Bayesian Persuasion with Lie Detection*

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Abstract

We consider a model of Bayesian persuasion in which the Receiver can detect lies with positive probability. We show that the Sender lies more when the lie detection probability increases. As long as the lie detection probability is sufficiently small the Sender's and the Receiver's equilibrium payoff are unaffected by the lie detection technology because the Sender simply compensates by lying more. When the lie detection probability is sufficiently high, the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability.

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1 Introduction

Lies are a pervasive feature of communication even when communication is subject to intense public and media scrutiny. For example, during his tenure as US President Donald Trump has made over 20,000 false or misleading claims.[1] But such lies are also detectable. Monitoring and fact-checking should constrain how much licence a sender of communication has when making false statements. But, interestingly, in the face of increased fact-checking and media focus the rate of Trump's lying has increased rather than decreased—a development that runs counter to this intuition.

In this paper we incorporate probabilistic lie detection in an otherwise standard model of Bayesian persuasion (Kamenica and Gentzkow, 2011; Kamenica, 2019). Two players, a Sender and a Receiver, engage in one round of communication. The Sender observes the binary state of nature and sends a message to the Receiver. To clearly define whether a message is a lie or not we assume that the message space and the state space are the same. The Receiver observes the message, and if the message is a lie, it is flagged as such with positive probability q. The Receiver then takes an action. Whereas the Sender prefers the Sender to take the "favorable" action regardless of the state of nature, the Receiver wants to match the action to the underlying state. Finally, payoffs are realized for both parties.

Our main assumption that lies—but not the underlying truth—are detectable is arguably a natural one in many applications. Facts may come to light that contradict the initial claim of the Sender. These facts do not necessarily reveal the payoff-relevant state, but only prove that the Sender has lied. For example, in job interviews or trial testimonies, the Sender may be required to provide details and arguments supporting his statements, and if he is lying, he may be at risk of producing an internally or externally inconsistent account, thereby revealing his lie. Liars may also exhibit physical reactions such as blushing, which reveal the fact of lying.

Our model delivers the following set of results. First, the Sender lies more frequently when the lie detection technology improves. Second, as long as the lie detection probability is sufficiently small the equilibrium payoffs of both players are unaffected by the lie detection technology because the Sender simply compensates by lying more frequently in the unfavorable state of nature by claiming that the state is favorable. That is to say, the lie detection technology changes the Sender's message strategy but does

[1] See https://www.washingtonpost.com/politics/2020/07/13/president-trump-has-made-more-than-20000-false-or-misleading-claims/ for a comprehensive analysis of this behavior.

not have an impact on utilities. Third, when the lie detection technology is sufficiently reliable, any further increase in the lie detection probability causes the Sender to also lie more frequently in the favorable state of nature. In the limit case of perfect lie detection ($q = 1$) the Receiver is perfectly informed and the Sender's messages are indeterminate. Fourth, when the lie detection technology is sufficiently reliable, the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability.

Three recent papers (Balbuzanov, 2019; Dziuda and Salas, 2018; Jehiel, 2019), also investigate the role of lie detectability in communication. The largest difference with respect to our paper lies in the commitment assumption of the Sender. In all those papers, the communication game takes the form of cheap talk (Crawford and Sobel, 1982) rather than Bayesian persuasion as in our paper. We defer a detailed comparison between these papers and our work to Section 4. Related theoretical work on lying in communication games also includes Kartik et al. (2006) and Kartik (2009) who do not consider lie detection but instead introduce an exogenous cost of lying tied to the size of the lie in a cheap talk setting. They find that most types inflate their messages, but only up to a point. In contrast to our results they obtain full information revelation follows for some or all types depending on the bounds of the type and message space.

A large and growing experimental literature (Gneezy, 2005; Hurkens and Kartik, 2009; Sánchez-Pagés and Vorsatz, 2009; Ederer and Fehr, 2017; Gneezy et al., 2018) examines lying in a variety of communication games Most closely related to our own work is Fréchette et al. (2019) who investigate models of cheap talk, information disclosure, and Bayesian persuasion, in a unified experimental framework. Their experiments provide general support for the strategic rationale behind the role of commitment and, more specifically, for the Bayesian persuasion model of Kamenica and Gentzkow (2011).

Finally, whereas we focus on an improvement of the Receiver's communication technology (i.e., lie detection), Gehlbach and Vorobyev (2020) analyze how improvements that benefit the Sender (e.g., censorship and propaganda) impact communication under Bayesian persuasion.

2 Model

2.1 Setup

Let $w \in \{0,1\}$ denote the state of the world and $\Pr(w=1)=\mu\in(0,1)$. The Sender (S, he) observes w and commits to send a message $m\in\{0,1\}$ to the Receiver (R, she). If the Sender lies, *i.e.* $m\neq w$, R is informed with probability $q\in[0,1]$ and learns w perfectly. Denote $d=\{lie,\neg lie\}$ as the outcome of the detection result. The detection technology is common knowledge. In a standard Bayesian persuasion setup this detection probability q is equal to 0, giving us an immediately comparable benchmark.

