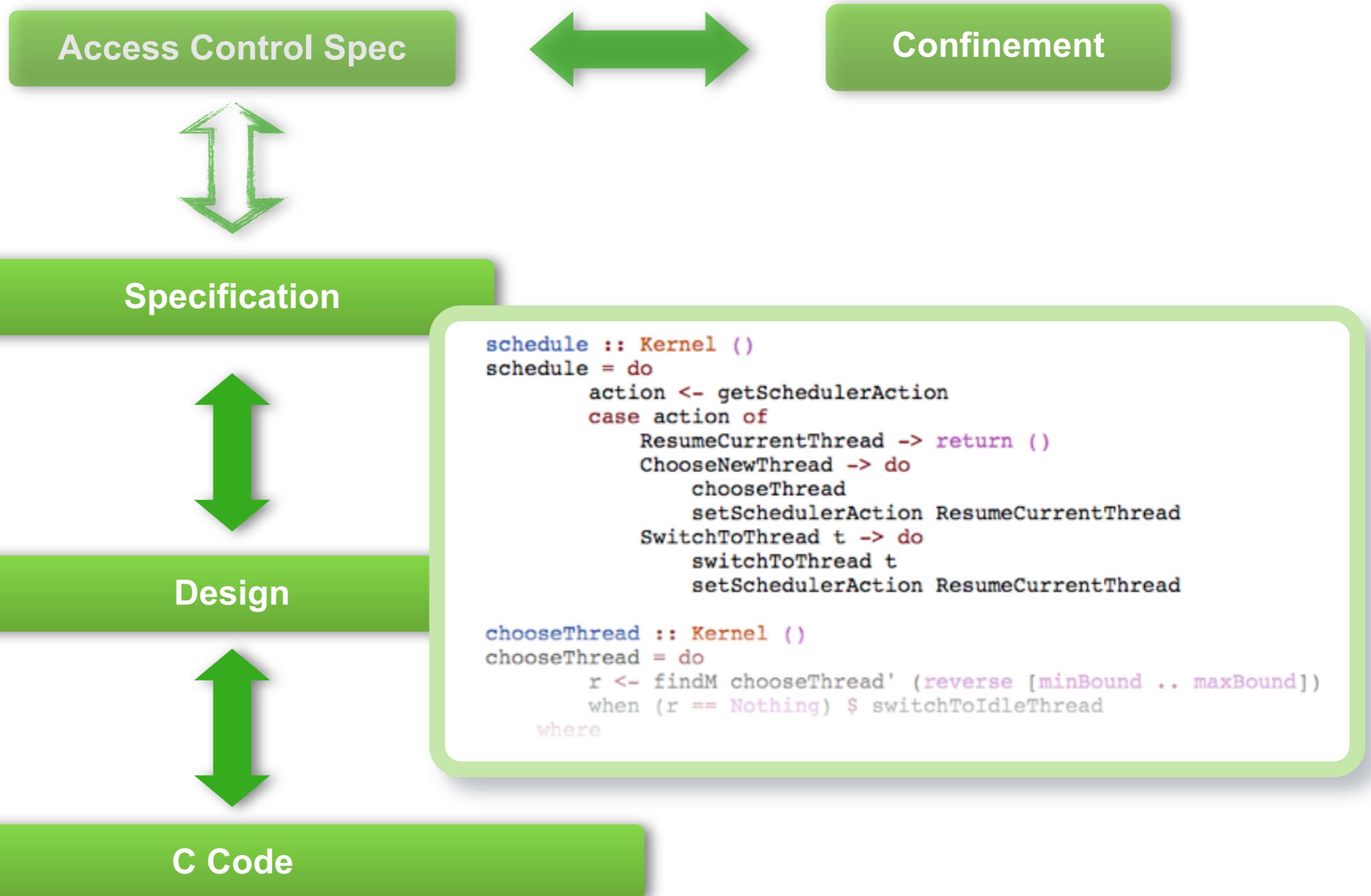
# Proof Architecture



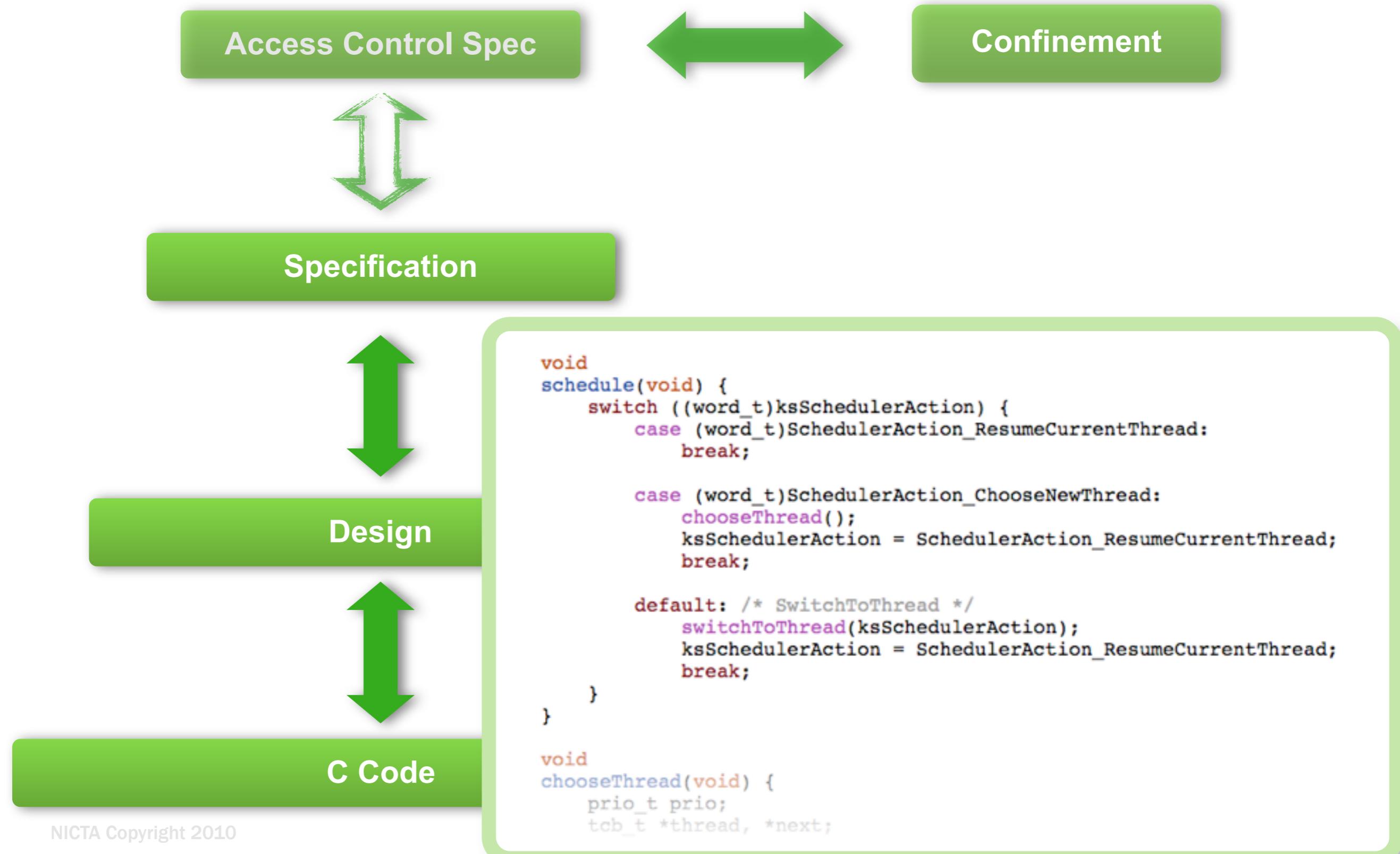
# Proof Architecture

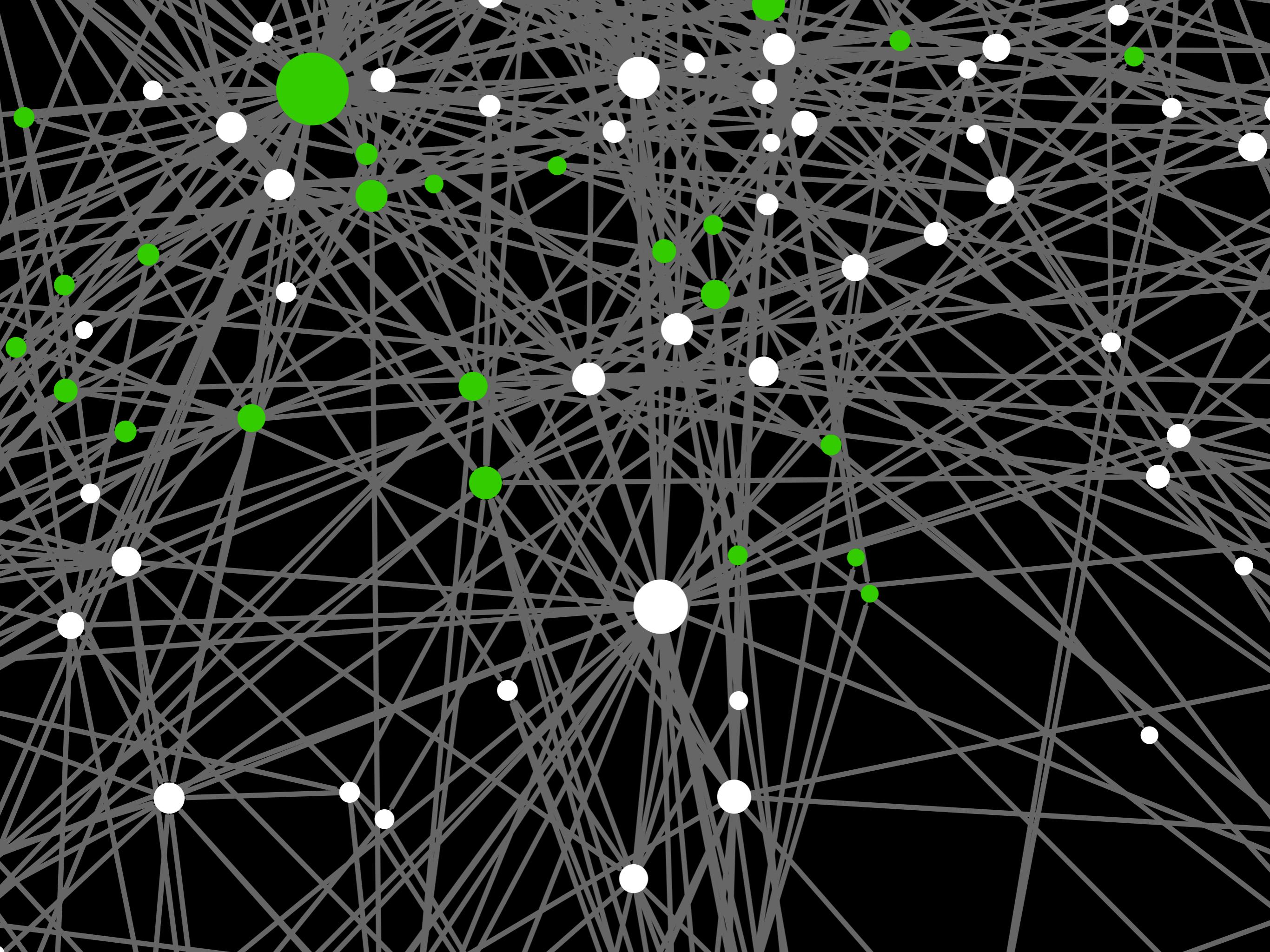


# Proof Architecture



# Proof Architecture







SeLA

# Did you find any Bugs?



## Bugs found

- in C: 160
- in design: ~150
- in spec: ~150

**460 bugs**

```
void
schedule(void) {
    switch ((word_t)ksSchedulerAction) {
        case (word_t)SchedulerAction_ResumeCurrentThread:
            break;

        case (word_t)SchedulerAction_ChoseNewThread:
            chooseThread();
            ksSchedulerAction = SchedulerAction_ResumeCurrentThread;
            break;
    }
}

void
chooseThread()
{
    prio_t
    tcb_t
    for(prio_t
        f
        if(!isRunnable(thread)) {
            next = thread->tcbSchedNext;
            tcbSchedDequeue(thread);
        }
        else {
            switchToThread(thread);
            return;
        }
    }
}

switchToThread(thread)
{
}
```

