Module EA4 – Éléments d'Algorithmique

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Rappel des épisodes précédents

Tri par insertion

- $\Theta(n^2)$ comparaisons au pire, $\Theta(n)$ au mieux,
- $\Theta(n^2)$ comparaisons en moyenne,
- trie en place

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Tri par fusion

- $\Theta(n \log n)$ comparaisons dans tous les cas,
- la constante cachée dans le Θ est importante,
- ne trie pas en place : complexité en espace $\in \Theta(n)$

Peut-on faire mieux ?

Coût minimal d'un tri par comparaison

Un tri par comparaison est un algorithme de tri qui n'utilise que des comparaisons entre les éléments d'entrée.

Contre-exemple : le tri par dénombrement n'est pas un tri par comparaison

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Un tri par comparaison est un algorithme de tri qui n'utilise que des comparaisons entre les éléments d'entrée.

Contre-exemple : le tri par dénombrement n'est pas un tri par comparaison

Théorème

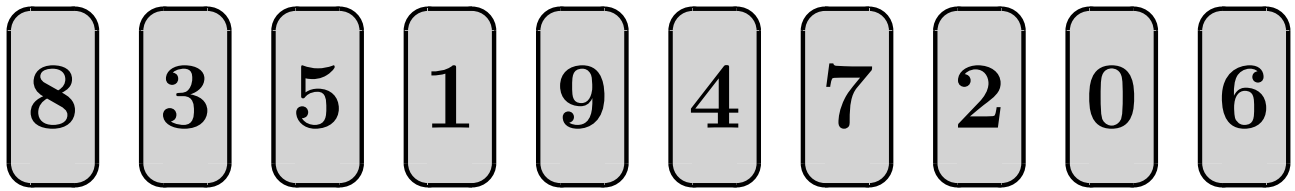
Tout tri par comparaison effectue, en moyenne sur les listes de longueur n, $\Omega(n \log n)$ comparaisons.

Corollaire

Le tri fusion est un tri par comparaison asymptotiquement optimal.

Tri rapide (*Quicksort*)

Exemple :



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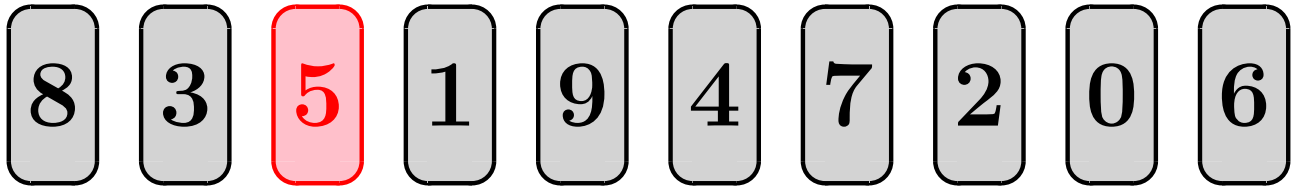

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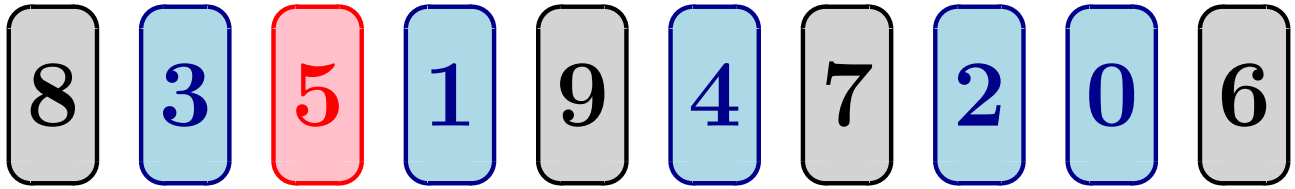
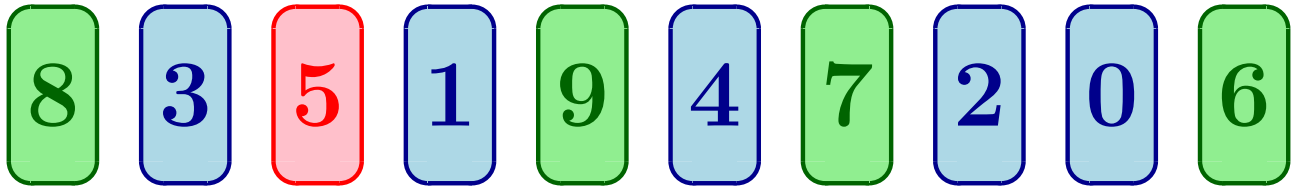


Tri rapide (*Quicksort*)

Exemple :



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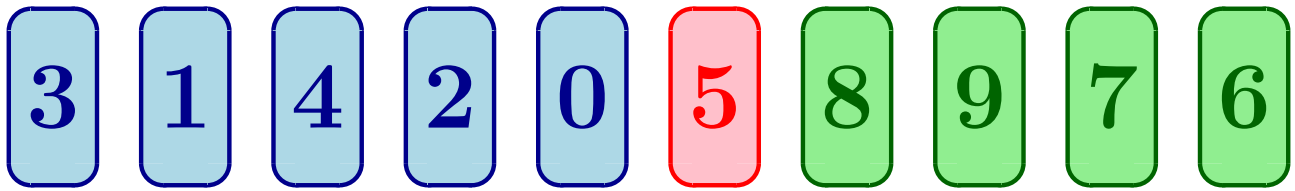

Tri rapide (*Quicksort*), version 1

```
def partition(T) :  # les éléments sont supposés distincts
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  gauche = [ elt for elt in T if elt < pivot ]
  droite = [ elt for elt in T if elt > pivot ]
  return pivot, gauche, droite
```

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  pivot = T[0]
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  return pivot, gauche, droite