Given both m and d, R takes an action $a\in\{0,1\}$, and the payoffs are realized. The payoffs are defined as follows.

$$u_S(a,w)=\mathbb{1}_{\{a=1\}} \tag{1}$$

$$u_R(a,w)=(1-t)\cdot\mathbb{1}_{\{a=w=1\}}+t\cdot\mathbb{1}_{\{a=w=0\}},\quad 0<t<1 \tag{2}$$

That is, the Sender wants the Receiver to always take the action $a=1$ regardless of the state, while the Receiver wants to match the state. The payoff from matching the state 0 may differ from the payoff from matching the state 1. Given the payoff function, the Receiver takes action $a=1$ if and only if $\Pr(w=1|m;d)\geq t$, so we could also interpret t as the threshold of the Receiver's posterior probability above which she takes $a=1$. Note that if $t\leq\mu$, there is no need to persuade because the Receiver will choose the Sender's preferred action $a=1$ even without a message. Therefore, assume $t\in(\mu,1)$ so that the problem is interesting.

2.2 Optimal Messages and Responses

As is common in the Bayesian persuasion literature we assume that the Sender's commitment to an information structure is binding.[2] The strategy of the Sender is a mapping $m:\{0,1\}\longrightarrow\Delta(\{0,1\})$, and the strategy of the Receiver is a mapping $a:\{0,1\}\times\{lie,\neg lie\}\longrightarrow\Delta(\{0,1\})$. Formally, the Sender is

[2]For a detailed discussion and relaxation of this assumption see Min (2017), Fréchette et al. (2019), Lipnowski et al. (2019), and Nguyen and Tan (2019). Titova (2020) shows that with binary actions and a sufficiently rich enough state space verifiable disclosure enables the sender's commitment solution as an equilibrium.

choosing $m(\cdot)$ to maximize

$$\mathbb{E}[u_S(a(m(w),d(m(w),w)),w)]$$

where $a(m,d)$ maximizes

$$\mathbb{E}[u_R(a,w)|m;d].$$

Due to the simple structure of the model, it is without loss of generality to assume that the Sender chooses only two parameters $p_0 = \Pr(m=0|w=0)$ and $p_1 = \Pr(m=1|w=1)$ to maximize $\Pr(a(m,d)=1)$ which we write as $\Pr(a = 1)$ henceforth for brevity of notation. We denote the optimal reporting probabilities of the Sender by p_0^* and p_1^*, and the ex-ante payoffs under this reporting probabilities as U_S and U_R. Given the Sender's reporting strategy, the Receiver could potentially see four types of events to which she needs to react when choosing action a.

First, the Receiver could observe the event $(m=0,d=lie)$ which occurs with probability $\mu(1-p_1)q$. Given the lie detection technology Receiver is certain that the message $m=0$ is a lie and therefore the state of the world w must be equal to 1, that is

$$\Pr(w=1|m=0,d=lie)=1. \tag{3}$$

As a result, the Receiver optimally chooses $a=1$.

Second, the event $(m=0,d=\neg lie)$ could occur with probability $\mu(1-p_1)(1-q)+(1-\mu)p_0$. In that case, the Receiver is uncertain about w because she does not know whether the Sender lied or not. Her posterior probability is given by

$$\Pr(w=1|m=0,d=\neg lie)=\frac{\mu(1-p_1)(1-q)}{\mu(1-p_1)(1-q)+(1-\mu)p_0}\equiv\mu_0. \tag{4}$$

Hence, the Receiver takes action $a=1$ if and only if $B\geq t$.

Third, $(m=1,d=lie)$ occurs with probability $(1-\mu)(1-p_0)q$. Because a lie was detected the Receiver is again certain about w and therefore her posterior probability is given by

$$\Pr(w=1|m=1,d=lie)=0, \tag{5}$$

which immediately implies the action $a=0$.

Fourth, $(m=1,d=\neg lie)$ occurs with probability $\mu p_1+(1-\mu)(1-p_0)(1-q)$. The Receiver is again uncertain about w. Her posterior is given by

$$\Pr(w=1|m=1,d=\neg lie)=\frac{\mu p_1}{\mu p_1+(1-\mu)(1-p_0)(1-q)}\equiv\mu_1 \tag{6}$$

and the Receiver takes action $a=1$ if and only if $\mu_1\geq t$.

Given these optimal responses by R, the relationships between the posteriors μ_1 and μ_2 and the posterior threshold t divide up the strategy space into four distinct regions which we denote by I, II, III, and IV respectively. Within each region, the Receiver's response as a function of (m,d) is the same, making it easy to find the region-optimal strategy. We are then left to pick the best strategy out of the four candidates for different values of q. These regions are defined as follows:

I. $\mu_1<t$, $\mu_2<t$: In this region, the Receiver only chooses $a=1$ if $(m=0,d=lie)$ and $a=0$ otherwise because the posteriors μ_1 and μ_2 are insufficiently high to persuade her to choose S's preferred action. Only if the Sender lies in state $w=1$ and his message is detected as a lie is R sufficiently convinced that $a=1$ is the right action. The probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\text{I}}(a=1)=\max_{p_0,p_1}\quad \mu(1-p_1)q \quad s.t.\,\mu_1<t,\mu_2<t \tag{7}$$

II. $\mu_1<t$, $\mu_2\geq t$: In this region, the Receiver chooses $a=1$ if $(m=0,d=lie)$ or $(m=0,d=\neg lie)$ and $a=0$ otherwise. The probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\text{II}}(a=1)=\max_{p_0,p_1}\quad \mu(1-p_1)+(1-\mu)p_0 \quad s.t.\mu_1<t,\mu_2\geq t \tag{8}$$

