Appendix H: Answers to Selected Exercises

Chapter 1

1.3 $B_\nu(T) = 2h\nu^3c^2/(e^{hc\nu/KT} - 1)$

1.4 Show that $(5 - x) = 5 \cdot e^{-x}$, where $x = hc/(K\lambda T)$, and find x

1.5 Insert $\lambda_m = \alpha/T$ into the Planck function

1.6 $\sim$300 K

1.7 7.52×10^{-23} J sec^{-1} m^{-2} sr^{-1}/μm; 81.2×10^{-3} J sec^{-1} m^{-2} sr^{-1}/(cm^{-1}); 8.57×10^{-17} J sec^{-1} m^{-2} sr^{-1}/Hz

1.8 5.22×10^2 J sec^{-1} m^{-2}; 4.96×10^2 J sec^{-1} m^{-2}; 9.36 μm

1.9 Both cases are about 0.4%

1.11 (b) $n = 1, 2$; $\lambda_{12} = 1216$ Å

1.12 1 J = 5.0345×10^{22} cm^{-1}

1.13 Use Planck's relation, $\Delta E = h\nu c$

1.14 Note: $\int e^{ax} \cos bx \, dx = \dfrac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$

1.15 Note: $\int_{-\infty}^{\infty} \dfrac{dx}{1 + x^2} = \pi; \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

1.17 (a) $\lim\limits_{x,y\to\infty} K(x, y) = \dfrac{1}{\pi} \dfrac{y}{x^2 + y^2}$;

(b) Define a delta function $f(x - t) = \dfrac{1}{\pi} \dfrac{y}{y^2 + (x - t)^2}$

1.18 (a) $n_1 b_{12} = n_2 b_{21}$; (b) $n_1 C_{12} u_{\tilde{\nu}} = n_2(C_{21} u_{\tilde{\nu}} + A_{21})$;

(c) $n_1(b_{12} + C_{12} u_{\tilde{\nu}}) = n_2(b_{21} + C_{21} u_{\tilde{\nu}} + A_{21})$

1.19 $\beta_\lambda = 0.1, 0.5$ m^{-1}; $\tau = 1, 5$

1.20 x(aerosol) = 48.9 km, x(dense fog) = 0.039 km

1.21 $I_\lambda(s) = I_\lambda(0)(1 - R_\lambda)^2 T_\lambda / \left(1 - R_\lambda^2 T_\lambda^2\right)$

1.23 $F_\nu(\tau = 0) = \pi B_\nu(T_s) 2E_3(\tau_*) + \pi B_\nu(T)[1 - 2E_3(\tau_*)]$

Chapter 2

2.1 At the poles, $\varepsilon = \delta$; at 60°, $\varepsilon = 30° + \delta$; at 45°$N$ and equinox, $2H = 12$ hr; at 45°N and at the solstice, $2H = 15.44$ hr

2.2 Use the equation for an ellipse involving the radius, the true anomaly, and eccentricity

2.4 (b) 102 min; (c) 35,865 km

2.5 5754 K

2.6 1.52991×10^{22} J

2.7 4.51×10^{-10}

2.8 (a) 1.14×10^{9} J sec^{-1}; (b) 1.82×10^{-2} J sec^{-1}

2.10 T(Venus) = 225.8 K

2.11 70.85 K (use the solid angle from Ex. 1.2)

2.12 4.2 K

2.13 (a) 556.60 W m^{-2}; (b) 434.81 W m^{-2} (daily mean)

2.14 Note: $\sin(-\varphi) = -\sin\varphi$; $\sin(-\varepsilon) = -\sin\varepsilon$

2.15 Note: $e << 1$ and $(1 - x)^{-2} = 1 + 2x - \cdots$

2.17 Note: Solar irradiance at TOA, $F_\lambda = \Omega B_\lambda(T)$, where Ω is a solid angle

2.18 $F_{0\lambda} = 22.31$ W m^{-2}, $T_\lambda = 0.68465$

Chapter 3

3.1 Note that the Boltzmann constant $K = MR$, where M is the molecular weight and R is the gas constant for air.

3.3 (a) Use Beer's law for the change of solar flux due to absorption from the outer edge of the atmosphere to the point P and (b) take an exponential variation of the density in the vicinity of the point P.