def tri_rapide(T) :
  if len(T) < 2 : return T
  pivot, gauche, droite = partition(T)
  return tri_rapide(gauche) + [pivot] + tri_rapide(droite)
```

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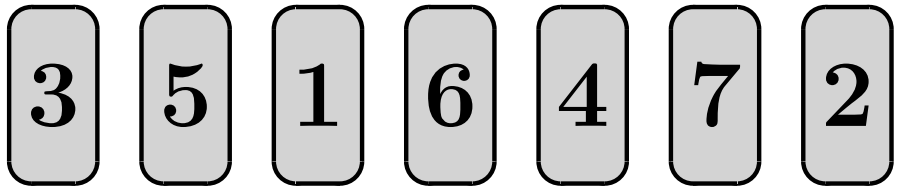

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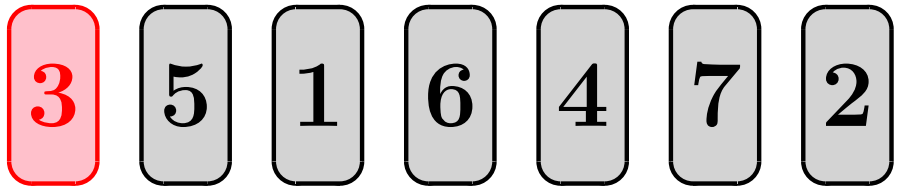

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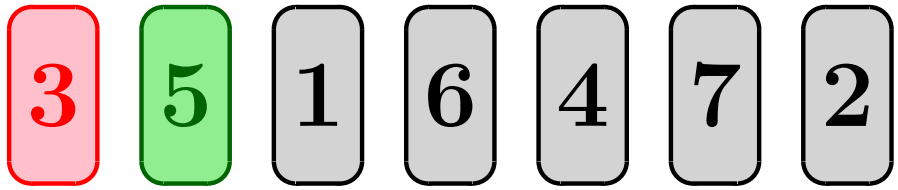

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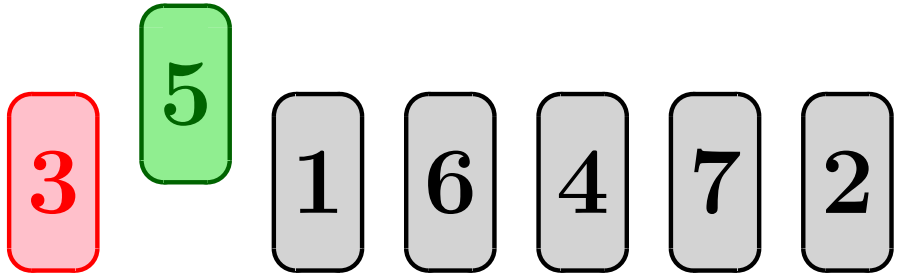

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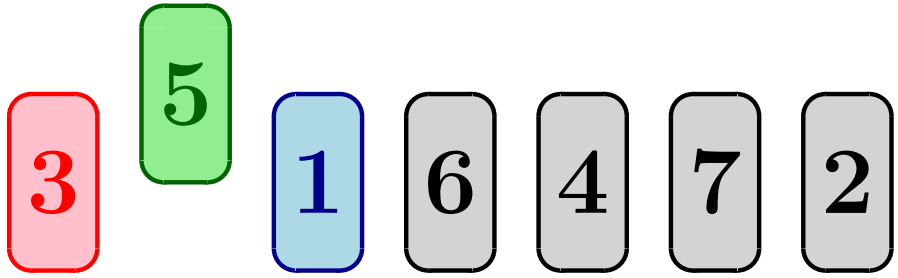

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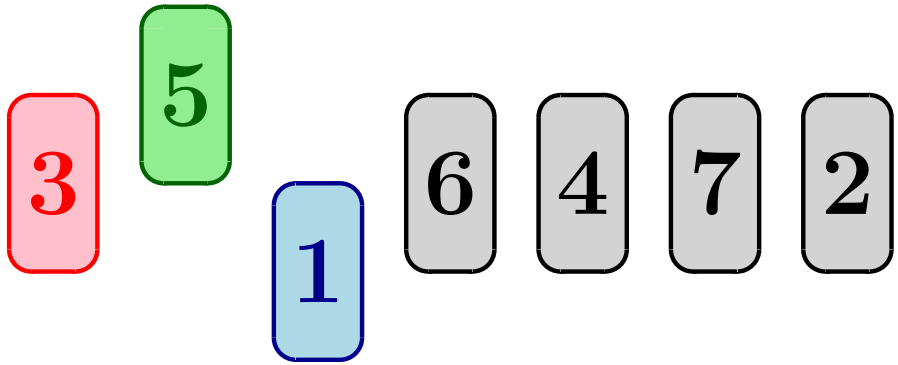

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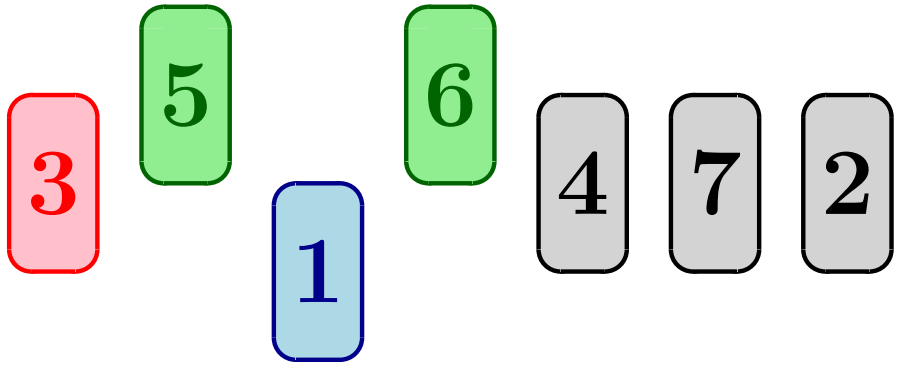

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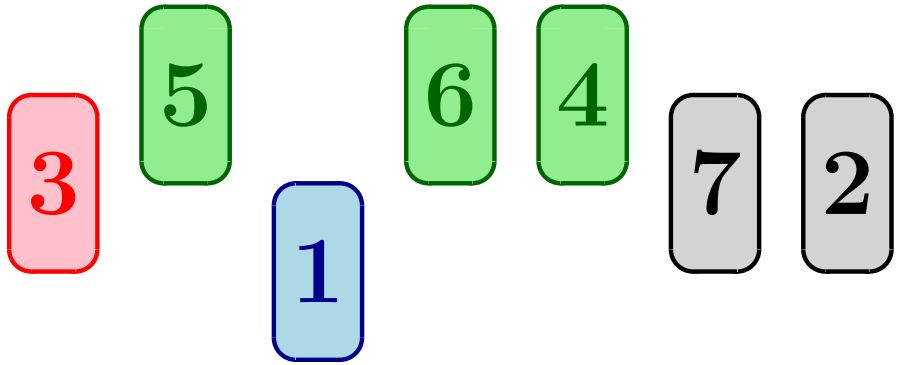

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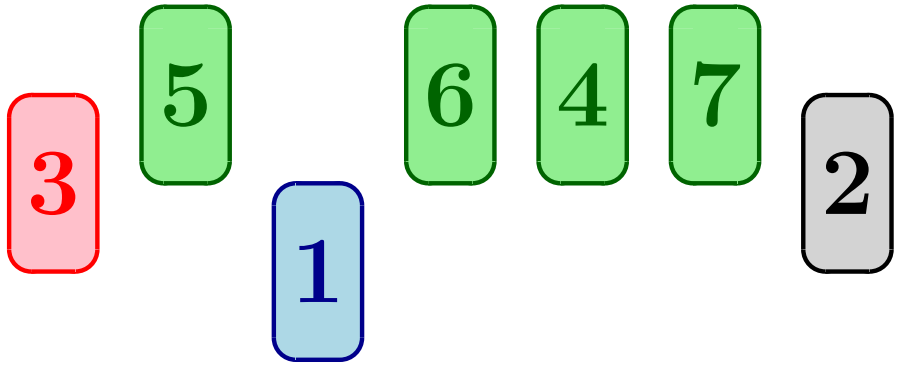

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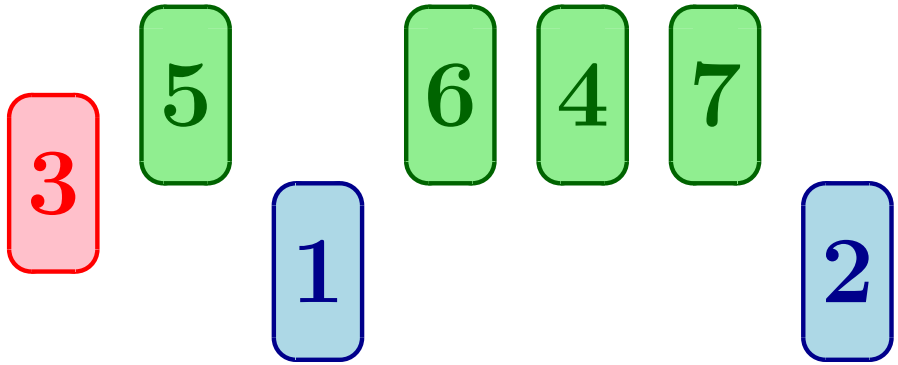

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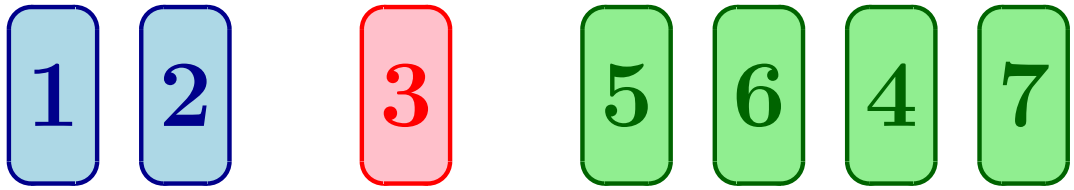