III. $\mu_1\geq t$, $\mu_2<t$: In this region, the Receiver chooses $a=1$ if $(m=0,d=lie)$ or $(m=1,d=\neg lie)$ and $a=0$ otherwise. The probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\text{III}}(a=1)=\max_{p_0,p_1}\quad \mu p_1+\mu(1-p_1)q+(1-\mu)(1-q)(1-p_0) \quad s.t.\,\mu_1\geq t,\mu_2<t \tag{9}$$

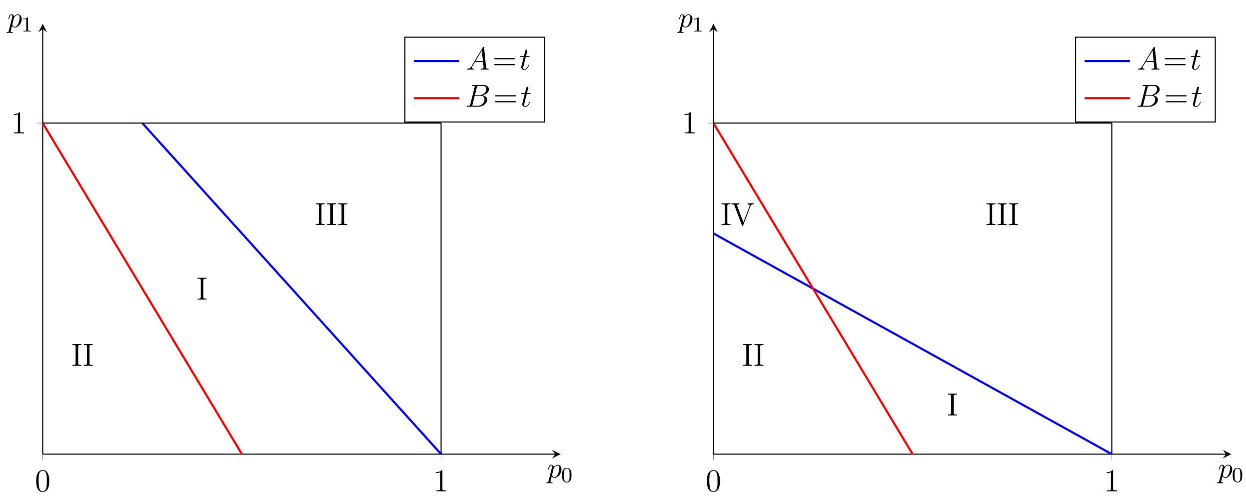
IV. $\mu_1 \geq t$, $\mu_2 \geq t$: In this region, the Receiver chooses $a=1$ if $(m=0, d=lie)$, $(m=0, d=\neg lie)$ or $(m=1, d=\neg lie)$. The probability that the Receiver chooses $a=1$ is given by

$$\Pr_{\mathrm{IV}}(a=1) = \max_{p_0, p_1} \quad 1-(1-\mu)(1-p_0)q \quad s.t. \mu_1 \geq t, \mu_2 \geq t \tag{10}$$

We are now ready to state the main proposition of our model.

Proposition 1. *Let $\overline{q} = 1 - \frac{\mu(1-t)}{t(1-\mu)} \in (0,1)$. If $q \leq \overline{q}$, the Sender's optimal strategy is in region* III*, in which the Sender always tells the truth under $w=1$, but lies with positive probability under $w=0$. If $q > \overline{q}$, the Sender's optimal strategy is in region* IV*, in which the Sender lies with positive probability under both states.*

In Figure 1 we graphically illustrate how these four regions are divided.



(a) $q \leq \overline{q}$

(b) $q > \overline{q}$

Figure 1: Equilibrium message strategies for different detection probabilities q.

The proof involves sequential comparisons between the four region-optimal strategies. First, the optimal strategy in region II is better than the optimal strategy in region I. To see this, consider a particular strategy $p_0 = p_1 = 0$ in region II (i.e., the Sender totally misreports the state). Following this strategy, the Receiver takes action $a=1$ if and only $w=1$, which occurs with probability μ. This strategy may not be optimal within region II, but it secures a lower bound μ for the value of the optimal strategy. This lower bound is sufficient to beat all strategies in region I since there $a=1$ only if $w=1$ and $(m=0, d=lie)$, which occurs with a probability less than μ.

Second, the optimal strategy in region III is better than the optimal strategy in region II. Notably, they should be equivalent if the lie detection technology is not available ($q=0$) since in that case the messages have no intrinsic meanings and we could always rename the messages so that the two maximization problems are identical in II and III. However, with the introduction of a lie detection technology, there is now an intrinsic meaning of the message the Sender uses. Regardless of the committed strategy by the Sender, an on-path message that was not detected as a lie always carries some credibility for the state to which it corresponds. The two regions differ in the presence of this additional credibility. In region II, the Sender wants an undetected lie $m=0$ to be indicative of $w=1$. This is more difficult because in that region the additional credibility is towards $w=0$. On the contrary, in region III, the Sender wants an undetected lie $m=1$ to be indicative of $w=1$, but this is easier because the additional credibility is towards the same direction. Therefore, both region I and region II are suboptimal and therefore we only need to focus on the comparison between region III and IV.

