# Bayesian Persuasion with Lie Detection*

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**Abstract**

We consider a model of Bayesian persuasion in which the Receiver can detect lies with positive probability. We show that the Sender lies more when the lie detection probability increases. As long as the lie detection probability is sufficiently small the Sender's and the Receiver's equilibrium payoff are unaffected by the lie detection technology because the Sender simply compensates by lying more. When the lie detection probability is sufficiently high, the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability.

JEL Codes: D83, D82, K40, D72

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# 1 Introduction

Lies are a pervasive feature of communication even when communication is subject to intense public and media scrutiny. For example, during his tenure as US President Donald Trump has made over 20,000 false or misleading claims.[1] But such lies are also detectable. Monitoring and fact-checking should constrain how much licence a sender of communication has when making false statements. But, interestingly, in the face of increased fact-checking and media focus the rate of Trump's lying has increased rather than decreased—a development that runs counter to this intuition.

In this paper we incorporate probabilistic lie detection in an otherwise standard model of Bayesian persuasion (Kamenica and Gentzkow, 2011; Kamenica, 2019). Two players, a Sender and a Receiver, engage in one round of communication. The Sender observes the binary state of nature and sends a message to the Receiver. To clearly define whether a message is a lie or not we assume that the message space and the state space are the same. The Receiver observes the message, and if the message is a lie, it is flagged as such with positive probability $q$. The Receiver then takes an action. Whereas the Sender prefers the Receiver to take the "favorable" action regardless of the state of nature, the Receiver wants to match the action to the underlying state. Finally, payoffs (or utilities which we use interchangeably) are realized for both parties.

Our main assumption that lies—but not the underlying truth—are detectable is arguably a natural one in many applications. Facts may come to light that contradict the initial claim of the Sender. These facts do not necessarily reveal the payoff-relevant state, but only prove that the Sender has lied. For example, in job interviews or trial testimonies, the Sender may be required to provide details and arguments supporting his statements, and if he is lying, he may be at risk of producing an internally or externally inconsistent account, thereby revealing his lie. Liars may also exhibit physical reactions such as blushing, which reveal the fact of lying.

Our model delivers the following set of results. First, the Sender lies more frequently when the lie detection technology improves. Second, as long as the lie detection probability is sufficiently small the equilibrium payoffs of both players are unaffected by the lie detection technology because the Sender simply compensates by lying more frequently in the unfavorable state of nature by claiming that the state is favorable. That is to say, the lie detection technology changes the Sender's message strategy

[1] See https://www.washingtonpost.com/politics/2020/07/13/president-trump-has-made-more-than-20000-false-or-misleading-claims/ for a comprehensive analysis of this behavior.

but does not have an impact on the utilities of both players. Third, when the lie detection technology is sufficiently reliable, any further increase in the lie detection probability causes the Sender to also lie more frequently in the favorable state of nature. In the limit case of perfect lie detection (i.e., $q=1$) the Receiver is perfectly informed and hence it is irrelevant what messages the Sender chooses to send. Fourth, when the lie detection technology is sufficiently reliable, the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability.

Two recent papers (Balbuzanov, 2019; Dziuda and Salas, 2018) also investigate the role of lie detectability in communication. The largest difference with respect to our paper lies in the commitment assumption of the Sender. In all those papers, the communication game takes the form of cheap talk (Crawford and Sobel, 1982) rather than Bayesian persuasion as in our paper. We defer a detailed comparison between these papers and our work to Section 4. In Jehiel (2019) lie detection arises endogenously in a setting with two rounds of communication and a Sender who in the second round cannot remember what lies she sent in the first round. As the state space becomes arbitrarily fine, the probability of lie detection goes to 1 because it is hard to guess exactly the same lie, and therefore only fully revealing equilibria arise.

Related theoretical work on lying in communication games also includes Kartik et al. (2006) and Kartik (2009) who do not consider lie detection but instead introduce an exogenous cost of lying tied to the size of the lie in a cheap talk setting. They find that most types inflate their messages, but only up to a point. In contrast to our results they obtain full information revelation follows for some or all types depending on the bounds of the type and message space.

A large and growing experimental literature (Gneezy, 2005; Hurkens and Kartik, 2009; Sánchez-Pagés and Vorsatz, 2009; Ederer and Fehr, 2017; Gneezy et al., 2018) examines lying in a variety of communication games Most closely related to our own work is Fréchette et al. (2019) who investigate models of cheap talk, information disclosure, and Bayesian persuasion, in a unified experimental framework. Their experiments provide general support for the strategic rationale behind the role of commitment and, more specifically, for the Bayesian persuasion model of Kamenica and Gentzkow (2011).

Finally, whereas we focus on an improvement of the Receiver's communication technology (i.e., lie detection), Gehlbach and Vorobyev (2020) analyze how improvements that benefit the Sender (e.g., censorship and propaganda) impact communication under Bayesian persuasion.

# 2 Model

## 2.1 Setup

Let $w \in \{0,1\}$ denote the state of the world and $\Pr(w=1)=\mu \in (0,1)$. The Sender ($S$, he) observes $w$ and commits to send a message $m \in \{0,1\}$ to the Receiver ($R$, she). If the Sender lies (i.e., $m \neq w$), the Receiver is informed with probability $q \in [0,1]$ that it is a lie and thus learns $w$ perfectly. With remaining probability $1-q$, she is not informed. If the sender does not lie (i.e., $m=w$), $R$ is not informed either. Formally, the detection technology can be described by the following relation

$$d(m,w)=\begin{cases} lie, & \text{with probability } q \text{ if } m \neq w \\ \neg lie, & \text{with probability } 1-q \text{ if } m \neq w \\ \neg lie, & \text{with probability 1 if } m=w \end{cases}$$

With a slight abuse of notation we denote $d=\{lie,\neg lie\}$ as the outcome of the detection result. The detection technology is common knowledge. In a standard Bayesian persuasion setup this detection probability $q$ is equal to 0, giving us an immediately comparable benchmark.