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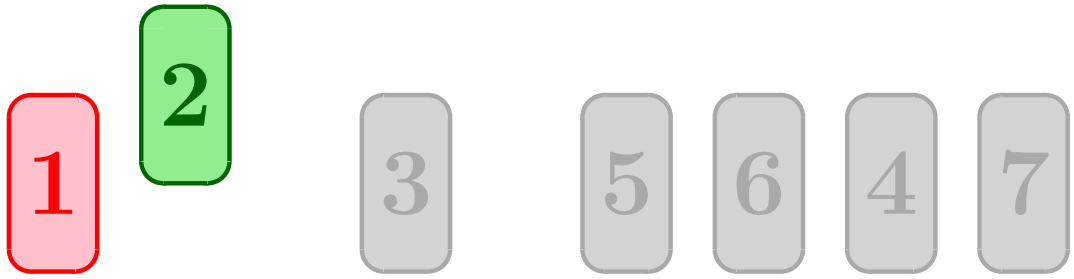

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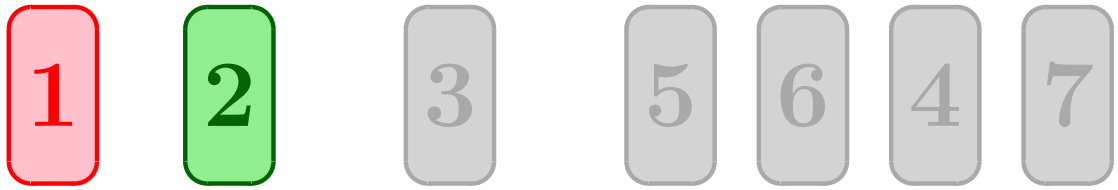

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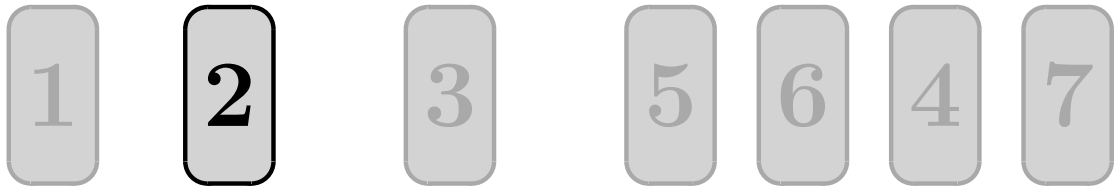

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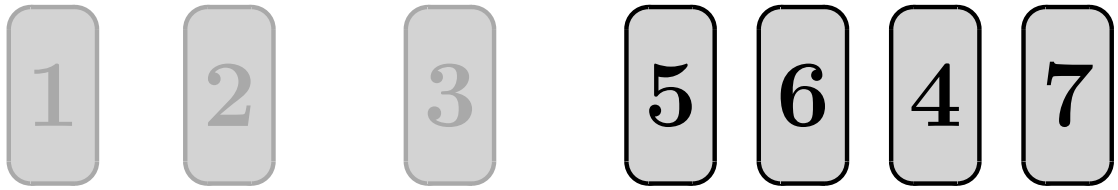

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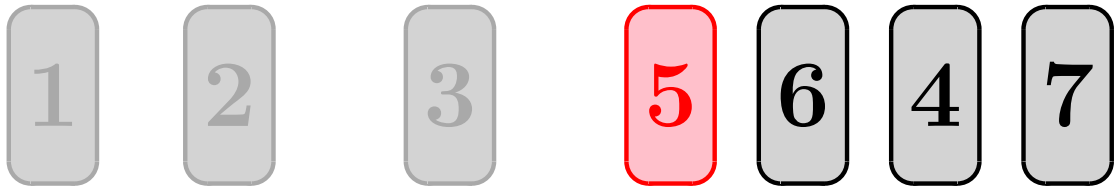

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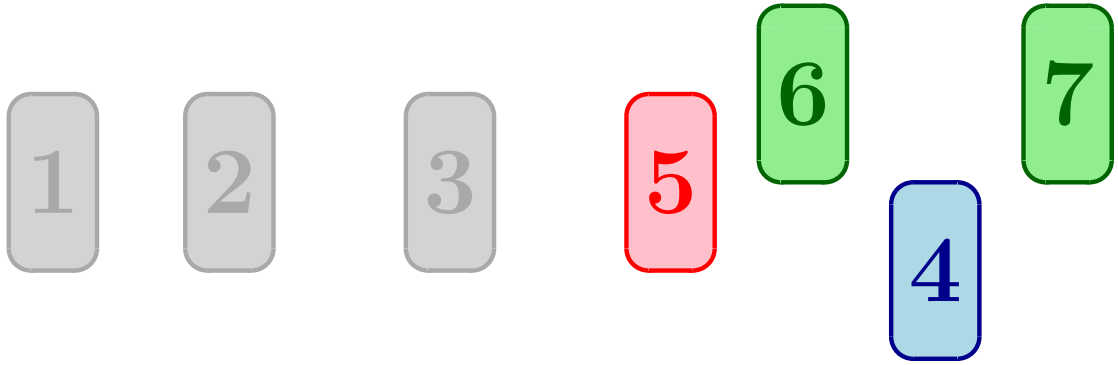

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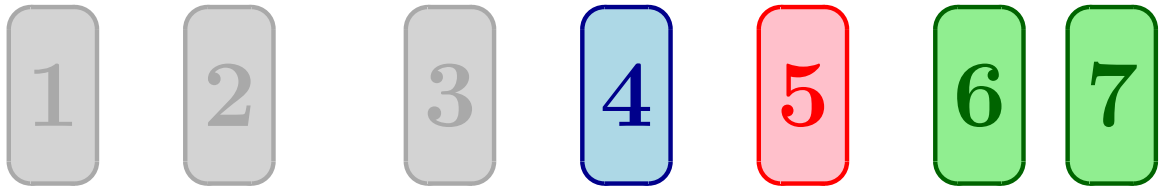

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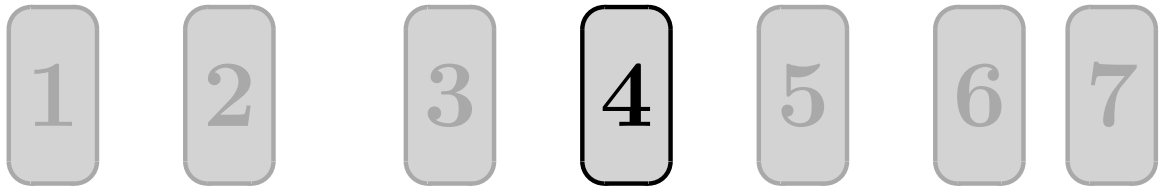

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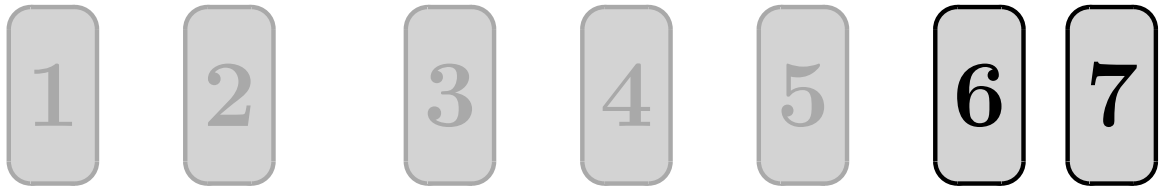

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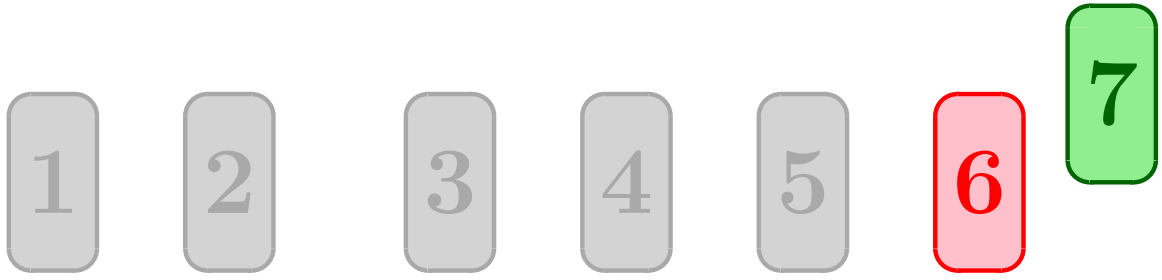

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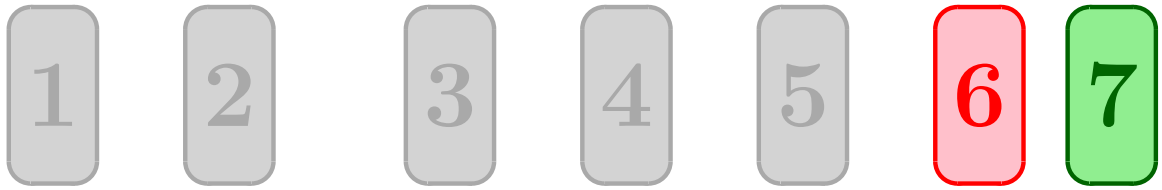

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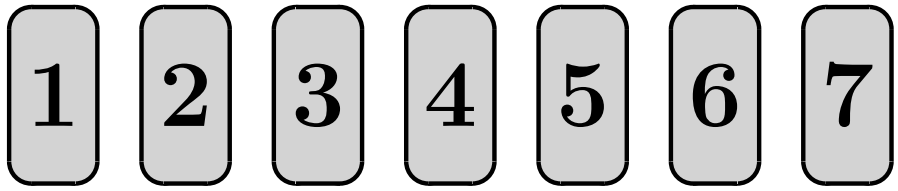

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Complexité de `tri_rapide` au pire : $\Theta(n^2)$ comparaisons