**Effort**

<b>Haskell design</b>	2 py
<b>First C impl.</b>	2 weeks
<b>Debugging/Testing</b>	2 months
<b>Kernel verification</b>	12 py
<b>Formal frameworks</b>	10 py
<b>Total</b>	25 py

From imagination to impact  
switchToThread(thread);

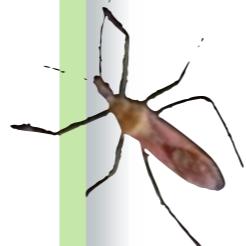
# Did you find any Bugs?



# Bugs found

- in C: 160
  - in design: ~150
  - in spec: ~150

# 460 bugs





		ead;
<b>Haskell design</b>	2	py
<b>First C impl.</b>	2	weeks
<b>Debugging/Testing</b>	2	months
<b>Kernel verification</b>	12	py
<b>Formal frameworks</b>	10	py
<b>Total</b>	25	py

```
    if (!ISRUNNABLE(thread)) {
        next = thread->tcbSchedNext;
        tcbSchedDequeue(thread);
    }
    else {
        switchToThread(thread);
        return;
    }
}
}

void chooseThread()
{
    word_t ksSchedulerAction;
    word_t prio_tcb_t;
    word_t currentThread;
    word_t nextThread;
    word_t resumeCurrentThread;
    word_t schedulerAction_chooseThread();
    word_t schedulerAction_resumeCurrentThread;
}
```

