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Matthew Rabin

Economics Department, University of California, Berkeley

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Abstract

Arrow (1971) shows that an expected-utility maximizer with a differentiable utility function will always want to take a sufficiently small stake in any positive-expected-value bet. That is, expected-utility maximizers are arbitrarily close to risk neutral when stakes are arbitrarily small. While most economists understand this formal limit result, fewer appreciate that the approximate risk-neutrality prediction holds not just for very small stakes, but for quite sizable and economically important stakes. Diminishing marginal utility of wealth is not a plausible explanation of people's aversion to risk on the scale of $10, $100, $1000 or even more. After illustrating and providing intuition for these claims, I shall argue that economists often reach misleading conclusions by invoking expected-utility theory to explain substantial risk aversion in contexts where the theory actually predicts virtual risk neutrality.

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Matthew Rabin
Department of Economics
University of California, Berkeley

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1. Introduction

Economists pervasively explain widespread risk aversion by the realistic assumption that people generally have diminishing marginal utility of wealth. Indeed, diminishing marginal utility of wealth probably explains much of our aversion to large-scale financial risk: We dislike vast uncertainty in lifetime wealth because the marginal value of a dollar when we are poor is higher than when we are rich.

Within the expected-utility framework, the concavity of the utility-of-wealth function is not only sufficient to explain risk aversion—it is also necessary: Diminishing marginal utility of wealth is the *sole* explanation for risk aversion. Unfortunately, it is an utterly implausible explanation for appreciable risk aversion, except when the stakes are very large. Any utility-of-wealth function that doesn't predict absurdly severe risk aversion over very large stakes predicts negligible risk aversion over modest stakes.

Arrow (1971, p. 100) shows that an expected-utility maximizer with a differentiable utility function will always want to take a sufficiently small stake in any positive-expected-value bet. That is, expected-utility maximizers are arbitrarily close to risk neutral when stakes are arbitrarily small. While most economists understand this formal limit result, fewer appreciate that the approximate risk-neutrality prediction holds not just for very small stakes, but for quite sizable and economically important stakes. Diminishing marginal utility of wealth is not a plausible explanation of people's aversion to risk on the scale of $10, $100, $1,000, or even more. After illustrating and providing intuition for these claims, I shall argue that economists often reach misleading conclusions by invoking expected-utility theory to explain substantial risk aversion in contexts where the theory actually predicts virtual risk neutrality.

2. Illustrations

The inability of expected-utility theory to provide a plausible account of risk aversion over modest stakes has been illustrated in writing in a variety of different contexts using standard utility functions.[1] For instance, Hansson (1988) notes that a person with constant absolute risk aversion who is indifferent between gaining $7 for sure and a 50-50 gamble of gaining either $0 or $21 prefers a sure gain of $7 to *any* lottery where the chance of gaining positive amounts of money is less than 40%—no matter how large the potential gain.

In Rabin (forthcoming), I provide a theorem showing that such implications are not restricted to particular contexts or to particular functional specifications of the utility function. Within the expected-utility framework, for *any* concave utility function, even very little risk aversion over modest stakes implies an absurd degree of risk aversion over large stakes. The theorem is "non-parametric", literally assuming nothing about the utility function except that it is increasing and concave.[2] Without invoking assumptions like constant absolute risk aversion, we can make statements of the form "If an expected-utility maximizer always turns down modest-stakes Gamble X, she will always turn down large-stakes Gamble Y."

Suppose that, from any initial wealth level, an expected-utility maximizer turns down gambles where she loses $100 or gains $110, each with 50% probability. Then she will turn down 50-50 bets of losing $1,000 or gaining *any* sum of money. An expected-utility maximizer who always turns down 50-50 lose $1,000/gain $1,050 bets would always turn down 50-50 bets of losing $20,000 or gaining any sum. An expected-utility maximizer who always turns down 50-50 lose $100/gain $101 bets would always turn down 50-50 bets losing $10,000 or gaining any sum. In each case, the aversion to the small bet realistically describes many people's risk attitudes, whereas the implied large-scale risk attitudes do not. Table 1 lists these and many more examples:

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[1] See, e.g., Epstein (1992), Epstein and Zin (1990), Kandel and Stambaugh (1991), Loomes and Segal (1994), and Segal and Spivak (1990).

[2] Many of the arguments and examples in this article closely match those in Rabin (forthcoming).