Tri rapide (*Quicksort*), version 1

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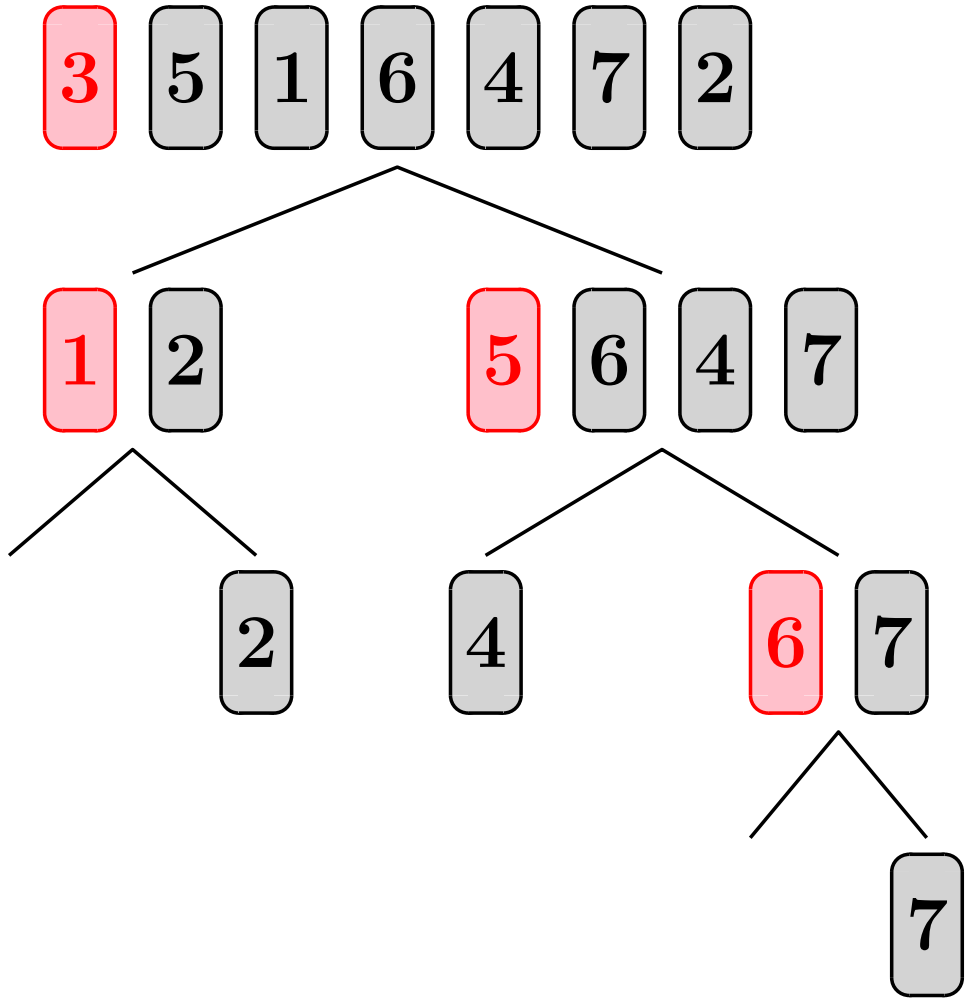

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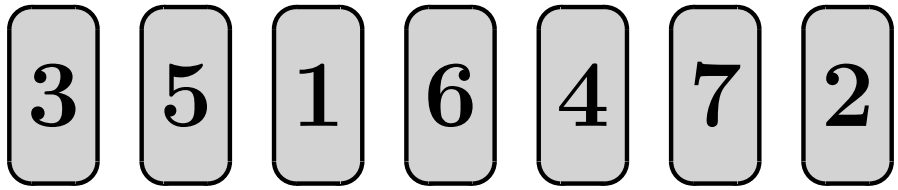

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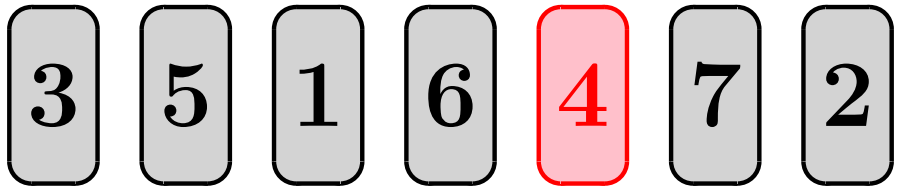

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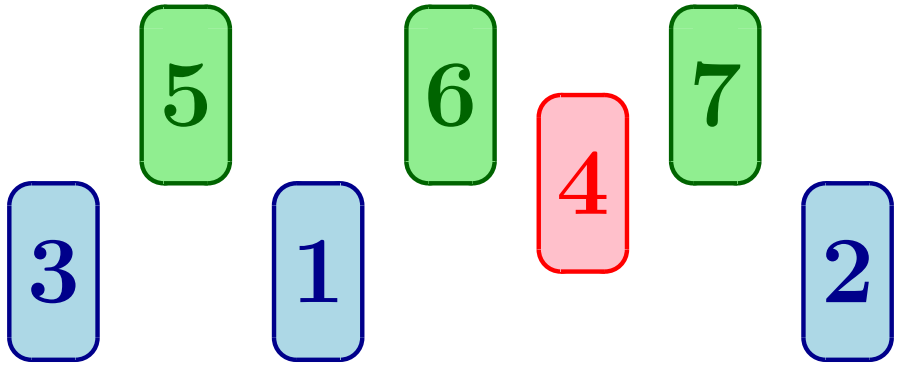

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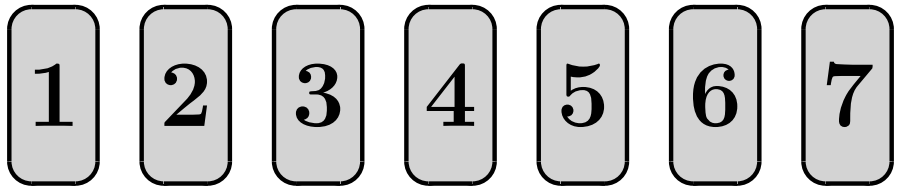

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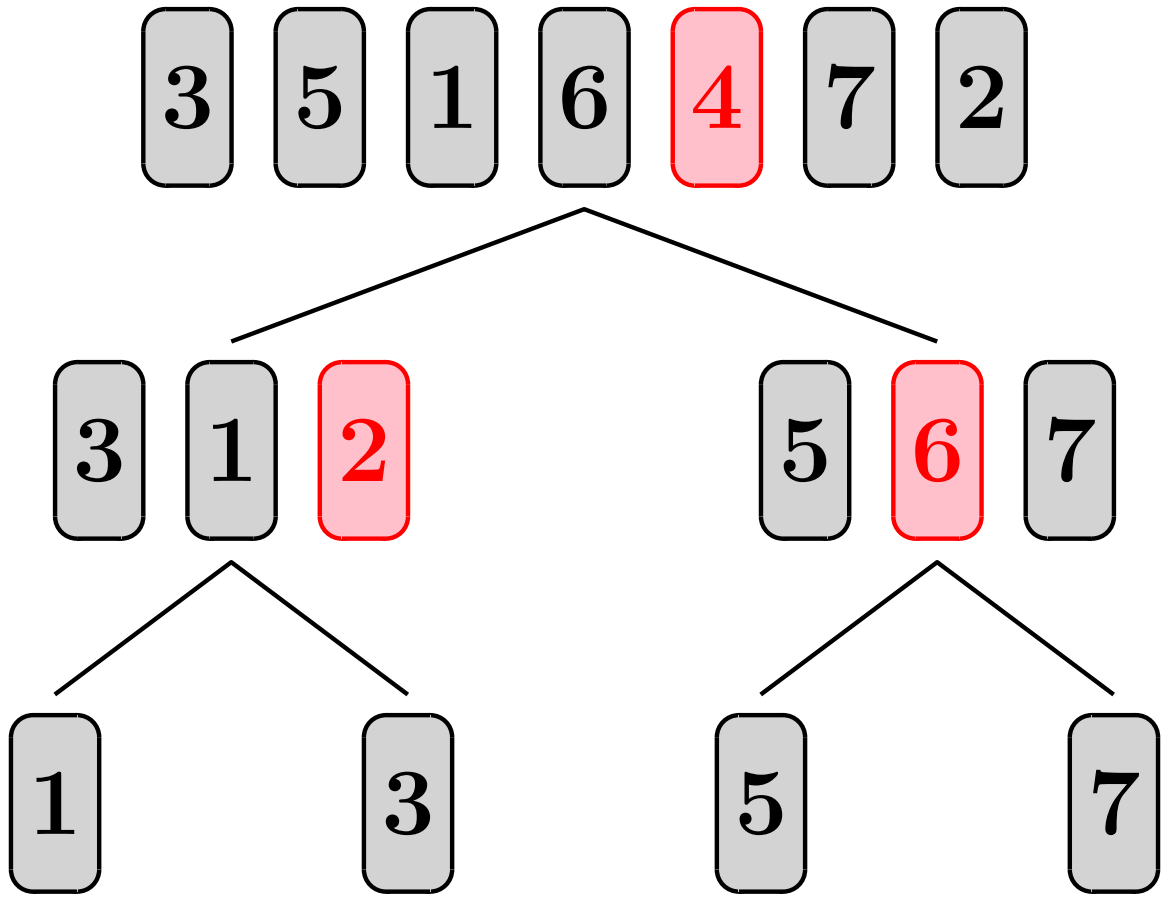


Tri rapide (*Quicksort*), version 1

Complexité de `tri_rapide` au pire : $\Theta(n^2)$ comparaisons

Complexité de `tri_rapide` dans le meilleur des cas : $\Theta(n \log n)$ comparaisons

Complexité de `tri_rapide` en moyenne (*admis*) : $\Theta(n \log n)$ comparaisons

Tri rapide (*Quicksort*), version 1

Inconvénients

- `partition` fait deux parcours, là où un seul suffit manifestement
- ne trie pas en place – même les éléments « bien placés » sont déplacés
- les mauvais cas sont des cas « assez probables » : tableaux triés ou presque, à l'endroit ou à l'envers

Tri rapide (*Quicksort*), version 2