Interestingly, as suggested by the two panels of Figure 1, region IV does not exist when q is small. It is the easiest to understand this result when $q=0$, which yields the standard Bayesian Persuasion benchmark. In that case, we know it is impossible to induce $a=1$ under all messages because by the martingale property, the posteriors after two messages must average to the prior, suggesting some posterior is lower than the prior and must induce $a=0$. However, the presence of lie detection extends the information from m to a couple (m,d). However, using this insight we just need the four posteriors' average over the prior. Furthermore, the posterior following $(1,lie)$ is 0. Therefore if q is sufficiently large, it is possible to support the two posteriors following $(1,\neg lie)$ and $(0,\neg lie)$ which are both higher than the prior and also higher than the threshold t. Combining this observation with the previous two arguments, we immediately obtain the first half of Proposition 1. In particular, if $q \leq \overline{q}$, the optimal strategy takes the following form:

$$p_0^* = \frac{\overline{q} - q}{1-q} \quad \text{and} \quad p_1^* = 1 \tag{11}$$

This is reminiscent of Kamenica and Gentzkow (2011), where the Receiver is indifferent between two actions when she takes the preferred action $a=1$, and certain of the state when she takes the less preferred action $a=0$.

If the detection probability $q>\overline{q}$, the optimal strategy in region III involves a corner solution since p_0 cannot drop below 0. In particular, S always claims $m=1$ regardless of the state, so that a message $m=0$ becomes off-path. Yet this is no more globally optimal as the region IV is now non-empty, and the optimal strategy in region IV is better than the one in region III described above. Specifically, the optimal strategy involves partial lying under both states ($0<p_0,p_1<1$) and is given by

$$p_0^*=\frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^*=\frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}}-(1-q)\right] \tag{12}$$

From the above equations it can easily be seen that $p_0^*<p_1^*$.

The reason for this perhaps puzzling behavior of lying in both states is as follows. There are two benefits for sending $m=0$ in the favorable state $w=1$. First, if the lie is detected which happens with probability q, it is a free way to convince R that the state is indeed $w=1$. Second, even if the lie is not detected, it renders $m=0$ more indicative of $w=1$. Thus, the Sender may have a chance to persuade the Receiver in this case. However, these two benefits come at the cost of reducing the credibility of $m=1$. When fewer truthful messages $m=1$ are sent when $w=1$, the Receiver may prefer to take action $a=0$ after seeing $(m=1,d=\neg lie)$. But with a sufficiently high detection probability q, this cost is equal to 0 because the constraint $\mu_1\geq t$ is now slack at the traditional Bayesian persuasion solution ($p_0=0,p_1=1$). Therefore, there is room to change p_0 and p_1 locally without affecting R's response after $(m=1,d=\neg lie)$.

3 Comparative Statics

We now consider the comparative statics of our model with respect to the central parameter of the lie detection probability q to show how the optimal communication and the utilities of the communicating parties changes as the lie detection technology improves.

3.1 Optimal Signals

Proposition 2 describes how the structure of optimal signal (p_0^*,p_1^*) changes as the detection probability varies. Figure 2 plots these optimal reporting probabilities as a function of q. For comparison, the probabilities p_0^{BP} and p_1^{BP} are the equilibrium reporting probabilities that would result in a standard Bayesian persuasion setup without lie detection.

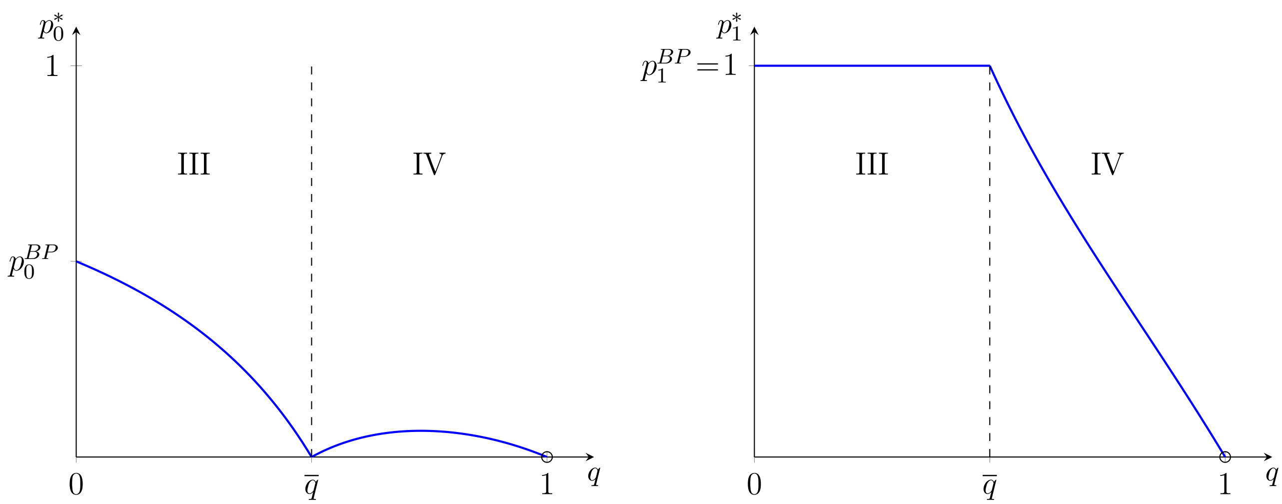
Proposition 2. *As the lie detection probability q increases,*

1. $p_0^* = Pr(m=0|w=0)$ *is decreasing over* $[0,\overline{q}]$*, and has an inverse U shape over* $(\overline{q},1]$.

2. $p_1^* = Pr(m=1|w=1)$ *is constant over* $[0,\overline{q}]$*, and decreases over* $(\overline{q},1]$.

If $q \leq \overline{q} = 1 - \frac{\mu(1-t)}{t(1-\mu)}$, p_0^* is decreasing in q and p_1^* is constant at 1. In this range of q, the Sender's optimal strategy lies in III which involves truthfully reporting the state $w=1$ (i.e., $p_1=1$), but progressively misreporting the state $w=0$ as the lie detection technology improves (i.e., $p_0<1$ and decreasing with q).

