

1 Emission: The Birth of Photons

This is the first of three foundation chapters supporting those that follow. The themes of these initial chapters are somewhat fancifully taken as the birth, death, and life of photons, or, more prosaically, emission, absorption, and scattering.

In this chapter and succeeding ones you will encounter the phrase “as if”, which can be remarkably useful as a tranquilizer and peacemaker. For example, instead of taking the stance that light *is* a wave (particle), then fiercely defending it, we can be less strident and simply say that it is *as if* light is a wave (particle). This phrase is even the basis of an entire philosophy propounded by Hans Vaihinger. In discussing its origins he notes that “The Philosophy of ‘As If’ . . . proves that consciously false conceptions and judgements are applied in all sciences; and . . . these scientific Fictions are to be distinguished from Hypotheses. The latter are assumptions which are probable, assumptions the truth of which can be proved by further experience. They are therefore verifiable. Fictions are never verifiable, for they are hypotheses which are known to be false, but which are employed because of their utility.”

1.1 Wave and Particle Languages

We may discuss electromagnetic radiation using two languages: wave or particle (photon) language. As with all languages, we sometimes can express ideas more succinctly or clearly in the one language than in the other. We use both, separately and sometimes together in the same breath. We need fluency in both. Much ado has been made over this supposedly lamentable duality of electromagnetic radiation. But no law requires physical reality to be described by a single language. We may hope for such a language, but Nature often is indifferent to our hopes. Moreover, we accept without protest or hand-wringing the duality of sound. We describe sound waves in air as continuous while at the same time recognizing that air, and hence sound, is composed of discrete particles (molecules) in motion.

How do we choose which language to use? Simplicity. Life is short. To understand nature we take the simplest approach consistent with accuracy. Although propagation of sound in air could be described as the motions of molecules, had this approach been taken acoustics would have floundered in a mathematical morass.

In the photon language a beam of radiation is looked upon as a stream of particles called photons with the peculiar property that they carry energy, linear momentum, and angular momentum but not mass. The mass of the photon often is said to be identically zero. But given the near impossibility of measuring zero in the face of inevitable errors and uncertainties, it would be more correct to say that the upper limit of the photon mass keeps decreasing, its present value being about 10^{-24} times the mass of the electron. If it bothers you that a particle

without mass can carry momentum this is because you are stuck on the notion that momentum is mass times velocity. Sometimes this is true (approximately), sometimes not. Momentum is momentum, a property complete in itself and not always the product of mass and velocity.

Photons are of one kind, differing only in their energy and momenta, whereas waves are of unlimited variety and often exceedingly complex, the simplest kind a plane harmonic wave characterized by a single (circular) frequency ω and direction of propagation (see Secs. 3.3 and 3.4). The dimensions of circular frequency are radians per unit time. You may be more familiar with just plain frequency, often denoted by ν (sometimes f), which has the dimensions of cycles per unit time. The unit of frequency is the hertz, abbreviated Hz, one cycle per second. Because one cycle corresponds to 2π radians, the relation between frequency and circular frequency is simple:

$$\omega = 2\pi\nu. \quad (1.1)$$

All electromagnetic waves propagate in free space (which does not strictly exist) with the same speed c , about $3 \times 10^8 \text{ m s}^{-1}$. A plane harmonic wave in free space can just as well be characterized by its wavelength λ , related to its frequency by

$$\lambda\nu = c. \quad (1.2)$$

You sometimes hear it said that frequency is more fundamental than wavelength. In a sense, this is correct, but wavelength is often more useful. When we consider the interaction of electromagnetic waves with chunks of matter, the first question we must ask ourselves is how large the waves are. Big and small have no meaning until we specify a measuring stick. For electromagnetic radiation the measuring stick is the wavelength. The mathematical expressions describing the interaction of such radiation with matter can be quite different depending on the size of the matter relative to the measuring stick.

How do we translate from wave to photon language? A plane harmonic wave with circular frequency ω corresponds to a stream of photons, each with energy

$$E = h\nu = \hbar\omega, \quad (1.3)$$

where h is Planck's constant ($6.625 \times 10^{-34} \text{ J s}$) and $\hbar = h/2\pi$. The frequency of visible electromagnetic radiation (light) is about 10^{14} Hz , and hence the photons that excite the sensation of vision have energies around 10^{-20} J . This isn't much energy; the kinetic energy of a golf ball as it slices through air is about 10^{13} times greater.

Understanding what happens when an electromagnetic wave is incident from air on the smooth surface of glass, say, is not especially difficult if one uses the wave language. The incident wave excites molecules in the glass to radiate secondary waves that combine to form (approximately) a net reflected wave given by the law of reflection and a net transmitted wave given by the law of refraction. There is no such thing as an absolutely smooth surface, so what is meant is smooth on the scale of the wavelength.

All this makes intuitive sense and causes no perplexity. But now consider what happens when we switch to photon language. If we look upon reflection as the rebound of photons at a surface and transmission as their penetration through it, then why, if all photons are identical, are some reflected and some transmitted? This is indeed puzzling; even more so is why

photons should be specularly (by which is meant mirror-like) reflected, because for photons imagined as particles of vanishingly small dimensions, all surfaces are rough.

This is not to say that one couldn't describe reflection and transmission at smooth interfaces in photon language, only that to do so would be exceedingly costly in mental effort. And the reverse sometimes is true. Many years ago one of the authors attended a colloquium entitled "The photoelectric effect without photons." By the photoelectric effect is usually meant the emission of electrons by a surface (often metallic) because of illumination by radiation (often ultraviolet). In photon language the photoelectric effect is simple to describe. When a photon of energy $h\nu$ is absorbed by the surface, the maximum kinetic energy E of the electrons thereby set free is

$$E = h\nu - p, \quad (1.4)$$

where p is the minimum energy an electron loses in breaking free of the surface. A single photon interacting with a single electron gives up its entire energy to that electron, which if sufficient enables the electron to break free of the forces binding it to the metal. According to this equation the energies of the emitted electrons are independent of the incident power whereas the photocurrent (rate and number of emitted electrons) is proportional to it, which accords with experiment. This simple equation, first written down by Einstein in 1905, is one of the keystones of the modern theory of radiation and matter. Yet the speaker at that colloquium years ago, in an effort to describe and explicate the photoelectric effect without photons, assailed the audience with dozens of complicated equations. And even at that, part way through his mathematical tour de force his mind and tongue betrayed him and he blurted out the forbidden word "photon". At that point, your author who was there leapt up from his seat and shouted, "Photons! Photons! You promised no photons."

A mirror illuminated by an incident beam gives rise to a reflected beam. Is this reflected beam redirected incident photons? Alas, we cannot do an experiment to answer this question. To determine if reflected photons are the same as incident photons would require us to be able to identify them. But photons are indistinguishable. We cannot tell one from another. We cannot tag a photon and follow its progress. Thus if you want to believe that reflected photons are the same as incident photons, you may do so. No one can prove you wrong. But you cannot prove you are right. When faced with an undecidable proposition, you may believe whatever you wish. Note that in the wave language we would not likely even ask if the reflected wave is the same as the incident wave.

It is not often acknowledged that there is a third language for talking about light, what might be called the who-gives-a-hoot-what-light-is? language. This is geometrical or ray optics, in which the nature of light isn't addressed at all. Fictitious rays are imagined to be paths along which the energy carried by light is transported, and these paths meander and bifurcate according to simple geometrical laws.

But which language is the more useful? In a letter to *American Journal of Physics*, M. Psi-mopoulos and T. Theocharis ask the rhetorical questions: "What new discoveries have (i) the particle or photon aspect of light, and (ii) the wave aspect of light, given rise to? Answer: (i) we are not aware of any; (ii) holography, laser, intensity interferometry, phase conjugation." To this list we add radar, all of interferometry, on which much of the science of measurement is based, and interference filters, which have many applications. The view of these authors is

extreme, but they also quote the more measured words of Charles Townes, a pioneer in masers and lasers: “Physicists were somewhat diverted by an emphasis in the world of physics on the photon properties of light rather than its coherent aspects.” That is, the photon language has been the more fashionable language among physicists, just as French was the fashionable language in the Imperial Russian court. When prestigious and munificent prizes began to be awarded for flushing “ons” (electron, positron, neutron, meson, and so on) from the jungle, shooting them, and mounting their stuffed heads on laboratory walls, the hunt was on, and slowed down only with the demise of the Superconducting Supercollider.

Although the wave language undoubtedly has been and continues to be more fruitful of inventions, the photon language is perhaps more soothing because photons can be incarnated, imagined to be objects we can kick or be kicked by. Waves extending through all space are not so easily incarnated. We can readily conceive of the photon as a thing. And yet an electromagnetic wave is just as much a thing as a photon: both possess energy and momentum (linear and angular) but not, it seems, mass.

1.2 Radiation in Equilibrium with Matter

We often are told that when bodies are heated they radiate or that “hot” bodies radiate. True enough, but it is just as true that when bodies are *cooled* they radiate and that “cold” bodies radiate. *All* matter – gaseous, liquid, or solid – at *all* temperatures emits radiation of *all* frequencies at *all* times, although in varying amounts, possibly so small at some frequencies, for some materials, and at some temperatures as to be undetectable with today’s instruments (tomorrow’s, who knows?). Note that there is no hedging here: all means all. No exceptions. Never. Even at absolute zero? Setting aside that absolute zero is unattainable (and much lower than temperatures in the depths of the Antarctic winter or in the coldest regions of the atmosphere), even at absolute zero radiation still would be associated with matter because of temperature fluctuations. Temperature is, after all, an average, and whenever there are averages there are fluctuations about them.

Radiation emitted spontaneously, as distinguished from scattered radiation (see Ch. 3), is not stimulated by an external source of radiation. Scattered radiation from the walls of the room in which you read these words may be stimulated by emitted radiation from an incandescent lamp. Turn off the lamp and the visible scattered radiation vanishes, but the walls continue to emit invisible radiation as well as visible radiation too feeble to be perceptible.

We are interested in the spectral distribution of radiation – how much in each wavelength interval – emitted by matter. Consider first the simpler example of an ideal gas in a sealed container held at absolute temperature T (Fig. 1.1). When the gas is in equilibrium its molecules are moving in all directions with equal probability, but all kinetic energies E are not equally probable. Even if all the molecules had the same energy when put into the container, they would in time have different energies because they exchange energy in collisions with each other and the container walls. A given molecule may experience a sequence of collisions in which it always gains kinetic energy, which would give it a much greater energy than average. But such a sequence is not likely, and so at any instant the fraction of molecules with kinetic energy much greater than the average is small. And similarly for the fraction of molecules with kinetic energy much less than the average. The distribution of kinetic energies is specified by

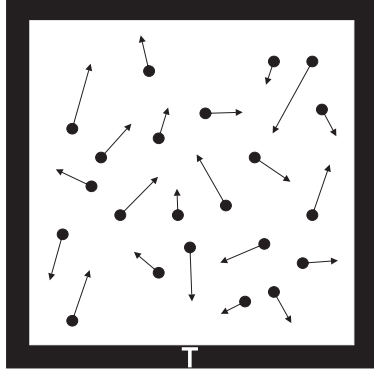


Figure 1.1: At equilibrium, ideal gas molecules in a closed container at absolute temperature T have a distribution of kinetic energies (Fig. 1.2) determined solely by this temperature.

a probability distribution function $f(E)$ which, like all distribution functions, is defined by its *integral* properties, that is,

$$\int_{E_1}^{E_2} f(E) dE \quad (1.5)$$

is the fraction of molecules having kinetic energies between any two energies E_1 and E_2 . Note that f does not specify *which* molecules have energies in a given interval, only the fraction, or probability, of molecular energies lying in this interval. If f is continuous and bounded then from the mean value theorem of integral calculus

$$\int_{E_1}^{E_2} f(E) dE = f(\bar{E})(E_2 - E_1), \quad (1.6)$$

where \bar{E} lies in the interval (E_1, E_2) . If we denote E_1 by E and E_2 by $E + \Delta E$ we have

$$f(E) = \lim_{\Delta E \rightarrow 0} \frac{1}{\Delta E} \int_E^{E+\Delta E} f(x) dx. \quad (1.7)$$

Because of Eq. (1.7) $f(E)$ is sometimes called a *probability density*. When the limits of the integral in Eq. (1.5) are the same (interval of zero width) the probability is zero. The probability that a *continuous* variable has exactly a particular value at any point over the interval on which it is defined is zero, as it must be, for if it were not the total probability would be infinite.

A distribution function such as $f(E)$ is sometimes defined by saying that $f(E) dE$ is the fraction (of whatever) lying in the range between E and $E + dE$. This is sloppy mathematics because although E represents a definite number dE does not. Moreover, this way of defining a distribution function obscures the fact that f is defined by its integral properties. As we shall see, failure to understand the nature of distribution functions can lead to confusion and

error. It would be better to say that $f(E) \Delta E$ is *approximately* the fraction of molecules lying between E and $E + \Delta E$, where the approximation gets better the smaller the value of ΔE .

You also often encounter statements that $f(E)$ is the fraction of molecules having energy E *per unit energy interval*. This can be confusing unless you recognize it as shorthand for saying that $f(E)$ must be multiplied by ΔE (or, better yet, integrated over this interval) to obtain the fraction of molecules in this interval. This kind of jargon is used for all kinds of distribution functions. We speak of quantities per unit area, per unit time, per unit frequency, etc., which is shorthand and not to be interpreted as meaning that the interval is one unit wide.

Gases within a sealed container held at constant temperature evolve to an equilibrium state determined solely by this temperature. In this state the distribution function for molecular kinetic energies is the *Maxwell–Boltzmann distribution*

$$f(E) = \frac{2\sqrt{E}}{\sqrt{\pi}(k_B T)^{3/2}} \exp(-E/k_B T), \quad (1.8)$$

where k_B , usually called *Boltzmann's constant*, is $1.38 \times 10^{-23} \text{ J K}^{-1}$, and f is normalized

$$\int_0^\infty f(E) dE = 1. \quad (1.9)$$

The limits of integration are symbolic: molecules have neither infinite nor zero kinetic energies; by zero is meant $\ll k_B T$ and by infinite is meant $\gg k_B T$. Because of Eq. (1.9) $f(E)$ is a probability distribution function.

The most probable kinetic energy E_m is that for which f is a maximum, the energy at which its derivative with respect to E is zero:

$$E_m = k_B T/2. \quad (1.10)$$

As the temperature of the gas increases so does the most probable kinetic energy of its molecules. Figure 1.2 shows f relative to its maximum as a function of E relative to E_m , a universal curve independent of temperature.

What does all this have to do with radiation? Because matter continuously emits radiation, a container with walls so thick that no photons leak from it will fill with a gas of photons (Fig. 1.3). The container is held at a fixed temperature T . At equilibrium the photons in the container, like gas molecules, do not all have the same energy (equivalently, frequency) but are distributed about a most probable value. The distribution function for the energies of photons in equilibrium with matter goes under various names and there are several versions of this function differing by a constant factor. Imagine a plane surface within the container. At equilibrium, the radiation field is isotropic, so regardless of how the surface is oriented the same amount of radiant energy crosses unit area in unit time. We consider only that radiant energy (photons) propagating in a hemisphere of directions either above or below the surface. The energy distribution function (or spectral distribution) is given by the *Planck distribution* (or *Planck function*)

$$P_e(\omega) = \frac{\hbar \omega^3}{4\pi^2 c^2} \frac{1}{\exp(\hbar \omega/k_B T) - 1}. \quad (1.11)$$

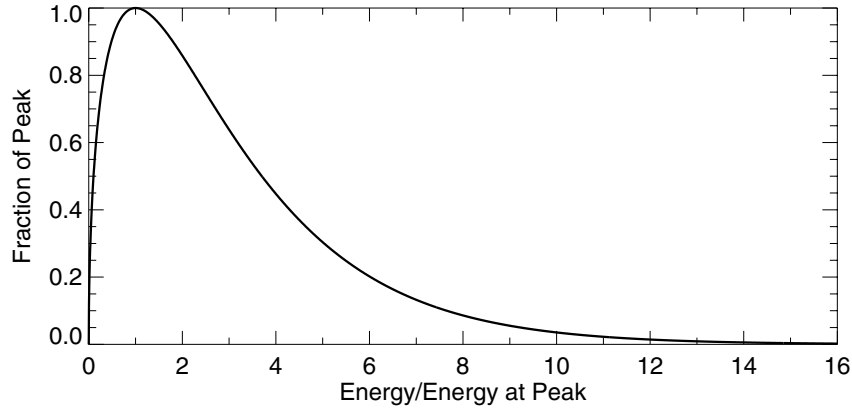


Figure 1.2: Distribution of kinetic energies of an ideal gas at equilibrium shown as a universal function independent of temperature. The kinetic energy relative to that at the peak of the distribution function, however, does depend on temperature.