Given both $m$ and $d$, $R$ takes an action $a \in \{0,1\}$, and the payoffs are realized. The payoffs are defined as follows.

$$u_S(a,w)=\mathbb{1}_{\{a=1\}}$$
$$u_R(a,w)=(1-t)\cdot\mathbb{1}_{\{a=w=1\}}+t\cdot\mathbb{1}_{\{a=w=0\}}, \quad 0<t<1$$

That is, the Sender wants the Receiver to always take the action $a=1$ regardless of the state, while the Receiver wants to match the state. The payoff from matching the state 0 may differ from the payoff from matching the state 1. Given the payoff function, the Receiver takes action $a=1$ if and only if

$$\Pr(w=1 \mid m,d) \geq t$$

and therefore one could also interpret $t$ as the threshold of the Receiver's posterior probability above

which she takes $a=1$. Note that if $t \leq \mu$, there is no need to persuade because the Receiver will choose the Sender's preferred action $a=1$ even without a message. Therefore, we assume $t \in (\mu,1)$.

## 2.2 Optimal Messages and Responses

As is common in the Bayesian persuasion literature we assume that the Sender's commitment to an information structure is binding.[2] The strategy of the Sender is a mapping $m:\{0,1\} \longrightarrow \Delta(\{0,1\})$, and the strategy of the Receiver is a mapping $a:\{0,1\} \times \{lie, \neg lie\} \longrightarrow \Delta(\{0,1\})$. Formally, the Sender is choosing $m(\cdot)$ to maximize

$$\mathbb{E}_{w,d,m}[u_S(a(m(w),d(m(w),w)),w)]$$

where $a(m,d)$ maximizes

$$\mathbb{E}_w[u_R(a,w) \,|\, m,d].$$

The two expectation signs are taken with respect to different variables. The expectation sign in the Sender's utility is taken with respect to both $w$, $d$ and perhaps $m$ if the strategy is mixed, whereas the expectation sign in the Receiver's utility is only taken with respect to both $w$. Due to the simple structure of the model, it is without loss of generality to assume that the Sender chooses only two parameters $p_0 = \Pr(m=0 \,|\, w=0)$ and $p_1 = \Pr(m=1 \,|\, w=1)$ to maximize $\Pr(a(m,d)=1)$ which we write as $\Pr(a=1)$ henceforth for brevity of notation. We denote the optimal reporting probabilities of the Sender by $p_0^*$ and $p_1^*$, and the ex-ante payoffs under this reporting probabilities as $U_S$ and $U_R$. Given the Sender's reporting strategy, the Receiver could potentially see four types of events to which she needs to react when choosing action $a$.

First, the Receiver could observe the event $(m=0,d=lie)$ which occurs with probability $\mu(1-p_1)q$. Given the lie detection technology Receiver is certain that the message $m=0$ is a lie and therefore the

[2]For a detailed discussion and relaxation of this assumption see Min (2017), Fréchette et al. (2019), Lipnowski et al. (2019), and Nguyen and Tan (2019). Titova (2020) shows that with binary actions and a sufficiently rich enough state space verifiable disclosure enables the sender's commitment solution as an equilibrium.

state of the world $w$ must be equal to 1, that is

$$\Pr(w=1\,|\,m=0,d=lie)=1.$$

As a result, the Receiver optimally chooses $a=1$.

Second, the event $(m=0,d=\neg lie)$ could occur with probability $\mu(1-p_1)(1-q)+(1-\mu)p_0$. In that case, the Receiver is uncertain about $w$ because she does not know whether the Sender lied or not. Her posterior probability is given by

$$\Pr(w=1\,|\,m=0,d=\neg lie)=\frac{\mu(1-p_1)(1-q)}{\mu(1-p_1)(1-q)+(1-\mu)p_0}\equiv\mu_0.$$

Hence, the Receiver takes action $a=1$ if and only if $\mu_0\geq t$. For brevity of notation we denote the posterior following this event by $\mu_0$ (and thus omitting the lie detection outcome $d=\neg lie$). When $p_0=0,p_1=1$, this event occurs with 0 probability, so the belief is off-path and not restricted by Bayesian updating. However, the off-path belief does not matter for the Sender, because if the Sender chooses the strategy that renders $(m=0,d=\neg lie)$ a zero probability event, he does not care about how Receiver responds to that event. For simplicity, define $\mu_0=0$ when $p_0=0,p_1=1$.

Third, $(m=1,d=lie)$ occurs with probability $(1-\mu)(1-p_0)q$. Because a lie was detected the Receiver is again certain about $w$ and therefore her posterior probability is given by

$$\Pr(w=1\,|\,m=1,d=lie)=0,$$

which immediately implies the action $a=0$.

Fourth, $(m=1,d=\neg lie)$ occurs with probability $\mu p_1+(1-\mu)(1-p_0)(1-q)$. The Receiver is again uncertain about $w$. Her posterior is given by

$$\Pr(w=1\,|\,m=1,d=\neg lie)=\frac{\mu p_1}{\mu p_1+(1-\mu)(1-p_0)(1-q)}\equiv\mu_1$$

and the Receiver takes action $a=1$ if and only if $\mu_1\geq t$. Analogously, for brevity of notation we denote the posterior following this event by $\mu_1$ (and thus omitting the lie detection outcome $d=\neg lie$). Similarly,

if $p_0=1, p_1=0$, then this event occurs with 0 probability and the belief $\mu_1$ is not well-defined, but again this does not matter for Sender. For simplicity, define $\mu_1=0$ when $p_0=1, p_1=0$.