If an expected-utility maximizer always turns down the 50/50 bet …	then she always turns down the 50/50 bet …
lose $10 / gain $10.10	**lose $800 / gain $3,494**
lose $10 / gain $10.10	**lose $1,000 / gain $∞**
lose $10 / gain $11	**lose $100 / gain $∞**
lose $100 / gain $101	**lose $8,000 / gain $34,940**
lose $100 / gain $101	**lose $10,000 / gain $∞**
lose $100 / gain $105	**lose $1,000 / gain $1,570**
lose $100 / gain $105	**lose $2,000 / gain $∞**
lose $100 / gain $110	**lose $1,000 / gain $∞**
lose $1,000 / gain $1,010	**lose $80,000 / gain $349,400**
lose $1,000 / gain $1,010	**lose $100,000 / gain $∞**
lose $1,000 / gain $1,050	**lose $20,000 / gain $∞**
lose $1,000 / gain $1,100	**lose $10,000 / gain $∞**
lose $1,000 / gain $1,250	**lose $6,000 / gain $∞**
lose $10,000 / gain $11,000	**lose $100,000 / gain $∞**
lose $10,000 / gain $12,500	**lose $60,000 / gain $∞**

The intuition for the implications illustrated in Table 1 is that within the expected-utility framework turning down a modest-stakes gamble means that the marginal utility of money must diminish very quickly. Suppose you have initial wealth of W, and that you reject a 50-50 lose $10/gain $11 gamble because of diminishing marginal utility of wealth. Then it must be that $U(W+11) - U(W) \leq U(W) - U(W-10)$. Hence, on average you value each of the dollars between W and $W+11$ by at most 10/11 as much as you on average value each of the dollars between W and $W-10$. By concavity, this implies that you value the dollar $W+11$ at most 10/11 as much as you value the dollar $W-10$. Iterating this observation, if you have the same aversion to the lose $10/gain $11 bet at wealth level $W+21$, then you value dollar $W+21+11 = W+32$ by at most 10/11 as you value dollar $W+21-10 = W+11$, which means you value dollar $W+32$ by at most 10/11 x 10/11 ≈ 5/6 as much as dollar $W-10$. You will value the $W+220^{th}$ dollar by at most 3/20 as much as dollar *W-10*, and the *W+880*th dollar by at most 1/2,000 as much as dollar *W-10*. This is an absurd rate for the value of money to deteriorate—and the rate of deterioration implied by expected-utility theory is actually quicker than this. This algebra shows that any attempt to explain attitudes to modest risk in terms of the utility of lifetime wealth would imply a paralyzing aversion to risks that everyone finds extremely attractive. It appears safe to conclude that aversion to modest-stakes risk has nothing to do with the diminishing marginal utility of wealth.

A similar argument can be developed without invoking the assumption that the person turns down the given gamble for all initial wealth levels. Suppose, for instance, we know a risk-averse person turns down 50-50 lose $100/gain $105 bets for any lifetime wealth level less than (say) $350,000, but know nothing about her utility function for wealth levels above $350,000, except that it is not convex. Then we know that from an initial wealth level of $340,000 the person will turn down a 50-50 bet of losing $4,000 and gaining $635,670. If we only know that a person turns down 50-50 lose $100/gain $125 bets when her lifetime wealth is below $100,000, we also know she will turn down a 50-50 lose $600/gain $36 billion bet beginning from a lifetime wealth of $90,000. If an expected-utility maximizer turns down a 50-50 lose $1,000/gain $1,050 gamble whenever her lifetime wealth is below $500,000, then from an initial wealth level of $400,000 she will turn down a 50-50 lose $40,000/gain $6,356,700 gamble.

The intuition is that the extreme concavity of the utility function between (say) $290,000 and $300,000 assures that the marginal utility at $300,000 is tiny compared to the marginal utility at wealth levels below $290,000; hence, even if the marginal utility does not diminish at all above $300,000, a person won't care nearly as much about money above $300,000 as she does below $290,000.

3. Misleading Economics from a Misleading Explanation

Expected-utility theory seems to be a useful and adequate model of risk aversion for many purposes, such as understanding the motivation for large-stakes insurance. It is attractive more generally in lieu of an equally tractable alternative model. But expected-utility theory is manifestly not close to the right explanation of risk attitudes over modest stakes. Moreover, when the specific structure of expected-utility theory is used to analyze situations involving modest stakes—such as in research that assumes that large-stake and modest-stake risk attitudes derive from the same utility-for-wealth function—it can be very misleading.

Some research methods that economists currently employ rely crucially on the expected-utility interpretation of modest-scale risk aversion. One example arises in experimental economics. In recent years, there has been extensive laboratory research in economics in which participants generate payoffs for themselves on the order of $10 to $20. Researchers are often interested in inferring subjects' beliefs from their behavior. Doing so often requires knowing the relative value subjects hold for different money prizes; if a person chooses $5 in event A over $10 in event B, we know that she believes A is at least twice as likely as B only if we can assume that she likes $10 at least twice as much as $5. Yet economic theory teaches us that, because of diminishing marginal utility of wealth, we should not assume people like $10 twice as much as $5. Experimentalists have developed a clever scheme to avoid this problem: Instead of prizes of $10 and $5, subjects are given prizes such as a 10% chance of winning $100 vs. a 5% chance of winning $100. Expected-utility theory says that, irrespective of the utility function, a subject values the 10% chance of a prize exactly twice as much as the 5% chance of winning the same prize.