The integral of this function over any frequency interval is the total radiant energy in that interval crossing unit area in unit time, called the *irradiance* (discussed in more detail in Sec. 4.2).

The Planck function is worthy of respect, if not awe, in that it contains not one, not two, but *three* fundamental (or at least believed to be so) constants of nature: the speed of light in a vacuum c , Planck's constant h , and Boltzmann's constant k_B . You can't get much more fundamental than that.

The most probable photon energy is obtained by setting the derivative of P_e with respect to ω equal to zero; the result is the transcendental equation

$$3(e^x - 1) = xe^x, \quad (1.12)$$

where $x = \hbar\omega/k_B T$, the solution to which (obtained quickly with a pocket calculator) is $x = 2.819$. Thus the most probable photon energy is

$$\hbar\omega_m = 2.819k_B T. \quad (1.13)$$

Note the similarity of Eq. (1.11) to Eq. (1.8) and Eq. (1.13) to Eq. (1.10), which is not surprising given that both are distribution functions for gases, although of a different kind. The most striking difference between a gas of molecules and a gas of photons is that the number of molecules in a sealed container is conserved (barring chemical reactions, of course) whereas the number of photons is not. As the temperature of the container, which is the source of the photons, increases, the number of photons within it increases. Photons are not subject to the same conservation laws as gas molecules, which are endowed with mass.

At frequencies for which $\hbar\omega \ll k_B T$ Eq. (1.11) can be approximated by

$$P_e(\omega) \approx \frac{k_B T \omega^2}{4\pi^2 c^2}. \quad (1.14)$$

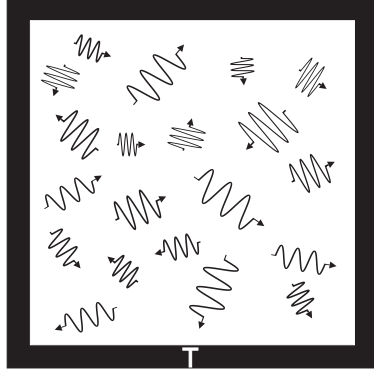


Figure 1.3: An opaque container at absolute temperature T encloses a gas of photons emitted by its walls. At equilibrium, the distribution of photon energies (Fig. 1.4) is determined solely by this temperature.

Folks interested in radiation of sufficiently low frequency (e.g., microwaves) sometimes express radiant power as a temperature. When first encountered this can be jarring until you realize that the Planck function is proportional to absolute temperature at such frequencies.

1.2.1 Change of Variable

We may express the Planck distribution as a function of frequency or wavelength. But in making a change of variables we have to be careful. The physical content of the Planck distribution is contained in its integral. According to the theorem for the change of variables in an integral

$$\int_{\omega_1}^{\omega_2} P_e(\omega) d\omega = \int_{\lambda_1}^{\lambda_2} P_e\{\omega(\lambda)\} \frac{d\omega}{d\lambda} d\lambda, \quad (1.15)$$

where $\omega(\lambda)$ is the transformation from circular frequency to wavelength and λ_j is the wavelength corresponding to ω_j . The derivative in the integral on the right side of this equation is called the *Jacobian* of the transformation. Equation (1.15) is *not* obtained by canceling the $d\lambda$ s, which is merely a way of remembering the theorem. The notation of calculus has evolved so as to make it easy to remember theorems, but notation should not cause us to forget that they all require proofs. No theorem can be proved by purely notational tricks.

According to Eq. (1.15) the Planck function expressed in wavelength terms is

$$P_e(\lambda) = P_e\{\omega(\lambda)\} \frac{d\omega}{d\lambda}, \quad (1.16)$$

where we use the same symbol P_e for both functions even though this is sloppy mathematics. The distinction between a function and its values is often blurred. We write

$$y = f(x) \quad (1.17)$$

to indicate that y is the value the function f assigns to x . Suppose that f is the function “square it”: $y = x^2$. If we transform from the variable x to $x = \sqrt{u}$, we obtain the new functional relation $y = u$. This is now a different function, and hence merits its own name (symbol). But to save having to invent more and more symbols, we are sloppy and write $y = f(x) = f\{x(u)\} = f(u)$, when we should write $y = f(x) = g(u)$. We often are even sloppier by confusing the value of the function with the function itself. That is, we write $y = y(x) = y\{x(u)\} = y(u)$. The fundamental rule of mathematical sloppiness is that you are allowed to be sloppy as long as you know how to do things correctly.

Although the Jacobian in Eq. (1.15)

$$\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2} \quad (1.18)$$

is negative, this does not mean that the radiant energy in the wavelength interval is negative. The upper limit on the right side of Eq. (1.15) is smaller than the lower limit, which by itself would make the integral negative, but the negative Jacobian makes the integral positive. So we write the Planck function as

$$P_e(\lambda) = P_e\{\omega(\lambda)\} \left| \frac{d\omega}{d\lambda} \right| = P_e\{\omega(\lambda)\} \frac{2\pi c}{\lambda^2} \quad (1.19)$$

and remember to reverse the limits of integration on the right side of Eq. (1.15). The Planck function expressed in wavelength terms is therefore

$$P_e(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}. \quad (1.20)$$

For $hc/\lambda k_B T \ll 1$, Eq. (1.20) is approximately

$$P_e(\lambda) \approx \frac{2\pi c k_B T}{\lambda^4}. \quad (1.21)$$

At temperatures around 300 K this equation is a good approximation (within about 1% or less) for wavelengths greater than about 250 μm . As we show in Section 8.1 the spectrum of skylight is approximately proportional to $1/\lambda^4$. As temperature increases without limit, therefore, the Planck function at visible wavelengths has approximately the same spectral dependence as the blue sky. So much for the notion that an exceedingly hot body is “white hot” or that blue is a “cold” color whereas red is a “warm” color.

The two forms of the Planck function presented here have the peculiar property that although the integral over any wavelength interval is equal to the integral over the corresponding frequency interval, the two functions do not peak at the same place. That is, if we find the frequency at which $P_e(\omega)$ is a maximum and transform that frequency into a wavelength, we do not obtain the wavelength at which $P_e(\lambda)$ is a maximum. To find this wavelength, differentiate Eq. (1.20) with respect to λ and set the result equal to zero. This yields the transcendental equation

$$5(e^x - 1) = xe^x, \quad (1.22)$$

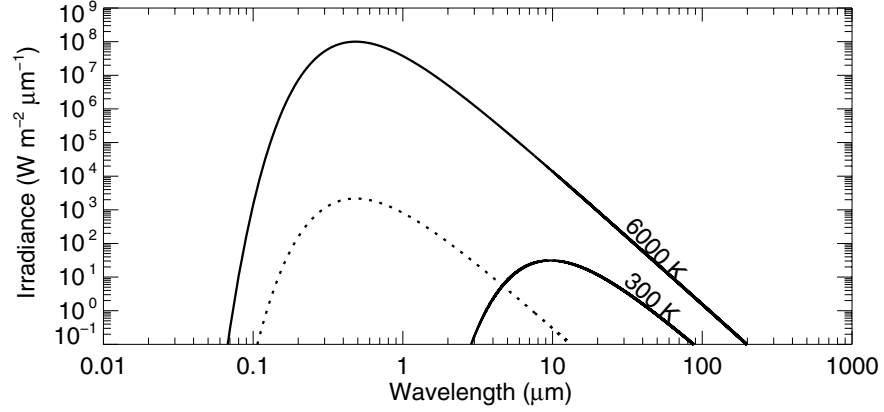


Figure 1.4: Planck function for 6000 K and 300 K. The dashed curve is the irradiance at the top of the atmosphere from a 6000 K blackbody at the Earth–sun distance, which approximates the solar irradiance.

where $x = hc/\lambda k_B T$, the solution to which is $x = 4.961$. From this we obtain *Wien’s displacement law* relating temperature to the wavelength λ_m at which $P_e(\lambda)$ is a maximum:

$$\lambda_m T = 2902 \mu\text{m K}. \quad (1.23)$$

For $T = 273 \text{ K}$ (0°C), $\lambda_m = 10.6 \mu\text{m}$. Equation (1.23) is called a displacement law because it determines how the Planck function is displaced as temperature increases. This displacement is evident in Fig. 1.4, which shows Eq. (1.20) for two temperatures, 6000 K and 300 K. Note also the huge difference in the amount of radiation emitted at these two temperatures.

But if we transform Eq. (1.13) into wavelength terms we obtain a different displacement law

$$\lambda_m T = 5107 \mu\text{m K}, \quad (1.24)$$

where λ_m is the wavelength corresponding to the frequency ω_m in Eq. (1.13). And this wavelength for 273 K is $18.7 \mu\text{m}$, quite a shift from $10.6 \mu\text{m}$. Which is correct? They both are. No law requires P_e to be plotted versus wavelength. This may be the custom in some fields, but not in others. Many spectroscopists plot spectra as a function of *wavenumber* (inverse wavelength, equivalent to frequency) and would consider doing otherwise an unnatural act. There is, in general, no invariant maximum for a distribution function. This may be unpalatable but it is a fact of life, in the nature of distribution functions. And yet this seems to be a difficult idea to get across. Once, after we had carefully discussed it in class, a student asked in all sincerity, “But where is the *real* maximum of the Planck function?” He thought we knew but were withholding it from the uninitiated, a secret to be revealed only on our deathbeds.

Failure to recognize that the maximum of a distribution function depends on how it is plotted has led and no doubt will continue to lead to errors. In a delightful paper Bernard Soffer

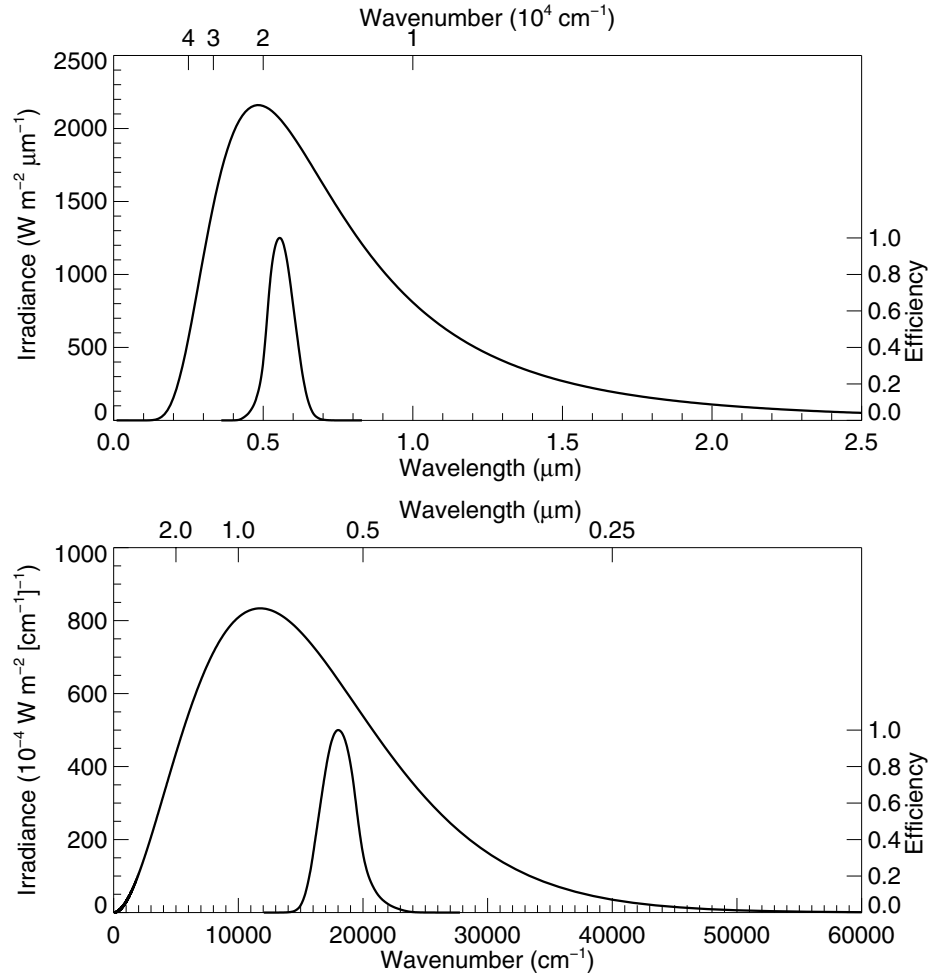


Figure 1.5: Luminous efficiency and Planck distribution at 6000 K (approximately the spectrum of solar radiation). The top figure shows the Planck distribution for wavelength as the independent variable; the bottom figure shows the Planck distribution for wavenumber (frequency). Note that although the peaks of the Planck distribution and luminous efficiency nearly coincide in the top figure, they are appreciably shifted in the bottom figure. These figures are similar to and were inspired by those presented by Soffer and Lynch (1999).

and David Lynch thoroughly demolish the widespread, but unsupportable, notion that the human eye is adapted to respond to the peak of the solar spectrum. If the *luminous efficiency* of the human eye (Sec. 4.1.7), the relative degree to which radiant energy of each visible wavelength is converted into the sensation of brightness, and the solar spectrum are plotted versus wavelength on the same figure, the peaks of the two curves roughly coincide. This is

shown in Fig. 1.5, the Planck function for a temperature of 6000 K, which is a good enough approximation to the solar spectrum to make the point, together with the luminous efficiency. This accidental coincidence of peaks has led countless biologists and vision scientists to leap to the unwarranted conclusion that the eye is optimized at the wavelength of the peak of the solar spectrum, a supposed triumph of evolution. We say “accidental” because if wavenumber (frequency) is taken as the independent variable (and why not?), the maxima of the two curves do not coincide, and by an appreciable amount. This is because the luminous efficiency is a *point function*, defined at each wavelength (or frequency), and hence its maximum doesn’t change when the independent variable is changed from wavelength to frequency or anything else. But this is not true of the solar spectrum, which, like the Planck function, is a spectral distribution function, its physical content specified by its integral. For its integral properties to be preserved, it has to change shape because equal frequency intervals do not correspond to equal wavelength intervals:

$$\omega_2 - \omega_1 = 2\pi c \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right). \quad (1.25)$$

Soffer and Lynch give other examples of errors resulting from the failure to distinguish between a point function and a distribution function. The moral of this story is that no reliable conclusions can be drawn, in general, from positions of the maxima of distribution functions.

1.2.2 Stefan–Boltzmann Law

The total radiant energy for the Planck function is obtained by integration over all frequencies:

$$\int_0^\infty \frac{\hbar \omega^3}{4\pi^2 c^2} \frac{1}{\exp(\hbar \omega / k_B T) - 1} d\omega. \quad (1.26)$$

Again, the limits of integration here are symbolic: no photon has zero or infinite energy. By transforming the variable of integration from ω to $x = \hbar \omega / k_B T$ this integral becomes

$$\frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{\exp(x) - 1} dx = \sigma T^4, \quad (1.27)$$

where

$$\sigma = \frac{k_B^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{\exp(x) - 1} dx = 5.669 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}. \quad (1.28)$$

This fourth-power law for total (spectrally integrated) irradiance is called the *Stefan–Boltzmann law* and σ is called the *Stefan–Boltzmann constant*.

1.3 Blackbody Radiation

Radiation in equilibrium with matter, as specified by the Planck function, often is called *blackbody radiation*. To understand the origins of this queer (as well as often confusing and

misleading) term requires postulating an idealized blackbody, which, like so many bodies in physics, does not exist. A blackbody cannot be excited to radiate, more than it would in isolation, by an external source of radiation of any frequency, direction, or state of polarization. The definition of a blackbody as one that absorbs all radiation incident on it contains a trap for the unwary. Notions about radiation being incident on bodies are valid only when they are much larger than the wavelength. We intuitively expect, based on our everyday experiences with objects large compared with visible wavelengths, that the radiant energy absorbed by an illuminated object is determined by its geometrical area. This expectation breaks down when the body is small compared with the wavelength, a restriction almost never mentioned although Planck recognized it clearly. On page 2 of his *Theory of Heat Radiation*, he says that he always assumes that the “linear dimensions of all parts of space considered, as well as the radii of curvature of all surfaces under consideration, are large compared with the wave lengths of the rays considered.” The concept of radiation incident on a body is from geometrical (or ray) optics, which is never strictly valid because all bodies are finite.

When one of the authors was interviewed many years ago for a position at Penn State he was asked if the *emissivity* (see following section) of a body can be greater than 1. He responded, “of course”, which may have saved a few lives (and possibly got him a job). At the time some meteorology graduate students were calculating emissivities of small particles at infrared wavelengths, and to their horror and dismay were obtaining values greater than 1. These students were almost suicidal because they had been brainwashed that emissivities cannot be greater than 1. Not so. Supposed upper limits on emissivities hold (approximately) only for objects much larger than the wavelength. In order to determine “all the radiation incident on” a body, what is meant by “incident” has to be well defined, but it is not. There are always departures from geometrical optics, although for sufficiently large bodies these departures may be negligible.