Given these optimal responses by $R$, the relationships between the posteriors $\mu_0$,$\mu_1$ and the posterior threshold $t$ divide up the strategy space into four regions which we denote by I, II, III, and IV respectively.[3] Within each region, the Receiver's response as a function of $(m,d)$ is the same, making it easy to find the region-optimal strategy. We are then left to pick the best strategy out of the four candidates for different values of $q$. These regions are defined as follows:

I. $\mu_0 \leq t$, $\mu_1 \leq t$: In this region, the Receiver only chooses $a=1$ if $(m=0,d=lie)$ and $a=0$ otherwise because the posteriors $\mu_0$ and $\mu_1$ are insufficiently high to persuade her to choose $S$'s preferred action. Only if the Sender lies in state $w=1$ and his message is detected as a lie is $R$ sufficiently convinced that $a=1$ is the right action. The maximal probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\mathrm{I}}(a=1) = \max_{p_0,p_1\in[0,1]} \quad \mu(1-p_1)q \quad \text{s.t.} \quad \mu_0 \leq t, \mu_1 \leq t \tag{1}$$

II. $\mu_0 \geq t$, $\mu_1 \leq t$: In this region, the Receiver chooses $a=1$ if $(m=0,d=lie)$ or $(m=0,d=\neg lie)$ and $a=0$ otherwise. The maximal probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\mathrm{II}}(a=1) = \max_{p_0,p_1\in[0,1]} \quad \mu(1-p_1)+(1-\mu)p_0 \quad \text{s.t.} \quad \mu_0 \geq t, \mu_1 \leq t \tag{2}$$

III. $\mu_0 \leq t$, $\mu_1 \geq t$: In this region, the Receiver chooses $a=1$ if $(m=0,d=lie)$ or $(m=1,d=\neg lie)$ and $a=0$ otherwise. The maximal probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\mathrm{III}}(a=1) = \max_{p_0,p_1\in[0,1]} \quad \mu p_1+\mu(1-p_1)q+(1-\mu)(1-q)(1-p_0) \quad \text{s.t.} \quad \mu_0 \leq t, \mu_1 \geq t \tag{3}$$

IV. $\mu_0 \geq t$, $\mu_1 \geq t$: In this region, the Receiver chooses $a=1$ if $(m=0,d=lie)$, $(m=0,d=\neg lie)$ or

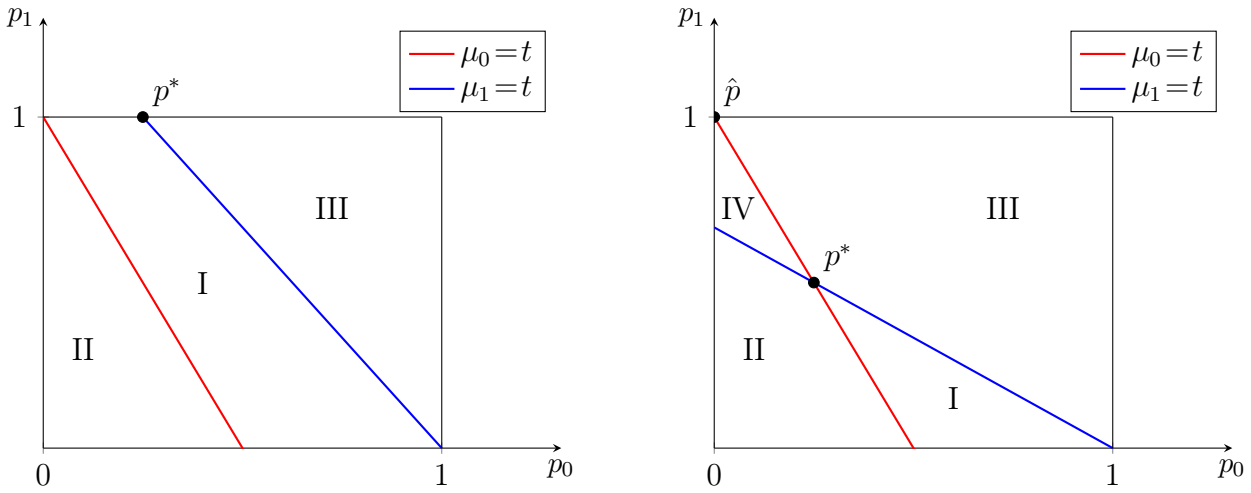
[3] The four regions are not mutually exclusive as some regions share a boundary. This is to ensure that the choice set in each region is closed so that the maximizer within each region exists.

$(m=1, d=\neg lie)$. The maximal probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\text{IV}}(a=1) = \max_{p_0,p_1\in[0,1]} \quad 1-(1-\mu)(1-p_0)q \quad \text{s.t.} \quad \mu_0 \geq t, \mu_1 \geq t \tag{4}$$

We are now ready to state the main proposition of our model.

**Proposition 1.** *Let* $\overline{q}=1-\frac{\mu(1-t)}{t(1-\mu)}\in(0,1)$. *If* $q\leq\overline{q}$, *the Sender's optimal strategy is in region* III, *in which the Sender always tells the truth under* $w=1$, *but lies with positive probability under* $w=0$. *If* $q>\overline{q}$, *the Sender's optimal strategy is in region* IV, *in which the Sender lies with positive probability under both states.*




(a) $q\leq\overline{q}$ (b) $q>\overline{q}$

Figure 1: Equilibrium message strategies for different detection probabilities $q$.

In Figure 1 we graphically illustrate how these four regions are divided. The proof involves sequential comparisons between the four region-optimal strategies. First, the optimal strategy in region II is better than the optimal strategy in region I. To see this, consider a particular strategy $p_0=p_1=0$ in region II (i.e., the Sender totally misreports the state). Following this strategy, the Receiver takes action $a=1$ if and only $w=1$, which occurs with probability $\mu$. This strategy may not be optimal within region II, but it secures a lower bound $\mu$ for the value of the optimal strategy. This lower bound is sufficient to beat all strategies in region I since there $a=1$ only if $w=1$ and $(m=0, d=lie)$, which occurs with a probability less than $\mu$.

