

$$1^{\circ} \quad V(R, h) = \pi R^2 h + \frac{2}{3} \pi R^3.$$

$$2^{\circ} \quad dV = \left( \frac{\partial V}{\partial R} \right)_h dR + \left( \frac{\partial V}{\partial h} \right)_R dh.$$

$$\begin{aligned} \bullet \left( \frac{\partial V}{\partial R} \right)_h &= 2\pi R h + 2\pi R^2 \\ &= 2\pi R (h + R) \end{aligned}$$

$$\bullet \left( \frac{\partial V}{\partial h} \right)_R = \pi R^2$$

donc :

$$dV = 2\pi R (h + R) dR + \pi R^2 dh.$$

$$3^{\circ} \quad \frac{dV}{dt} = 2\pi R (h + R) \frac{dR}{dt} + \pi R^2 \frac{dh}{dt}.$$

$$4^{\circ} \quad \text{A.N: } \begin{cases} \frac{dR}{dt} = 1 \text{ cm/min} \\ \frac{dh}{dt} = 2 \text{ cm/min} \end{cases}$$

$$\begin{aligned} \frac{dV}{dt} &= 2\pi \times 10 \times 30 \times 1 + \pi \times 100 \times 2 \\ &= 600\pi + 200\pi \\ &= 800\pi \text{ cm}^3 \cdot \text{min}^{-1} \\ &= \frac{800\pi}{60} \\ &= \frac{40\pi}{3} \text{ cm}^3 \cdot \text{s}^{-1} \end{aligned}$$