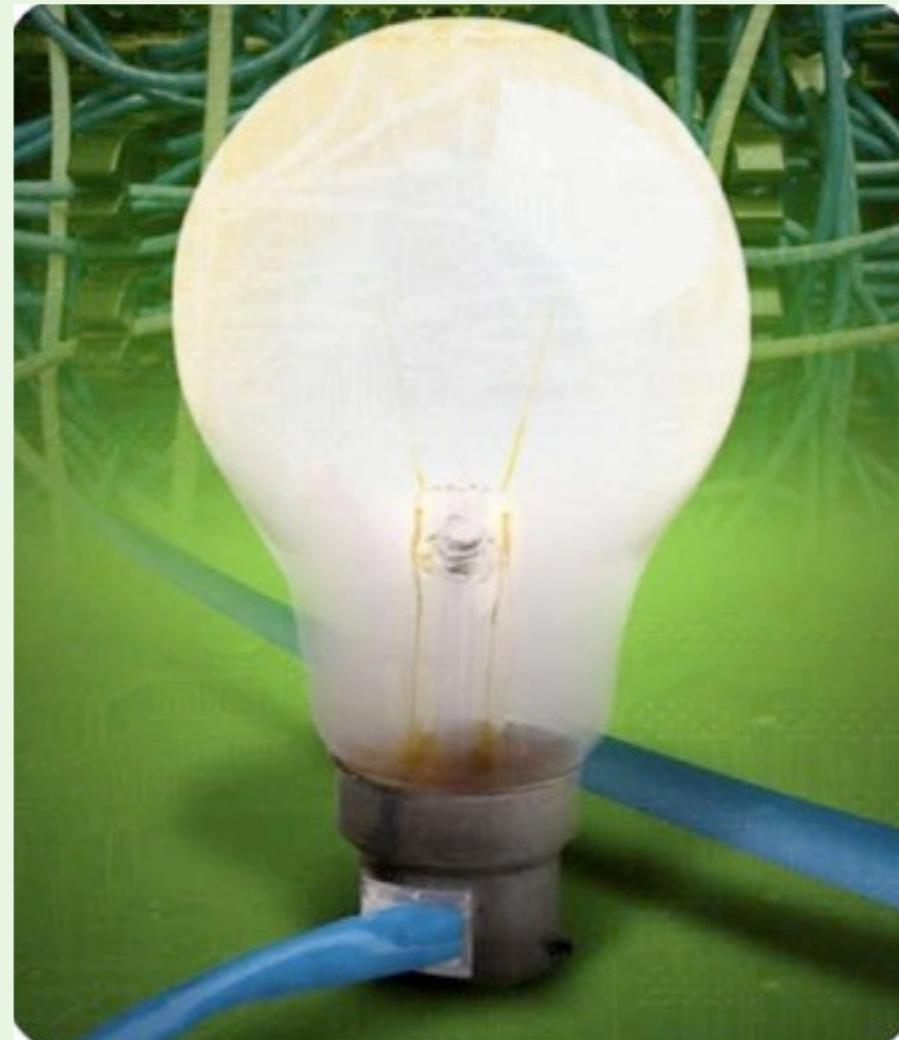
# Effort

<b>Haskell design</b>	2	py
<b>First C impl.</b>	2	weeks
<b>Debugging/Testing</b>	2	months
<b>Kernel verification</b>	12	py
<b>Formal frameworks</b>	10	py
<b>Total</b>	25	py