```
def tri_rapide(T, debut, fin) : # trie T[debut:fin]
  if fin - debut < 2 : return
  indice_pivot = partition(T, debut, fin)
  tri_rapide(T, debut, indice_pivot)
  tri_rapide(T, indice_pivot + 1, fin)
```

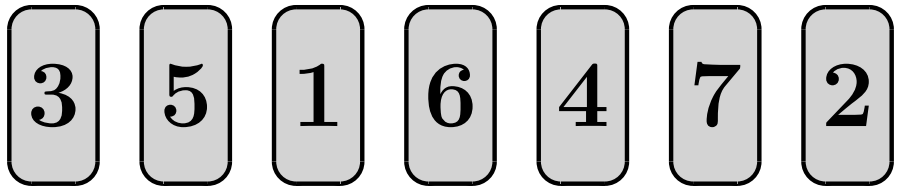
Tri rapide (*Quicksort*), version 2

```
def tri_rapide(T, debut, fin) : # trie T[debut:fin]
  if fin - debut < 2 : return
  indice_pivot = partition(T, debut, fin)
  tri_rapide(T, debut, indice_pivot)
  tri_rapide(T, indice_pivot + 1, fin)

def partition(T, debut, fin) :
  pivot, gauche, droite = T[debut], debut + 1, fin - 1
  while gauche < droite :
    while T[gauche] < pivot : gauche += 1
    while T[droite] > pivot : droite -= 1
    if gauche < droite :
      T[gauche], T[droite] = T[droite], T[gauche]
      gauche, droite = gauche + 1, droite - 1
  T[debut], T[droite] = T[droite], pivot
  return droite
```

Tri rapide (*Quicksort*), version 2

Exemple :



