MATH1120-LS-2022 W2

Marking scheme for written-answer question

0 Variant List

Variant	nth term	Radius
Variant 1	$n\,(1/2)^n\,\left(3x+2\right)^n$	2/3
Variant 2	$n\,(1/2)^n\,\left(4x+2\right)^n$	1/2
Variant 3	$n\,(1/2)^n\,\left(5x+2\right)^n$	2/5
Variant 4	$n\,(1/3)^n\,\left(2x+2\right)^n$	3/2
Variant 5	$n\,(1/3)^n\,\left(4x+2\right)^n$	3/4
Variant 6	$n\,(1/3)^n\,\left(5x+2\right)^n$	3/5
Variant 7	$n\,(1/4)^n\,\left(3x+2\right)^n$	4/3
Variant 8	$n\,(1/4)^n\,\left(5x+2\right)^n$	4/5
Variant 9	$n\,(1/5)^n\,\left(2x+2\right)^n$	5/2
Variant 10	$n\,(1/5)^n\,\left(3x+2\right)^n$	5/3
Variant 11	$n\,(1/5)^n\,\left(4x+2\right)^n$	5/4

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

1 Variant 1

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/2)^n\,\big(3x+2\big)^n$$

[For office use only: V1]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/2)^{n+1}\,\big(3x+2\big)^{n+1}}{n\,(1/2)^n\,\big(3x+2\big)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/2)(3x+2) \right| \\ &= \left|(1/2)(3x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/2)(3x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+4/3| < 2/3.$$

Thus, the radius of convergence is $R = 2/3$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

2 Variant 2

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/2)^n \left(4x+2\right)^n$$

[For office use only: V2]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/2)^{n+1}\left(4x+2\right)^{n+1}}{n\,(1/2)^n \left(4x+2\right)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/2)(4x+2) \right| \\ &= \left|(1/2)(4x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/2)(4x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+1| < 1/2.$$

Thus, the radius of convergence is $R = 1/2$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

3 Variant 3

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/2)^n\,\big(5x+2\big)^n$$

[For office use only: V3]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/2)^{n+1}\,\big(5x+2\big)^{n+1}}{n\,(1/2)^n\,\big(5x+2\big)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/2)(5x+2) \right| \\ &= \left|(1/2)(5x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/2)(5x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+4/5| < 2/5.$$

Thus, the radius of convergence is $R = 2/5$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

4 Variant 4

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/3)^n\,\left(2x+2\right)^n$$

[For office use only: V4]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/3)^{n+1}\,\left(2x+2\right)^{n+1}}{n\,(1/3)^n\,\left(2x+2\right)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/3)(2x+2) \right| \\ &= \left|(1/3)(2x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/3)(2x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+3| < 3/2.$$

Thus, the radius of convergence is $R = 3/2$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

5 Variant 5

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/3)^n \left(4x+2\right)^n$$

[For office use only: V5]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/3)^{n+1} \left(4x+2\right)^{n+1}}{n\,(1/3)^n \left(4x+2\right)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/3)(4x+2) \right| \\ &= \left|(1/3)(4x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/3)(4x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+3/2| < 3/4.$$

Thus, the radius of convergence is $R = 3/4$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

6 Variant 6

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/3)^n\,\big(5x+2\big)^n$$

[For office use only: V6]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/3)^{n+1}\,\big(5x+2\big)^{n+1}}{n\,(1/3)^n\,\big(5x+2\big)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/3)(5x+2) \right| \\ &= \left|(1/3)(5x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/3)(5x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+6/5| < 3/5.$$

Thus, the radius of convergence is $R = 3/5$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

7 Variant 7

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/4)^n\,\big(3x+2\big)^n$$

[For office use only: V7]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty}\left|\frac{(n+1)\,(1/4)^{n+1}\,\big(3x+2\big)^{n+1}}{n\,(1/4)^n\,\big(3x+2\big)^n}\right| \\ &= \lim_{n\to\infty}\left|\frac{n+1}{n}(1/4)(3x+2)\right| \\ &= \left|(1/4)(3x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/4)(3x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+8/3| < 4/3.$$

Thus, the radius of convergence is $R = 4/3$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

8 Variant 8

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/4)^n\,\left(5x+2\right)^n$$

[For office use only: V8]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/4)^{n+1}\,\left(5x+2\right)^{n+1}}{n\,(1/4)^n\,\left(5x+2\right)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/4)(5x+2) \right| \\ &= \left|(1/4)(5x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/4)(5x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+8/5| < 4/5.$$

Thus, the radius of convergence is $R = 4/5$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

9 Variant 9

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/5)^n\,\left(2x+2\right)^n$$

[For office use only: V9]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/5)^{n+1}\,\left(2x+2\right)^{n+1}}{n\,(1/5)^n\,\left(2x+2\right)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/5)(2x+2) \right| \\ &= \left|(1/5)(2x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/5)(2x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+5| < 5/2.$$

Thus, the radius of convergence is $R = 5/2$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

10 Variant 10

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/5)^n\,\left(3x+2\right)^n$$

[For office use only: V10]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty}\left|\frac{(n+1)\,(1/5)^{n+1}\,\left(3x+2\right)^{n+1}}{n\,(1/5)^n\,\left(3x+2\right)^n}\right| \\ &= \lim_{n\to\infty}\left|\frac{n+1}{n}(1/5)(3x+2)\right| \\ &= \left|(1/5)(3x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/5)(3x+2)| < 1 \qquad \Longleftrightarrow \qquad |x + 10/3| < 5/3.$$

Thus, the radius of convergence is $R = 5/3$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

11 Variant 11

Compute the radius of convergence of the following power series. Show all your work.

$$f(x) = \sum_{n=0}^{\infty} n\,(1/5)^n\,\big(4x+2\big)^n$$

[For office use only: V11]

Solution.

We use the ratio test. Let

$$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{(n+1)\,(1/5)^{n+1}\,\big(4x+2\big)^{n+1}}{n\,(1/5)^n\,\big(4x+2\big)^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{n+1}{n}(1/5)(4x+2) \right| \\ &= \left|(1/5)(4x+2)\right|. \end{aligned}$$

Now the power series will converge if $L < 1$, i.e.

$$|(1/5)(4x+2)| < 1 \qquad \Longleftrightarrow \qquad |x+5/2| < 5/4.$$

Thus, the radius of convergence is $R = 5/4$.

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
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- 3 marks: The student demonstrates a good understanding and obtains the correct answer.