This seemingly heretical assertion about emissivities greater than 1, when cast in the language of antenna engineers, would be considered almost trivial. Any antenna engineer knows that the effective area of a receiver can be much larger than its geometrical area.

Although no strict blackbodies exist, some bodies are approximately black over a limited range of frequencies, directions, and polarization states of the exciting radiation.

Suppose that a hypothetical blackbody is placed inside an opaque cavity the walls of which are held at constant temperature. This blackbody is bathed in equilibrium radiation, which is isotropic (the same in all directions) and unpolarized (see Sec. 7.1). The rate at which radiant energy of all frequencies is absorbed by this blackbody is (subject to the restriction stated by Planck)

$$A \int_0^\infty P_e(\omega) d\omega, \quad (1.29)$$

where A is the total surface area of the blackbody. Let $\mathcal{E}_b(\omega)$ be the spectral emittance, the power emitted per unit frequency in a hemisphere of directions per unit area of the blackbody; the total emittance is the integral of the spectral emittance. If the blackbody is in thermal equilibrium (its temperature does not change), the total rates of absorption and emission must be equal:

$$\int_0^\infty P_e(\omega) d\omega = \int_0^\infty \mathcal{E}_b(\omega) d\omega. \quad (1.30)$$

Because of this equation we might be tempted to set the integrands equal to each other:

$$P_e(\omega) = \mathcal{E}_b(\omega). \quad (1.31)$$

Although Eq. (1.31) is *sufficient* to ensure that the blackbody is in thermal equilibrium, it is not *necessary* unless supplemented by an additional physical argument. According to the *principle of detailed balance*, Eq. (1.31) is both necessary and sufficient for thermal equilibrium. That is, at each frequency the rate of emission by the blackbody must be equal to the rate of absorption. Underlying this principle is a fundamental symmetry property of nature called *time-reversal symmetry*: the equations of the electromagnetic field have the same form when time is run backwards. To make this clearer consider a simpler example, the equation of motion of a point mass m acted on by a force \mathbf{F} at position \mathbf{x} :

$$m \frac{d^2 \mathbf{x}}{dt^2} = \mathbf{F}. \quad (1.32)$$

If \mathbf{F} depends only on the position of m , Eq. (1.32) is unchanged if we make the transformation $t \rightarrow -t$, the consequence of which is that if m follows a path to a point where its velocity is \mathbf{v} and then launched from this point with velocity $-\mathbf{v}$, m will retrace its original path.

In the context of the problem of interest here, the principle of detailed balance (time-reversal symmetry) yields the result that absorption and emission are inverse processes. That is, if time were to reverse, absorbed photons would become emitted photons and vice versa.

According to Eq. (1.31) the emission spectrum of a blackbody at temperature T is the Planck spectrum, the spectrum of radiation in equilibrium with matter. Hence such radiation is often called blackbody radiation even though its existence does not hinge on that of a blackbody. Defining equilibrium radiation by something that doesn't exist is unsatisfying and also deflects attention from its physical nature.

1.4 Absorptivity and Emissivity

Suppose that a real body is placed inside a cavity held at constant temperature. Such a body is uniformly illuminated by blackbody radiation. The spectral *absorptivity* $\alpha(\omega)$ is defined as that number which when multiplied by the irradiance $P_e(\omega)$ gives the rate of absorption per unit area of radiation per unit frequency by the body. Because blackbody (cavity) radiation is unpolarized and isotropic, α is the absorptivity for such radiation. Real bodies (large compared with the wavelength) are characterized by (dimensionless) absorptivities less than or equal to 1.

We define the spectral *emissivity* $\varepsilon(\omega)$ as that number which when multiplied by the Planck function gives the rate of emission by the real body in all hemispherical directions (again, per unit area and frequency). For the real body to be in thermal equilibrium inside the cavity requires that total absorption be balanced by total emission:

$$\int_0^\infty \alpha(\omega) P_e(\omega) d\omega = \int_0^\infty \varepsilon(\omega) P_e(\omega) d\omega. \quad (1.33)$$

Again, a sufficient condition for Eq. (1.33) to be satisfied is

$$\alpha(\omega) = \varepsilon(\omega), \quad (1.34)$$

and by the principle of detailed balance is also a necessary condition. Thus the spectral emissivity is equal to the spectral absorptivity, which again points to absorption and emission as inverse processes.

Alas, this simple and unambiguous equation has been translated, unnecessarily, by some textbook writers into a mantra, “a good absorber is a good emitter”, which is as useless as it is misleading. Whether something is good or bad cannot be determined until criteria for goodness or badness are specified. Steam pipes in power plants are wrapped with aluminum sheeting, and insulation for houses is coated with aluminum foil. Metals such as aluminum, especially when highly polished, have low spectral emissivities over the range of frequencies that encompass most of the integrated Planck function at typical terrestrial temperatures. To reduce the rate of radiative cooling of the contents of pipes (or of anything) they are wrapped with a “bad” emitter, which is “good.” Do you wrap hot sandwiches with aluminum foil? If so, you are making use of the “good” properties of a “bad” emitter (low emissivity).

Equation (1.34) often is called *Kirchhoff's law*, but is a restricted form of this law. In general, the absorptivity (and hence emissivity) of a body depends on the direction and state of polarization of the incident radiation as well as its frequency, caveats often omitted. We would not have to worry about this if all bodies were always illuminated by blackbody radiation (unpolarized and isotropic). But alas, this is not true, and real bodies when removed from cavities are illuminated by radiation that usually is not the same in all directions and may be partially polarized (see Sec. 7.1).

1.4.1 Blackbody Radiation without a Blackbody

Blackbody radiation is radiation in equilibrium with matter. We discuss this by way of a container (cavity) at a fixed temperature. You sometimes find textbook treatments of blackbody radiation in which it is stated (or implied) that the walls of the cavity must be black. This is not true. All that is required for a cavity to be filled with blackbody radiation is that the cavity be opaque and have a nonzero emissivity (and hence absorptivity) at all wavelengths. Thus if we fashion a cavity from a material that is neither black nor 100% reflecting at any wavelength, the radiation contained therein is still blackbody radiation. How can this be? How can blackbody radiation be obtained without a blackbody?

To answer this question, consider two opaque parallel plates, large compared with their separation (Fig. 1.6); the spectral emissivity of each plate is ε and both plates are at the same temperature. We want to determine the amount of radiation that crosses unit area (between the plates and parallel to them) in all upward directions. For simplicity we ignore that ε strictly depends on direction and polarization state.

The radiation between the plates is a consequence of radiation emitted *and* reflected (because they are not necessarily black) by them. Because the plates are opaque (no transmission by them) the *reflectivity* (fraction of incident radiation reflected) of a plate is $1 - \alpha$, which from Kirchhoff's law is $1 - \varepsilon$.

Consider first the lower plate. An amount of radiation ε times the Planck function is emitted by this plate. We may omit this function because it is a common factor and simply say that an amount of radiation (per unit frequency) ε is emitted upward by the lower plate. A fraction $1 - \varepsilon$ of this radiation is reflected downward by the upper plate, and then a fraction $1 - \varepsilon$ is reflected upward by the lower plate, which contributes to the upward radiation. Some

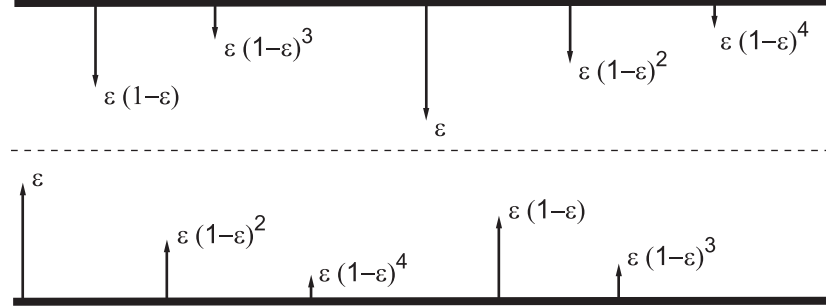


Figure 1.6: The radiation field between two identical opaque plates with emissivity ε is a consequence of emission and reflection by them. The total upward radiation field can be expressed as the sum of upward emitted radiation, downward emitted radiation reflected once, three times, and so on, as well as upward emitted radiation reflected twice, four times, and so on.

of this radiation is reflected twice again to contribute further to the upward radiation, and so on *ad infinitum*. The total contribution of the lower plate to the upward radiation is therefore an infinite series

$$\varepsilon + \varepsilon(1 - \varepsilon)^2 + \varepsilon(1 - \varepsilon)^4 + \dots \quad (1.35)$$

which can be written

$$\varepsilon(1 + x + x^2 + \dots), \quad (1.36)$$

where $x = (1 - \varepsilon)^2$. The series in parentheses pops up on so many diverse occasions that its sum is worth knowing.

Consider the finite sum

$$S_n = 1 + x + x^2 + \dots + x^n. \quad (1.37)$$

Multiply both sides of this equation by x and add 1 to the resulting equation:

$$xS_n + 1 = S_n + x^{n+1} \quad (1.38)$$

and hence

$$S_n = \frac{1 - x^{n+1}}{1 - x}. \quad (1.39)$$

The sum of the infinite series in Eq. (1.36) is the limit of Eq. (1.39) as n goes to infinity. If $|x| < 1$, the limit of x^{n+1} for infinite n is zero. With this restriction on x , the sum S of the infinite series is

$$S = 1 + x + x^2 + \dots = \frac{1}{1 - x}. \quad (1.40)$$

With this result, the contribution to the upward radiation from emission by the lower plate and multiple reflections between the two plates is

$$\frac{\varepsilon}{1 - (1 - \varepsilon)^2} = \frac{1}{2 - \varepsilon}. \quad (1.41)$$

But we are not finished. The upper plate contributes to the upward radiation indirectly by way of multiple reflections. An amount of radiation ε is emitted downward by the upper plate, and a fraction $1 - \varepsilon$ is reflected upward by the lower plate, thereby contributing to the upward radiation. A fraction of this radiation is reflected downward by the upper plate, then upward by the lower plate. And so on *ad infinitum*. We obtain another infinite series

$$\varepsilon(1 - \varepsilon) + \varepsilon(1 - \varepsilon)^3 + \dots \quad (1.42)$$

the sum of which is

$$\frac{1 - \varepsilon}{2 - \varepsilon}. \quad (1.43)$$

The total upward radiation is the sum of Eqs. (1.41) and (1.43):

$$\frac{1}{2 - \varepsilon} + \frac{1 - \varepsilon}{2 - \varepsilon} = 1. \quad (1.44)$$

Now multiply this by the Planck function to obtain the amount of radiation in the upward direction between the plates. As far as the radiation field is concerned, it is *as if* the emissivity of the lower (or upper) plate were 1. A cavity with walls that are not black nevertheless fills with blackbody radiation because of emission and multiple reflections. High reflectivity compensates for low emissivity. The only condition we imposed on the cavity is that it be opaque at the frequencies of interest.

Before moving on to averages of emissivities and absorptivities we critically discuss the concept of an average. The following section is applicable to many branches of science.

1.4.2 Averages: A Critical Look

One of the most misleading, if not outright dangerous, yet widespread concepts is that of the unqualified average of a set of values of some physical variable. Not only are there an infinite number of possible averages, not all of them, nor indeed any of them, may be of any use whatsoever. Consider a set of values x_j ($j = 1, 2, \dots, N$) of anything (temperature, pressure, height, weight, etc.). The *arithmetic average* (or *arithmetic mean*) of this set is defined as

$$\langle x \rangle = \frac{1}{N} \sum_{j=1}^N x_j. \quad (1.45)$$

This is one number that in some sense (a point we return to) represents the entire set of values. But a number that might be just as good at representing the set is the *median*, defined as that value of x such that 50% of the values in the set lie above it and 50% below. And then there

is the most probable value (or *mode*), defined as the value of x that occurs most frequently. So we already have before us three equally acceptable ways of characterizing a set of values by a single number. But we've hardly scratched the surface of the universe of averages. The arithmetic average is an equally weighted average, and there is no reason why we cannot weight values differently depending on some expectation of their relative importance. That is, we form the average

$$\sum_{j=1}^N w_j x_j, \quad (1.46)$$

where the weights satisfy

$$\sum_{j=1}^N w_j = 1. \quad (1.47)$$

For the arithmetic average, all the weights are $1/N$. The weights w_j could, but need not, be probabilities of occurrence of the values x_j . Because there is no end to weighting functions, there is no end to possible averages. But matters are even worse. A perfectly respectable (and sometimes useful) average is the *root-mean-square average*, the square root of

$$\sum_{j=1}^N w_j x_j^2. \quad (1.48)$$

But why stop at the square root of the average of squares? Why not take the n^{th} root of

$$\sum_{j=1}^N w_j x_j^n, \quad (1.49)$$

where n can be *any* number, not just an integer? This is a perfectly good average in the sense that it is one number that characterizes a distribution. We can go further still. Consider any function $f(x)$. Denote the inverse of this function as f^{-1} , defined by

$$f^{-1}\{f(x)\} = x. \quad (1.50)$$

For example, the inverse of the square is the square root; the inverse of the exponential function is the natural logarithm. Now we can generalize the two previous averages as

$$f^{-1}\left\{\sum_{j=1}^N w_j f(x_j)\right\}. \quad (1.51)$$

We can generalize this to a continuous distribution of values x :

$$f^{-1}\left\{\int w(x)f(x) dx\right\}, \quad (1.52)$$

where

$$\int w(x) dx = 1. \quad (1.53)$$

The limits of integration (not shown) are whatever we choose them to be. Given the indefinite number of possible functions f and w , the number of possible averages is boundless. Implicit in Eq. (1.52) is the requirement that the inverse function f^{-1} be single-valued, which is not true for every function. All of these averages are different, in general, and all of them tell different stories, have different uses, or may be useless if not outright dangerous.

First consider a dangerous average. Shoe sizes are distributed over a population. Not everyone wears the same size. Suppose that a shoe manufacturer were to determine the arithmetic average shoe size of a large pool of potential customers and then make shoes only of that size. A lot of customers would go barefoot. Indeed, depending on the distribution of shoe sizes all customers might go without shoes. Suppose, for example, that the population were composed of two groups, those with big feet and those with little feet. The (arithmetic) average shoe size would therefore lie in the intermediate range, and shoes of this size would fit no one. The shoe manufacturer would have lots of unhappy customers and would soon go broke. Here is an example in which one number characterizing a distribution is less than worthless. The shoe manufacturer needs to know the entire distribution of sizes, not just one number obtained from it.

If this example of shoe sizes seems far-fetched consider a region of the planet dominated by marine boundary layer stratus and cirrus. The distribution of cloud-base heights would be peaked around heights in the boundary layer and near the tropopause. The average of these cloud-base heights would produce a mean value far from both peaks in the distribution of heights.

Now let's turn to an average that does have a meaning, although a limited one. Cloud droplets are distributed in size. As we show in Chapter 3 the total power scattered by a cloud droplet depends on its size. Here we can calculate (in principle) an average total scattered power per droplet. The weighting function is the droplet-size probability distribution. The total power scattered by a fictitious cloud of droplets of identical size, all with the same particular average total scattered power per droplet, is the same as that by a real cloud with droplets distributed in size, where the total number of droplets is the same in both clouds. All well and good – but there's a catch (there always is). What size should be associated with the identical droplets in the fictitious cloud? We assumed that there is one and only one average droplet size that gives rise to the average total scattered power per droplet even though this is not always true (see Fig. 3.11). This is not the only problem. Absorption by cloud droplets is, in general, a *different* function of droplet size than scattering by droplets. This means that if we base our calculations of average size on the dependence of absorption on size, we end up, in general, with a different average. And that's not the end of our problems. The angular dependence of scattering is yet another function of droplet size. Although we may be able to determine average sizes that separately have physical meaning for (total) scattered power, absorption, and angular scattering, each one of these averages is different, and thus the question, What is *the* average size of a cloud droplet?, should be greeted with a horselaugh.

Global mean temperature is another example of a dubious concept and for more than one reason. As we have seen, infinitely many mean temperatures are possible, and each one is

different. But more important, a single number for an entire planet cannot possibly capture the consequences of temperature changes to human health, wealth, and happiness. As with shoe sizes, one needs the entire distribution, which in this context means everything related to weather: spatial and temporal distributions of temperature, rainfall amount and distribution in time and space, winds, duration, timing, and strength of storms – the list goes on and on. If you live in Minneapolis and were to choose your clothing every day on the basis of the global mean temperature you'd likely be uncomfortable most of the time (or possibly even perish).

1.4.3 Average Emissivity and Absorptivity

Kirchhoff's law was derived for a body illuminated by equilibrium radiation. This naturally leads to the question, Does this law still hold for arbitrary illumination? Although several years ago this question generated some controversy, it now seems safe to say that the dust has settled and we should not hesitate to apply Kirchhoff's law to bodies even when they are not illuminated by equilibrium (blackbody) radiation (see Sec. 2.8). But where we can get into trouble is by misapplying Kirchhoff's law to *averages*.