3.5 $[\mathrm{O}]^2 = J_2[\mathrm{O}_2]/(K_{11}[M] + K'_{11})$

3.6 $A_{\bar{\nu}}(u/\mu_0) = 2\sqrt{Su\alpha/\mu_0}$

3.7 $\sigma_s(0.7\,\mu\mathrm{m}) = 1.71 \times 10^{-27}$ cm^2

3.8 $\tau(0.5\,\mu\mathrm{m}) \cong 0.15$

3.9 At $z = 10$ km; $\rho = 0.414 \times 10^{-3}$ g cm^{-3}, $(m_r - 1) \times 10^{-3} = 0.099$

3.10 $F \cong F_0 l \cdot 1.37 \times 10^{-11}$ (at 10 km)

3.11 For $\lambda = 10$ cm; $\beta_\pi \cong 9.76 \times 10^{-10}$ km^{-1}, 8.66×10^{-10} km^{-1}

3.12 The total deviation from the original direction is $\theta' = 2(\theta_i - \theta_t) + 2(p - 1) \cdot (\pi/2 - \theta_t)$, where $p = 1$ denotes two refractions and $p \geq 2$ denotes internal reflections. The first rainbow ($p = 2$) is located at the 137° scattering angle.

3.13 The deviation from the original ray is $\theta' = (\theta_i - \theta_t) + (\theta_i' - \theta_t')$. Note that $\theta_i' = \theta_t, \theta_t' = \theta_t$ and $\theta_t = A/2$. The 22° and 46° halos are defined by A = 60° and 90°, respectively.

3.14 At about 24.1° from the sun

3.15 An aerosol particle size of about 0.45 to 0.48 μm (use blue and red light in the calculations).

3.17 Note that for $\tilde{\omega} = 1, k = 0$ and $\gamma_1 = \gamma_2$

3.18 $\chi = -[(\gamma_1 - \gamma_2)(1 - 2\gamma_3) + 1/\mu_0]\tilde{\omega}\, F_\odot$

Chapter 4

4.2 $x \to 0,\ L(x) \cong x; x \to \infty,\ L(x) \cong 2\sqrt{x/2\pi}$ (prove)

4.3 $f(k) = \dfrac{1}{k}\dfrac{1}{\pi}\left(\dfrac{2k\delta}{S}\coth\beta - \dfrac{k^2\delta^2}{S^2} - 1\right)^{-1/2}$

4.4 $(d)\ f(k) = L^{-1}(e^{-au}) = \delta(k - a)$;

$f(k) = L^{-1}(\exp[-(au)^{1/2}]) = \frac{1}{2}\left(\dfrac{a}{\pi}\right)^{1/2} k^{-3/2}\exp(-a/4k)$

4.5 $\dfrac{\overline{W}}{\delta} = \dfrac{2}{\pi}(b_{\bar{\nu}}y)^{1/2}\tan^{-1}\left(\dfrac{\delta}{\pi\nu_0}(b_{\bar{\nu}}y)^{1/2}\right) + \dfrac{2\nu_0 C_y}{1 + \nu_0 Cy(\delta/\nu_0)},\ y = a_{\bar{\nu}}u$

4.6 Note: $du = q\, dp/g$

4.7 $T_{\bar{\nu}} = F_{\bar{\nu}}/F_{0,\bar{\nu}}$

4.8 To obtain $c_{\bar{\nu}}$ and $d_{\bar{\nu}}$, use strong- and weak-line approximations

4.9 Note: $\displaystyle\int_0^\infty e^{-a^2x^2}\cos bx\, dx = \frac{\sqrt{\pi}}{2a}e^{-b^2/4a^2};\ \int_0^\infty \frac{\cos ax}{1 + x^2}dx = \frac{\pi}{2}e^{-a}$