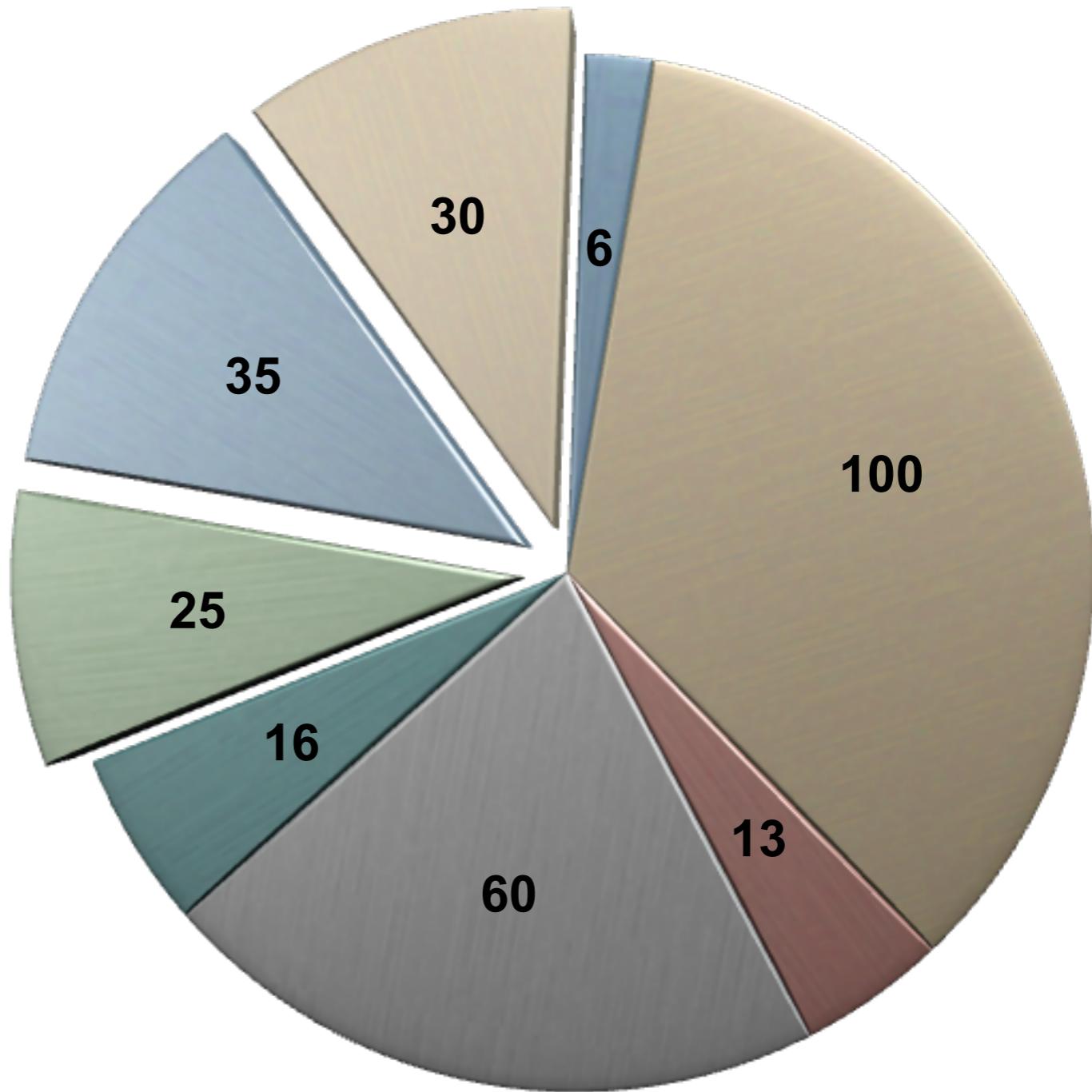
# Proofs and Libraries



# Proofs and Libraries



# Main Proof Components



# Theory Inclusion Graph

---



# Theory Inclusion Graph



# Main Libraries



- **Libraries**
  - Word library
  - Enums
  - Haskell library support
  - Monads
  - Monad VCG + case splitter + strengthening
  - Refinement on Monads
  - Submonads and rewriting under refinement
  - Refinement between Monads and C + tools
  - SIMPL + imperative VCG
  - Crunch
  - LemmaBucket



# Main Libraries



- **Libraries**
  - Word library
  - Enums
  - Haskell library support
  - Monads
  - Monad VCG + case splitter + strengthening
  - Refinement on Monads
  - Submonads and rewriting under refinement
  - Refinement between Monads and C + tools
  - SIMPL + imperative VCG
  - Crunch
  - LemmaBucket



# Main Libraries

- Libraries
  - Word library
  - Enums
  - Haskell library support
  - Monads
  - Monad VCG + case splitter + strengthening
  - Refinement on Monads
  - Submonads and rewriting under refinement
  - Refinement between Monads and C + tools
  - SIMPL + imperative VCG
  - Crunch
  - LemmaBucket



# Tools

- Productivity tools
  - theorem search (`find_thms`)
  - theorem moving (Levity)
  - finding definitions (`locate`)
  - theorem web search (`www_find`)
  - numeral syntax (hex, oct, binary)
- Isabelle
  - Record package
  - Proof cache
  - attributes like “rotated”
  - automated simp rules for case construct

# Tools

- Productivity tools
  - theorem search (find\_thms)
  - theorem moving (Levity)
  - finding definitions (locate)
  - theorem web search (www\_find)
  - numeral syntax (hex, oct, binary)
- Isabelle
  - Record package
  - Proof cache
  - attributes like “rotated”
  - automated simp rules for case construct



# What we thought would work

---



- Automation
  - First order provers (vampire)
  - SAT for word problems
  - Termination proofs
- What we thought might be a problem
  - Performance
  - Memory
  - Lemmas/goals/proofs too big to parse

# Problems

---



# Problems

---



- What turned out to be a problem

# Problems

---



- What turned out to be a problem
  - Isabelle updates

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently
  - Image rebuilds, theory merges and context

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently
  - Image rebuilds, theory merges and context
  - Avoiding duplication and wheel reinvention

# Problems

---

- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently
  - Image rebuilds, theory merges and context
  - Avoiding duplication and wheel reinvention
  - Annoying word proofs

# Problems

---



- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently
  - Image rebuilds, theory merges and context
  - Avoiding duplication and wheel reinvention
  - Annoying word proofs
  - Developing/maintaining libraries + frameworks

# Problems

---

- What turned out to be a problem
  - Isabelle updates
  - Performance, but not as bad
    - theory merges
    - defining large locales + entering large contexts
    - large records
    - waiting for large goals to be printed
  - Memory, but only recently
  - Image rebuilds, theory merges and context
  - Avoiding duplication and wheel reinvention
  - Annoying word proofs
  - Developing/maintaining libraries + frameworks
    - Hard to get over good enough

# Wish List

---



- Better name space management
- Better performance (time + memory)
- Better parallelisation
  - run nitpick, quickcheck, sledgehammer etc in background
  - run proofs faster interactively
- Better dependency handling
  - faster turnaround for deep definition changes
  - faster image rebuilding

