Fundamentals of Atmospheric Radiation
Craig F. Bohren

8 Meteorological Optics: The Reward

The subtitle of this chapter promises that mastering concepts in previous chapters provides the means to understand – even to see in the first place – a rich assortment of atmospheric displays, some strikingly beautiful, some subtle, often remarkably detailed and full of surprises. A day hardly goes by anywhere without something in the sky worth looking, even marveling, at. From sunrise to sunset and beyond, a free show is open to those whose minds have been prepared to see it. Scientific understanding enhances, not diminishes, one's pleasure from scanning the sky. We often have begun a lecture to a large class by asking if anyone has ever seen a sun dog or a halo. Usually, not one student has. But within a week after we have discussed and explained them, excited students come to tell us that what they formerly had been blind to they now have seen. The atmosphere is not different, only the minds of the students.

We use the term *meteorological optics* here, in preference to *atmospheric optics*, to emphasize that our subject does *not* include, say, transmission of laser light through the atmosphere. Here we are interested in what the human observer sees when looking skyward.

Meteorological (or atmospheric) optics is nearly synonymous with light scattering, the only restriction being that the scatterers – molecules or particles – inhabit the atmosphere and the primary source of their illumination is the sun. We know of only a few exceptions in which absorption plays an important role in what is observed. In previous sections, for example that on polarization of skylight (Sec. 7.3), we touch on various topics in meteorological optics. And the chapters on scattering (Ch. 3), radiometry and photometry (Ch. 4), and multiple scattering (Ch. 5) lay the foundations on which we build this chapter.

8.1 Color and Brightness of the Molecular Atmosphere

A few years ago, Spain's leading newspaper, *El País*, published an interview with Manuel Toharia, Director of the Prince Felipe Museum of Sciences, in which he averred that science "is the response to man's curiosity. And this curiosity leads him to raise big and small questions. To me the question of why the sky is blue is not smaller than the question of where we come from and where we are going. Both are big."

Why is the sky blue? is one of the most frequently asked scientific questions. Although not inherently difficult, an answer that is not misleading, incomplete, or outright wrong is almost impossible to find.

One of our former graduate students, Cliff Dungey, taught a game to his daughter when she was three. In a gathering of people he would ask her, "Caryn, Why is the sky blue?" She would answer proudly, "Because of Rayleigh scattering" – and we'd all laugh. This story has

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two morals. One is that invoking the name of a theory doesn't explain anything. And the other is that anything a child can parrot is not a good explanation. Missing from Caryn's childish mantra is the agent responsible for the blue sky. If Lord Rayleigh had never lived, this agent would be the same. Theories come and go, agents endure.

Water is essential for life, truly a vital fluid. Early in our schooling we are told that it is anomalous in its properties. With time, water becomes not merely vital and anomalous but magical. We remind you of two scientific scandals that received worldwide attention, Polywater and Cold Fusion, in which the minds of otherwise sober scientists were clouded by water.

Given the magical status water has acquired, no wonder it often is taken as the cause of the blue sky. Leonardo's explanation was that "the blueness we see in the atmosphere... is caused by warm vapour evaporated in minute and insensible atoms on which the solar rays fall, rendering them luminous against the infinite darkness of the fiery sphere which lies beyond." Newton invoked "Globules of water." Clausius proposed scattering by minute bubbles. All water-based theories – and all wrong. Scattering of sunlight by air molecules is the cause of the blue sky. Which ones? The most numerous: nitrogen and oxygen. Water plays no essential role. Only about one air molecule out of every 100 is a water molecule, and molecule for molecule water vapor scatters *less* visible light than either nitrogen or oxygen. Water vapor does scatter sunlight, and hence does contribute to skylight, but because of its relatively low abundance and smaller scattering per molecule, its contribution is negligible. Nitrogen and oxygen are the agents causing the blue sky because they are selective or wavelength-dependent scatterers. Air is a scattering filter.

Scattering of visible light by air molecules is described to good approximation by Rayleigh's scattering law, according to which scattering is inversely proportional to the wavelength of the illumination to the fourth power (see Sec. 3.2). Reality is a bit more complicated. The power in Rayleigh's law is not on the same footing as, say, that in the law of universal gravitational attraction, which is 2 to a staggering number of digits. Scattering by air molecules over the visible spectrum is described more accurately as inversely proportional to wavelength to the power 4.09. By simple dimensional arguments, Rayleigh captured most but not all of the power law applicable to scatterers small compared with the wavelength of the illumination.

A scattering law with a limited domain of validity was one of Rayleigh's major contributions, but he was not the first to recognize the origins of the blue sky. Indeed, he begins his 1871 paper with the assertion "It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course. On this point the experiments of Tyndall with precipitated clouds seem quite decisive." One can go back even further than Tyndall, almost a century, to de Saussure who in 1789 wrote that "air is not perfectly transparent; its elements always reflect some rays of light, especially blue rays. It is these reflected rays which produce the blue color of the sky. The purer the air, and the greater the extent of this pure air, the darker the blue color."

In 1871 Rayleigh confessed that he could not say with certainty which "small suspended particles" are responsible for the blue sky. He opined that they might be common salt, but recognized that his law could neither prove nor disprove this because it is valid for any matter if sufficiently finely divided. This loose end must have bothered him because he returned to

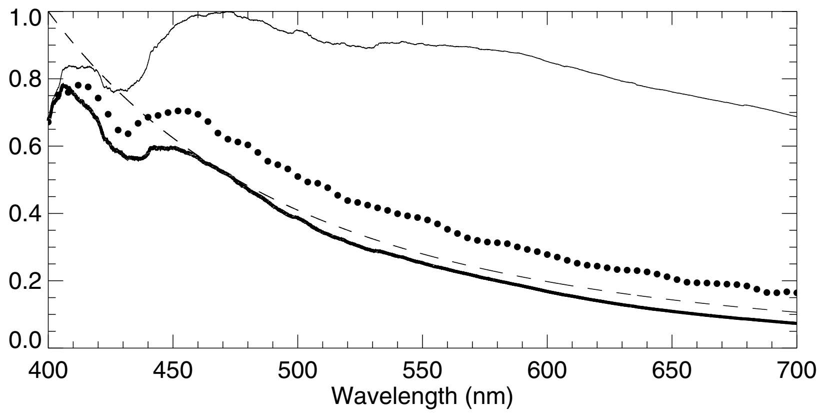

Figure 8.1: Overhead skylight around noon measured in spring in Central Pennsylvania (circles). The thick solid curve is the product of Rayleigh's scattering law (dashes) normalized to one at its greatest value and the solar spectrum (thin solid curve) outside Earth's atmosphere normalized to one at its peak.

the problem of the blue sky 28 years later, beginning his 1899 paper with the assertion "I think that even in the absence of foreign particles we should still have a blue sky."

Are we done? No, we have just begun. In particular, we have to criticize the way Rayleigh's scattering law is almost invariably used to explain the blue of the sky. It is obvious from this law that blue light is scattered more than red, and hence, we are told, the sky is blue. Misusing Rayleigh's law in this way is an example of cutting and bending a theory to fit an observation. It is just as obvious from Rayleigh's law that violet light is scattered even more than blue, and hence by the same logic the sky should be violet. But as we show in Section 4.3, there is no necessary connection between the maximum of a spectrum and the color it evokes.

Evidence for the essential correctness of Rayleigh's explanation is agreement between the product of his scattering law and the solar spectrum outside the atmosphere with the measured zenith skylight spectrum (Fig. 8.1). But again, we are not done. The clear sky is neither uniformly nor inevitably blue.

8.1.1 Variation of Sky Color and Brightness

Selective scattering by molecules is necessary but not sufficient for a blue sky. The atmosphere also must be optically thin, at least for most zenith angles. The blackness of space as a backdrop is taken for granted but also is necessary, as Leonardo recognized. Figure 8.2 shows the normal scattering optical thickness versus wavelength for the Standard Atmosphere.

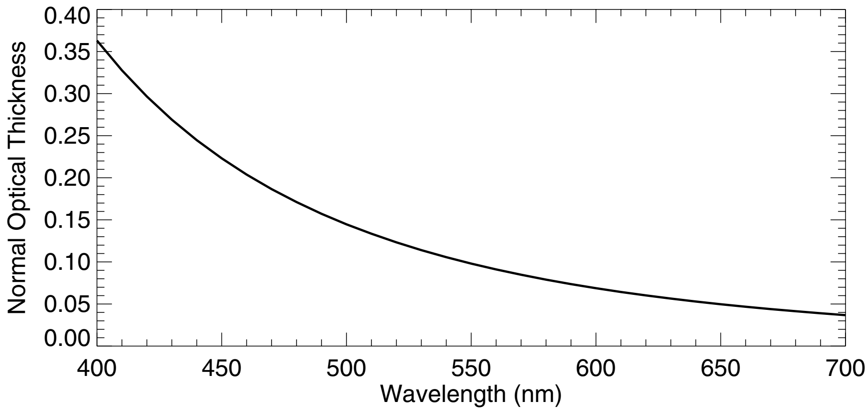

Figure 8.2: Normal scattering optical thickness for the Standard Atmosphere. From Penndorf (1957).

From the two-stream approximation [Eq. (5.66)] with $g = 0$ (molecular scattering) the diffuse downward irradiance $D_\downarrow$ of overhead skylight at the surface is

$$\frac{D_\downarrow}{F_0} = \frac{1}{1 + \tau_\mathrm{n}/2} - \exp(-\tau_\mathrm{n}), \tag{8.1}$$

where F_0 is the incident irradiance and τ_n is the normal optical thickness of the atmosphere. For $\tau_\mathrm{n} \ll 1$ this approximates to

$$\frac{D_\downarrow}{F_0} \approx \frac{\tau_\mathrm{n}}{2}. \tag{8.2}$$

Equations (8.1) and (8.2) are for a black underlying surface (zero reflectivity). What about the other extreme, a white underlying surface (a reflectivity of 1)? We can answer this by solving Eq. (5.49) subject to equal downward and upward irradiances at the surface. But it is better for our souls (i.e., our physical intuition) if we guess that it must be approximately twice that given by Eq. (8.2) because the air is illuminated by two approximately equal sources: direct and reflected sunlight. From Fig. 8.2 it follows that the condition for the validity of Eq. (8.2) is satisfied. Thus the spectrum of skylight for a molecular atmosphere should be the solar spectrum modulated by Rayleigh's scattering law, as indeed it is for the overhead sky (Fig. 8.1). What about the other extreme, the horizon sky? To answer this leads us to consider *airlight*.

The only real distinction between airlight and skylight is that the backdrop for airlight can be finite objects at a finite distance, whereas the backdrop for skylight is nearly empty, boundless space. Because of airlight, light scattered by all the molecules and particles along the line of sight from observer to object, even an intrinsically black object is luminous. Consider a horizontally uniform line of sight uniformly illuminated by sunlight (Fig. 8.3). Denote by L_0 the solar radiance illuminating the line of sight. The irradiance in the direction of the sun is

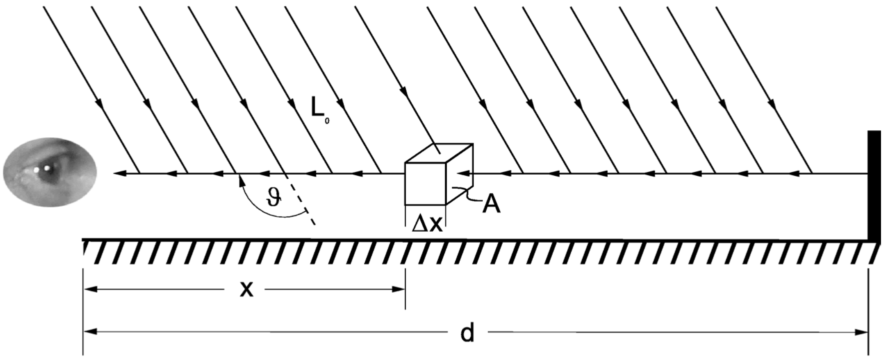

Figure 8.3: An observer looking at a distant black object nevertheless receives some light because of scattering by all the molecules and particles along the line of sight.

$L_0\Omega_s$, where Ω_s is the solid angle subtended by the sun. Assume negligible reflection by the ground. The light scattered toward the observer per unit solid angle by all the molecules and particles in a small volume $A\Delta x$ at a distance x is therefore $L_0\Omega_s\beta\Delta x A p(\vartheta)$, where β is the scattering coefficient and $p(\vartheta)$ is the probability per unit solid angle of scattering in the direction ϑ. Divide by A, which is perpendicular to the line of sight, to obtain the contribution to the radiance from $A\Delta x$. This is the radiance scattered at x *toward* the observer, which has to be multiplied by the transmissivity $\exp(-\beta x)$ to obtain the fraction of this radiance received *by* the observer. The total radiance as a consequence of scattering by everything along a line of sight between the observer and a black object at a distance d is the integral

$$L = L_0\Omega_s p(\vartheta)\beta \int_0^d \exp(-\beta x)\, dx = L_0 G\{1 - \exp(-\tau)\}, \tag{8.3}$$

where $\tau = \beta d$ is the optical thickness along the path d and the two geometrical factors are lumped into a single factor $G = \Omega_s p(\vartheta)$. Underlying Eq. (8.3) is the assumption that light scattered out of the line of sight is not scattered again in this direction, which is a good assumption if the optical thickness in directions lateral to the line of sight is small (which it is for the clear atmosphere but not for fog).

Only in the limit $\tau \rightarrow 0$ is $L = 0$ and a black object seen to be black. For $\tau \ll 1$, $L \approx L_0 G\tau$. In a purely molecular atmosphere τ varies with wavelength according to Rayleigh's law, and hence the distant black object is perceived to be bluish. As τ increases so does L but not proportionately: the longer the path, the greater the number of scatterers, but also the greater the attenuation. The limiting value of L ($\beta d \gg 1$) is $L_0 G$, and the radiance spectrum is that of the source illumination on the line of sight *regardless* of the wavelength dependence of β . This result ought to put an end to blather about the white horizon sky infallibly signaling scattering by "big particles."

Although the molecular optical thickness in the visible of Earth's atmosphere is small along a radial path, this is no longer true for paths near or along the horizon. The optical

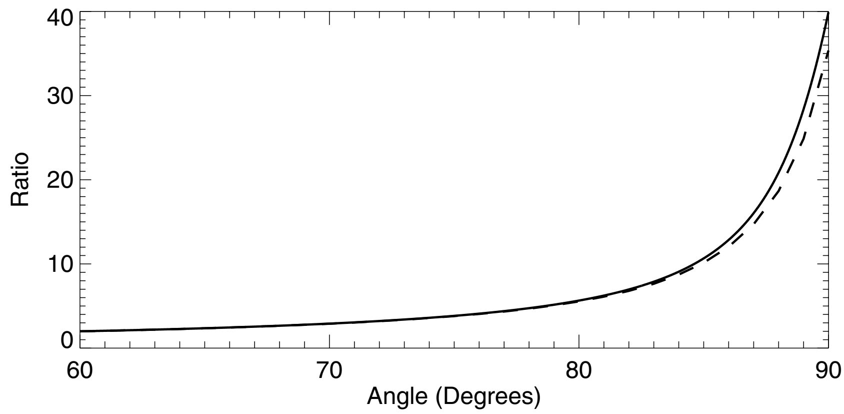

Figure 8.4: Scattering optical thickness of a pure molecular atmosphere with scale height 8 km on Earth relative to the normal optical thickness for a range of zenith angles near the horizon. The solid line is the uniform atmosphere approximation; the dashed line is for an exponentially decreasing scattering coefficient.

thickness along any path is an integral:

$$\tau = \int \beta \, ds. \tag{8.4}$$

For a path from the surface making a constant zenith angle Θ with the vertical direction in an atmosphere with an exponentially decreasing density of scatterers, Eq. (8.4) is

$$\tau = \int_0^\infty \beta_0 \exp\left\{ \frac{R - \sqrt{s^2 + 2sR\cos\Theta + R^2}}{H} \right\} ds, \tag{8.5}$$

where R is Earth's radius, H the scale height for molecular number density (i.e., the rate at which number density decreases exponentially with height), and β_0 the scattering coefficient at sea level. For a radial (normal) path ($\Theta = 0$) Eq. (8.5) can be integrated to obtain

$$\tau_{\mathrm{n}} = \beta_0 H. \tag{8.6}$$

Thus the normal optical thickness of an atmosphere in which the number density of scatterers decreases exponentially with height is the same as that for a uniform atmosphere of finite thickness H.

Although Eq. (8.5) cannot be integrated analytically for arbitrary zenith angle, the uniform, finite atmosphere approximation

$$\frac{\tau}{\tau_{\mathrm{n}}} = \sqrt{\frac{R^2}{H^2}\cos^2\Theta + \frac{2R}{H} + 1} - \frac{R}{H}\cos\Theta \tag{8.7}$$

is surprisingly good right down to the horizon ($\Theta = \pi/2$), as shown in Fig. 8.4. Taking the exponential decrease of molecular number density into account yields an optical thickness at most 10% lower. A flat Earth is one with infinite R, for which Eq. (8.7) yields the expected relation

$$\lim_{R\to\infty} \frac{\tau}{\tau_{\mathrm{n}}} = \frac{1}{\cos\Theta}. \tag{8.8}$$

The tangential (horizon) optical thickness ($\Theta = \pi/2$) from Eq. (8.7) is to good approximation

$$\frac{\tau_{\mathrm{t}}}{\tau_{\mathrm{n}}} = \sqrt{\frac{2R}{H}} \tag{8.9}$$

because $2R/H \gg 1$. For $R = 6400$ km and $H = 8$ km, $\tau_{\mathrm{t}} = 40\tau_{\mathrm{n}}$.

The variation of brightness and color of dark objects with distance was called *aerial perspective* by Leonardo. By means of it we unconsciously estimate distances to objects of unknown size, such as mountains. Aerial perspective is similar to the variations of color and brightness of the sky with zenith angle. Although the optical thickness along a horizon path is not infinite, it is sufficiently large (Figs. 8.2 and 8.4) that GL_0 is a good approximation for the radiance of the horizon sky. For isotropic scattering, a condition almost satisfied by molecules (see Sec. 7.3), G is about 10^{-5}, the ratio of the solid angle subtended by the sun to the solid angle of all directions (4π). Thus the horizon sky is not nearly so bright as direct sunlight.

Unlike in the milk experiment described in Section 5.2, what one observes when looking at the horizon sky is *not* (much) multiply scattered light. Both the whiteness of milk and that of the horizon sky have their origins in multiple scattering but manifested in different ways. Milk is white because it is weakly absorbing and optically thick, and hence all components of incident white light are multiply scattered to the observer even though the violet and blue components traverse a shorter average path in the milk than the orange and red components. White horizon light is that which has *escaped* being multiply scattered, although multiple scattering is why this light is white (strictly, has the spectrum of the source). More light at the short-wavelength end of the spectrum than at the long-wavelength end is scattered *toward* the observer, as evidenced by β in Eq. (8.3). But long-wavelength light has the greater probability of being transmitted to the observer without being scattered *out* of the line of sight, as evidenced by $\exp(-\beta x)$ in Eq. (8.3). For a sufficiently long optical path, these two processes compensate, resulting in a horizon radiance that of the source.

With Eq. (8.3) in hand we can make a stab at estimating the ratio of the horizon radiance to the zenith (overhead) radiance. If we take the incident sunlight to be nearly directly overhead the horizon (tangential) radiance is approximately

$$L_{\mathrm{t}} = L_0\Omega_{\mathrm{s}}p(90^\circ)\{1 - \exp(-\tau_{\mathrm{t}})\} \approx L_0\Omega_{\mathrm{s}}p(90^\circ) \tag{8.10}$$

and the zenith radiance is approximately

$$L_{\mathrm{n}} = L_0\Omega_{\mathrm{s}}p(0^\circ)\{1 - \exp(-\tau_{\mathrm{n}})\} \approx L_0\Omega_{\mathrm{s}}p(0^\circ)\tau_{\mathrm{n}}, \tag{8.11}$$

where p is the phase function for molecular scattering and L_0 is the radiance outside the atmosphere. All we need is the ratio of phase functions for the two scattering directions,

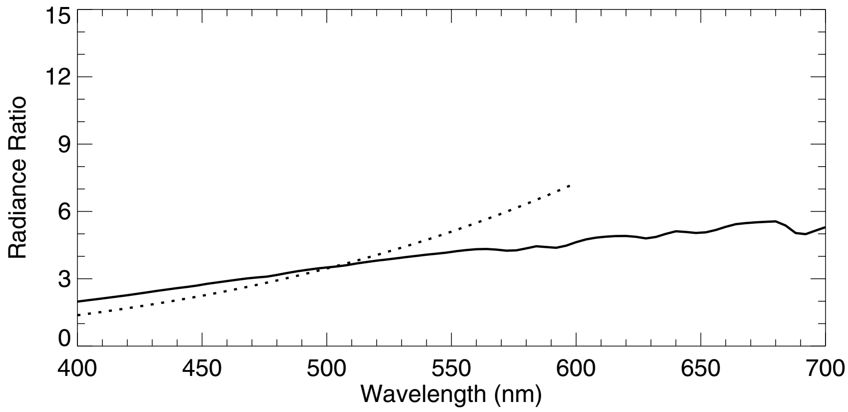

Figure 8.5: Measured ratio (solid line) of the horizon radiance to the radiance directly overhead with the sun high in the sky on a clear day in State College, Pennsylvania. The dotted line is this ratio predicted by simple theory for a pure molecular atmosphere.