Total emission by a body at temperature T and with spectral emissivity $\varepsilon(\omega)$ is

$$\int \varepsilon(\omega) P_e(\omega; T) d\omega. \quad (1.54)$$

We can't do any damage to this integral by multiplying and dividing it by the same (nonzero) quantity, the integral of P_e , which from Eq. (1.27) is σT^4 . Thus we may write Eq. (1.54) as

$$\sigma T^4 \int_0^\infty \varepsilon(\omega) p(\omega; T) d\omega = \langle \varepsilon \rangle \sigma T^4, \quad (1.55)$$

where the (normalized) weighting function p is defined by

$$p(\omega; T) = \frac{P_e(\omega; T)}{\int_0^\infty P_e(\omega; T) d\omega} \quad (1.56)$$

and the average emissivity is

$$\langle \varepsilon \rangle = \int_0^\infty \varepsilon(\omega) p(\omega; T) d\omega. \quad (1.57)$$

The spectral emissivity, like the weighting function, could depend on temperature.

What about absorption? Total absorption by a body is the integrated product of its spectral absorptivity and the spectral distribution of the illumination $F(\omega)$:

$$\int_0^\infty \alpha(\omega) F(\omega) d\omega. \quad (1.58)$$

We may define an average absorptivity by

$$\langle \alpha \rangle = \frac{\int_0^\infty \alpha(\omega) F(\omega) d\omega}{\int_0^\infty F(\omega) d\omega}, \quad (1.59)$$

but this average absorptivity is not, in general, equal to the average emissivity even though $\alpha = \varepsilon$ because the weighting functions for the two averages are, in general, different.

If we assume that the average emissivity Eq. (1.57) is equal to the average absorptivity Eq. (1.59), we should do so knowing that this is not strictly true except for a body at temperature T illuminated by blackbody radiation for this temperature. Equality of the two averages may be a good approximation under some circumstances, such as if the spectral quantities are nearly independent of frequency over the range of frequencies for which the weighting functions have their greatest values.

1.4.4 Brightness and Color Temperature

Suppose we have an instrument that can measure radiant power over some range of frequencies anywhere in the electromagnetic spectrum. For simplicity we assume a narrow field of view for the instrument, but this is not necessary. If we were to point the instrument in a particular direction at a source of radiation, which could, but need not, be a measurably emitting body, the instrument would dutifully measure a radiant power. Now we can ask, What temperature must a blackbody have in order for the instrument reading to be the same? This temperature is called the *brightness temperature* of the source, not to be confused with the ordinary (or thermodynamic) temperature. Even if the radiation measured is mostly or entirely emitted (as opposed to reflected) by a body, its brightness temperature is not the same as its temperature unless we happen to choose a frequency range over which the emissivity of the body is almost 1.

To show that a brightness temperature always exists consider the integral of the Planck function Eq. (1.11) over any range of frequencies:

$$\frac{\hbar}{4\pi^2 c^2} \int_{\omega_1}^{\omega_2} \frac{\omega^3}{\exp(\hbar\omega/k_B T) - 1} d\omega. \quad (1.60)$$

This integral approaches 0 as $T \rightarrow 0$ and ∞ as $T \rightarrow \infty$, and its derivative with respect to T is always positive. Thus whatever our instrument reads, we can always find one and only one temperature such that Eq. (1.60) matches it. But keep in mind that this temperature depends on the frequency interval and possibly the direction (unless the source is isotropic). And if the instrument is equipped with a polarizing filter, and we were to rotate it, the brightness temperature might change (unless the source is unpolarized.)

Although the concept of brightness temperature is not restricted, *color temperature* is. The color temperature of a source of (necessarily) visible radiation is the temperature of a blackbody with the same perceived color. As we show in Section 4.3, the gamut of colors accessible to the human observer can be represented as a set of points in a two-dimensional space, whereas the possible colors of blackbodies lie on a curve in this space. Thus blackbodies of all temperatures can match only a small sample of possible colors. Nevertheless, color temperatures can be useful as long as we recognize their limitations. The color temperature of average daylight (sunlight plus skylight) is around 6500 K; that of an ordinary incandescent (tungsten) lamp around 3000 K. Color temperatures of skylight are 10,000–40,000 K. Perhaps it is fortunate that we can't touch the sky. If we could, we'd surely burn our fingers.

1.4.5 A Few Comments on Terminology

No single term, not misleading or faintly ludicrous, for radiation emitted by matter at typical terrestrial temperatures is adequate to distinguish it uniquely and unambiguously from solar radiation. Despite the title of Planck's book (the German title of which may not have the same connotations as its English translation), there is no such thing as "heat radiation" in the sense of a special kind of radiation emitted only by "heated bodies". According to the Planck function, radiation is emitted by *all* bodies regardless of temperature, although to varying degrees at different frequencies. Yves Le Grand calls radiant heat a "meaningless term", and adds, "to say that the sun, for instance, radiates heat is naïve". Moreover, there is no special range of frequencies such that radiation within this range and only within it is capable of raising the temperature of bodies that absorb it. A sufficiently intense source of radiation of almost any frequency can heat a body suitably chosen for its absorption properties at that frequency. For example, lasers at many different frequencies are commercially available for cutting and welding. And although the absorptivity of snow is close to 1 at wavelengths beyond a few micrometers (Fig. 5.16), a single crystal of ordinary salt is transparent from visible wavelengths far into the infrared (to around 60 μm). The heating power of a source of radiation is not uniquely determined by its spectrum but depends as much on its magnitude and the properties of what it illuminates.

Sometimes radiation from the sun is called *shortwave radiation*, whereas that from objects at terrestrial temperatures is called *longwave radiation*. If everyone understands exactly what is meant, no ambiguity arises, but these terms are inherently ambiguous because short and long depend on an arbitrary boundary. The solar spectrum, for example, can be divided into those wavelengths shorter than a certain wavelength and those longer.

If we call the radiation emitted by objects at typical terrestrial temperatures infrared radiation we face some problems. As we show in the following subsection, about half the radiation from the sun lies in the infrared (beyond the red). Moreover, the infrared is a spectral region extending more than three decades, from about 0.7 μm to 1000 μm . Prefixing the qualifiers near, middle, and far to infrared may not help because these terms mean different things to different people. Most important, if we call terrestrial radiation infrared radiation we hobble our thinking. Some people make their living measuring *microwave radiation* emitted by terrestrial objects, and other people make their living measuring microwave radiation from the sun. If we believed in the literal truth of assertions that terrestrial objects emit infrared radiation and that the solar spectrum peters out beyond 2.5 μm we would have to conclude that these people are either charlatans or self-deluded. But they are neither. They simply have risen to the never-ending challenge of measuring today what was not measurable yesterday.

The term *terrestrial radiation* can mislead people into thinking that this is radiation emitted only by solid bodies, ones that you can kick. And, believe it or not, we have stumbled upon conflicting assertions that only solids have emissivities or that only gases have them.

The term *thermal radiation* at least signifies that the emitted radiation is a consequence solely of a body's temperature, in contrast with, say, luminescence, which is more or less independent of temperature, or laser radiation, which has nothing fundamental to do with temperature. But radiation from the sun has just as much right to be called thermal radiation as does radiation emitted by snow, soils, rocks, and cabbages.

The only sure way to avoid confusion and error is to give the source and either a wavelength (or range of wavelengths) or some other clue that will leave no doubts as to what is meant. A good example of ambiguity is the unqualified term *infrared imagery*, which could mean imagery based on solar infrared radiation scattered by objects or on terrestrial infrared radiation emitted by them. Ambiguity vanishes when the source and wavelength are specified. We call radiation originating from the sun *solar* radiation, that from the Earth *terrestrial* radiation, and give the appropriate wavelengths if necessary.

1.4.6 The Solar Spectrum

In preceding paragraphs we criticized the various terms used to distinguish radiation emitted by the sun from that emitted by objects at typical terrestrial temperatures. But there is no disputing the existence of two distinct spectral regions, only what to call them. Figure 1.4 shows the spectral irradiance of a 6000 K blackbody, with radius equal to that of the sun, at the Earth–sun distance, which approximates the solar irradiance (see Fig. 1.7 and Prob. 1.27), together with that of a 300 K blackbody. There is little overlap between these two spectra. A distinction must be made between the solar irradiance and the irradiance of the sun. At the top of Earth’s atmosphere, about 1369 W of solar radiant energy (all wavelengths) crosses each square meter oriented with its normal toward the sun (strictly, this is an average because the Earth–sun distance varies slightly over a year). This quantity, denoted as S_0 , is now usually called the *solar irradiance*, formerly the *solar constant*, an inappropriate name given the interest in and effort devoted to measuring its changes. The irradiance of the sun, however, is that *at* the sun. Although the spectra of the two irradiances have the same shape, the magnitude of the solar irradiance is uniformly lower than the irradiance of the sun by the square of the ratio of the sun’s radius to the Earth–sun distance. Figure 1.4 also compares the (approximate) solar irradiance with the irradiance of a blackbody at a typical terrestrial temperature. These two spectral irradiances are well separated, whereas the (approximate) irradiance of the sun towers over that of the 300 K blackbody. Where the two curves intersect may be interpreted as the wavelength at which a photon is equally likely to be of solar as of terrestrial origin.

Solar radiation is not blackbody radiation because the sun is not enclosed in an opaque container, but the solar spectrum does approximate that of a blackbody. A detailed solar spectrum is shown in Fig. 1.7, on which the boundaries between ultraviolet, visible, and infrared are marked. About 51% of the solar irradiance lies in the infrared, 42% in the visible, and 7% in the ultraviolet. These numbers, easy to remember, are not exact if for no other reason than that the boundaries between these regions are somewhat arbitrary.

1.4.7 Imaging and Spectral Dependence of Contrast

A good rule of thumb is that most terrestrial objects (e.g., snow, soil, water, vegetation) are approximately black (have spectral emissivities near 1) over the range of frequencies that encompass much of the Planck function for typical terrestrial temperatures. Because of this, the brightness temperature (for infrared frequencies) of terrestrial objects is a good approximation to their thermodynamic temperature. This provides a means for determining temperatures remotely: measure the amount of radiation (in some frequency interval) emitted by an object and convert this radiation into a temperature by way of the Planck function. We can do even

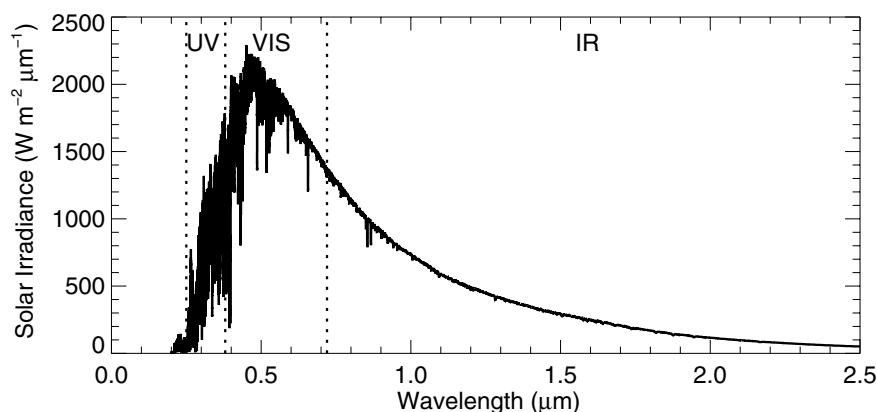


Figure 1.7: Solar spectrum (after Kurucz and Clough).

more. Temperature differences give rise to *contrast*, which allows for the possibility of imaging radiation we cannot see but that an instrument can.

The *image* of any object is a one-to-one transformation: every point on the object corresponds to one and only one point on the image. This transformation is the function of lenses in cameras, in slide projectors, and in your eyes. Remove the lens from a camera and light from objects still illuminates the film, but a single point on the film receives light from many object points. Remove the lens from a slide projector and it still projects light, but no image, onto a screen. A pinhole is the simplest kind of imaging device. One and only one line can be drawn from any point on an object, through the pinhole, to an image point.

Our eyes are imaging devices that respond to different amounts of radiation coming from different directions. We can get about in the world because of these (relative) differences, which is called *contrast*. A whiteout is the absence of contrast. You can experience whiteouts in blizzards or while descending through thick clouds in an airplane (see Sec. 5.2). If you have ever been in a whiteout you know that it can be frightening. You can't tell up from down, right from left. You are lost in a field of radiation the same in all directions.

Our eyes form images using visible radiation, the (indirect) source of which is the sun or lamps. That is, we usually image *scattered* light (we rarely look directly at the sun or at light bulbs). But imaging devices are not restricted to visible frequencies. The radiation emitted by terrestrial objects at different temperatures provides contrast between them, the greater the temperature difference, the greater the contrast. All imaging devices have a contrast threshold below which they cannot distinguish one object from an adjacent one.

Several years ago we got a telephone call from a very frustrated scientist. While poring over infrared images of sea ice he had noticed that the contrast was better in images at shorter wavelengths. He wanted to know why. So he called remote sensing experts and asked them. They all agreed that contrast was indeed better at the shorter infrared wavelengths. But that wasn't his question. He wanted to know *why*, but ran into a blank wall. No one disagreed with his observation but no one could explain it. And the more incomprehension he encountered,

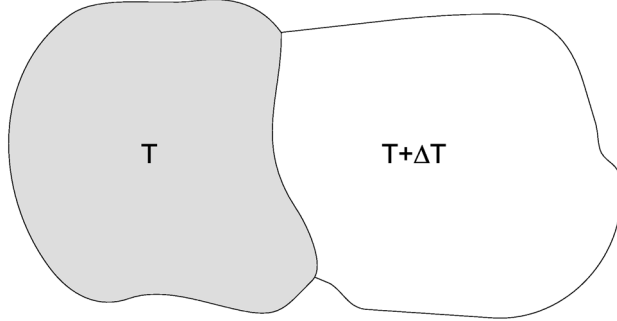


Figure 1.8: Two adjacent objects at different temperatures can be distinguished one from the other if the relative difference in the radiation emitted by them is sufficiently large. This contrast depends on the relative temperature difference $\Delta T/T$ and the chosen frequency of the radiation.

the more frustrated he became. The following is our answer to his question, which is a simple application of the Planck function and also illustrates the importance of beginning with fundamentals when faced with a problem.

Suppose that we have two objects, side by side, at different temperatures T and $T + \Delta T$ (Fig. 1.8). We may define the contrast between these two objects (assumed to be nearly black over the frequency range of interest) as

$$\frac{P_e(T + \Delta T) - P_e(T)}{P_e(T)}. \quad (1.61)$$

The first term in the numerator can be expanded in a Taylor series and truncated to yield an approximation for the contrast:

$$\frac{1}{P_e} \frac{\partial P_e}{\partial T} \Delta T \approx \frac{\Delta T}{T} \frac{x}{1 - e^{-x}}, \quad (1.62)$$

where $x = \hbar\omega/k_B T$. Not surprisingly, the contrast between the two objects depends on their relative temperature difference, but also on frequency by way of the quantity x . The contrast enhancement

$$\frac{x}{1 - e^{-x}} \quad (1.63)$$

is approximately 1 for small x and increases approximately linearly with increasing x . Thus, all else being equal (always an important caveat), greater contrast is obtained for higher frequencies (shorter wavelengths). At typical terrestrial temperatures (~ 300 K) $x \gg 1$ at wavelengths in the range $4 - 40 \mu\text{m}$ and so the contrast is approximately

$$\frac{\Delta T}{T} \left(\frac{\hbar\omega}{k_B T} \right) = \frac{\Delta T}{T} \left(\frac{hc}{\lambda k_B T} \right). \quad (1.64)$$

If inherent contrast were the *only* criterion for choosing one infrared frequency over another, the highest possible frequency would be the best choice. Of course, at sufficiently high frequencies there might not be enough emitted radiation to image.

1.5 Emission by Clouds

All else being equal, we expect lower air temperatures on mornings after clear nights than after cloudy nights. The correct explanation of this fairly common observation, however, is not widely known. What is widespread is an *incorrect* explanation: radiation emitted by the ground is reflected by clouds. Do clouds reflect radiation? Of course they do. Any fool can see that clouds are bright. And so they may be – at visible wavelengths, which is all that any fool can see. But because we cannot see the kind of radiation emitted by the ground at typical temperatures, we are on thin ice extrapolating from visible to infrared wavelengths. Metals (clean) such as silver and aluminum are highly reflecting at visible wavelengths. As it happens, these metals are just as highly reflecting at infrared, microwave, and even radio wavelengths. But what is true for silver and aluminum is not true for clouds. Although thick clouds may have high reflectivities for visible solar radiation, these same clouds are nearly black at the infrared wavelengths of radiation emitted by the ground (see Fig. 5.15). Our eyes deceive us about clouds. And not only clouds. For example, black and white (to our eyes) paints have nearly identical emissivities, as does black and white skin, over the Planck spectrum at typical terrestrial temperatures.

