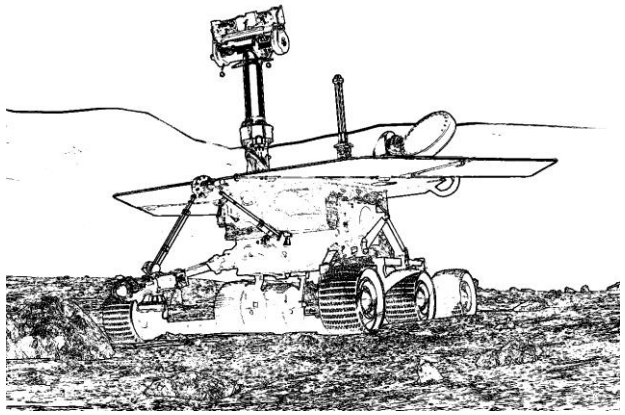


CSC477
Introduction to Mobile Robotics

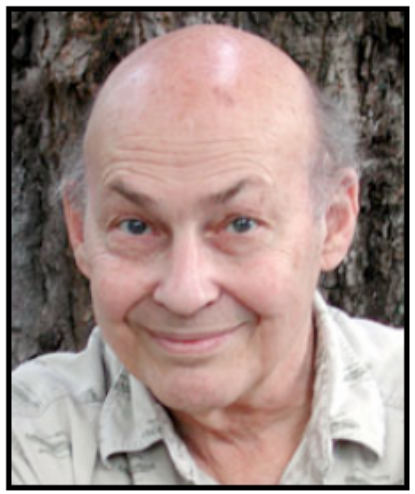
Florian Shkurti

Weeks #11 & #12: Multiview Geometry, Visual Odometry, Visual SLAM

Motivation

- We have already seen quite successful SLAM methods based on laser sensors. Why bother with vision?
  - Camera technology cheap and ubiquitous
  - Camera is a passive sensor, lower energy
  - Some environments/platforms can’t support laser
  - Vision is quite a “rich” source of information

How hard is computer vision?



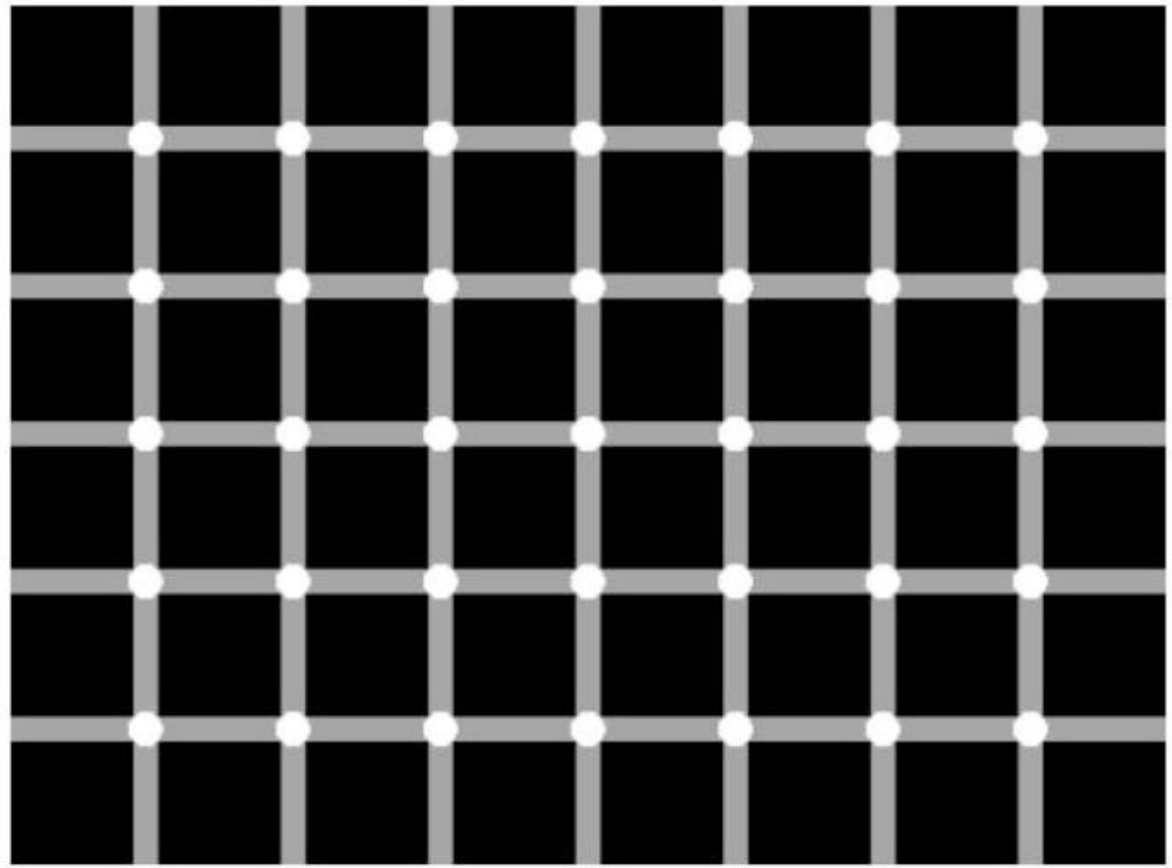
Marvin Minsky, MIT
Turing award,1969

"In 1966, Minsky hired a first-year undergraduate (JS) student and assigned him a problem to solve over the summer: connect a television camera to a computer and get the machine to describe what it sees."

Crevier 1993, pg. 88

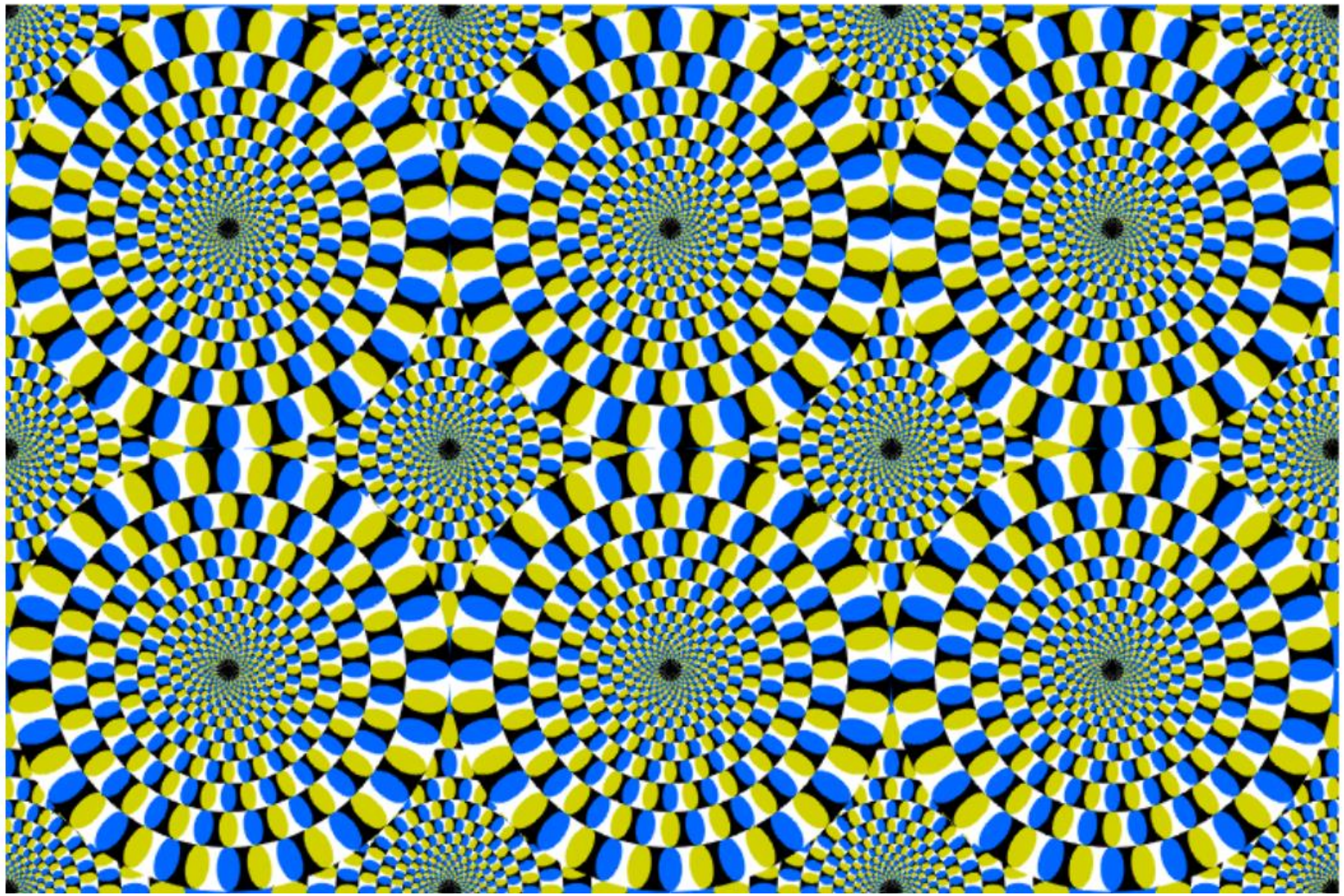
Depth perception can be ambiguous from just a single image

What do humans see?



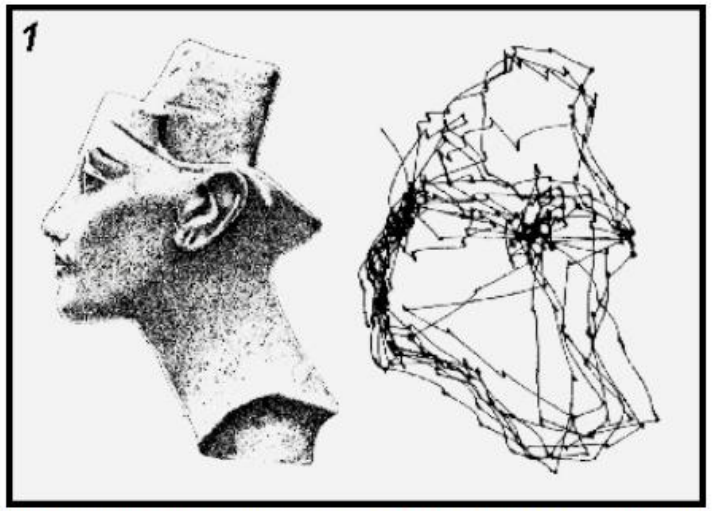
Count the black dots! :o)

Peripheral drift illusion

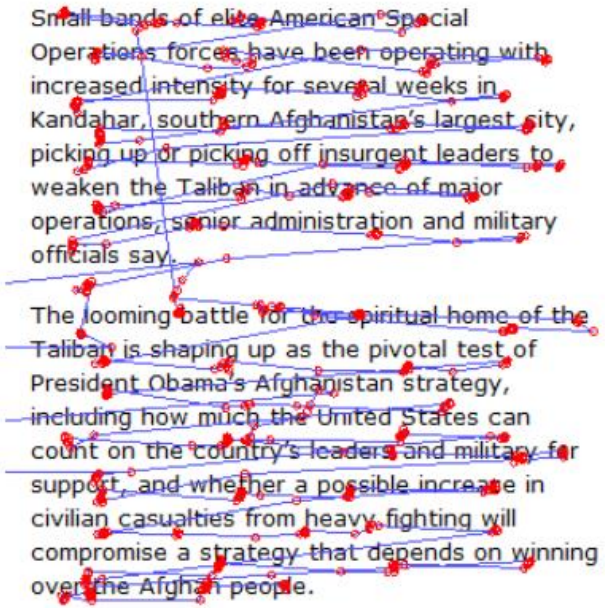


Where do humans fixate?

Top down or bottom up?



"Eye Movements and Vision" by A. L. Yarbus; Plenum Press, New York; 1967



Visual saccades

Camera obscura: dark room

- Known during classical period in China and Greece (e.g., Mo-Ti, China, 470BC to 390BC)

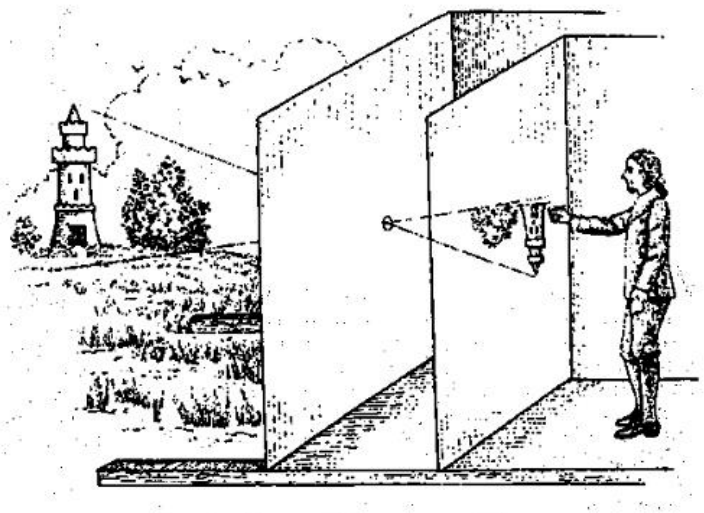
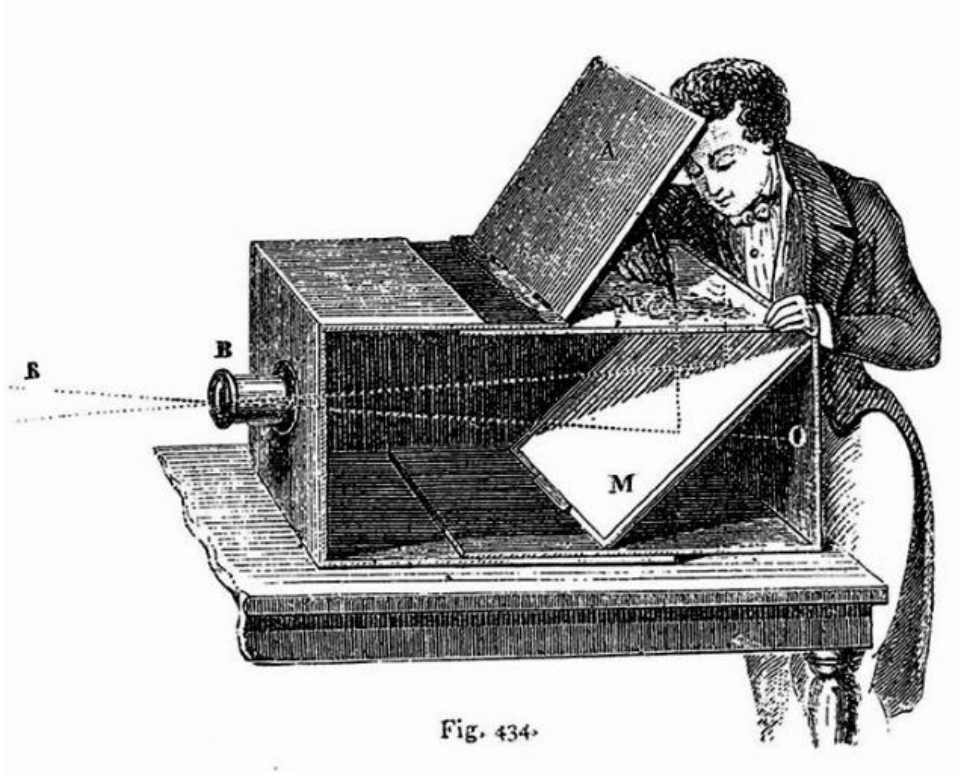


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys



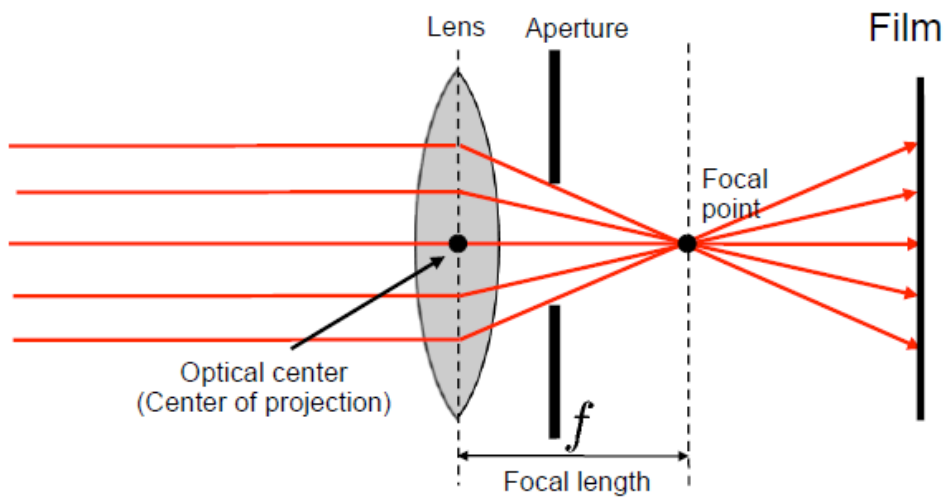
Lens Based Camera Obscura, 1568

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

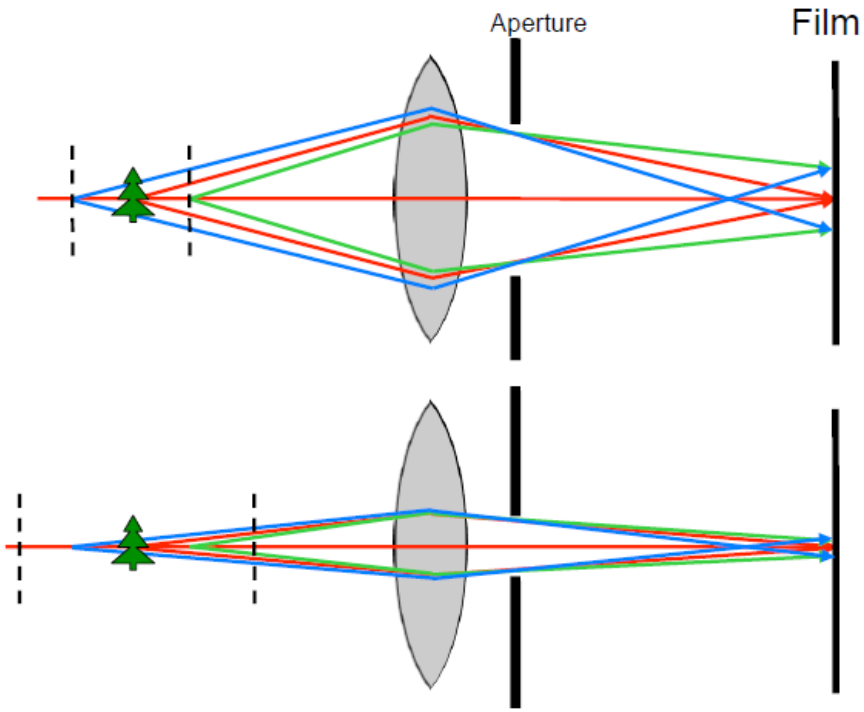
Lenses



A lens focuses parallel rays onto a single focal point

- focal point at a distance *f* beyond the plane of the lens
  - *f* is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
  - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)

Depth of field



f / 5.6

f / 32

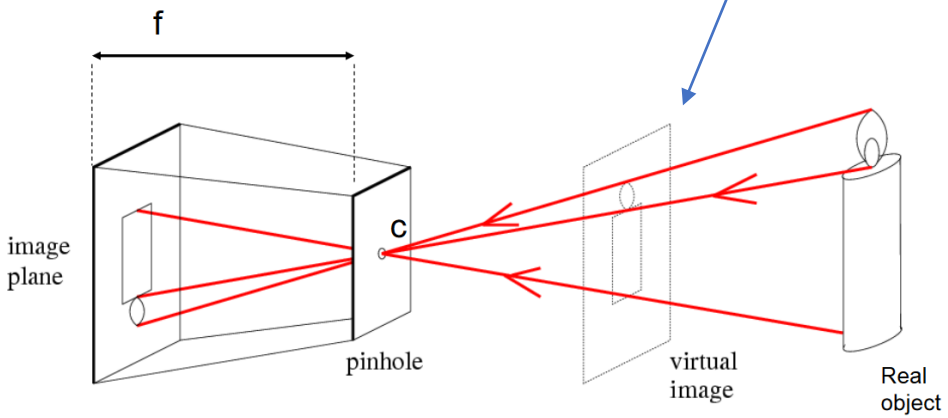
Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

To avoid thinking
about image inversion

Pinhole camera model

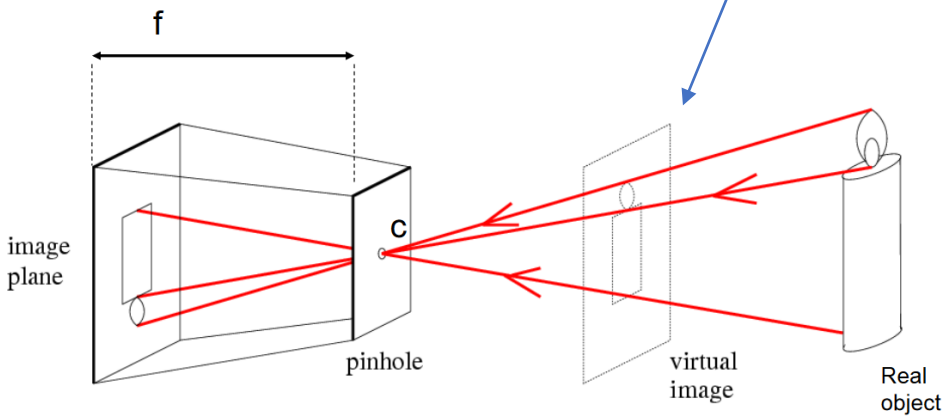


f = Focal length
c = Optical center of the camera

Point aperture → nearly every pixel in the image is in focus

To avoid thinking about image inversion

Pinhole camera model

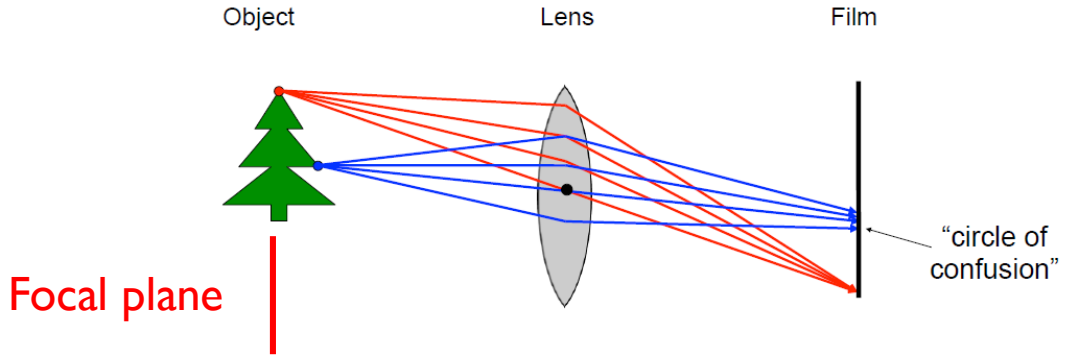


f = Focal length
c = Optical center of the camera

Point aperture → nearly every pixel in the image is in focus → almost infinite depth of field

Adding a lens

Some times called the thin-lens model

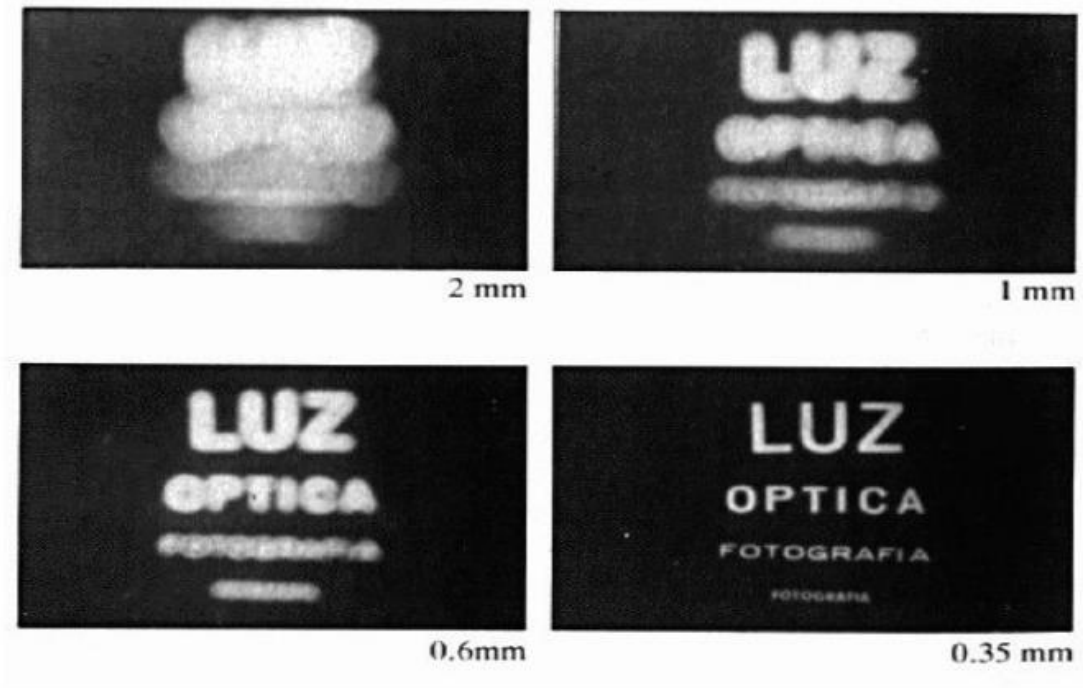


A lens focuses light onto the film

- There is a specific distance at which objects are "in focus"
  - other points project to a "circle of confusion" in the image
- Changing the shape of the lens changes this distance

Aperture of nonzero diameter → only pixels corresponding to objects on the focal plane are in focus → narrow depth of field

Shrinking the aperture

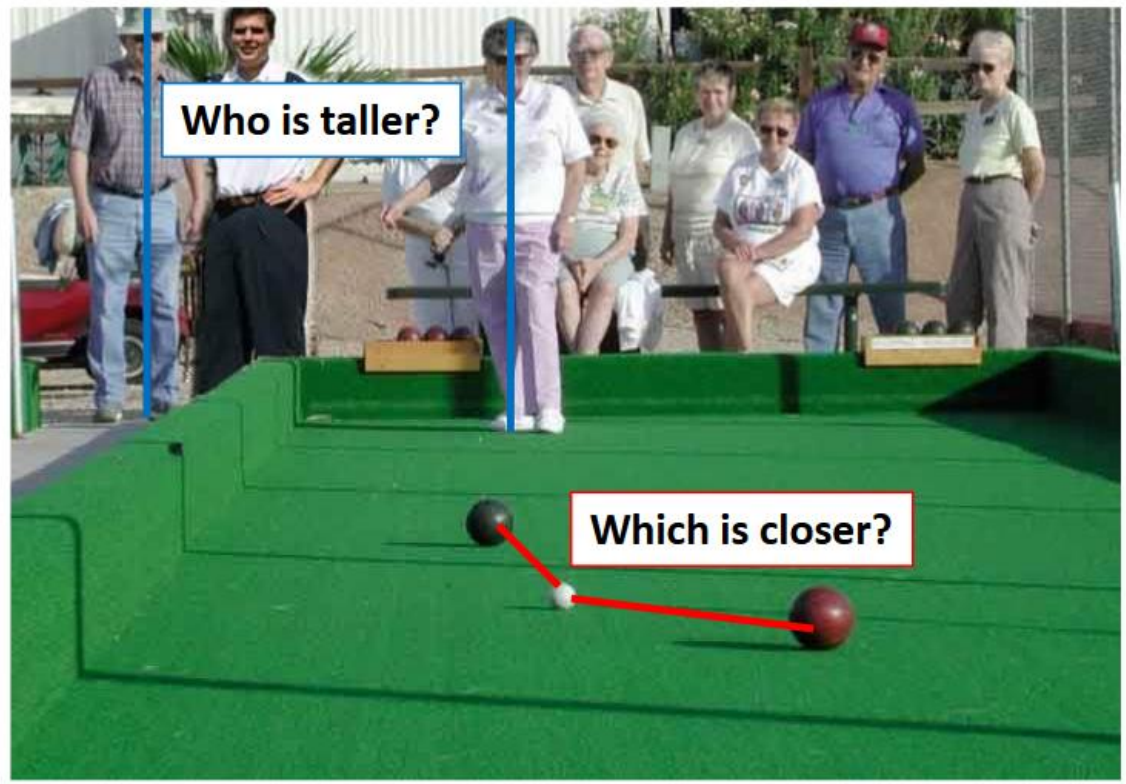


Why not make the aperture as small as possible?

- Less light gets through
- *Diffraction* effects...

Projective Geometry

Length (and so area) is lost.