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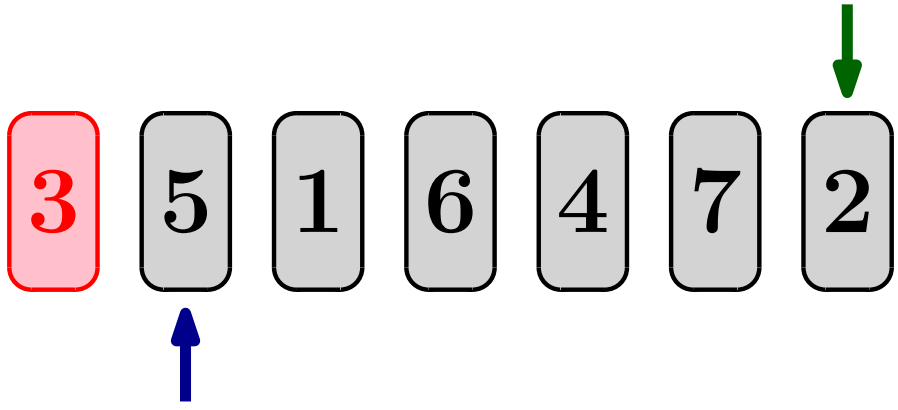

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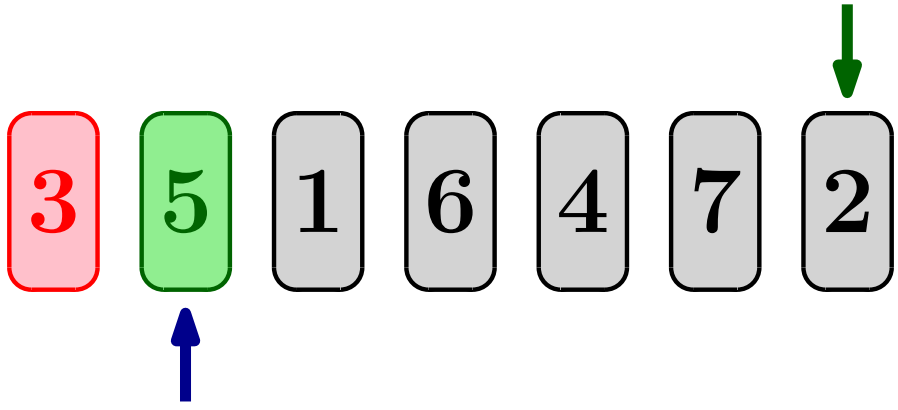

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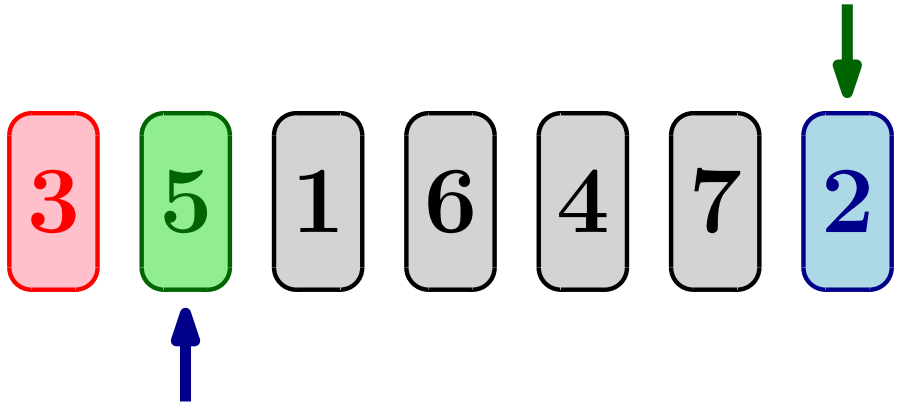

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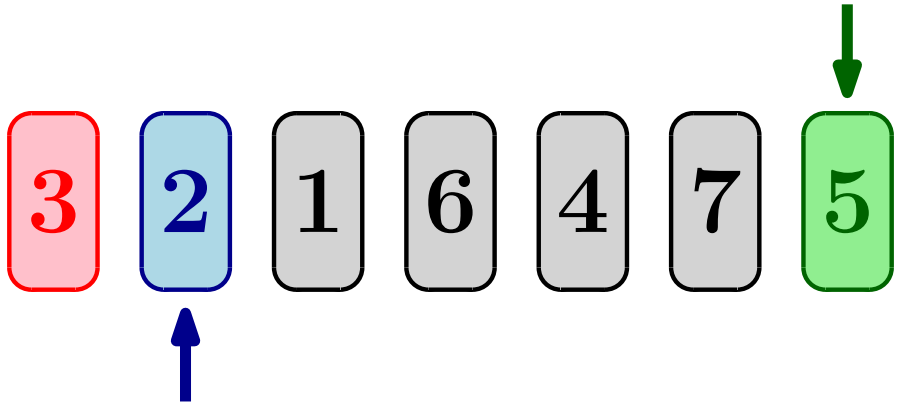

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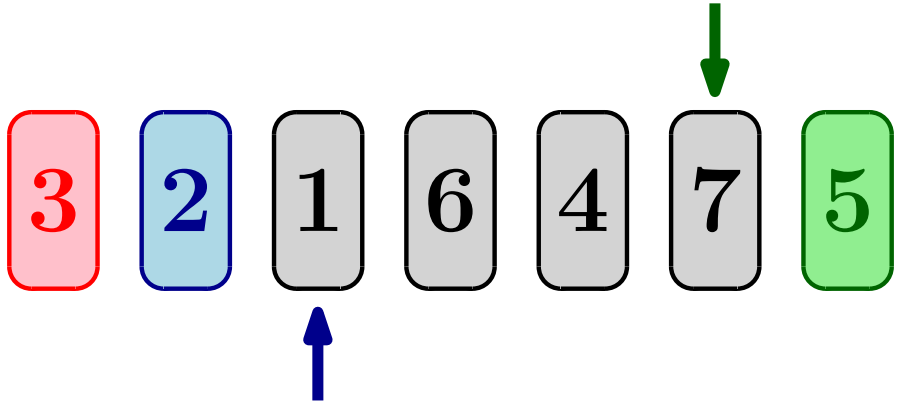
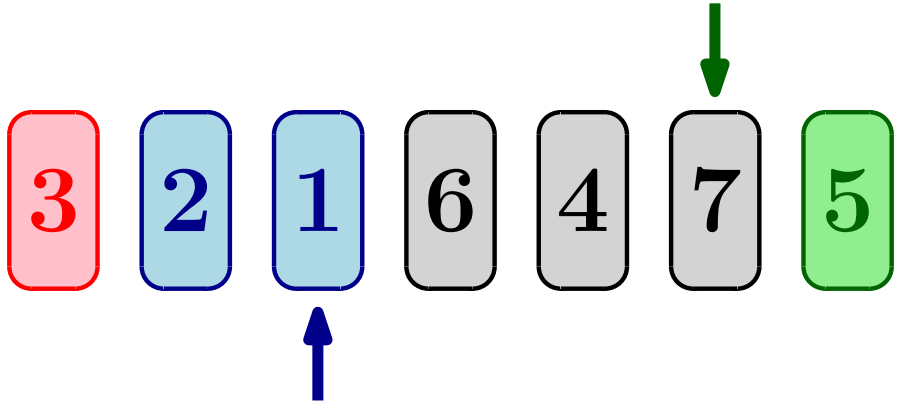


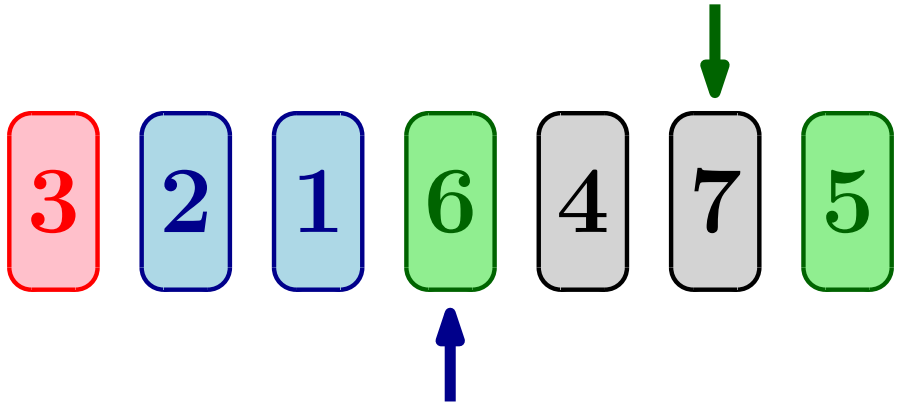
Tri rapide (*Quicksort*), version 2

Exemple :



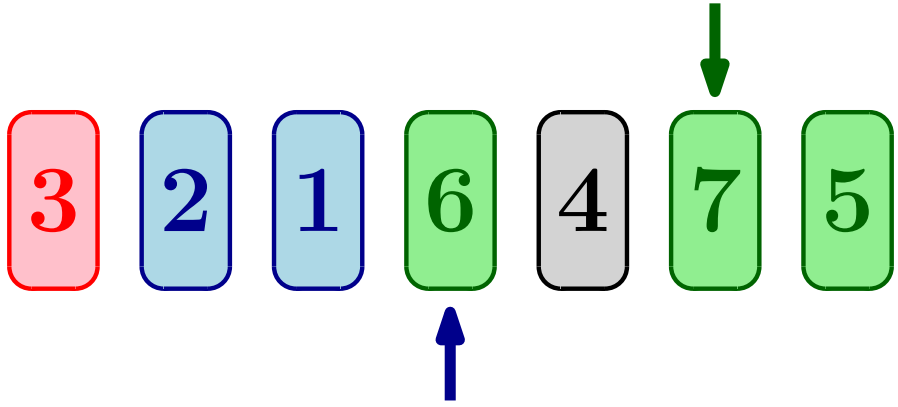
Tri rapide (*Quicksort*), version 2

Exemple :



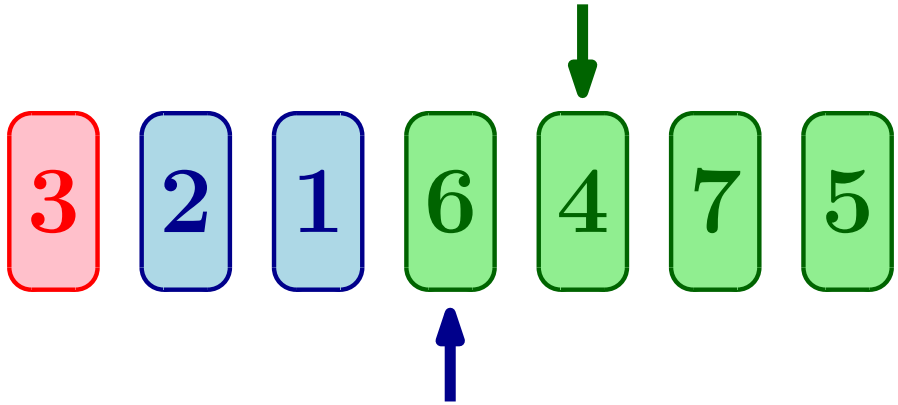
Tri rapide (*Quicksort*), version 2

Exemple :



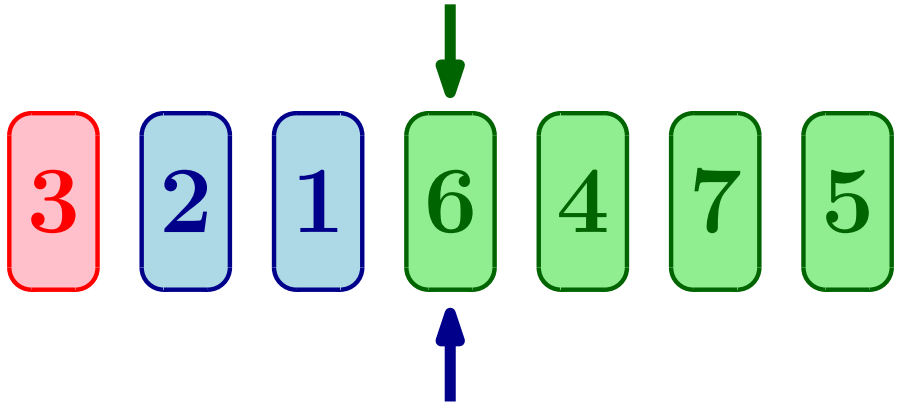
Tri rapide (*Quicksort*), version 2

Exemple :



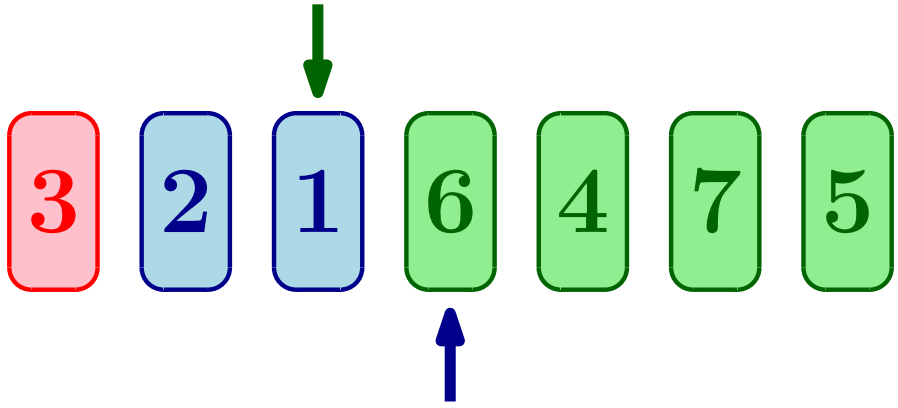
Tri rapide (*Quicksort*), version 2