Suppose that some of the water vapor in clear air condenses into a cloud of water droplets; the total amount of water substance does not change. Suppose also that the temperature (assumed uniform) is the same for the clear as for the cloudy air. What has changed radiatively?

On a partly cloudy spring day we pointed an infrared thermometer at a patch of clear overhead sky. Although the air temperature measured with an ordinary thermometer was 20 °C, the temperature recorded by the infrared thermometer was a frigid –50 °C, not even close to air temperature. This is because an infrared thermometer measures brightness temperatures, which are lower or at most equal to terrestrial thermodynamic temperatures. When we shifted the thermometer's field of view from clear sky to an adjacent patch of cloudy sky the brightness temperature shot up to –3 °C. It is not plausible that two adjacent patches of sky differed in temperature by 47 °C, so the only possible explanation is that the emissivities of the two patches were different given that the reflectivity of clouds (Fig. 5.15) and air for terrestrial radiation is small. We have done this simple but dramatic experiment many times, always with the same result: clear sky is always radiatively much colder than adjacent cloudy sky. This difference lies mostly in the markedly different spectral emissivities of water vapor and of liquid water. Later that same day, after sundown, as clouds thickened, the overhead brightness temperature had increased to 2–3 °C even though air temperatures had dropped.

On a summer day with broken clouds we pointed an infrared thermometer at the zenith and measured the changing brightness temperature (Fig. 1.9); the state of the sky when the measurements were taken is shown in Fig. 1.10. As clouds flitted in and out of the thermometer's field of view, the brightness temperature fluctuated by about 40 °C. One of the *greguerías* of Ramón Gómez de la Serna is a charming way of looking at broken clouds: “*La curiosidad del cielo por ver la tierra abre muchas veces el nublado*” (The sky's curiosity to see the earth often opens clouds).

To further support the hypothesis that the difference between clear and cloudy skies lies in their emissivities, consider Figure 1.11. This figure shows the (calculated) *normal emissivity* over a range of infrared wavelengths (about 12–16 μm) of a uniform layer of moist air at a total pressure of 1 atmosphere, temperature 20 °C, and 1 cm of precipitable water. Normal

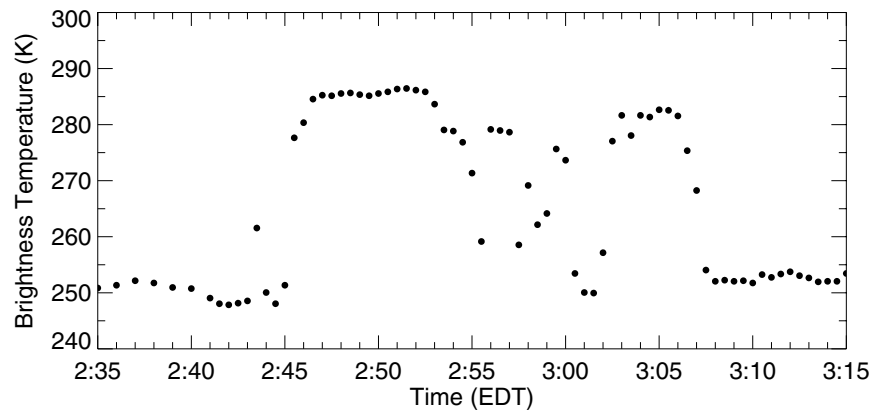


Figure 1.9: Brightness temperature of the zenith sky on a summer day with broken clouds. Measurements were made with the radiation thermometer pointed vertically. See Fig. 1.10 for an all-sky photograph of the cloud cover.

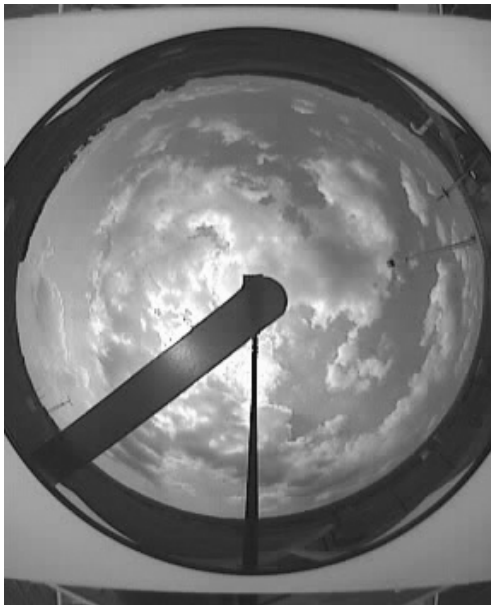


Figure 1.10: All-sky photograph of a partly cloudy sky on a summer day in State College, Pennsylvania. A time-series of the brightness temperatures of the zenith sky is shown in Fig. 1.9.

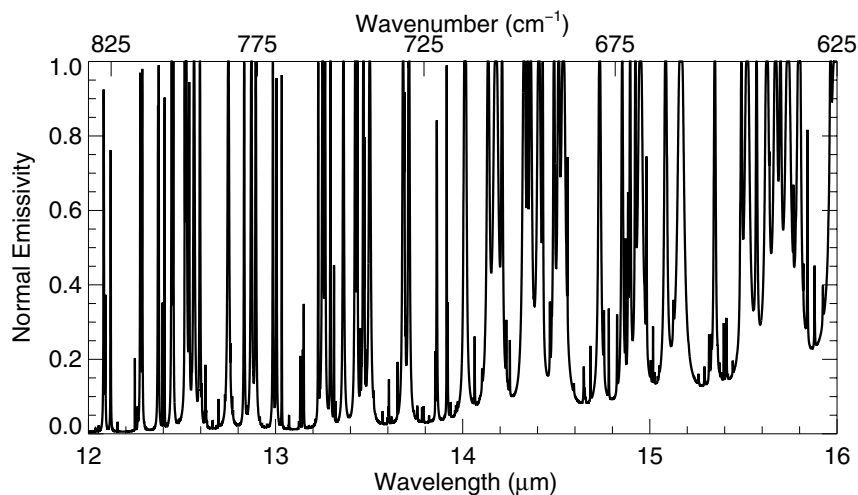


Figure 1.11: Normal emissivity of a uniform layer of moist air at 20 °C, total pressure of 1 atmosphere, and 1 cm of precipitable water. To resolve details only a fairly narrow range of wavelengths is shown; the emissivity spectrum is similar over the entire infrared spectrum.

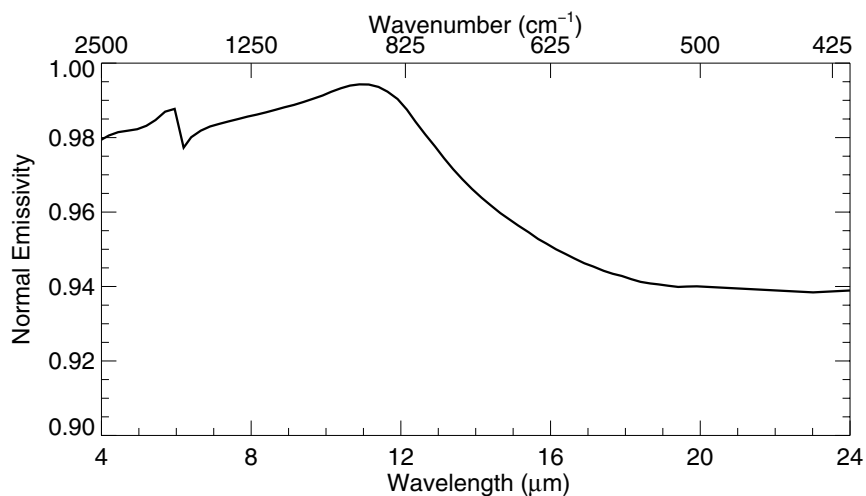


Figure 1.12: Normal emissivity of a layer of pure water sufficiently thick that very little incident radiation is transmitted by it.

emissivity is that in the direction perpendicular to the layer (remember, emissivity, in general, depends on direction). Precipitable water of 1 cm means that if all the water vapor were condensed the result would be a layer of liquid water 1 cm thick. The spectral emissivity of water vapor is a series of sharp peaks and deep valleys as a consequence of the *absorption*

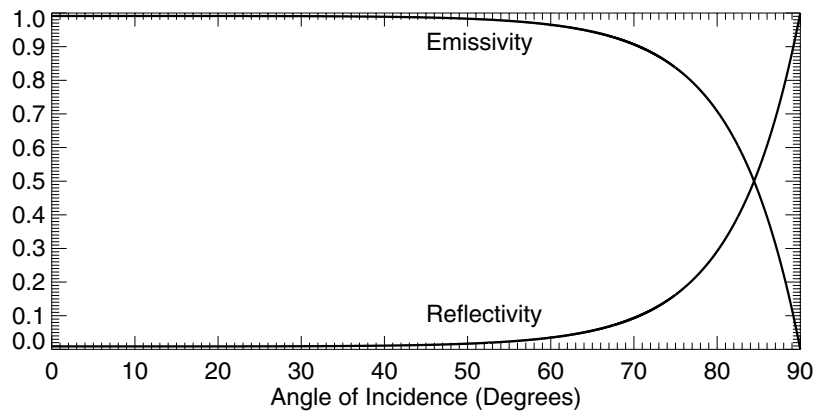


Figure 1.13: Reflectivity for incident unpolarized radiation ($\lambda = 10 \mu\text{m}$) and the corresponding emissivity for pure water.

lines of water vapor (see Secs. 2.6 and 2.7), narrow spectral regions over which absorption is high. At many wavelengths, the emissivity is close to 1, but at just as many other wavelengths it is smaller, and total emission depends on the *average* emissivity [see Eqs. (1.55) and (1.57)]. Contrast the normal emissivity of water vapor with that of liquid water for a layer sufficiently thick (less than a millimeter) that transmission by it is negligible (Fig. 1.12). This emissivity is a more or less smooth function of wavelength and departs only slightly from 1 over the range 4–24 μm . You don't have to do any calculations to recognize that the average emissivity of the liquid water layer is higher than that of the water vapor. And the emissivity of a thick cloud of water droplets is essentially the same as that of a liquid water layer (see Prob. 2.11). The rate of nocturnal cooling of the ground depends on the difference between its rate of emission and the rate at which it absorbs radiation emitted downward from the sky. The lower this net radiation, all else being equal, the greater the cooling rate. Because emission by clouds, all else being equal, is greater than emission by clear sky, radiative cooling is greater on cloudless nights.

1.5.1 Directional Emissivity

Emissivity depends on the direction of emission (see Sec. 1.4), which is why we qualified the emissivities of interest in the preceding discussion as *normal* emissivities. For an opaque body, the absorptivity for radiation in a particular direction is 1 minus the reflectivity, which by Kirchhoff's law is the emissivity. Because reflectivity depends on direction, so does emissivity. Figure 1.13 shows reflectivity of pure water versus angle of incidence for unpolarized incident radiation ($\lambda = 10 \mu\text{m}$) and the corresponding emissivity calculated using the Fresnel coefficients (Sec. 7.2). At this wavelength, a layer of water only a few millimeters thick is opaque. If emissivity depends on direction, so must brightness temperature.

The brightness temperature T_b in a direction ϑ is defined by

$$\int \varepsilon(\omega, \vartheta) P_e(\omega, T) d\omega + \int \{1 - \varepsilon(\omega, \vartheta)\} P_e(\omega, T_s) d\omega = \int P_e(\omega, T_b) d\omega, \quad (1.65)$$

where the range of integration is determined by the instrument, $\varepsilon(\omega, \vartheta)$ is the spectral emissivity in the direction ϑ , T is the (thermodynamic) temperature, and T_s is the brightness temperature of the sky (which also depends on direction; see Sec. 2.2). The first integral on the left side of Eq. (1.65) is emission; the second integral is reflection of radiation from the sky. As evidenced by the weak dependence of the (normal) emissivity of water on frequency over a wide range (Fig. 1.12), we can ignore the frequency dependence of the directional emissivity in Eq. (1.65). Because our infrared thermometer responds to a narrow range (about $2 \mu\text{m}$) of wavelengths around $10 \mu\text{m}$, we can approximate Eq. (1.65) by

$$\varepsilon(\vartheta) P_e(\omega, T) \approx P_e(\omega, T_b), \quad (1.66)$$

where the frequency corresponds to $10 \mu\text{m}$. We also assume that reflection is negligible compared with emission, which is not true for near-glancing angles. At this wavelength, and for T around 300 K, the exponential term in the Planck function Eq. (1.11) is much greater than 1, and hence we can approximate Eq. (1.66) by

$$\varepsilon \exp(\hbar\omega/k_B T_b) \approx \exp(\hbar\omega/k_B T) \quad (1.67)$$

Take the natural logarithm of both sides and rearrange terms to obtain

$$\frac{\hbar\omega}{k_B T_b} \approx \frac{\hbar\omega}{k_B T} - \ln \varepsilon. \quad (1.68)$$

If we write $T_b = T - \Delta T$ and assume that $\Delta T/T \ll 1$, we can further approximate Eq. (1.68) as

$$\frac{T - T_b}{T} \approx -\frac{k_B T}{\hbar\omega} \ln \varepsilon. \quad (1.69)$$

Thus we predict that at a fixed temperature T , the relative difference between thermodynamic temperature T and brightness temperature T_b for any direction is a linear function of the negative of the natural log of the emissivity in that direction.

To verify the correctness of Eq. (1.69) we measured the brightness temperature of water in a $30 \text{ cm} \times 40 \text{ cm}$ pan about 2 cm deep at four elevation angles (60° , 40° , 25° , and 16°). Because an infrared thermometer cannot distinguish emitted from reflected radiation, the water was heated to maximize emission, and measurements were made on a cool, clear day to minimize incident radiation from the sky. The emissivity was obtained from Fig. 1.13. In Eq. (1.69) T is the temperature of the *surface* of the water, which, because of evaporative cooling, is not what one measures by immersing an ordinary thermometer into the water. As a good estimate for T we used the (average) brightness temperature at 60° ; the emissivity is close to 1 for this direction (Fig. 1.13). Emission from the overhead sky corresponded to a brightness temperature less than about 218 K on the day measurements were made (see Sec. 2.2 for more on the variation of brightness temperature of the sky with direction). The

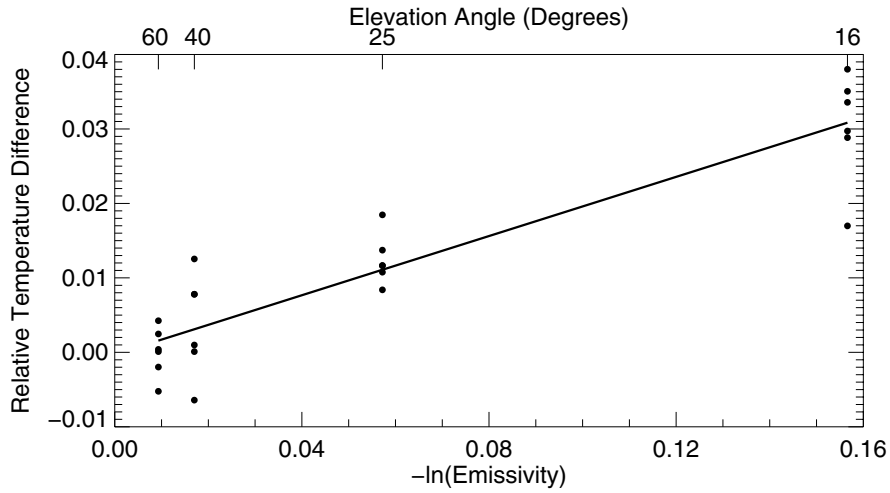


Figure 1.14: Measured differences between thermodynamic temperature and brightness temperature relative to thermodynamic temperature for pure water at four elevation angles with six measurements per angle. The straight line is a linear least-squares fit to the measurements. The chi-square goodness-of-fit statistic is 0.00065 implying a correlation coefficient r^2 of 0.83.

reflectivity of water for radiation from the overhead sky is small (~ 0.01). We made six sets of measurements for the four directions. The average surface temperature of the water was about 338 K, for which the slope in Eq. (1.68) is about 0.22. Figure 1.14 shows a least-squares fit to all the measurements, with slope 0.22, in agreement with the predicted slope even given all the approximations underlying it and the difficulty of making the measurements.

We have more to say about directional emissivity in Section 2.2.

1.6 Emissivity and Global Warming

One cannot open a newspaper or magazine these days without encountering warnings about impending global warming because of the ill-named greenhouse effect. As a consequence of this extraordinary publicity, explanations of the greenhouse effect have been designed more with journalists and politicians in mind than scientists. Even those who publish more frequently in scientific journals than in the popular press either assert that the greenhouse effect is the result of “closing the atmospheric window”, thereby “trapping” radiation, or that it is the result of increased emission from the atmosphere. When adherents to both explanations clash, the result is indeed warming, although local rather than global.