Length and area are not preserved

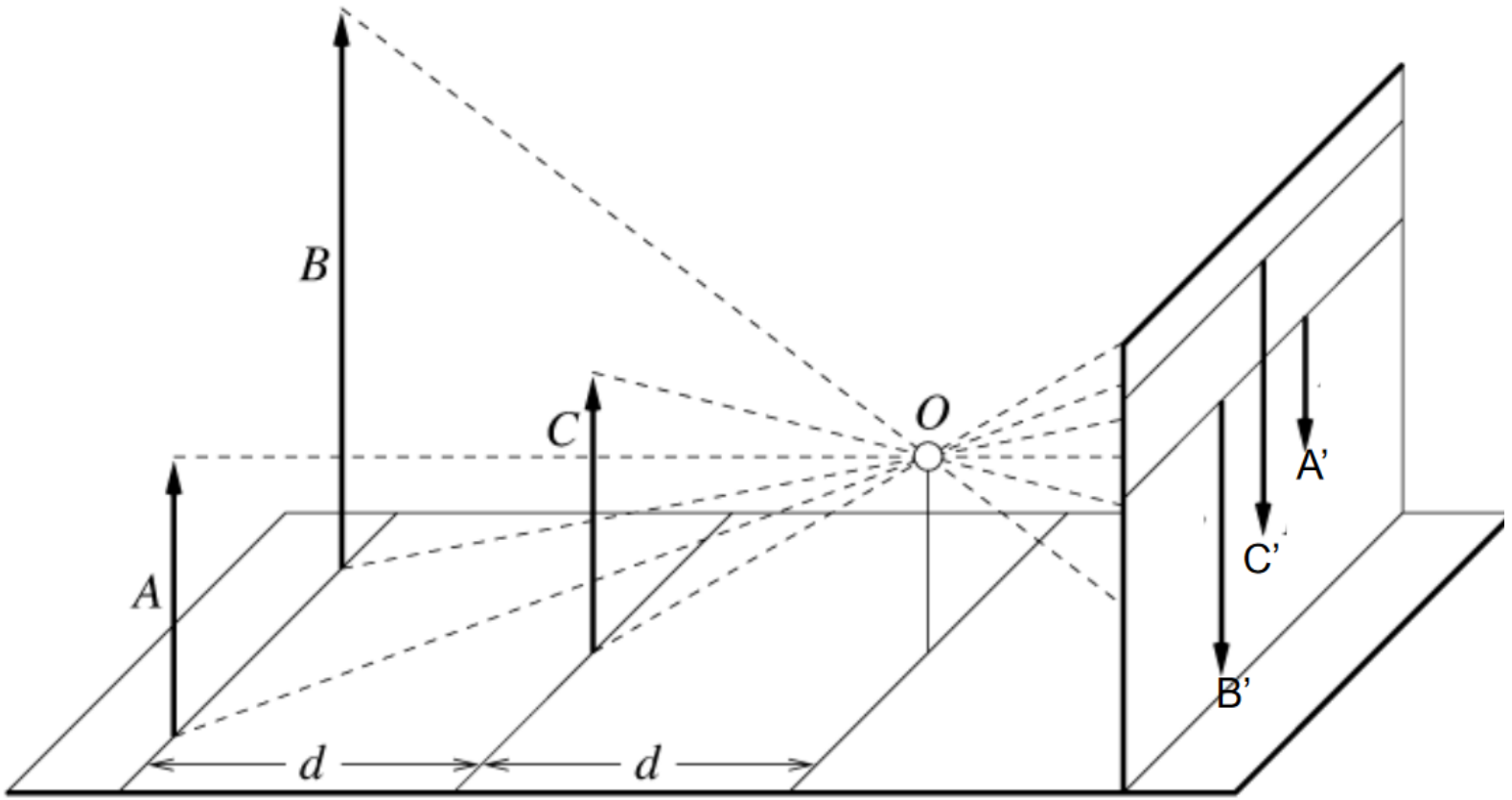
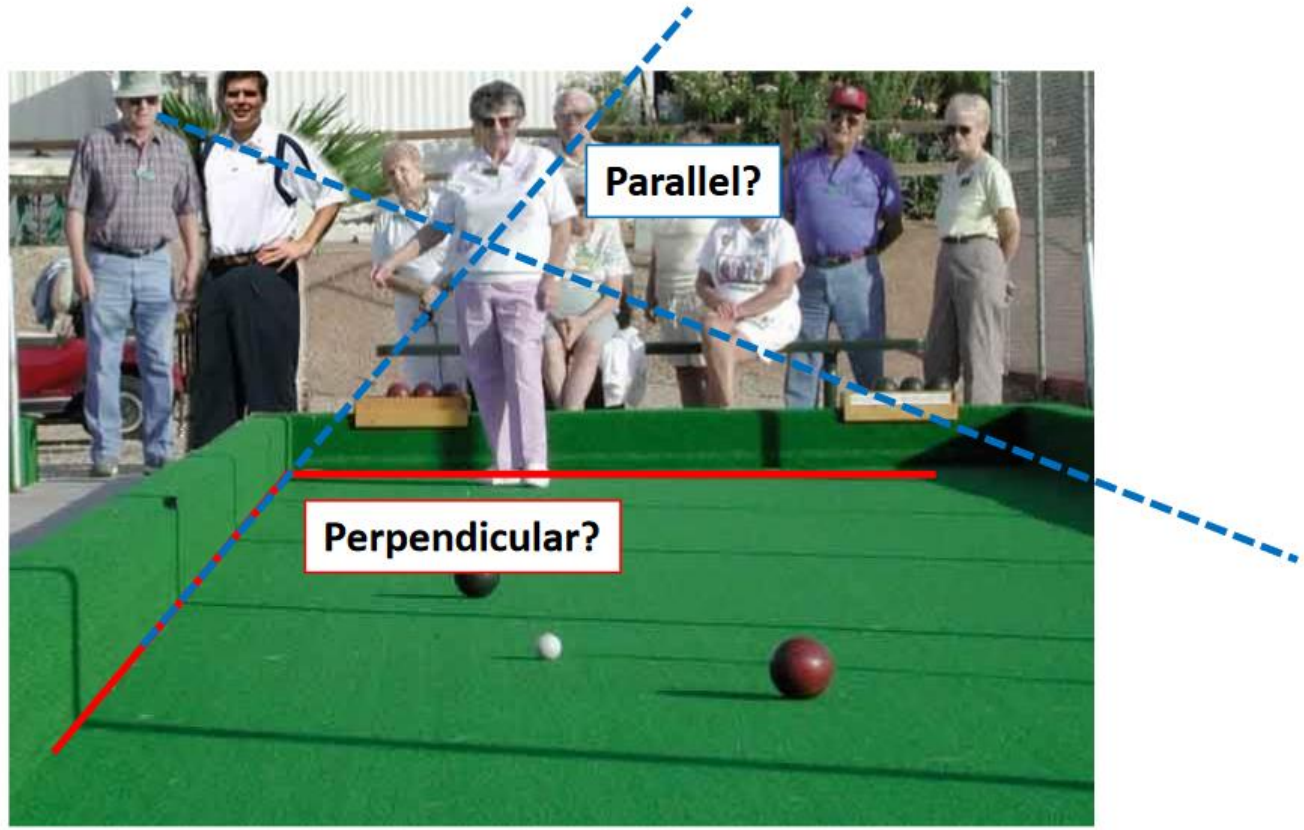


Figure by David Forsyth

Projective Geometry

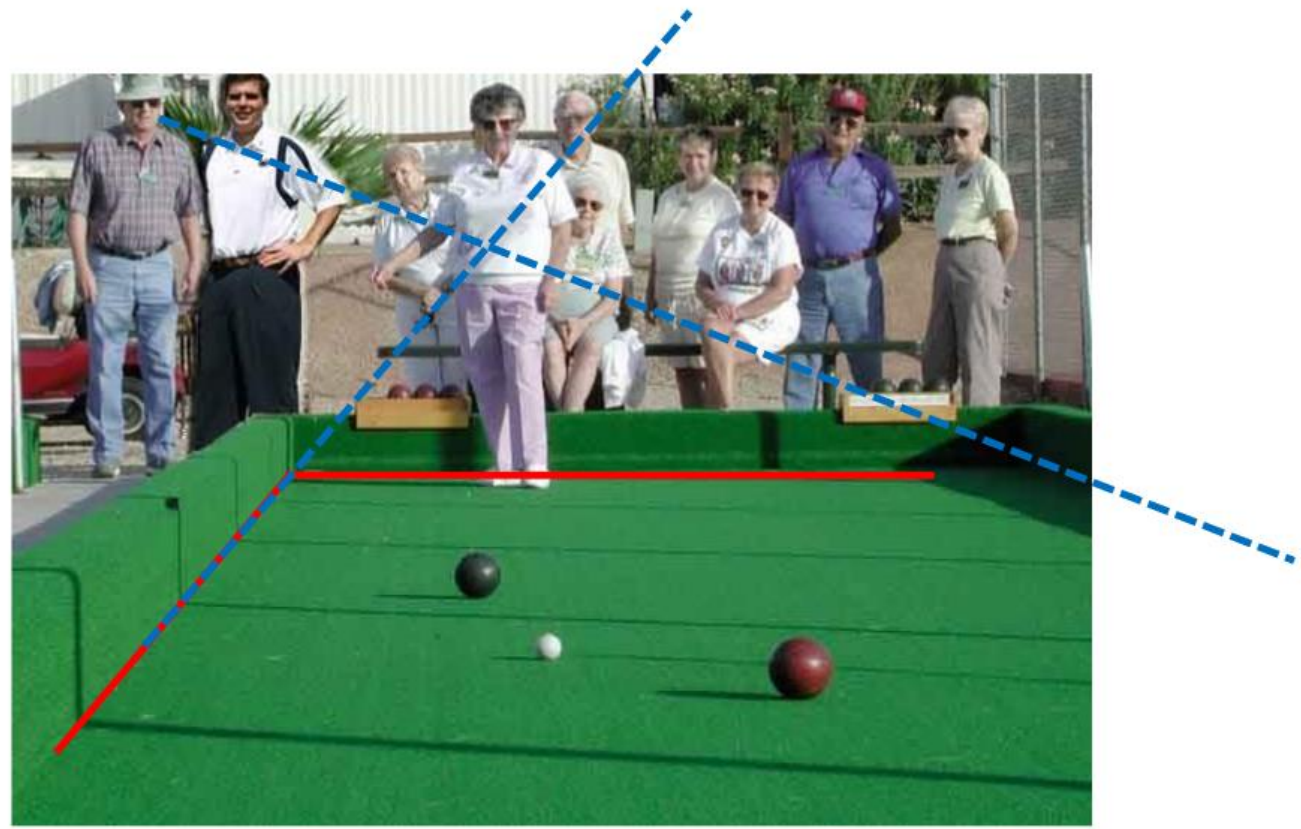
Angles are lost.



Projective Geometry

What is preserved?

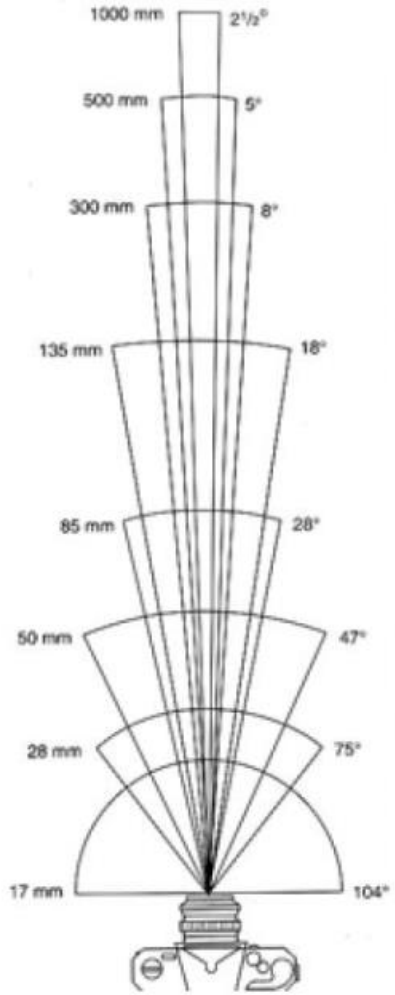
- Straight lines are still straight.



Chromatic aberration

Failure of a lens to focus all colors to the same convergence point.
Due to difference wavelengths having different refractive indeces

Field of View (Zoom, focal length)



17mm

28mm

50mm

85mm

From London and Upton

Camera parameters

Focus – Shifts the depth that is in focus.

Focal length – Adjusts the zoom, i.e., wide angle or telephoto lens.

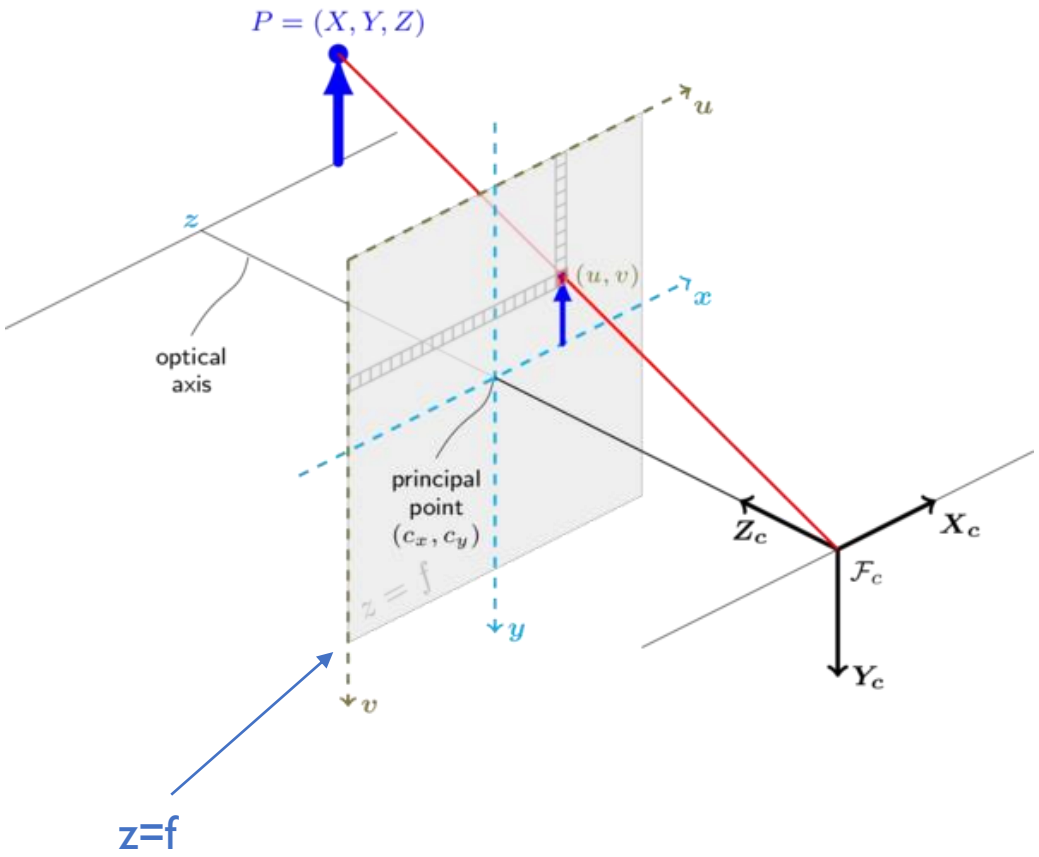
Aperture – Adjusts the depth of field and amount of light let into the sensor.

Exposure time – How long an image is exposed. The longer an image is exposed the more light, but could result in motion blur.

ISO – Adjusts the sensitivity of the “film”. Basically a gain function for digital cameras. Increasing ISO also increases noise.

How do we project 3D points to pixels?
What is the measurement model?

From 3D points to pixels: pinhole camera



(1) Perspective projection $\begin{bmatrix} x \\ y \end{bmatrix} = \pi(X, Y, Z)$

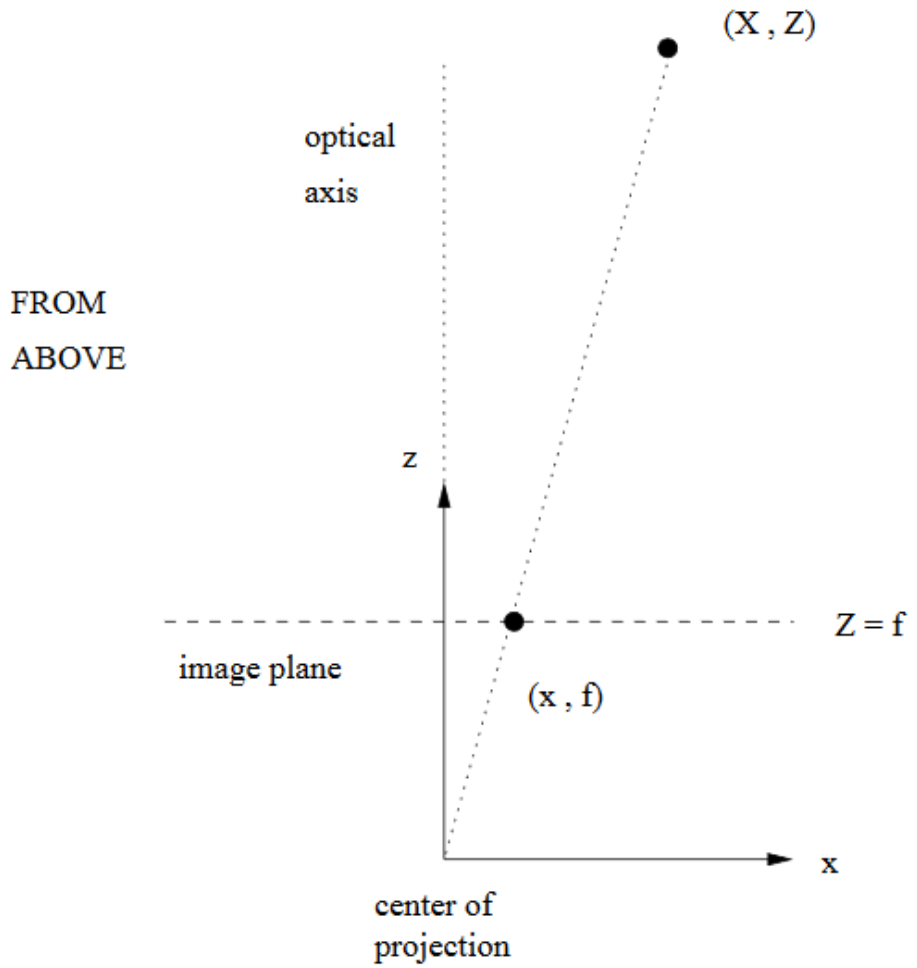
(2) Conversion from metric to pixel coordinates

$$u = m_x x + c_x$$

$$v = m_y y + c_y$$

m_x, m_y represent number of pixels per mm for the two axes

Perspective projection
$[x,y] = \pi(X,Y,Z)$



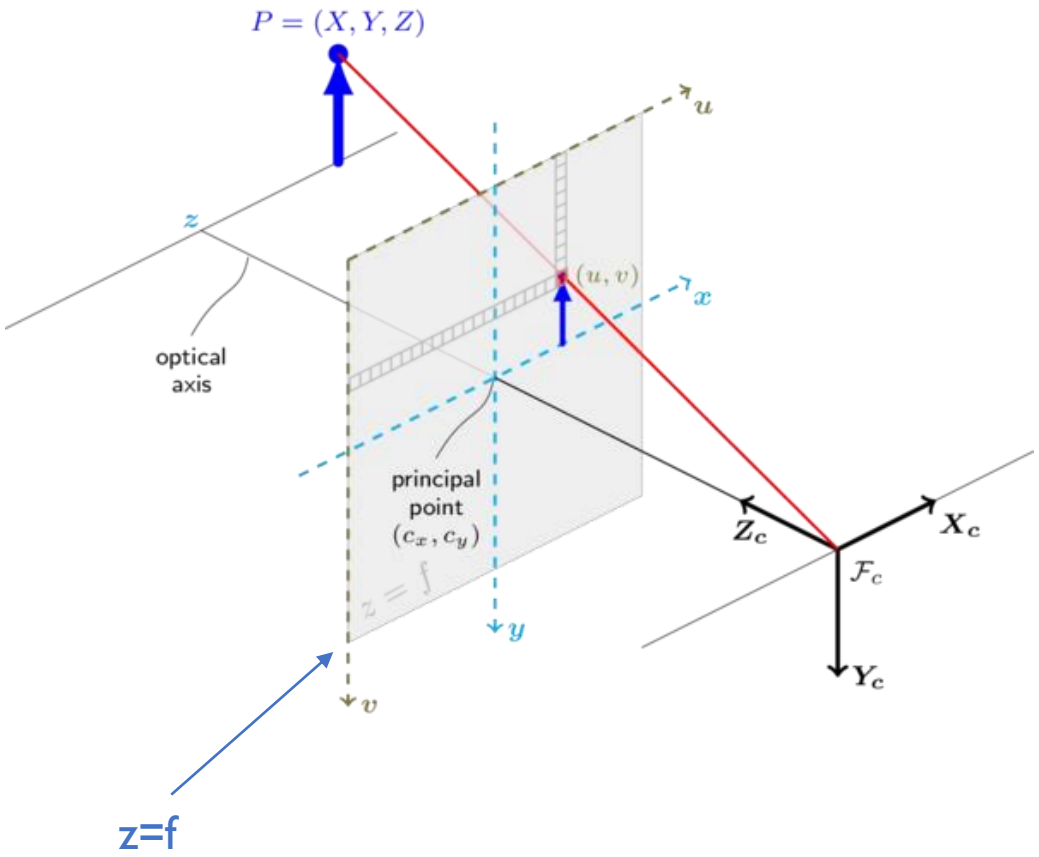
By similar triangles: $x/f = X/Z$

So, $x = f * X/Z$ and similarly $y = f * Y/Z$

Problem: we just lost depth (Z) information by doing this projection, i.e. depth is now uncertain.

http://www.cim.mcgill.ca/%7Elanger/558.html

From 3D points to pixels: pinhole camera



(1) Perspective projection $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \end{bmatrix} = \pi(X, Y, Z)$

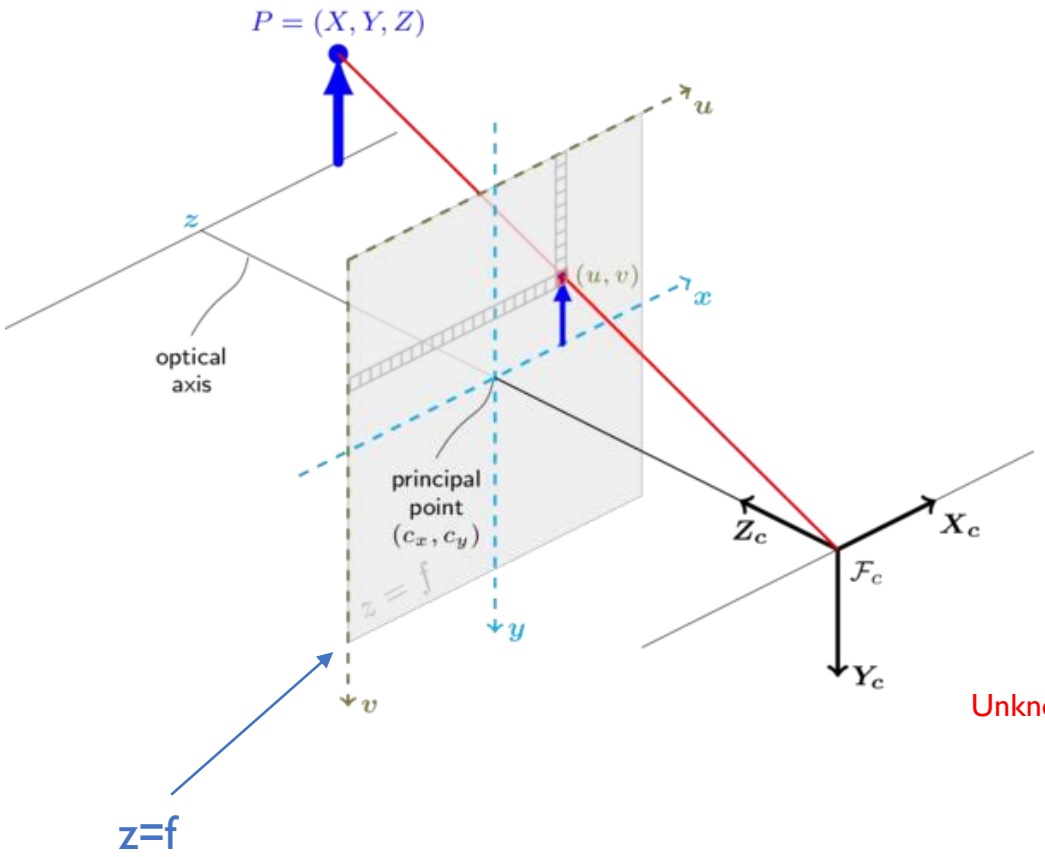
(2) Conversion from metric to pixel coordinates

$$u = m_x x + c_x$$

$$v = m_y y + c_y$$

$$h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + \text{noise in pixels}$$

From 3D points to pixels: pinhole camera



(1) Perspective projection $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \end{bmatrix} = \pi(X, Y, Z)$

(2) Conversion from metric to pixel coordinates

$$u = m_x x + c_x$$

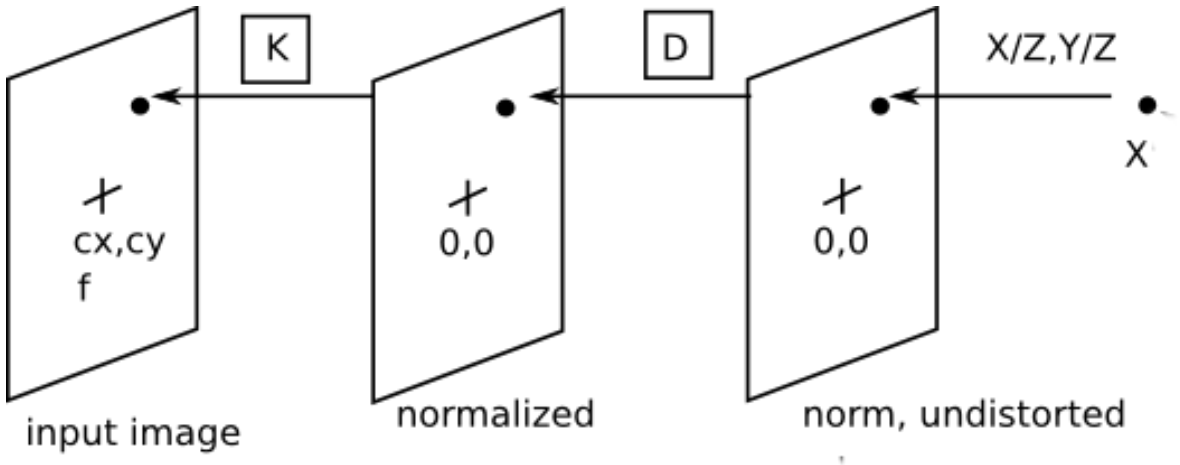
$$v = m_y y + c_y$$

Unknown depth/scale

Camera calibration matrix

Usually presented as $s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} fm_x & 0 & c_x \\ 0 & fm_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

From 3D points to pixels: thin lens camera



(1) Perspective projection $\begin{bmatrix} x \\ y \end{bmatrix} = \pi(X, Y, Z)$

(2) Lens distortion

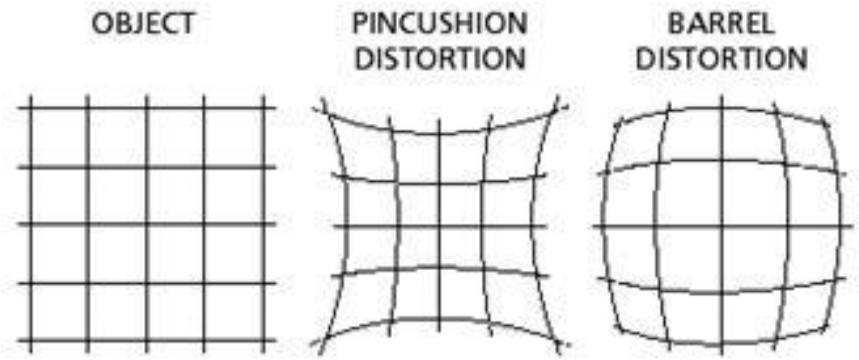
$$[x^*, y^*] = D(x, y)$$

(3) Conversion from metric to pixel coordinates

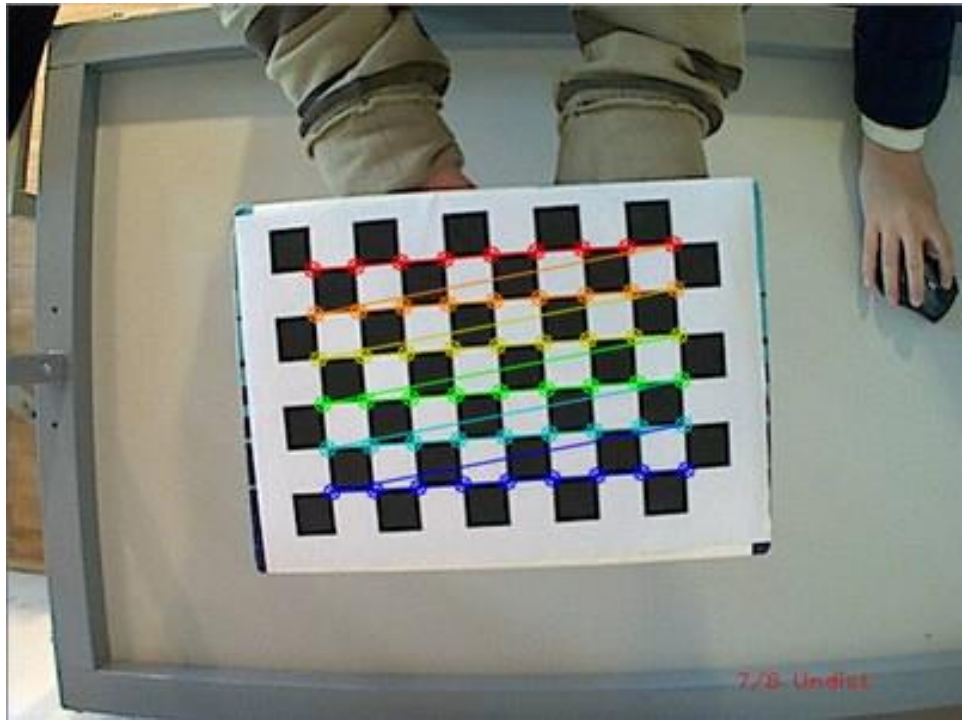
$$u = m_x x^* + c_x$$
$$v = m_y y^* + c_y$$

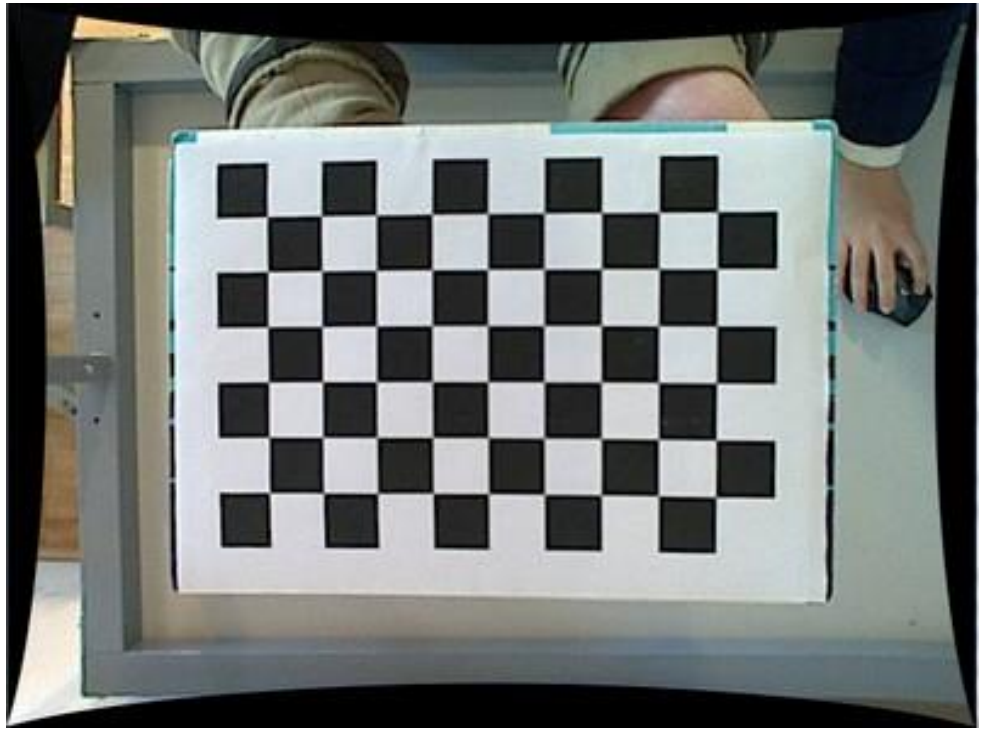
(2) Lens distortion

$[x^*, y^*] = D(x,y)$



(2) Estimating parameters of lens distortion: [x*, y*] = D(x,y)





$$x^* = x\frac{1 + k_1r^2 + k_2r^4 + k_3r^6}{1 + k_4r^2 + k_5r^4 + k_6r^6} \quad \text{where} \quad r = x^2 + y^2$$

$$y^* = y\frac{1 + k_1r^2 + k_2r^4 + k_3r^6}{1 + k_4r^2 + k_5r^4 + k_6r^6} \quad \text{where} \quad r = x^2 + y^2$$