Exemple :



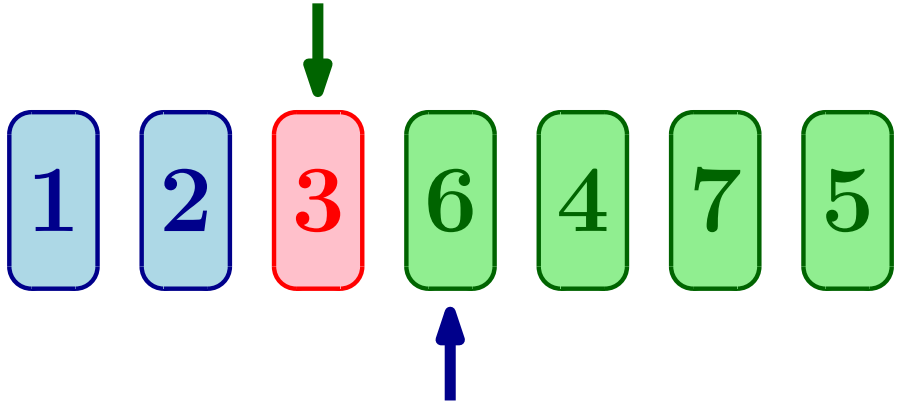
Tri rapide (*Quicksort*), version 2

Exemple :



Tri rapide (*Quicksort*), version 2

Exemple :



Tri rapide (*Quicksort*), version 2

Exemple :

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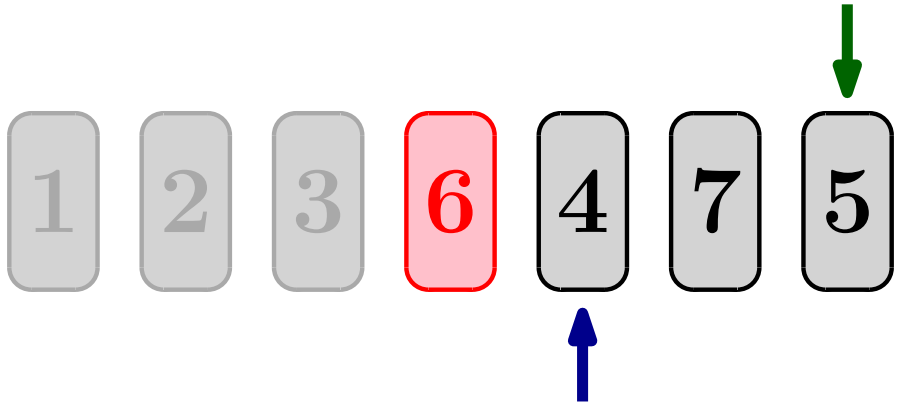
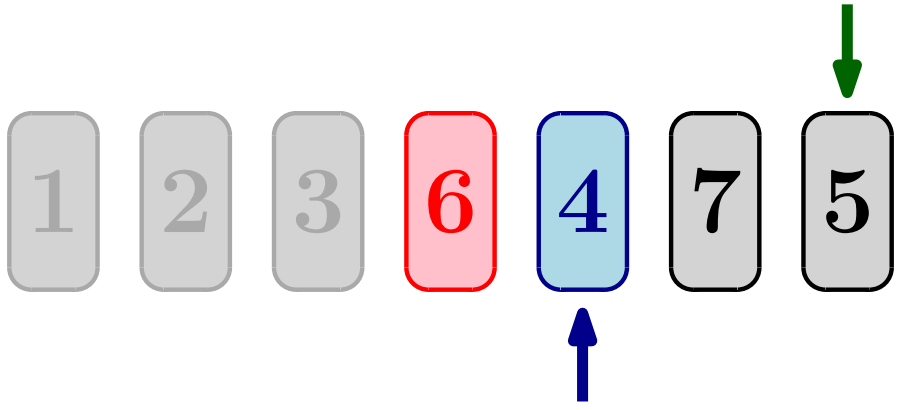


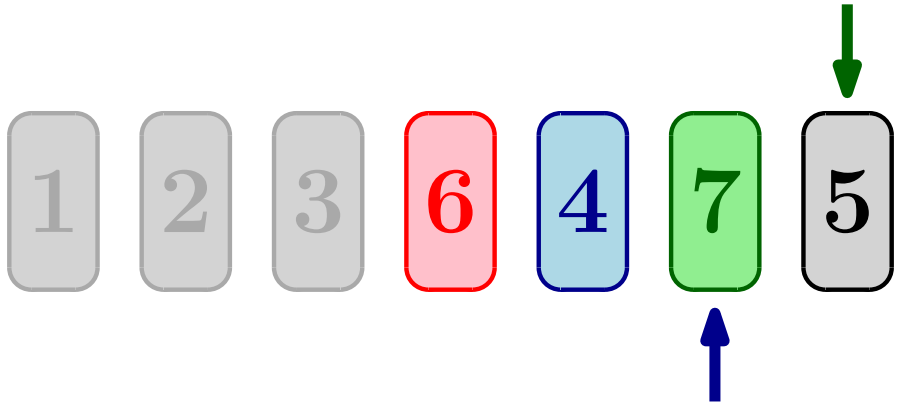
Tri rapide (*Quicksort*), version 2

Exemple :



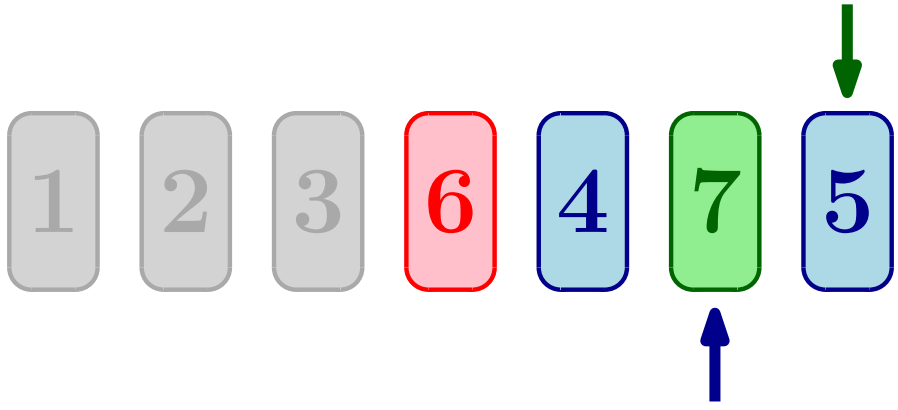
Tri rapide (*Quicksort*), version 2

Exemple :



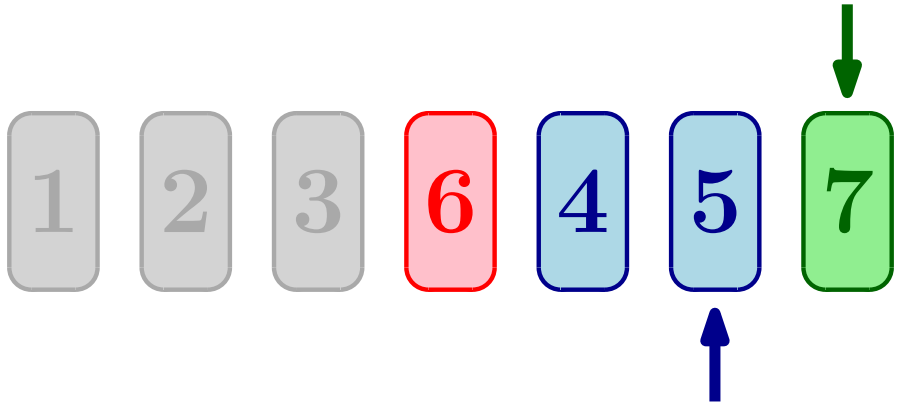
Tri rapide (*Quicksort*), version 2

Exemple :



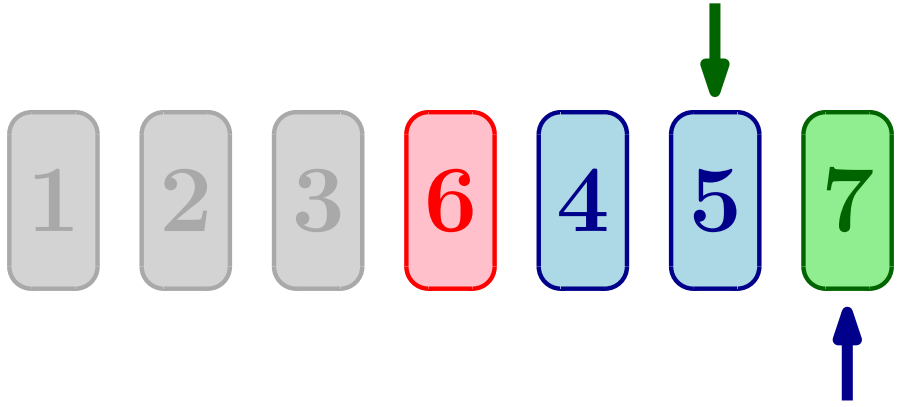
Tri rapide (*Quicksort*), version 2

Exemple :



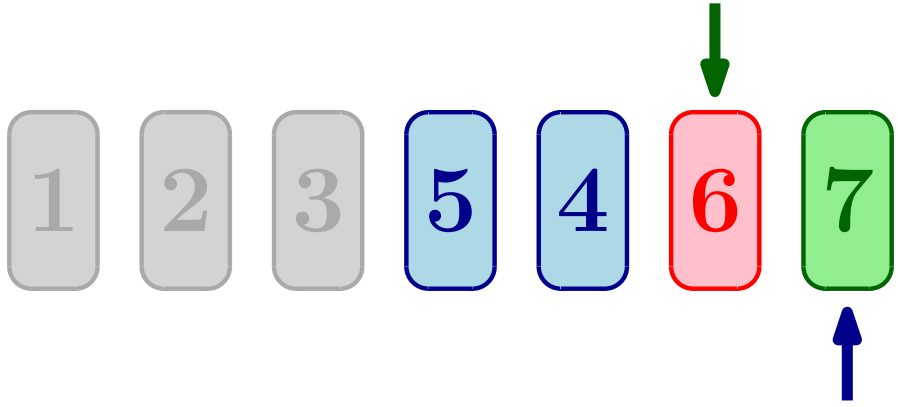
Tri rapide (*Quicksort*), version 2

Exemple :



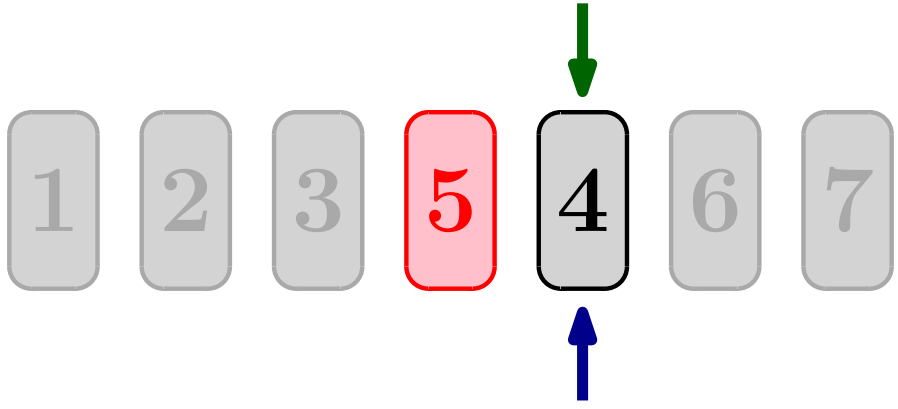
Tri rapide (*Quicksort*), version 2

Exemple :



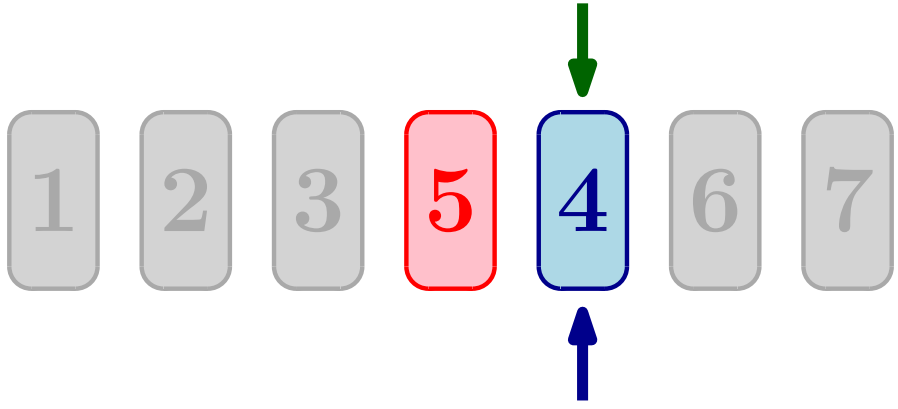