The problem with this lottery procedure is that it is known to be sufficient for neutralizing risk aversion only when we maintain the expected-utility hypothesis. But then it is not necessary—since expected-utility theory tells us that people will be virtually risk neutral in decisions on the scale of laboratory stakes. On the other hand, if we think that subjects in experiments are risk averse, then we know they are not expected-utility maximizers. Hence the lottery procedure, which is motivated by expected-utility theory's assumptions that preferences are linear in probabilities and that risk attitudes come only from the curvature of the utility-of-wealth function, has little presumptive value in "neutralizing" risk aversion. In a sense, this cumbersome procedure exists because economists have interpreted the expected-utility hypothesis literally—but not seriously.

Indeed, the observation that diminishing marginal utility of wealth is irrelevant in laboratory experiments raises questions about interpreting experimental tests of the adequacy of expected-utility theory. For instance, while showing that existing alternative models fit experimental data better than does expected-utility theory, Harless and Camerer (1994) show that expected-utility theory fits experimental data better than does "expected-value theory"—risk-neutral expected-utility theory. But because expected-utility theory implies that laboratory subjects should be risk neutral, such evidence that expected-utility theory explains behavior better than expected-value theory is evidence *against* expected-utility theory.

Expected-utility theory's presumption that risk aversion over modest-scale and large-scale risks derive from the same utility-of-wealth function relates to a widely-discussed implication of the theory: That people have approximately the same risk attitude towards an aggregation of independent, identical gambles as they do towards each of the independent gambles. This observation was introduced in a famous article by Samuelson (1963), who showed that expected-utility theory implies that if (for some sufficiently wide range of initial wealth levels) a person turns down a particular gamble, then he should also turn down an offer to play $n > 1$ of those gambles. Hence, in the example Samuelson discussed, if when close to his current wealth level his colleague is unwilling to accept a 50-50 lose $100/gain $200 gamble, then he also should be unwilling to accept 100 of those gambles taken together.

This result seems counter-intuitive: While many people might reject the single 50-50 lose $100/gain $200 bet, virtually everybody would find the aggregated gamble of 100 50-50 lose $100/gain $200 bets attractive. It has an expected yield of $5,000, with negligible risk: There is only a 1/2,300 chance of losing any money and merely a 1/62,000 chance of losing more than $1,000. A good lawyer could have you declared legally insane for turning down this gamble. In fact, the theorem in Rabin (forthcoming) implies that, under exactly the same assumptions invoked by Samuelson, somebody who turns down a 50-50 lose $100/gain $200 gamble will turn down a 50-50 lose $200/gain $20,000 gamble. This has an expected return of $9,900—with zero chance of losing more than $200. Even a *lousy* lawyer could have you declared legally insane for turning down *this* bet.

While Samuelson's insight thirty-five years ago might naturally have been used to reach the same conclusion I am reaching here—that people's aversion to modest-scale risks does not derive from diminishing marginal utility of wealth—economists have not used the insight in this way. In fact, Samuelson seemed to hypothesize that the mistake people were making was to treat probabilities on the scale of 1/2,300 or 1/62,000 as negligible. He invents a colorful "virtual quote" from Gertrude Stein—"Epsilon ain't zero"—to argue that the discrepancy between people's attitudes and the predictions of expected-utility theory is due to people's sloppiness in treating probabilities such as these as negligible. But given the limited nature of the risk in this example, 1/2,300 and 1/62,000 plainly *are* negligible.[3] In one sense, Samuelson's intuition was right: The utility-of-wealth function needed to reconcile his colleague's aversion to the 50/50 lose $100/gain $200 gamble with expected-utility theory would indeed render 1/62,300 far from negligible—because it would imply that the colleague values a dollar 800,000,000,000,000,000,000 times as much when $10,000 poorer than at his current wealth. In this case, economists have cleverly identified the logical consequences of applying the expected-utility model to modest-scale risk, but rather than recognize the ludicrousness of those consequences, economists have insisted that we live by them. Expected-utility theory's prescription about how people who are (for whatever reason) risk averse over modest stakes should feel about these risks when amalgamated is not counter-intuitive because people have poor intuitions about near-zero probabilities. It is counterintuitive because it is crazy.