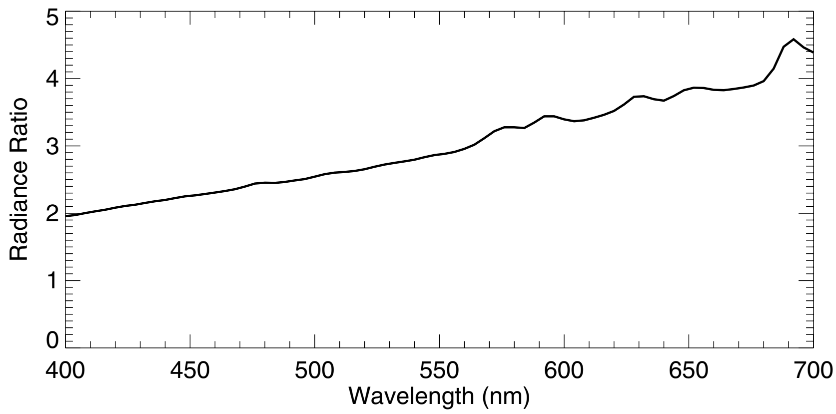

Figure 8.6: Measured ratio of the spectral radiance of magnesium oxide (MgO) powder illuminated by daylight to the spectral radiance of the horizon sky on a clear day in State College, Pennsylvania.

which we get from Eq. (7.117). The result is

$$\frac{L_{\mathrm{t}}}{L_{\mathrm{n}}} \approx \frac{1}{2\tau_{\mathrm{n}}}. \tag{8.12}$$

Although attenuation of sunlight illuminating the line of sight is neglected in Eqs. (8.10) and (8.11), when attenuation is included Eq. (8.12) is unchanged. Also $p(0^\circ)$ does not mean that

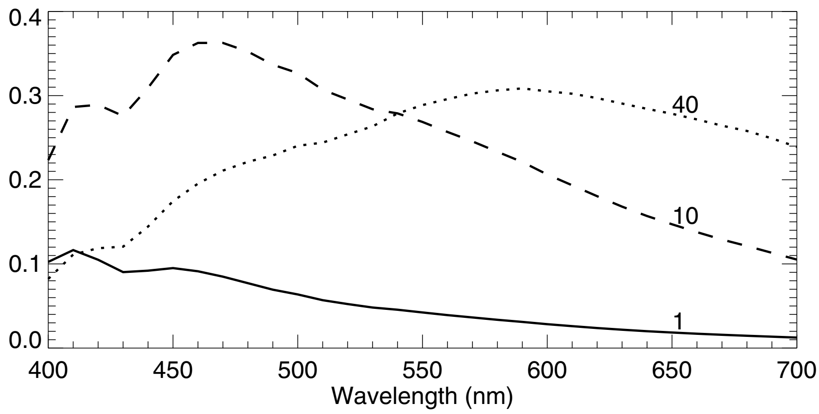

Figure 8.7: Spectra of overhead skylight from the two-stream theory for a molecular atmosphere with the present optical thickness (solid line), 10 times this thickness (dashed line), and 40 times this thickness (dotted line).

the sun is directly overhead and the line of sight is directly toward the sun but rather that the sun is high in the sky and the scattering angle is, say, less than 10–20°. Evidence for the validity of Eq. (8.12) is shown in Fig. 8.5, the ratio of measured radiances of the horizon and overhead skies, with the sun high in the sky, on a clear day. Agreement between measured ratios and those calculated with Eq. (8.12) using the normal optical thickness in Fig. 8.2 is surprisingly good. Moreover, the disagreement at longer wavelengths is in the expected direction: the normal optical thickness is almost always greater than that for a pure molecular atmosphere even in a very clean environment.

The optical thickness through the atmosphere along a horizon path is essentially infinite even in clear air. The source of illumination of this path is sunlight. The optical thickness of a large cumulus cloud is also essentially infinite, the source of illumination for which is also sunlight. Yet the radiance of the brightest cumulus cloud is larger, by roughly a factor of four, than that of the clear horizon sky. This is shown in Fig. 8.6, the ratio of the measured spectral radiance of magnesium oxide powder, which simulates a thick cloud, illuminated by daylight (i.e., direct sunlight and skylight) to the radiance of the horizon.

Although Eq. (8.12) is strictly valid only for small ($\ll 1$) normal optical thicknesses, it does suggest that with increasing optical thickness the gradient (in angle) of skylight radiance should decrease. And, in fact, this is what is observed: on murky days the sky is more nearly uniformly bright. Moreover, Eq. (8.12) also suggests that as one ascends in the atmosphere, and hence τ_n of everything above one's elevation decreases, the gradient of skylight should increase; this can be observed from an airplane.

It follows from the plot of Eq. (8.1) in Fig. 5.13 and the molecular optical thickness spectrum that a blue sky is not inevitable. For optical thicknesses less than about 2.2, skylight irradiance relative to the solar irradiance increases with increasing optical thickness. Because the optical thickness of the molecular atmosphere increases with decreasing wavelength, and over the visible spectrum is less than about 0.36 (Fig. 8.2), skylight irradiance increases with decreasing wavelength. But for optical thicknesses greater than about 2.2, skylight irradiance decreases with increasing optical thickness. The smallest molecular optical thickness in the visible is about 0.04 (at 700 nm). Thus if the atmosphere were about 50 times thicker skylight irradiance would *decrease* with decreasing wavelength. Figure 8.7 shows calculated spectra of the zenith sky over black ground for a molecular atmosphere with the present normal optical thickness as well as for hypothetical atmospheres 10 and 40 times thicker. What we take to be inevitable is accidental: if Earth's atmosphere were much thicker the sky would not only be brighter, its color would be quite different from what it is now.

By showing that the blue sky is not inevitable, we hope to have given you a taste for thinking the unthinkable. We emphasized that the white horizon sky is not a consequence of "big particles" but occurs in a purely molecular atmosphere. Now we are going to turn this on its head and assert that "big particles" not only are not necessary for a white horizon sky, they can make it bluer than it would be otherwise.

Equation (8.3) for airlight has the same form for an atmosphere populated by molecules and particles because scattering coefficients and optical thicknesses are additive. For sufficiently large total optical thickness along a horizon path, the horizon radiance is

$$L = L_0 \Omega_{\mathrm{s}} \overline{p}(\vartheta), \tag{8.13}$$

where $\overline{p}$ is the weighted average phase function for molecules (m) and particles (p):

$$\overline{p}(\vartheta) = \frac{\beta_{\mathrm{m}}}{\beta_{\mathrm{m}} + \beta_{\mathrm{p}}} p_{\mathrm{m}}(\vartheta) + \frac{\beta_{\mathrm{p}}}{\beta_{\mathrm{m}} + \beta_{\mathrm{p}}} p_{\mathrm{p}}(\vartheta). \tag{8.14}$$

To understand the observable consequences of Eqs. (8.13) and (8.14) consider a few limiting cases. Suppose that both scattering toward the observer and total scattering are dominated by particles ($\beta_{\mathrm{p}} p_{\mathrm{p}} \gg \beta_{\mathrm{m}} p_{\mathrm{m}}$ and $\beta_{\mathrm{p}} \gg \beta_{\mathrm{m}}$):

$$L \approx L_0 \Omega_{\mathrm{s}} p_{\mathrm{p}}(\vartheta). \tag{8.15}$$

If the particles are big in the sense that angular scattering by them is independent of wavelength for scattering angles ϑ of interest, the airlight spectrum is white (i.e., that of the source L_0). No surprise here.

Now suppose that both scattering toward the observer and total scattering are dominated by molecules ($\beta_{\mathrm{m}} p_{\mathrm{m}} \gg \beta_{\mathrm{p}} p_{\mathrm{p}}$ and $\beta_{\mathrm{m}} \gg \beta_{\mathrm{p}}$):

$$L \approx L_0 \Omega_{\mathrm{s}} p_{\mathrm{m}}(\vartheta). \tag{8.16}$$

Again the radiance is that of the source because the molecular phase function is to good approximation independent of wavelength. Equation (8.14) also predicts a white horizon if the particles are sufficiently small that scattering by them has the same wavelength dependence as molecular scattering.

If molecules dominate total scattering whereas particles dominate scattering toward the observer ($\beta_{\mathrm{m}} \gg \beta_{\mathrm{p}}$ and $\beta_{\mathrm{p}} p_{\mathrm{p}} \gg \beta_{\mathrm{m}} p_{\mathrm{m}}$)

$$L \approx L_0 \frac{\beta_{\mathrm{p}} p_{\mathrm{p}}(\vartheta)}{\beta_{\mathrm{m}}}. \tag{8.17}$$

Here the horizon radiance is *inversely* related to the molecular scattering coefficient, and hence the airlight is reddish. This is a variation on the theme of distant reddish clouds discussed in the following section on colors at sunrise and sunset. Particles distributed along the entire line of sight play the same role as localized clouds.

When we consider the converse of the previous limiting case, namely total scattering dominated by particles but scattering toward the observer dominated by molecules ($\beta_{\mathrm{p}} \gg \beta_{\mathrm{m}}$ and $\beta_{\mathrm{m}} p_{\mathrm{m}} \gg \beta_{\mathrm{p}} p_{\mathrm{p}}$), we obtain a surprising result:

$$L \approx L_0 \Omega_{\mathrm{s}} \frac{\beta_{\mathrm{m}} p_{\mathrm{m}}(\vartheta)}{\beta_{\mathrm{p}}}. \tag{8.18}$$

For this example, the horizon airlight is bluish. To understand this perhaps contra-intuitive result we return to the example of a pure molecular atmosphere for which the horizon sky is white even though the scatterers are selective. Scattering of sunlight toward the observer favors light at the short wavelength end of the visible spectrum. If this light were transmitted without attenuation, the airlight would be bluish. But it is impossible for molecules to scatter light toward the observer without also scattering some of this light out of the line of sight. This selective attenuation of light scattered toward the observer favors the long wavelength end of the spectrum. For a sufficiently long optical path selective scattering toward the observer is exactly balanced by selective scattering out of the line of sight.

Now we can better understand why big particles can, contrary to what might be expected, make the horizon sky bluer than it would otherwise be. Given the assumptions underlying Eq. (8.18) we can write the airlight radiance as

$$L \approx L_0 \Omega_{\mathrm{s}} \beta_{\mathrm{m}} p_{\mathrm{m}}(\vartheta) \int_0^d \exp\{-(\beta_{\mathrm{m}} + \beta_{\mathrm{p}})x\}\, dx. \tag{8.19}$$

The factor $\beta_{\mathrm{m}} p_{\mathrm{m}}$ is the wavelength-dependent scattering toward the observer and is the same everywhere along the line of sight. The integral is an attenuation function; the exponential term in the integral is the probability that light scattered at x toward the observer will not be scattered again in traversing this distance. Although light scattered at all points on the line of sight contributes to the radiance, most of the contribution comes from scattering at distances less than about $3/(\beta_{\mathrm{m}} + \beta_{\mathrm{p}})$, which is approximately $3/\beta_{\mathrm{p}}$ if $\beta_{\mathrm{p}} \gg \beta_{\mathrm{m}}$. Over such distances, however, molecular scattering does not greatly redden the transmitted light (i.e., $\exp\{-\beta_{\mathrm{m}} x\} \approx 1$ for $x < 3/\beta_{\mathrm{p}}$). Thus the color balance is not restored by attenuation as it was for a pure molecular atmosphere.

In his famous book, *The Nature of Light and Color in the Open Air*, Marcel Minnaert notes that "to this day there are scientists who do not consider the problem of the blue sky as being definitively solved…On very exceptional days, occurring perhaps not even once a year, the sky is beautifully blue right down to the horizon. Observations on days like these should be carefully recorded and described…for according to the theory of scattering, such a phenomenon is impossible: with layers of such thickness, the air ought to appear white." Yet the "theory of scattering" [Eq. (8.18)] does show why the sky can be beautifully blue right down to the horizon, although Minnaert was correct in saying that this is "exceptional." The concentration of particles has to be high enough that *total* scattering is dominated by them, but sufficiently low that *differential* scattering is dominated by molecules. This is possible because scattering by molecules does not vary much with scattering angle whereas scattering by particles comparable with or larger than the wavelength is highly peaked in the forward direction and drops by several decades toward the backward direction (see Sec. 3.5).

8.1.2 Sunrise and Sunset

If short-wavelength light is preferentially scattered *out* of direct sunlight, long-wavelength light must be preferentially transmitted *in* the direction of sunlight. Transmission is exponential if multiple scattering is negligible (see Sec. 5.2):

$$L = L_0 \exp(-\tau), \tag{8.20}$$

where L is the radiance in the direction of the sun, L_0 is that of sunlight outside the atmosphere, and τ is the optical thickness along the line of sight. If the wavelength dependence of τ follows Rayeigh's scattering law, transmitted sunlight is reddened, comparatively richer at the long-wavelength end of the visible spectrum than the incident light. But to say that sunlight is reddened is not to say that it is red. The perceived color can be yellow, orange, or red depending on the magnitude of the optical thickness. Equation (8.20) applies to the radiance only in the direction of the sun. Yet oranges and reds can be seen in other directions because reddened sunlight illuminates scatterers that are not on the line of sight to the sun. A striking example of this is a horizon sky tinged with oranges and pinks in the direction *opposite* the sun.

In an atmosphere free of all particles the optical thickness along a path from the sun, even on or below the horizon, is not sufficient to give perceptually red transmitted light. Although selective scattering by molecules yields a blue sky, reds are not possible in a molecular atmosphere, only yellows and oranges. Although this can be proven by the kind of colorimetric analysis in Section 4.3, Nature itself provides the proof. On exceptionally clear days the horizon sky at sunrise or sunset may be tinged with yellow or orange but not red.

The color and brightness of the sun changes as it arcs across the sky because the optical thickness along the line of sight to it changes with solar zenith angle Θ. If Earth were flat, as some still aver, the transmitted solar radiance would be

$$L = L_0 \exp(-\tau_{\mathrm{n}}/\cos\Theta). \tag{8.21}$$

This equation is a good approximation except near the horizon. On a flat Earth, the optical thickness is infinite for horizon paths. On a spherical Earth, all optical thicknesses are finite although much larger for horizon than for vertical paths (Fig. 8.4).

Variations on the theme of reds and oranges at sunrise and sunset can be seen even when the sun is overhead. The radiance at an observer an optical distance τ from a horizon cloud is the sum of transmitted cloudlight and airlight:

$$L = L_0 G\{1 - \exp(-\tau)\} + L_0 G_{\mathrm{c}} \exp(-\tau), \tag{8.22}$$

which is an extension of Eq. (8.3). If the cloud is approximated as an isotropic reflector with reflectivity R and illuminated at an angle Φ from the normal to it, the cloud geometrical factor G_{c} is $\Omega_{\mathrm{s}} R \cos\Phi$. If $G_{\mathrm{c}} > G$ the observed radiance is redder than the incident radiance, but if

$G_\mathrm{c} < G$ the observed radiance is bluer than the incident radiance. Thus distant horizon clouds can be reddish if they are bright or bluish if they are dark.

Underlying Eq. (8.22) is the implicit assumption that the line of sight is uniformly illuminated by sunlight. The first term in this equation is airlight; the second is transmitted cloudlight. Suppose, however, that the line of sight is shadowed from direct sunlight by clouds that do not occlude the distant clouds. This may reduce the first term in Eq. (8.22) so that the second term dominates. Thus under a partly overcast sky, distant horizon clouds may be reddish even when the sun is high in the sky.

Small particles affect the color of the low sun out of proportion to their normal optical thickness because they are concentrated more toward the surface. The scale height for molecules is about 8 km whereas that for particles is typically 1–2 km. Subject to the approximations underlying Eq. (8.7), the ratio of the tangential (horizon) optical thickness for particles τ_tp to that for molecules τ_tm is

$$\frac{\tau_\mathrm{tp}}{\tau_\mathrm{tm}} = \frac{\tau_\mathrm{np}}{\tau_\mathrm{nm}}\sqrt{\frac{H_\mathrm{m}}{H_\mathrm{p}}}, \tag{8.23}$$

where m denotes molecules and p particles. Because of the incoherence of scattering by atmospheric molecules and particles, scattering coefficients are additive, and hence so are optical thicknesses. Even for equal normal optical thicknesses, the tangential optical thickness for particles is more than twice that for molecules. As we noted previously, molecules by themselves cannot give red sunsets and sunrises. Molecules need the help of small particles, and for a fixed normal optical thickness for particles, their tangential optical thickness is greater the more they are concentrated near the surface.

For equal normal optical thicknesses, particles also disproportionately increase the rate at which transmitted radiance decreases with angle near the horizon:

$$\left(-\frac{1}{L}\frac{\partial L}{\partial \Theta}\right)_{\Theta=\pi/2} = \left(\frac{\partial \tau_\mathrm{m}}{\partial \Theta} + \frac{\partial \tau_\mathrm{p}}{\partial \Theta}\right)_{\Theta=\pi/2} = \tau_\mathrm{np}\frac{R}{H_\mathrm{p}} + \tau_\mathrm{nm}\frac{R}{H_\mathrm{m}}, \tag{8.24}$$

which follows from Eq. (8.7) and Eq. (8.20) with $\tau = \tau_\mathrm{m} + \tau_\mathrm{p}$. The rate of decrease because of particles can be so great that the color of the setting sun varies across its diameter, from yellow at its top, to red at its bottom.

8.1.3 Ozone and the Twilight Sky

No theory ought to be accepted until it has been pushed to its limits. The theory of the blue sky discussed in previous sections gives cause for doubts when we consider the overhead sky at or around sunset. Even without doing any calculations we ought to suspect a discrepancy between theory and observations. The source of light illuminating molecules and particles along a vertical path at sunset is sunlight that has been appreciably reddened (i.e., its spectrum skewed toward the long-wavelength end of the spectrum) over long tangential atmospheric paths. Selective scattering by molecules does not create blue light: if red light illuminates molecules the scattered light is red. So how can the overhead sky be blue when the sun is low on the horizon? Solely by scattering it cannot. To show this quantitatively we appeal to the uniform, finite atmosphere approximation (Fig. 8.8).

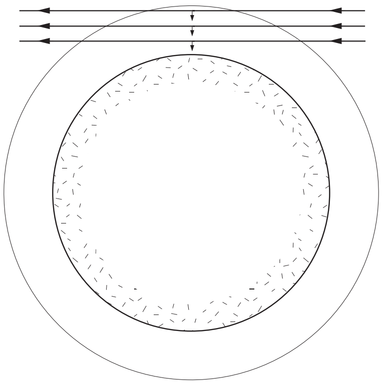

Figure 8.8: With the sun on the horizon, an observer looking straight overhead sees light scattered along a vertical path, where the source of this light is sunlight attenuated by different amounts along horizontal paths. The scale here is greatly distorted.

If we neglect multiple scattering, the total overhead radiance as a consequence of scattering of attenuated sunlight is

$$L = L_0 G \int_0^H \beta \exp\{-\beta(d+z)\}\, dz = L_0 G \int_0^H \beta \exp\{-\tau(z)\}\, dz, \tag{8.25}$$

where G is a geometrical factor of no concern here, d is the path length of sunlight incident at height z above the surface, and τ is the optical thickness. Because the radius R of Earth is much greater than the scale height H, to good approximation

$$d \approx \sqrt{2R(H-z)}. \tag{8.26}$$

Even without evaluating Eq. (8.25) it should be evident that L does not correspond to bluish light. Although the scattering coefficient β is greater at the short-wavelength end of the spectrum, the wavelength dependence of the exponential function is just the reverse. And because the optical thickness lies between τ_{n} (normal) and τ_{t} (tangential), the exponential function dominates. If the variable of integration is transformed to $H-u$, H neglected in comparison with R, the variable of integration transformed to u/H and finally to u^2, Eq. (8.25) becomes

$$L \approx 2L_0 G \exp\{-\tau_{\mathrm{n}}\}\tau_{\mathrm{n}} \int_0^1 \exp\{-\tau_{\mathrm{t}} u\}\, u\, du. \tag{8.27}$$

The integral in this equation is approximately $1/\tau_{\mathrm{t}}^2$ because $\exp\{-\tau_{\mathrm{t}}\} \ll 1$. The final result is the tidy expression

$$L \approx L_0 G \frac{H}{R} \frac{\exp\{-\tau_{\mathrm{n}}\}}{\tau_{\mathrm{n}}}. \tag{8.28}$$

As a check on the correctness of this result note that $L \to 0$ as $H/R \to 0$. That is, when the atmosphere doesn't exist ($H = 0$) or Earth is flat, and hence the horizon path length is

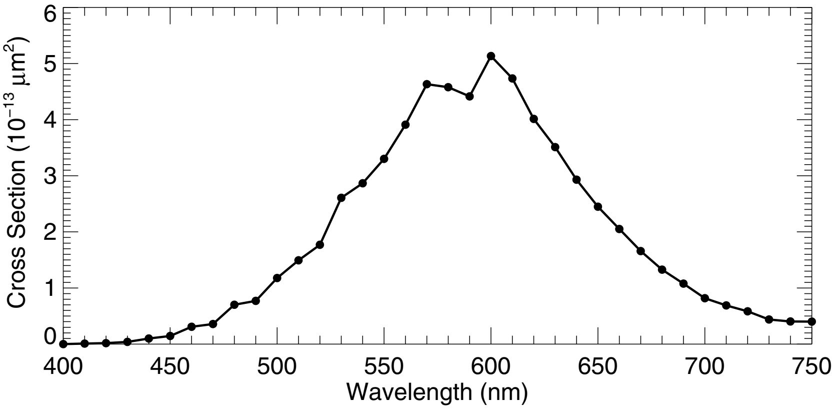

Figure 8.9: Absorption cross section of ozone for 238 K and 10 mb total pressure obtained from the line-by-line code developed by Clough *et al.* (1992) and cited at the end of Chapter 2. The Chappuis bands of ozone, originating from electronic transitions, do not depend strongly on temperature and pressure.

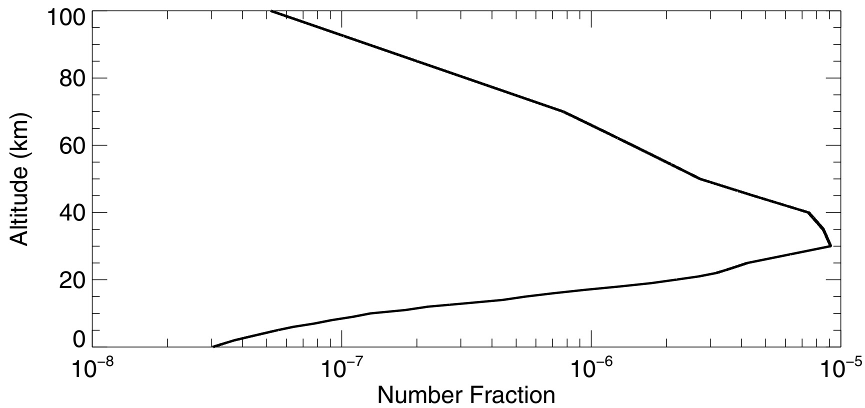

Figure 8.10: Mid-latitude summer ozone profile taken from McClatchey *et al.* (1972) cited at the end of Chapter 2.

infinite, the overhead radiance must vanish. Equation (8.28) is just the inverse of the blue sky: wavelength to the fourth power (approximately) instead of the inverse fourth power.

What is missing from this analysis is absorption by ozone. We briefly mention the Chappuis bands of ozone in Section 2.8.2, which are shown in Fig. 2.12. Figure 8.9 shows these bands in more detail on a linear wavelength scale. Note the broad absorption peak in the middle of the visible spectrum. Because photo-dissociation of oxygen into atomic oxygen

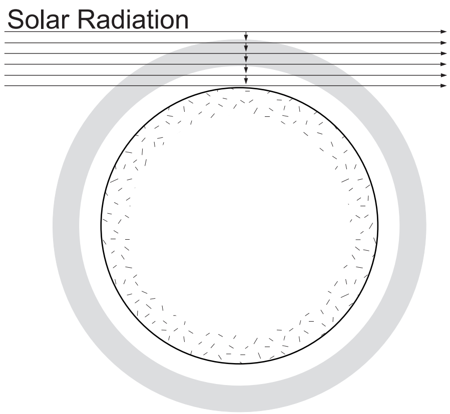

Figure 8.11: Path lengths, and hence absorption optical thicknesses, through an ozone layer concentrated mostly between about 20 km and 40 km (shaded) are much greater when the sun is on the horizon than when overhead. The scale of this figure is greatly distorted

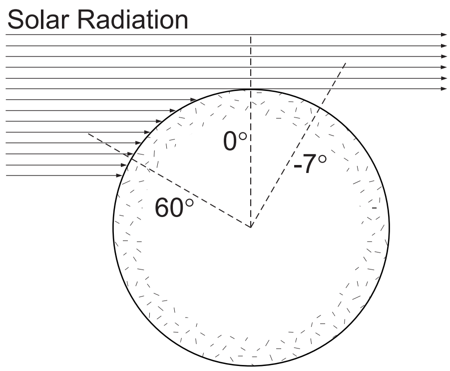

Figure 8.12: Dashed lines intersect Earth at points for which the sun elevation is 60°, on the horizon, and below the horizon. The angle below the sun is exaggerated (calculations were done for -7°) to show that much of the observer's line of sight is not illuminated by direct sunlight.

and subsequent combination of atomic and molecular oxygen is the source of ozone in the atmosphere, excluding lightning and anthropogenic sources, its concentration is sharply peaked between about 20 km and 40 km (Fig. 8.10). Figure 8.11 depicts this layer of ozone, path lengths through which depend on the solar elevation angle: the lower the sun, the greater the path lengths and hence absorption optical thicknesses.