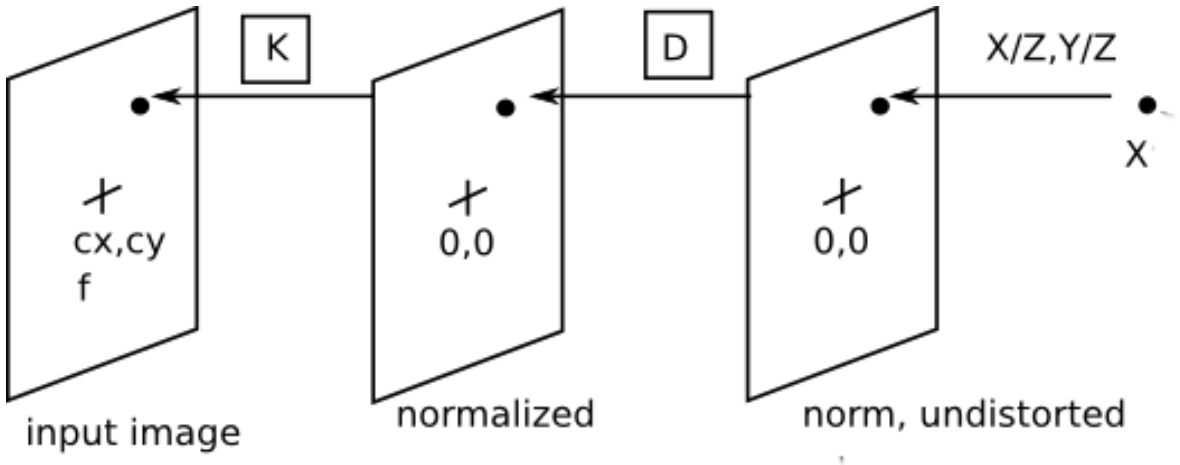
Correcting radial distortion

Barrel distortion

Corrected

from Helmut Dersch

From 3D points to pixels: thin lens camera



(1) Perspective projection $\begin{bmatrix} x \\ y \end{bmatrix} = \pi(X, Y, Z)$

(2) Lens distortion

$$[x^*, y^*] = D(x, y)$$

(3) Conversion from metric to pixel coordinates

$$u = m_x x^* + c_x$$

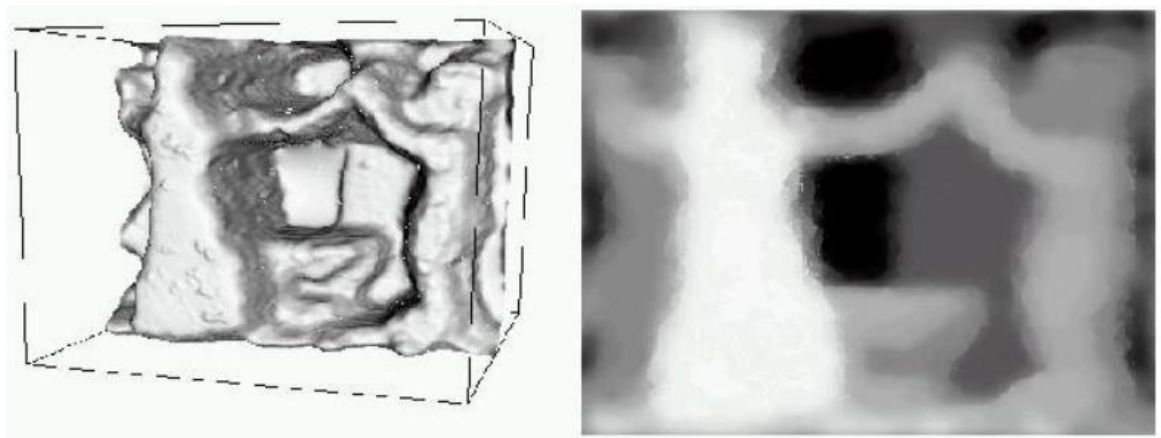
$$v = m_y y^* + c_y$$

If we have access to camera calibration parameters
we can undo the lens distortion, and treat the measurement
model as in the pinhole camera → single-camera image rectification

What visual or physiological cues help us to perceive 3D shape and depth?

Focus/defocus

Images from same point of view, different camera parameters



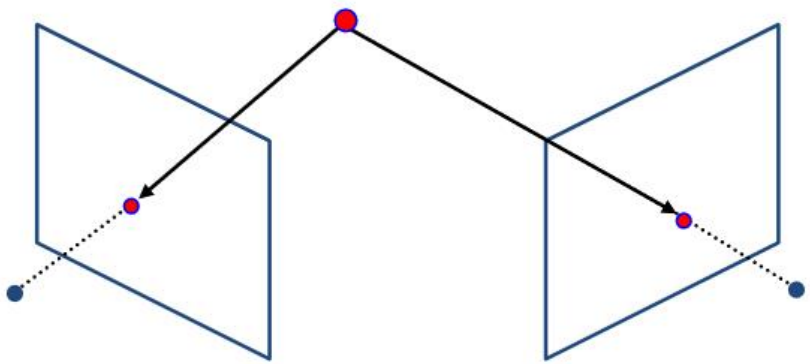
3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

Perspective effects

Image credit: S. Seitz

Stereo



Slides: James Hays and Kristen Grauman

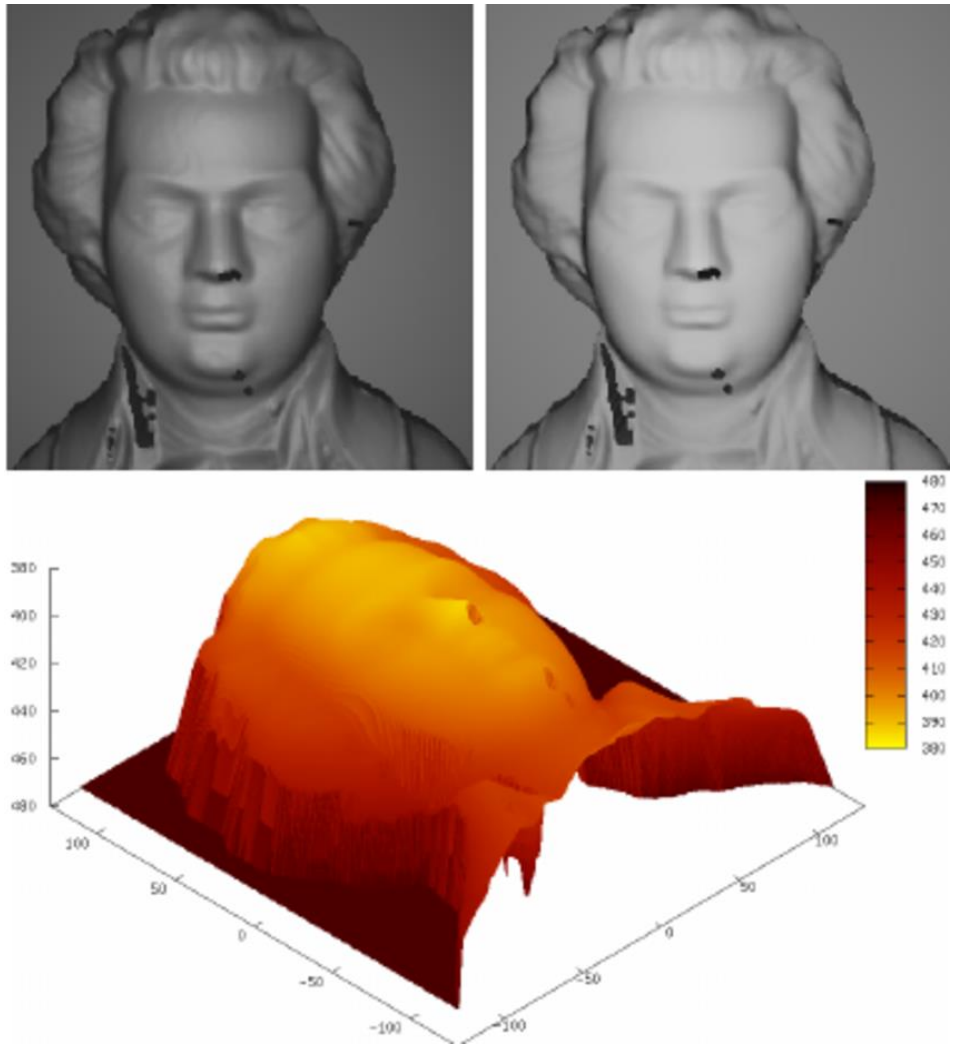
Why multiple views?

Structure and depth can be ambiguous from single views...

Images from Lana Lazebnik

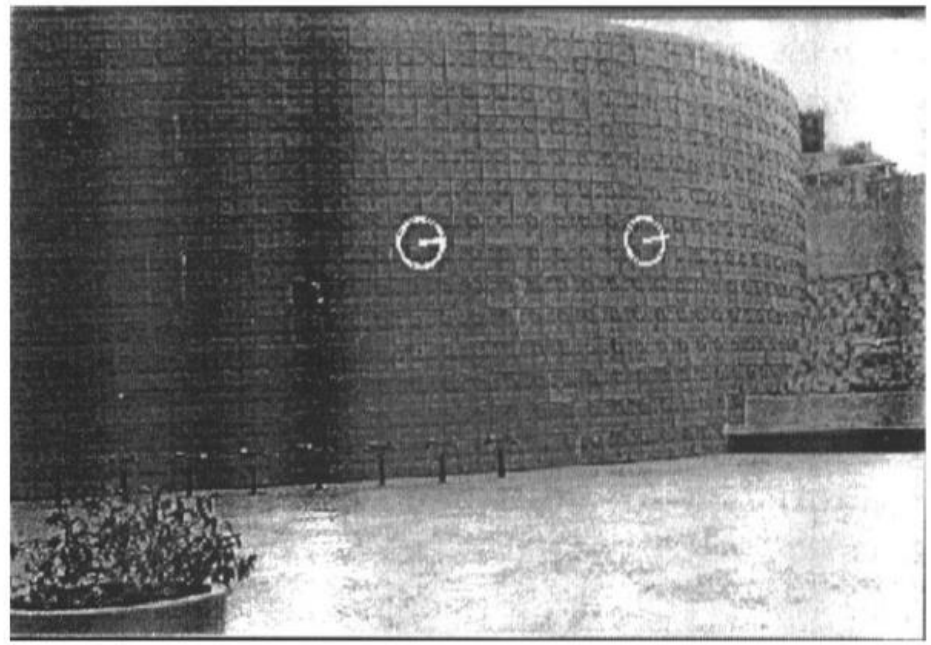
If stereo were critical for depth perception, navigation, recognition, etc., then rabbits would never have evolved.

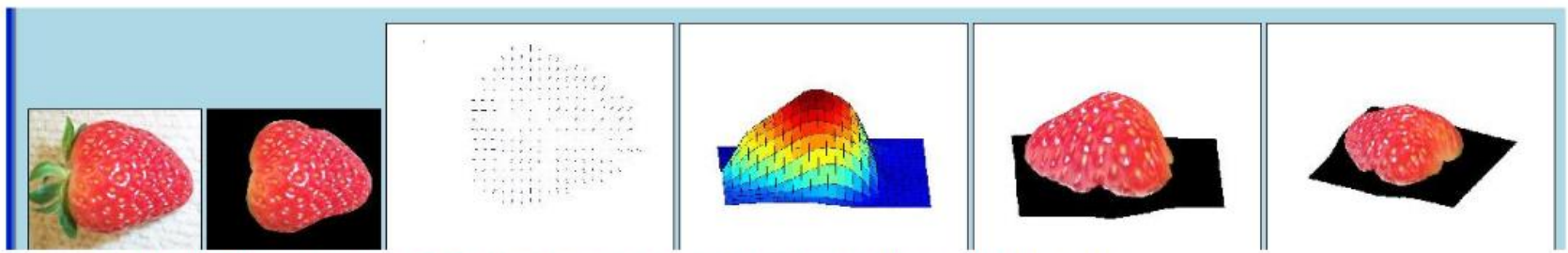
Shape from shading



"Numerical schemes for advanced reflectance models for Shape from Shading", Vogel, Cristiani

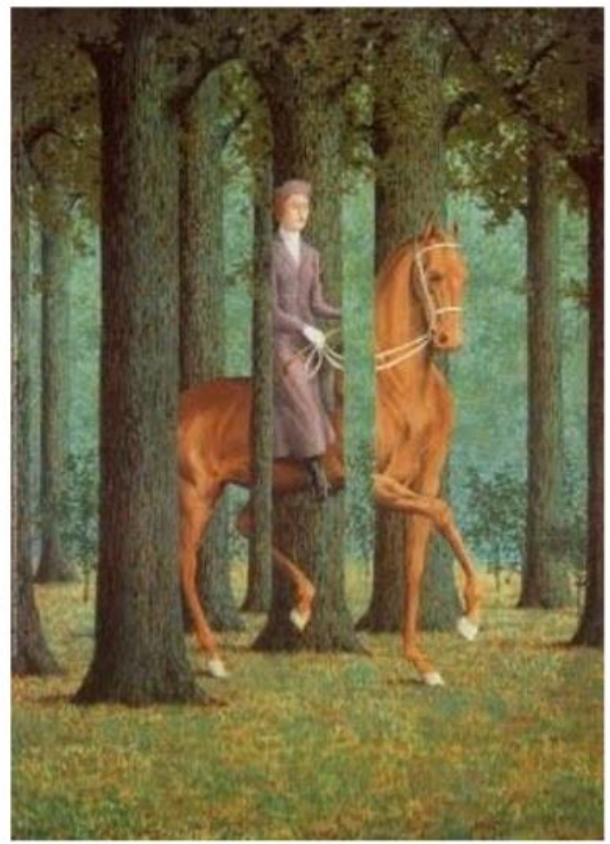
Texture





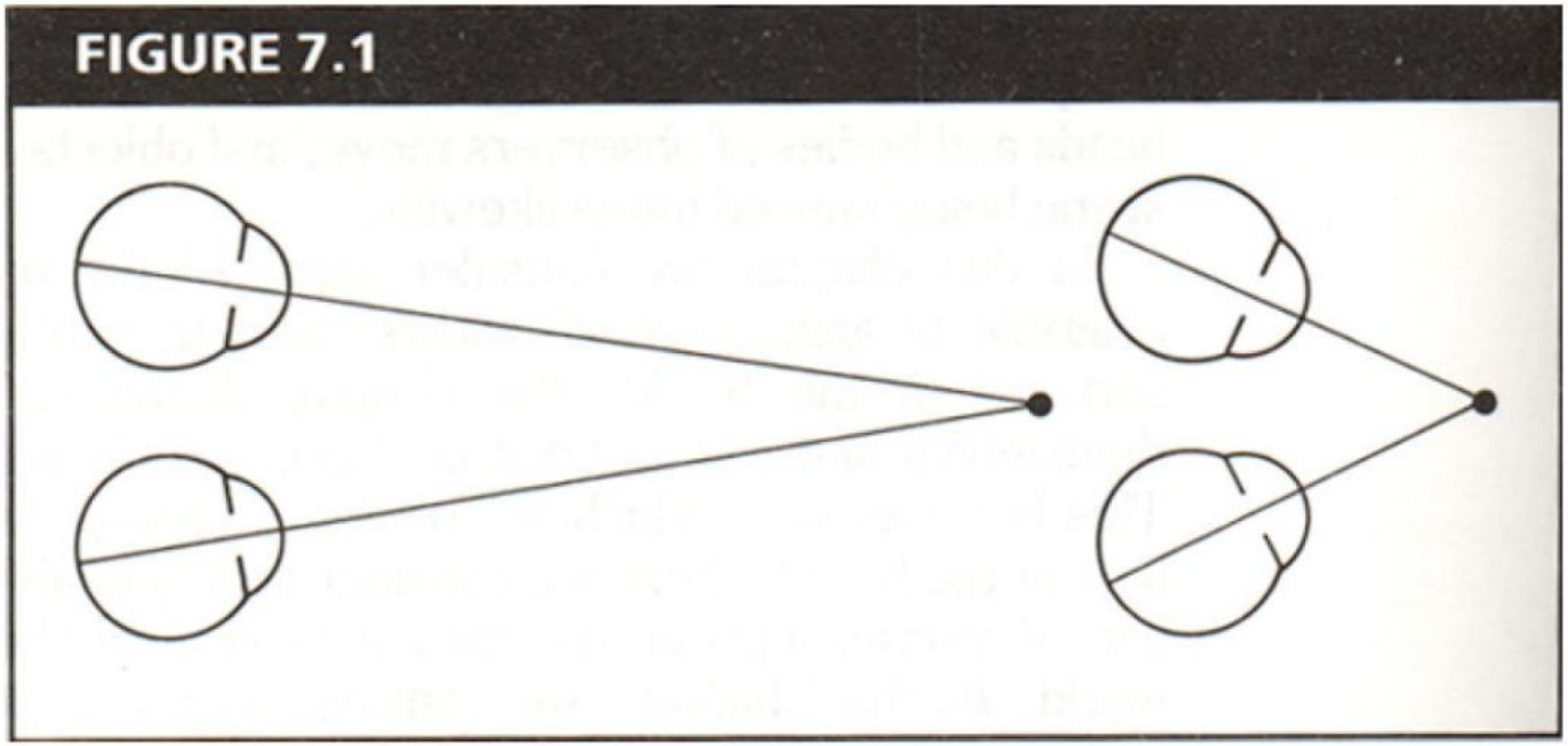
[From A.M. Loh. The recovery of 3-D structure using visual texture patterns. PhD thesis]

Occlusion



Rene Magritt'e famous painting *Le Blanc-Seing* (literal translation: "The Blank Signature") roughly translates as "free hand" or "free rein".

Human stereopsis



From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Human eyes **fixate** on point in space – rotate so that corresponding images form in centers of fovea.

Structure from Motion

"SFMedu: A Structure from Motion System for Education", Jianxiong Xiao

Many depth from X methods. We are going to focus on
structure from motion and stereo → part of multiple-view geometry

- Visual SLAM
  - Localization and mapping with measurements usually coming from tracking image features:
    - keypoints/corners
    - edges
    - image intensity patches
  - Can use one or more cameras

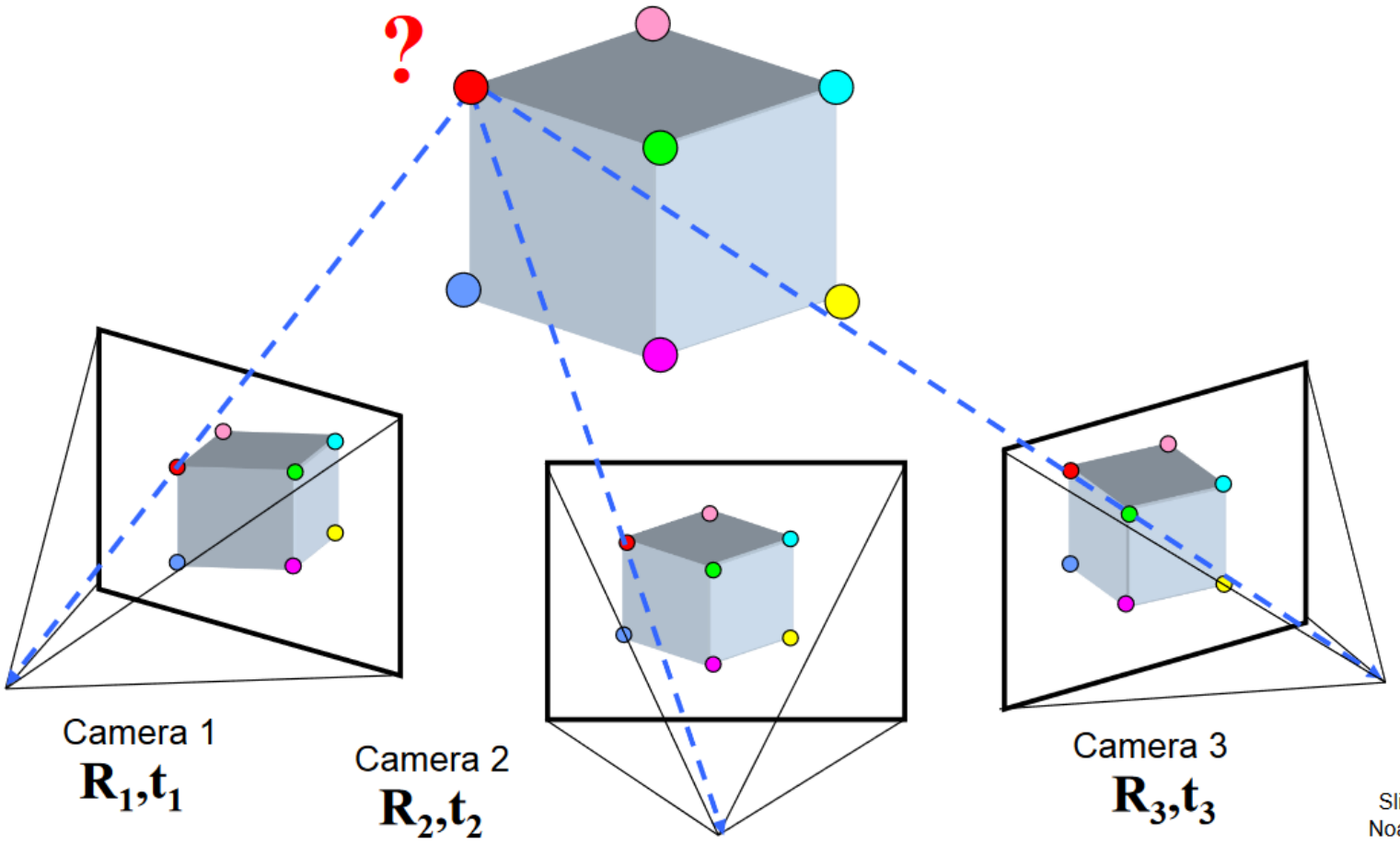
- Visual Odometry
  - Real-time localization with measurements usually coming from tracking image features:
    - keypoints/corners
    - edges
    - image intensity patches
  - Can use one or more cameras

Multi-view geometry problems

A.k.a. mapping

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3D point coordinates are unknown and to be estimated



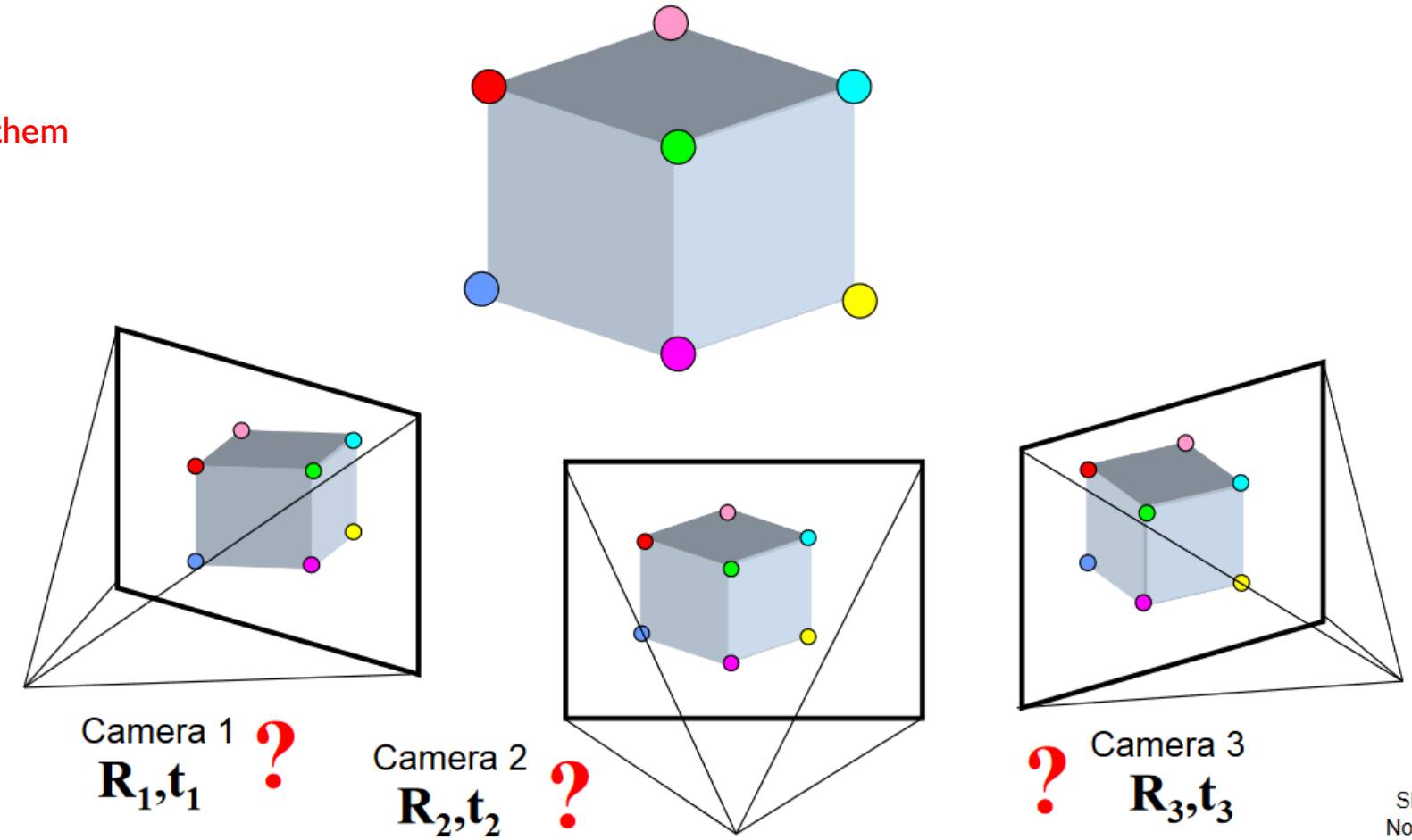
Camera frame transformations are known

Slide credit: Noah Snavely

Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters

3D point coordinates are unknown, but we won't try to estimate them



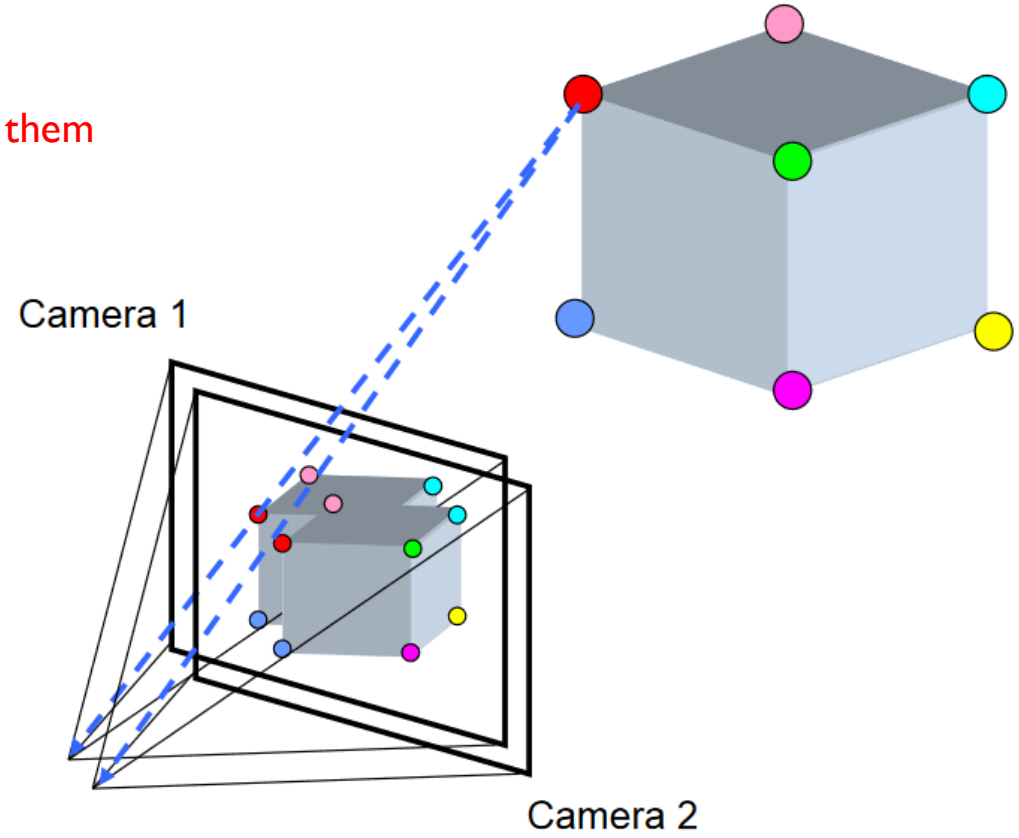
Camera frame transformations are unknown and to be estimated

Slide credit: Noah Snavely

Multi-view geometry problems

- **Optical flow:** Given two images, find the location of a world point in a second close-by image with no camera info.