4. Explaining Modest-Scale Risk Aversion

Many alternatives to expected-utility theory seem to provide a more plausible account of modest-scale risk attitudes, and can reconcile substantial risk aversion over modest stakes and non-ridiculous risk aversion over large stakes. Indeed, a direct explanation for modest-scale risk aversion is provided by what is empirically the most firmly established feature of risk preferences. *Loss aversion*, first highlighted by Kahneman and Tversky (1979), says that people are significantly more averse to losses relative to the status quo than they are attracted by gains, and more generally that people's utilities are determined by changes in wealth rather than absolute levels. Preferences incorporating loss aversion can reconcile significant modest-scale risk aversion with reasonable degrees of large-scale risk aversion. A loss-averse person may, for

[3] In any event, as indicated by the evidence presented in Thaler, Tversky, Kahneman, and Schwartz (1997) and Gneezy and Potter (1997), the more accurately people see the amalgamated gamble, the *more* apt they are to prefer it.

instance, turn down one 50/50 lose $100/gain $200 gamble but surely accept one hundred such gambles pooled together.

Kahneman and Lovallo (1993), Benartzi and Thaler (1995), and Read, Loewenstein, and Rabin (forthcoming) note that an additional departure from standard assumptions is implicated in many risk attitudes: People tend to assess risky choices in isolation, and as a result behave differently than if they assessed these risks jointly. We might reject one 50/50 lose $100/gain $200 gamble on each of 100 days, but accept all of these gambles if they are offered at the same time. Benartzi and Thaler (1995) argue that a related type of myopia is an explanation for the "equity premium puzzle"—the mystery about the curiously large premium on returns that investors seem to demand as compensation for the riskiness associated with investing in stocks. Such risk aversion can be explained with plausible loss-averse preferences—if investors are assumed to assess gains and losses over a short-run (yearly) horizon rather than the longer-term horizon for which they are actually investing. This is but one illustration of how recognizing that the expected-utility model is fundamentally flawed and massively miscalibrated can lead us to consider behaviorally realistic alternatives that permit us to improve economic analysis.

References

Arrow, K. (1971), Essays in the Theory of Risk-Bearing, Markham Publishing Company.

Benartzi, S. and Thaler, R. (1995), "Myopic Loss Aversion and the Equity Premium Puzzle," Quarterly Journal of Economics 110(1), 73-92.

Camerer, C. F. (1992), "Recent Tests of Generalized Utility Theories," in Utility Theories: Measurement and Applications, ed. W. Edwards, Cambridge University Press.

Epstein, L. G. (1992), "Behavior Under Risk: Recent Developments in Theory and Applications," in Advances in Economic Theory, vol. II, edited by Jean-Jacques Laffont, Cambridge University Press, 1-63.

Epstein, L. G. and S. E. Zin (1990), "'First-Order' Risk Aversion and the Equity Premium Puzzle," Journal of Monetary Economics 26(3), 387-407.

Gneezy, U. and J. Potters (1997), "An Experiment on Risk Taking and Evaluation Periods," Quarterly Journal of Economics 112(2), 631-645.

Hansson, B. (1988), "Risk Aversion as a Problem of Conjoint Measurement," in Decision, Probability, and Utility, eds. P. Gardenfors and N.-E. Sahlin, Cambridge University Press, 136-158.

Harless, D. W. and C. F. Camerer (1994), "The Predictive Utility of Generalized Expected Utility Theories," Econometrica 62(6), 1251-1289.

Kahneman, D., Knetsch, J., and Thaler, R. (1991), "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias," Journal of Economic Perspectives 5, Winter, 193-206.

Kahneman, D., and D. Lovallo (1993), "Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking," Management Science 39(1), 17-31.

Kahneman, D., and A. Tversky (1979), "Prospect Theory: An Analysis of Decision Under Risk," Econometrica 47, March, 263-291.

Kandel, S. and R. F. Stambaugh (1991), "Asset Returns, Investment Horizons, and Intertemporal Preferences," Journal of Monetary Economics, February, 39-71.

Loomes, G. and U. Segal (1994), "Observing Different Orders of Risk Aversion," Journal of Risk and Uncertainty 9(3), 239-256.

Machina, M. (1987), "Choice Under Uncertainty: Problems Solved and Unsolved," Journal of Economic Perspectives 1, 121-154.

Mankiw, N. G. and Zeldes, S. (1991), "The Consumption of Stockholders and Nonstockholders," Journal of Financial Economics 29(1), 97-112.

Rabin, M. (forthcoming), "Risk Aversion and Expected-Utility Theory: A Calibration Theorem," Econometrica, forthcoming.

Read, D., Loewenstein, G., and M. Rabin (forthcoming), "Choice Bracketing," Journal of Risk and Uncertainty, forthcoming.

Samuelson, P. (1963), "Risk and Uncertainty: A Fallacy of Large Numbers," Scientia 98, 108-113.

Segal, U. and A. Spivak (1990), "First Order Versus Second Order Risk Aversion," Journal of Economic Theory 51(1), 111-125.

Thaler, R., Tversky, A., Kahneman, D., and A. Schwartz (1997), "The Effect of Myopia and Loss Aversion on Risk Taking: An experimental test," Quarterly Journal of Economics 112(2), 647-661.