Second, the optimal strategy in region III is better than the optimal strategy in region II. Notably, they should be equivalent if the lie detection technology is not available $(q=0)$ since in that case

the messages have no intrinsic meanings and we could always rename the messages such that the two maximization problems are identical in II and III. However, with the introduction of a lie detection technology, there is now an intrinsic meaning of the message the Sender uses. Regardless of the committed strategy by the Sender, an on-path message that was not detected as a lie always carries some credibility for the state to which it corresponds. The two regions differ in the presence of this additional credibility. In region II, the Sender wants an undetected lie $m=0$ to be indicative of $w=1$. This is more difficult because in that region the additional credibility is towards $w=0$. On the contrary, in region III, the Sender wants an undetected lie $m=1$ to be indicative of $w=1$, but this is easier because the additional credibility is towards the same direction. Therefore, both region I and region II are suboptimal and therefore we only need to focus on the comparison between region III and IV.

Interestingly, as suggested by the two panels of Figure 1, region IV does not exist when $q$ is small, where the proof is given in Appendix A. When $q=0$ our setup yields the standard Bayesian Persuasion benchmark. In that case, we know it is impossible to induce $a=1$ under all messages because by the martingale property, the posteriors after two messages must average to the prior, suggesting some posterior is lower than the prior and must induce $a=0$. However, the presence of lie detection extends the information from $m$ to a couple $(m,d)$, and the martingale property only requires the four posteriors' average over the prior. Furthermore, the posterior following $(1,lie)$ is 0. Therefore if $q$ is sufficiently large, it is possible to support the two posteriors following $(1,\neg lie)$ and $(0,\neg lie)$ which are both higher than the prior and also higher than the threshold $t$. Combining this observation with the previous two arguments, we immediately obtain the first half of Proposition 1. In particular, if $q\leq\overline{q}$, the optimal strategy takes the following form:

$$p_0^*=\frac{\overline{q}-q}{1-q} \quad \text{and} \quad p_1^*=1 \tag{5}$$

This is reminiscent of Kamenica and Gentzkow (2011), where the Receiver is indifferent between two actions when she takes the preferred action $a=1$, and certain of the state when she takes the less preferred action $a=0$.

If the detection probability $q>\overline{q}$, the optimal strategy in region III involves a corner solution since $p_0$ cannot drop below 0. In particular, S always claims $m=1$ regardless of the state such that a message $m=0$ becomes an off-path message. Yet this is no longer globally optimal as the region IV is now

non-empty, and the optimal strategy in region IV is better than the one in region III described above. Specifically, the optimal strategy involves partial lying under both states ($0<p_0,p_1<1$) and is given by

$$p_0^*=\frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^*=\frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}}-(1-q)\right] \tag{6}$$

From the above equations it can easily be seen that $p_0^*<p_1^*$. That is to say, the Sender always lies more in the unfavorable state than in the favorable state.

In fact, not only is the optimal strategy in region III, which is denoted as $\hat{p}$ in Figure 1 (b), no longer the globally optimal strategy, but it is actually the worst strategy in region IV. In other words, any strategy in region IV is better than $\hat{p}$ whenever region IV exists. To this end, we decompose the Sender's expected payoff into two parts, one in the favorable state $w=1$ and the other in the unfavorable state $w=0$. The Sender's expected payoff in the favorable state $w=1$ is the same between $\hat{p}$ and any strategy $p$ in region IV because $m=0$ and $m=1$ are equally effective in inducing $a=1$. If $m=0$ is flagged as a lie, the Receiver is informed the state is $w=1$ and hence takes $a=1$. Otherwise, the Receiver also takes action $a=1$ because by construction, $\mu_0\geq t$ in region IV. However, the Sender's expected payoff in the unfavorable state $w=0$ is larger given $p$ because the Sender sometimes induces $a=1$ via sending $m=0$ which is a better way than sending $m=1$, because the former message will never be flagged as a lie in the unfavorable state. Hence, we conclude that $\hat{p}$ is the worst strategy within IV. As we see, the main benefit of $p$ relative to $\hat{p}$ is that the more effective message $m=0$ is sent more frequently in $p$, so the optimal strategy within the region IV must involve the highest $p_0$, or the least lying in the unfavorable state, which is given by $p^*$ in Figure 1 (b). The Sender however also has to lie in the favorable state to maintain the condition required by the optimal strategy in region IV given by $\mu_0\geq t$.

Finally, the region-switching threshold $\overline{q}$ is decreasing in $\mu$ and increasing in $t$. To see the intuition for this result fix the lie detection probability $q\in(0,1)$. If a weak signal is sufficient to persuade the Receiver (i.e., the prior $\mu$ is already close to the threshold $t$), the outcome is more likely to be in region IV. On the other hand, if the signal has to be very convincing to persuade the Receiver (i.e., the threshold $t$ is much larger than the prior $\mu$), the outcome is more likely to be in region III.

# 3 Comparative Statics

We now consider the comparative statics of our model with respect to the central parameter of the lie detection probability $q$ to show how the optimal communication and the utilities of the communicating parties changes as the lie detection technology improves.

## 3.1 Optimal Signals

Proposition 2 describes how the structure of optimal signal $(p_0^*, p_1^*)$ changes as the detection probability varies. Figure 2 plots these optimal reporting probabilities as a function of $q$. For comparison, the probabilities $p_0^{BP}$ and $p_1^{BP}$ are the equilibrium reporting probabilities that would result in a standard Bayesian persuasion setup without lie detection.

**Proposition 2.** *As the lie detection probability $q$ increases,*

1. $p_0^* = Pr(m=0|w=0)$ *is decreasing over* $[0,\overline{q}]$*, and has an inverse U shape over* $(\overline{q},1]$.