If we set aside criticism of the term greenhouse effect (the function of greenhouses is primarily to suppress convection by enclosing a space heated by solar radiation), we are still left with the task of trying to square two apparently irreconcilable physical explanations.

To do so we turn to a simple example of radiative equilibrium, that for a system consisting of two uniform slabs, one above the other (Fig. 1.15). The absolute temperature of the top slab

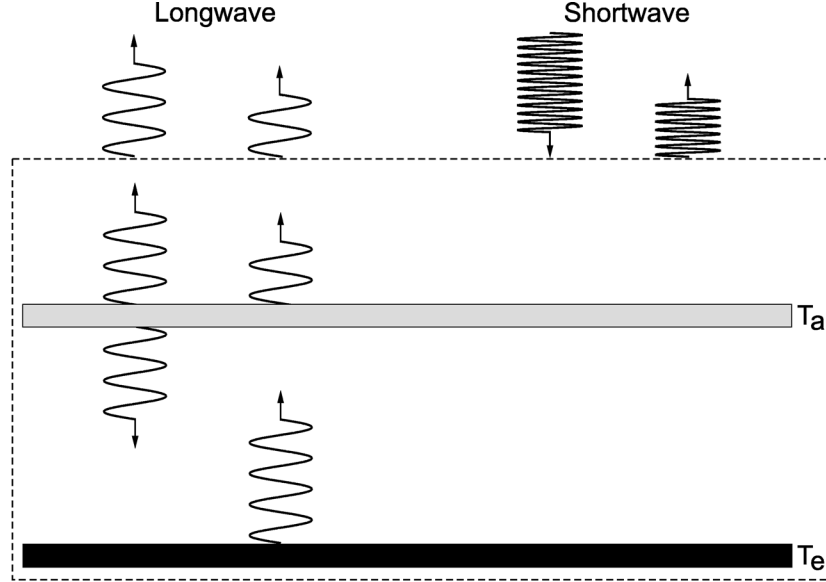


Figure 1.15: Radiative equilibrium between two uniform slabs, infinite in lateral extent. The top slab is transparent to incident shortwave radiation, whereas both slabs emit and absorb longwave radiation. The bottom slab is black to longwave radiation.

is T_a ; that of the bottom slab is T_e . The absorptivity of the top slab averaged over the spectrum of the radiation emitted by the bottom slab (assumed to be a blackbody) is α . We assume that the emissivity ε of the top slab (averaged over its emission spectrum) is equal to α , which is not strictly true unless the two temperatures are equal or the emissivity and absorptivity are independent of frequency. We also assume that the top slab does not reflect radiation from the bottom slab, and hence the transmissivity τ (fraction of incident radiation transmitted) of the top slab is $1 - \alpha = 1 - \varepsilon$. The top slab is transparent to radiation from a source outside the system illuminating it from above, where S is the net *irradiance* (net radiant energy crossing unit area in unit time) of the source. The source spectrum hardly overlaps the emission spectra of the slabs.

Consider first the two slabs taken as a single system. If it is in radiative equilibrium, the net radiant energy input to it must be balanced by the radiation emitted by the bottom slab and transmitted by the top slab plus the radiation emitted by the top slab:

$$S = \sigma T_e^4(1 - \varepsilon) + \varepsilon \sigma T_a^4. \quad (1.70)$$

A radiative energy balance applied to the top slab yields

$$2\varepsilon \sigma T_a^4 = \alpha \sigma T_e^4 = \varepsilon \sigma T_e^4. \quad (1.71)$$

The term on the left is total emission by the top slab, which emits both upward and downward (hence the factor 2); the term on the right is absorption by the top slab of radiation emitted by the bottom slab.

We can solve these two equations to obtain

$$T_e^4 = \frac{S}{\sigma(1 - \varepsilon/2)} \quad (1.72)$$

for the radiative equilibrium temperature of the bottom slab. There are no other modes of energy transfer; the slabs are suspended in a vacuum.

Because of the assumption $\alpha = \varepsilon = 1 - \tau$, we can rewrite Eq. (1.72) as

$$T_e^4 = \frac{2S}{\sigma(1 + \tau)}. \quad (1.73)$$

Note that the temperature of the slabs is a consequence of an external source, but for fixed S the equilibrium temperature depends on the radiative properties (ε or, equivalently, τ) of the top slab.

What does this all have to do with the atmosphere, specifically global warming? Take the top slab to represent the atmosphere, the bottom slab to represent the ground. The total amount of solar radiant energy intercepted by Earth is $S_0 \pi R_e^2$, where R_e is its radius. The total surface area, however, is $4\pi R_e^2$, and so the solar irradiance spread uniformly over the entire globe is $(1369/4) \text{ W m}^{-2}$. But a fraction of this radiant energy is reflected, as evident from satellite photos, which (at visible wavelengths) show a mostly dark Earth (the oceans) brightened here and there by clouds. An estimate for the total fraction of reflected solar radiation, the planetary *albedo* a , is 0.30 (30%). This yields a net solar irradiance $S = S_0(1 - a)/4 = 240 \text{ W m}^{-2}$ (albedo is more or less synonymous with reflectivity, although albedo is usually applied to radiation reflected in a hemisphere of directions whereas reflectivity may apply to radiation reflected in a particular direction or in a hemisphere of directions). This is the net solar radiant energy input to Earth spread out evenly over the entire planet. If we take this value for S in Eq. (1.72) we obtain different temperatures T_e depending on ε . For $\varepsilon = 1$, $T_e = 303 \text{ K}$ (30°C), whereas for $\varepsilon = 0$, $T_e = 255 \text{ K}$ (-18°C). These temperatures are at least in line with typical air temperatures near Earth's surface, neither ridiculously higher nor lower.

Although nitrogen and oxygen are the numerically dominant atmospheric gases, they are not radiatively dominant. The contribution to the emissivity (over the Planck spectrum for typical terrestrial temperatures) of Earth's atmosphere is mostly from comparatively small amounts of certain *infrared-active* gases (see Ch. 2), water vapor being by far the most abundant, although still less than about 1% of the atmosphere. Carbon dioxide is another infrared-active gas, with an abundance of around 360 ppm. This gas would be present in the atmosphere even if Earth were devoid of human inhabitants, but has been increasing since the beginning of the Industrial Revolution, presumably because of increased burning of fossil fuels.

The emissivity of the present atmosphere is, say, 0.8, which if substituted in Eq. (1.72) with $S = 240 \text{ W m}^{-2}$ yields $T_e = 289 \text{ K}$ (16°C). We note that this is *not* the global average temperature. In the first place, there is no such thing as *the* average temperature (or *the* average anything) but rather infinitely many possible averages depending on the function of temperature averaged and how it is weighted. And what exactly does T_e correspond to? Is it the temperature of the ground? If so, this is not the air temperature, which varies with height, especially near the ground. All we can say for certain is that $T_e = 289 \text{ K}$ is the radiative equilibrium temperature of a slab that absorbs about 240 W m^{-2} of radiant energy, above which

is another slab with $\varepsilon = 0.8$ but which does not absorb any solar radiation. In addition, the lower slab is black to the radiation emitted by the upper slab, and radiation is the *only* form of energy transfer (this is what is meant by *radiative equilibrium*).

It would be a bit of a stretch to say that the simple system consisting of two slabs shown in Fig. 1.15 is a model of the atmosphere. It is at best an analogue, useful for helping us understand some basic physics, possibly to frame testable hypotheses, even to estimate relative changes if used judiciously. For example, if ε of the atmosphere is increasing because of increased amounts of infrared-active gases, this suggests that temperatures in the lower atmosphere could increase.

Equation (1.73) is the basis for interpreting global warming as the result of “closing the window”. As the transmissivity of the (analogue) atmosphere decreases, the radiative equilibrium temperature T_e increases. Equation (1.72) is the basis for interpreting global warming as the result of increased emission. As the emissivity increases, so does the radiative equilibrium temperature. Which interpretation is correct? One interpretation cannot be right and the other wrong if they are based on the same theory. The two interpretations are merely two different ways of saying the same thing.

Although one interpretation cannot be right and the other wrong, one may be less misleading, more felicitous than the other. We prefer the increased emission interpretation for a few reasons. According to this interpretation we are warmed at the surface of Earth by *two* sources of radiation: the sun and the atmosphere. With this interpretation the atmosphere is actually doing something (emitting) whereas according to the other interpretation it only prevents something from happening. Moreover, the notion that the atmosphere traps radiation is at best a bad metaphor, at worst downright silly. In the emission interpretation the atmosphere is a source of radiation, not a photon trap that corrals wayward photons and sends them back to Earth just as a truant officer returns wayward children to school. A truant officer can return children to school because they are distinguishable, whereas photons are not. If this doesn’t bother you, what about the fact that the spectrum of the radiation emitted by the ground to the atmosphere is not the same as the spectrum of radiation emitted to the ground by the atmosphere?

To further bolster the emission interpretation, consider the following thought experiment. Quickly paint the entire globe with a highly conducting metallic paint, thereby reducing the emissivity of the surface to near zero. Because the surface no longer emits radiation, none can be “trapped” by the atmosphere. Yet the atmosphere keeps radiating as before, oblivious to the absence of radiation from the surface (at least initially; as the temperature of the atmosphere drops, its emission rate drops). Of course, if the surface doesn’t emit radiation but continues to absorb solar radiation, the surface temperature rises and no equilibrium is possible until the emission spectrum shifts to regions for which the emissivity is not zero.

A more physically relevant quantity than temperature is the downward emission of radiation, which from Eqs. (1.71) and (1.72) is

$$F_{\downarrow} = \frac{\varepsilon S}{2 - \varepsilon}. \quad (1.74)$$

This equation is Earth’s radiation budget in a nutshell. It tells us that emission from the atmosphere to the surface is a consequence of a radiant energy *transformation*. Solar radiation (S) is transformed, because of absorption by the surface, into longer wavelength radiation

F_{\downarrow} from the atmosphere. But this transformation cannot occur unless the atmosphere has a nonzero emissivity. As ε increases, the downward radiation increases, and the rate of increase is greatest at $\varepsilon = 1$. S is the *net* incoming solar radiation to the planet, which is affected by the radiative properties of clouds and their spatial extent as well as the output of the sun and the Earth–sun distance. Although the slabs in our analogy were uniform, the atmosphere is not. The sky can be cloudy or clear or both at the same time, and we have seen that the emissivity of clouds is greater than that of clear air. This points to the opposing roles of clouds in reducing F_{\downarrow} because of greater reflection (reduced S) and increasing it because of greater emission.

The total amount of radiant energy of all wavelengths absorbed by the bottom (surface) slab is

$$F_{\downarrow} + S = \frac{S}{1 - \varepsilon/2}, \quad (1.75)$$

which lies between S ($\varepsilon = 0$) and $2S$ ($\varepsilon = 1$).

An average (or better yet, effective) temperature that does have an unambiguous physical meaning is the effective radiative equilibrium temperature T_{eff} of Earth defined as the temperature of a blackbody with a total emission equal to the net solar radiation received by Earth averaged over its entire surface:

$$\sigma T_{\text{eff}}^4 = S = \frac{S_0}{4}(1 - a). \quad (1.76)$$

For $S = 240 \text{ W m}^{-2}$, $T_{\text{eff}} = 255 \text{ K}$. This is the equivalent blackbody temperature an observer on the moon would infer for Earth looked upon as an infrared sun. Just as we on Earth say that the sun is equivalent to a 6000 K blackbody (based on the solar irradiance), an observer on the moon would say that Earth is equivalent to a 255 K blackbody (based on the terrestrial irradiance). Note that the effective temperature defined by Eq. (1.76) in no (direct) way depends on the emissive properties of Earth's atmosphere.

In general, energy (or power) is a more relevant physical quantity than temperature. Energies are additive, temperatures are not; energy is conserved, temperature is not. Energy fluxes drive atmospheric processes. But W m^{-2} is banned from American newspapers, both because it is an SI quantity and because it is much too scientific for readers of even the most pretentious newspapers in the land. Similarly, instead of energy fluxes we get the wind-chill temperature, which obscures the fact that energy fluxes, not temperatures, kill people by hypothermia. Canadian meteorologists at one time, perhaps still, gave energy fluxes (rates of energy transfer from bodies), and we recall once seeing a value of 1000 W m^{-2} during the winter, a number that makes you shiver if you understand energy fluxes.

The total amount of radiation emitted by the atmosphere over a given time and a given area is a definite number. It may be difficult to measure but it is not an average.

Equation (1.74) contains a trap, not a radiation trap but one for the unwary, and it has caught some big fish. At first glance it would seem that the worst that could happen would be for ε to reach 1, and hence the maximum irradiance from the atmosphere to the surface would be pegged at S . The addition of any more infrared-active gases would have no effect. This dubious conclusion illustrates the perils of pushing an analogy too far. We made a simplifying assumption likely to become invalid as the concentration of infrared-active gases increases

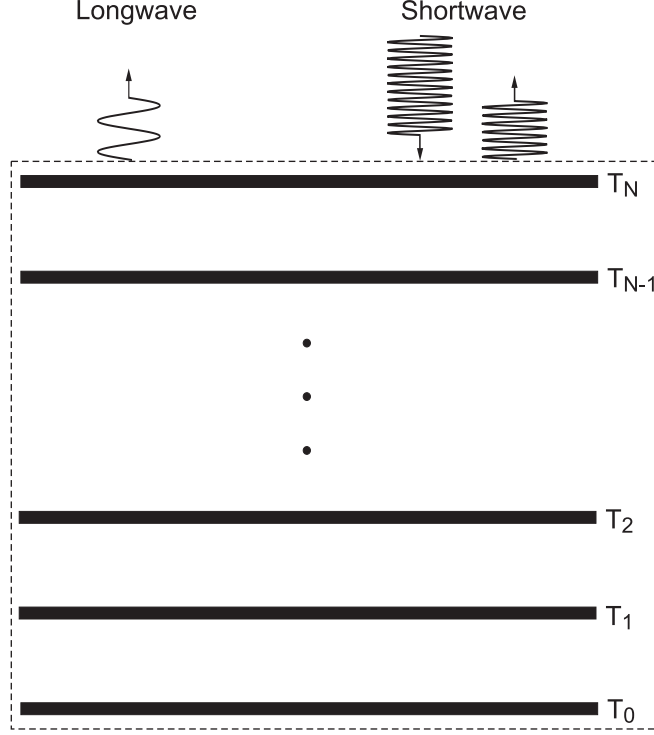


Figure 1.16: A set of N slabs, infinite in lateral extent, each of which is black to longwave radiation. All the slabs except the bottom one are transparent to incident shortwave radiation.

without limit, namely, that the atmosphere can be represented as a single slab (temperature). Let's see how matters change if we consider a set of N slabs, each of which is black to longwave radiation (Fig. 1.16). As before all slabs are transparent to shortwave radiation except the bottom one. Radiative energy balances for the entire system, then for each slab in succession from the top downward yield

$$S = \sigma T_N^4, \quad 2\sigma T_N^4 = \sigma T_{N-1}^4, \quad 2\sigma T_{N-1}^4 = \sigma T_N^4 + \sigma T_{N-2}^4, \quad \dots \quad (1.77)$$

from which it follows that

$$\sigma T_{N-j}^4 = (j+1)S \quad (j = 0, 1, \dots, N). \quad (1.78)$$

The downward radiation to the bottom slab (surface)

$$F_{\downarrow} = NS \quad (1.79)$$

therefore increases without limit as N increases. Of course, this simple analogy would at some point break down, but it does show that downward radiation to the surface could increase with increasing concentration of infrared-active gases, accompanied by higher atmospheric (tropospheric) temperatures.

Does the atmosphere “act like a blanket”? No, not really. Blankets, like almost all insulation, suppress convection. This is why so many insulating materials bear a family resemblance. What do wool, down, cork, felt, hair, glass wool, foam, earth, snow, etc. have in common? They all are porous materials. When subjected to temperature differences, air moves, therefore transporting energy. But the air in pores, if they are sufficiently small, doesn’t move much. So the function of blankets is to suppress the movement of air by enclosing it in small pores. If you want a blanket to also reduce net radiation, wrap the blanket with aluminum foil, which has a low emissivity. This is why the insulation in houses or around pipes is coated with foil. But note that this is just the opposite of what happens in the atmosphere. As the emissivity of the atmosphere increases, we expect downward radiation from it to increase (all else being equal). This is yet another reason why assertions about the “atmosphere acting like a blanket” are absurd.

Now a parting shot about the term “greenhouse effect”. The only difference between a greenhouse and an ordinary house is where their furnaces are located: outside (the sun) for a greenhouse, inside for an ordinary house. This is why the walls of a greenhouse are (partly) transparent to solar radiation. These same walls are more or less opaque to the infrared radiation emitted by the interior. But so are the walls of ordinary houses, and no one says that they “trap” radiation. You can demonstrate for yourself just exactly what houses do by opening all the doors and windows of your house on a cold and blustery winter day. The walls “trap” just as much radiation as when the house was shut tightly, but now the furnace is heating the great out of doors.