If $q>\overline{q}$, p_0^* initially increases and then decreases. In contrast, p_1^* decreases over the entire range of $[\overline{q},1]$. In this range, the Sender's optimal strategy lies in IV which involves misreporting both states of the world.



(a) $p_0^* \equiv \Pr(m=0 \,|\, w=0)$ (b) $p_1^* \equiv \Pr(m=1 \,|\, w=1)$

Figure 2: Equilibrium reporting probabilities p_0^* and p_1^* as a function of q for $\mu = \frac{1}{3}$ and $t = \frac{1}{2}$

For $q=0$ we have the Bayesian benchmark. Recall from Kamenica and Gentzkow (2011) that if an optimal signal induces a belief that leads to the worst action for the Sender ($a=0$ in our case), the Receiver is certain of her action at this belief. In addition, if the optimal signal induces a belief that leads to the best action for the Sender ($a=1$ in our case) the Receiver is indifferent between the two actions at this belief.

Now consider the addition of a lie detection technology. As the lie detection probability q increases, $(m=1, d=\neg lie)$ becomes more indicative of the favorable state $w=1$ and therefore the Receiver would strictly prefer to take the favorable action $a = 1$. As a response, the Sender would like to send the message $m=1$ more often while still maintaining that $(m=1, d=\neg lie)$ sufficiently persuades the Receiver to take the action $a=1$. Because the Sender already sends the message $m=1$ with probability 1 under $w=1$, the only way to increase the frequency of $m=1$ is to send such a message more often in the unfavorable state $w=0$ (i.e., lie more frequently if $w=0$). In other words, the Sender increases the frequency of lying just enough about the unfavorable state ($w=0$) to make the Receiver indifferent when choosing the favorable action $a=1$.

Recall that in the canonical Bayesian persuasion setup the Receiver is just held to her outside utility of getting no information whatsoever. When lie detection q goes up the Receiver is now more certain that $(m=1, d=\neg lie)$ means $w=1$ and would therefore obtain a larger surplus from the improvement in the lie detection technology. However, as long as p_0^* is greater than 0 the Sender can simply undo this improvement by lying more about $w=0$ (i.e., reduce p_0^* even further) thereby "signal-jamming" the information obtained by the Receiver.

However, the intuition for the comparative statics of the optimal messages with respect to the lie detection probability in region IV has to be considered together because p_0 and p_1 are jointly determined. On the one hand, when the detection probability just passes the switching threshold from region III to IV at $\overline{q}$, p_1 is close to 1. If p_0 were to remain at 0, then a message $m=0$ could only ever come from the Sender if $w=1$, and is thus surely a lie. Hence $m=0$ could successfully persuade the Receiver to choose $a=1$, regardless of the lie detection outcome. Then the Sender will be tempted to increase p_0 as long as the posterior of $m=0$ is still higher than the belief threshold t. This explains why p_0 is positive for $q > \overline{q}$. On the other hand, when the detection probability is close to 1, (i.e., there is almost perfect lie detection technology) p_1 is close to 0. Then $m=1$ is very likely to be a lie. To make sure that $w=1$ sufficiently persuades the Receiver to choose $a=1$, the Sender has to decrease p_0 proportionally along with the decrease of p_1.

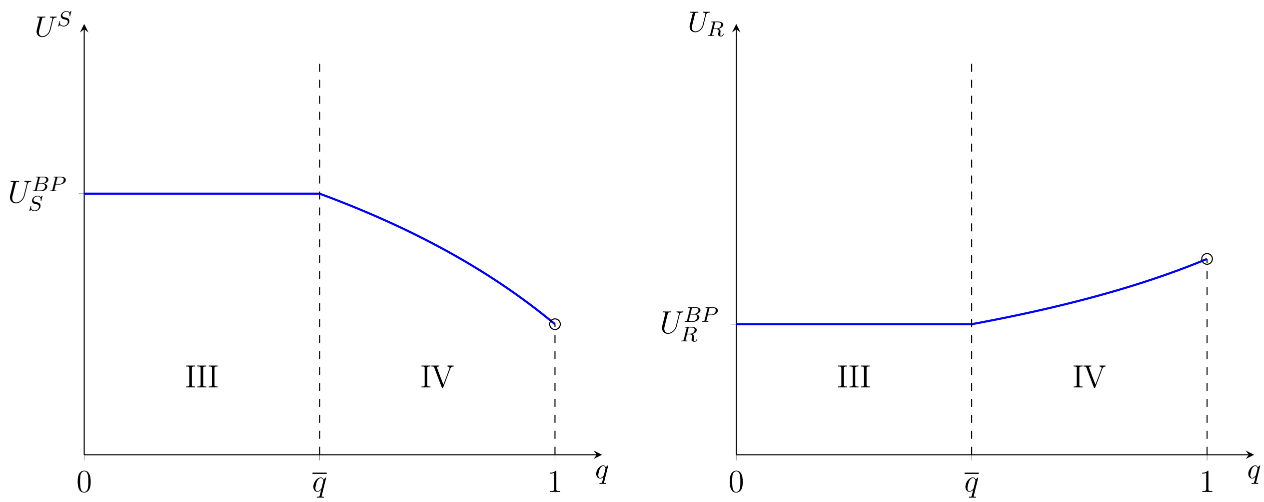
3.2 Utilities

We denote the equilibrium payoffs of the Sender and the Receiver by U_S and U_R and investigate how U_S and U_R are affected by improvements in the lie detection technology. The results are summarized in Proposition 3 and graphically depicted in Figure 3. For comparison, the utilities U_S^{BP} and U_R^{BP} are the equilibrium utilities that would result in a standard Bayesian persuasion setup without lie detection.