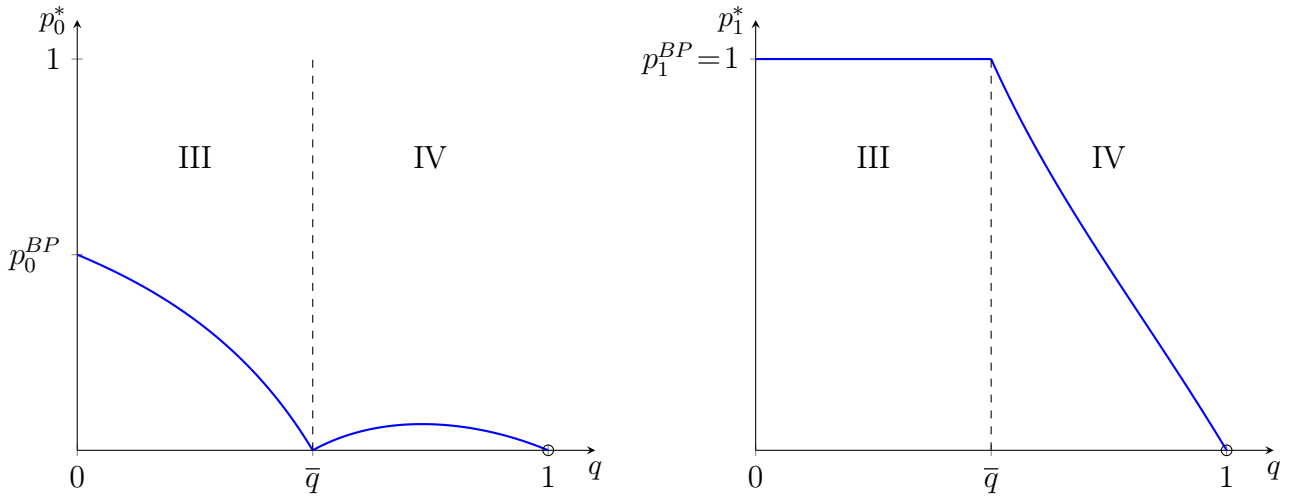
2. $p_1^* = Pr(m=1|w=1)$ *is constant over* $[0,\overline{q}]$*, and decreases over* $(\overline{q},1]$.

If $q \leq \overline{q} = 1 - \frac{\mu(1-t)}{t(1-\mu)}$, $p_0^*$ is decreasing in $q$ and $p_1^*$ is constant at 1. In this range of $q$, the Sender's optimal strategy lies in III which involves truthfully reporting the state $w=1$ (i.e., $p_1 = 1$), but progressively misreporting the state $w=0$ as the lie detection technology improves (i.e., $p_0 < 1$ and decreasing with $q$).

If $q > \overline{q}$, $p_0^*$ initially increases and then decreases. In contrast, $p_1^*$ decreases over the entire range of $[\overline{q},1]$. In this range, the Sender's optimal strategy lies in IV which involves misreporting both states of the world.

For $q=0$ we have the Bayesian benchmark. Recall from Kamenica and Gentzkow (2011) that if an optimal signal induces a belief that leads to the worst action for the Sender ($a=0$ in our case), the Receiver is certain of her action at this belief. In addition, if the optimal signal induces a belief that leads to the best action for the Sender ($a=1$ in our case) the Receiver is indifferent between the two actions at this belief.

Now consider the addition of a lie detection technology. As the lie detection probability $q$ increases, $(m=1, d=\neg lie)$ becomes more indicative of the favorable state $w=1$ and therefore the Receiver would strictly prefer to take the favorable action $a=1$. As a response, the Sender would like to send the message $m=1$ more often while still maintaining that $(m=1, d=\neg lie)$ sufficiently persuades the Receiver




(a) $p_0^* \equiv \Pr(m=0\,|\,w=0)$ (b) $p_1^* \equiv \Pr(m=1\,|\,w=1)$

Figure 2: Equilibrium reporting probabilities $p_0^*$ and $p_1^*$ as a function of $q$ for $\mu=\frac{1}{3}$ and $t=\frac{1}{2}$

to take the action $a=1$. Because the Sender already sends the message $m=1$ with probability 1 under $w=1$, the only way to increase the frequency of $m=1$ is to send such a message more often in the unfavorable state $w=0$ (i.e., lie more frequently if $w=0$). In other words, the Sender increases the frequency of lying just enough about the unfavorable state ($w=0$) to make the Receiver indifferent when choosing the favorable action $a=1$.

Recall that in the canonical Bayesian persuasion setup the Receiver is just held to her outside utility of getting no information whatsoever. When lie detection $q$ goes up the Receiver is now more certain that ($m=1$,$d=\neg lie$) means $w=1$ and would therefore obtain a larger surplus from the improvement in the lie detection technology. However, as long as $p_0^*$ is greater than 0 the Sender can simply undo this improvement by lying more about $w=0$ (i.e., reduce $p_0^*$ even further) thereby "signal-jamming" the information obtained by the Receiver.

However, once the detection probability $q$ rises above $\overline{q}$ it is no longer possible for the Sender to just lie about the unfavorable state because he already maximally lies about it at $\overline{q}$. His optimal messaging strategy is now in region IV when $q>\overline{q}$. In region IV the Receiver only ever takes the unfavorable action $a=0$ if he receives a message $m=1$ that is flagged as a lie. This is because in region IV the Receiver has access to such a good lie detection technology that a lie involving the message $m=1$ is sufficiently likely to be detected as a lie and will then induce the unfavorable action $a=0$. At the same time the Receiver is also

very likely to be notified of a lie involving the message $m=0$ which the Sender can use to his advantage to ensure that the Receiver chooses the favorable action $a=1$. At $q=\overline{q}$ the Sender therefore wants to increase the frequency of the message $m=0$ which he achieves by both increasing $p_0$ and decreasing $p_1$. However, when the detection probability is close to 1, (i.e., the lie detection technology is almost perfect) $p_1$ is close to 0 and any message $m=1$ is very likely to be a lie. To make sure that a message $m=1$ which is not detected as a lie still sufficiently persuades the Receiver to choose $a=1$ (i.e., does not violate the constraints $\mu_0 \geq t$ and $\mu_1 \geq t$ required in region IV), the Sender also has to decrease $p_0$ while decreasing $p_1$.