References and Suggestions for Further Reading

The quotation in the opening paragraph is from Hans Vaihinger, 1935: *The Philosophy of ‘As if’*, 2nd ed. Barnes & Noble, p. xlii.

We owe the insight about the duality of sound to Fritz Bopp, 1957: The principles of the statistical equations of motion in quantum theory, in *Observation and Interpretation in the Philosophy of Physics*, S. Körner, Ed., Dover, p. 190.

For the latest experimental limits on the photon mass see Jun Luo, Liang-Cheng Tu, Zhong-Kun Hu, and En-Jie Luan, 2003: New experimental limits on the photon rest mass with a rotating torsion balance. *Physical Review Letters*, Vol. 90, pp. 081801 (1–4). The authors cite a string of papers, published during the past 33 years, reporting ever-decreasing upper limits on the photon mass. If it were known to be identically zero, many physicists have been wasting their time and the taxpayers’ money.

For an English translation of Einstein’s 1905 paper giving a theoretical interpretation of the photoelectric effect (the official reason for his 1921 Nobel prize) see the collection edited by Henry A. Boorse and Lloyd Motz, 1966: *The World of the Atom*, Vol. I, Basic Books, pp. 544–57, an invaluable two-volume collection of and commentaries on the most important papers on atomism from antiquity to the age of quarks and other strange particles.

Despite its age, the treatise by Arthur Llewelyn Hughes and Lee Alvin Dubridge, 1932: *Photoelectric Phenomena*, McGraw-Hill is still worth reading, especially the first two chapters on the history of Eq. (1.4) and its experimental verification, which required considerable effort and ingenuity. The authors assert (p. 5) that there “are but few more interesting developments in all of physics than the growth of the surface photoelectric effect from an obscure physical phenomenon to one of profound theoretical significance, and, finally, to one of surpassing commercial importance.”

Heinrich Hertz is usually credited with the (1887) experimental discovery of the (surface) photoelectric effect, although it is much less well known that he invented the thermodynamic diagrams used by meteorologists (Elizabeth Garber, 1976: *Thermodynamics and meteorology. Annals of Science*, Vol. 33, pp. 51–65.)

The history of attempts to interpret the photoelectric effect without photons is recounted by Roger H. Stuewer, 1970: Non-Einsteinian interpretations of the photoelectric effect. *Minnesota Studies in the Philosophy of Science*, Vol. 5. *Historical and Philosophical Perspectives of Science*, Roger H. Stuewer, Ed., pp. 246–63, University of Minnesota Press. Section V is a refutation of the oft-repeated mantra that it is impossible to understand the photoelectric effect without invoking photons.

Arguments for the greater usefulness of the wave language for light are in a letter by M. Psi-mopolous and T. Theocharis, 1986: “... To see it as it is. ... to know it as it isn’t...” *American Journal of Physics*, Vol. 54, p. 969.

Charles H. Townes’s statement about physicists being “diverted” is in his 1984 paper Ideas and stumbling blocks in quantum electronics. *IEEE Journal of Quantum Electronics*, Vol. 20, pp. 547–50.

For an excellent critical discussion of the photon concept, including its long and controversial history, see Richard Kidd, James Ardin, and Anatol Anton, 1989: Evolution of the modern photon. *American Journal of Physics*, Vol. 57, pp. 27–35.

For a simple experimental demonstration that unilluminated bodies we cannot see may emit enough visible light to be detected, see Craig F. Bohren, 1987: *Clouds in a Glass of Beer*, John Wiley & Sons, pp. 74–5.

Minimum temperatures for a blackbody to be visible are given by Leo Levi, 1974: Blackbody temperature for threshold visibility. *Applied Optics*, Vol. 13, p. 221.

Derivations of the Maxwell–Boltzmann distribution, Eq. (1.8), are given in treatises on the kinetic theory of gases such as Earle H. Kennard, 1938: *Kinetic Theory of Gases*, McGraw-Hill, Ch. II; Leonard B. Loeb, 1961: *The Kinetic Theory of Gases*, Dover, Ch. IV; and Sir James Jeans, 1982: *An Introduction to the Kinetic Theory of Gases*, Ch. IV. Loeb (pp. 130–8) and Jeans (pp. 124–30) discuss some of the early experimental verifications of this distribution. Deriving it by detailed consideration of molecular energy exchanges by collisions is

not trivial, but plausible arguments (not a rigorous derivation) for the distribution of molecular speeds (from which follows that for energies) is given by Craig F. Bohren and Bruce A. Albrecht, 1998: *Atmospheric Thermodynamics*, Oxford University Press, pp. 61–4.

The Planck distribution, Eq. (1.11), is not easy to derive. If it were, Planck would not be almost a household name. One of the best derivations we've seen is by David Bohm, 1989: *Quantum Theory*, Dover, Ch. 1. To follow this derivation requires a good grounding in electromagnetic theory, statistical mechanics, and classical mechanics.

Nowadays we blithely write down the Planck distribution forgetting that the theoretical and experimental path to it was long and arduous. Imagine how difficult it was to make absolute spectral measurements more than 100 years ago. The history of the Planck distribution is recounted in Hans Kangro, 1976: *Early History of Planck's Radiation Law*, Taylor & Francis. For a history of the role that blackbody radiation played in the evolution of quantum mechanics see Thomas S. Kuhn, 1978: *Black-Body Theory and the Quantum Discontinuity 1894–1912*, Oxford University Press.

The equilibrium distribution of molecular energies in a closed container can come about only because of interactions (energy exchanges) between molecules within the gas and between gas molecules and the walls of the container. The Maxwell–Boltzmann distribution applies strictly to an ideal gas in which molecules exert no forces on each other (or the time over which they do exert forces is zero, which also is physically unrealistic). Photons do not interact with each other, so the equilibrium distribution of a photon gas can come about only because of interactions of photons with the walls of the container. Radiation in a perfectly reflecting container (which does not exist) would never evolve to the equilibrium distribution if it did not initially have this distribution. These points are made by F. E. Irons, 2004: Reappraising Einstein's 1909 application of fluctuation theory to Planckian radiation. *American Journal of Physics*, Vol. 72, pp. 1059–67.

Planck's *The Theory of Heat Radiation*, an English translation of the second edition of which (1913) was published by Dover in 1959, is full of insights and qualifiers that have been forgotten over the years. For example, Planck recognized that “the surface of a body never emits rays, but rather it allows parts of the rays coming from the interior to pass through” (p. 4), that a “finite amount of energy... is emitted only by a finite... volume, not by a single point” (p. 5), that all matter is inhomogeneous at some scale (p. 8), that “in nature there is no such thing as absolutely parallel light or an absolutely plane wave front” (p. 14), and that the maximum of a distribution function depends on how it is expressed (p. 16).

Proof of the theorem for the change of variables in integration is given by R. Creighton Buck, 1978: *Advanced Calculus*, 3rd ed., McGraw-Hill, pp. 182–3. This excellent textbook is rigorous without being oppressive. Buck recognizes that theorems must be proven by a sequence of logical steps, not by notational tricks. But he does recognize that notation “serves as a guide in the correct application of theorems.”

The consequences of the failure to recognize that a distribution function does not have a unique maximum are discussed by Bernard H. Soffer and David K. Lynch, 1999: Some paradoxes, er-

rors, and resolutions concerning the spectral optimization of human vision. *American Journal of Physics*, Vol. 67, pp. 946–58.

A discussion of the principle of detailed balance in the context of proving Kirchhoff's law is given by F. Reif, 1965: *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, pp. 382–4.

A concise but good discussion of time-reversal symmetry is given by P. C. W. Davies, 1976: *The Physics of Time Asymmetry*, University of California Press, pp. 22–7. The time-reversal symmetry of fundamental dynamical laws (e.g., Newton's laws, electromagnetic theory, quantum mechanics) is to be distinguished from what Davies calls the “asymmetry of the world with respect to time” (i.e., we all grow older, alas, even though growing younger is not forbidden by dynamical laws).

We are by no means the first to ridicule the notion that there are “good” and “bad” emitters. In an obituary notice for John Henry Poynting (whose eponymous vector appears in Chapter 4), Oliver Lodge noted that Poynting's “rebellion against an excessive anthropomorphism... some substances being praised as good radiators while others are stigmatized as bad... though doubtless more than half humorous was in itself wholesome” (*Collected Scientific Papers by John Henry Poynting*, 1920, Cambridge University Press, p. xii).

Kirchhoff's law in Section 1.4 is not his original version, for which see William Francis Magie, 1965: *A Source Book of Physics*, Harvard University Press, pp. 354–60, a superb collection of excerpts from more than 100 papers of great historical significance preceded by short biographies of their authors. For example, the empirical (i.e., obtained by curve fitting) discovery of the radiation law, Eq. (1.27), later derived by Boltzmann, is given in Stefan's own words on pp. 378–81.

For a brief biography of Gustav Robert Kirchhoff and the significance of his work see the entry by L. Rosenfeld in *Dictionary of Scientific Biography*, Vol. VII, pp. 379–83. Other than original papers, the *DSB* is the first place to look if you want to know what our illustrious predecessors really did and said. The last place to look is in textbooks, which are notorious spreaders of rumors, half-truths, and outright errors. Ask any historian of science. Better yet, read Tony Rothman's (2003) delightful *Everything's Relative and Other Fables from Science and Technology*, John Wiley & Sons.

The controversy over the validity of Kirchhoff's law is reviewed by H. P. Baltes, 1976: On the validity of Kirchhoff's law of heat radiation for a body in a nonequilibrium environment. *Progress in Optics*, Vol. 13, pp. 1–25.

The term *emittance* is sometimes used for what we call emissivity. Indeed, it has been suggested that emissivity be used for bodies that are “pure and smooth”, emittance for bodies that are not. For a criticism of this suggestion see the letter by William L. Wolfe, 1982: A proclivity for emissivity. *Applied Optics*, Vol. 21, p. 1. Wolfe's parting shot is “On reflection, I like reflectivity and emissivity”. So do we, but for a contrary view see Joseph C. Richmond's

rebuttal following Wolfe's letter. If the term *emittance* is to be used at all it is best reserved as an abbreviation for *emitted irradiance* (or *radiance*). Note in Chapter 4 that radiometric and photometric dimensional quantities (radiance, irradiance, luminance, etc.) all end in *ance*, and hence in the same spirit *emittance* ought to be *emissivity times the Planck function*.

The assertion on page 22 that radiant heat is a "meaningless term" is from Yves Le Grand, 1957: *Light, Colour, and Vision*, John Wiley & Sons, p. 4.

Luminescence (mentioned in Sec. 1.4.5) is the term first used by Eilhardt Wiedemann (1888) for "all those phenomena of light which are not solely conditioned by the rise in temperature." Luminescence, or "cold light", is contrasted with incandescence, or "hot light". Examples of luminescence are the dim light of phosphorous, the light of fireflies, and the light emitted by substances excited by various kinds of electromagnetic radiation or subatomic particles. By light is usually meant visible or near-visible.

Fluorescence originally was applied to light that ceases immediately upon removal of its source of excitation, in contrast with phosphorescence, which persists. But this distinction is not absolute, long-lived fluorescence merging continuously into short-lived phosphorescence. Moreover, what is meant by "long-lived" and "short-lived" depends on the sensitivity of instruments. Some observable consequences of fluorescence are presented in Section 4.1.7.

In all examples of luminescence electrons in atoms and molecules are excited by some means into higher energy levels. At a later time these electrons decay to lower energy levels, with the attendant emission of photons having energies equal to the differences in the energy levels. In fluorescence, the time between excitation and decay may range from a second to as low as 10^{-9} s, which is still a long time compared with the inverse frequency of visible radiation.

For a history of luminescence see the monumental treatise by Edmund Newton Harvey, 1957: *A History of Luminescence From the Earliest Times Until 1900*, American Philosophical Society. See especially Chapter XI on fluorescence. This term was coined by Sir George Gabriel Stokes, a central figure in Chapter 7. For Stokes's abstract of his lengthy paper on fluorescence see Magie, pp. 344–52.

The solar spectrum in Figure 1.7 is derived from a spectrum developed by Robert L. Kurucz of the Harvard-Smithsonian Center for Astrophysics. The best reference for this spectrum is his unpublished article *The Solar Irradiance by Computation* (November 25, 1997). We obtained our copy from Kurucz. Tony Clough averaged Kurucz's spectrum to 1 cm^{-1} resolution. We started with Clough's spectrum and averaged it further to 20 cm^{-1} resolution. The best way to obtain the Kurucz solar irradiance values is by downloading the Line-by-Line Radiative Transfer Model (LBLRTM) developed by Tony Clough and his co-workers. Both the original Kurucz spectrum and Clough's 1 cm^{-1} resolution spectrum come bundled with LBLRTM, which is available through Atmospheric and Environmental Research (AER), Tony Clough's employer.

For an older set of tables of the solar irradiance at intervals of 10 nm or 20 nm over most of the spectrum, which may be more convenient for calculations and yet still adequate for many

purposes, see M. P. Thekaekara and A. J. Drummond, 1971: Standard values for the solar constant and its spectral components. *Nature Physical Science*, Vol. 229, pp. 6–9.

For simple experiments to demonstrate that emission by aluminum foil is quite different from that of bodies painted white or black, and that despite appearances to the contrary, some (visibly) black and white bodies are nearly identical at infrared wavelengths, see Craig F. Bohren, 1991: *What Light Through Yonder Window Breaks?*, John Wiley & Sons, Ch. 7. See also Richard A. Bartels, 1990: Do darker objects really cool faster? *American Journal of Physics*, Vol. 58, pp. 244–8.

The *greguería* on page 26 is from Ramón Gómez de la Serna, 1994: *Greguerías*, Clasicos Castilla, p. 69.

Problems

1.1. We note in Section 1.2.1 that the Planck distribution can be expressed as a function of frequency or wavelength. But this doesn't exhaust the possibilities: any single-valued function of frequency is a suitable independent variable. Moreover, other distributions related to the Planck distribution are also possible. For example, the spectral number density n (number per unit volume) of photons in a cavity at a fixed temperature is proportional to the Planck function divided by the photon energy (see Prob. 4.51), and n can be expressed as photons per unit frequency $n(\omega)$ or per unit wavelength $n(\lambda)$. Do so and then find the corresponding displacement laws for the maxima of these distributions.

1.2. The inverse of a function might not be single valued (i.e., the inverse might not exist). To convince yourself of this, and to obtain a better understanding of what is meant by the inverse of a function, sketch a simple function with a multi-valued inverse. This requires thinking carefully about what is meant by a mathematical function and its inverse. A simple sketch is all that is needed.

1.3. All raindrops are not identical but are distributed in size. The *Marshall–Palmer distribution*

$$N(D) = N_o e^{-\Lambda D} \quad (1.80)$$

often has been used for raindrops, where N_o and Λ are constants. $N(D)$ has the property that

$$\int_{D_1}^{D_2} N(D) dD \quad (1.81)$$

is the number of raindrops per unit volume with diameters between D_1 and D_2 . From this distribution one can obtain the mean diameter, call it $\langle D \rangle$. But the scattering of sunlight (see Sec. 3.5) by raindrops is proportional to D^2 , so as far as light scattering is concerned the root-mean-square diameter, the square root of the average of D^2 , is of physical significance.

The mass of a raindrop is proportional to the cube of its diameter, so the relevant average diameter here is the cube root of the average of D^3 . The contribution of a drop to rainfall

rate depends on the product of its volume and its terminal velocity, which is approximately proportional to diameter. Here the relevant mean diameter is the fourth root of the average of D^4 . Finally, the radar backscattering cross section (see Sec. 3.5.2) is proportional to the sixth power of diameter.

Find the relative values of the root-mean-sixth, root-mean-fourth, root-mean-cube, root-mean-square, and mean diameters for the Marshall–Palmer distribution (for D ranging from 0 to ∞). What do you conclude from this?

1.4. A radiation thermometer infers temperatures of objects (at typical terrestrial temperatures) by measuring infrared radiation from them. We have done the following demonstration. We put some water and ice cubes in a shiny, thin-walled metal container with an open top (a tin can with the label removed will do) and stirred the water thoroughly. Then we pointed the thermometer at the water surface. The temperature reading was close to 0°C , but when we immediately pointed the thermometer at the side of the container, the temperature reading increased. Then we replaced the ice–water mixture with hot water. The temperature of the water measured at its top surface was a certain value, but when we pointed the thermometer at the side of the container, the temperature reading decreased. Explain.

1.5. Estimate plausible minimum and maximum values (W m^{-2}) at Earth’s surface of radiation emitted downward by the atmosphere.