Calculations of the radiance spectrum of the overhead sky for a molecular atmosphere with and without ozone are shown in Fig. 8.13 for solar elevation angles 60°, 0° (sun on the horizon), and -7° (sun below the horizon). For these calculations the uniform atmosphere approximation was not used because most ozone is well above the scale height $H \approx 8\,\mathrm{km}$. Figure 8.12 depicts the position of the observer for these three solar elevations.

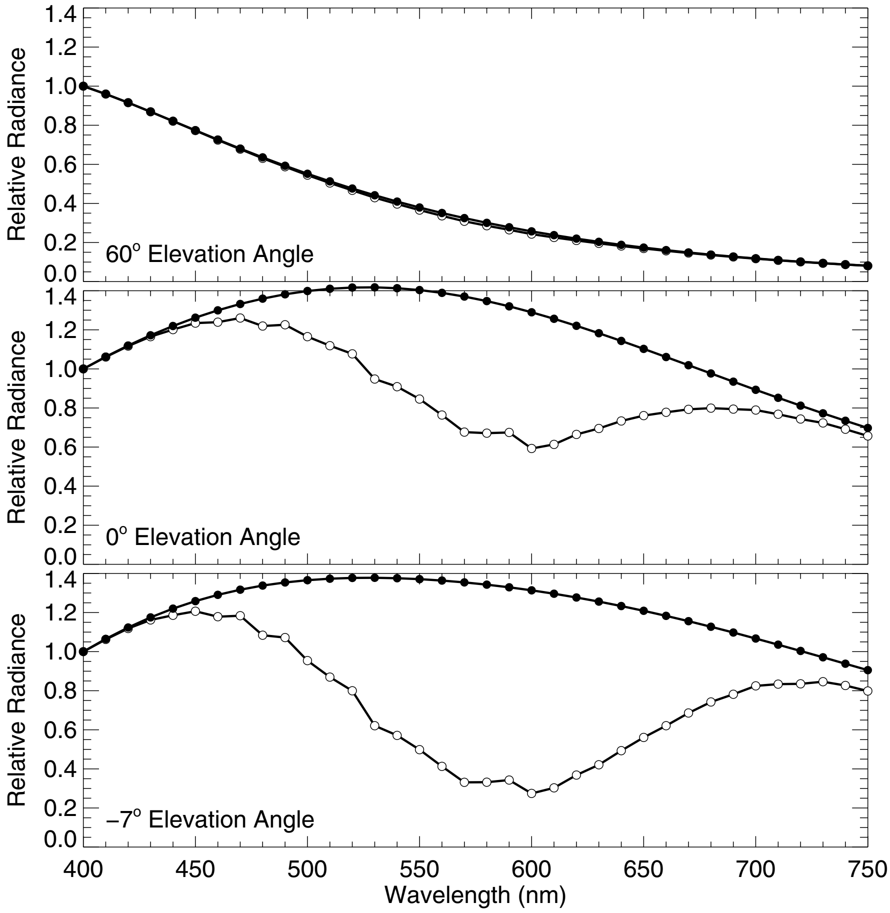

Figure 8.13: Calculated radiance spectra (normalized to 1 at 400 nm) of the overhead sky for a molecular atmosphere with (open circles) and without (closed circles) absorption by ozone for three different solar elevation angles. The spectrum of the illumination is a 6000 K blackbody, which approximates the solar spectrum outside Earth's atmosphere.

When the sun is 60° above the horizon, radiance spectra with and without absorption by ozone are indistinguishable (Fig. 8.13). And this is more or less true even when the sun is as low as 10° above the horizon. Thus over much of the day ozone plays no essential role in the blue of the sky. But when the sun is near or below the horizon, the overhead sky would not be blue without ozone. Absorption by ozone takes a big bite out of the middle of the radiance spectrum. But there is more to this story than just the *amount* of ozone: the radiance spectrum depends on *where* the ozone resides. Calculated radiance spectra (Fig. 8.14) for the sun on the horizon and for a fixed integrated amount of ozone but uniformly distributed between the surface and 15 km, between 20 km and 35 km, and between 85 km and 100 km

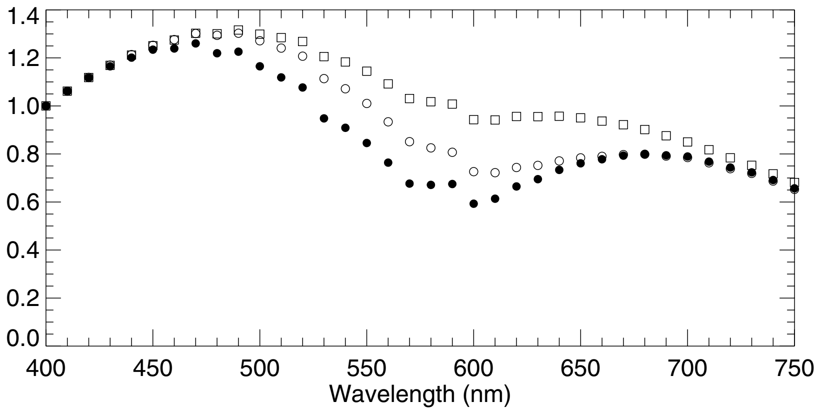

Figure 8.14: Calculated radiance spectrum (normalized to 1 at 400 nm) of the overhead sky with the sun on the horizon for a fixed total amount of ozone but distribted differently: a uniform layer between 20 km and 35 km (solid circles), a uniform layer from the surface to 15 km (open circles), and a uniform layer from 85 km to 100 km (squares). The spectrum of the illumination is a 6000 K blackbody, which approximates the solar spectrum outside Earth's atmosphere.

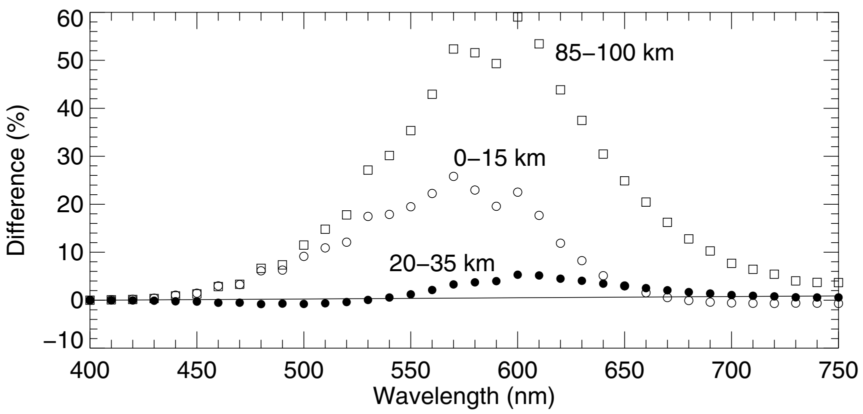

Figure 8.15: Relative difference between the radiance spectrum of the overhead sky with the sun on the horizon for a fixed total amount of ozone in a uniform layer between 20 km and 35 km (solid circles), between the surface and 15 km (open circles), and between 85 km and 100 km (squares), and the radiance spectrum corresponding to the non-uniform distribution shown in Fig. 8.10. The spectrum of the illumination is a 6000 K blackbody, which approximates the solar spectrum outside Earth's atmosphere.

are different than that for ozone distributed as shown in Fig. 8.10. The difference is greatest for the uniform 85–100 km layer, least for the 20–35 km layer, which roughly corresponds to the actual non-uniform distribution (Fig. 8.15).

This result strengthens our assertion that the blue sky is accidental rather than inevitable even though scattering by individual air molecules obeys Rayleigh's scattering law. A blue overhead sky for Earth's atmosphere from dawn to dusk requires not only a sufficiently small optical thickness but ozone in just the right amount and in the right place.

8.2 Atmospheric Visual Range

On a clear day can we really see forever? If not, how far can we see? To answer this question requires qualifying it by restricting viewing to more or less horizontal paths during daylight. Stars at staggering distances can be seen at night, partly because there is little skylight to reduce contrast, partly because stars overhead are seen in directions for which attenuation by the atmosphere is least.

We are careful to distinguish between visibility, a *quality*, and visual range, a *quantity*. One can say that visibility is good or poor but not that it is 10 km or 100 km. W. E. Knowles Middleton, whose book *Vision in the Atmosphere* is the standard work on atmospheric visibility, inveighed against the careless confutation of these two terms. He didn't have much effect, and neither will we, so all we can do is express our scorn for folks, especially learned doctors of science, so devoid of linguistic sense that they can't distinguish a quality from a quantity.

The radiance in the direction of a black object is not zero because of airlight (Sec. 8.1). At sufficiently large distances, this airlight is indistinguishable from the horizon sky. An example is a phalanx of parallel dark ridges, each ridge brighter than those in front of it (Fig. 8.16). The farthest ridges blend into the horizon sky. Beyond some distance we cannot see ridges because of insufficient contrast.

Figure 8.16: The brightness of each of these ridges, all covered with the same dark vegetation, increases with increasing distance, and hence their contrast with the horizon sky decreases.

Equation (8.3) gives the airlight radiance L, from which we obtain the airlight luminance B, which is what humans sense, by integrating over the visible spectrum:

$$B = K \int V(\lambda) L(\lambda)\ d\lambda, \tag{8.29}$$

where V is the luminous efficiency of the human eye and K is a constant of no concern here (see Sec. 4.1). The *contrast* C between any object, with luminance B, and the horizon sky is

$$C = \frac{B - B_\infty}{B_\infty}, \tag{8.30}$$

where B_∞ is the horizon luminance. For a uniformly illuminated line of sight of length d, uniform in its scattering properties, and over black ground, the contrast obtained from Eq. (8.3) is

$$C = -\frac{\int GVL_0 \exp(-\beta d)\ d\lambda}{\int GVL_0\ d\lambda}, \tag{8.31}$$

where we assume an infinite horizon optical thickness. This ratio of integrals defines an average (over the visible spectrum) optical thickness

$$\langle \tau \rangle = -\ln |C| \tag{8.32}$$

and a corresponding scattering coefficient

$$\langle \beta \rangle = \frac{\langle \tau \rangle}{d}. \tag{8.33}$$

Equation (8.32) for contrast reduction with optical thickness is formally, but not physically, identical to the expression for exponential attenuation at any wavelength of radiance [Eq. (8.20)], which perhaps is responsible for the misconception that atmospheric visibility is reduced because of attenuation. But if there is no light from a black object to be attenuated, its finite visual range cannot be a consequence of attenuation.

The distance beyond which a black object cannot be distinguished from the horizon sky depends on the *contrast threshold*, the smallest absolute value of contrast detectable by a human observer. Although this depends on the particular observer, the angular size of the object observed, the presence of nearby objects, and the absolute luminance, a contrast threshold of 0.02 is often taken as a typical value. To find the visual range for this threshold we have to evaluate the integrals in Eq. (8.31) numerically for various values of d to find the one for which the right side of this equation is -0.02. But if β is independent of wavelength the solution is simply

$$\beta d = 3.9. \tag{8.34}$$

This equation is often called Koschmieder's law, although W. E. Knowles Middleton notes that "there can be no doubt that Bouguer was quite clear about the main factors determining the horizontal visual range, and that he effectively stated the law which has lately been called Koschmieder's law."

The scattering coefficient for molecules, however, is not independent of wavelength, although the geometrical factor G is. The function V is fairly sharply, and symmetrically, peaked around 550 nm. Because the molecular scattering coefficient decreases, and hence the exponential function in Eq. (8.31) increases with increasing wavelength, the average molecular scattering coefficient must correspond to wavelengths somewhat greater than 550 nm. We numerically solved Eq. (8.31) for a pure molecular atmosphere at standard pressure (1013 mb) and temperature (0 °C) to obtain $d = 330$ km, which corresponds to a wavelength of about 560 nm. Because the wavelength dependence of scattering is steepest for molecules and particles small compared with the wavelength, we usually can estimate the visual range from Eq. (8.34) using the scattering coefficient at or around 560 nm.

According to this analysis, therefore, "forever" is around 330 km, the maximum visual range at which a black object at sea level can be distinguished from the horizon sky for a purely molecular atmosphere, assuming a contrast threshold of 0.02 and also ignoring the curvature of the Earth. This maximum is more than twice the visual range considered exceptionally high. From this we conclude that almost always visual range is limited by particles.

We also observe contrast between elements of the same scene, a hillside mottled with stands of trees and forest clearings, for example. The extent to which we can discern details in such a scene depends on sun angle as well as distance. The airlight radiance for a black object is given by Eq. (8.3), whereas that for a reflecting object is given by Eq. (8.22). These two equations can be combined to give the contrast between adjacent reflecting and non-reflecting objects

$$C = \frac{-\Gamma \exp(-\tau)}{1 + (\Gamma - 1)\exp(-\tau)}, \tag{8.35}$$

where

$$\Gamma = \frac{R \cos \Phi}{\pi p(\Theta)}. \tag{8.36}$$

Although Eq. (8.35) specifies the contrast of radiance, this equation is a good approximation to the luminance contrast if we take τ to be in the middle of the visible spectrum.

All else being equal, contrast decreases as p increases. And as we show in Section 3.5, p is more sharply peaked in the forward direction the larger the scatterer. Thus we expect the details of a distant scene to be less distinct when looking toward than away from the sun if the optical thickness of the line of sight has an appreciable component contributed by particles comparable with or larger than the wavelength. Indeed, on many occasions we have observed marked improvements in contrast on a distant ridge or mountain to the east from morning to late afternoon despite no obvious change in particle concentration.

The misconception that water vapor is a powerful scatterer of sunlight is probably largely a consequence of the common observation that on humid, hazy days, visibility is often depressingly poor. But haze is not water vapor, rather water that has *ceased* to be vapor. At high relative humidities, but still well below 100%, small soluble particles in the atmosphere accrete liquid water to become solution droplets. Although these droplets are much smaller than cloud droplets, they markedly diminish visual range because of the sharp increase in scattering with particle size (see Fig. 3.11). Because of coherence, the same number of water molecules when aggregated in haze scatter vastly more than when apart, increasing the scattering coefficient and therefore decreasing visual range.

8.3 Atmospheric Refraction

Atmospheric refraction is a consequence of molecular scattering, which is rarely stated given the historical accident that before light and matter were well understood refraction and scattering were locked in separate compartments and subsequently have been sequestered more rigidly than monks and nuns in neighboring cloisters.

Consider a beam of light propagating in an optically homogeneous medium. Light is scattered laterally to the beam, weakly but observably, and more strongly in the same direction as the beam (i.e., the forward direction). The observed beam is a coherent superposition of incident light and forward-scattered light excited by the incident light. Although real refractive indices are often defined by ratios of phase velocities (see Sec. 3.5.1), we may also look upon the real refractive index as a parameter that specifies the phase shift between an incident beam and forward-scattered light. The connection between incoherent scattering and refraction, coherent scattering, can be divined from the expression for the refractive index n of a gas and that for the scattering cross section σ_{s} of a gas molecule:

$$n = 1 + \frac{1}{2}\alpha N, \tag{8.37}$$

$$\sigma_{\mathrm{s}} = \frac{k^4}{6\pi}\left|\alpha\right|^2, \tag{8.38}$$

where N is the number (not mass) density of gas molecules, k is the wavenumber of the incident light, and α is the polarizability of the molecule (i.e., induced dipole moment per unit incident inducing electric field). The appearance of the polarizability in Eq. (8.37) but its square in Eq. (8.38) is the clue that refraction is associated with electric fields whereas scattering is associated with electric fields squared. Scattering without qualification often means incoherent scattering in all directions. Refraction, in a nutshell, is coherent scattering in a particular direction.

8.3.1 Terrestrial Mirages

Mirages are not illusions, any more so than are reflections in a pond. Reflections of plants growing at its edge are not interpreted as plants growing into the water. If the water is ruffled by wind, the reflected images may be so distorted that they are no longer recognizable as those of plants. Yet we would not call such distorted images illusions. And so it is with mirages. They are images noticeably different from what they would be in the absence of atmospheric refraction, creations of the atmosphere, not of the mind. An example of a true illusion is the *moon illusion*, a moon seen to be larger on the horizon than overhead. This seemingly enlarged moon is a creation of the mind, not the atmosphere. And yet the moon illusion is still often attributed to atmospheric refraction even though this has been known not to be true for at least 1000, possibly 2000 years, and can be verified by simple measurements of the angular size of the moon at different elevations.

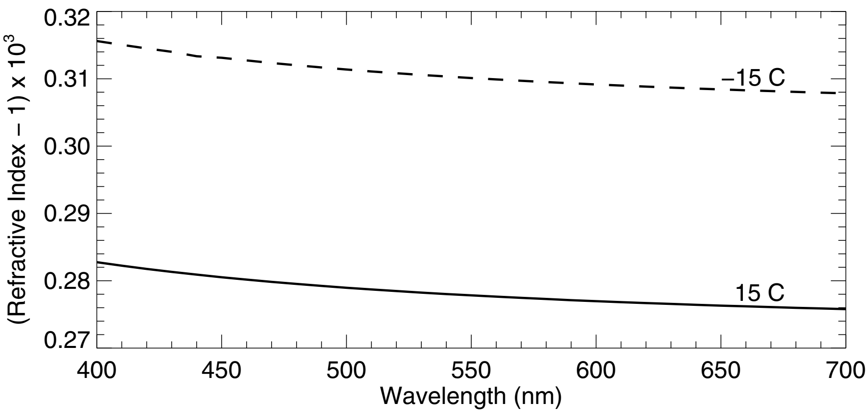

Figure 8.17: Refractive index of dry air at a pressure of one atmosphere and for the two temperatures noted. From the compilation by Penndorf (1957).

Mirages are vastly more common than is realized. Look and you shall see them. Contrary to popular opinion, they are not unique to deserts. Mirages can be seen frequently even over ice-covered landscapes and highways flanked by deep snowbanks. Temperature *per se* is not what produces mirages but rather temperature *gradients*.

Because air is a mixture of gases, the polarizability for air in Eq. (8.37) is an average over all its molecular constituents, although their individual polarizabilities are about the same at visible and near-visible wavelengths. The vertical refractive index gradient can be written so as to show its dependence on pressure p and absolute temperature T by way of the ideal gas law $p = Nk_{\mathrm{B}}T$ and Eq. (8.37):

$$\frac{d}{dz}\ln(n-1) = \frac{1}{p}\frac{dp}{dz} - \frac{1}{T}\frac{dT}{dz}. \tag{8.39}$$

Pressure decreases approximately exponentially with height [i.e., $\exp(-z/H)$], where the scale height H is about 8 km. The first term on the right side of Eq. (8.39) is therefore about $0.1\ \mathrm{km}^{-1}$. Temperature usually decreases with height in the atmosphere. An average lapse rate of temperature (i.e., its decrease with height) is about $6\ °\mathrm{C\ km}^{-1}$. A characteristic temperature in the troposphere, within about 15 km of the surface, is 280 K. Thus the magnitude of the second term in Eq. (8.39) is about $0.02\ \mathrm{km}^{-1}$. On average, therefore, the refractive index gradient is dominated by the vertical pressure gradient. But within a few meters of the surface, conditions are far from average. On a sun-baked highway your feet may be touching asphalt at 50 °C while your nose is breathing air at 35 °C, which corresponds to a lapse rate thousands of times the average. Moreover, temperature near the surface can increase with height. In shallow surface layers, in which pressure is nearly constant, the temperature gradient dominates the refractive index gradient. In such shallow layers mirages, which are caused by refractive index gradients, are seen.

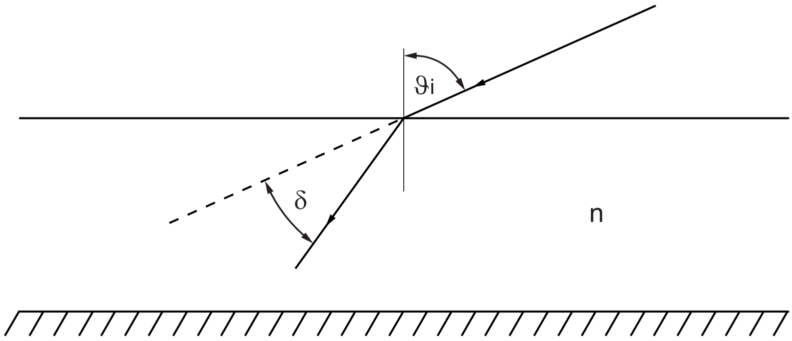

Figure 8.18: Deviation of incident light because of refraction by a uniform slab with refractive index n.

Cartoonists with fertile imaginations unfettered by science and careless textbook writers have engendered the notion that atmospheric refraction can work wonders, lifting images of ships, for example, from the sea high into the sky. A back-of-the-envelope calculation dispels such notions. The refractive index of air at sea level is about 1.0003 (Fig. 8.17). Light from free space incident on a uniform slab (Fig. 8.18) with this refractive index is displaced from where it would have been in the absence of refraction by an angle δ given by Snel's law

$$\sin\vartheta_\mathrm{i} = n\sin\vartheta_\mathrm{t} = n\sin(\vartheta_\mathrm{i} - \delta) = n(\sin\vartheta_\mathrm{i}\cos\delta - \sin\delta\cos\vartheta_\mathrm{i}), \tag{8.40}$$

which at glancing incidence ($\vartheta_\mathrm{i} = 90\ °$) yields

$$\cos\delta = \frac{1}{n}. \tag{8.41}$$

Because $n \approx 1$, and hence $\delta \ll 1$, we can approximate Eq. (8.41) as

$$\delta \approx \sqrt{2(n-1)}. \tag{8.42}$$

For $n - 1 = 0.0003$, Eq. (8.42) gives an angular displacement of about $1.4°$, which is a rough upper limit.

Trajectories of light rays in nonuniform media can be expressed in different ways. According to Fermat's principle of least time, which ought to be *extreme* time, the actual path taken by a ray between two points is such that the path integral

$$\int_1^2 n\ ds \tag{8.43}$$

is an extremum; strictly, this integral is *stationary*, which includes the possibility of a point of inflection. That is, of all possible paths between 1 and 2, that taken by a light ray is such that Eq. (8.43) is either a minimum (least time) or a maximum (greatest time). Why time?