## 3.2 Utilities

We denote the equilibrium payoffs of the Sender and the Receiver by $U_S$ and $U_R$ and investigate how $U_S$ and $U_R$ are affected by improvements in the lie detection technology. The results are summarized in Proposition 3 and graphically depicted in Figure 3. For comparison, the utilities $U_S^{BP}$ and $U_R^{BP}$ are the equilibrium utilities that would result in a standard Bayesian persuasion setup without lie detection.

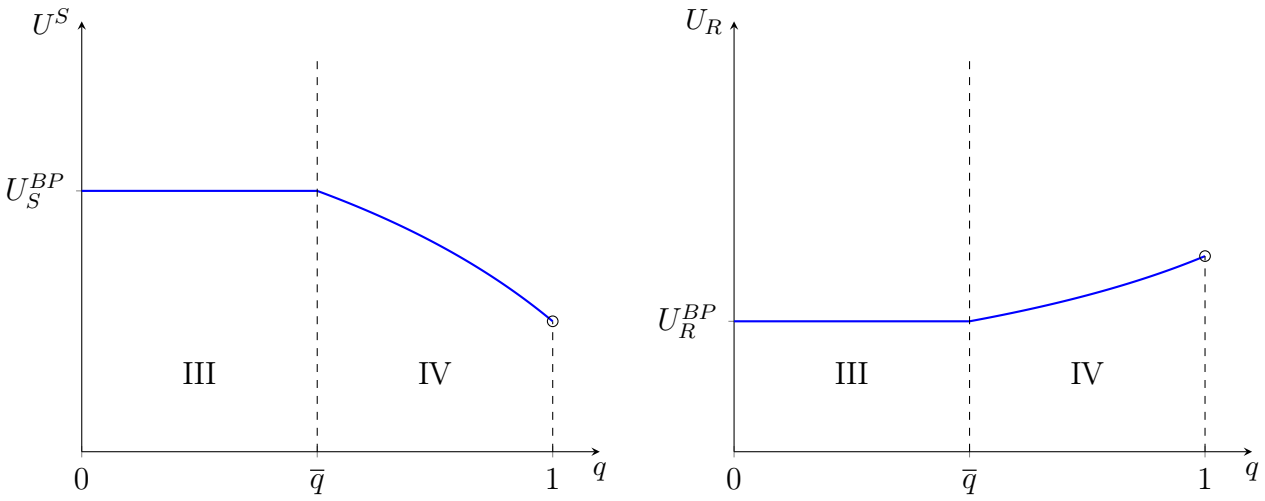
**Proposition 3.** *As the lie detection probability $q$ increases,*

1. *$U_S$ is constant over $[0,\overline{q}]$, and decreases over $(\overline{q},1]$.*

2. *$U_R$ is constant over $[0,\overline{q}]$, and increases over $(\overline{q},1]$.*

The Sender's equilibrium payoff does not change for $q \leq \overline{q}$ and decreases with $q$ for $q > \overline{q}$. As long as $q \leq \overline{q}$ the Sender receives exactly the same utility that he would receive under the Bayesian Persuasion benchmark. Any marginal improvement in the lie detection technology (i.e., increase in $q$) is completely offset by less truthful reporting when $w=0$ (i.e., decrease in $p_0^*$). However, for $q > \overline{q}$ any further improvements reduce the Sender's utility. In the limit case where $q=1$ the Sender has no influence anymore and the action $a=1$ in only implemented when the state is actually $w=1$ which occurs with probability $\mu$.

Analogously for the case of the Sender's utility, the Receiver's utility is also constant at the Bayesian persuasion benchmark as long as $q \leq \overline{q}$ and then increases with $q$ for $q > \overline{q}$ as the lie detection technology starts to bite. In the limit, the Receiver is just as well off as she would be under perfect information.




(a) Sender's Equilibrium Payoff

(b) Receiver's Equilibrium Payoff

Figure 3: Equilibrium Payoffs as a function of $q$ for $\mu = \frac{1}{3}$, $t = \frac{1}{2}$

# 4 Discussion

Balbuzanov (2019) and Dziuda and Salas (2018) also study strategic communication in the presence of a lie detection technology but in a cheap talk setting. The largest difference between these two papers and ours therefore lies in the commitment power of the Sender. Although it is debatable whether the extreme cases of full commitment (as in Bayesian persuasion) or no commitment (as in cheap talk) constitute more plausible assumptions about real-life communication setting, we believe our model is an important step towards studying the communication games with lie detection under (partial) commitment.

Our paper also differs from Balbuzanov (2019) in the payoff functions. In Balbuzanov (2019) the Sender and the Receiver have some degree of common interest whereas in our model there is no common interest. Due to this difference the Sender's type-dependent preferences in Balbuzanov (2019) permit fully revealing equilibria in some cases as it allows the Receiver to tailor message-specific punishment actions. In particular, fully revealing equilibria exist for some intermediate degree of lie detectability if the Sender's bias is small. However, the Sender in our model never reveals the state perfectly due to the conflict in payoffs.

Dziuda and Salas (2018) do not allow for common interest and therefore, like in our paper, fully revealing equilibria are impossible in their paper. In their continuous state model there are many off-path

beliefs to be specified. To discipline these off-path beliefs, they impose two refinements and show that in all remaining equilibria, the lowest types lie, while *some* higher types tell the truth. Our model violates the second refinement in Dziuda and Salas (2018) and thus irrespective of the commitment power of the Sender our paper is not nested by theirs.

Both papers feature a type of non-monotonicity result with respect to the relationship between lie detection and lying that is similar to that in our paper. In Balbuzanov (2019), the set of detection probabilities that permits fully revealing equilibrium obtains for low lie detection probabilities, but not for higher ones. Although in the baseline model of Dziuda and Salas (2018) a lower lie detection probability leads to less truth-telling, in an extension of their model they show that if the Sender is given an opportunity to make costly investments in decreasing the lie detectability, then for an intermediate region of the cost, the mass of liars can increase with the investment cost.