# Archive of Formal Proofs



# Archive of Formal Proofs



- Repository of Isabelle proofs
  - Open Source (BSD + LGPL)
  - Maintained
  - Archived, version controlled
  - Quality controlled
- Hosted on SourceForge
  - [<http://afp.sf.net>]



# Screen Shot



The Archive of Formal Proofs

http://afp.sourceforge.net/ Google Google

Google SMH Spiegel Slashdot L4v Wiki Bugzilla ERTOS 2 Wiki



## THE ARCHIVE OF FORMAL PROOFS

The Archive of Formal Proofs is a collection of proof libraries, examples, and larger scientific developments, mechanically checked in the theorem prover [Isabelle](#). It is organized in the way of a scientific journal and has an ISSN: 2150-914x. Submissions are refereed. The preferred citation style is available [\[here\]](#). A [development version](#) of the archive is available as well.

<b>Home</b>
<b>About</b>
<b>Submission Guidelines</b>
<b>Updating entries</b>
<b>Search</b>
<b>Index</b>
<b>Download</b>

### 2010

2010-06-24: [Free Groups](#)

Author: Joachim Breitner

2010-06-20: [Category Theory](#)

Author: Alexander Katovsky

2010-06-17: [Executable Matrix Operations on Matrices of Arbitrary Dimensions](#)

Author: Christian Sternagel and René Thiemann

2010-06-14: [Abstract Rewriting](#)

Author: Christian Sternagel and René Thiemann

2010-05-28: [Verification of the Deutsch-Schorr-Waite Graph Marking Algorithm using Data Refinement](#)

Author: Viorel Preoteasa and Ralph-Johan Back

# Index by Topic



The Archive of Formal Proofs – Index by Topic

<http://afp.sourceforge.net/topics.shtml>

Google Google

Google SMH Spiegel Slashdot L4v Wiki Bugzilla ERTOS 2 Wiki

The Isabelle logo features a 3D cube composed of smaller colored cubes (yellow, red, blue, purple) with letters A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z on their faces.

## INDEX BY TOPIC

## Computer Science

### Automata and Formal Languages

[Regular-Sets](#) [Presburger-Automata](#) [Functional-Automata](#) [Tree-Automata](#)

### Algorithms

[DPT-SAT-Solver](#) [Depth-First-Search](#) [FFT](#) [GraphMarkingIBP](#)  
[SATSolverVerification](#) [MuchAdoAboutTwo](#) **Distributed:** [DiskPaxos](#)  
[GenClock](#) [ClockSyncInst](#)

### Data Structures

[AVL-Trees](#) [BDD](#) [BinarySearchTree](#) [FinFun](#) [Collections](#) [FileRefinement](#)  
[List-Index](#) [Matrix](#) [Huffman](#) [Lazy-Lists-II](#)

### Functional Programming

[Coinductive](#) [Stream-Fusion](#)

### Programming Languages

**Language Definitions:** [CoreC++](#) [FeatherweightJava](#) [Jinja](#) [JinjaThreads](#)  
**Locally-Nameless-Sigma** [POPLmark-deBruijn](#) [Simpl](#) **Type**  
**Systems:** [MiniML](#) [VolpanoSmith](#) **Logics:** [Simpl](#) [Abstract-Hoare-Logics](#)  
[BytecodeLogicJmlTypes](#) [DataRefinementIBP](#) [SIFPL](#) **Compiling:** [Compiling-](#)