3D point coordinates are unknown, but we won't try to estimate them

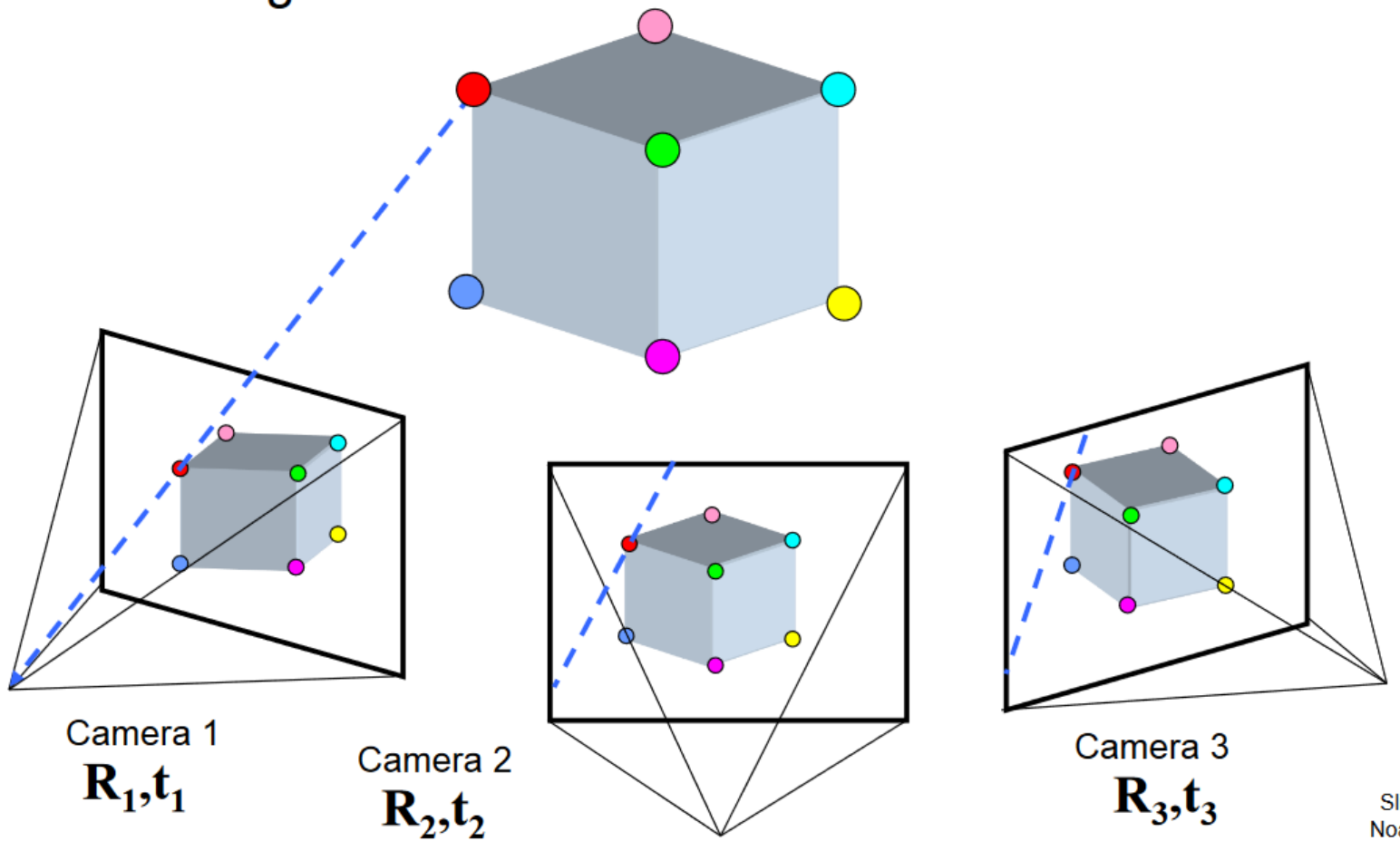


We are estimating pixel displacement from one image to the next

Camera frame transformations are unknown, but we won't try to estimate them

Multi-view geometry problems

- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?



Slide credit:
Noah Snavely

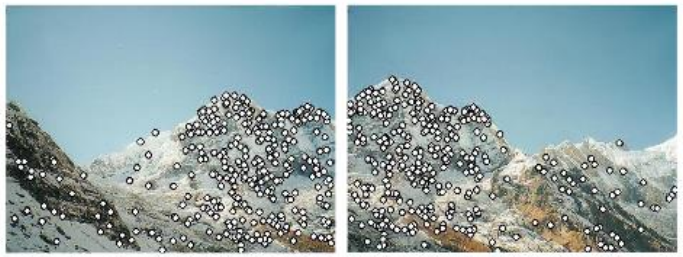
Basic underlying component in many of these problems:

keypoint detection and

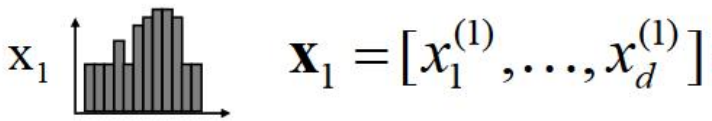
matching across images

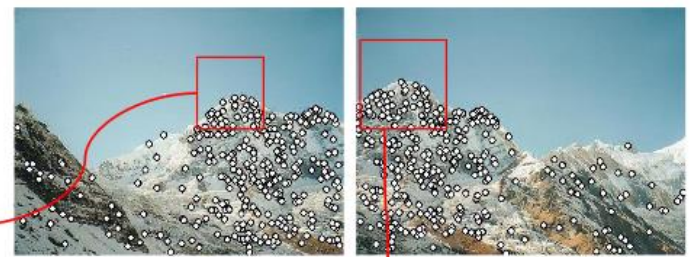
Local features: main components

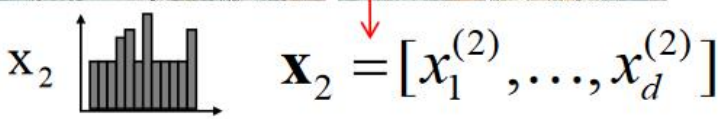
1) Detection:
Find a set of distinctive key points.



2) Description:
Extract feature descriptor around each interest point as vector.

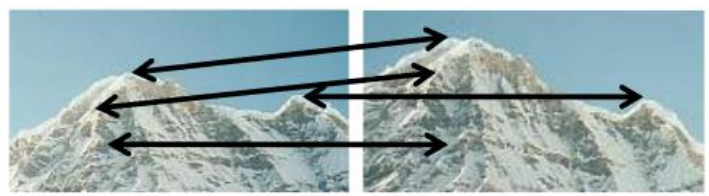






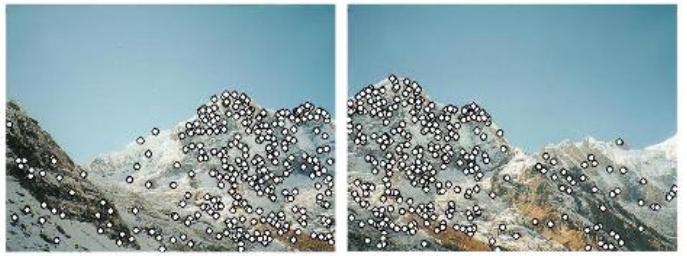
3) Matching:
Compute distance between feature vectors to find correspondence.

$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$

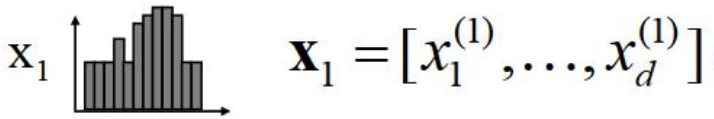


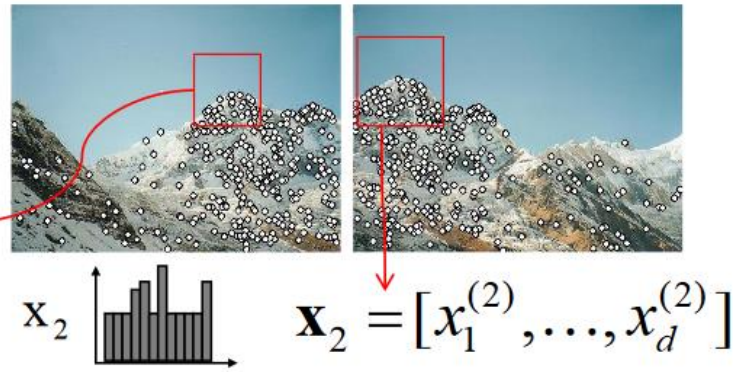
Local features: main components

1) **Detection:**
Find a set of distinctive key points.



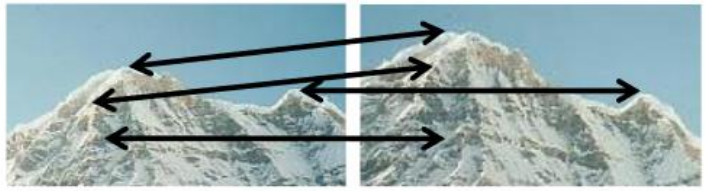
2) **Description:**
Extract feature descriptor around each interest point as vector.





3) **Matching:**
Compute distance between feature vectors to find correspondence.

$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$



Ideally, we want the descriptor to be invariant (i.e. little to no change) when there are

- viewpoint changes (small rotation or translation of the camera)

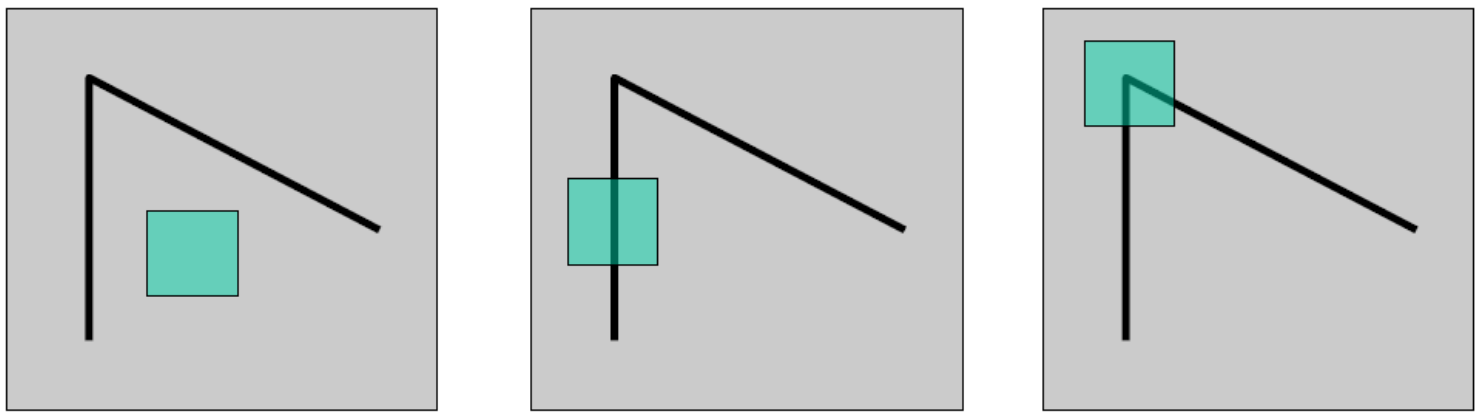
- scale-changes

- illumination changes

Local measures of uniqueness

Suppose we only consider a small window of pixels

- What defines whether a feature is a good or bad candidate?

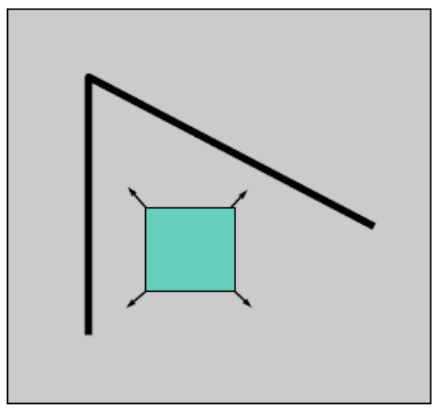


Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

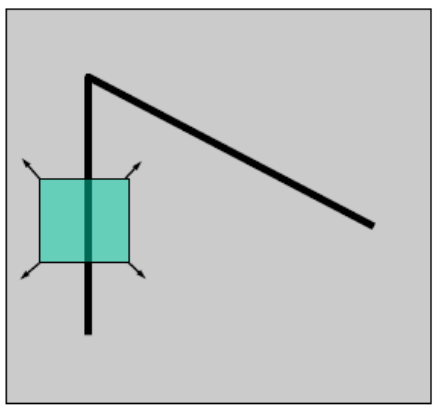
Feature detection

Local measure of feature uniqueness

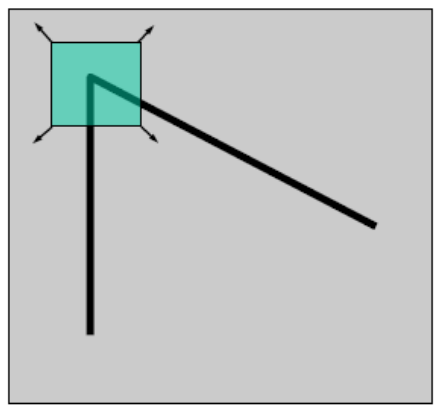
- How does the window change when you shift by a *small amount*?



"flat" region:
no change in all directions



"edge":
no change along the edge direction



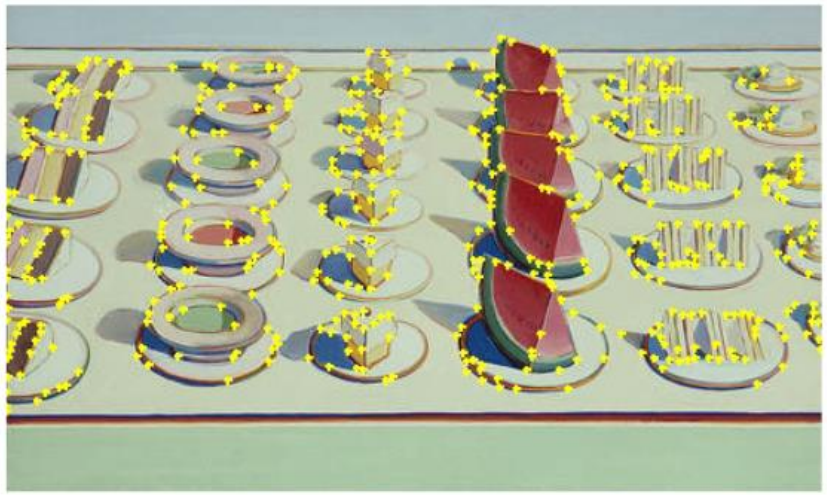
"corner":
significant change in all directions

Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

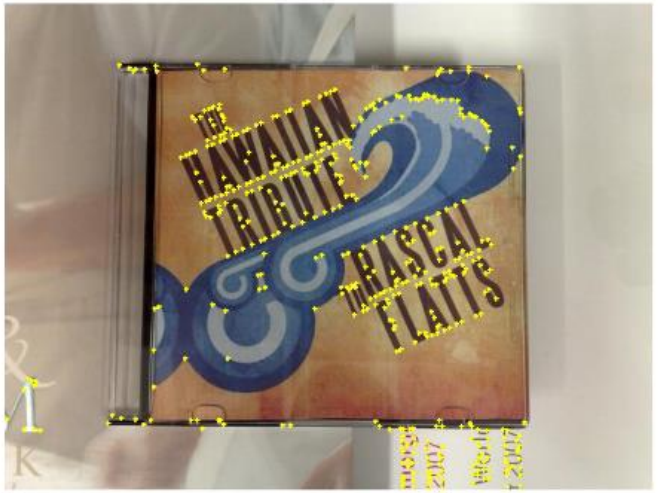
Corner detectors

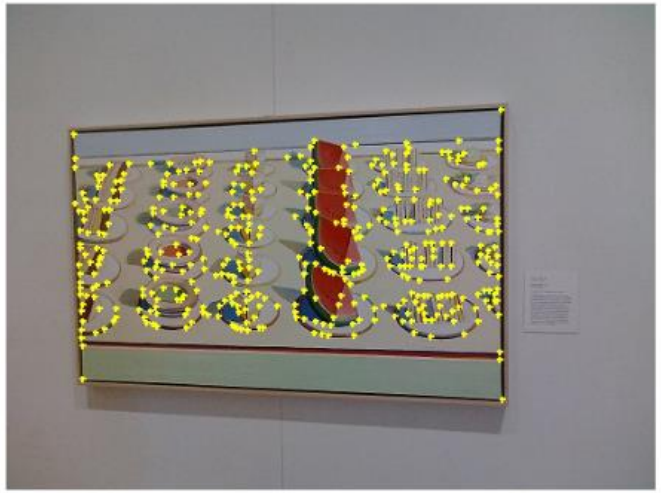
- Harris
- FAST
- Laplacian of Gaussian detector
- SUSAN
- Forstner

Superimposed Harris keypoints

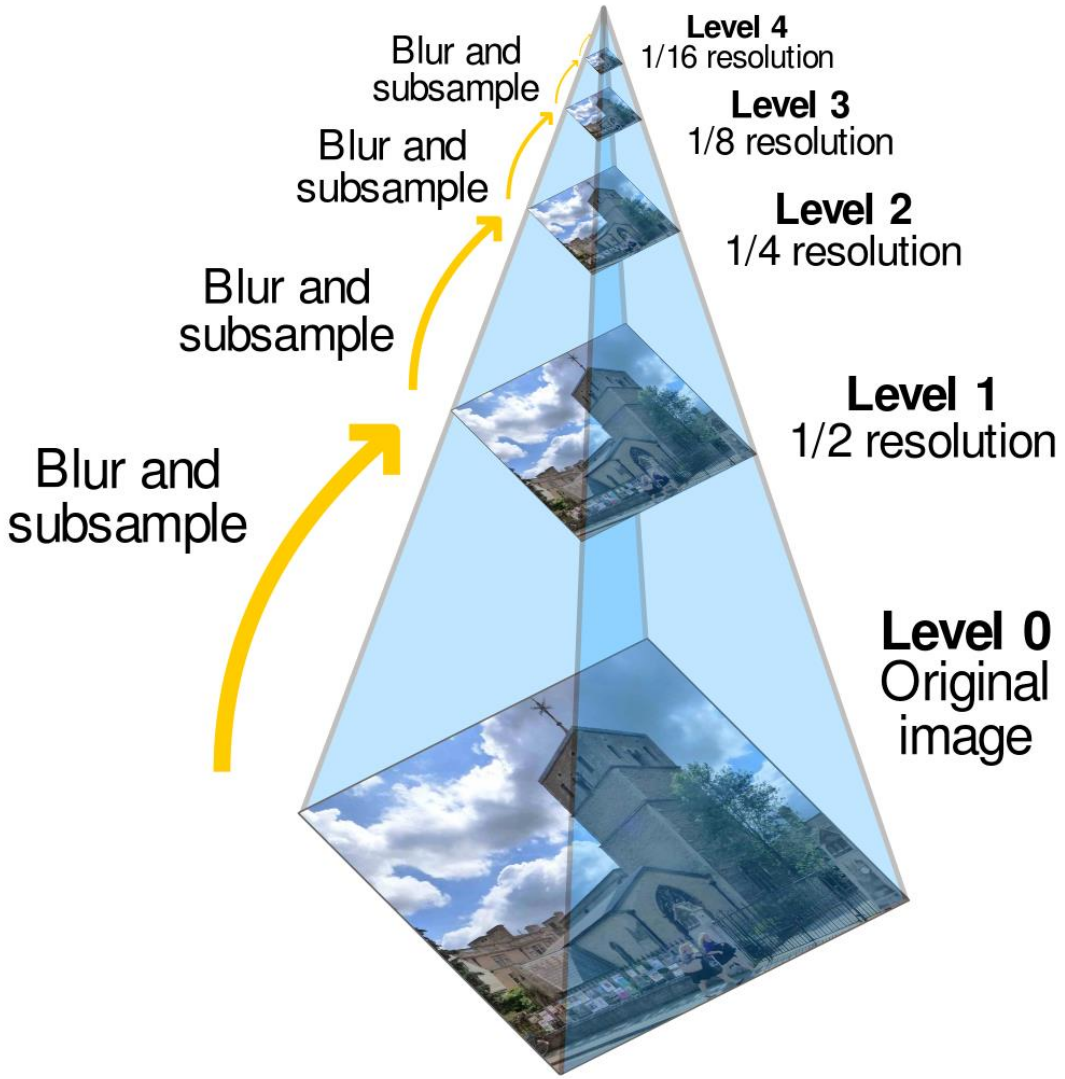


500 strongest keypoints





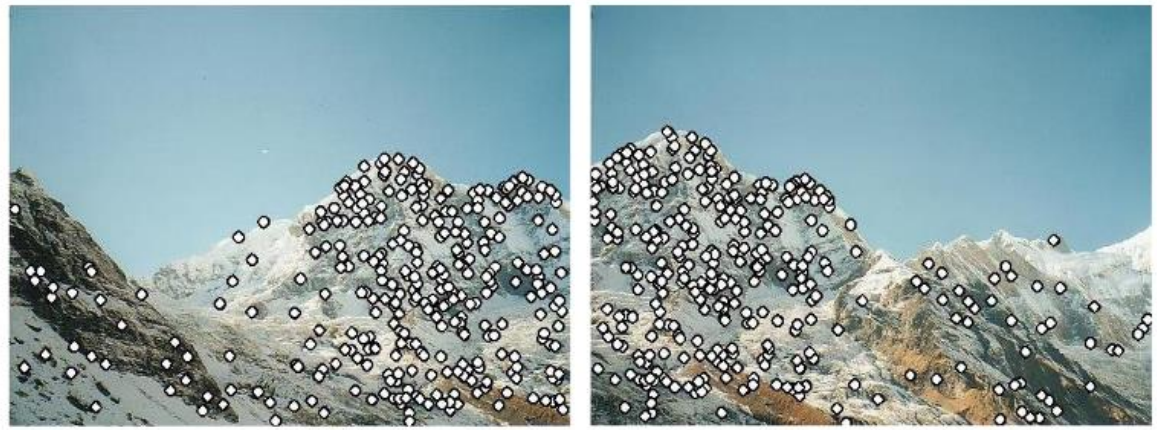
Scale-space representation



Feature detection:

search for “corners”/keypoints
across many scales, and return
a list of (x, y, scale) keypoints

Characteristics of good features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

SIFT descriptor formation

- Compute on local 16 x 16 window around detection.
- Rotate and scale window according to discovered orientation Θ and scale σ (gain invariance).
- Compute gradients weighted by a Gaussian of variance half the window (for smooth falloff).

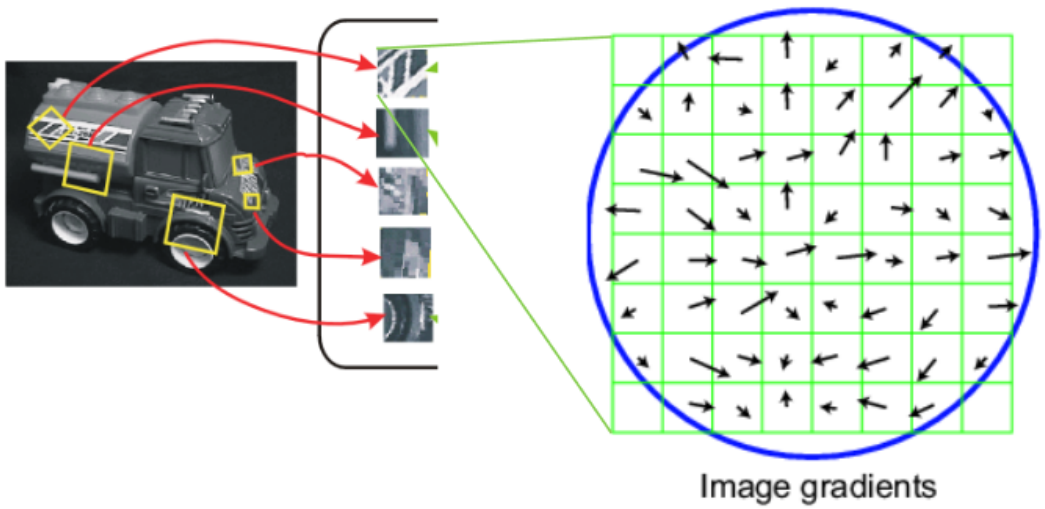
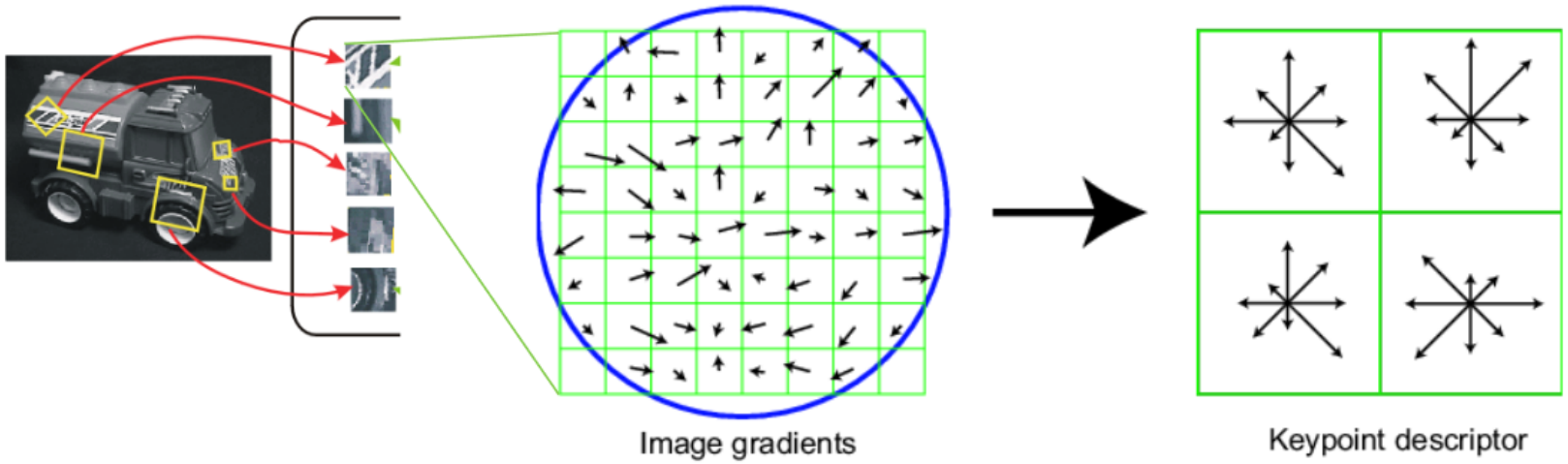


Image gradients

Actually 16x16, only showing 8x8

SIFT vector formation

- 4x4 array of gradient orientation histograms weighted by gradient magnitude.
- Bin into 8 orientations x 4x4 array = 128 dimensions.

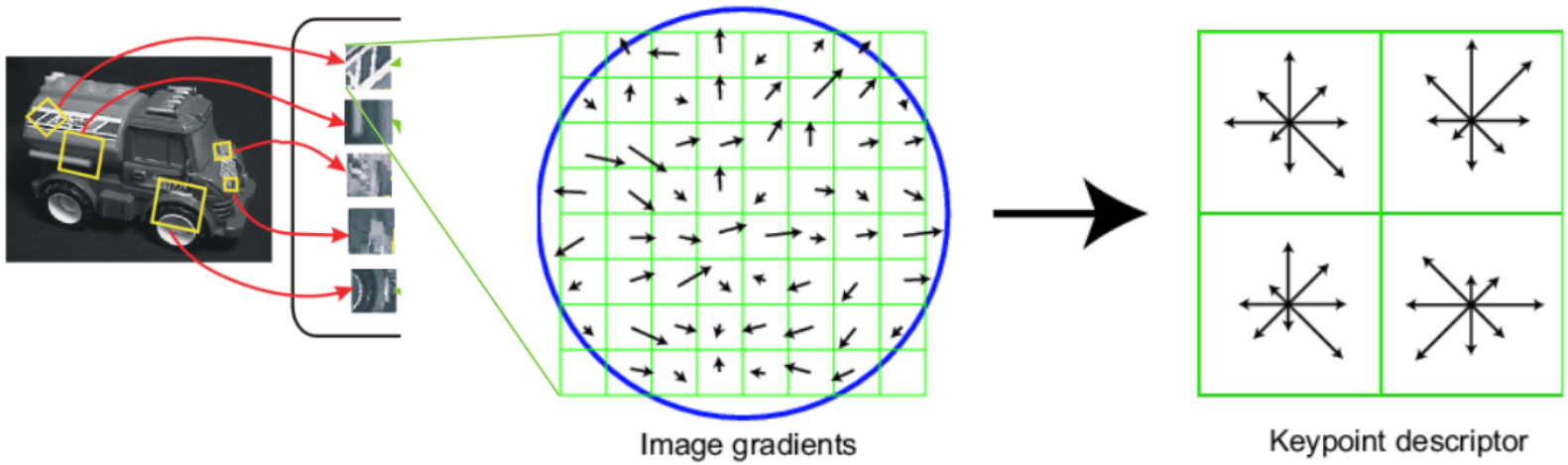


Showing only 2x2 here but is 4x4

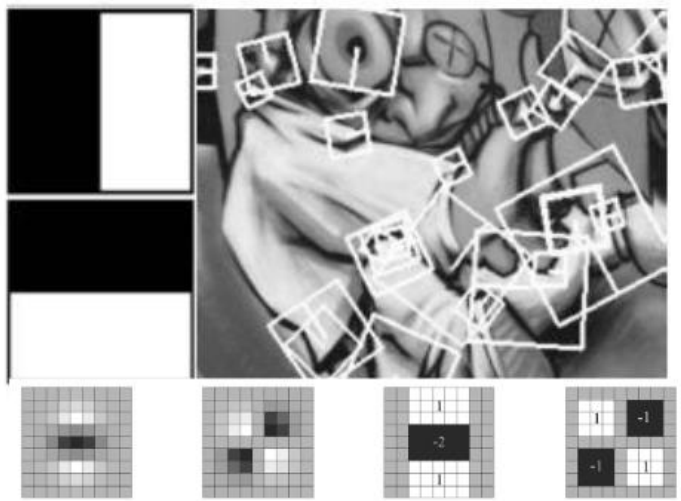
James Hays

Reduce effect of illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - After normalization, clamp gradients > 0.2
  - Renormalize



Local Descriptors: SURF



Fast approximation of SIFT idea

Efficient computation by 2D box filters & integral images

⇒ 6 times faster than SIFT

Equivalent quality for object identification

GPU implementation available

Feature extraction @ 200Hz
(detector + descriptor, 640×480 img)