Proposition 3. *As the lie detection probability q increases,*

1. *U_S is constant over $[0,\overline{q}]$, and decreases over $(\overline{q},1]$.*

2. *U_R is constant over $[0,\overline{q}]$, and increases over $(\overline{q},1]$.*



(a) Sender's Equilibrium Payoff

(b) Receiver's Equilibrium Payoff

Figure 3: Equilibrium Payoffs as a function of q for $\mu = \frac{1}{3}$, $t = \frac{1}{2}$

The Sender's equilibrium payoff does not change for $q \leq \overline{q}$ and decreases with q for $q > \overline{q}$. As long as $q \leq \overline{q}$ the Sender receives exactly the same utility that he would receive under the Bayesian Persuasion benchmark. Any marginal improvement in the lie detection technology (i.e., increase in q) is completely offset by less truthful reporting when $w = 0$ (i.e., decrease in p_0^*). However, for $q > \overline{q}$ any further improvements reduce the Sender's utility. In the limit case where $q = 1$ the Sender has no influence anymore and the action $a = 1$ in only implemented when the state is actually $w = 1$ which occurs with probability μ.

Analogously for the case of the Sender's utility, the Receiver's utility is also constant at the Bayesian persuasion benchmark as long as $q \leq \overline{q}$ and then increases with q for $q > \overline{q}$ as the lie detection technology starts to bite. In the limit, the Receiver is just as well off as she would be under perfect information.

4 Discussion

Balbuzanov (2019) and Dziuda and Salas (2018) also study strategic communication in the presence of a lie detection technology but in a cheap talk setting. The largest difference between these two papers and ours therefore lies in the commitment power of the Sender. Although it is debatable whether the extreme cases of full commitment (as in Bayesian persuasion) or no commitment (as in cheap talk) constitute more plausible assumptions about real-life communication setting, we believe our model is an important step towards studying the communication games with lie detection under (partial) commitment.

In addition, our paper also differs from Balbuzanov (2019) in the payoff functions. In Balbuzanov (2019) the Sender and the Receiver have some degree of common interest whereas in our model there is no common interest. Due to this difference the Sender's type-dependent preferences in Balbuzanov (2019) permit fully revealing equilibria in some cases as it allows the Receiver to tailor message-specific punishment actions. In particular, fully revealing equilibria exist for some intermediate degree of lie detectability if the Sender's bias is small. However, the Sender in our model never reveals the state perfectly due to the conflict in payoffs.

Dziuda and Salas (2018) do not allow for common interest and therefore, like in our paper, fully revealing equilibria are impossible in their paper. In their continuous state model there are many off-path beliefs to be specified. To discipline these off-path beliefs, they impose two refinements and show that in all remaining equilibria, the lowest types lie, while *some* higher types tell the truth. Our model violates the second refinement in Dziuda and Salas (2018) and thus irrespective of the commitment power of the Sender our paper is not nested by theirs.

In addition to these two papers Jehiel (2019) considers two rounds of communication and a Sender who in the second round cannot remember what lies she sent in the first round. Since the Sender remembers only the unconditional distribution of first-round lies in equilibrium, lie detection probability is endogenous to equilibrium and the structure of the equilibria is similar to Dziuda and Salas (2018). As the state space becomes arbitrarily fine, the probability of lie detection goes to 1 because it is hard to guess exactly the same lie, and therefore only fully revealing equilibria arise.

Interestingly, all three papers feature a type of non-monotonicity result with respect to the relationship between lie detection and lying that is similar to that in our paper. In Balbuzanov (2019), the set of detection probabilities that permits fully revealing equilibrium obtains for low lie detection probabilities, but not for higher ones. Although in the baseline model of Dziuda and Salas (2018) a lower lie detection probability leads to less truth-telling, in an extension of their model they show that if the Sender is given an opportunity to make costly investments in decreasing the lie detectability, then for an intermediate region of the cost, the mass of liars can increase with the investment cost.

5 Conclusion

In this paper we analyze the role of probabilistic lie detection in a model of Bayesian persuasion between a Sender and a Receiver. We show that the Sender lies more when the lie detection probability increases. As long as the lie detection probability is sufficiently small the Sender's and the Receiver's equilibrium payoff are unaffected by the lie detection technology because the Sender simply compensates by lying more. Once the lie detection probability is sufficiently high, the Sender is no longer able to maximally lie about the unfavorable state and the Sender's (Receiver's) equilibrium payoff decreases (increases) with the lie detection probability. Our model rationalizes that a sender of communication chooses to lie more frequently when it is more likely that their false statements will be flagged as lies.

Our paper explores the impact of lie detection on communication in a setting with complete commitment to a communication strategy by the sender and thereby establishes a useful benchmark relative to the diametrically opposed assumption of no commitment in existing cheap talk models with lie detection. However, how does lie detection influence communication behavior in intermediate settings with partial commitment? And what role does lie detection play under Bayesian persuasion with a richer state and message space? We leave these and other interesting questions to future research.