From imagination to impact

**Home**

**About**

**Submission Guidelines**

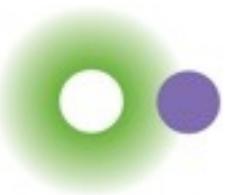
**Updating entries**

**Search**

**Index**

**Download**

# An Entry



Archive of Formal Proofs

<http://afp.sourceforge.net/entries/SATSolverVerification.shtml>

Google SMH Spiegel Slashdot L4v Wiki Bugzilla ERTOS 2 Wiki



## FORMAL VERIFICATION OF MODERN SAT SOLVERS

<b>Title:</b>	Formal Verification of Modern SAT Solvers
<b>Author:</b>	<a href="#">Filip Maric</a>
<b>Submission date:</b>	2008-07-23
<b>Abstract:</b>	<p>This document contains formal correctness proofs of modern SAT solvers. Following (Krstic et al, 2007) and (Nieuwenhuis et al., 2006), solvers are described using state-transition systems. Several different SAT solver descriptions are given and their partial correctness and termination is proved. These include:</p> <ul style="list-style-type: none"><li>• a solver based on classical DPLL procedure (using only a backtrack-search with unit propagation),</li><li>• a very general solver with backjumping and learning (similar to the description given in (Nieuwenhuis et al., 2006)), and</li><li>• a solver with a specific conflict analysis algorithm (similar to the description given in (Krstic et al., 2007)).</li></ul> <p>Within the SAT solver correctness proofs, a large number of lemmas about propositional logic and CNF formulae are proved. This theory is self-contained and could be used for further exploring of properties of CNF based SAT algorithms.</p>

[Home](#)

[About](#)

[Submission Guidelines](#)

[Updating entries](#)

[Search](#)

[Index](#)

[Download](#)

SOURCEFORGE.NET

[project summary](#)

[Proof outline](#)

[Proof document](#)

[Browse theories](#)

[Download this entry](#)

# Statistics

- **Numbers**
  - started 2003
  - now 77 entries
  - 473 kloc of proof scripts
  - 22,000 lemmas
  - runs 4-6h daily
- **Typical entry**
  - belongs to a paper
  - nice proof document
  - small, 1-4 kloc
  - some big developments



# What works well

---



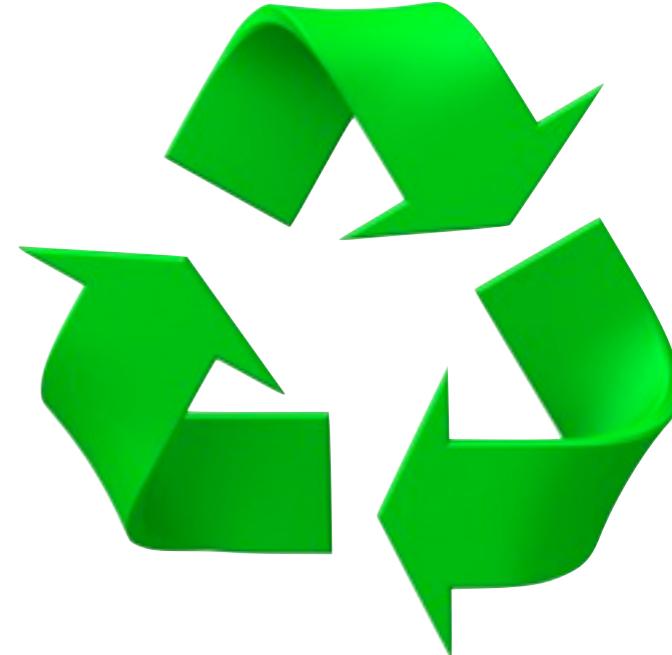
- **Archival and Maintenance**
  - everything kept up to date by Isabelle team
  - new release with every Isabelle release
  - good number of submissions
  - mostly good quality submissions
  - some authors keep improving their entries
    - (but need better high-level change tracking, versioning)
  - good test bed for Isabelle team
  - good place to publish proofs for paper authors
  - authors can get repository access



# Reuse?

---

- Not much reuse:
  - Few entries designed as libraries
  - Few entries used in others (3)
- Reasons
  - Cultural (mostly papers)
  - Good libraries go directly into Isabelle distribution
  - Technical hurdles
    - not enough instruction on how to include
    - could give more built-in support, name spaces again
    - can't import from more than one image
    - dependencies? hackage site?



# Summary



# Summary



# Summary



## Proof Libraries are like Program Libraries

- **L4.verified:**
  - 200kloc proof
  - lots of libraries
  - some are reusable, costs more work, usually worth it

- **Archive of Formal Proofs**
  - 77 entries, 470kloc proof in total
  - works well for papers
  - not so well for libraries yet, hope to get more in the future



NICTA

# Thank You

A large, dynamic graphic element composed of several thick, glowing green lines that curve and overlap, creating a sense of motion and depth across the bottom half of the slide.