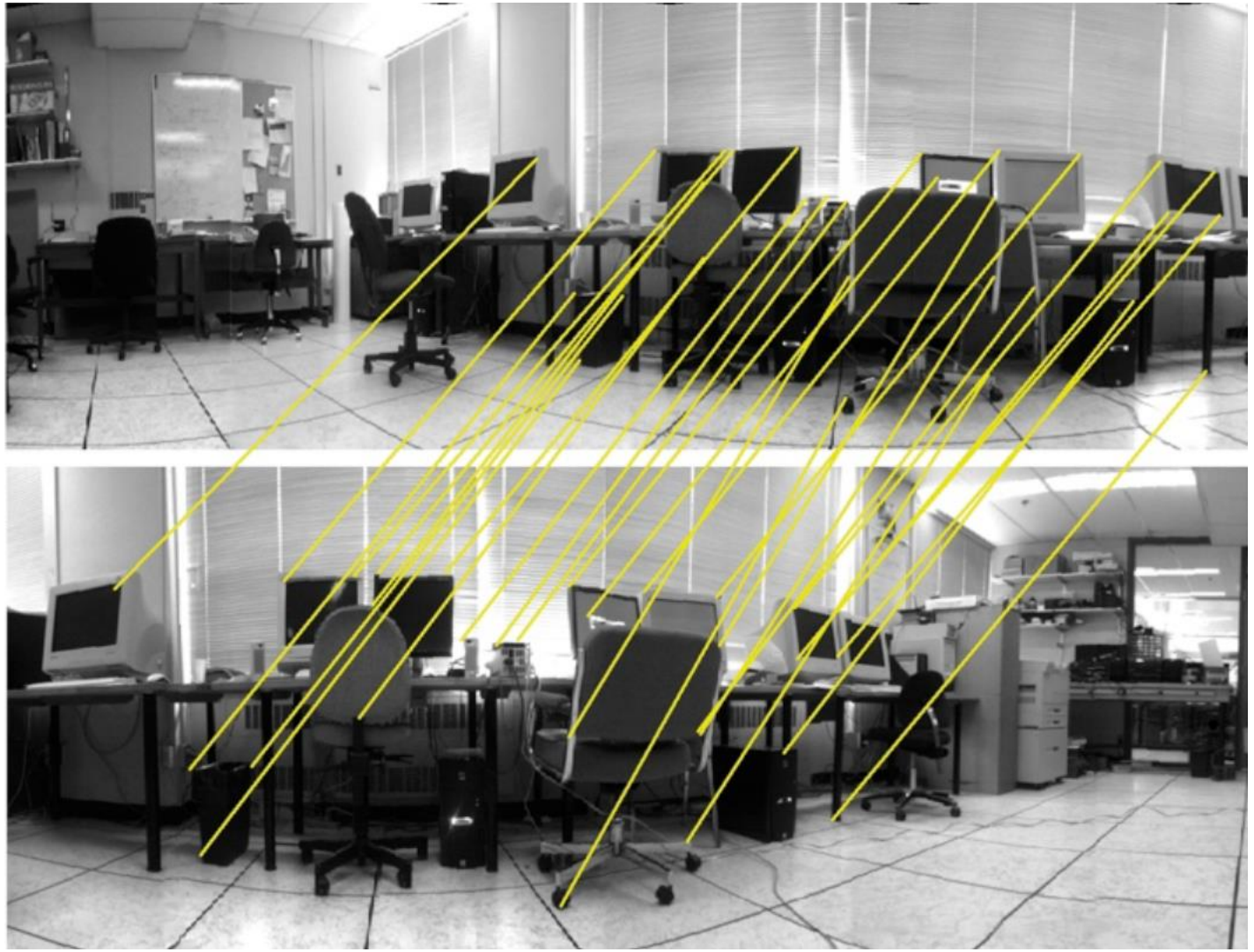
http://www.vision.ee.ethz.ch/~surf

[Bay, ECCV'06], [Cornelis, CVGPU'08]

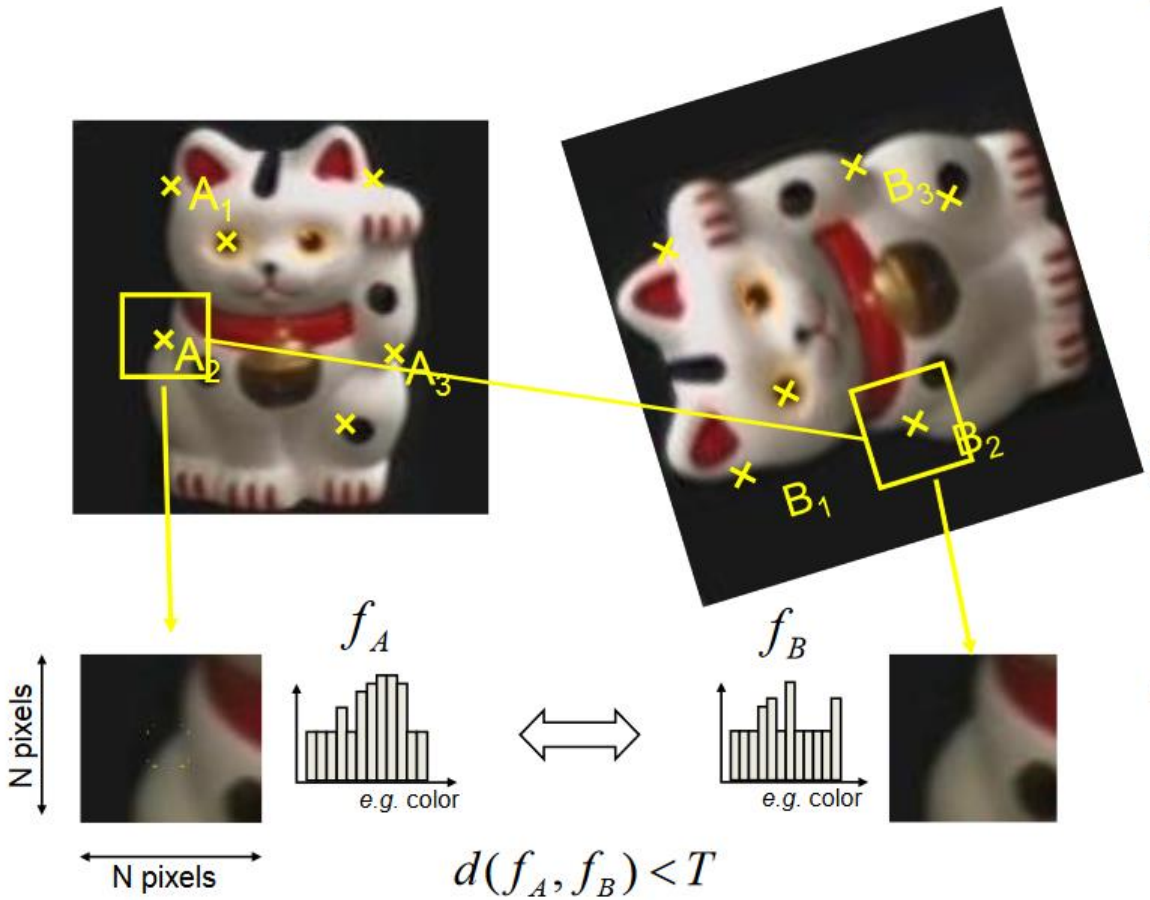
Many other local descriptors

- ORB
- BRIEF
- FREAK
- RootSIFT-PCA

Feature matching



Overview of Keypoint Matching



1. **Find a set of distinctive key-points**
2. **Define a region around each keypoint**
3. **Extract and normalize the region content**
4. **Compute a local descriptor from the normalized region**
5. **Match local descriptors**

K. Grauman, B. Leibe

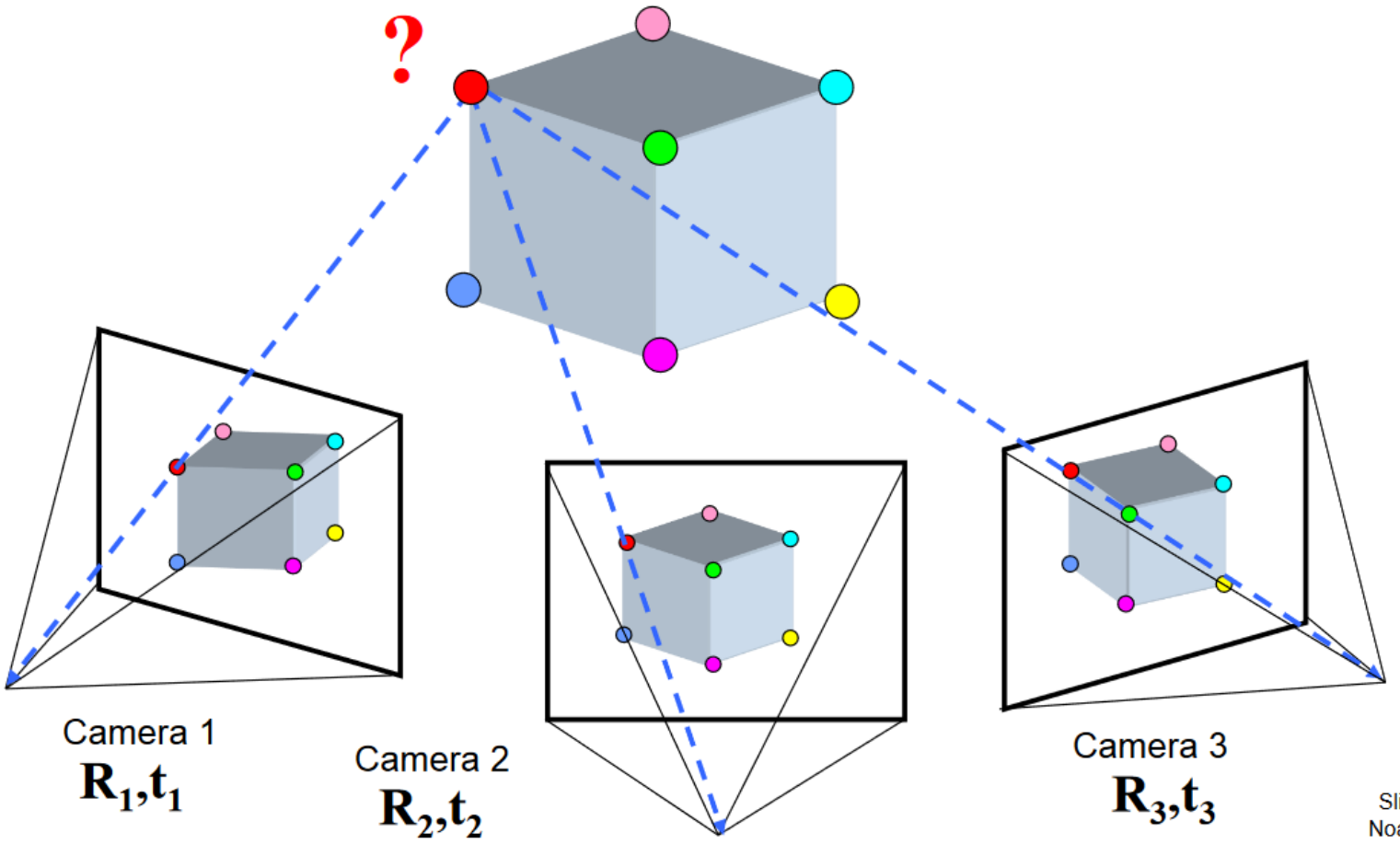
Problem #1: landmark triangulation

Multi-view geometry problems

A.k.a. mapping

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3D point coordinates are unknown and to be estimated



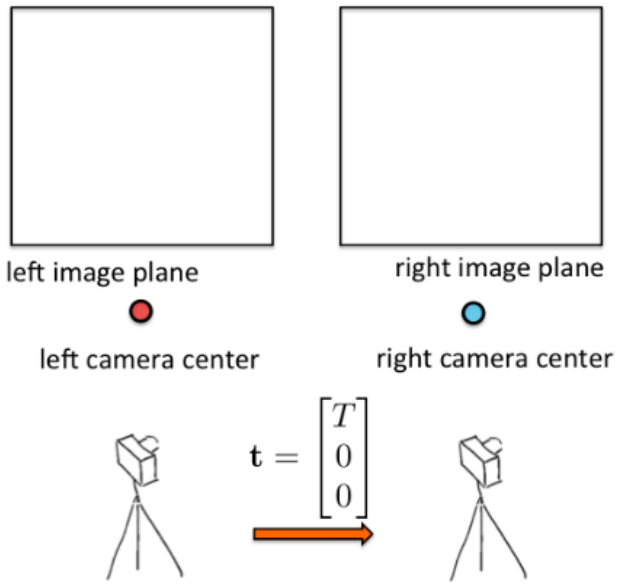
Camera frame transformations are known

Slide credit: Noah Snavely

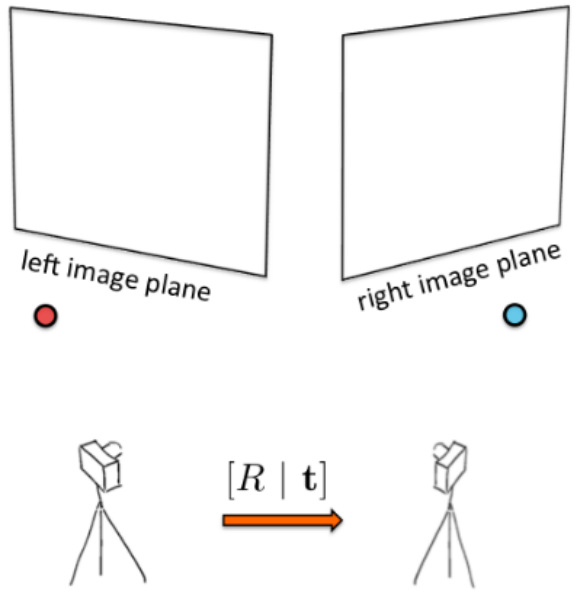
Epipolar geometry

- Case with two cameras with parallel optical axes ← **First this**
- General case

Parallel stereo cameras:

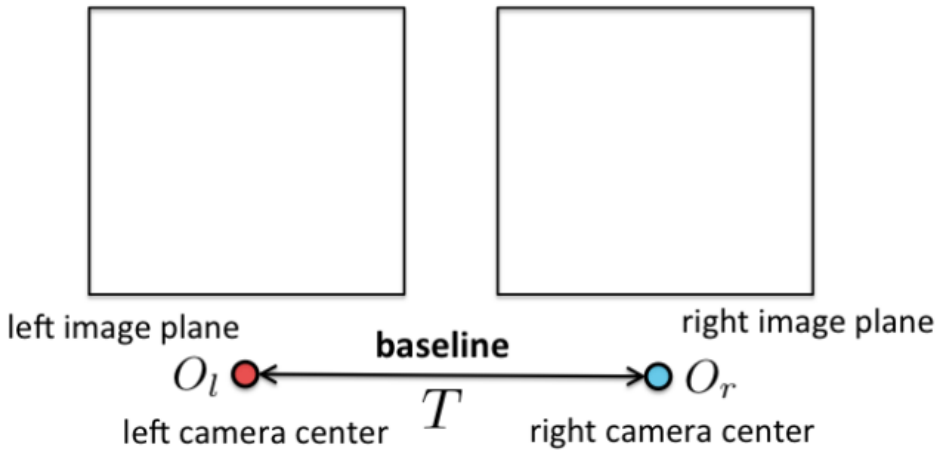


General stereo cameras:



Stereo: Parallel Calibrated Cameras

- We assume that the two calibrated cameras (we know intrinsics and extrinsics) are parallel, i.e. the right camera is just some distance to the right of left camera. We assume we know this distance. We call it the **baseline**.

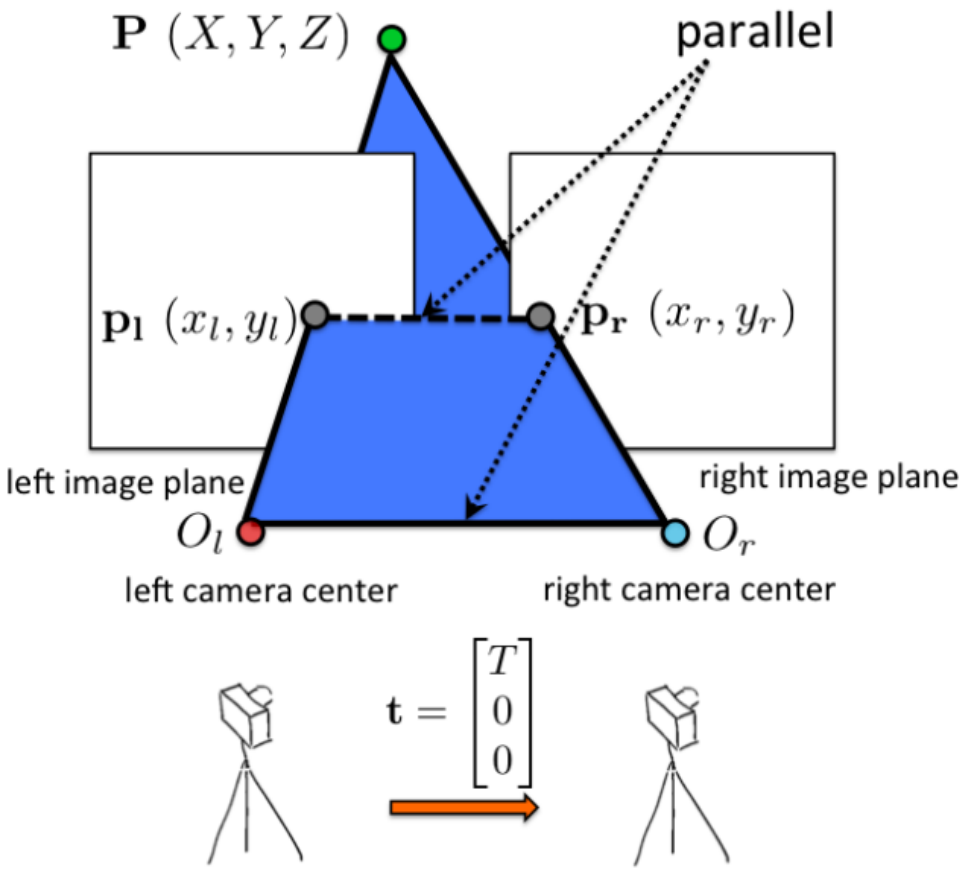


$$\mathbf{t} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}$$

The right camera is shifted to the right in X direction

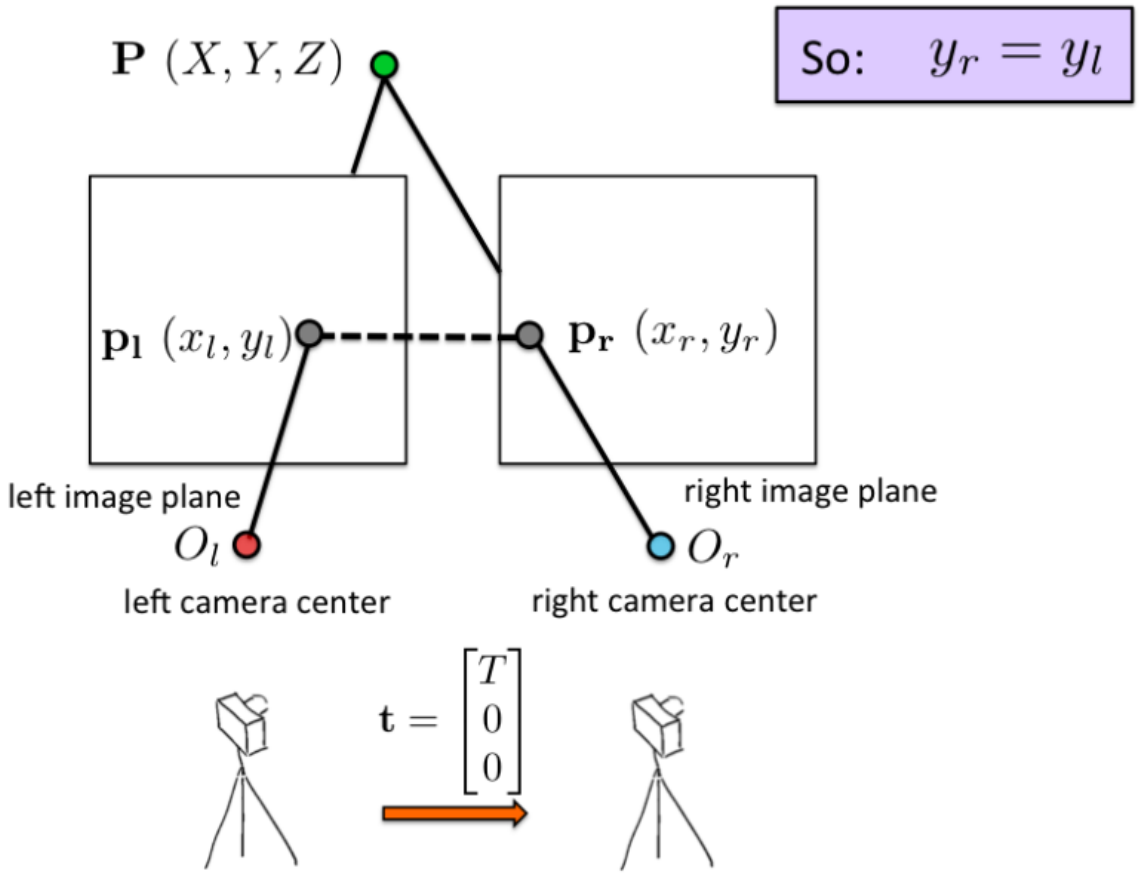
Stereo: Parallel Calibrated Cameras

- Points $\mathbf{O_l}$, $\mathbf{O_r}$ and $\mathbf{P}$ (and $\mathbf{p_l}$ and $\mathbf{p_r}$) lie on a plane. Since two image planes lie on the same plane (distance f from each camera), the lines $\mathbf{O_l O_r}$ and $\mathbf{p_l p_r}$ are parallel.



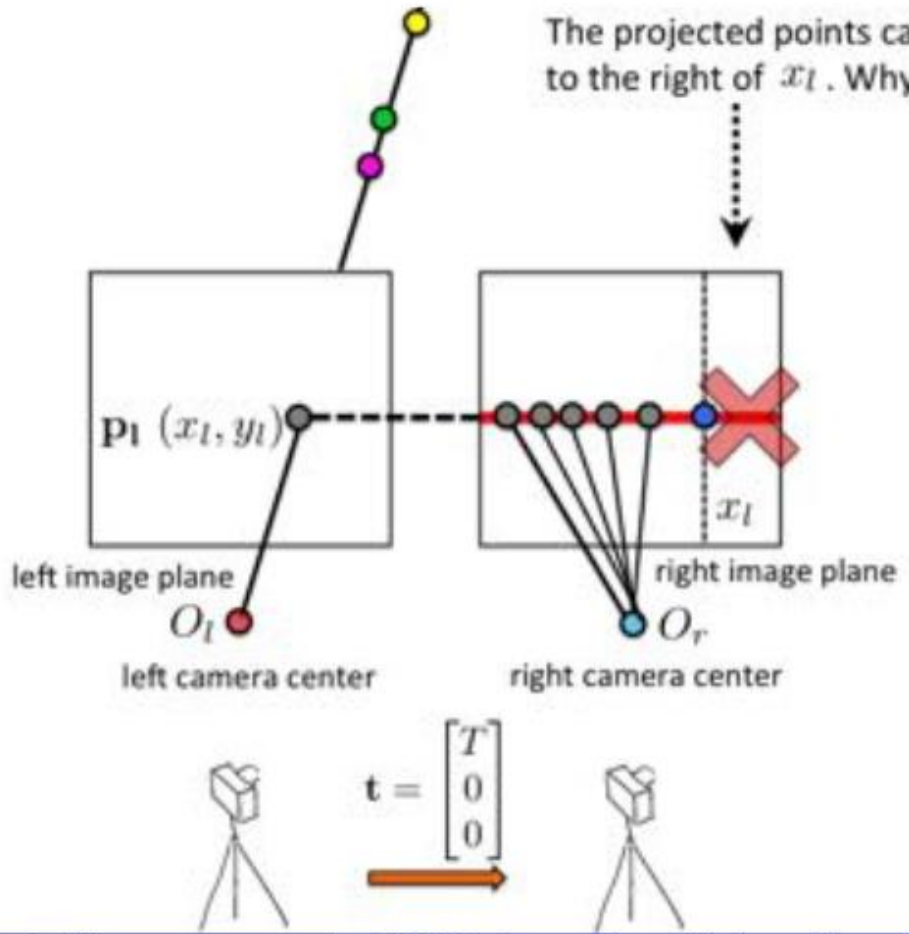
Stereo: Parallel Calibrated Cameras

- Since lines $\mathbf{O_l O_r}$ and $\mathbf{p_l p_r}$ are parallel, and $\mathbf{O_l}$ and $\mathbf{O_r}$ have the same y, then also $\mathbf{p_l}$ and $\mathbf{p_r}$ have the same y: $y_r = y_l$!



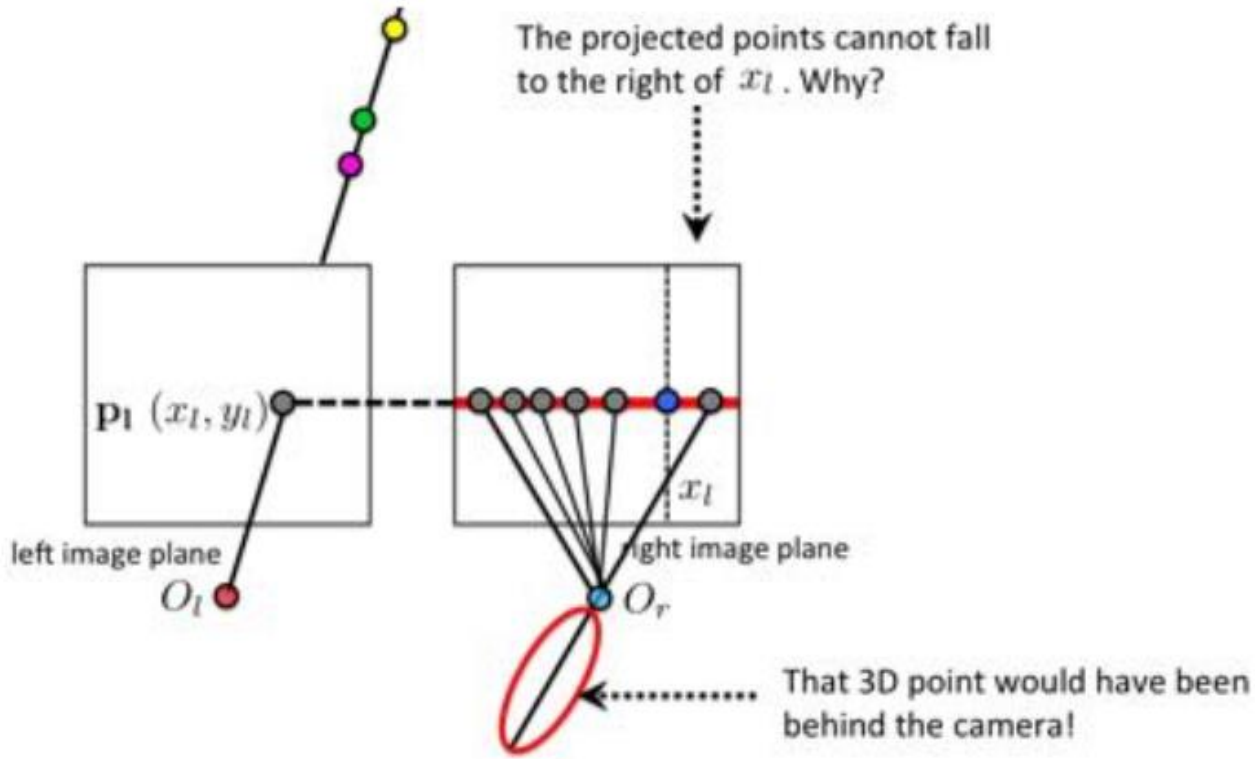
Stereo: Parallel Calibrated Cameras

- Another observation: No point from $\mathbf{O_l p_l}$ can project to the right of x_l in the right image. **Why**?



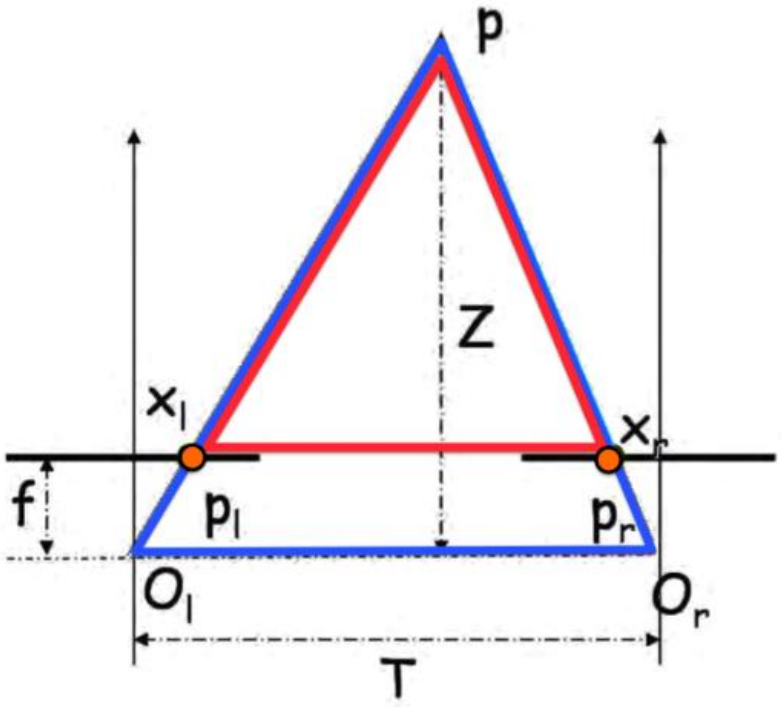
Stereo: Parallel Calibrated Cameras

- Because that would mean our image can see behind the camera...



Stereo: Parallel Calibrated Cameras

- We can then use similar triangles to compute the depth of the point P



Similar triangles:

$$\frac{T}{Z} = \frac{T + x_r - x_l}{Z - f}$$

$$Z = \frac{f \cdot T}{x_l - x_r}$$

baseline

focal length

disparity

In metric, not in pixel coordinates. To convert to pixel coordinates need to use elements of the camera calibration matrix.

[Adopted from: J. Hays]

Conclusion: if you have a well-calibrated and rectified (parallel) stereo camera you do not need to do least squares triangulation.

You can estimate depth via the disparity map.