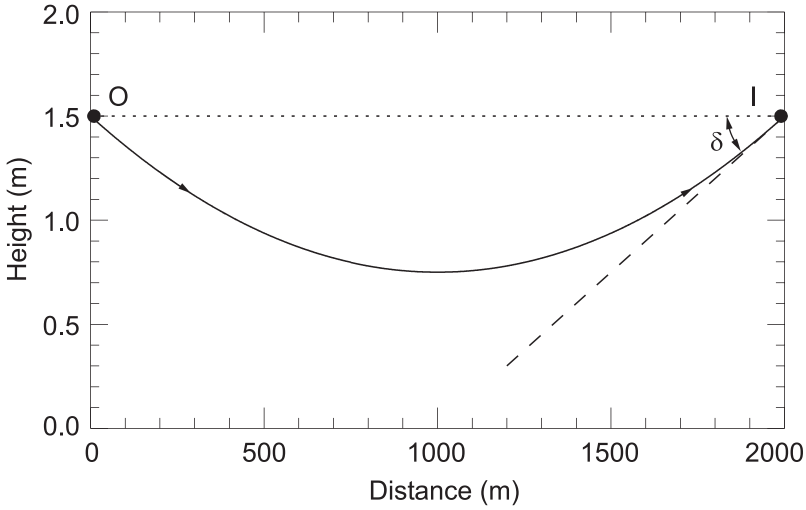

Figure 8.19: Ray trajectory from object point O to image point I in air with a temperature decreasing at a rate more than 100 times the average rate in Earth's atmosphere. To an observer at I it is as if the light from O comes from an object displaced downward from the line of sight OI by an angle δ. Note that the horizontal and vertical scales differ by a factor of about 600, which creates the impression that δ can be much larger than it is in reality ($\sim 1^\circ$).

Because n is the ratio c/v, where c is a universal constant (the free-space speed of light) and v is the phase speed, and hence except for the constant factor c, Eq. (8.43) has the dimensions of time. But this time is *not* the time it would take a signal to propagate from 1 to 2 except in a non-dispersive medium (see Sec. 3.5). The principle of least time has inspired piffle about the alleged efficiency of nature, which directs light over routes that minimize travel time, presumably giving light more time to attend to important business at its destination.

The scale of terrestrial mirages is such that in analyzing them we may pretend that Earth is flat. On such a planet, with an atmosphere in which the refractive index varies only in the vertical, Fermat's principle yields a generalization of Snel's law:

$$n \sin \vartheta = \text{constant} = C, \tag{8.44}$$

where ϑ is the angle between the ray and the vertical direction. We could have bypassed Fermat's principle to obtain this result.

A ray in such a medium is a curve $z = f(y)$, where the yz-plane is the plane of incidence and z is the vertical coordinate. The slope of this curve is

$$\frac{dz}{dy} = \tan(\pi/2 - \vartheta) = \frac{1}{\tan \vartheta}. \tag{8.45}$$

Square Eqs. (8.44) and (8.45) and combine them to obtain

$$\left(\frac{dz}{dy}\right)^2 = \frac{n^2 - C^2}{C^2}. \tag{8.46}$$

Take the derivative with respect to y of both sides:

$$\frac{d^2z}{d^2y} = n\frac{dn}{dz}\frac{1}{C^2}. \tag{8.47}$$

Here $n \approx 1$, and if we restrict ourselves to nearly horizontal rays (i.e., $\vartheta \approx \pi/2$), we can set both n and C equal to 1 in Eq. (8.47) to obtain the approximate differential equation satisfied by nearly horizontal rays:

$$\frac{d^2z}{d^2y} = \frac{dn}{dz}. \tag{8.48}$$

This equation shows that terrestrial mirages are a consequence of vertical refractive index gradients: if this gradient is zero, ray paths are straight lines.

For a constant refractive index gradient, which to good approximation occurs for a constant temperature gradient, the solution to Eq. (8.48) is a parabola. One such parabola, for a constant lapse rate more than 100 times the average, is shown in Fig. 8.19. Note the greatly different horizontal and vertical scales. If we had plotted the parabola to uniform scale its curvature would not have been noticeable. The image is displaced downward from what it would be in the absence of the atmosphere, strictly in the absence of a vertical refractive index gradient. That is, an observer at I sees light that originated from O coming from a direction below (in angle) the straight line between O and I: hence the designation *inferior mirage*. This is the familiar mirage seen over highways warmer than the air above them. The downward angular displacement is

$$\delta = \frac{1}{2}s\frac{dn}{dz}. \tag{8.49}$$

This was obtained by solving Eq. (8.48), then determining the two constants of integration by requiring the ray to go through the points (h, 0) and (h, s), where h is the height and s the horizontal distance between object (O) and image (I). The displacement is then

$$\tan\delta = \left(\frac{dz}{dy}\right)_{y=s} \approx \delta. \tag{8.50}$$

Even for temperature gradients 1000 times the average lapse rate, angular displacements of mirages are less than a degree at distances of a few kilometers.

If temperature increases with height, as it might, for example, in air over a colder sea, the resulting mirage is called a *superior mirage*. The refractive index gradient in Eq. (8.49) changes sign, as does δ. Inferior and superior are not designations of lower and higher castes but rather of displacements downward and upward.

For a constant temperature gradient, one and only one parabolic ray trajectory connects an object point to an image point. Multiple images therefore are not possible. But temperature gradients close to the ground are rarely linear. The upward transport of energy from a hot surface occurs by molecular conduction through a stagnant boundary layer of air. Somewhat above the surface, however, energy is transported by air in motion. As a consequence the temperature gradient steepens near the ground if the energy flux is constant. This variable gradient can lead to two consequences: magnification and multiple images.

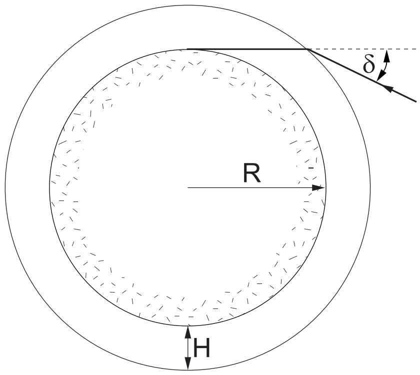

Figure 8.20: When the sun is at an angle δ (a fraction of a degree) below the horizon it still can be seen because of atmospheric refraction. The relative value of the scale height H to Earth's radius R is greatly exaggerated in this figure.

According to Eq. (8.49) all image points at a given horizontal distance are displaced downward by the same amount proportional to the constant refractive index gradient. This suggests that the closer an object point is to a surface, where the temperature is greatest, the greater the downward displacement of the corresponding image point. Thus nonlinear vertical temperature profiles may magnify images. Magnification in the optical sense is an increase in angular size, which is all that human observers directly perceive. Transforming angular sizes into linear sizes (lengths) is a complicated perceptual process. There is far more to seeing than the formation of images on retinas.

Multiple images are seen frequently on highways. What often appears to be water on a highway but evaporates before the water is reached is the inverted image of either the horizon sky or horizon objects brighter than the highway.

8.3.2 Extraterrestrial Mirages

When we turn from mirages of terrestrial objects to those of extraterrestrial bodies, most notably the sun and moon, we can no longer pretend that Earth is flat. But we can pretend that its atmosphere is uniform and bounded with a constant refractive index equal to the surface value n_0. The integrated refractive index of a vertical ray from the surface to infinity is the same in an atmosphere with an exponentially decreasing molecular number density as in a hypothetical atmosphere with a uniform density equal to the surface value up to the scale height H.

A ray refracted along a horizon path by this hypothetical atmosphere and originating from outside it (Fig. 8.20) must have been incident on it from an angle δ below the horizon. From Snel's law we have

$$\sin\vartheta_{\mathrm{i}} = \sin(\vartheta_{\mathrm{t}} + \delta) = n_0 \sin\vartheta_{\mathrm{t}} = \sin\vartheta_{\mathrm{t}}\cos\delta + \cos\vartheta_{\mathrm{t}}\sin\delta, \tag{8.51}$$

from which it follows that

$$\sin\delta = \tan\vartheta_{\mathrm{t}}(n_0 - \cos\delta), \tag{8.52}$$

where

$$\tan\vartheta_{\mathrm{t}} = \frac{R}{\sqrt{2HR + H^2}} \approx \sqrt{\frac{R}{2H}}. \tag{8.53}$$

The radius R of Earth is much greater than the scale height of its atmosphere. Given that we expect $\delta \ll 1$ because $n \approx 1$, we can approximate $\sin\delta$ by δ and $\cos\delta$ by 1 in Eq. (8.51) to obtain

$$\delta \approx (n_0 - 1)\sqrt{\frac{R}{2H}}. \tag{8.54}$$

A slightly more accurate (about 10%) but more cumbersome expression can be obtained from Eq. (8.51) by truncating the series expansion for the cosine after the second term rather than the first.

According to Eq. (8.54) when the sun or moon is seen to be on the horizon it is actually more than halfway below it, δ being about 0.34°, whereas the angular width of the sun or moon is about 0.5°.

Extraterrestrial bodies seen near the horizon also are vertically compressed. The simplest way to estimate the amount of compression is from the rate of change of angle of refraction with angle of incidence for a uniform slab, which from Eq. (8.51) is

$$\frac{d\vartheta_{\mathrm{t}}}{d\vartheta_{\mathrm{i}}} = \frac{\cos\vartheta_{\mathrm{i}}}{\sqrt{n_0^2 - \sin^2\vartheta_{\mathrm{i}}}} = \sqrt{\frac{1 - \sin^2\vartheta_{\mathrm{i}}}{n_0^2 - \sin^2\vartheta_{\mathrm{i}}}}, \tag{8.55}$$

where the angle of incidence is taken to be that for a curved but uniform atmosphere such that the refracted ray is horizontal:

$$\sin\vartheta_{\mathrm{t}} = \frac{R}{R + H}. \tag{8.56}$$

Equations (8.55) and (8.56) combined yield

$$\frac{d\vartheta_{\mathrm{t}}}{d\vartheta_{\mathrm{i}}} \approx \sqrt{1 - \frac{R}{H}(n_0 - 1)} \tag{8.57}$$

if we neglect terms of order $(H/R)^2$ and approximate $n_0 + 1$ as 2 and n_0 as 1 when it is a multiplicative factor. According to Eq. (8.57) the sun or moon near the horizon is distorted into an ellipse with aspect ratio about 0.87. We are unlikely to notice this distortion, however, because we expect the sun and moon to be circular, and hence we see them that way. But if we compare two photographs of a low sun or moon taken at the same moment, one rotated by 90° relative to the other, the elliptical shape may become obvious.

Our conclusions about the downward displacement and distortion of the sun were based on a refractive-index profile determined mostly by the pressure gradient. That is, the average

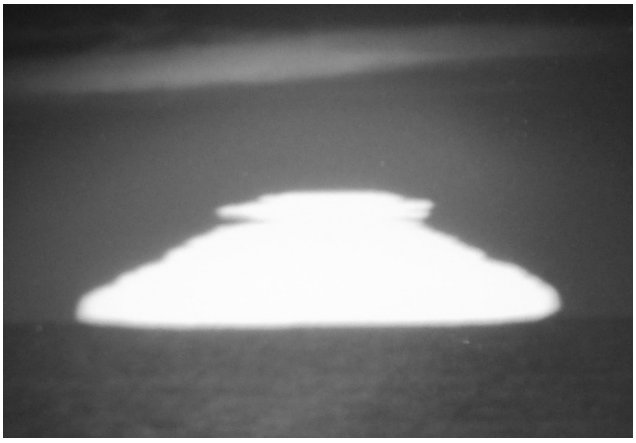

Figure 8.21: Atmospheric refraction transformed this low sun into nearly a triangle. The serrations are a consequence of horizontal variations in the atmospheric refractive index.

refractive index gradient for a uniform slab of thickness H is $(1 - n_0)/H$, which is the same as Eq. (8.39) with $n = n_0$ if the temperature gradient term is negligible. But as we noted, near the surface the temperature gradient is the prime determinant of the refractive-index gradient, as a consequence of which the horizon sun can take on shapes more striking than a mere ellipse. For example, Fig. 8.21 shows a nearly triangular sun with serrated edges. Assigning a cause to these serrations provides a lesson in the perils of jumping to conclusions. Obviously, the serrations are the result of sharp changes in the temperature gradient – or so one might think. Setting aside how such changes could be produced and maintained in a real atmosphere, a theorem by Alistair Fraser gives pause for thought: "In a horizontally (spherically) homogeneous atmosphere it is impossible for more than one image of an extraterrestrial object (sun) to be seen above the astronomical horizon [horizontal direction determined by a bubble level]." These serrations on the sun are multiple images. But if the refractive index varies only vertically (i.e., along a radius), no matter how sharply, multiple images are not possible. Thus the serrations must owe their existence to *horizontal* variations of the refractive index, which Fraser attributes to gravity waves propagating along a temperature inversion.

8.3.3 The Green Flash

Compared to the rainbow, the green flash is not a rare phenomenon. Before you dismiss this assertion as the ravings of lunatics, consider that rainbows require raindrops as well as sunlight to illuminate them, and yet the clouds that are the source of these raindrops often completely obscure the sun. Moreover, the sun must be below about 42° (see Sec. 8.4). As a consequence, rainbows do not occur frequently (at least not in many parts of the world), but when they do occur, they are difficult *not* to see. And they are seen often enough to be considered the paragon of color variation ("all the colors of the rainbow" is a cliché). Yet tinges of green on the upper rim of the sun can be seen every day at sunrise or sunset given a sufficiently low horizon and a cloudless sky. Thus the conditions for seeing a green flash are met more often than those for seeing a rainbow. Why then is the green flash considered to be so rare ("the rare green flash" is another cliché)? The distinction here is that between a rarely *observed* phenomenon (the green flash) and a rarely *observable* phenomenon (the rainbow). To see the green flash requires knowing when and where and how to look whereas even people who go through life in a daze do occasionally trip over rainbows.

The green flash is not without its commercial uses, although much fewer than rainbows. Several green flash restaurants and bars can be found near beaches, including in the Caribbean, and even a Green Flash Brewing Company in California.

To understand the origins of the green flash we may consider the sun to be an infinite set of overlapping discs, one for each visible wavelength. When the sun is overhead, all these discs coincide and we see the sun as white. But as it descends in the sky, atmospheric refraction displaces the discs in angle by slightly different amounts, the red less than the violet (see Fig. 8.17). Most of each disc overlaps all the others except for the discs at the extremes of the visible spectrum. As a consequence, the upper rim of the low sun is violet or blue, its lower rim red, whereas its interior, the region in which all discs overlap, is still white.

At least this is what would happen in the absence of lateral scattering of sunlight. But refraction and lateral scattering go hand in hand; one cannot occur without the other even in an atmosphere completely free of particles. Selective incoherent scattering by atmospheric molecules and particles causes the spectrum of transmitted sunlight to shift toward longer wavelengths, and hence the perceived color of the sun to change. In particular, the violet-bluish upper rim can be transformed to green.

According to Eq. (8.55) and the refractive indices in Fig. 8.17 the angular separation between the violet and red solar discs when the sun is on the horizon is about 0.01°, which is too small to be resolved by the human eye. You can verify this yourself by drawing two black parallel lines, say 4–6 mm apart, on a white piece of paper and observing them at increasingly greater distances. At a certain distance, they will merge into one, and this distance corresponds to an angular separation of around 1 minute of arc (0.3 mrad). You can make one of the lines red and the other green to convince yourself that different colors do not change the resolving power of the human eye. Thus in order to see the upper green rim of the sun requires binoculars or a telescope. But depending on the temperature profile, the atmosphere itself can magnify this rim and yield a second image of it, thereby enabling it to be seen by the naked eye. Green rims, which require artificial magnification, can be seen more frequently than green flashes, which require natural magnification. Yet both can be seen often by those who know what to look for and when and are willing to look.

Although the green flash is objectively real, Andrew Young argues that there is "compelling evidence that adaptation in the visual system strongly affects the perceived color of most green flashes." He notes that "photography shows that there is a real green flash in some sunsets. Green flashes are not afterimages. Nevertheless. . . physiological effects in the visual system must usually make the preceding yellow stage of a sunset flash appear green to an attentive observer." But green flashes at sunrise are a horse of a different color: "green flashes are also seen at sunrise, when the eye has not been previously exposed to bright light. . . Sunrise flashes are therefore seen more nearly in their intrinsic colors."

8.4 Scattering by Single Water Droplets

All the colored atmospheric displays that result when water droplets or ice crystals are illuminated by sunlight have the same underlying cause: light is scattered in different amounts in different directions by particles larger than the wavelength, and the directions in which scattering is greatest depend on wavelength. Thus when such particles are illuminated by sunlight, the result can be angular separation of colors even if scattering integrated over all directions is independent of wavelength, as it is for droplets and ice crystals. This description, although correct, is too general to be completely satisfying. We need something more specific, more quantitative, which requires theories of scattering.

Because superficially different theories have been used to describe different optical phenomena, the notion has become widespread that they are caused by these theories. For example, coronas are said to be caused by diffraction and rainbows by refraction. Yet both the corona and rainbow can be described quantitatively to high accuracy with Mie theory (Sec. 3.5.2) in which diffraction and refraction do not appear explicitly. As we noted in Section 3.1, no impenetrable barrier separates scattering from specular reflection, refraction, and diffraction. Because these terms came into general use and were entombed in textbooks before the nature of light and matter were well understood, we are stuck with them. But if we insist that diffraction, for example, is somehow different from scattering, we do so at the expense of shattering the unity of the seemingly disparate observable phenomena that result when light interacts with matter. What is observed depends on the composition and disposition of matter, not on which approximate theory in a hierarchy is used for quantitative description.

Atmospheric optical phenomena are best classified by the direction in which they are seen and the agents that cause them. Accordingly, the following sections are arranged in order of scattering direction, from forward to backward.

When a *single* water droplet is illuminated by white light and the scattered light projected onto a screen, the result is a set of colored rings. But this same set of rings in the sky is a mosaic to which a thin cloud of *many* droplets contributes. Light from each direction is that scattered by a different set of droplets in each patch of sky. Thus for a complete mosaic droplets must be present in sufficient number and illuminated by sunlight, and the cloud must be sufficiently thin that multiple scattering does not wash out the mosaic.

8.4.1 Coronas and Iridescent Clouds

A cloud of droplets narrowly distributed in size and thinly veiling the sun or moon can yield a striking series of concentric colored rings around it. This *corona* is most easily described quantitatively by the Fraunhofer diffraction theory, a simple approximation valid for particles large compared with the wavelength for scattering near the forward direction. According to this approximation, which must break down because it is oblivious to the polarization state of the incident light and the composition of the scatterer, the differential scattering cross section of a spherical droplet of radius a illuminated by light of wavenumber k is $|S|^2/k^2$, where the scattering amplitude is

$$S = x^2 \frac{1+\cos\vartheta}{2} \frac{J_1(x\sin\vartheta)}{x\sin\vartheta}. \tag{8.58}$$

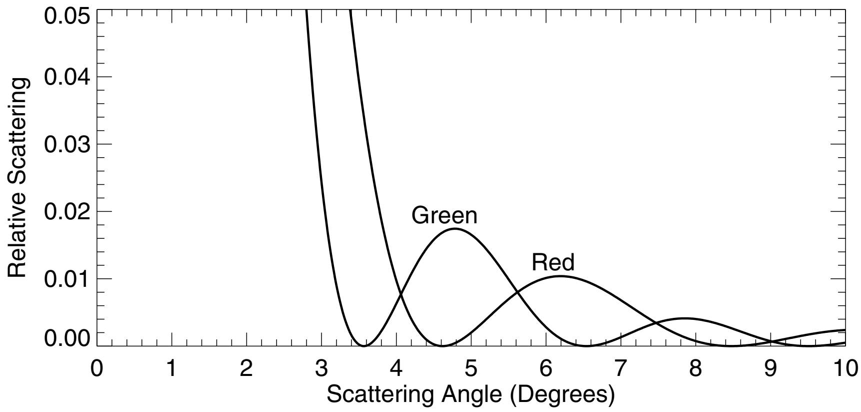

Figure 8.22: Differential scattering cross section calculated by the Fraunhofer approximation for a sphere of diameter 10 μm. Green corresponds to 510 nm, red to 660 nm. Both cross sections are normalized to the value for green at 0°.

J_1 is the Bessel function of first order and the size parameter $x = ka$. The quantity $(1 + \cos\vartheta)/2$ usually is approximated as 1 because only near-forward scattering angles ϑ are of interest.

This differential scattering cross section, which determines the angular distribution of scattered light, has maxima for

$$x \sin\vartheta = 5.137,\ 8.417,\ 11.62, \dots \tag{8.59}$$

Thus the dispersion in the position of the first maximum is

$$\frac{d\vartheta}{d\lambda} \approx \frac{0.817}{a}, \tag{8.60}$$

and is greater for higher-order maxima. This dispersion determines the upper limit on droplet size such that a corona can be observed. For the total angular dispersion over the visible spectrum to be greater than the angular width of the sun (0.5°), the droplets cannot be larger than about 60 μm in diameter. Drops in rain, even drizzle, are appreciably larger, which is why coronas are not seen through rainshafts. But scattering by a droplet of diameter 10 μm (Fig. 8.22), a typical cloud droplet size, gives sufficient dispersion to yield colored coronas.

Suppose that the first angular maximum for a droplet of radius a occurs at a wavelength λ. For a droplet of radius $a + \Delta a$, the position of this maximum is the same at a wavelength $\lambda + \Delta\lambda$, where from Eq. (8.59)

$$\frac{\Delta a}{a} \approx \frac{\Delta\lambda}{\lambda}. \tag{8.61}$$

If we take $\Delta\lambda$ to be half the width of the visible spectrum (about 0.15 μm) and λ to be in the middle of the spectrum (0.55 μm), $\Delta a/a \approx 0.3$. Thus coronas require fairly narrow size distributions: if droplets are distributed in size with a relative variance much greater than 30–40%, color separation is not possible.

Because of the stringent requirements for complete coronas (a thin veil of droplets narrowly distributed in size extending within 10–20° of the sun), they are not observed often in the sky, although you might see them in clouds formed on your breath, the source of illumination street lights or automobile lights. Or you might see them as a result of scattering by water droplets condensed onto a window or the windshield of a car. Of greater occurrence are the corona's cousins, iridescent clouds, which display patches of colors but usually not arranged in any obvious geometrical pattern. You may miss iridescence because it is seen toward the sun and so may be dazzled by sunlight. And yet on many partly cloudy days the thin edges of even thick clouds may be tinged with red and green. To enhance your ability to see these colored patches you can reduce the luminance with sunglasses or by reflection by a window or a puddle. Alistair Fraser used to give students in his observing class black glass tiles, about 10 cm on a side, to be used for scanning the sky for iridescence. When you know where and how to look, you can see it frequently.

Coronas are not the unique signature of spherical scatterers. Randomly oriented ice columns and plates give similar near-forward scattering patterns according to Fraunhofer theory. As a practical matter, however, most coronas probably are caused by droplets. Many clouds at temperatures well below freezing contain subcooled water droplets. Only if a corona were seen through a cloud at a temperature known to be lower than $-40\,°\mathrm{C}$ could one assert with confidence that it must be an ice-crystal corona. Ken Sassen and his collaborators have made what appear to be incontrovertible observations of occasional coronas and, even more rarely, iridescence in clouds composed of ice crystals. But the rarity of this is evident from the title of one of Sassen's papers, "Cirrus cloud iridescence: a rare case study".

