Exercise 7

Deadline: 12.06.2017, 2:15 pm
Regulations: You should hand in the exercises in groups of two or three persons. Please send a *compressed* (!) directory or file containing your solutions including all graphics, descriptions and source code to *thorsten.beier@iwr.uni-heidelberg.de*. The subject line of this email should start with [MLCV17][EX07] followed by the full names of all group members. Please cross-reference your code files in your writeup, such that it is clear which file has to be run for each exercise.

1 Gaussian Graphical Model (20 points)

$$p(x|\mu) \propto \underbrace{\exp\left(-\frac{1}{2\sigma^2} \cdot (x-\mu)^T(x-\mu)\right)}_{\text{"unaries"}} \cdot \underbrace{\exp\left(-\frac{1}{2}x^T Q x\right)}_{\text{"second order"}} \tag{1}$$

A large value of σ^2 indicated that there is a lot of uncertainty in the measurements μ.

The energy is given by the negative log of the probabilities.

$$E(x) = \frac{1}{2\sigma^2} \underbrace{\left(\sum_i x_i^2 - 2\mu_i x_i + \mu_i^2\right)}_{\text{"unaries"}} + \underbrace{\left(\frac{1}{2}\sum_i \sum_j q_{ij} x_{ij}\right)}_{\text{"second order"}} \tag{2}$$

To find argmin we set the derivative equal to zero

$$\frac{\partial E(x)}{\partial x} = -\frac{1}{\sigma^2}(x-\mu) + Qx \stackrel{!}{=} 0 \tag{3}$$

This yields this sparse system of equations which can be solved for x.

$$\mu = x\left(I + \sigma^2 Q\right) \tag{4}$$

Consider a gaussian model model with a grid structure.

The Q matrix of a gaussian graphical model encodes the graph structure of the problem. In this exercise we will set up Q such that any pixel is connected with its four nearest neighbors.

We will try two different types of entries for Q. In the first part, all non-zeros **off-diagonal** values of Q should have a constant value. What sign should the non-zero value have?

Next, we try non constant values $\alpha \cdot \exp(-\gamma||c_i - c_j||)$ where c_i and c_j are the gray/RGB value of pixel i and j and $\gamma > 0$. What sign should alpha have?

What is good choice for the values on the diagonal of Q?

1.1 Implementation 20 p

Take an rgb image as `skimage.data.astronaut()` [1].

Add gaussian noise with some strength s to each pixel. Set up the Q matrix, find the argmin by solving the sparse system of linear equations.

Do this for different noise levels s . You can measure the performance of your algorithm by taking the sum squared differences between the image without noise and the result.

How do the parameters α, γ, s and σ effect the results / performance?

What is effect of the values q_{ii} on the diagonal?

1.2 Denoising Bonus 10 p

Try all denoising algorithms on `http://www.ipol.im/` . Each algorithm should have a demo section where you can run the algorithm in your browser. Compare the results of the different algorithms. How well does your code compare against these solutions?

[1] If this image is to large / your computer to small, feel free to a) resize the image b) take a crop of the image.