Stereo: Parallel Calibrated Cameras

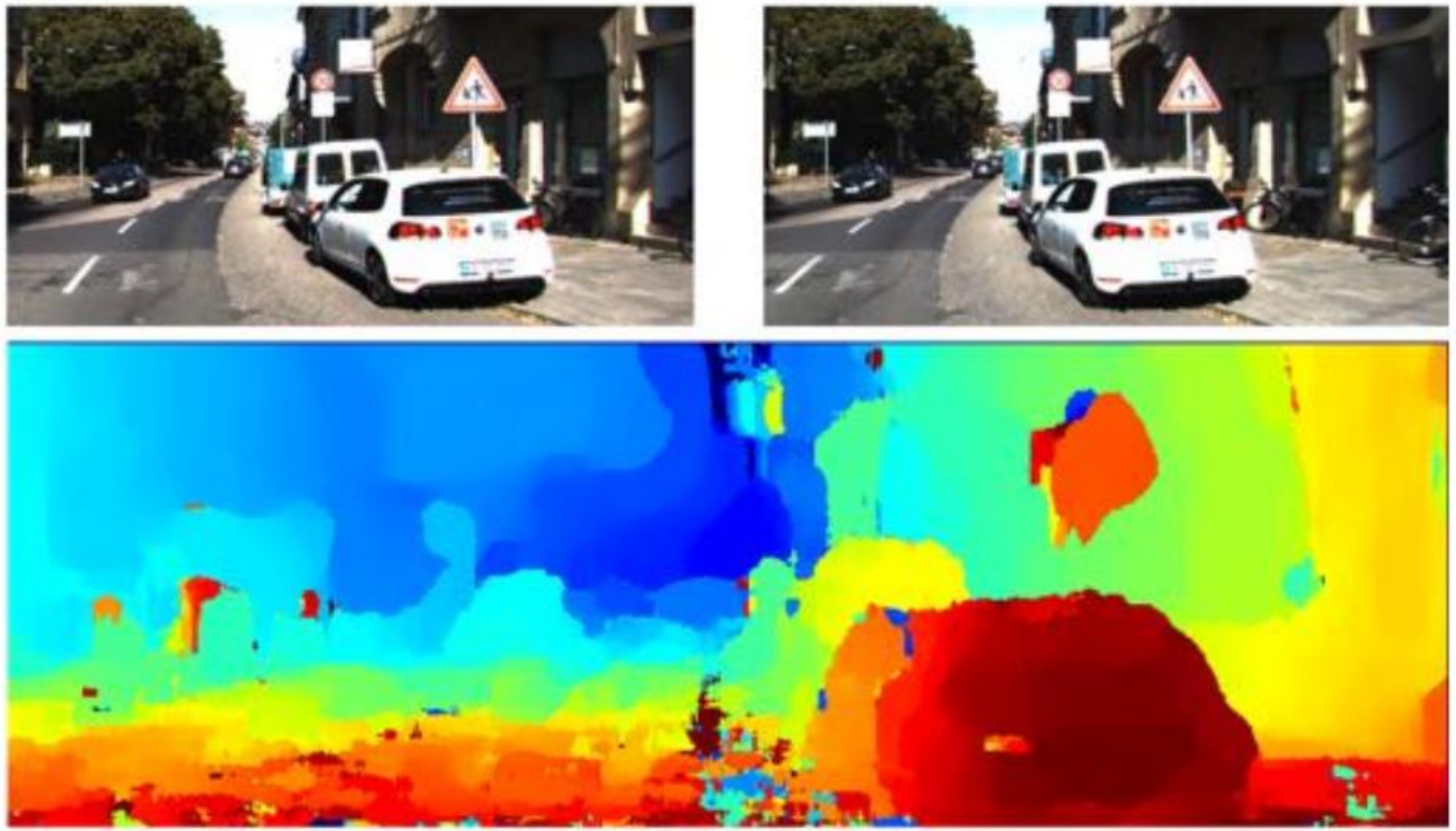
- For each point $\mathbf{p_l} = (x_l, y_l)$, how do I get $\mathbf{p_r} = (x_r, y_r)$? By matching. Patch around (x_r, y_r)) should look similar to the patch around (x_l, y_l).

left image

Do this for all the points in the left image!

Stereo: Parallel Calibrated Cameras

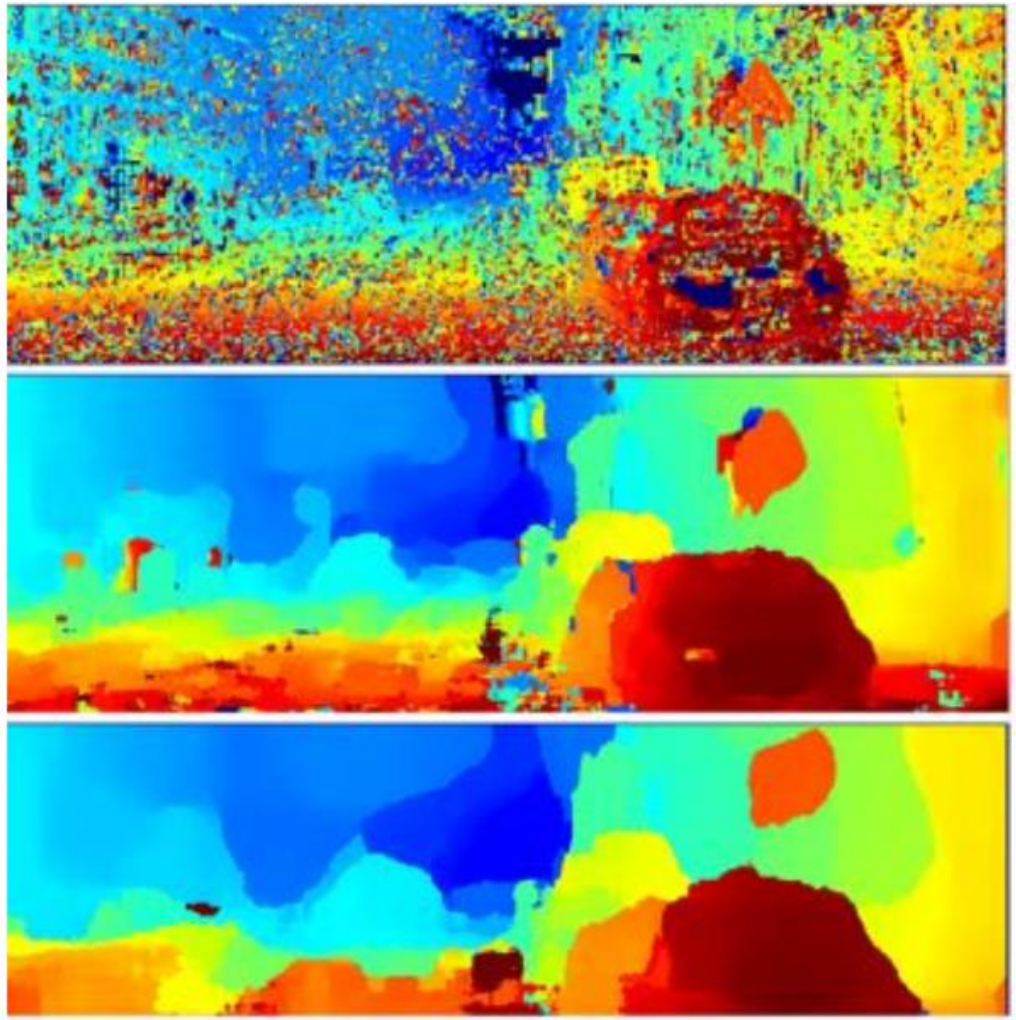
- We get a disparity map as a result



Result: **Disparity map**
(red values large disp., blue small disp.)

Stereo: Parallel Calibrated Cameras

- Smaller patches: more detail, but noisy. Bigger: less detail, but smooth



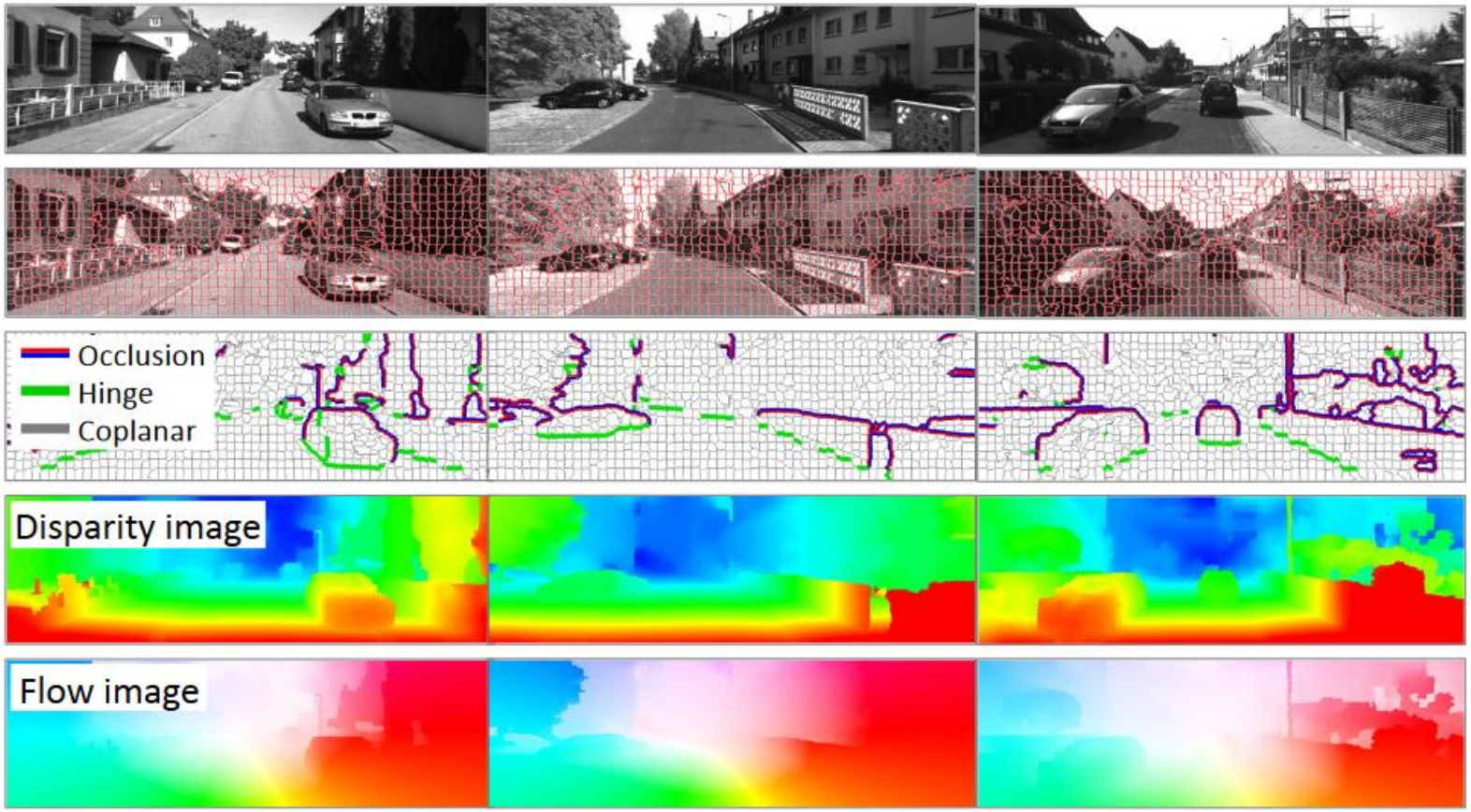
patch size = 5

patch size = 35

patch size = 85

You Can Do It Much Better...

[K. Yamaguchi, D. McAllester and R. Urtasun, ECCV 2014]



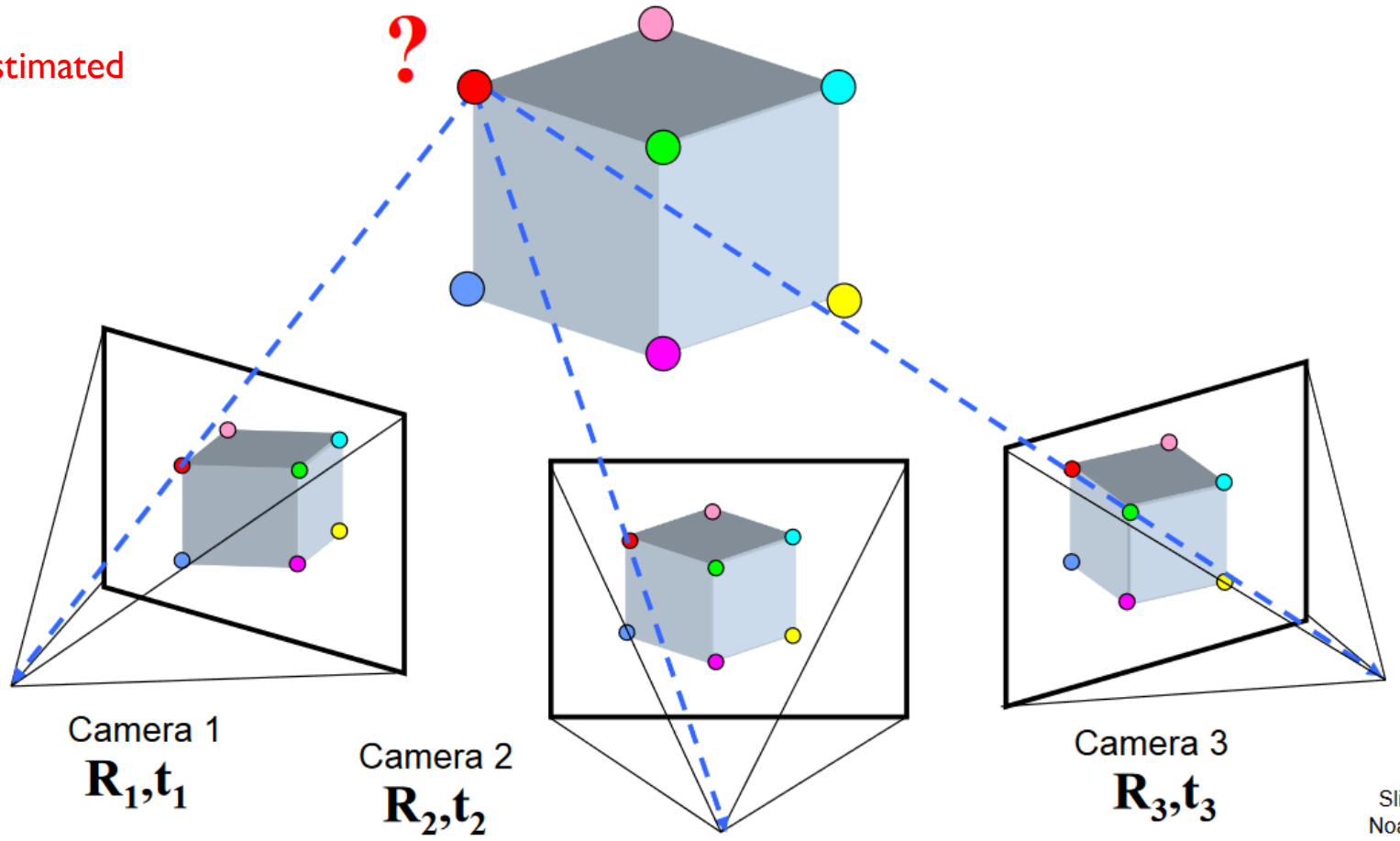
Multi-view geometry problems

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3 x 7 = 21 variables to be estimated

7 pixel observations in each camera, so 21 pixel observations across all cameras

→ 42 constraints in total



Slide credit: Noah Snavely

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

Actual pixel observation of a keypoint by camera frame k

Expected pixel observation of 3D point ${}^{w}p$ by camera frame k

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

Enumerate all cameras that observed the keypoint.

The only term to be optimized. The rest are known.

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

3D point expressed in the frame of camera k

$$ {}^{k}p = {}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw} $$

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

Reprojection error of point

$${}^{k}p = {}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw}$$

into camera k's frame

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$
\begin{aligned}
{}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* &= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 \\
&= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1, {}^{w}p \rightarrow {}^{k}p}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}p)\,]\, ||^2
\end{aligned}
$$

$$
h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)}
$$

Triangulation as a least squares problem

$$
\begin{aligned}
{}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* &= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 \\
&= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1, {}^{w}p \to {}^{k}p}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}p)\,]\, ||^2 \\
&= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1, {}^{w}p \to {}^{k}p}^{K} || \begin{bmatrix} \bar{u}^{(k)} \\ \\ \bar{v}^{(k)} \end{bmatrix} - \begin{bmatrix} \frac{f m_x {}^{k}X}{{}^{k}Z} + c_x \\ \\ \frac{f m_y {}^{k}Y}{{}^{k}Z} + c_y \end{bmatrix} ||^2
\end{aligned}
$$

$$
h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)}
$$

Triangulation as a least squares problem

$$
\begin{aligned}
{}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* &= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 \\
&= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1, {}^{w}p \to {}^{k}p}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}p)\,]\, ||^2 \\
&= \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1, {}^{w}p \to {}^{k}p}^{K} || \begin{bmatrix} \bar{u}^{(k)} \\ \\ \bar{v}^{(k)} \end{bmatrix} - \begin{bmatrix} \frac{f m_x {}^{k}X}{{}^{k}Z} + c_x \\ \\ \frac{f m_y {}^{k}Y}{{}^{k}Z} + c_y \end{bmatrix} ||^2
\end{aligned}
$$

Note: unconstrained optimization does not guarantee that the solution will be in the camera's field of view. For example, it could happen that it returns ${}^{k}Z < 0$ which is an invalid solution (i.e. behind the camera)

$$
h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)}
$$

Potential pitfalls with triangulation: near parallel rays

"point at infinity"

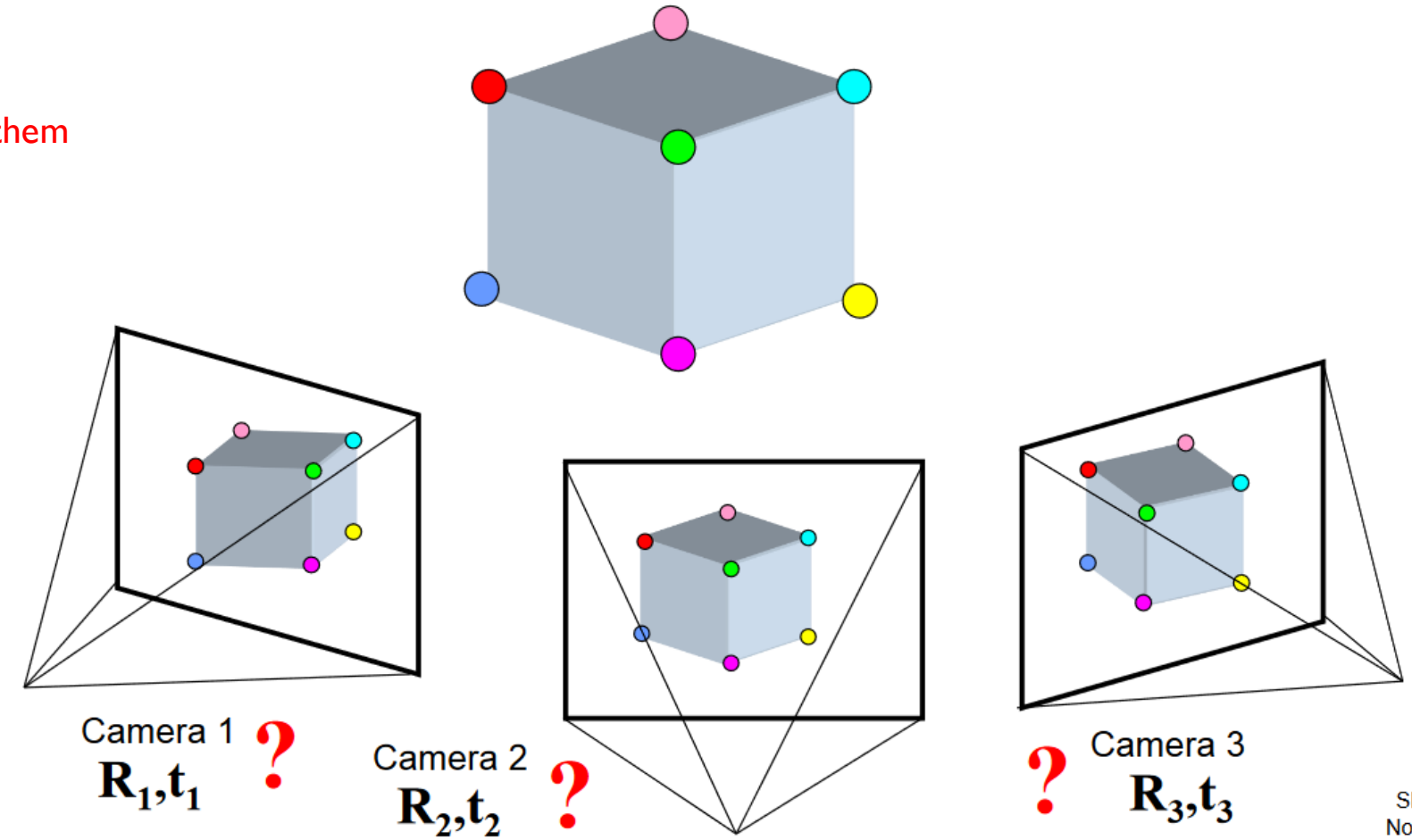
Intersection point is too far away, dominated by noise and insufficient image resolution.
Triangulating these points is typically impossible without sufficient baseline between camera frames.

Problem #2:
camera localization/visual odometry

Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters

3D point coordinates are unknown, but we won't try to estimate them



Camera frame transformations are unknown and to be estimated

Slide credit: Noah Snavely

Camera localization as a least squares problem?

$$ {}^k_w R^*, {}^k t^*_{kw} = \underset{{}^k_w R, {}^k t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k_w R^w p + {}^k t_{kw})\,]\, ||^2 $$

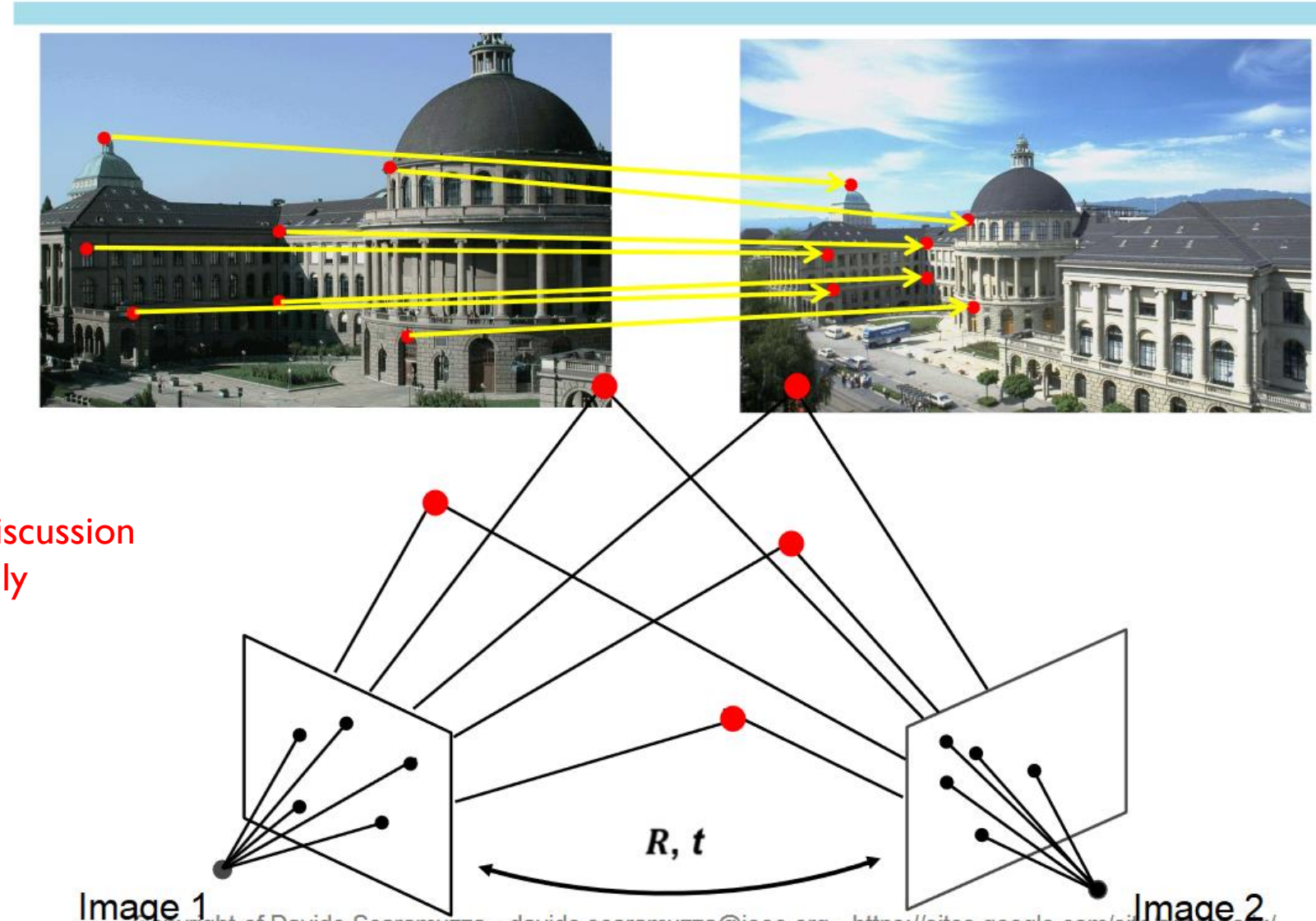
The only terms to be optimized.

But, 3D position is unknown!

So, we cannot solve the problem using the reprojection error unless we know the 3D position corresponding to the keypoint.

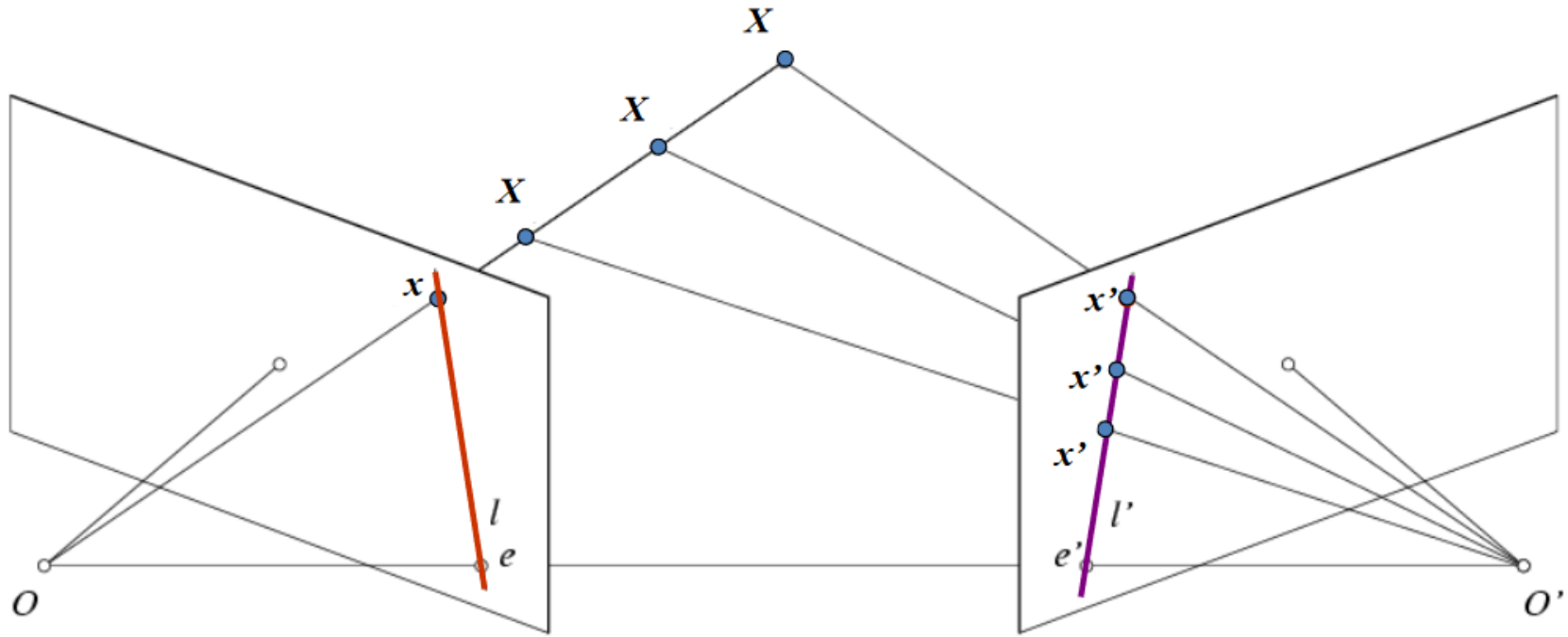
$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Working Principle



Let's restrict the discussion to two cameras only

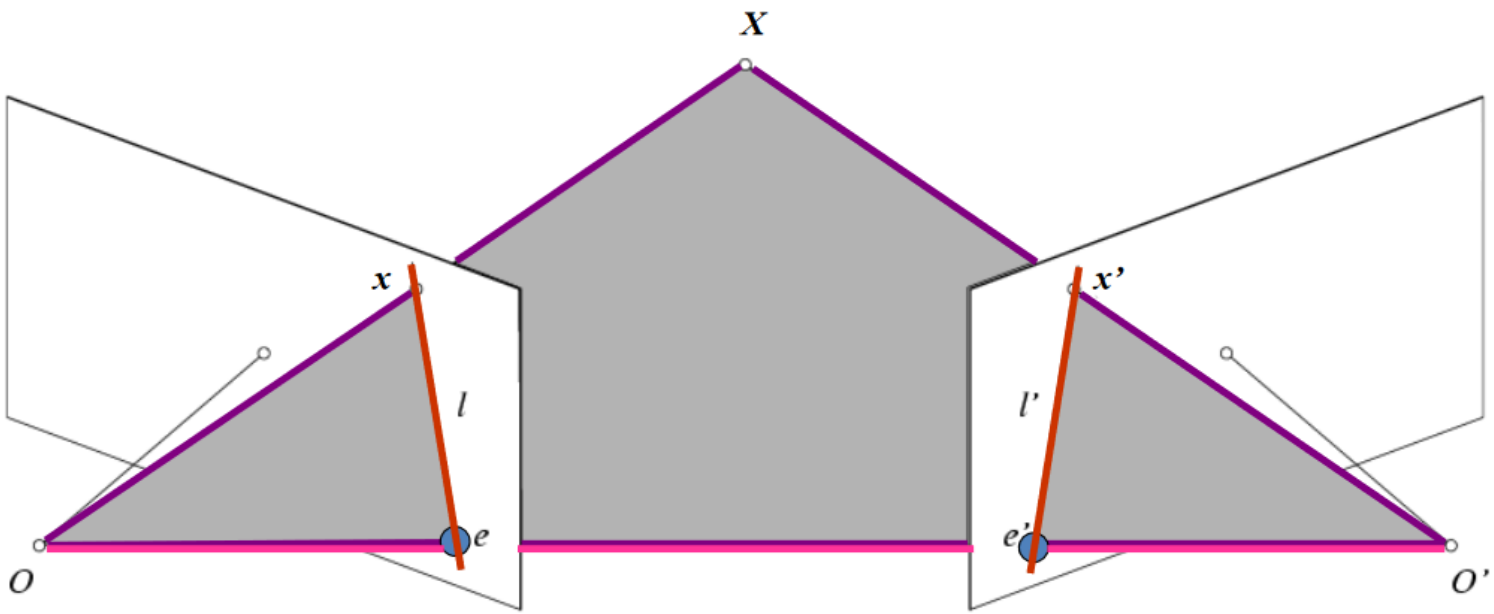
Key idea: Epipolar constraint



Potential matches for *x'* have to lie on the corresponding line *l*.

Potential matches for *x* have to lie on the corresponding line *l'*.