Another exception to the assertion about most coronas being caused by droplets is elliptical coronas. These have been observed, photographed, simulated, and attributed to scattering by more or less spheroidal pollen grains oriented by aerodynamic forces as they fall through air.

8.4.2 Rainbows

In contrast with coronas, which are seen toward the sun, rainbows are seen away from it, and caused by water drops much larger than those that give coronas. To describe the rainbow quantitatively we pretend that light incident on a transparent sphere much larger than the wavelength is composed of individual rays, each of which suffers a different fate determined only by the laws of specular reflection and refraction. Each incident ray splinters into an infinite number of scattered rays: reflected at the first interface, transmitted without internal reflection, transmitted after one, two, and so on internal reflections. For any scattering angle ϑ, each splinter contributes to the scattered light and hence to the differential scattering cross section (see Sec. 3.5). Consider a small area $a^2 \sin\vartheta_\mathrm{i} \Delta\vartheta_\mathrm{i} \Delta\varphi_\mathrm{i}$, defined by co-latitudes between ϑ_i and $\vartheta_\mathrm{i} + \Delta\vartheta_\mathrm{i}$ and azimuthal angles between φ_i and $\varphi_\mathrm{i} + \Delta\varphi_\mathrm{i}$, on a sphere of radius a. If ϑ_i is the angle an incident ray makes with this small area, the radiant energy intercepted by it is proportional to $a^2 \sin\vartheta_\mathrm{i} \cos\vartheta_\mathrm{i} \Delta\vartheta_\mathrm{i} \Delta\varphi_\mathrm{i}$, a fraction of which is reflected, a fraction transmitted without internal reflection, and so on. The solid angle of the corresponding set of scattered rays is $\sin\vartheta\Delta\vartheta\Delta\varphi$, and hence the contribution to the differential scattering cross section by a splinter has the form

$$\mathcal{T}\frac{a^2\sin\vartheta_{\mathrm{i}}\cos\vartheta_{\mathrm{i}}\Delta\vartheta_{\mathrm{i}}\Delta\varphi_{\mathrm{i}}}{\sin\vartheta\Delta\vartheta\Delta\varphi}, \tag{8.62}$$

where $\mathcal{T}$ is a transmissivity (in general, a product of various transmissivities and reflectivities obtained from the Fresnel coefficients discussed in Sec. 7.2). For our purposes here all we need to know about $\mathcal{T}$ is that it is finite and nonzero and hence can be ignored.

Because of the azimuthal symmetry of a sphere, $\Delta\varphi = \Delta\varphi_{\mathrm{i}}$, whereas ϑ is a *different* function of ϑ_{i}, or conversely, for every splinter. In the limit $\Delta\vartheta \to 0$ Eq. (8.62) becomes

$$a^2\frac{\cos\vartheta_{\mathrm{i}}\sin\vartheta_{\mathrm{i}}}{\sin\vartheta}\frac{d\vartheta_{\mathrm{i}}}{d\vartheta}, \tag{8.63}$$

where we omit $\mathcal{T}$. This can be written more compactly by way of the *impact parameter* $b = a\sin\vartheta_{\mathrm{i}}$:

$$\frac{b}{\sin\vartheta}\frac{db}{d\vartheta}. \tag{8.64}$$

The differential scattering cross section is an infinite series of terms of the form of Eq. (8.63) or Eq. (8.64). Singularities, or caustics, in the differential scattering cross section occur at scattering angles for which

$$\frac{d\vartheta_{\mathrm{i}}}{d\vartheta} \to \infty,\ \frac{\cos\vartheta_{\mathrm{i}}\sin\vartheta_{\mathrm{i}}}{\sin\vartheta} \neq 0, \tag{8.65}$$

or, equivalently,

$$\frac{db}{d\vartheta} \to \infty,\ \frac{b}{\sin\vartheta} \neq 0. \tag{8.66}$$

According to geometrical (i.e., ray) optics, at a caustic the differential scattering cross section is infinite. In reality, it is finite for all scattering angles, so geometrical optics can at best only point out the approximate positions of spikes. For our purposes, Eq. (8.65) is more convenient for determining caustics and the condition on the derivative is written more conveniently as

$$\frac{d\vartheta}{d\vartheta_{\mathrm{i}}} = 0. \tag{8.67}$$

Equation (8.67) defines *rainbow angles*. No rainbow angle is associated with splinters externally reflected and transmitted without internal reflection, but there may be rainbow angles for transmission after one internal reflection (*primary* rainbow), after two internal reflections (*secondary* rainbow) and so on. Consider first splinters transmitted after one internal reflection (Fig. 8.23). The scattering or deviation angle is

$$\vartheta = (\vartheta_{\mathrm{i}} - \vartheta_{\mathrm{t}}) + (\pi - 2\vartheta_{\mathrm{t}}) + (\vartheta_{\mathrm{i}} - \vartheta_{\mathrm{t}}) = 2\vartheta_{\mathrm{i}} - 4\vartheta_{\mathrm{t}} + \pi. \tag{8.68}$$

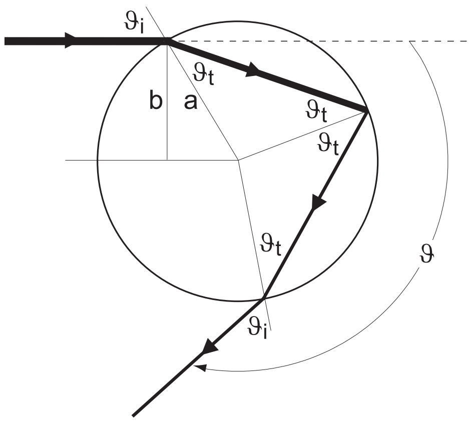

Figure 8.23: Any of the infinitude of rays imagined to be incident on a large, compared with the wavelength, transparent water sphere can be transmitted into it and reflected once before being transmitted out. Only one ray is shown here, its angle of incidence ϑ_i.

The curve of ϑ as a function of ϑ_i for a water sphere exhibits a minimum (Fig. 8.24). The angle of incidence corresponding to this minimum follows from Eqs. (8.67) and (8.68) and Snel's law:

$$\sin\vartheta_\mathrm{i} = \sqrt{\frac{4-n^2}{3}}, \tag{8.69}$$

where n is the real refractive index. Because $\sin\vartheta_\mathrm{i}$ must be a real number less than or equal to 1, primary rainbows require drops with refractive index less than 2.

In the middle of the visible spectrum (550 nm) the primary rainbow angle is 137.7°, but this angle is better expressed as its complement (42.3°) because the primary rainbow is seen at about 42° from the *antisolar point*, the direction directly opposite the sun (Fig. 8.25). Because n varies with wavelength, so does the rainbow angle, the angular spread from violet (425 nm) to red (650 nm) about 2.3°. This is appreciably greater than the angular width of the sun, which is why color separation can be seen in natural rainbows. Because the refractive index of water is least in the red, so is the corresponding rainbow angle, and hence the red bow is highest above the antisolar point.

Contrary to appearances, rainbows are not palpable objects lying in a vertical plane. Nor are they even caused solely by raindrops in a plane. All raindrops in space lying on a cone with its apex at the observer and an apex angle of about 84° contribute to a rainbow (Fig. 8.25). Moreover, because the rainbow angle is an angle of minimum deviation, light is scattered in all directions within this cone. Thus the rainbow is the bright outer edge of a luminous disc. But why do we see only part of this disc? The facile answer is that the ground gets in the way, but a more satisfying answer is that the optical thickness along paths that intersect the ground is in general much less than along paths that do not. We show this schematically in Fig. 8.25.

To our surprise and delight, the noted adventure photographer Galen Rowell responded to a challenge to photograph a complete rainbow. He writes on page 52 of *Galen Rowell's Vision:*

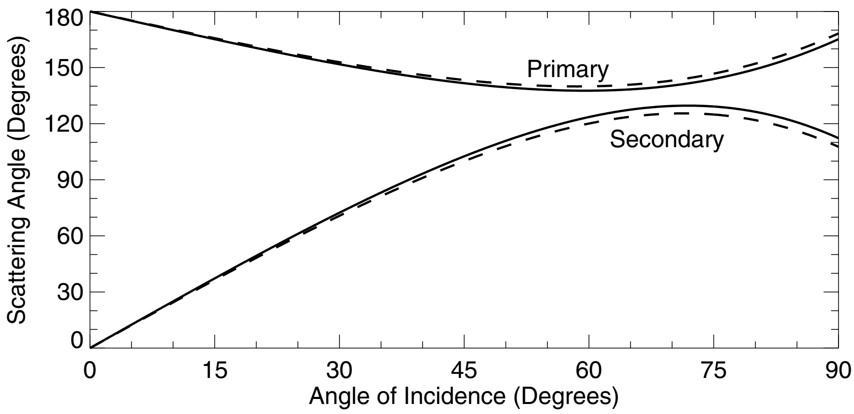

Figure 8.24: The scattering angle for incident rays undergoing one internal reflection by a large, transparent sphere is a minimum for a primary rainbow, whereas that for incident rays undergoing two internal reflections is a maximum for a secondary rainbow. The solid curve is for a wavelength of 650 nm, the dashed curve for 425 nm.

The Art of Adventure Photography: "When my wife, Barbara flew her single-engine Cessna to Patagonia, I was able to photograph... the extremely rare 360-degree double rainbow... that appeared in a sunlit rainstorm along the coast of Mexico. Atmospheric physicist Craig F. Bohren says in his 1987 book, *Clouds in a Glass of Beer*, 'to the best of my knowledge, one has never been photographed'." He continues with "a suggestion for anyone who would like a bit of fame and fortune: photograph a complete rainbow. You will need an airplane... You will also have to persuade the pilot to fly in stormy weather. If you survive your flight you will have acquired something rare indeed."

But what is this "double rainbow" that Rowell mentions? This is a term for the primary and secondary rainbows seen together. The secondary rainbow often is missed because it is less bright than the primary, but if you know where to look you often can see at least parts of the secondary bow. The deviation angle for splinters that have been reflected internally twice before transmission (Fig. 8.26) is

$$\vartheta = 2\vartheta_{\mathrm{i}} - 6\vartheta_{\mathrm{t}} + 2\pi. \tag{8.70}$$

Note that this is the *total* deviation angle. To obtain from it the corresponding scattering angle, which by definition lies between 0 and π, requires subtracting Eq. (8.70) from 2π. This scattering angle exhibits a *maximum*, the total deviation a minimum, for a water drop (Fig. 8.24). The corresponding angle of incidence follows from Eqs. (8.67) and (8.70) and Snel's law:

$$\sin\vartheta_{\mathrm{i}} = \sqrt{\frac{9-n^2}{8}}. \tag{8.71}$$

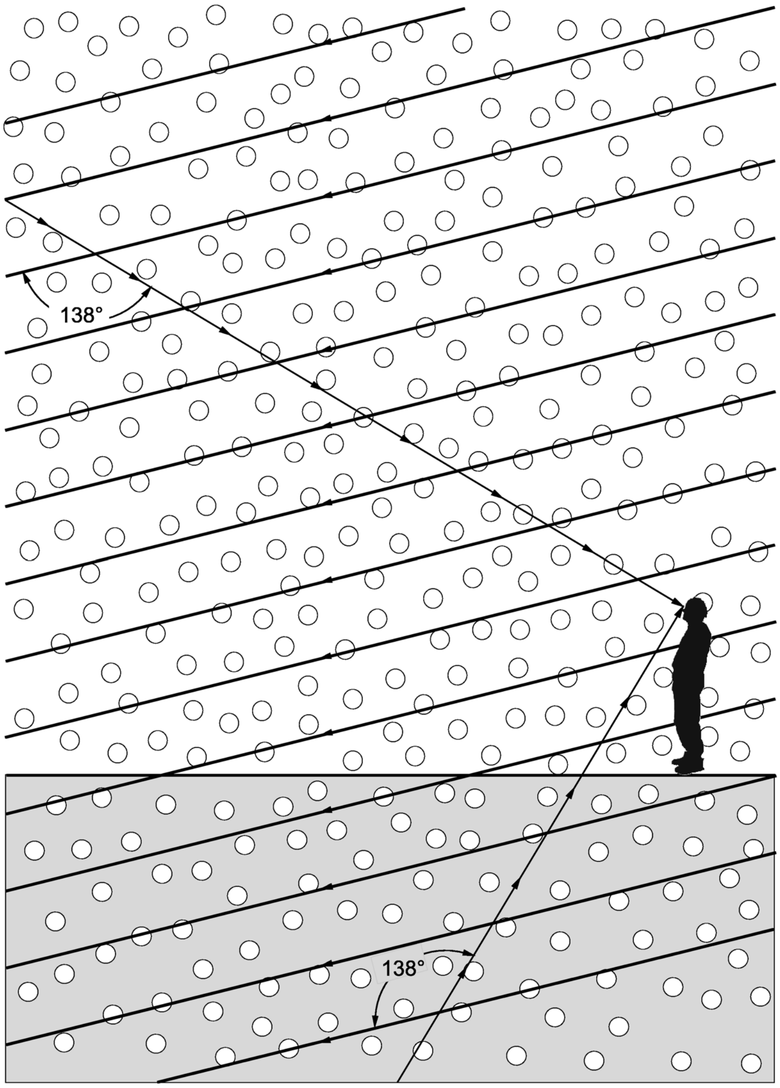

Figure 8.25: The primary rainbow is a consequence of strong scattering by all illuminated drops lying on a cone with its apex at the eye of the observer and with apex angle approximately 84°.

If rain were composed of titanium dioxide ($n \approx 2.7$), a common constituent of paints, primary rainbows would be absent from the sky and we would have to be content with secondary rainbows.

In the middle of the visible spectrum the secondary rainbow angle is 128.3°, but again its complement (51.7°) is more relevant to an observer: the secondary rainbow lies about 52° above the antisolar point, and hence about 9° above the primary rainbow. Indeed, we drew

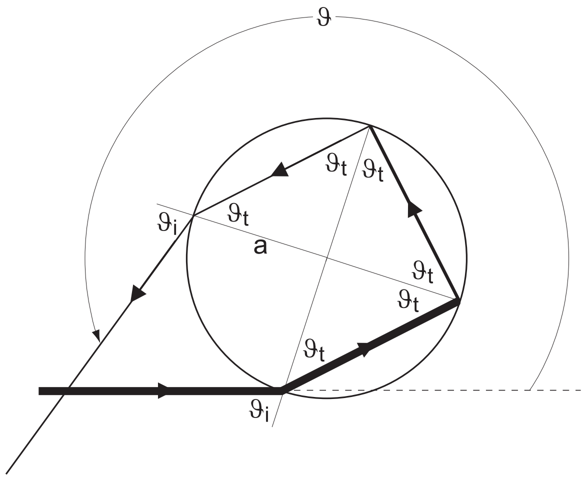

Figure 8.26: Any of the infinitude of rays imagined to be incident on a large, compared with the wavelength, transparent water sphere can be transmitted into it and reflected twice before being transmitted out. Only one ray is shown here, its angle of incidence ϑ_i corresponding to the secondary rainbow angle for which the scattering angle ϑ is a maximum.

Figs. 8.23 and 8.26 differently in order to make this clear. The angular spread in the secondary rainbow from violet to red is about $4.1°$. Because the scattering angle corresponding to the deviation angle determined by Eqs. (8.70) and (8.71) is greatest in the red, the angular color order in the secondary bow is reversed. It is sometimes said that this is because the second internal reflection turns the rainbow inside out. But the different color order of the two bows is an accident of the magnitude of the refractive index of water. Because the primary rainbow angle is a minimum scattering angle and the secondary rainbow angle is a maximum scattering angle, geometrical optics predicts a dark band, called *Alexander's dark band*, between the primary and secondary bow, which indeed is observed.

The colors of rainbows are a consequence of sufficient dispersion of the refractive index of water over the visible spectrum to give a spread of rainbow angles that appreciably exceeds the angular width of the sun. As we noted, the width of the primary bow is about $2.3°$ and that of the secondary bow about $4.1°$, although these values depend on the choice of end points of the visible spectrum and the values of the refractive index. Because of its band of colors arcing across the sky, the rainbow has become the paragon of colors, the standard against which all other colors are compared. Raymond Lee and Alistair Fraser, however, have challenged this status of the rainbow, pointing out that even the most vivid rainbows are colorimetrically far from pure (see Sec. 4.3). Rainbows are almost invariably discussed as if they occurred literally in a vacuum. But real rainbows, as opposed to the pencil-and-paper variety, are necessarily observed in an atmosphere, the molecules and particles of which scatter sunlight that adds to the light from rainbows but subtracts from their purity of color.

According to geometrical optics, the differential scattering cross section is infinite at all rainbow angles. But in reality, scattering by raindrops is everywhere finite, and the scattered light at each successive rainbow angle diminishes because of ever decreasing transmission.

Secondary rainbows are less bright than primary rainbows. Tertiary and higher-order rainbows are drowned in natural environments by background illumination (e.g., skylight). In a darkened laboratory, with a laser illuminating a single droplet, caustics of remarkably high order can be observed, but it is a stretch to call them rainbows.

Although geometrical optics yields the positions, widths, and color separation of rainbows, it yields little else. For example, it is blind to *supernumerary bows*, a series of narrow bands sometimes seen below the primary bow. These bows cannot be described without *explicitly* invoking interference. (As noted in Secs. 3.1 and 7.2, specular reflection and refraction are implicit interference patterns.) Except at the rainbow angle, a horizontal line intersects the curve of scattering angle versus angle of incidence in two points (Fig. 8.24). The corresponding two rays therefore have the same direction but follow different trajectories in a drop. If we look upon ray trajectories as specifying (approximately) the transmission paths of waves, the two waves corresponding to the two rays in the same direction are different in phase and hence interfere. Moreover, this interference depends on drop size unlike the positions of rainbow angles, which according to geometrical optics are independent of size. The interference interpretation of supernumerary bows has been seized upon despite the embarrassing fact that it is puzzling to anyone who knows about rain. Raindrops are widely distributed in size (see Prob. 1.3) so how can supernumerary bows be seen in rain showers? In a nice piece of detective work, Alistair Fraser answered this question.

Raindrops falling in a vacuum are spherical. Those falling in air are distorted by aerodynamic forces, not, despite the depictions of countless artists, into teardrops but rather into more nearly oblate spheroids with their axes approximately vertical. Fraser argues that supernumerary bows are caused by drops of diameter about 0.5 mm, for which the angular position of the first and second supernumerary bow is a minimum. Interference causes the angular position of the supernumerary bow to increase with decreasing size whereas drop distortion causes it to increase with increasing size. Supernumerary patterns contributed by drops on either side of the minimum cancel, leaving only the contribution from drops at the minimum. This cancellation occurs only near the tops of rainbows, where supernumerary bows are seen. In the vertical parts of a rainbow, a horizontal slice through a distorted drop is more or less circular, and hence these drops do not exhibit a minimum supernumerary angle.

According to geometrical optics, all spherical drops, regardless of size, yield the same rainbow. But it is not necessary for a drop to be spherical for it to yield rainbows independent of its size. This merely requires that the plane defined by the incident and scattered rays intersect the drop in a circle. Even distorted drops satisfy this condition in the vertical part of a bow. As a consequence, the absence of supernumerary bows there is compensated for by more vivid colors of the primary and secondary rainbows. Smaller drops are more likely to be spherical, but the smaller the drop, the less light it scatters. Thus the dominant contribution to the luminance of rainbows is from the larger drops. At the top of the bow, the plane defined by the incident and scattered rays intersects the large, distorted drops in an ellipse, yielding a range of rainbow angles varying with the amount of distortion, and hence a pastel rainbow. To the careful observer rainbows are no more uniform in color and brightness than is the sky.

Although geometrical optics predicts that all rainbows are equal, neglecting background light, real rainbows do not slavishly follow the dictates of this approximate theory. Rainbows in nature range from nearly colorless fog bows, or cloud bows, to the vividly colorful vertical portions of rainbows likely to have inspired myths about pots of gold.

8.4.3 The Glory

Continuing our sweep of scattering directions, from forward to backward, we arrive at the end of the arc to the *glory*. Because it is most easily seen from airplanes it sometimes is called the *pilot's bow*. Another name is *anticorona*, a corona around the antisolar point. Although glories and coronas share some common characteristics, there are differences between them other than direction of observation. Unlike coronas, which may be caused by nonspherical ice crystals, glories require spherical cloud droplets (but see the references at the end of the chapter for a possible exception). And a greater number of colored rings may be seen in glories because the decrease in luminance away from the backward direction is not as steep as that away from the forward direction. To see a glory from an airplane, look for colored rings around its shadow cast on clouds below. This shadow is not an essential part of the glory, merely a way of finding the antisolar point.

Like the rainbow, the glory may be looked upon as a singularity in the differential scattering cross section. Equation (8.65) gives one set of conditions for a singularity; the second set is

$$\sin\vartheta = 0,\ b(\vartheta) \neq 0. \tag{8.72}$$

That is, the differential scattering cross section is infinite for nonzero impact parameters, corresponding to incident rays that do not intersect the center of the sphere, that give forward (0°) or backward (180°) scattering. The forward direction is excluded because intense scattering in this direction is accounted for by the Fraunhofer theory.

For one internal reflection, Eqs. (8.68) and (8.72) and Snel's law yield the condition

$$\sin\vartheta_{\mathrm{i}} = \frac{n}{2}\sqrt{4-n^2}, \tag{8.73}$$

which is satisfied only for refractive indices between 1.414 and 2, the lower refractive index corresponding to a grazing-incidence ray. The refractive index of water lies outside this range. Although a condition similar to Eq. (8.73) is satisfied by four or more internal reflections, insufficient radiant energy is associated with such rays. Thus it seems that we have reached an impasse: the theoretical condition for a glory cannot be met by water droplets. Not so says Henk van de Hulst. He argues that 1.414 is close enough to 1.33 given that geometrical optics is, after all, an approximation. Cloud droplets are large compared with the wavelengths of visible light, but not so large that geometrical optics is an infallible guide to their optical behavior. Support for the van de Hulstian interpretation of glories was provided by Bryant and Cox, who showed that the dominant contribution to the glory is from the last term in the exact series for scattering by a sphere. Each successive term in this series is associated with ever-larger impact parameters. Thus the terms that give the glory are indeed those corresponding to grazing rays. Further unraveling of the glory and vindication of van de Hulst's conjecture about the glory were provided by Nussenzveig.

It sometimes is said that geometrical optics is incapable of treating the glory. Yet the same can be said about the rainbow. Geometrical optics explains rainbows only in the limited sense that it predicts singularities for scattering in certain directions (i.e., rainbow angles). But it can predict only the angles of intense scattering, not the amount of light scattered. Indeed, the error is infinite. Geometrical optics also predicts a singularity in the backward direction but is powerless to predict more. Results from geometrical optics for both rainbows and glories are not the end but rather the beginning, an invitation to take a closer look with more powerful magnifying glasses.