4.10 Note: $\displaystyle\int_0^\infty\int_{k''}^\infty \to \int_0^\infty\int_0^k$

4.11 Let $M = I^\uparrow + I^\downarrow$, and $N = I^\uparrow - I^\downarrow$

Chapter 5

5.1 Perform $mk\,\nabla \times \mathbf{N}_\psi = \nabla \times \nabla \times \mathbf{M}_\psi$

5.2 Take a trial solution: $rv^i = \dfrac{1}{k}\displaystyle\sum_{n=1}^\infty \beta_n\,\psi_n(kr)\,P_n^1(\cos\theta)\sin\phi$, and determine β_n

5.3 (a) Note: $\nabla \times \nabla\psi = 0$ and $\nabla \cdot \nabla \times \mathbf{A} = 0$;

(b) $\mathbf{M}_\psi = \nabla \times (\mathbf{a}_z\psi) = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\phi}\mathbf{a}_r - \dfrac{\partial\psi}{\partial r}\mathbf{a}_\phi$

5.4 Let $\psi(r, \phi, z) = e^{-i\omega t}R(r)\Phi(\phi)Z(z)$

5.5 See Eq. (5.4.24)

5.6 $\theta_i = \cos^{-1}\sqrt{1/(1+m^2)}$

5.7 (a) Red outside and violet inside, $a \cong 3.5\,\mu$m;

(b) Primary cloudbow, 137.78° (red) and 139.49° (violet);

(c) 22° halo $(A = 60°)$, $\Delta\theta = 0.76°$; 46° halo $(A = 90°)$, $\Delta\theta = 2.17°$

5.8 Let $m\cos\theta_t = u + iv$ in Eq. (5.3.23a)

5.9 See also Eq. (5.2.113)

5.10 (c) $\phi_i = A/2$, $\phi_i = (A + \theta_h')/2$, where θ_h' is the minimum deviation angle projected on the horizontal plane, and $\sin\theta'/2 = \cos\varepsilon_i \sin\theta_h'/2$ (prove) where ε_i is the elevation angle; θ (red) = 24.54°

5.14 The average cross section of randomly oriented hexagonal crystals is given by
$$\overline{G} = \frac{6}{\pi}\int_0^{\pi/6}\int_0^{\pi/2} G(\alpha,\beta)\cos\alpha\, d\alpha\, d\beta$$

Chapter 6

6.1 $\tilde{\omega}(\tilde{\nu}) = (1+x^2)/(2+x^2)$, where $x = (\nu - \nu_0)/\alpha$, $R(\mu,\mu_0)$
$$= \frac{\widetilde{\omega}}{4(\mu+\mu_0)} \times \frac{(1+\sqrt{3}\mu)(1+\sqrt{3}\mu_0)}{[1+\mu\sqrt{3(1-\widetilde{\omega})}][1+\mu_0\sqrt{3(1-\widetilde{\omega})}]}$$

6.2 $r(\mu_0) = F^{\uparrow}(\mu_0)/\mu_0 F_{\odot}$; $\tilde{\omega} = 0.8$, $r(\mu_0 = 1) = 0.32$, $\bar{r} = 0.35$

6.3 Follow the procedures outlined in Section 6.3.2

6.4 Note: $1 + x + x^2 + \cdots = 1/(1-x)$

6.6 Let $F_{\odot} = \pi$ and $\phi_0 = 0°$. For $\tau = 0.1$, $I(\mu = 1, \phi = 0°) = 0.028$

6.7 $I_2(0,\mu,\phi)$
$$= \frac{\mu_0 F_{\odot}\tilde{\omega}^2}{4}\int_0^{2\pi}\int_{-1}^{1} P(\mu,\phi;\mu'\phi')P(\mu',\phi',-\mu_0,\phi_0)g(\mu,\mu_0,\mu')\,d\mu'\,d\phi',$$
where
$$g(\mu,\mu_0,\mu') = \frac{1}{4(\mu_0+\mu')}\left[\frac{\mu_0}{\mu_0+\mu}\left\{1-\exp\left[-\tau_1\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\right]\right\}\right.$$
$$+\frac{\mu'}{\mu-\mu'}\left\{\exp\left[-\tau_1\left(\frac{1}{\mu_0}+\frac{1}{\mu'}\right)\right]\right.$$
$$\left.\left.-\exp\left[-\tau_1\left(\frac{1}{\mu_0}+\frac{1}{\mu}\right)\right]\right\}\right],\ \mu \neq \mu'$$