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A Proofs

A.1 Proof of Proposition 1

We now show that strategies I and II are suboptimal because the resulting implementation probabilities $\Pr_{\mathrm{I}}(a=1)$ and $\Pr_{\mathrm{II}}(a=1)$ are dominated by the probability $\Pr_{\mathrm{III}}(a=1)$ resulting from III. To see this, note first that

$$\Pr_{\mathrm{I}}(a=1) \leq \mu \leq \Pr_{\mathrm{II}}(a=1) \tag{13}$$

The second inequality holds because $p_0=p_1=0$ is feasible in II and gives value μ. In fact, Within region II, it is optimal to set $p_1=0$ because this loosens both constraints, but it does not have a direct effect on the objective of maximizing the probability. Given this, we have $A=0<t$ so the optimum requires $B=t$. Therefore, p_0^* solves the following equation

$$\frac{\mu(1-q)}{\mu(1-q)+(1-\mu)p}=t \tag{14}$$

and hence

$$\Pr_{\mathrm{II}}(a=1)=\mu+\left(\frac{\mu}{t}-\mu\right)(1-q) \tag{15}$$

For $\Pr_{\mathrm{III}}(a=1)$ it is optimal to set $p_1=1$ for similar reasons. This yields $B=0<t$ which at the optimum requires p_0 to be small enough, but still ensures that $A \geq t$. Define $\overline{q} \equiv 1-\frac{\mu(1-t)}{t(1-\mu)} \in (0,1)$, then there are two cases to consider.

- $\frac{\mu}{\mu+(1-\mu)(1-q)} \leq t$ or $q \leq \overline{q}$. In this case, there exists p_0^* s.t. $A=t$, that is $\frac{\mu}{\mu+(1-\mu)(1-p_0^*)(1-q)}=t$. Therefore, $\Pr_{\mathrm{III}}(a=1)=\frac{\mu}{t}$.

- $\frac{\mu}{\mu+(1-\mu)(1-q)} > t$ or $q > \overline{q}$. In this case, $A \geq t$ can never bind. So the best option is to set $p=0$ which implies $\Pr_{\mathrm{III}}(a=1)=\mu+(1-\mu)(1-q)$.

Clearly, in either case, $\Pr_{\mathrm{III}}(a=1) > \Pr_{\mathrm{II}}(a=1)$, so both strategies I and II are suboptimal. It therefore remains to compare $\Pr_{\mathrm{III}}(a=1)$ and $\Pr_{\mathrm{IV}}(a=1)$.

(1) If $\frac{\mu}{\mu+(1-\mu)(1-q)}\leq t$, Region IV does not exist, *i.e.*, there is no way to choose p_0,p_1 such that $A\geq t$ and $B\geq t$. If that were the case we would have $\frac{\mu p_1}{\mu p_1+(1-\mu)(1-p_0)(1-q)}\geq t$ and $\frac{\mu(1-p_1)(1-q)}{\mu(1-p_1)(1-q)+(1-\mu)p}\geq t$ which would imply

$$\frac{\mu p_1+\mu(1-p_1)}{\mu p_1+\mu(1-p_1)+(1-\mu)(1-p_0)(1-q)+(1-\mu)\frac{p}{1-q}}\geq t \tag{16}$$

and therefore

$$t\leq\frac{\mu}{\mu+(1-\mu)(1-p_0)(1-q)+(1-\mu)\frac{p}{1-q}}\leq\frac{\mu}{\mu+(1-\mu)(1-q)} \tag{17}$$

where the last inequality is binding if $q=0$ or $p=0$. This in turn yields $t<\frac{\mu}{\mu+(1-\mu)(1-q)}$ which is a contradiction. Hence, if $\frac{\mu}{\mu+(1-\mu)(1-q)}\leq t$, $\Pr_{III}(a=1)$ is optimal with

$$p_0^*=1-\frac{\frac{\mu(1-t)}{t(1-\mu)}}{1-q} \quad \text{and} \quad p_1^*=1 \tag{18}$$

Alternatively,

$$p_0^*=\frac{\overline{q}-q}{1-q} \quad \text{and} \quad p_1^*=1 \tag{19}$$

(2) If $\frac{\mu}{\mu+(1-\mu)(1-q)}>t$, then it is possible to induce $A\geq t,B\geq t$. In particular, the constraints can be rewritten as two lines (half spaces) where the coordinates are p_0 and p_1. In particular, we have

$$A\geq t\Leftrightarrow(1-t)\mu p_1\geq t(1-\mu)(1-p_0)(1-q) \tag{20}$$

which passes through (1,0) and $\left(0,\frac{t(1-\mu)(1-q)}{(1-t)\mu}\right)$ where $\frac{t(1-\mu)(1-q)}{(1-t)\mu}<1$ by assumption. We also have

$$B\geq t\Leftrightarrow\mu(1-t)(1-p_1)\geq t(1-\mu)\frac{p}{1-q} \tag{21}$$

which passes through (0,1) and $\left(\frac{\mu(1-t)(1-q)}{t(1-\mu)},0\right)$ where $\frac{\mu(1-t)(1-q)}{t(1-\mu)}<1$ because $t>\mu$.