8.5 Scattering by Single Ice Crystals

Scattering by spherical particles in the atmosphere can result in three distinct displays: coronas, rainbows, and glories. If the particles depart somewhat from perfect spheres, the displays are not greatly changed. An entirely new and more varied set of displays arises when the particles are ice crystals, which are far from spherical and, because of their shape, can be oriented by aerodynamic forces. As with rainbows the gross features of ice-crystal displays can be described simply, but approximately, by following the various trajectories of rays incident on crystals. Colorless displays (e.g., the subsuns touched on in Sec. 4.1) are generally associated with reflected rays, colored displays (e.g., sun dogs and halos) with refracted rays. Because of the wealth of ice-crystal displays, we cannot treat all of them, but one example should point the way toward understanding many of them.

8.5.1 Sun Dogs and Halos

Because of its hexagonal crystalline structure, ice can form as hexagonal plates in the atmosphere. The stable position of a plate falling in air is with its face more or less horizontal, which can be demonstrated with an ordinary business card. When dropped with its face vertical, the supposedly aerodynamic position that many people choose instinctively, the card somersaults in a helter-skelter path to the ground. But when dropped with its face horizontal, the card gently rocks back and forth in descent.

A hexagonal ice plate falling through air and illuminated by a low sun is like a 60° prism illuminated normally to its sides (Fig. 8.27). Because there is no mechanism for orienting a plate within a horizontal plane, all plate orientations in this plane are equally probable. Stated another way, all angles of incidence on a fixed plate are equally probable. Yet all deviation (i.e., scattering) angles of rays refracted into and out of the plate are not equally probable. Let $p(\vartheta_{\mathrm{i}})$ be the uniform probability distribution for incident angles ϑ_{i} and $P(\vartheta)$ that for deviation angles ϑ, where ϑ is a function of ϑ_{i}. As with all probability distributions, the integral of $P(\vartheta)$ over any interval is the probability that the deviation angle lies in that interval. From the theorem in Section 1.2 for transforming from one distribution to another (i.e., transforming variables of integration)

$$P(\vartheta)\left|\frac{d\vartheta}{d\vartheta_{\mathrm{i}}}\right| = p(\vartheta_{\mathrm{i}}), \tag{8.74}$$

which is more illuminating when written as

$$P(\vartheta) = \frac{p(\vartheta_{\mathrm{i}})}{|d\vartheta/d\vartheta_{\mathrm{i}}|}. \tag{8.75}$$

Note that Eq. (8.75) does not give the radiant energy in the twice-refracted radiation. To obtain this would require including the Fresnel transmission coefficients for the two interfaces.

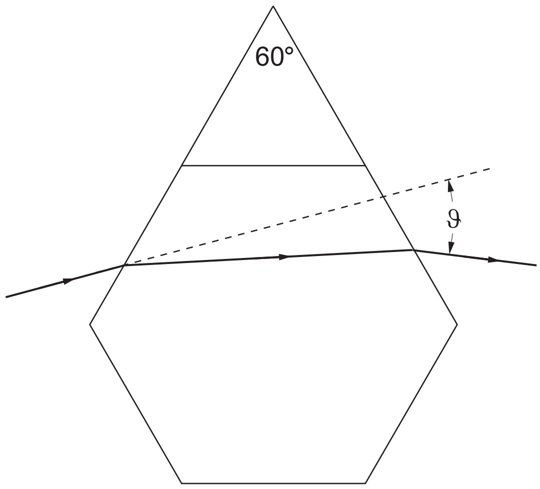

Figure 8.27: Deviation (i.e., scattering) of incident light by an angle ϑ because of refraction by a 60° prism that is part of a hexagonal plate.

Figure 8.28 shows deviation angle as a function of incidence angle for a 60° ice prism that is part of a hexagonal plate. For angles of incidence less than about 13° the transmitted ray is totally internally reflected at the second interface; for angles of incidence greater than about 70° reflection rises sharply (e.g., see Fig. 7.6) and hence transmission plunges. Thus most incident rays of consequence lie in this range. According to Eq. (8.75) the probability density $P(\vartheta)$ becomes infinite if the deviation angle has a minimum, which it does for $\vartheta_{\mathrm{i}} \approx 40°$; the corresponding deviation angle is about 22°. The observable manifestation of this singularity, or caustic, at the angle of minimum deviation for a 60° ice prism is a bright spot about 22° from either or both sides of a sun low in the sky. These bright spots are called *sun dogs*, because they accompany the sun, or *parhelia* or *mock suns*.

The minimum deviation angle ϑ_{m}, and hence the angular position of sun dogs, depends on the prism angle Δ and refractive index n (see Prob. 8.20):

$$\vartheta_{\mathrm{m}} = 2 \sin^{-1} \left(n \sin \frac{\Delta}{2} \right) - \Delta. \tag{8.76}$$

Because n varies with wavelength, the separation between the angles of minimum deviation for red (650 nm) and violet (430 nm) light is about 0.7° (Fig. 8.28), slightly greater than the angular width of the sun. As a consequence, sun dogs may be tinged with color, most noticeably toward the sun. Because the refractive index of ice is least at the red end of the spectrum, the red components of a sun dog are closest to the sun. Moreover, for any wavelength, except that corresponding to red, a horizontal line tangent at the minimum angle to the curve of deviation angle versus incident angle intersects curves for other wavelengths. Because of this overlap of deviation angles, red is the purest color in sun dogs, which fade into white away from their red inner edges.

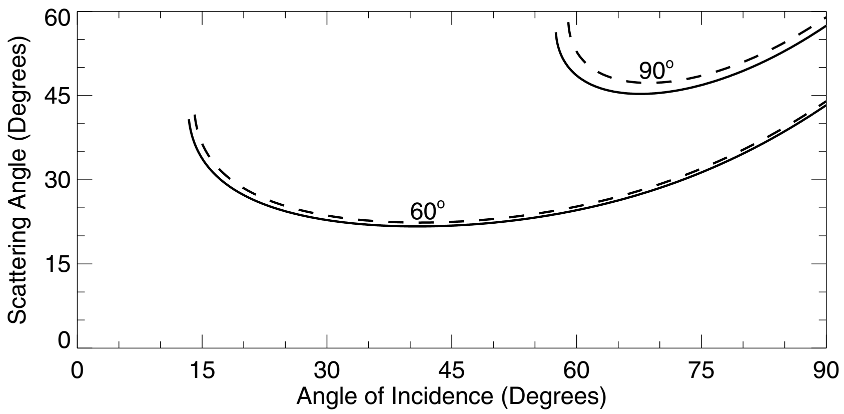

Figure 8.28: Deviation (i.e., scattering) angle versus angle of incidence for a 60° ice prism and a 90° ice prism. The solid line is for a wavelength of 650 nm, the dashed line for 430 nm.

With increasing solar elevation, sun dogs move away from the sun. A falling ice plate is approximately equivalent to a prism the angle of which increases with solar elevation. From Eq. (8.76) it follows that the angle of minimum deviation, hence the position of the sun dog, also increases. Why is only the 60° prism portion of a hexagonal plate singled out for attention? According to Fig. 8.27, a hexagonal plate could be considered to be made up of 120° prisms. For a ray to be refracted twice, its angle of incidence at the second interface must be less than the critical angle [see Eq. (4.63)], which imposes limitations on the prism angle. For $n \approx 1.31$ (ice at visible wavelengths), all incident rays are totally internally reflected by prisms with angles greater than about 99.5°.

A close relative of the sun dog is the 22° halo, a bright ring approximately 22° from the sun. Lunar halos and moon dogs are also possible, but because of their low brightness may not be noticed as frequently as their solar counterparts. Until Alistair Fraser analyzed halos the conventional wisdom had been that they obviously were the result of randomly oriented crystals, yet another example of jumping to conclusions. By combining optics and aerodynamics, he showed that ice crystals small enough to be randomly oriented by Brownian motion are too small to yield sharp scattering patterns.

But partially oriented larger plates can produce halos, especially ones of non-uniform brightness. Each part of a halo is contributed to by a plate with a different tip angle, angle between the normal to the plate and the vertical. The transition from oriented plates with zero tip angle to randomly oriented plates occurs over a narrow range of sizes. In the transition region, plates can be small enough to be partially oriented yet large enough to give a distinct contribution to the halo. Moreover, the mapping between tip angles and azimuthal angles on the halo depends on solar elevation. When the sun is near the horizon, plates can give a distinct halo over much of its azimuth. When the sun is high in the sky, hexagonal plates cannot give a sharp halo but hexagonal columns – another possible form of atmospheric ice particles – can. The stable position of a falling column is with its long axis horizontal. When the sun is directly overhead, such columns can give a uniform halo even if they all lie in the horizontal plane. When the sun is not overhead but well above the horizon, columns also can give halos.

A corollary of Fraser's analysis is that halos are caused by crystals with sizes in the range 12–40 μm. Larger crystals are oriented, smaller crystals too small to yield distinct scattering patterns. More or less uniformly bright halos with the sun neither high nor low in the sky could be caused by mixtures of hexagonal plates and columns or by clusters of bullets (rosettes). Fraser opines that the latter is more likely.

One of the by-products of his analysis is an understanding of the relative rarity of the 46° halo. Rays can be transmitted through two sides of a hexagonal column ($\Delta = 60°$) or through one side and an end ($\Delta = 90°$). For $n = 1.31$ and $\Delta = 90°$ Eq. (8.76) yields a minimum deviation angle of 46° (see Fig. 8.28). Although 46° halos are possible, they are seen much less frequently than 22° halos. Plates cannot give distinct 46° halos although columns can. But they must be solid and most columns have hollow ends. Moreover, the range of sun elevations is restricted.

Like the green flash, ice-crystal phenomena are not intrinsically rare. Halos and sun dogs can be seen frequently once you know what to look for, where, and when. Hans Neuberger reports that halos were observed in State College, Pennsylvania an average of 74 days a year over a 16-year period, with extremes of 29 and 152 halos a year. Although the 22° halo was by far the most frequently seen display, ice-crystal displays of all kinds were seen, on average, more often than once every four days at a location not especially blessed with clear skies. Although thin clouds are necessary for ice-crystal displays, clouds thick enough to obscure the sun are their bane.

8.6 Clouds as Givers and Takers of Light

Despite their apparent solidity, clouds are so flimsy as to be almost nonexistent – except optically. The fraction of cloud volume occupied by water substance, liquid or solid, is about 10^{-6} or less, and hence the mass density of clouds is a small fraction of the density of sea-level air. And yet their optical thickness per unit physical thickness is much greater because scattering by a water molecule when part of a coherent array (e.g., water droplet) is vastly greater than that by a single, isolated molecule (see Fig. 3.11).

Clouds seen by passengers in an airplane flying above cloud can be dazzling, but if the airplane were to descend through the cloud these passengers might describe the sky overhead as gloomy. Clouds are both givers and takers of light. Their dual role is exemplified in Fig. 5.13, which shows the calculated diffuse downward irradiance below clouds of varying optical thickness. On an airless planet the sky would be black in all directions except directly toward the sun. But if the sky were to be filled from horizon to horizon with thin clouds, the brightness overhead would markedly increase. As so often happens, however, more is not always better. Beyond a certain cloud optical thickness, the diffuse irradiance decreases, and for a sufficiently thick cloud the sky overhead can be darker than the clear sky (see Probs. 5.25 and 5.26).

Why are clouds bright? Why are they dark? No inclusive one-line answer can be given to these questions. Better to ask, Why is that particular cloud bright? Why is that particular cloud dark? Each observation must be treated individually; generalizations are risky. Moreover, we must keep in mind the difference between brightness and luminance when addressing the queries of human observers (see Sec. 4.1). If the luminance of an object is appreciably greater than that of its surroundings, we call the object bright; if appreciably less, we call it dark. But these are relative, not absolute terms. Two clouds, identical in all respects, including illumination, still may appear different because they are seen against different backgrounds, a cloud seen against the horizon sky appearing darker than when seen against the zenith sky. Of two clouds under identical illumination, the optically smaller will be less bright. If an even larger cloud were to appear, the cloud formerly described as white might be demoted to gray. With the sun below the horizon, two identical clouds at markedly different elevations might be quite different in brightness, the lower cloud being shadowed from direct illumination by sunlight. A striking example of dark clouds sometimes can be seen well after sunset. Low-lying clouds not illuminated by direct sunlight may be inky blotches staining the faint twilight sky.

Because the dark objects of our everyday lives usually owe their darkness to absorption, nonsense about dark clouds is rife: they are filled with pollution. Yet of all the reasons why clouds sometimes are seen to be dark or even black, absorption is not among them. But there is at least one example in which absorption by clouds of pure water droplets or ice crystals can result in observable consequences, which we discuss in the following section.

8.6.1 Green Thunderstorms

Blue or red skies, clear or cloudy, are unremarkable, but green is the color of ground-level objects: grass and trees. A green sky therefore gets your attention, a source of wonder, even fear, an indication that the world has gone topsy-turvy. And yet green thunderstorms are seen from time to time. Their existence is not in doubt, although explanations of them are. Where tornadoes occur frequently, they are said to be the cause of green thunderstorms, although the mechanism is not specified. Where hail occurs, it is said to be the cause of green thunderstorms. Both of these so-called explanations exemplify one of the most widespread forms of defective reasoning: if two events occur in succession or nearly simultaneously, hail and green thunderstorms, for example, one causes the other. We go beyond folklore and offer two rational explanations, variations on previous themes, of green thunderstorms.

Equation (8.3) is a simple expression for the airlight radiance L seen in the direction of a black object at an optical distance τ:

$$L = L_0 G\{1 - \exp(-\tau)\}, \tag{8.77}$$

where G is a geometrical factor of no importance here and L_0 is the radiance on the uniformly illuminated line of sight. If $\tau \ll 1$, L is approximately proportional to τ, which, if that for scattering by molecules and small particles, results in bluish airlight. But underlying this assertion, which is consistent with many observations, is an implicit assumption: L_0 is for sunlight that is not greatly attenuated. For sufficiently long optical paths through the atmosphere, we cannot ignore attenuation of the source illumination. So let us rewrite Eq. (8.77) more carefully:

$$L = L_\mathrm{s} \exp(-\tau_\mathrm{s}) G\{1 - \exp(-\tau)\}, \tag{8.78}$$

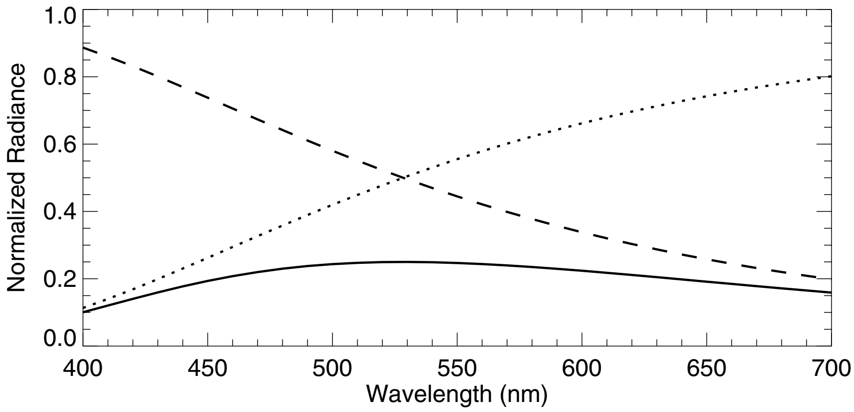

Figure 8.29: Airlight radiance due to an unattenuated source (dashed curve) and the radiance of attenuated solar radiation (dotted curve), both normalized by the solar radiance outside Earth's atmosphere, for an optical thickness 6 times the normal optical thickness of the molecular atmosphere. The product of these two curves is the solid curve.

where $L_{\rm s}$ is the solar radiance above Earth's atmosphere and $\tau_{\rm s}$ is the optical thickness through the atmosphere to the line of sight. Thus the airlight radiance is a product of two spectral functions. One function, $1-\exp(-\tau)$, *decreases* steadily from violet to red over the visible if τ is that for scattering by molecules and small particles; the other function, $\exp(-\tau_{\rm s})$, *increases* steadily from violet to red. The product of a function that yields red attenuated sunlight and a function that yields blue airlight can yield the intermediate green. Figure 8.29 shows $1-\exp(-6\tau_{\rm n})$, $\exp(-6\tau_{\rm n})$, and their product, where $\tau_{\rm n}$ is the normal molecular optical thickness. This corresponds to a black object some tens of kilometers away and the sun low in the sky. The result is an airlight spectrum with a broad peak in the green. This figure underscores a point made in our discussion of Fig. 8.7: for an atmosphere composed only of nonabsorbing molecules over black ground, blue is possible, as is red and everything in between.

Suppose that the distant black object is a thunderstorm, so thick that little sunlight is transmitted by it. For $\tau_{\rm s}$ sufficiently large, the corresponding airlight can be green. The thunderstorm is not the source of this green light, merely the dark backdrop against which it is seen. But it is possible that the thunderstorm itself is green, which we turn to next.

We stated in Section 5.3 that the bottoms of very thick clouds can have a bluish cast because water, liquid or solid, has an absorption minimum in the blue. But we omitted the implicit assumption that the source of illumination is sunlight that has not been greatly attenuated. Sunlight is reddened by atmospheric attenuation, with the result that its chromaticity coordinates (see Sec. 4.3) move to the right of the achromatic point, which we may take to be that of unattenuated sunlight. If reddened sunlight, by which we do not necessarily mean perceptually red, illuminates an intrinsically blue object, such as a cloud, the transmitted light can be perceptually green. An almost childishly simple demonstration shows this (see Prob. 4.44).

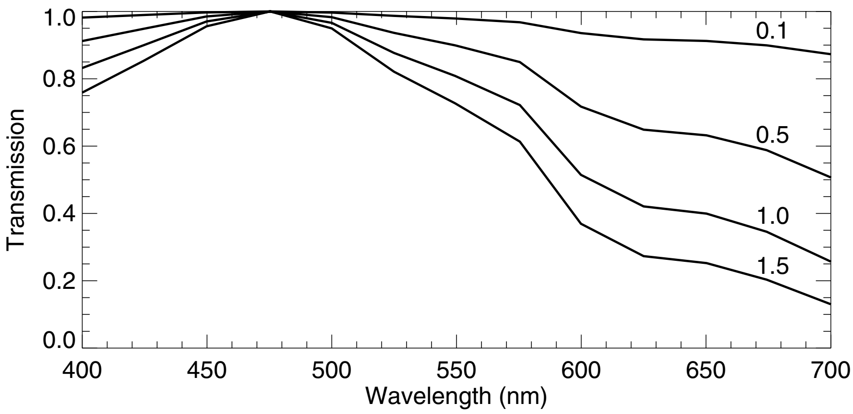

Figure 8.30: Transmission normalized to the maximum at around 470 nm for water droplet clouds with liquid water paths ranging from 0.1 cm to 1.5 cm.

Add some yellow food coloring to a glass of water, some blue food coloring to another glass of water. Then put the glass of bluish water in front of the glass of yellowish water and view the latter through the former. You will see green. This demonstration serves more than one purpose. It shows very simply a mechanism for green thunderstorms and also puts more nails in the coffin containing nonsense about yellow objects transmitting only yellow light, blue objects only blue light, and so on. If this were true, the demonstration would be impossible because the yellow water would be a source of only yellow light, whatever that is, and hence the blue water could not transmit green.

Green thunderstorms are associated with exceedingly thick clouds and seem to be observed mostly at sundown, when the illumination is reddened sunlight. Can this combination of red illumination of an intrinsically blue object be the cause of green thunderstorms? To answer this we have to be more precise about what is meant by "very thick clouds." By solving the two-stream Eq. (5.69) subject to the boundary conditions $F_{\downarrow}(0) = F_0$ and $F_{\uparrow}(\overline{\tau}) = 0$ we obtain the transmissivity of a normally illuminated finite cloud:

$$\frac{F_{\downarrow}(\overline{\tau})}{F_0} = \frac{(1 - R_{\infty}^2)\exp(-K\overline{\tau})}{1 - R_{\infty}^2 \exp(-2K\overline{\tau})}, \tag{8.79}$$

where $\overline{\tau}$ is the total optical thickness, K is given by Eq. (5.70), and R_{∞} is the reflectivity of the corresponding infinite medium. From Eqs. (5.73), (5.83), and (5.86) we have the approximation

$$K\overline{\tau} \approx h_{\mathrm{w}}\sqrt{\frac{3\kappa_{\mathrm{w}}(1-g)}{d}}, \tag{8.80}$$

where h_{w} is the liquid water path of the cloud, κ_{w} is the bulk absorption coefficient of water, g is the asymmetry parameter and d is a mean droplet diameter. Because R_{∞} is more or less flat across the visible spectrum (Fig. 5.15) and we are interested in clouds that are thick in

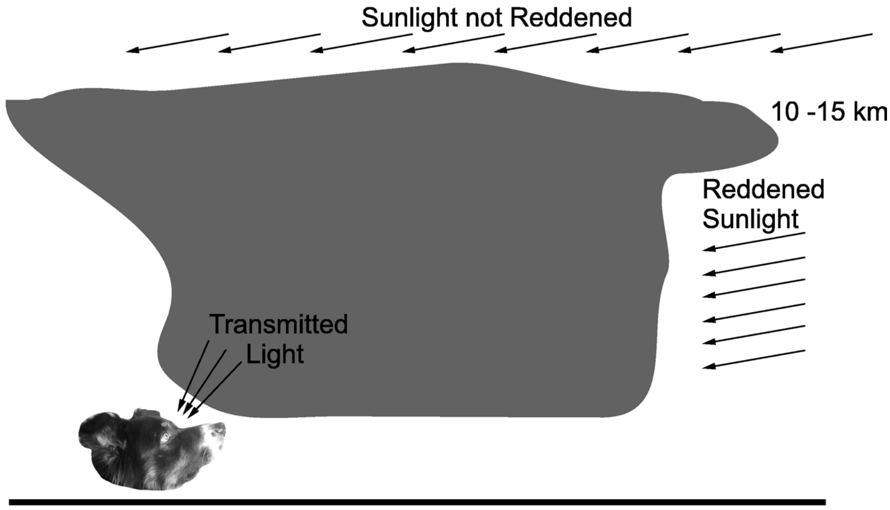

Figure 8.31: With the sun low in the sky the top of a thick cloud is illuminated by sunlight that has not been reddened, whereas the side of the cloud is illuminated by reddened sunlight. An observer on the ground receives transmitted light from both sources (top and side illumination).

the sense that $K\overline{\tau} > 1$, we plot $\exp(-K\overline{\tau})$ relative to its peak value versus wavelength in Fig. 8.30 for various liquid water paths instead of the transmissivity [Eq. (8.79)]. We take the asymmetry parameter $g = 0.85$ and mean droplet diameter $d = 10\ \mathrm{\mu m}$, although their precise values are irrelevant given that they are factors of h_{w} in Eq. (8.80). Only when the liquid water path of a cloud exceeds about 0.5 cm does transmission dip markedly in the red. Such a value is high but not unrealistically so, although it does correspond to clouds 10–15 km thick.

If a sufficiently thick cloud is illuminated by reddened sunlight, the result can be perceptually green transmitted light. But herein lies a problem. The cloud to which Eq. (8.79) applies is illuminated from above, and if thick clouds are needed, their tops must be high. Above altitudes of 10 km to 15 km, there is insufficient atmosphere to appreciably redden sunlight even with the sun low in the sky. So the cloud top illumination is essentially unattenuated sunlight. What is needed is reddened light illuminating the sides of two-dimensional, perhaps even three-dimensional clouds (Fig. 8.31). The one-dimensional analysis therefore does not prove that thick clouds illuminated at their tops give green thunderstorms but rather that long paths traversed by reddened sunlight in clouds can yield green light. Such clouds must be thick both vertically and horizontally.