6.8 $$I^{\uparrow}(0,\mu_1) = \frac{1}{2}\left\{\frac{\mu_0}{\mu_1}\left[2S^- - 2b\frac{\mu_0}{\mu_1}(S^+ + S^-)\right] + \frac{K\mu_1}{2b} + H\right\},$$ where
$$K = -\frac{2\mu_0}{\mu_1}\left[\left(S^- - \frac{b\mu_0}{\mu_1}\right)e^{-\tau_1/\mu_0} + S^+ + \frac{b\mu_0}{\mu_1}(S^+ + S^-)\right]\Big/\left(\frac{\tau_1+\mu_1}{b}\right),$$

$$H = \frac{2\mu_0}{\mu_1}\left[S^+ + \frac{b\mu_0}{\mu_1}(S^+ + S^-)\right] + \frac{K\mu_1}{2b}$$

6.12 $(E_l/a_l)^2 + (E_r/a_r)^2 - 2(E_l/a_l)(E_r/a_r)\cos\delta = \sin^2\delta$

6.15 (b) Let $I_0 = 1$, $\left[2 - \frac{1}{2}\, 0\, \frac{1}{2}\right]$, $I = 2$, $P = \frac{1}{2\sqrt{2}}\%$, $I_r = \frac{5}{4}$; $(c)\chi = 0°$, $\beta = 22.5°$; (d)$\frac{1}{4}[(4-\sqrt{2})\,(2\sqrt{2}-1)\,0\,(2\sqrt{2}-1)]$, $\frac{1}{4}[(4-\sqrt{2})\,(1-2\sqrt{2})\,0\,(1-2\sqrt{2})]$; (e)$\frac{1}{4}[(4+\sqrt{2})\,(2\sqrt{2}-3)\,0\,(1+2\sqrt{2})]$ for right-hand polarization

6.17 $\nabla^2 I_0^0 - k^2\beta_e^2 I_0^0 = -\chi\beta_e^2 e^{-\tau_s}$; where $\chi = 3\tilde{\omega}F_\odot(1 + g - \tilde{\omega}g)/4\pi$

Chapter 7

7.1 (b) $\nu^* = 3.13$

7.2 (a) $g(k) = -\frac{12}{k^2}e^{-k} + \left(\frac{24}{k^2} - \frac{8}{k} + 2\right)\left[\frac{1}{k^2} - \left(\frac{1}{k} + \frac{1}{k^2}\right)e^{-k}\right]$, $i = 10$, $k = 5$, $g(k) = 0.0490$; $(b)\, i = 10$, $k = 5$, $g(k) = 0.0489$

7.3

$$\text{Second difference, } \mathbf{H} = \begin{bmatrix} 1 & -2 & 1 & & & & & & & 0 \\ -2 & 5 & -4 & 1 & & & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & & \\ & 1 & -4 & 6 & -4 & 1 & & & & \\ & & & & & & \cdots & & & \\ & & & & & & 1 & -4 & 5 & -2 \\ 0 & & & & & & & 1 & -2 & 1 \end{bmatrix}$$

7.5 $\tau \sim \mathrm{LWC}^{2/3} N^{1/3}$

Chapter 8

8.1 302 K

8.2 $\bar{\varepsilon} = 0.88$, $T_a = 250\,\mathrm{K}$; $\bar{\varepsilon} = 0.9$, $T_a = 249\,\mathrm{K}$

8.3 Note: $1 + y + y^2 + \cdots = 1/(1-y)$

8.4 $\partial T_s/\partial F_{ir} = T_s/4F_{ir}$

8.5 $T(x_i = 0.95) \cong -7°C$; $x = [a - b/(Q/Q_0)]^{1/2}$, where $a \cong 1.71646$ and $b \cong 0.81396$

8.6 (c) $x_i^4 - ax_i^3 - bx_i^2 + cx_i - d + [e + f(\triangle Q/Q_0)]/(1 + \triangle Q/Q_0) = 0$, where *a, b, c, d, e,* and *f* are certain coefficients

8.7 $F_2(6D'' + 1) = QH_2(x_i)$, find D$''$