1.6. Try the following demonstration. Hold your arms out from your sides, level with your shoulders, hands open, for about 10 seconds. Then suddenly bring your arms together so that your palms are a centimeter or less apart *facing each other but not touching*. Hold your hands in this position for perhaps 10 seconds or more. Describe what you experience and explain it.

1.7. For the previous problem students have mentioned radiation from the air. By simple physical arguments you should be able to show that radiation from air in a room is negligible. HINT: A good estimate for the infrared brightness temperature of the sky is about 250 K.

1.8. Estimate the temperature of the filament of a 100 W electric light bulb. State all assumptions. Estimate the uncertainty in your estimate because of uncertainties in the quantities needed for your calculations.

HINT: If you can find a burned-out bulb, smash it (carefully) and examine the filament. If you can’t find such a bulb, try to find a clear light bulb so that you can estimate the dimensions of a filament. Be sure to ask yourself if your estimate is reasonable (300 K is not, nor is 30,000 K).

1.9. If ozone in the upper atmosphere is a “good absorber” of solar ultraviolet radiation, why isn’t it also a “good emitter” of such radiation, thereby undoing the good it is said to do?

1.10. Suppose that you have a friend, a Texan, say, who tells you that in Texas it once was so hot that his spit boiled when it hit the pavement. Is it possible for water to boil when placed on a surface heated by sunlight (no lenses please)? What is wanted here is a simple, quantitative argument showing that spit boiling on hot pavement either is possible or is not. If possible, is it plausible?

HINT: At the top of the atmosphere, the amount of solar radiation incident on every square meter (pointed toward the sun) is about 1369 W, but, of course, is less at the surface.

1.11. The following was taken from a book on radar systems: “It is known from the theory of blackbody radiation that any body which absorbs energy radiates the same amount of energy that it absorbs.” Discuss.

1.12. How might heights of clouds above the surface of Earth be estimated from observations made from a satellite well above them? The satellite is not equipped with a laser or any other source of radiation. State all assumptions.

1.13. What fraction of electricity bills for lighting with incandescent lamps is wasted because they emit invisible as well as visible radiation? You may take the color temperature of incandescent lamps to be 2500 K and assume constant emissivity of the filament.

HINT: You can do this problem analytically by making judicious approximations or you can write a computer program for numerically integrating the relevant integral. If you are in a hurry, you can content yourself with an upper limit on this fraction.

1.14. Estimate the maximum temperature of the illuminated surface of the moon. You may take 1369 W m^{-2} as the solar irradiance at the top of Earth's atmosphere. State all assumptions. After you have made your estimate, try to find out how it squares with any measurements you can find.

1.15. Sketch the spectral absorptivity, from ultraviolet well into the infrared, of the best solar energy collector. By "best" is meant that the temperature it reaches is as high as possible. Take a stab at estimating this temperature. You may take the solar spectrum to extend from $0.25 \mu\text{m}$ to $2.5 \mu\text{m}$ and 1000 W m^{-2} as an estimate for the maximum solar irradiance at the ground.

You can't determine this temperature on the back of an envelope but will have to write a computer program. At the very least, write down the equation that has to be solved and give some thought to how you might do so.

1.16. Several years ago a newspaper advertisement described a new kind of radiant heating system. The radiating panels were said to be "hot to the touch but they do not burn." The advertisement went on to say "the panels radiate long infrared rays which are most readily absorbed by the objects, including the human body. In fact, they are the same rays that warm us from the sun." Discuss.

1.17. When one of the authors and his wife lived in Wales they had in their bedroom a heating panel similar to that described in Problem 1.16. Now they have a small electric heater in their bedroom, and it seems to do about as good a job as the panel. This heater has 6 strips, each about 20 cm long and 0.5 cm wide. When the heater is operating, the strips glow red. On the basis of this information you should be able to estimate the size of the panel in Wales. You may take the panel to be square. State all assumptions and, as always, be sure that your answer makes sense.

1.18. Why do temperature inversions (temperature increasing with height) often form at night immediately above the tops of clouds?

1.19. In the simple two-slab analogue to the Earth-atmosphere system, the top slab representing the atmosphere was assumed to emit the same amount of radiation to space as to the ground. Is this true for the real atmosphere? Why or why not?

1.20. If a warm object is wrapped with aluminum foil the rate of cooling of the object decreases (by "warm" is meant that the temperature of the object is higher than that of its surroundings). You can verify this by heating some water, pouring it into a jar, then measuring the rate at which the water temperature decreases with time. Do this with a bare jar and then with one wrapped in aluminum foil.

Aluminum foil has a lower emissivity than glass, and hence the foil reduces emission. But at the same time, the foil has a lower absorptivity, which means that the foil decreases the absorbed radiation from the surroundings. Because the foil gives and the foil takes away why isn't its net effect zero? What is the essential condition for the cooling rate to decrease because of wrapping the jar with foil? Another version of this question is, Under what conditions would wrapping an object with foil increase the rate of cooling?

You may assume that Kirchhoff's law is valid even for spectrally averaged emissivity and absorptivity.

1.21. The following statement is in an opening paragraph in a paper (Carl G. Ribbing, 1990: Beryllium oxide: a frost preventing insulator. *Applied Optics*, Vol. 15, pp. 882–84) on low emissivity coatings for dew and frost prevention: "The possibility for wavelength-selective condensation prevention is based on the existence of atmospheric transmittance windows. It is only by emission of radiation in these wavelength intervals that a body can cool to a substantially lower temperature than the immediately surrounding air. . ."

Atmospheric windows are spectral regions in the terrestrial infrared where the atmosphere has a comparatively high transmissivity.

Discuss the statement enclosed in quotation marks. It can be restated as follows: To develop coatings with spectral emissivities that reduce net radiative cooling, one need consider only the atmospheric window regions of the terrestrial infrared spectrum.

What implicit assumptions underlie this statement? Do you agree with it? You can give either mathematical or physical arguments or both.

HINT: It may help to divide the terrestrial infrared spectrum into two regions: window and non-window. Don't hesitate to make simplifying assumptions that enable you to concentrate on what is essential rather than get bogged down in details.

1.22. One of the authors lived in Arizona on a former dude ranch in the desert outside Tucson. One year he painted the roof of his cottage (not air-conditioned) with a metallic paint. The aim was to increase the reflectivity of the roof for solar radiation, thereby reducing the temperature inside the cottage. If your aim is to keep the interior of a house cooler in a region of intense solar illumination would you paint the roof of the house with metallic or white paint? Explain your choice. This is an open-ended question without a right or wrong answer. For measurements relevant to this problem see Dena G. Russell and Richard A. Bartels, 1989: The temperature of various surfaces exposed to solar radiation: an experiment, *The Physics Teacher*, Vol. 27, pp. 179-81.

1.23. Photons have energy and momentum (linear and angular). We gave an expression for the energy $h\nu$ of a photon but not its momentum. But from this expression you should be able to guess the expressions for photon linear momentum and angular momentum (to within dimensionless constants) by simple dimensional arguments. Don't look up the answer in a physics book. If you know the answer, don't do this problem because it won't teach you anything.

1.24. Estimate the maximum irradiance of a towering inferno (e.g., a huge forest fire) relative to that of the sun on a clear day at noon.

1.25. The September 30, 1995 issue of *Science News* contained the following in an article on engineering solutions to global warming: "One of the more expensive options would be to

install a giant solar deflector, built from materials on the moon, at a point 1.5 million miles from Earth in the direction of the sun. Thinner than a human hair, the diaphanous sheet would stretch 2,000 kilometers across and deflect 2 percent of the radiation headed toward Earth. Estimated cost: \$1 trillion to \$10 trillion.”

You might think that thousands of hours of computer time went into computing the value “2 percent”. In fact, you can obtain this number (approximately) in a few minutes without a calculator. The purpose of the “diaphanous sheet” is to compensate for the increased emissivity of the atmosphere as a consequence of doubling the concentration of CO₂. Assume that the increase in the radiative equilibrium temperature of the simple slab model of the Earth–atmosphere system at the surface because of this increased CO₂ is 1.5 K and that the present radiative equilibrium temperature is 288 K. This is all you need in order to do this problem. Huge computers, elaborate global circulation models and radiative transfer codes are not necessary. The analysis underlying a project estimated to cost more than \$1 trillion can be done on the back of an envelope – if you know what you are doing.

1.26. Estimate the total rate of emission of radiation from a (naked) human. A good estimate for skin temperature is 33°C. The surface area S (in cm²) of a human can be estimated from the DuBois formula

$$S = 71.84 W^{0.425} L^{0.725},$$

where W is body mass in kg and L is height in cm. This is the total surface area, about 80% of which is the effective radiating area. To good approximation human skin and hair have an emissivity close to 1 over the relevant range of infrared wavelengths. The DuBois formula is in Max Kleiber, 1975: *The Fire of Life*, 2nd ed., Robert E. Krieger, p. 184. This very readable book is filled with fascinating data such as the weight loss of the fasting dog Oscar (p. 28) and the time evolution of the rectal temperature of pigs in air at different temperatures (p.177).

Express your result in kilocalories (food calories) per day. One calorie is 4.184 J and a kilocalorie is 1000 cal. On the basis of this calculation what do you conclude about your necessary daily food intake if there were no radiation from the environment? Suppose that there were no such radiation. What strategy would humans likely have had to adopt in order to survive? What class of animals would we resemble?

Now determine your *net* infrared radiation, in kcal/day, in an environment at room temperature (20°C). Estimate the fraction of your daily food intake (in kcal) that is needed just to balance net radiation.

How does clothing alter your numbers and conclusions? To answer this you either have to estimate the temperature drop from skin to clothing or measure this drop with an infrared thermometer.

You often hear the assertion (but not from us) that “most body heat is lost from your head.” Can you make any sense out this, its implication being that your head is some kind of super-radiator? Yet the emissivity of skin and hair on your head is no different from that on the rest of your body, and head temperatures are not greatly different from those on other areas of your body. Moreover, you have more hair on your head than other parts of your body (unless you are bald). You can make your answer quantitative by estimating the contribution of your head area to total body area.

1.27. What is the temperature of a blackbody with an emission spectrum that peaks at the same wavelength as the solar spectrum (outside Earth’s atmosphere)? Suppose that the sun

were replaced by a blackbody of the same radius and distance to Earth. What is the temperature of this blackbody such that the solar irradiance is its presently accepted value (about 1369 W m^{-2})? The angular width of the sun is about 0.5° .

1.28. For what angle is the reflectivity of water at $10 \mu\text{m}$ such that emission by it at 100°C equals reflection of clear-sky radiation of the same wavelength? You may take the radiative temperature of the sky to be 250 K. This problem is related to the directional emissivity experiment discussed in Section 1.5.1.

1.29. We show in Section 1.2.1 that the wavelength at which the Planck function has a maximum (for a given temperature) is not an absolute quantity but depends on the independent variable chosen, which is arbitrary. Mark Heald (2003) in an article in *American Journal of Physics* (Where is the “Wien peak”?, Vol. 71, pp. 1322–23) agrees but notes that a peak wavelength can be defined unambiguously as a *median wavelength*: the wavelength that divides the Planck function into two regions of equal area (integrated irradiance). First, convince yourself that this median wavelength is independent of the variable in the Planck function and why. Then determine the displacement law for the median wavelength.

HINT: The second part of this problem requires numerically evaluating an integral.

1.30. Brightness temperature is equal to thermodynamic temperature if the emissivity of the object of interest is 1 at the wavelengths detected by the radiation thermometer and the detected radiation is solely that emitted by this object. Assume that the second condition is satisfied and derive a simple expression for the relative difference between brightness temperature (at, say, a wavelength of $10 \mu\text{m}$ or thereabouts) and thermodynamic temperature as a function of the departure of the emissivity from 1 for typical terrestrial temperatures.

HINT: Expand the brightness temperature in a Taylor series around $\varepsilon = 1$.

1.31. The result of the previous problem might lead you to believe that errors in brightness temperatures because of departures of emissivity from 1 are large. Measurements of the brightness temperature of objects are always made in a particular environment. In what kind of environment would the error be greatest? In what kind of environment would the error be least (possibly zero) even if the emissivity of the object of interest were appreciably less than 1? To help you answer these questions consider measuring the brightness temperature of a wall inside a house. Even though the emissivity of a wall probably is less than 1, the radiation thermometer reading is more or less correct. Why?

1.32. Because photons possess linear momentum, light can exert *radiation pressure*, that is, transfer momentum to illuminated objects. Determine the radiation pressure of direct solar radiation illuminating a surface with reflectivity 1 at all solar wavelengths. Take the irradiance to be 1000 W m^{-2} . Express your answer relative to sea-level atmospheric pressure. What is the radiation pressure on a surface that is black at all solar wavelengths?

HINTS: Problems 1.1 and 1.23 should be helpful. The relation between radiation pressure and irradiance is so simple that you could obtain it (to within a constant factor) by dimensional analysis.

1.33. It has been stated countless times that no body can emit more radiation in any wavelength interval than a blackbody at the same temperature. Yet there are plenty of examples in which bodies can emit thousands of times more than blackbodies. Give a few simple examples. You don’t have to go far to find them.

1.34. We have hit on a sure-fire weight loss scheme, the Bohren and Clothiaux Radiation Diet. According to Einstein's famous mass energy equation, which now is such a commonplace that it is a staple of cartoons, if a body loses energy its mass decreases. So we are planning construction of a world-wide chain of weight-loss rooms. The walls of a room are black to the kind of infrared radiation emitted at body temperature and cooled to the temperature of dry ice (-78.5°C) to maximize net radiation from clients in the room. While sitting in it they will radiate their excess weight without exercise or diet at a cost of only \$10 an hour. We are offering exclusive franchises to readers of this book for the absurdly low fee of only \$50,000. How can you lose? Send us money now and take advantage of this advance offer.

1.35. Estimate how long it would take in a completely darkened room to photograph an object at room temperature. Assume that with the fastest film available and the aperture as large as possible (smallest f-stop) you can photograph the coils of an electric stove (estimated temperature of, say, 1500 K) with an exposure time of 1/100 s. All that is wanted here is a crude estimate. Would it take hours, weeks, months, years, centuries?

1.36. Green vegetation is more absorbing of solar radiation with wavelengths less than about 700 nm than of radiation with greater wavelengths (note the steeply rising radiance of grass beyond about 690 nm in Fig. 4.13). Can you think of reasons why plants might have evolved to have this spectral absorption property?

1.37. Radiation literally could be trapped in a container with perfectly reflecting (100%) walls. Photons injected into such a container would rattle around forever. But perfection is not of this world. Estimate how much departure from perfection (what reflectivity $< 100\%$) could be tolerated in a container with linear dimensions of order 10 cm such that radiant energy trapped within it would decrease by no more than, say, $1/e$ of its initial value in an hour.

1.38. Photons emitted by a body cannot be interrogated as to their origins: luminescence or thermal emission. All that can be measured is total emission. But it is possible to determine if part of the emitted radiation is a consequence of luminescence and even how much. If total emission (at a given frequency) exceeds that of a blackbody, the excess can be attributed to luminescence (almost by definition). But a body can emit less than a blackbody (at the same temperature) and yet still be luminescent. What do you need to know in order to determine the contribution of luminescence to total emission? Obtain an expression for this contribution.

1.39. Show that in principle one can measure the temperature of a blackbody by measuring only the ratio of radiation emitted by it at two different wavelengths. How should these wavelengths be chosen and how does your choice depend on the expected range of temperatures?

HINT: Sketching a few functions should help.

1.40. Show that the form of the Stefan–Boltzmann law, Eq. (1.27), can be derived by applying the first and second laws of thermodynamics to a photon gas (in a cavity) given two results for such a gas: its pressure p is equal to one-third its energy density u , which depends on T only.

HINT: Consider a reversible expansion (or compression) of the photon gas. The total internal energy of the gas is uV , where V is its volume.

1.41. What is the temperature of the air near the surface and just above the top layer in the two-slab model developed in Section 1.6?

HINT: Place a thin layer in each location and assume that it does not affect the overall energy balance of the two-slab model. This question is motivated by Section 3.8 in Dennis L. Hartmann, 1994: *Global Physical Climatology*, Academic Press, pp. 61–3.

1.42. Meteorology students often are taught that the maximum daytime temperature (of what we are left to guess) occurs when the net radiation balance (again, of what we must guess) is zero. Critically discuss this.

1.43. Peter W. Huber and Mark P. Mills, 2005: *The Bottomless Well*, Basic Books, p. 6, assert that the amount of solar energy reaching the “surface from above” is “roughly ten thousand times as much as humanity consumes in the form of fossil fuels...Green plants seize and temporarily store a tiny fraction of it. During the night, the dark side of the Earth radiates all the rest, along with the geothermal heat, back out into the black depths of the cosmos.” Critically discuss.

1.44. Why is the photon language of almost no use to most radio engineers? There is a simple yet fundamental reason that has essentially nothing to do with phase differences. A related question, which is also a hint, is what must the photon number density be such that a detector with area A senses a beam as a more or less discrete set of photons?