Both explanations of green thunderstorms, which are not mutually exclusive, require illumination by reddened light, which seems to demand a low sun such as at sunrise or sunset. And, in fact, most green thunderstorms seem to be seen late in the day. But this by itself is not definitive because late in the day is when most thunderstorms occur. If a midday green thunderstorm is ever reported, we have a possible explanation for it. Suppose that sunlight shines through a gap in exceedingly thick clouds (Fig. 8.32). Illuminated air and particles at the bottom of the gap are the source of light scattered toward the observer, but only that frac-

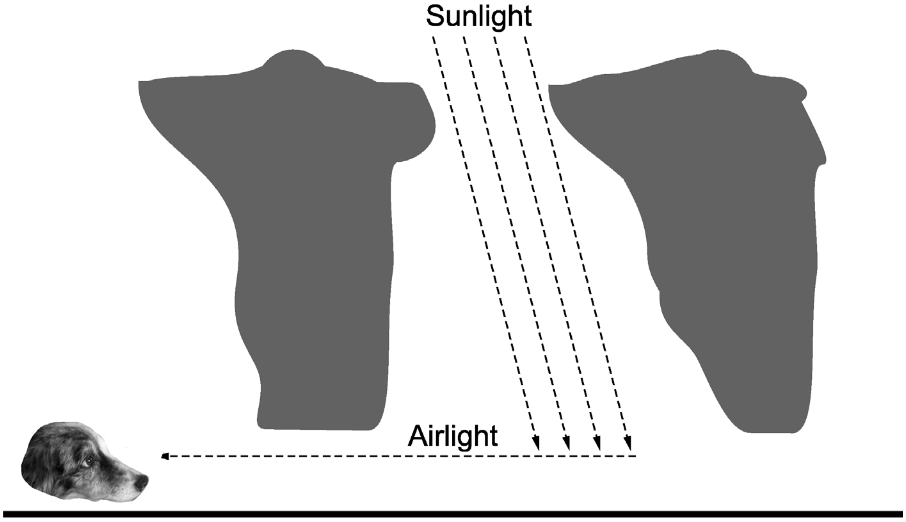

Figure 8.32: Scattered light results mostly from illumination by sunlight of an open patch of air between the two thick clouds whereas this scattered light is attenuated along the entire line of sight to an observer. Depending on the relative lengths of the paths below cloud and in the clear air, the spectrum of the transmitted light can be perceptually green: scattering shifts the spectrum toward shorter wavelengths, transmission shifts it toward longer wavelengths.

tion of it not scattered again along a path beneath the cloud is seen by an observer. Depending on the size of the gap and its distance from the observer, the transmitted light can be perceptually green. The mathematical analysis is identical to Eq. (8.78): the light within the gap is proportional to $1 - \exp(-\tau_g)$, where τ_g is the horizontal optical thickness of the gap, whereas the fraction of this light transmitted beneath the cloud is $\exp(-\tau_c)$, where τ_c is the optical thickness of the path beneath the cloud to the observer. If the cloud is thick, this path receives little illumination. Again, green light is possible with just the right combination of bluing by scattering (τ_g) and reddening by transmission (τ_c); it makes no difference in which order this occurs. We would not call this green light seen under thick clouds a green thunderstorm but rather green light seen in conjunction with a thunderstorm. A thunderstorm might be called green simply because it occurs simultaneously with green light.

Green in the twilight sky is a consequence of a similar mechanism but different geometry. Space is the black backdrop against which light at the short wavelength end of reddened sunlight is preferentially scattered toward the observer. What about reflection by grass as the source of green thunderstorms? Although theory [Eq. (5.68)] and measurements cast doubts on this explanation, the best refutation of it is observational: green thunderstorms have been reported over water and over parched ground nearly devoid of vegetation.

References and Suggestions for Further Reading

This chapter is based on Craig F. Bohren, 1995: Optics, atmospheric, *Encyclopedia of Applied Physics*, Vol. 12, pp. 405–34, subsequently reprinted (with minor changes and the title changed to atmospheric optics) in two other encyclopedic works: 2003: *Handbook of Weather, Climate, and Water*, Thomas D. Potter and Bradley R. Coleman, Eds., Wiley-Interscience, pp. 453–500, and 2004: *The Optics Encyclopedia* Vol. 1, Th. G. Brown, K, Creath, H. Kogelnik, M. A. Kriss, J. Schmit, M. J. Weber, Eds., Wiley-VCH, pp. 53–91.

Many of the seminal papers in atmospheric optics are more readily accessible in the collection compiled by Craig F. Bohren, Ed., 1989: *Selected Papers on Scattering in the Atmosphere*, SPIE Optical Engineering Press than in the original journals. An asterisk (*) before the name of authors in the citations that follow indicates that they are reprinted in this compendium.

Over the years special issues of Optical Society of America journals have been devoted to papers presented at Topical Meetings on Meteorological Optics (later changed to Topical Meetings on Light and Color in the Open Air): *Journal of the Optical Society of America*, Vol. 69, pp. 1051–198 (1979); Vol. 73, pp. 1622–64 (1983), Vol. A4, pp. 558–620 (1987); *Applied Optics*, Vol. 30, pp. 3381–552 (1991); Vol. 33, pp. 4535–760 (1994); Vol. 37, pp. 1425–1588 (1998); and Vol. 42, pp. 307–525 (2003).

The title of these Topical Meetings was inspired by Marcel Minnaert, 1954: *The Nature of Light and Color in the Open Air*. Dover. This is the bible of atmospheric optics. Minnaert inspired a few successors: Robert Greenler, 1980: *Rainbows, Halos, and Glories*. Cambridge University Press; R. A. R. Tricker, 1970: *Meteorological Optics*, Elsevier; Earl J. McCartney, 1976: *Optics of the Atmosphere*, Wiley; David K. Lynch and William Livingston, 1995: *Color and Light in Nature*. Cambridge University Press. The books by Greenler and by Lynch and Livingston are especially recommended for their many beautiful color plates.

Although not devoted exclusively to atmospheric optics William Jackson Humphreys, 1964: *Physics of the Air*. Dover contains a few chapters on the subject.

For an excellent history of polywater see Felix Franks, 1981: *Polywater*, MIT Press; for cold fusion see Frank Close, 1991: *Too Hot to Handle: The Race for Cold Fusion*, Princeton University Press.

Lord Rayleigh's On the light from the sky, its polarization and colour (1871) and On the transmission of light through an atmosphere containing small particles in suspension and the origin of the blue of the sky (1899) were originally published in *Philosophical Magazine*, but are more accessible in his *Scientific Papers*, Cambridge University Press, Vol. I, 1899, pp. 88–103 and Vol. III, 1902, pp. 397–405, respectively, or in the compendium *Selected Papers on Scattering in the Atmosphere*.

Not only does invoking Rayleigh scattering explain nothing, the term itself is fraught with different meanings. See *Andrew T. Young, 1982: Rayleigh scattering. *Physics Today*, January, pp. 2–8.

For a good history of the blue sky see Pedro Lilienfeld, 2004: A blue sky history. *Optics and Photonics News*, June, pp. 32–9. Evidence that he takes the history of science seriously is that he is one of the few authors to correctly credit Bouguer with the law of exponential attenuation (see Sec. 2.1).

For a detailed discussion of how the eye processes skylight containing more light that is not blue than is blue and yet yields the sensation blue, see Glenn S. Smith, 2005: Human color vision and the unsaturated blue color of the daytime sky. *American Journal of Physics*, Vol. 73, pp. 590–7.

Problem 4.10 asks for an estimate of the exposure time necessary to photograph the moonlit blue night sky. For a photograph of the night sky together with exposure times for various speed films and apertures see Joseph A. Shaw: 1996: What color is the night sky? *Optics and Photonics News*, November, pp. 54–5. Joe subsequently photographed the night sky with a digital camera and sent us some striking photos. He tells us that there are some "interesting differences" (e.g., shorter exposure times with digital) between digital and film night-sky photography, which he hopes to elucidate in a future article.

For more on daylight, especially the history of daylight spectroscopy, see S. T. Henderson, 1977: *Daylight and its Spectrum*, 2$^{\text{nd}}$ ed. John Wiley & Sons.

The (molecular) normal optical thickness (Fig. 8.2) and refractive index of air (Fig. 8.9) were taken from the compilation by *Rudolf Penndorf, 1957: Tables of the refractive index for standard air and the Rayleigh scattering coefficient for the spectral region between 0.2 and 20.0 μ and their application to atmospheric optics. *Journal of the Optical Society of America*, Vol. 47, pp. 176–82.

The heretical if not outright dangerous idea that big particles are not only unnecessary for a white horizon, they can make the horizon sky bluer is put forth by Craig F. Bohren and Clifton E. Dungey, 1992: Colors of the sky: can big particles make the sky bluer? *Contributions to Atmospheric Physics (Beiträge zur Physik der Atmosphäre)*, Vol. 64, pp. 329–34.

For an expository article on sky colors, including a proof that red sunsets and sunrises are not possible in a purely molecular atmosphere, see Craig F. Bohren and Alistair B. Fraser, 1985: Colors of the sky. *The Physics Teacher*, Vol. 23, pp. 267–72.

It seems that the first to recognize that molecular scattering is insufficient to explain the blue of the zenith sky when the sun is low was *E. O. Hulburt, 1953: Explanation of the brightness and color of the sky, particularly the twilight sky. *Journal of the Optical Society of America*, Vol. 42, pp. 113–18.

For a detailed treatment of the twilight sky see Georgii Vladimirovich Rozenberg, 1965: *Twilight: A Study in Atmospheric Optics*. Plenum.

The standard work on atmospheric visibility is by William Edgar Knowles Middleton, 1952: *Vision Through the Atmosphere*. University of Toronto Press.

For a simple demonstration that the contrast threshold depends on angular size, see Craig F. Bohren, 1987: *Clouds in a Glass of Beer*. John Wiley & Sons, Fig. 16.1.

With a bit of effort Eq. (8.34) can be extracted from Eq. (5.33) in *Harald Koschmieder, 1924: Theorie der horizontalen Sichtweite. *Beiträge zur Physik der freien Atmosphäre*, Vol. 12, pp. 33–54.

The quoted statement in Section 8.2 about Bouguer is in *William Edgar Knowles Middleton, 1960: Random reflections on the history of atmospheric optics. *Journal of the Optical Society of America*, Vol. 50, pp. 97–100.

There is more to contrast than visual range. Coming to grips with contrast may save your life. The senior author always has had white vehicles. In *Vision and Highway Safety* (Chilton Book Company, 1970) Merrill J. Allen asserts (p. 138) that "Studies have shown that up to ten times as many accidents happen to black cars as to white. Other data indicate white cars are up to forty times more visible than black." This is backed up by measurements of "relative visibility" of automobiles of difference colors.

Mirages, no matter how complicated, are readily explicable by fairly simple physics. This cannot be said of the moon illusion, a full understanding of which awaits (perhaps forever) a thorough understanding of how the human brain constructs a world out of raw visual data. The compendium by Maurice Hershenson, Ed., 1989: *The Moon Illusion*. Lawrence Erlbaum, is the result of its editor soliciting contributions from authors who have proposed (since 1962) explanations – there are at least nine – of the moon illusion, and commentaries from researchers in visual perception. The multiplicity of opinions in this compendium is evidence of how difficult it is to state *the* cause of the moon illusion. For a good treatise on the moon illusion see Helen E. Ross and Cornelis Plug, 2002: *The Mystery of the Moon Illusion: Exploring Size Perception*. Oxford University Press.

One of the best ever expository articles on mirages is by Alistair B. Fraser and William H. Mach, 1976: Mirages. *Scientific American*, Vol. 234(1), pp. 102–11. A mathematician's view of mirages, but without equations, is by Walter Tape, 1985: The topology of mirages. *Scientific America*. June, pp. 120–9.

For a putative explanation of the sun standing still because of a superior mirage, see Dario Camuffo, 1990: A meteorological anomaly in Palestine 33 centuries ago: How did the sun stop? *Theoretical and Applied Climatology*, Vol. 41, pp. 81–5.

The theorem about the impossibility of multiple images of astronomical objects above the horizon is in *Alistair B. Fraser, 1975: The green flash and clear air turbulence. *Atmosphere*, Vol. 13, pp. 1–10.

For a discussion of controversy and confusion over the role of the air temperature profile near the surface in astronomical refraction (angular displacement of astronomical objects) see Andrew T. Young, 2004: Sunset science IV: Low-altitude refraction. *The Astronomical*

Journal, Vol. 127, pp. 3622–37. He concludes that “At and below the astronomical horizon, the refraction depends primarily on atmospheric structure *below* the observer and varies so much (tens of minutes, or even several degrees) that only very crude predictions can be made. The observed time of sunset at a sea horizon often varies by a few minutes from day to day, and the variations increase with height above the sea.”

For many color photographs of the low sun and references to papers on the green flash (but not much in the way of explanation) see D. J. K. O’Connell, 1958: *The Green Flash and Other Low Sun Phenomena*. North Holland.

A detailed colorimetric analysis of the rim of the horizon sun was done by *Glenn E. Shaw, 1973: Observations and theoretical reconstruction of the green flash. *Pure and Applied Geophysics*, Vol. 102, pp. 223–35. His calculations include attenuation of sunlight by molecular scattering, scattering by particles, and absorption by ozone. Unfortunately, he did not do calculations in which ozone or particles were omitted, thus leaving unanswered questions about the degree to which they contribute to the green flash. We leave it as a problem (Prob. 8.61) for you to determine if absorption by ozone is an essential ingredient in the green flash.

A simple demonstration of the green flash (strictly, the green rim of the sun) using a slide projector as the source of illumination and a pan of slightly milky water with a mirror under the water is in Craig F. Bohren, 1982: The green flash. *Weatherwise*, Vol. 35, pp. 271–5. A rewritten version was published as Chapter 13 in Craig F. Bohren, 1987: *Clouds in a Glass of Beer*. John Wiley & Sons without, alas, the color plates in the original article.

For a discussion of the role of physiology in the perception of the green flash see Andrew T. Young, 2000: Sunset science. III. Visual adaptation and green flashes. *Journal of the Optical Society of America A*, Vol. 17, pp. 2129–39.

One of the classic papers on coronas is by *George C. Simpson, 1912: Coronae and iridescent clouds. *Quarterly Journal of the Royal Meteorological Society*, Vol. 38, pp. 291–301. Many years later Simpson “developed glaucoma and used his extensive knowledge of atmospheric haloes to analyze ocular haloes.” These “haloes” are what we would call coronas, which can originate from scattering within the eye. For more on this see David Miller and George Benedek, 1973: *Intraocular Light Scattering: Theory and Clinical Applications*. Charles C. Thomas, Ch. IV.

Near-forward scattering by randomly oriented hexagonal columns and plates is shown to be similar to that for spheres by Y. Takano and S. Asano, 1983: Fraunhofer diffraction by ice crystals suspended in the atmosphere. *Journal of the Meteorological Society of Japan*, Vol. 61, pp. 281–300. A figure showing the similarity between near-forward scattering by randomly oriented circular cylinders and spheres, according to the Fraunhofer approximation, is in Hendrik C. van de Hulst, 1957: *Light Scattering by Small Particles*. John Wiley & Sons, Fig. 16.

For observational evidence that coronas and iridescence are on rare occasions the result of scattering by nonspherical ice crystals see Kenneth Sassen, 2003: Cirrus cloud iridescence:

a rare case study. *Applied Optics*, Vol. 42, pp. 486–91; Kenneth Sassen, Gerald G. Mace, John Hallett, and Michael R. Poellet, 1998: Corona-producing ice clouds: a case study of a cold mid-latitude cirrus layer. *Applied Optics*, Vol. 37, pp. 1477–85; Kenneth Sassen, 1991: Corona-producing cirrus cloud properties derived from polarization lidar and photographic analysis. *Applied Optics*, Vol. 30, pp. 3421–8. Evidence for coronas and iridescence caused by scattering by ice particles in mountain wave clouds is put forward by Joseph A. Shaw and Paul J. Neiman, 2003: Coronas and iridescence in mountain wave clouds. *Applied Optics*, Vol. 42, pp. 476–85. They argue that ice-particle iridescence is more common than is believed, although only in wave clouds, not cirrus clouds.

For more about elliptical coronas see Pekka Parviainen, Craig F. Bohren, and Veikko Mäkelä, 1994: Vertical elliptical coronas caused by pollen. *Applied Optics*, Vol. 33, pp. 4548–51; Eberhard Tränkle and Bernd Mielke, 1994: Simulation and analysis of pollen coronas. *Applied Optics*, Vol. 33, pp. 4552–62.

The rainbow book to end all rainbow books is by Raymond Lee, Jr. and Alistair B. Fraser: 2001: *The Rainbow Bridge: Rainbows in Art, Myth, and Science*. The Pennsylvania State University Press and SPIE Press.

For a detailed discussion of supernumerary bows see *Alistair B. Fraser, 1983: Why can supernumerary bows be seen in a rain shower? *Journal of the Optical Society of America*, Vol. 73, pp. 1626–8. For a popular account, accompanied by superb photographs and illustrations, see Alistair B. Fraser, 1983: Chasing rainbows: numerous supernumeraries are super. *Weatherwise*, Vol. 36, pp. 280–9.

Investigation of reports of tertiary and higher-order rainbows in nature almost always reveal that they are something else, especially given that to many people any splash of color in the sky is a rainbow just as to other than bird watchers any small brown bird is a sparrow. One of the few exceptions is the observation by David E. Pedgley, 1986: A tertiary rainbow. *Weather*, Vol. 41, p. 401. Pedgley is a sufficiently careful observer to be believed. But to date no one seems to have photographed a tertiary rainbow in nature. In the laboratory is another matter. See Jearl D. Walker, 1976: Multiple rainbows from single drops of water and other liquids. *American Journal of Physics*, Vol. 44, pp. 421–33. For a photograph of an infrared rainbow see *Robert G. Greenler, 1971: Infrared rainbow. *Science*, Vol. 173, pp. 1231–2.

For a theory of the glory and its confirmation by detailed calculations see *Hendrik C. van de Hulst, 1947: A theory of the anti-coronae. *Journal of the Optical Society of America*, Vol. 37, pp. 16–22, *H. C. Bryant and A. J. Cox, 1966: Mie theory and the glory. *Journal of the Optical Society of America*, Vol. 56, pp. 1529–32, and *H. Moysés Nussenzveig, 1979: Complex angular momentum theory of the rainbow and the glory. *Journal of the Optical Society of America*, Vol. 69, pp. 1068–79.

For evidence of a possible ice-crystal glory see Kenneth Sassen, W. Patrick Arnott, Jennifer M. Barnett, and Steve Aulenbach, 1998: Can cirrus clouds produce glories? *Applied Optics*, Vol. 37, pp. 1427–33.

The backscattering direction is a source of delights for the eye and mind: the glory, coherent backscattering (see Probs. 6.28 and 6.29), the heiligenschein, and its fairly recently discovered cousin the *sylvanshine* (Alistair B. Fraser, 1994: The sylvanshine: retroreflection from dew-covered trees. *Applied Optics*, Vol. 33, pp. 4539–47). Keep in mind that these are distinguished by the agents responsible for them, not the theories used to describe them: no theory causes anything.

For a collection of stunning ice-crystal displays see Walter Tape, 1994: *Atmospheric Haloes*. American Geophysical Union. The display on the cover takes your breath away. Tape photographed many displays, some at the South Pole, and at the same time collected the crystals responsible for them. Several of the displays he photographed are compared with computer simulations. For an earlier treatise on haloes but without color photographs see R. A. R. Tricker, 1979: *Ice Crystal Haloes*. Optical Society of America.

Arguments that halos need not be caused by randomly oriented crystals are set forth by *Alistair B. Fraser, 1979: What size of ice crystals causes the halos? *Journal of the Optical Society of America*, Vol. 69, pp. 1112–18.

For a discussion of the consequences of multiple scattering to halos see *Eberhard Tränkle and Robert G. Greenler, 1987: Multiple-scattering effects in halo phenomena. *Journal of the Optical Society of America A*, Vol. 4, pp. 591–9.

The frequency of ice-crystal phenomena seen in Central Pennsylvania during a 16-year period is reported by Hans Neuberger, 1951: *Introduction to Physical Meteorology*. School of Mineral Industries, Pennsylvania State College, p. 174. (The School of Mineral Industries is now The College of Earth and Mineral Sciences, and Pennsylvania State College is now Pennsylvania State University).

It is unusual for proponents of different theories of the same phenomenon to publish them jointly in the same paper, but this was done by Craig F. Bohren and Alistair B. Fraser, 1993: Green thunderstorms. *Bulletin of the American Meteorological Society*, Vol. 74, pp. 2185–93. Observational evidence, by way of spectral measurements, that green thunderstorms are not "optical illusions" (whatever they are) is in Frank W. Gallagher III, William H. Beasley, and Craig F. Bohren, 1996: Green thunderstorms observed. *Bulletin of the American Meteorological Society*, Vol. 77, pp. 2889–97.

For evidence that green thunderstorms are not a consequence of reflection by green ground see Frank W. Gallagher III, 2001: Ground reflection and green thunderstorms. *Journal of Applied Meteorology*, Vol. 40, pp. 776–82.

Understanding atmospheric optical phenomena is enhanced by at least some knowledge of the particles responsible for them. To this end, the following are recommended: Hans R. Pruppacher and James D. Klett, 1980: *Microphysics of Clouds and Precipitation*. Dordrecht; Sean A. Twomey, 1977: *Atmospheric Aerosols*. Elsevier.

Problems

8.1. How might it be possible to determine the spectral solar irradiance at the top of the atmosphere and the normal optical thickness (clear sky) of the atmosphere by making spectral irradiance measurements at the surface of Earth? Devise a scheme for doing so.

HINT: Make measurements on a clear day. Assume exponential attenuation of direct solar radiation. Assume that your instrument is capable of measuring absolute irradiance of the (attenuated) sun.

HINT: Consider the variable $1/\cos\vartheta$, where ϑ is the solar zenith angle, to vary from 0 to infinity even though you can measure this quantity only for values greater than 1.

HINT: If you plot your data in the right way, it should be evident how to obtain the optical thickness and the irradiance at the top of the atmosphere.

HINT: The normal optical thickness varies from day to day and from place to place.

8.2. The clear sky is blue even though violet light is scattered more than blue light and the peak of the skylight spectrum is in the violet. Yet we sometimes see violet light in rainbows. Explain this apparent contradiction.

8.3. Estimate the largest zenith angle (smallest elevation angle) of the moon such that it cannot be seen against the daylight sky. You may take the reflectivity of the moon to be 0.06 over the visible spectrum. You may take the contrast threshold to be 0.02. You may take the normal optical thickness of the atmosphere (molecules and particles) to be 0.3 in the middle of the visible spectrum.

HINTS: In determining the contrast between the moon and the surrounding sky, be careful to ask yourself what light you see when you look at the moon through the atmosphere. For this problem you may assume that scattering by the molecules and particles is isotropic (i.e., the probability of scattering per unit solid angle in any direction is $1/4\pi$). Be sure that your answer makes sense in light of your own observations of the moon.