Epipolar geometry: notation



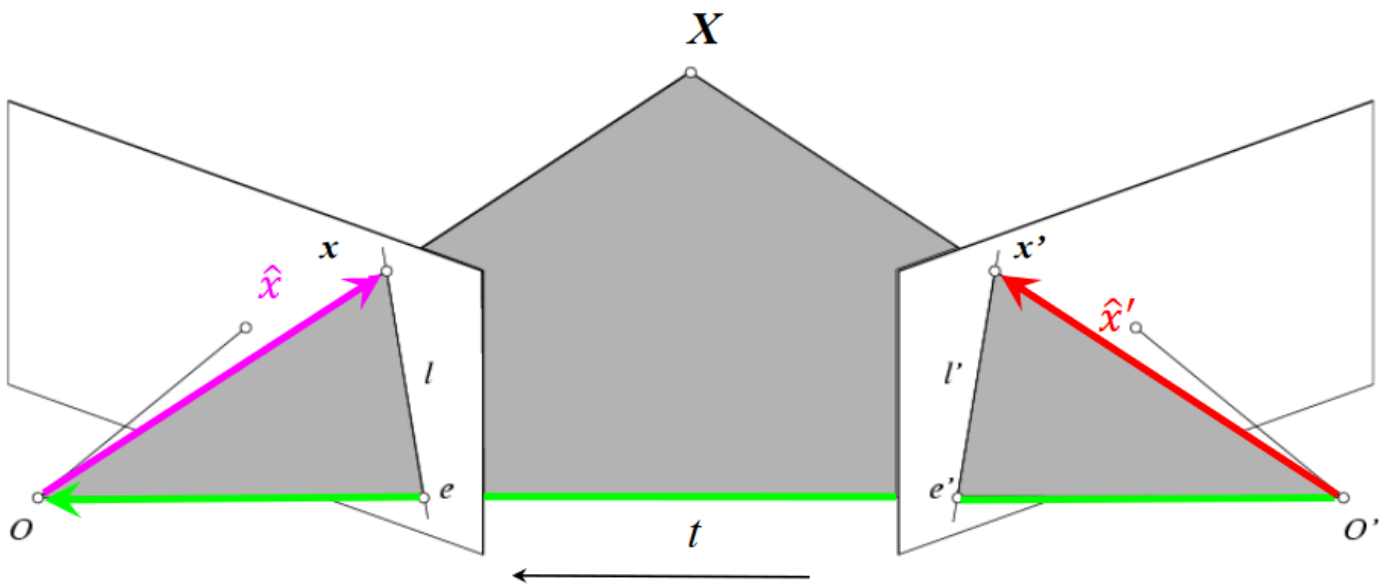
- **Baseline** – line connecting the two camera centers
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Epipolar constraint: Calibrated case



$$\hat{x} = K^{-1}x = X$$

Homogeneous 2d point (3D ray towards X)

2D pixel coordinate (homogeneous)

3D scene point

$$\hat{x}' = K'^{-1}x' = X'$$

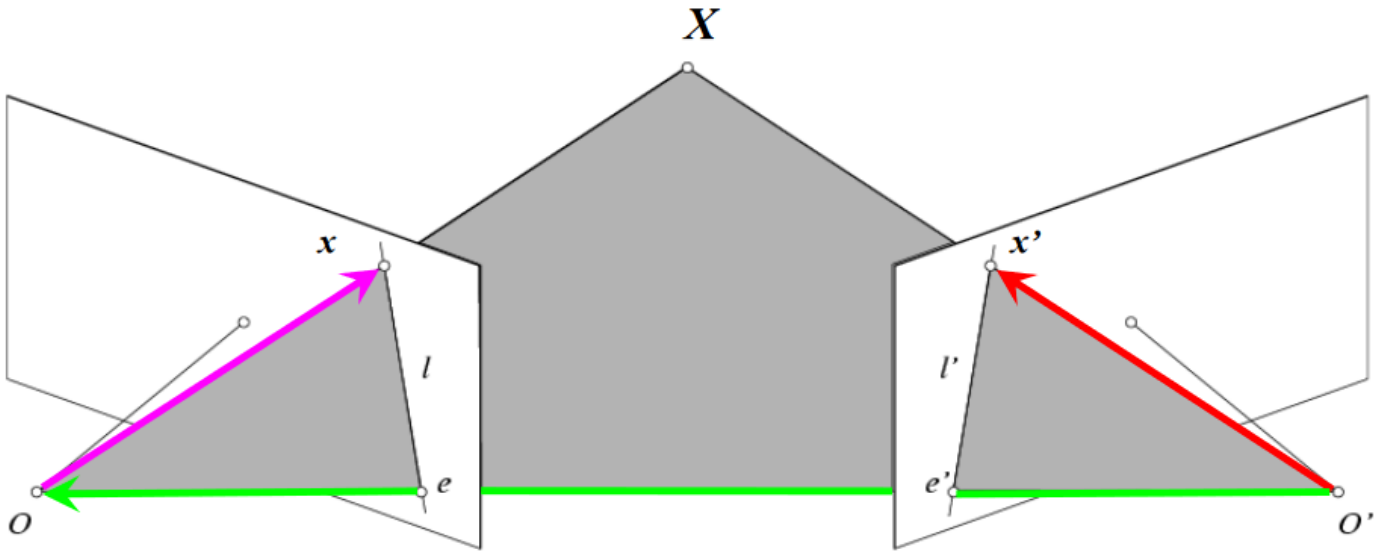
3D scene point in 2nd camera's 3D coordinates

$$\hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because $\hat{x}, R\hat{x}'$, and t are co-planar)

Essential matrix

"5-point algorithm" by David Nister computes essential matrix and then decomposes it into rotation and translation.



$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Essential Matrix
(Longuet-Higgins, 1981)

E is a 3x3 matrix which relates corresponding pairs of normalized homogeneous image points across pairs of images – for *K* calibrated cameras.

Estimates relative position/orientation.

Note: $[t]_\times$ is matrix representation of cross product

$$[t]_\times = \begin{bmatrix} 0 & -t_2 & t_1 \\ t_2 & 0 & -t_0 \\ -t_1 & t_0 & 0 \end{bmatrix}$$

After estimating the essential matrix, we extract t, R.

However, the translation t, is only estimated up to a multiplicative scale.

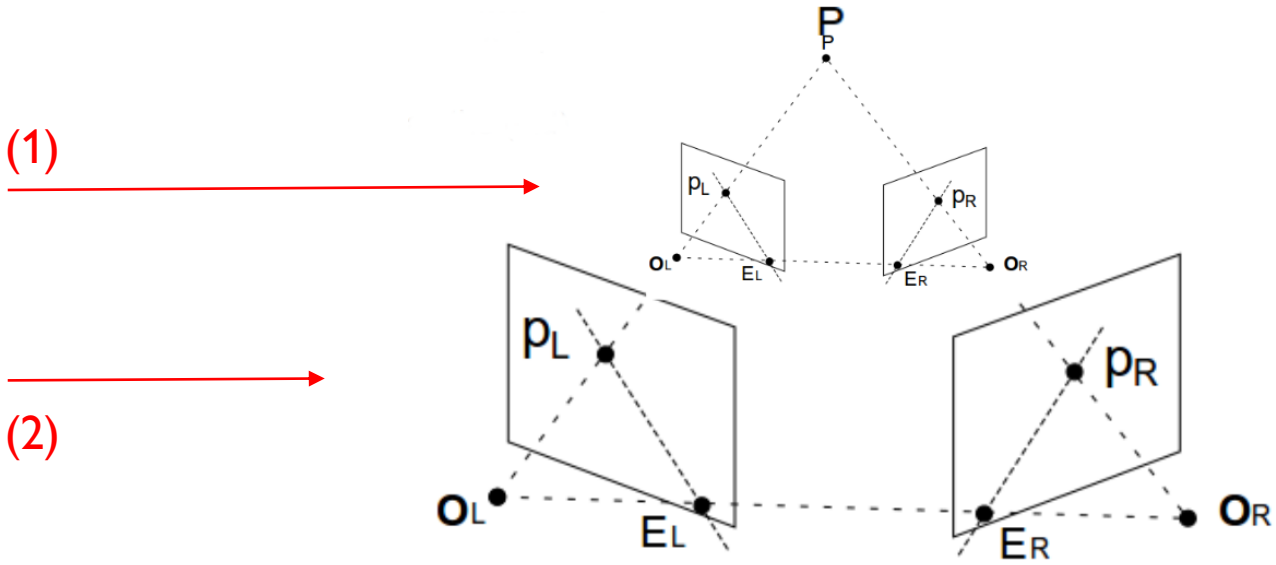
→ Translation is not fully observable with a single camera.

→ To make it observable we need stereo

Visual odometry with a single camera: translation is recovered only up to a scale

- Scale = multiplier between real-world metric distance units and estimated map distance units

Camera placements (1) and (2) generate the same observation of P. In fact, infinitely many possible placements of the two camera frames along their projection rays could have generated the same measurement.

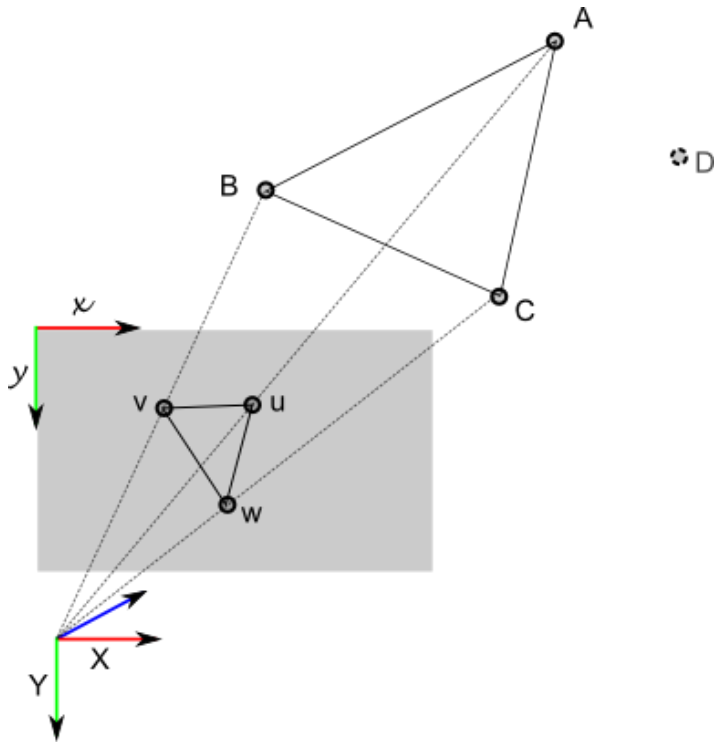


Visual odometry with a single camera: translation is recovered only up to a scale

- Scale = multiplier between real-world metric distance units and estimated map distance units

Q: Is there a way to obtain true metric distances only with a single camera?

A: The only way is to have an object of known metric dimensions in the observed scene. For example if you know distances AB, BC, CA then you can recover true translation. This is commonly referred to as the Perspective-3-Point (P3P), or in General, the Perspective-n-Point (PnP) problem.

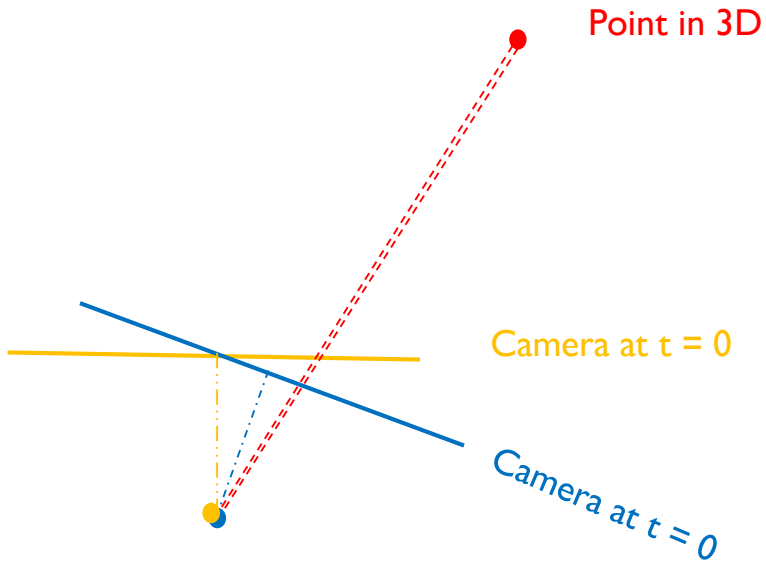


Visual odometry with a single camera: translation is recovered only up to a scale

- Scale = multiplier between real-world metric distance units and estimated map distance units

Q: Does scale remain constant throughout the trajectory of a single camera?

A: No, there is **scale drift,** which is most apparent during in-place rotations (i.e. pure rotation, no translation), because depth estimation for 3D points is unconstrained, so it is easily misestimated.



2D-to-2D Algorithm

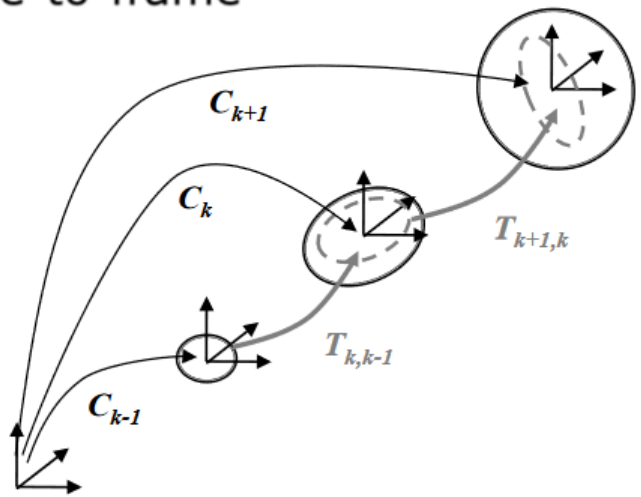
Motion estimation		
2D-2D	3D-3D	3D-2D

Algorithm 1: VO from 2D-to-2D correspondeces

1 Capture new frame I_k
2 Extract and match features between I_{k-1} and I_k,
3 Compute essential matrix for image pair I_{k-1}, I_k
4 Decompose essential matrix into R_k and t_k, and form T_k
5 Compute relative scale and rescale t_k accordingly
6 Concatenate transformation by computing $C_k = C_{k-1}T_k$
7 Repeat from 1

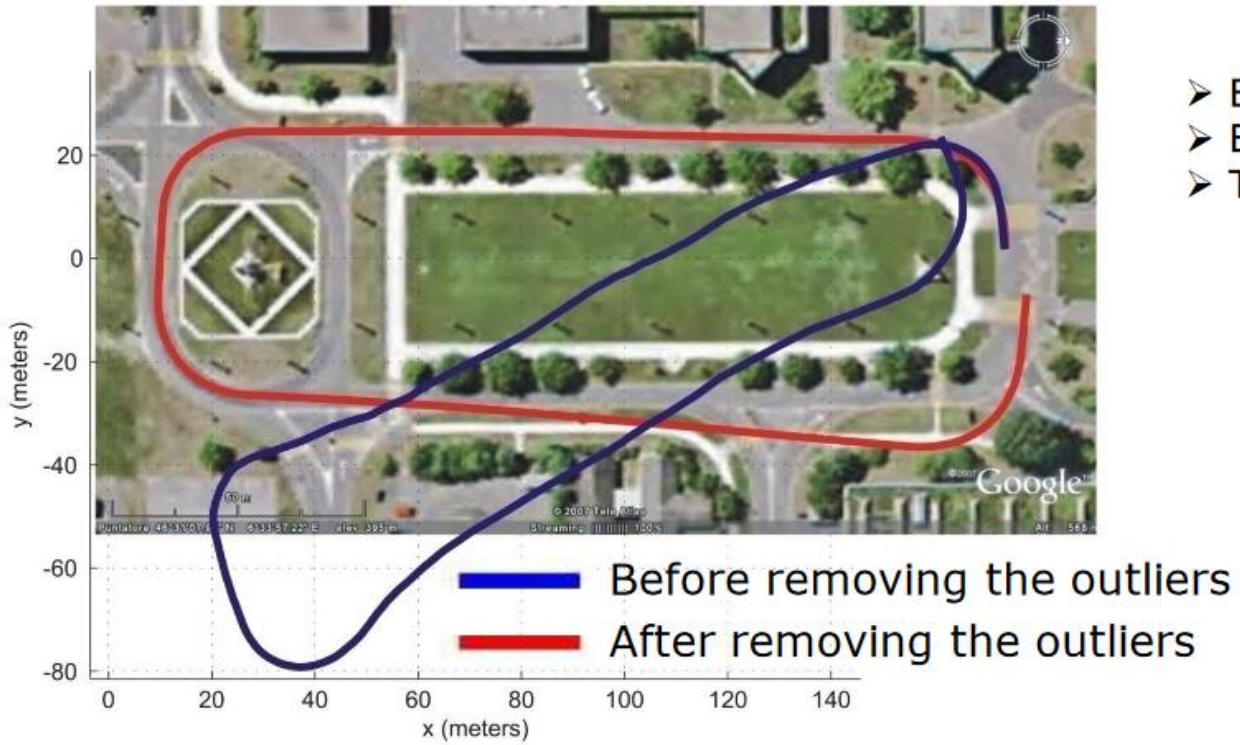
VO Drift

- The errors introduced by each new frame-to-frame motion accumulate over time
- This generates a drift of the estimated trajectory from the real path



The uncertainty of the camera pose at C_k is a combination of the uncertainty at C_{k-1} (black solid ellipse) and the uncertainty of the transformation $T_{k,k-1}$ (gray dashed ellipse)

Influence of Outliers on Motion Estimation



- Error at the loop closure: 6.5 m
- Error in orientation: 5 deg
- Trajectory length: 400 m

Are points-at-infinity useful for localization?

Points-at-infinity
can help estimate
the camera's rotation,
similarly to how we use stars
for navigation, without
estimating how far they are.

For estimating translation, most likely no. For estimating rotation, yes.
Look up "Inverse Depth Parameterization for Monocular SLAM" for more info.

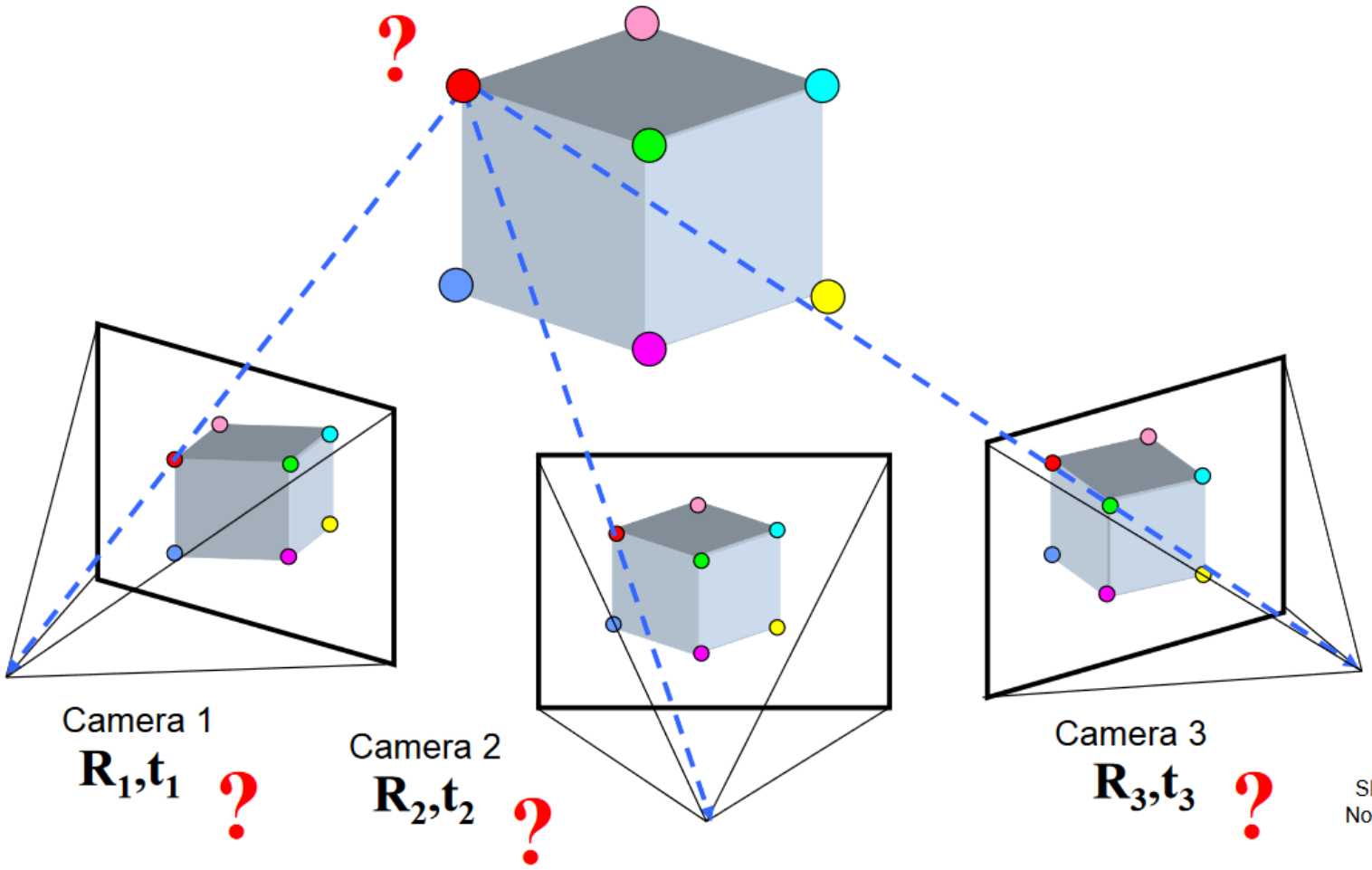
Problem #3: Visual SLAM

Structure from Motion

How can we estimate both 3D point positions and the relative camera transformations?

Sometimes also called bundle adjustment

Q: Why is it different than SLAM?



Slide credit: Noah Snavely

Structure from Motion

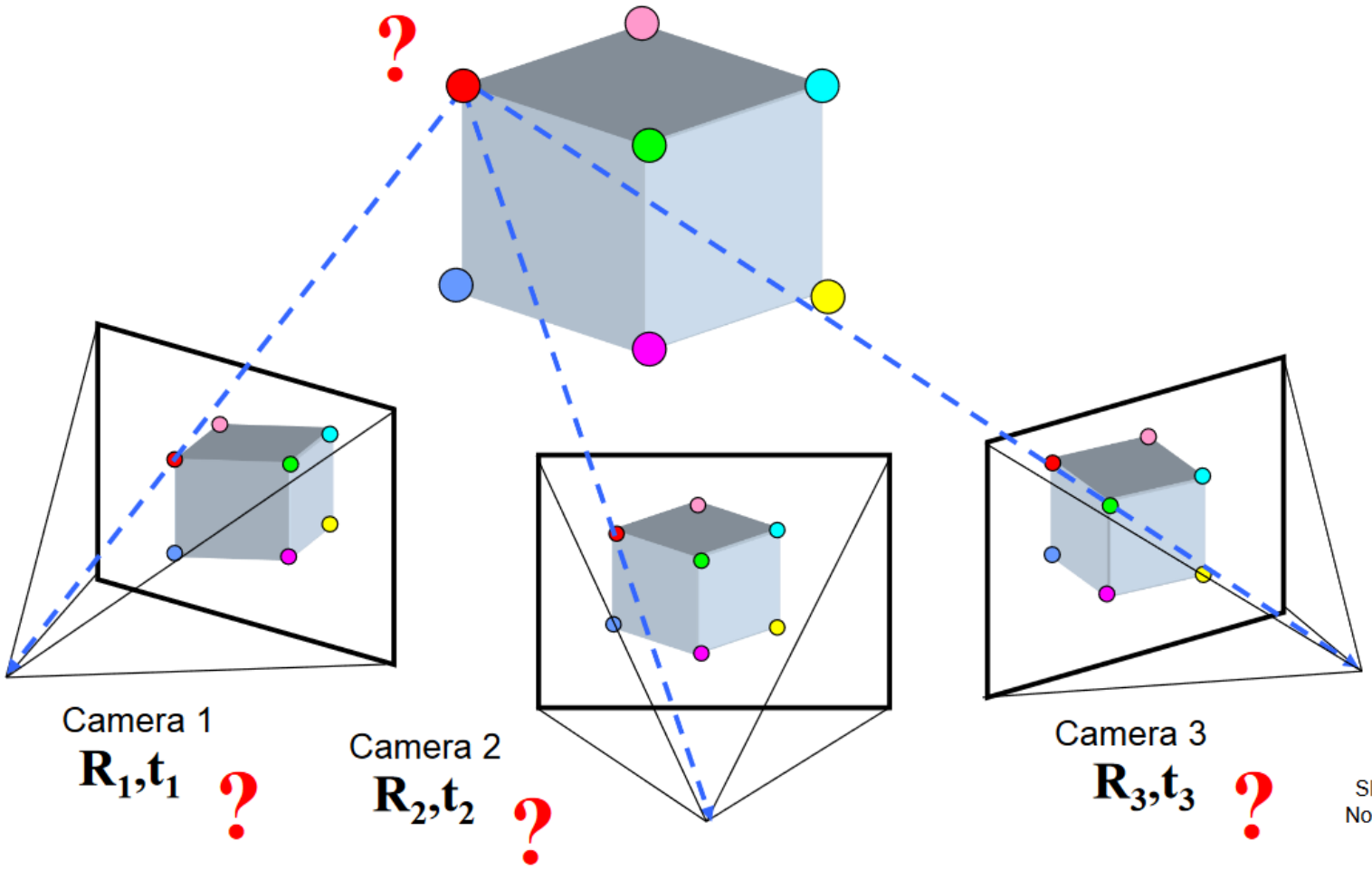
How can we estimate both 3D point positions and the relative camera transformations?

Sometimes also called bundle adjustment

Q: Why is it different than SLAM?

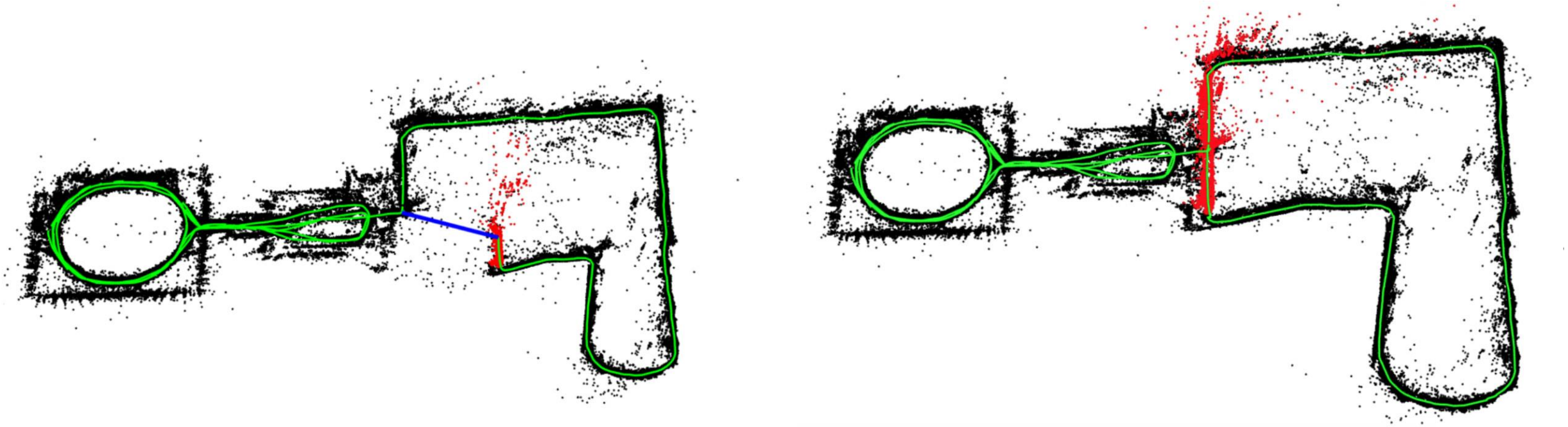
A: SLAM potentially includes

- loop closure
- dynamics constraints
- velocities, accelerations



Slide credit: Noah Snavely

Loop Closure in Visual SLAM



ORB-SLAM, Mur-Artal, Tardos, Montiel, Galvez-Lopez

Bundler (bundle adjustment/structure from motion)

Structure from Motion as Least Squares

$$\boxed{{}^k_w R^*, {}^k t^*_{kw}}, \boxed{{}^w X^*, {}^w Y^*, {}^w Z^*} = \underset{{}^w p = [{}^w X, {}^w Y, {}^w Z],\ {}^k_w R,\ {}^k t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_w R\, {}^w p + {}^k t_{kw})\ ]\ ||^2$$

Indicates the frame of the k-th camera.

$$h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)}$$

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ] ||^2 $$

Actual pixel measurement of 3D point ${}^wp$ from camera k

Expected pixel projection of 3D point ${}^wp$ onto camera k

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{fm_xX}{Z} + c_x \\ \frac{fm_yY}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Structure from Motion as Least Squares

$$ {}^{k}_{w}R^{*}, {}^{k}t^{*}_{kw}, {}^{w}X^{*}, {}^{w}Y^{*}, {}^{w}Z^{* } = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z],\ {}^{k}_{w}R,\ {}^{k}t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\ ]\ ||^{2} $$

Q: Is the scale of these two estimates accurate/unambiguous when measurements are done from a monocular (single) camera in motion? I.e. is it observable?

Note: **scale** = relationship between real-world metric distances and estimated map distances. I.e. relationship between distance units.

Structure from Motion as Least Squares

$$ {}^{k}_{w}R^{*}, {}^{k}t^{*}_{kw}, {}^{w}X^{*}, {}^{w}Y^{*}, {}^{w}Z^{*} = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z],\ {}^{k}_{w}R,\ {}^{k}t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^{k}_{w}R\,{}^{w}p + {}^{k}t_{kw})\ ]\ ||^{2} $$

Q: Is the scale of these two estimates accurate/unambiguous when measurements are done from a monocular (single) camera in motion? I.e. is it observable?

A: No, regardless of how many common keypoints are matched in between camera frames. Without external reference distance, e.g. stereo baseline, or real size of observed object, the scale is ambiguous and unobservable, just as it was in Visual Odometry.

Structure from Motion as Least Squares

$$ {}^{k}_{w}R^{*}, {}^{k}t^{*}_{kw}, {}^{w}X^{*}, {}^{w}Y^{*}, {}^{w}Z^{* } = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z],\ {}^{k}_{w}R,\ {}^{k}t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\ ]\ ||^{2} $$

Q: Is the scale of these two estimates constant during the entire experiment, if we use a monocular camera?

Structure from Motion as Least Squares

$$ {}^{k}_{w}R^{*}, {}^{k}t^{*}_{kw}, {}^{w}X^{*}, {}^{w}Y^{*}, {}^{w}Z^{* } = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z],\ {}^{k}_{w}R,\ {}^{k}t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\ ]\ ||^{2} $$

Q: Is the scale of these two estimates constant during the entire experiment, if we use a monocular camera?

