MATH1120-S2-2021 Week-8

Marking scheme for written-answer question

0 Variant List

Variant	A	Solution
Variant 1	$\begin{pmatrix} -1 & -2 \\ 1 & -4 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
Variant 2	$\begin{pmatrix} -4 & 1 \\ -2 & -1 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
Variant 3	$\begin{pmatrix} -1 & 2 \\ -1 & -4 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$
Variant 4	$\begin{pmatrix} -4 & -1 \\ 2 & -1 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
Variant 5	$\begin{pmatrix} -4 & 2 \\ -1 & -1 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
Variant 6	$\begin{pmatrix} -1 & -1 \\ 2 & -4 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
Variant 7	$\begin{pmatrix} -4 & -2 \\ 1 & -1 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
Variant 8	$\begin{pmatrix} -1 & 1 \\ -2 & -4 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
Variant 9	$\begin{pmatrix} -2 & -2 \\ -2 & 1 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
Variant 10	$\begin{pmatrix} 1 & -2 \\ -2 & -2 \end{pmatrix}$	$C_1e^{-3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

1 Variant 1

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -x_1 - 2x_2$$
$$\frac{dx_2}{dt} = x_1 - 4x_2$$

[For office use only: V 1]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -1 & -2 \\ 1 & -4 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 1 & -2 \\ 1 & -\lambda - 4 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} 2 & -2 & 0 \\ 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

2 Variant 2

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -4x_1 + x_2$$
$$\frac{dx_2}{dt} = -2x_1 - x_2$$

[For office use only: V 2]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -4 & 1 \\ -2 & -1 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 4 & 1 \\ -2 & -\lambda - 1 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ -2 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} -2 & 1 & 0 \\ -2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

3 Variant 3

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -x_1 + 2x_2$$
$$\frac{dx_2}{dt} = -x_1 - 4x_2$$

[For office use only: V 3]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -1 & 2 \\ -1 & -4 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 1 & 2 \\ -1 & -\lambda - 4 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{rr|r} 2 & 2 & 0 \\ -1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{rr|r} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{rr|r} 1 & 2 & 0 \\ -1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{rr|r} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

4 Variant 4

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -4x_1 - x_2$$
$$\frac{dx_2}{dt} = 2x_1 - x_2$$

[For office use only: V 4]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -4 & -1 \\ 2 & -1 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 4 & -1 \\ 2 & -\lambda - 1 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 2 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} -2 & -1 & 0 \\ 2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

5 Variant 5

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -4x_1 + 2x_2$$
$$\frac{dx_2}{dt} = -x_1 - x_2$$

[For office use only: V 5]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -4 & 2 \\ -1 & -1 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 4 & 2 \\ -1 & -\lambda - 1 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} -1 & 2 & 0 \\ -1 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} -2 & 2 & 0 \\ -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

6 Variant 6

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -x_1 - x_2$$
$$\frac{dx_2}{dt} = 2x_1 - 4x_2$$

[For office use only: V 6]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -1 & -1 \\ 2 & -4 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 1 & -1 \\ 2 & -\lambda - 4 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

7 Variant 7

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -4x_1 - 2x_2$$
$$\frac{dx_2}{dt} = x_1 - x_2$$

[For office use only: V 7]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -4 & -2 \\ 1 & -1 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 4 & -2 \\ 1 & -\lambda - 1 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ 1 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Longrightarrow \mathbf{v}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} -2 & -2 & 0 \\ 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Longrightarrow \mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} -2 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

8 Variant 8

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -x_1 + x_2$$
$$\frac{dx_2}{dt} = -2x_1 - 4x_2$$

[For office use only: V 8]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -1 & 1 \\ -2 & -4 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 1 & 1 \\ -2 & -\lambda - 4 \end{vmatrix} = (\lambda + 4)(\lambda + 1) + 2 = (\lambda + 3)(\lambda + 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = -2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ -2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

For $\lambda_2 = -2$, we solve the system

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ -2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

9 Variant 9

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = -2x_1 - 2x_2$$
$$\frac{dx_2}{dt} = -2x_1 + x_2$$

[For office use only: V 9]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} -2 & -2 \\ -2 & 1 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda - 2 & -2 \\ -2 & -\lambda + 1 \end{vmatrix} = (\lambda + 2)(\lambda - 1) - 4 = (\lambda + 3)(\lambda - 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = 2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ -2 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = 2$, we solve the system

$$\left(\begin{array}{cc|c} -4 & -2 & 0 \\ -2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.

10 Variant 10

Solve the following system of coupled linear differential equations, showing all your work:

$$\frac{dx_1}{dt} = x_1 - 2x_2$$
$$\frac{dx_2}{dt} = -2x_1 - 2x_2$$

[For office use only: V 10]

Solution.

The coefficient matrix is

$$A = \begin{pmatrix} 1 & -2 \\ -2 & -2 \end{pmatrix}.$$

We determine its eigenvalues and eigenvectors:

$$0 = \det(A - \lambda I) = \begin{vmatrix} -\lambda + 1 & -2 \\ -2 & -\lambda - 2 \end{vmatrix} = (\lambda + 2)(\lambda - 1) - 4 = (\lambda + 3)(\lambda - 2).$$

Thus $\lambda_1 = -3$, and $\lambda_2 = 2$. Next, we determine the corresponding eigenvectors.

For $\lambda_1 = -3$, we solve the system

$$\left(\begin{array}{cc|c} 4 & -2 & 0 \\ -2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

For $\lambda_2 = 2$, we solve the system

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ -2 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\implies \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

The final solution is thus

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

Marking Scheme:

- 1 mark: The student demonstrates a partial understanding of how to do the problem.
- 2 marks: The student demonstrates a good understanding of how to do the problem (some minor errors permitted).
- 3 marks: The student demonstrates a good understanding and obtains the correct answer.