8.4. We noted in Section 8.1.1 that the radiance of the brightest cumulus cloud is larger, by roughly a factor of four, than that of the clear horizon sky. Please explain why. The phase function of cloud particles is largely irrelevant to the radiance of a cloud (if it is optically thick); scattering by air molecules is roughly isotropic.

Although we say "roughly a factor of four", this figure can be made more precise by using the result of Problem 7.18.

8.5. Although the blue sky still is sometimes attributed to water vapor it is not difficult to show that scattering of visible light by water vapor is less and by how much, per molecule, than that by the dominant components of air. You'll need the following data, taken from the 47^{th} edition of the *Handbook of Chemistry and Physics*, for refractive indices at the sodium D line (about in the middle of the visible spectrum) at 0 °C and pressure of one atmosphere: oxygen, 1.000271–1.000272; nitrogen, 1.000296–1.000298; water vapor, 1.000249–1.000259.

8.6. This problem is related to the previous one. Suppose that Earth's atmosphere were composed entirely of helium. What would the clear sky look like? How would the maximum degree of polarization of skylight change? To answer the second question requires knowing just a bit about the helium atom. The refractive index of helium at the sodium D line at 0 °C and one atmosphere is 1.000036.

8.7. David E. H. Jones used to write a regular column for *New Scientist*, under the pen name Daedalus, in which he would float highly imaginative and original scientific schemes, ostensibly plausible, sometimes outrageous, always interesting. These columns have been collected in two compendiums *The Inventions of Daedalus* and *The Further Inventions of Daedalus*. One such "plausible scheme" is the "optically flat Earth." Daedalus notes that if the refractive index gradient were such that the curvature of rays matched that of Earth it would be optically flat. Daedalus implicitly assumed an isothermal atmosphere, or, at the very least, that the refractive index gradient is dominated by the pressure gradient. In all that follows you may make this same assumption even though it is not true near the surface.

First, determine the condition for an optically flat Earth. Strictly speaking, this problem should be done by deriving the equation for ray trajectories in spherical coordinates, but you can use Eq. (8.48) to approximate the amount a ray initially horizontal at the surface descends vertically for a given (small) horizontal distance. Then match this with the same distance Earth's surface is below a horizontal plane. From this expression, determine the condition that the refractive index gradient must satisfy, then the radius of Earth such that a ray will follow its curvature. A quick approach is to simply guess, by dimensional analysis, the condition on the refractive index gradient. Although we obtained essentially the same expression as Daedalus, he claims that the radius of the optically flat Earth is only 13 km less than its actual value, whereas we obtain a radius almost 5 times larger. So we conclude that Daedalus made a computational error.

After doing this problem we noticed an inconsistency in the analysis. We (and Daedalus) assume a fixed scale height, whereas it depends on g, the acceleration due to gravity at the surface, which in turn depends on Earth's radius (assuming constant density). Given that the scale height is inversely proportional to g, determine the radius of the optically flat Earth (you'll need to know or find out how g depends on radius for fixed density).

But after doing this we noted another inconsistency. We assume a fixed refractive index at the surface. But if the total number of molecules in the atmosphere is fixed, the surface number density changes with scale height and radius. You can determine this dependence by integrating the number density (assumed exponential) over a spherical atmosphere or, to save effort, you can guess the correct form by dimensional analysis. If number density is not independent, neither is refractive index. Take this into account and determine the radius of the optically flat Earth. The result may surprise you.

Daedalus claimed that if Earth were optically flat, "people would not have realized the Earth was round until they discovered that, with a good telescope, you could see the back of your head." He ends with the assertion that "if the Earth were only 13 km smaller in radius, we could see round it." Discuss the physical correctness of this assertion, paying no heed to the fact that 13 km is incorrect.

8.8. The diffuse downward irradiance at the surface [Eq. (8.2)] for a molecular atmosphere was obtained from the two-stream approximation. But it can be obtained by direct integration assuming only single scattering. Take the incident sunlight to be directly overhead. You'll need the phase function for molecular scattering (see Prob. 7.18).

HINTS: You'll need to integrate over a laterally infinite slab of physical thickness h. This integration is done most easily in cylindrical polar coordinates. Ignore attenuation of light along the path from the point it is scattered to the point at which the irradiance is to be determined.

8.9. What is the normal (radial) optical thickness of the molecular atmosphere above the altitude at which commercial airlines fly (relative to the value from the surface to infinity)? Suppose that you were to determine this by using the uniform, finite atmosphere approximation. How much error would you make? What does this tell you about the limitations of this approximation?

8.10. Many years ago the niece of one of the authors pointed to the sunset sky and asked him why low scattered clouds were tinged with red whereas high clouds were white. He gave her an answer, which satisfied her. A few days later they were driving together in the early morning. This time she noted that low clouds were gray whereas those higher were reddish. She therefore challenged the previous explanation. Answer both of her questions.

HINT: It might be possible to answer this question without diagrams but it would be foolish to try.

8.11. We calculated “forever” to be about 330 km. This is the greatest distance at which a black object can be seen against the horizon sky at sea level in a molecular atmosphere for a contrast threshold of 0.02 (ignoring Earth’s curvature). This result was based on the assumption that the horizon optical thickness is essentially infinite (i.e., the horizon radiance is negligibly different from its asymptotic value). Although this assumption is a good one for even a clean atmosphere (for which particles increase the optical thickness), it is not quite true for an atmosphere *completely* free of particles. Estimate by how much “forever” is changed for a finite horizon optical thickness.

8.12. The parents of one of our students made the interesting observation that crescent moons are rarely, if ever, red or orange. Explain

8.13. If you look at distant clouds through a polarizing filter on a clear day, with the sun high in the sky, you are likely to notice a perceptible change in color of the clouds, from white to red, as you rotate the filter. Explain.

8.14. By how much is the visual range (at sea level) in an atmosphere free of particles expected to vary because of (realistic) variations in surface air temperature? Where we live, in Central Pennsylvania, we have noticed that exceptionally clear days often are also cold. Does this exceptional clarity have anything to do with temperature *per se*? Support your answer quantitatively.

8.15. In deriving an expression for the visual range we implicitly assumed that the ground was everywhere black. That is, the source illuminating the line of sight is only sunlight. Suppose that the ground were everywhere white (diffusely reflecting with a reflectivity close to 1). All else being equal, would the visual range increase, decrease, or remain the same? Explain your answer.

8.16. This problem is related to the previous one. Consider a distant black object, a line of hills, for example. Suppose that the ground were black everywhere from the observer to the object, then white everywhere beyond the object. Would the contrast between the object and the horizon sky go up, down, or stay the same? Now suppose that the ground were white everywhere from the observer to the object, then black everywhere beyond the object. Would the contrast go up, down, or stay the same?

HINT: You can answer these questions without doing detailed and complicated derivations of the airlight.

8.17. Solve Eq. (5.49) to verify our assertion that the diffuse downward irradiance (clear sky) for an atmosphere overlying a white surface should be approximately twice that for an atmosphere overlying a black surface [Eq. (8.2)].

8.18. Derive Eq. (8.8).

HINT: A binomial expansion is helpful.

8.19. We derived an expression for visual range in an atmosphere containing scatterers but not absorbers. Does the visual range change, and if so does it increase or decrease, when the (uniform) line of sight is characterized by an absorption coefficient κ as well as a scattering coefficient β? Before tackling a detailed solution, try to obtain it by simple physical reasoning.

8.20. Derive Eq. (8.76), the angle of minimum deviation for an arbitrary prism. First show that deviation is least when the incident ray and exit rays make the same angle with the prism face.

HINTS: You need Snel's law, simple trigonometry, and implicit differentiation

$$\frac{d}{dx}f\{y(x)\} = \frac{df}{dy}\frac{dy}{dx}$$

is useful.

8.21. Show that all rays incident at less than about $13°$ on a $60°$ ice prism are totally internally reflected (see Sec. 8.5.1).

8.22. Show that all incident rays are total internally reflected by ice prisms with prism angle greater than about $99.5°$ (see Sec. 8.5.1).

HINT: First convince yourself that the internal angle of incidence is *least* for an external angle of incidence of $90°$.

8.23. Misconceptions about what Newton did are widespread. For example, Newton is said to have broken up a beam of white light into a spectrum using a prism, then passed this spectrum through another prism to obtain the original beam of white light. Show that in general it is not possible to use two and only two identical prisms to transform a beam of white light back into a beam of white light. Show that this is possible with two identical slabs. Show that there is one special instance in which two prisms can restore a beam of white light. Find out what Newton really did by reading his *Optiks*. This problem was inspired by two notes in *American Journal Physics*. The first by Henry Perkins (Common misunderstanding of Newton's synthesis of light, Vol. 9, 1941, pp. 188–9), the second (The synthesis of light, Vol. 12, 1944, p. 232), in which Albert E. Hennings asserts that Perkins misunderstood Newton's synthesis of light. We think that they both are wrong, although Hennings not as wrong as Perkins.

HINT: Consider a single ray of white light to be a superposition of rays corresponding to a continuous set of wavelengths, all propagating along the same line. We can say that such a ray is broken up into its spectral components if interaction with an optical element results in rays separated in space or direction or both according to wavelength. To restore the original ray of white light requires that all these separate rays again propagate together in the same direction along the same line. The reversibility of refraction also should be helpful.

8.24. Estimate the degree of polarization of a sun dog. You need the Fresnel coefficients for transmission (Sec. 7.2) and the Mueller matrix for transmission by a prism. Ignore the birefringence of ice and take the (visible) refractive index of ice to be 1.31.

8.25. Estimate the degree of polarization of the primary rainbow. This is not a complicated problem. A sketch and a few simple calculations are sufficient. Concentrate on essentials, don't get bogged down in details. Figure 7.5 is helpful. Before tackling this problem ask yourself why the primary rainbow might be partially polarized.

8.26. Estimate the solar elevation above which a sun dog is not possible. You can do this in one line. Lengthy calculations are not needed.

8.27. Show that a zeroth-order rainbow is not possible. By zeroth-order is meant associated with rays transmitted after zero internal reflections.

8.28. Under what (plausible) conditions might it be possible to see a secondary but not a primary rainbow in nature?

8.29. We note in Section 8.4.2 that the reversed (angular) color order in the primary and secondary rainbows is an accident of the refractive index of water, not a necessary consequence of the different number of internal reflections. Show this.

HINT: The easiest way is by combining a bit of analysis with trial and error.

8.30. Prove the laws of specular reflection and refraction for a planar interface between two optically homogeneous negligibly absorbing media using Fermat's principle. That is, show that of all possible ray trajectories connecting any two points above the interface, subject to the constraint that a trajectory intersect it, the trajectory that *minimizes* Eq. (8.43) is the one for which the law of specular reflection holds. For the law of refraction, the ray trajectories must intersect an arbitrary fixed point in the medium from which they are incident and a fixed point inside the second medium.

To show that Fermat's principle is an extremum principle, not a minimum principle, consider a spherical concave mirror. Take the mirror to be a hemisphere. Show that of all possible ray trajectories that connect two points above the mirror and intersect it, the actual trajectory is the one that *maximizes* Eq. (8.43). For simplicity, take the two points to lie in the plane that intersects the hemisphere in a circle with radius equal to that of the hemisphere and equidistant from the center.

8.31. We showed that "forever" is about 330 km, the maximum distance at which a black object can be distinguished from the horizon sky, in a molecular atmosphere at sea level, assuming a contrast threshold of 0.02. Suppose, however, that you lived on the Tibetan Plateau at an altitude of about 4 km. By how much would "forever" be increased? You may take the scale height of the molecular atmosphere to be 8 km.

8.32. What is the maximum error in the optical thickness of a slant path through the molecular atmosphere, for zenith angles less than 85° (i.e., at least 5° above the horizon), as a consequence of assuming a flat Earth?

8.33. Derive Eq. (8.79) and make a stab at giving it a physical interpretation.

8.34. Show quantitatively that hail is irrelevant to green thunderstorms.

HINT: Consider the origin of hailstones and their size relative to cloud droplets.

8.35. Equation (8.80) does not include scattering by air, and yet clouds are not suspended in a vacuum. How does the air within a cloud affect any conclusions drawn from this equation used in Eq. (8.79)? Could scattering by air within a cloud result in reddening that can yield green light transmitted by a sufficiently thick cloud?

HINT: Before trying to answer this equation by modifying Eq. (8.80) so that it includes scattering by air, try to answer these questions by simple physical reasoning.

8.36. People who have witnessed green thunderstorms sometimes assert that the sky suddenly glowed bright green. Is this consistent with the two explanations of green thunderstorms discussed in Section 8.6.1? If not, what do you make of such assertions?

HINT: This is as much a problem in psychology and physiology as in physics.

8.37. Although the sky is blue, it is far from being pure blue, and atmospheric particles small compared with the wavelengths of sunlight can only make the sky a less pure blue. But because of such particles, sunsets can be orange or even red of much higher purity than the blueness of the sky. Explain this difference in purity.

8.38. Why was no mention made in this chapter of supernumerary sun dogs, the counterpart of supernumerary rainbows? Both sun dog and rainbow angles are caustics, and the curves of deviation angle versus angle of incidence are similar.

HINT: To understand why supernumerary sun dogs are conspicuous by their absence you have to understand why supernumerary rainbows are seen in rain showers.

8.39. Several years ago we were at a scientific meeting devoted to meteorological optics. While saying not a word, one speaker showed a slide that stunned the audience. Most of us understood the slide immediately and gasped. It showed a rainbow caused partly by rain, partly by spray from ocean waves crashing against a rocky coast. What caused us to gasp was a kink in the rainbow. Explain. Devise a simple demonstration of a kinky rainbow.

HINT: This has nothing to do with drop size. One of our students did this demonstration at the cost of a few dollars spent in the garden section of a hardware store.

8.40. The Old Testament contains an account of Joshua (10, 12–14) commanding the sun to stand still so that he would have more light for killing his enemies: "So the sun stood still in the midst of heaven, and hasted not to go down about a whole day. And there was no day like that before it or after it". This account has been attributed to a superior mirage as a consequence of hailstones on the ground (mentioned in this passage). Discuss. In particular, address such questions as, Could atmospheric refraction raise the sun to "the midst of heaven"? It if could, how would the sun appear? How much longer is the day because of atmospheric refraction?; Could this increase have been measured, or likely even been noticed, 33 centuries ago? And so on. This is an open–ended question.

8.41. Show that the generalization of Eq. (8.44) to a radially homogeneous atmosphere on a spherical Earth is $n \sin\vartheta / r = \mathrm{const.}$, where ϑ is the angle between a ray and the normal to a spherical surface at r.

HINT: Consider two adjacent, concentric spherical shells, sufficiently thin that the refractive index within each can be taken to be a (different) constant, and use Snel's law and the law of sines.

8.42. In deriving Eqs. (8.10) and (8.11) the source of radiation illuminating each point of the line of sight was taken to be unattenuated sunlight. How are these equations modified if attenuation is taken into account? Show that doing so does not affect Eq. (8.12).

HINT: Use the uniform, finite atmosphere approximation.

8.43. Do Problem 8.42 again but this time account for the variable number density of molecules with height. That is, take the scattering coefficient to be $\beta_0 \exp(-z/H)$, where β_0 is the scattering coefficient at $z = 0$ and H is the scale height.

HINT: A first step is to determine the optical thickness of the atmosphere from z to infinity.

8.44. Estimate the distance d from a black object such that the airlight spectral radiance (horizontal line of sight) is approximately the same as that of the overhead sky for a molecular atmosphere. How does your estimate change if the atmosphere also contains particles that are small compared with the wavelength? How does your estimate change if the atmosphere also contains particles that are comparable with or larger than the wavelength?

8.45. Leonardo introduced a fine spray of water into a darkened chamber illuminated by sunlight. The result was "blue rays", as a consequence of which he attributed the blue of the sky to the "particles of moisture which catch the rays of the sun." The only problem with this interpretation is that it is impossible to make a spray of water droplets (i.e., using a nozzle) much smaller than the wavelengths of visible light. If Leonardo did indeed see blue in his darkened chamber when he sprayed water into it, what is a physically plausible explanation for his observation?

HINT: To answer this question you need to know the rudiments of cloud formation, especially the distinction between cloud and haze droplets.

8.46. If a rainbow really did exhibit "all the colors of the rainbow", what would the curve of colorimetric coordinates on a transect through a bow (points along a radial line perpendicular to the bow) look like on the CIE chromaticity diagram (see Sec. 4.3.1)?

8.47. Problem 3.14 addressed the *existence* of the refractive index of a cloud (at sufficiently long wavelengths). Estimate its *value*.

HINTS: You will need Eq. (8.37) and Problem 8.5. For simplicity assume that the polarizability of a water droplet at the wavelengths determined in the previous problem is the polarizability of a single water molecule at visible wavelengths times the number of water molecules in the droplet. This is a crude approximation which can be made better by digging up the refractive index of water vapor at the wavelengths calculated in Problem 3.14. Also estimate the refractive index of the snowpack in Problem 3.14.

8.48. A correspondent wrote to us that "water in motion, when it freezes, is clear rather than ice cubes (or in clouds) which are cloudy (white)." This statement embodies an understandable misconception, namely, that the individual particles in clouds, water droplets or ice, have the same optical properties as the clouds themselves. What visual observations can you adduce to argue that cloud particles are not cloudy? What fairly simple laboratory experiment might you do in support of the observational evidence?

HINT: For your experiment a laser would be handy, although not absolutely necessary, and an eyedropper or hypodermic syringe.

8.49. One clear and dry evening many years ago the senior author and Sean Twomey emerged from a pub in Tucson, Arizona into the parking lot behind the pub. As they did their eyes were drawn toward an intense green lamp on a pole perhaps a dozen meters distant, the only source of illumination in the lot. The following conversation ensued. "Do you see it?" "Yep." "It's not real is it?" "Nope." And then both nearly simultaneously made a simple test to confirm their supposition. The "it" here was a green halo around the lamp, perhaps a few degrees from it. What simple test did they make and what was the source of this halo?

8.50. We once received a letter from a retired Canadian commercial airline pilot who told us that in all his years of flying he had never once seen the horizon from cruising altitude. Do a rough calculation to show that this is exactly what is to be expected even in a very clean atmosphere.

HINT: The uniform, finite atmosphere approximation is adequate here. For more on this problem see Craig F. Bohren and Alistair B. Fraser, 1986: At what altitude does the horizon cease to be visible? *American Journal of Physics*, Vol. 54, pp. 222–7.

8.51. Why does the agreement between the measured radiance ratio in Fig. 8.5 and that calculated using Eq. (8.12) with a molecular optical thickness get worse for longer wavelengths?

8.52. In 1899 Lord Rayleigh published a derivation of the scattering coefficient of air based on the assumption that the radiant power scattered by N molecules is N times scattering by one molecule ("phases are entirely distributed at random"). The wavelength dependence of this scattering coefficient (inverse fourth power) is in accord with the observed color of the clear sky. But this observation is qualitative. There remains the question of the correct magnitude of the scattering coefficient. What observations (without using a radiometer) might Rayleigh have made to verify the correct magnitude of his scattering coefficient? Or, stated another way, what observations would you make?

8.53. Approximately how much (what fraction) of the total mass density of a cloud is contributed by cloud particles? What is the ratio of this contribution to that by water vapor only?

8.54. Over what range of wavelengths are rainbows possible?

8.55. Attenuation of beams of radiation (also sound) is often expressed in decibels (dB), defined as 10 times the logarithm (base 10) of the ratio of the radiance at any distance to that at the source. What is the attenuation of a beam of near-infrared radiation (say 1 $\mu\mathrm{m}$) that travels completely around Earth (at sea-level) in an atmosphere free of particles? What thickness of fog gives the same attenuation? Fog liquid water contents are typically $0.1\ \mathrm{g\,m^{-3}}$ or less, and a representative fog droplet diameter might be 2 $\mu\mathrm{m}$. Ignore multiple scattering.

8.56. For some purposes one can take the refractive index of air to be identically equal to 1 without introducing appreciable error. But for other purposes this would result in infinite error. Give examples. You might want to review Sections 3.5 and 7.2.

8.57. Although it is difficult to make a complete indoor rainbow, you can make indoor sundogs with a $60°$ prism mounted with its vertical axis perpendicular to a base that can be rotated. A slide projector and a black slide with a hole in it serves as a monodirectional source of white light. Illuminate the prism while it is rapidly rotating and two colored spots will be projected onto a wall behind the prism. The only difference between these artificial sun dogs and those in nature is the angle. What is the angle for the artificial sun dogs? Suppose that you were intent on duplicating the exact $22°$ sundog with a glass prism. What would be the required prism angle?

8.58. Strictly, Eq. (8.37) applies to an ideal gas composed of a single species. Stated another way, the polarizability in this equation is an average. Generalize this equation to a mixture of ideal gases, and hence verify the assertion made in the references at the end of Chapter 3 that the refractive index of air is the volume-weighted average of its gaseous components.

HINT: You have to be careful to specify exactly what you mean by a volume-weighted average.

8.59. Students of the atmosphere who understand the conditions under which Rayleigh's scattering law is valid should also be able to state why scattering by tiny (pure) water droplets cannot be the origin of the blue sky. Please do so.

8.60. Estimate the absorption optical thickness through the ozone layer, at the peak of the Chappuis bands, for a horizontal path (sun on the horizon) tangential to Earth. Make any reasonable approximations.

8.61. With the results of the previous problem, determine if the Chappuis bands of ozone markedly affect the green flash.

8.62. In Section 4.3.1 we assert that colors in our everyday lives are far from pure and support this with measurements (Fig. 4.22). But there is at least one exception: the often high purity of orange and red sunsets and sunrises. Explain.

8.63. A student once showed us a puzzling photograph of a low sun several degrees above a lake. The sun was yellowish but its specular reflection in the lake was reddish. As evidenced by Fig. 3.8, specular reflection by water does not depend on wavelength over the visible spectrum. Moreover, anything dissolved in the water will not change this. Explain the difference in the photographed color of the sun and its reflection. You may take the water to be pure and free of any suspended matter. What observation would you try to make in order to support your explanation?

8.64. We state in Section 8.4.2 that light scattered by atmospheric molecules and particles reduces the purity of the colors of rainbows. Suppose that by some magic the atmosphere were to disappear. Would the rainbow colors now be pure? Assume that geometrical optics is exact. That is, this question is not about the appropriateness of a theory but about one of the fundamental reasons why rainbow colors are not pure.

8.65. Steven Greenberg, a graduate student in meteorology at Penn State University, asked us the following question: "The attached picture shows a mid-visible (660 nm) satellite image of the northern Alaska area. During the period of this satellite image, the lidar (532 nm) is unable to penetrate through the persistent liquid-topped boundary layer clouds (as expected). My question is... if both the satellite and lidar operate at visible wavelengths, then why are we able to see the geography beneath the cloud layer? Wouldn't the cloud be opaque for both instruments?" Answer his question. Point Barrow, the northernmost tip of Alaska, is in the middle of the image. The Brooks Range is the rugged topography in the lower right. Sea ice can be seen through cloud cover at the top right. The ground is covered with snow, but the Arctic Ocean to the north and west has yet to freeze.