Tri rapide (*Quicksort*), version 2

Exemple :



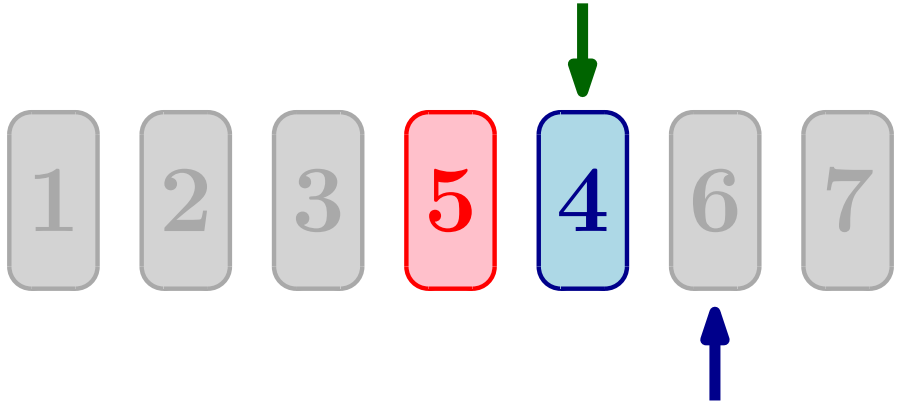
Tri rapide (*Quicksort*), version 2

Exemple :



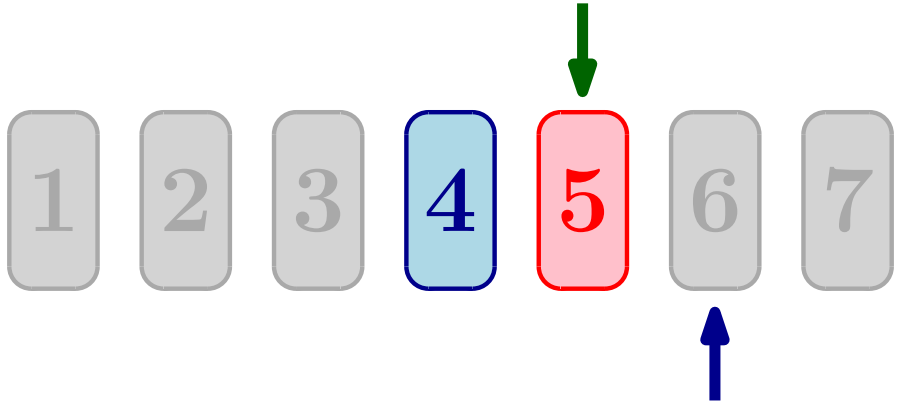
Tri rapide (*Quicksort*), version 2

Exemple :



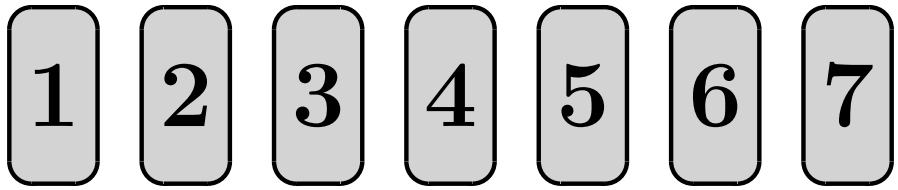
Tri rapide (*Quicksort*), version 2

Exemple :



Tri rapide (*Quicksort*), version 2

Exemple :



Tri rapide, version *randomisée*

Il reste le cas problématique des tableaux (presque) triés

```
def partition(T, debut, fin) :
  alea = random.randint(debut, fin - 1)
  T[debut], T[alea] = T[alea], T[debut]
  pivot, gauche, droite = T[debut], debut + 1, fin - 1
  while gauche < droite :
    while T[gauche] < pivot : gauche += 1
    while T[droite] > pivot : droite -= 1
    if gauche < droite :
      T[gauche], T[droite] = T[droite], T[gauche]
      gauche, droite = gauche + 1, droite - 1
  T[debut], T[droite] = T[droite], pivot
  return droite
```