A: No. During in-place rotations there is not enough baseline between camera frames to triangulate new points. So, error in structure and in motion accumulates → **scale drift**

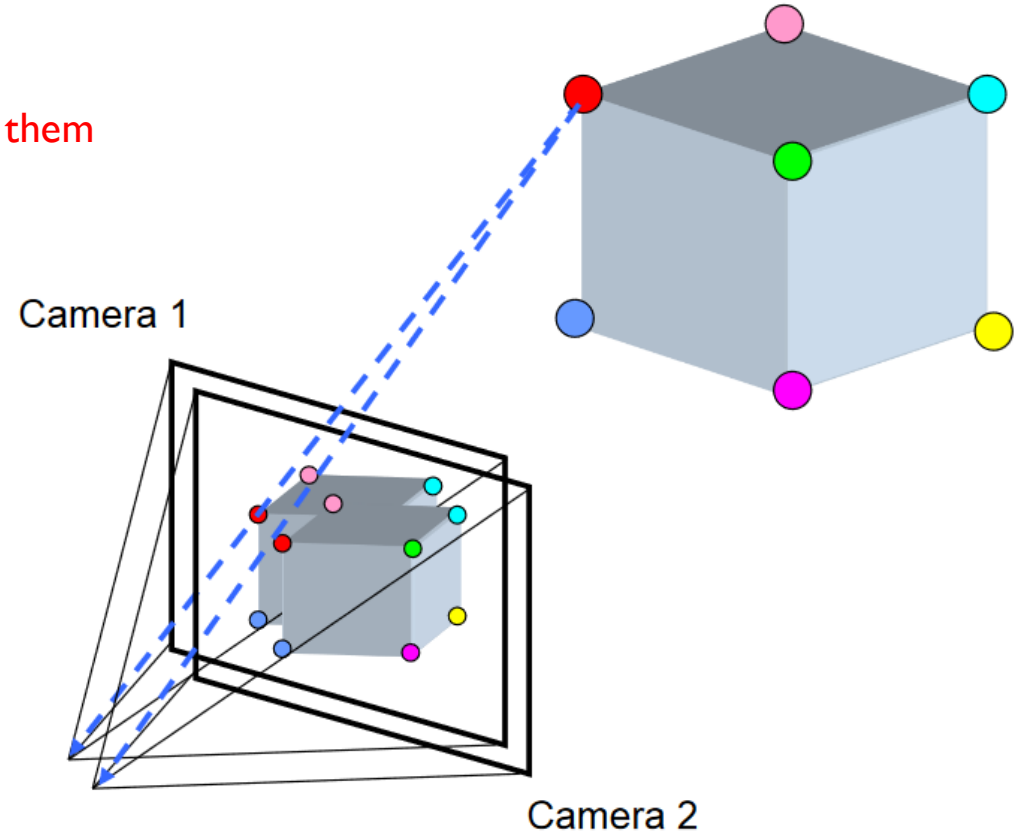
Similarly to visual odometry with a single camera.

Problem #4: optical flow

Multi-view geometry problems

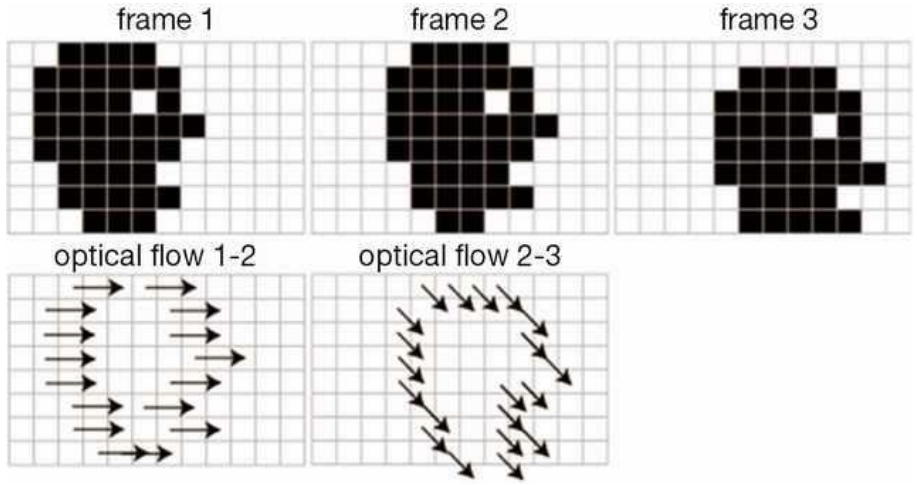
- **Optical flow:** Given two images, find the location of a world point in a second close-by image with no camera info.

3D point coordinates are unknown, but we won't try to estimate them



Camera frame transformations are unknown, but we won't try to estimate them

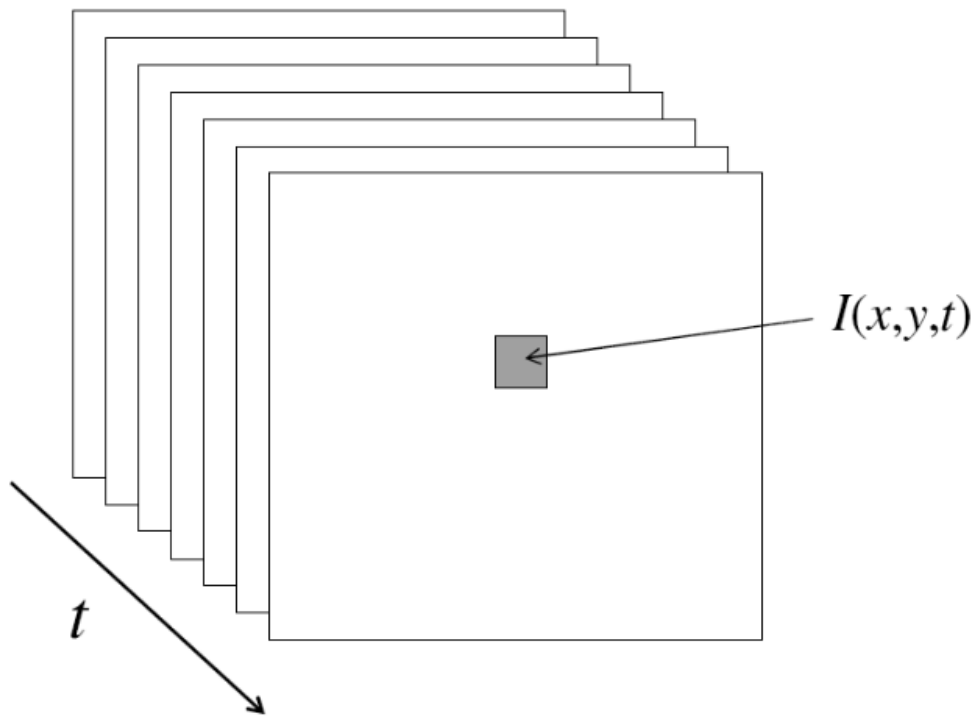
We are estimating pixel displacement from one image to the next



http://tcr.amegroups.com/article/viewFile/3200

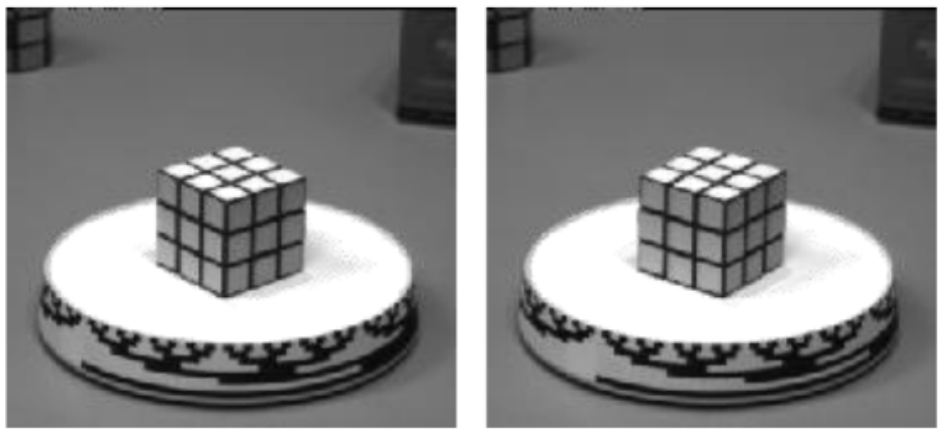
Video

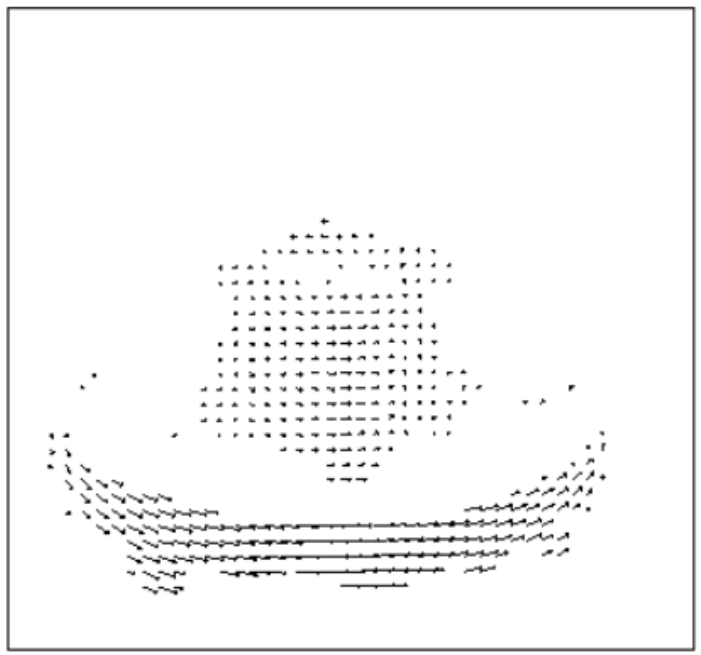
- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Motion estimation: Optical flow

Optic flow is the **apparent** motion of objects or surfaces





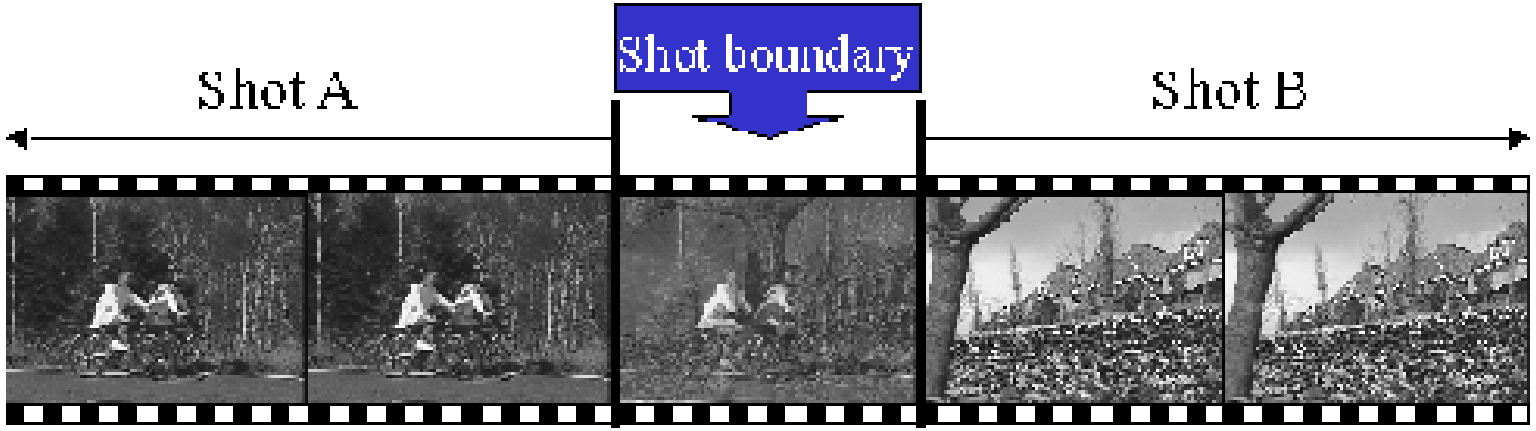
Will start by estimating motion of each pixel separately
Then will consider motion of entire image

Motion Applications: Segmentation of video

- Background subtraction
  - A static camera is observing a scene
  - Goal: separate the static *background* from the moving *foreground*

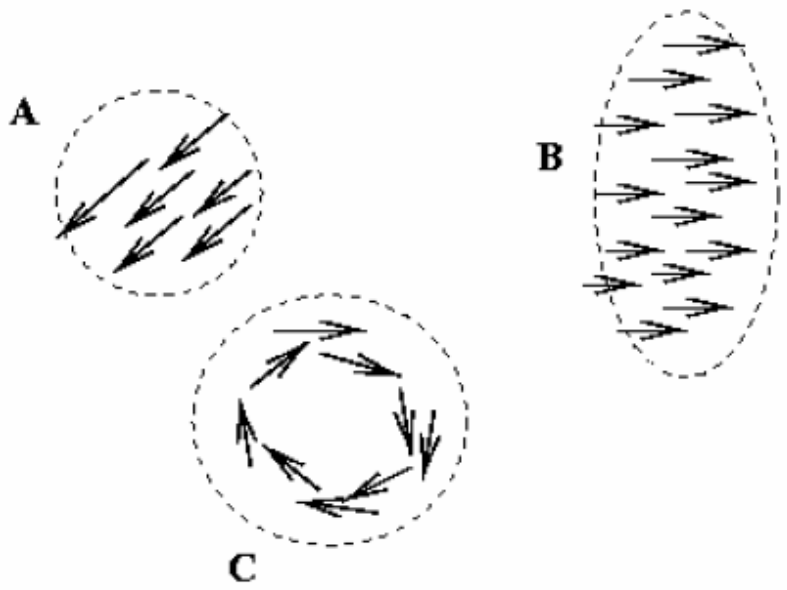
Motion Applications: Segmentation of video

- Shot boundary detection in edited video
  - Edited video is usually composed of *shots* or sequences showing the same objects or scene
  - Goal: segment video into shots for summarization and browsing (each shot can be represented by a single keyframe in a user interface)
  - Difference from background subtraction: the camera is not necessarily stationary



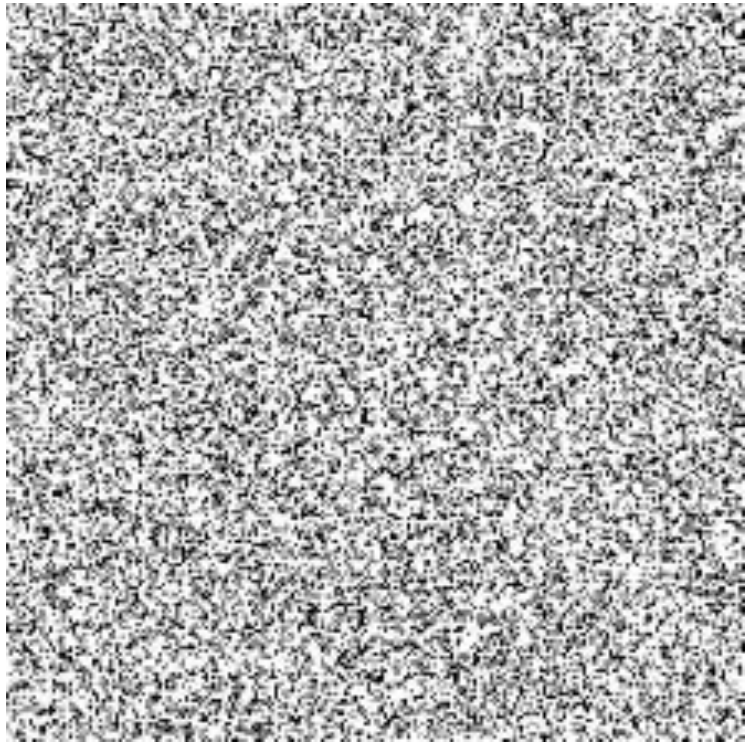
Motion Applications: Segmentation of video

- Background subtraction
- Shot boundary detection
- Motion segmentation
  - Segment the video into multiple coherently moving objects



Motion and perceptual organization

- Sometimes, motion is the only cue



Motion and perceptual organization

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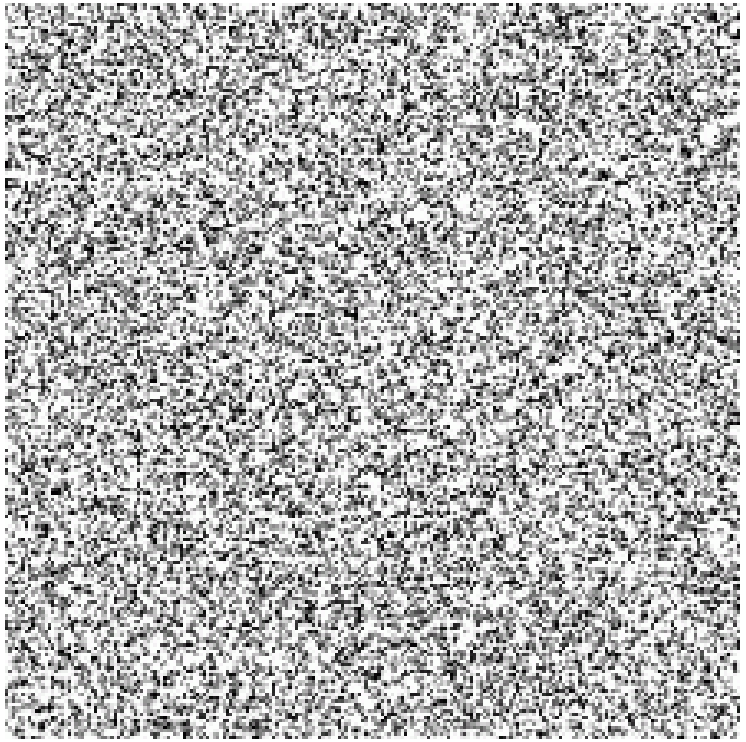
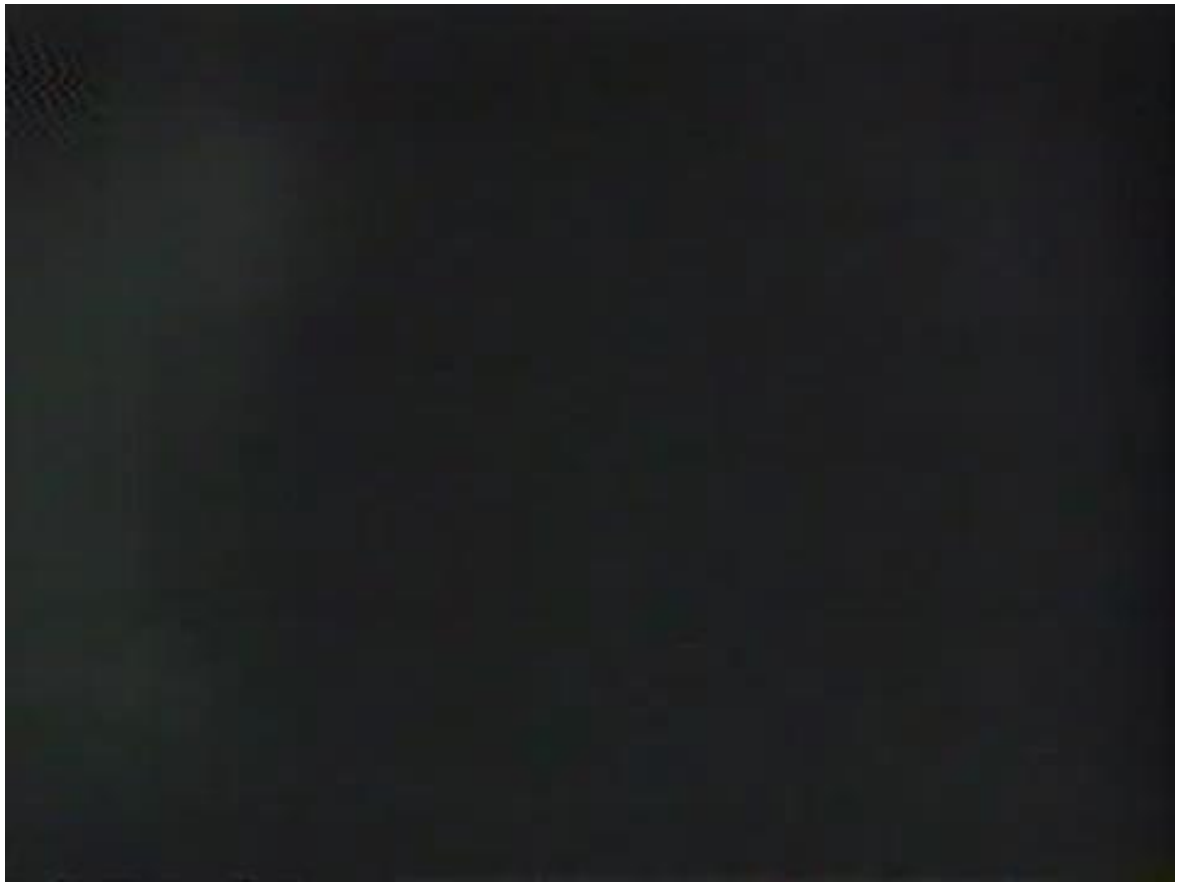
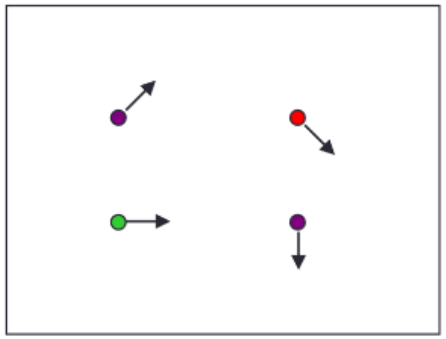


Motion and perceptual organization

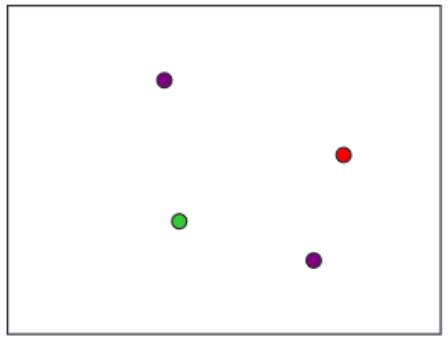


Experimental study of apparent behavior.
Fritz Heider & Marianne Simmel. 1944

Problem definition: optical flow



$I(x, y, t)$



$I(x, y, t+1)$

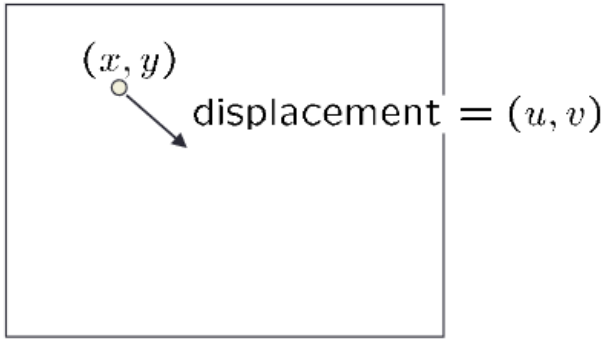
How to estimate pixel motion from image $I(x,y,t)$ to $I(x,y,t+1)$?

- Solve pixel correspondence problem
  - Given a pixel in I(x,y,t), look for nearby pixels of the same color in *I(x,y,t+1)*

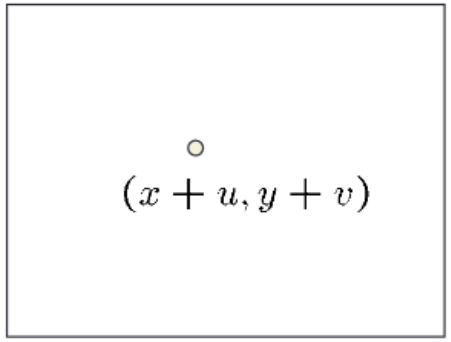
Key assumptions

- Small motion: Points do not move very far
- Color constancy: A point in *I(x,y,t)* looks the same in *I(x,y,t+1)*
  - For grayscale images, this is brightness constancy

Optical flow constraints (grayscale images)



$I(x, y, t)$



$I(x, y, t+1)$

- Let's look at these constraints more closely
  - Brightness constancy constraint (equation)

$$I(x, y, t) = I(x+u, y+v, t+1)$$

  - Small motion: (u and v are less than 1 pixel, or smooth)

From Taylor expansion of I

$$I(x+u, y+v, t+1) = I(x, y, t+1) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

Optical flow equation

- Combining these two equations

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x,y,t) \\
&\approx I(x,y,t+1) + I_x u + I_y v - I(x,y,t)
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

Optical flow equation

- Combining these two equations

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x, y, t) \\
&\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t) \\
&\approx [I(x, y, t+1) - I(x, y, t)] + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

Optical flow equation

- Combining these two equations

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x,y,t) \\
&\approx I(x,y,t+1) + I_x u + I_y v - I(x,y,t) \\
&\approx [I(x,y,t+1) - I(x,y,t)] + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

In the limit as u and v go to zero, this becomes exact

Brightness constancy constraint equation

$$I_x\, u + I_y\, v + I_t = 0$$

The brightness constancy constraint

Can we use this equation to recover image motion (u,v) at each pixel?

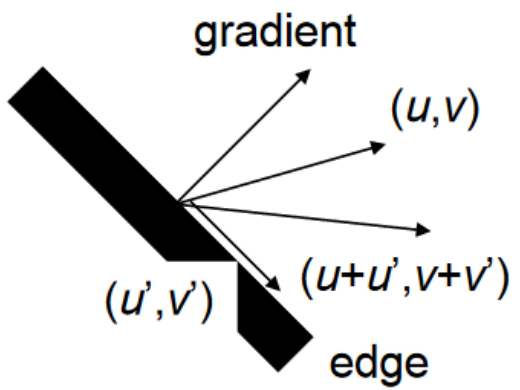
$$0 = I_t + \nabla I \cdot <u, v> \quad \text{or} \quad I_x u + I_y v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns (u,v)

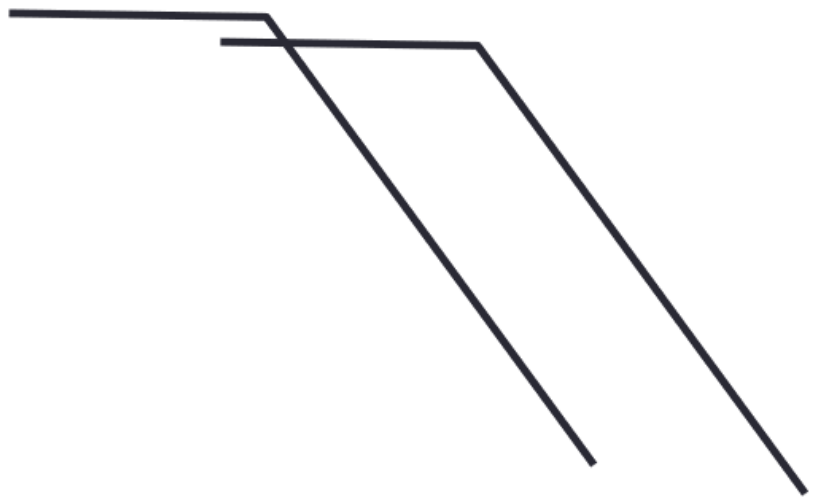
The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

If (u, v) satisfies the equation, so does (u+u', v+v') if

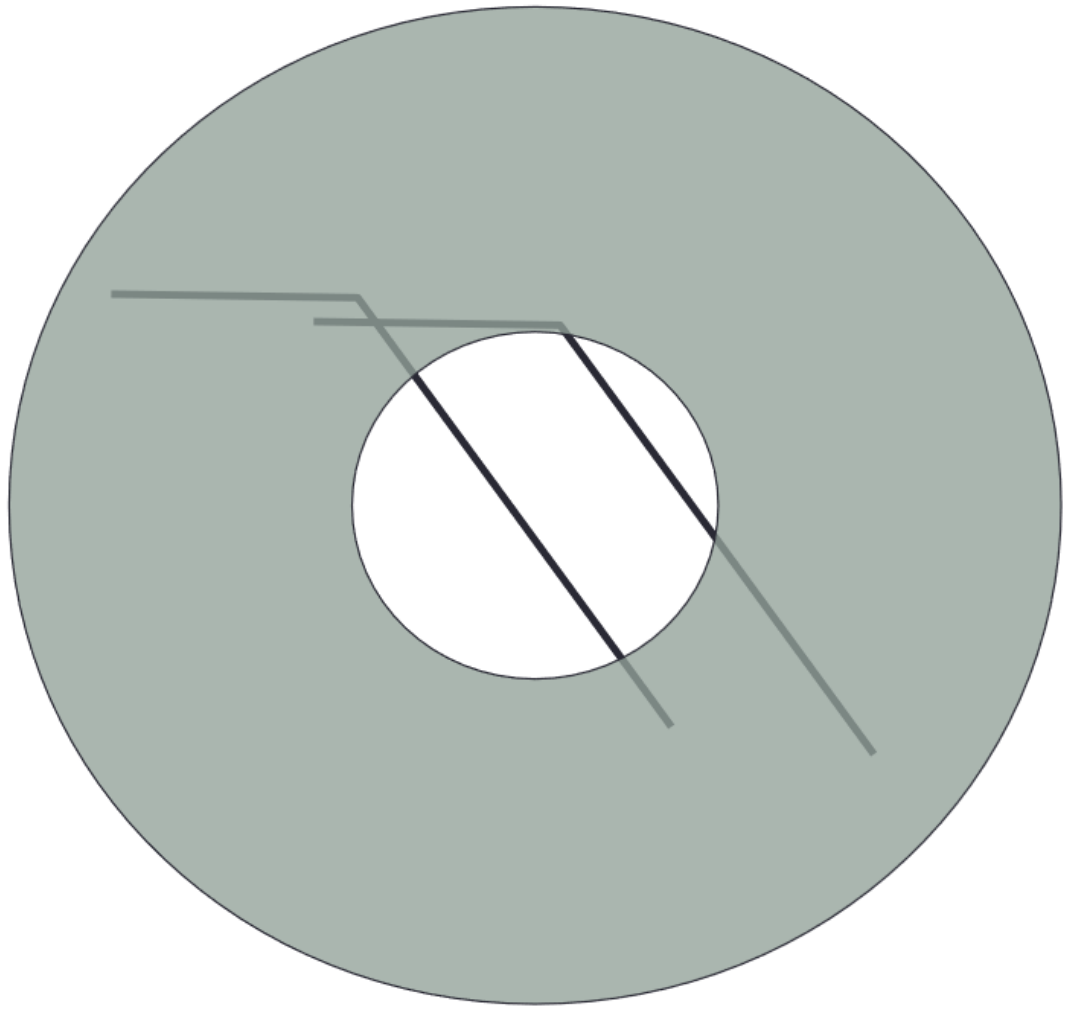
$$\nabla \mathrm{I} \cdot \left[\mathrm{u'}\ \mathrm{v'}\right]^{\mathrm{T}} = 0$$



Aperture problem



Aperture problem



Solving the ambiguity…

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

- How to get more equations for a pixel?
- **Spatial coherence constraint**
- Assume the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u\ v]$$

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

Solving the ambiguity…

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

$$\underset{25\text{x}2}{A} \quad \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$