# 5 Conclusion

In this paper we analyze the role of probabilistic lie detection in a model of Bayesian persuasion between a Sender and a Receiver. We show that the Sender lies more when the lie detection probability increases. As long as the lie detection probability is sufficiently small the Sender's and the Receiver's equilibrium payoff are unaffected by the lie detection technology because the Sender simply compensates by lying more. Once the lie detection probability is sufficiently high, the Sender is no longer able to maximally lie about the unfavorable state and the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability. Our model rationalizes that a sender of communication chooses to lie more frequently when it is more likely that their false statements will be flagged as lies.

Our paper explores the impact of lie detection on communication in a setting with complete commitment to a communication strategy by the sender and thereby establishes a useful benchmark relative to the diametrically opposed assumption of no commitment in existing cheap talk models with lie detection. However, how does lie detection influence communication behavior in intermediate settings with partial commitment? And what role does lie detection play under Bayesian persuasion with a richer state and message space? We leave these and other interesting questions to future research.

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# A Proofs

## A.1 Proof of Proposition 1

We now show that strategies I and II are suboptimal because the resulting implementation probabilities $\Pr_{\mathrm{I}}(a=1)$ and $\Pr_{\mathrm{II}}(a=1)$ are dominated by the probability $\Pr_{\mathrm{III}}(a=1)$ resulting from III. To see this, note first that

$$\Pr_{\mathrm{I}}(a=1) \leq \mu \leq \Pr_{\mathrm{II}}(a=1) \tag{7}$$

The second inequality holds because $p_0=p_1=0$ is feasible in II and gives value $\mu$. In fact, Within region II, it is optimal to set $p_1=0$ because this loosens both constraints, but it does not have a direct effect on the objective of maximizing the probability. Given this, we have $\mu_1=0<t$ and hence the optimum requires $\mu_0=t$. Therefore, $p_0^*$ solves the following equation

$$\frac{\mu(1-q)}{\mu(1-q)+(1-\mu)p}=t \tag{8}$$

and hence

$$\Pr_{\mathrm{II}}(a=1)=\mu+\left(\frac{\mu}{t}-\mu\right)(1-q) \tag{9}$$

For $\Pr_{\mathrm{III}}(a=1)$ it is optimal to set $p_1=1$ for similar reasons. This yields $\mu_0=0<t$ which at the optimum requires $p_0$ to be small enough, but still ensures that $\mu_1 \geq t$. Define $\overline{q} \equiv 1-\frac{\mu(1-t)}{t(1-\mu)} \in (0,1)$, then there are two cases to consider.

- $\frac{\mu}{\mu+(1-\mu)(1-q)} \leq t$ or $q \leq \overline{q}$. In this case, there exists $p_0^*$ s.t. $\mu_1 = t$, that is $\frac{\mu}{\mu+(1-\mu)(1-p_0^*)(1-q)} = t$. Therefore, $\Pr_{\mathrm{III}}(a=1)=\frac{\mu}{t}$.

- $\frac{\mu}{\mu+(1-\mu)(1-q)} > t$ or $q > \overline{q}$. In this case, $\mu_1 \geq t$ can never bind. Thus, the best option is to set $p=0$ which implies $\Pr_{\mathrm{III}}(a=1)=\mu+(1-\mu)(1-q)$.

Clearly, in either case we have $\Pr_{\mathrm{III}}(a=1) > \Pr_{\mathrm{II}}(a=1)$ and therefore both strategies I and II are suboptimal. It therefore remains to compare $\Pr_{\mathrm{III}}(a=1)$ and $\Pr_{\mathrm{IV}}(a=1)$.

(1) If $\frac{\mu}{\mu+(1-\mu)(1-q)} \leq t$, Region IV does not exist, *i.e.*, there is no way to choose $p_0$,$p_1$ such that $\mu_1 \geq t$ and $\mu_0 \geq t$. If that were the case we would have $\frac{\mu p_1}{\mu p_1+(1-\mu)(1-p_0)(1-q)} \geq t$ and $\frac{\mu(1-p_1)(1-q)}{\mu(1-p_1)(1-q)+(1-\mu)p} \geq t$ which would imply

$$\frac{\mu p_1+\mu(1-p_1)}{\mu p_1+\mu(1-p_1)+(1-\mu)(1-p_0)(1-q)+(1-\mu)\frac{p}{1-q}} \geq t \tag{10}$$

and therefore

$$t \leq \frac{\mu}{\mu+(1-\mu)(1-p_0)(1-q)+(1-\mu)\frac{p}{1-q}} \leq \frac{\mu}{\mu+(1-\mu)(1-q)} \tag{11}$$

where the last inequality is binding if $q=0$ or $p=0$. This in turn yields $t < \frac{\mu}{\mu+(1-\mu)(1-q)}$ which is a contradiction. Hence, if $\frac{\mu}{\mu+(1-\mu)(1-q)} \leq t$, $\Pr_{III}(a=1)$ is optimal with

$$p_0^* = 1 - \frac{\frac{\mu(1-t)}{t(1-\mu)}}{1-q} \quad \text{and} \quad p_1^* = 1 \tag{12}$$

Alternatively,

$$p_0^* = \frac{\overline{q}-q}{1-q} \quad \text{and} \quad p_1^* = 1 \tag{13}$$

(2) If $\frac{\mu}{\mu+(1-\mu)(1-q)} > t$, then it is possible to induce $\mu_1 \geq t$,$\mu_0 \geq t$. In particular, the constraints can be rewritten as two lines (half spaces) where the coordinates are $p_0$ and $p_1$. In particular, we have

$$\mu_1 \geq t \Leftrightarrow (1-t)\mu p_1 \geq t(1-\mu)(1-p_0)(1-q) \tag{14}$$

which passes through (1,0) and $\left(0, \frac{t(1-\mu)(1-q)}{(1-t)\mu}\right)$ where $\frac{t(1-\mu)(1-q)}{(1-t)\mu} < 1$ by assumption. We also have

$$\mu_0 \geq t \Leftrightarrow \mu(1-t)(1-p_1) \geq t(1-\mu)\frac{p}{1-q} \tag{15}$$

which passes through (0,1) and $\left(\frac{\mu(1-t)(1-q)}{t(1-\mu)}, 0\right)$ where $\frac{\mu(1-t)(1-q)}{t(1-\mu)} < 1$ because $t > \mu$.