Since the objective is to maximize $1-(1-\mu)(1-p_0)q$, we want to find the point in region IV with the largest value of p_0. Clearly, this point is O at the intersection of the two lines in Figure 1. At the optimum, this point is given by

$$p_0^* = 1 - \frac{1-(1-q)\frac{\mu(1-t)}{t(1-\mu)}}{(2-q)q} \quad \text{and} \quad p_1^* = 1 - \frac{1-(1-q)\frac{t(1-\mu)}{\mu(1-t)}}{(2-q)q} \tag{22}$$

where $\frac{\mu(1-t)}{t(1-\mu)} \in (1-q,1)$ by assumption. Alternatively,

$$p_0^* = \frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^* = \frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}} - (1-q)\right] \tag{23}$$

As a result, we have $\Pr_{\text{III}}(a=1) < \Pr_{\text{IV}}(a=1)$ because the following inequality holds

$$\Pr_{\text{III}}(a=1) = \mu + (1-\mu)(1-q) = 1-(1-\mu)q < 1-(1-\mu)q(1-p_0^*) = \Pr_{\text{IV}}(a=1). \tag{24}$$

Therefore, the optimal strategy for S involves lying in both states, that is $p_0^* \in (0,1)$ and $p_1^* \in (0,1)$. Even though S wants the action $a=1$ to be taken, he may want to say $m=0$ when $w=1$.

A.2 Proof of Proposition 2

- If $q \leq \overline{q}$,

$$p_0^* = \frac{\overline{q}-q}{1-q} \quad \text{and} \quad p_1^* = 1 \tag{25}$$

Clearly, $p_0^* = 1 - \frac{1-\overline{q}}{1-q}$ decreases in q and p_1^* is constant in q.

- If $q > \overline{q}$,

$$p_0^* = \frac{1-q}{(2-q)q}(q-\overline{q}) \quad \text{and} \quad p_1^* = \frac{1-q}{(2-q)q}\left[\frac{1}{1-\overline{q}} - (1-q)\right] \tag{26}$$

This implies

$$\frac{\partial p_0^*}{\partial q} = \frac{(-2q+1+\overline{q}) \cdot (2-q)q - (2-2q)(1-q)(q-\overline{q})}{(2-q)^2 q^2} \tag{27}$$

$$= \frac{-q^2 + (q^2 - 2q + 2)\overline{q}}{(2-q)^2 q^2} \tag{28}$$

Therefore,

$$\frac{\partial p_0^*}{\partial q} \geq 0 \Longleftrightarrow \frac{1}{\overline{q}} \leq \frac{q^2 - 2q + 2}{q^2} = 1 + \frac{2-2q}{q^2} \tag{29}$$

RHS decreases in q, meaning the sign of the derivative at most changes one time. Since the derivative is positive at $q = \overline{q}$, but negative at $q = 1$, we conclude that p_0^* is first increasing and then decreasing in q over $(\overline{q}, 1]$.

On the other hand, p_1^* can be written as a product of $\frac{(1-q)}{(2-q)}$ and $\frac{\frac{1}{1-\overline{q}} - (1-q)}{q}$. Each term decreases in q, the it follows that p_1^* decreases in q over $(\overline{q}, 1]$.

A.3 Proof of Proposition 3

The expected payoff of the Sender is $\Pr(a=1)$. There are two cases depending on whether $q > \overline{q}$.

- If $q \leq \overline{q}$, then the Receiver chooses $a=1$ whenever $m=1, d=\neg lie$ or $m=0, d=lie$. But the latter occurs with probability 0 in the equilibrium. So,

$$U_S = \mu + (1-\mu)(1-p_0)^*(1-q) = \frac{\mu}{t} \tag{30}$$

which is constant in q.

- If $q > \overline{q}$, then the Receiver chooses $a=1$ always unless $m=1, d=lie$. So,

$$U_S = 1 - (1-\mu)(1-p_0^*)q = 1 - \frac{t(1-\mu) - \mu(1-t)(1-q)}{t(2-q)} \tag{31}$$

which is decreasing in q as

$$\begin{aligned}
\frac{\partial U_S}{\partial q} &= \frac{-\mu(1-t)t(2-q) - t[t(1-\mu) - \mu(1-t)(1-q)]}{t^2(2-q)^2} && (32)\\
&= \frac{-\mu(1-t) - t(1-\mu)}{t(2-q)^2} && (33)\\
&< 0 && (34)
\end{aligned}$$

The expected payoff of the Receiver is $t \cdot \Pr(a=w=0) + (1-t) \cdot \Pr(a=w=1)$. Again, there are two cases.

- If $q \leq \overline{q}$, then the Receiver matches the state $w=0$ correctly if $(w=0,m=0)$ or if $(w=0,m=1,d=lie)$, and matches the state $w=1$ correctly if $w=1$. In sum,

$$
\begin{aligned}
U_R &= (1-\mu)t \cdot [p_0^* + (1-p_0^*)q] + \mu(1-t) && (35)\\
&= (1-\mu)t \cdot [1-(1-p_0^*)(1-q)] + \mu(1-t) && (36)\\
&= (1-\mu)t \cdot \left[1 - \frac{\mu(1-t)}{t(1-\mu)}\right] + \mu(1-t) && (37)\\
&= (1-\mu)t && (38)
\end{aligned}
$$

which is constant in q.

- If $q > \overline{q}$, then the Receiver matches the state $w=0$ correctly if $(w=0,m=1,d=lie)$, and matches the state $w=1$ correctly if $w=1$. In sum,

$$
\begin{aligned}
U_R &= (1-\mu)t \cdot (1-p_0^*)q + \mu(1-t) && (39)\\
&= (1-\mu)t \cdot \frac{1-(1-q)\frac{\mu(1-t)}{t(1-\mu)}}{2-q} + \mu(1-t) && (40)\\
&= \frac{(1-\mu)t + t(1-\mu)}{2-q} && (41)
\end{aligned}
$$

which is increasing in q.