Since the objective is to maximize $1-(1-\mu)(1-p_0)q$, we want to find the point in region IV with

the largest value of $p_0$. Clearly, this point is at the intersection of the two lines in Figure 1. At the optimum, this point is given by

$$p_0^*=1-\frac{1-(1-q)\frac{\mu(1-t)}{t(1-\mu)}}{(2-q)q} \quad \text{and} \quad p_1^*=1-\frac{1-(1-q)\frac{t(1-\mu)}{\mu(1-t)}}{(2-q)q} \tag{16}$$

where $\frac{\mu(1-t)}{t(1-\mu)}\in(1-q,1)$ by assumption. Alternatively,

$$p_0^*=\frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^*=\frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}}-(1-q)\right] \tag{17}$$

As a result, we have $\Pr_{\text{III}}(a=1)<\Pr_{\text{IV}}(a=1)$ because the following inequality holds

$$\Pr_{\text{III}}(a=1)=\mu+(1-\mu)(1-q)=1-(1-\mu)q<1-(1-\mu)q(1-p_0^*)=\Pr_{\text{IV}}(a=1). \tag{18}$$

Therefore, the optimal strategy for $S$ involves lying in both states, that is $p_0^*\in(0,1)$ and $p_1^*\in(0,1)$. Even though $S$ wants the action $a=1$ to be taken, he may want to say $m=0$ when $w=1$.

## A.2 Proof of Proposition 2

- If $q\leq\overline{q}$,

$$p_0^*=\frac{\overline{q}-q}{1-q} \quad \text{and} \quad p_1^*=1 \tag{19}$$

Clearly, $p_0^*=1-\frac{1-\overline{q}}{1-q}$ decreases in $q$ and $p_1^*$ is constant in $q$.

- If $q>\overline{q}$,

$$p_0^*=\frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^*=\frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}}-(1-q)\right] \tag{20}$$

This implies

$$\frac{\partial p_0^*}{\partial q}=\frac{(-2q+1+\overline{q})\cdot(2-q)q-(2-2q)(1-q)(q-\overline{q})}{(2-q)^2q^2} \tag{21}$$

$$=\frac{-q^2+(q^2-2q+2)\overline{q}}{(2-q)^2q^2} \tag{22}$$

Therefore,

$$\frac{\partial p_0^*}{\partial q} \geq 0 \Longleftrightarrow \frac{1}{\overline{q}} \leq \frac{q^2-2q+2}{q^2} = 1+\frac{2-2q}{q^2} \tag{23}$$

RHS decreases in $q$, meaning the sign of the derivative at most changes one time. Since the derivative is positive at $q=\overline{q}$, but negative at $q=1$, we conclude that $p_0^*$ is first increasing and then decreasing in $q$ over $(\overline{q},1]$.

On the other hand, $p_1^*$ can be written as a product of $\frac{(1-q)}{(2-q)}$ and $\frac{\frac{1}{1-\overline{q}}-(1-q)}{q}$. Each term decreases in $q$, the it follows that $p_1^*$ decreases in $q$ over $(\overline{q},1]$.

## A.3 Proof of Proposition 3

The expected payoff of the Sender is $\Pr(a=1)$. There are two cases depending on whether $q>\overline{q}$.

- If $q \leq \overline{q}$, then the Receiver chooses $a=1$ whenever $m=1,d=\neg lie$ or $m=0,d=lie$. But the latter occurs with probability 0 in the equilibrium. So,

$$U_S = \mu+(1-\mu)(1-p_0)^*(1-q) = \frac{\mu}{t} \tag{24}$$

which is constant in $q$.

- If $q>\overline{q}$, then the Receiver chooses $a=1$ always unless $m=1,d=lie$. So,

$$U_S = 1-(1-\mu)(1-p_0^*)q = 1-\frac{t(1-\mu)-\mu(1-t)(1-q)}{t(2-q)} \tag{25}$$

which is decreasing in $q$ as

$$\frac{\partial U_S}{\partial q} = \frac{-\mu(1-t)t(2-q)-t[t(1-\mu)-\mu(1-t)(1-q)]}{t^2(2-q)^2} \tag{26}$$

$$= \frac{-\mu(1-t)-t(1-\mu)}{t(2-q)^2} \tag{27}$$

$$< 0 \tag{28}$$

The expected payoff of the Receiver is $t\cdot\Pr(a=w=0)+(1-t)\cdot\Pr(a=w=1)$. Again, there are two

cases.

- If $q \leq \overline{q}$, then the Receiver matches the state $w=0$ correctly if $(w=0,m=0)$ or if $(w=0,m=1,d=lie)$, and matches the state $w=1$ correctly if $w=1$. In sum,

$$U_R = (1-\mu)t \cdot [p_0^* + (1-p_0^*)q] + \mu(1-t) \tag{29}$$

$$= (1-\mu)t \cdot [1-(1-p_0^*)(1-q)] + \mu(1-t) \tag{30}$$

$$= (1-\mu)t \cdot \left[1 - \frac{\mu(1-t)}{t(1-\mu)}\right] + \mu(1-t) \tag{31}$$

$$= (1-\mu)t \tag{32}$$

  which is constant in $q$.

- If $q > \overline{q}$, then the Receiver matches the state $w=0$ correctly if $(w=0,m=1,d=lie)$, and matches the state $w=1$ correctly if $w=1$. In sum,

$$U_R = (1-\mu)t \cdot (1-p_0^*)q + \mu(1-t) \tag{33}$$

$$= (1-\mu)t \cdot \frac{1-(1-q)^{\frac{\mu(1-t)}{t(1-\mu)}}}{2-q} + \mu(1-t) \tag{34}$$

$$= \frac{(1-\mu)t + t(1-\mu)}{2-q} \tag{35}$$

  which is increasing in $q$.