Exercise 02 Raphael Michel and Florian Stoertz

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0.1 Exercise 1

In [87]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy import ndimage, misc, stats
from skimage import img_as_float
%matplotlib inline
```

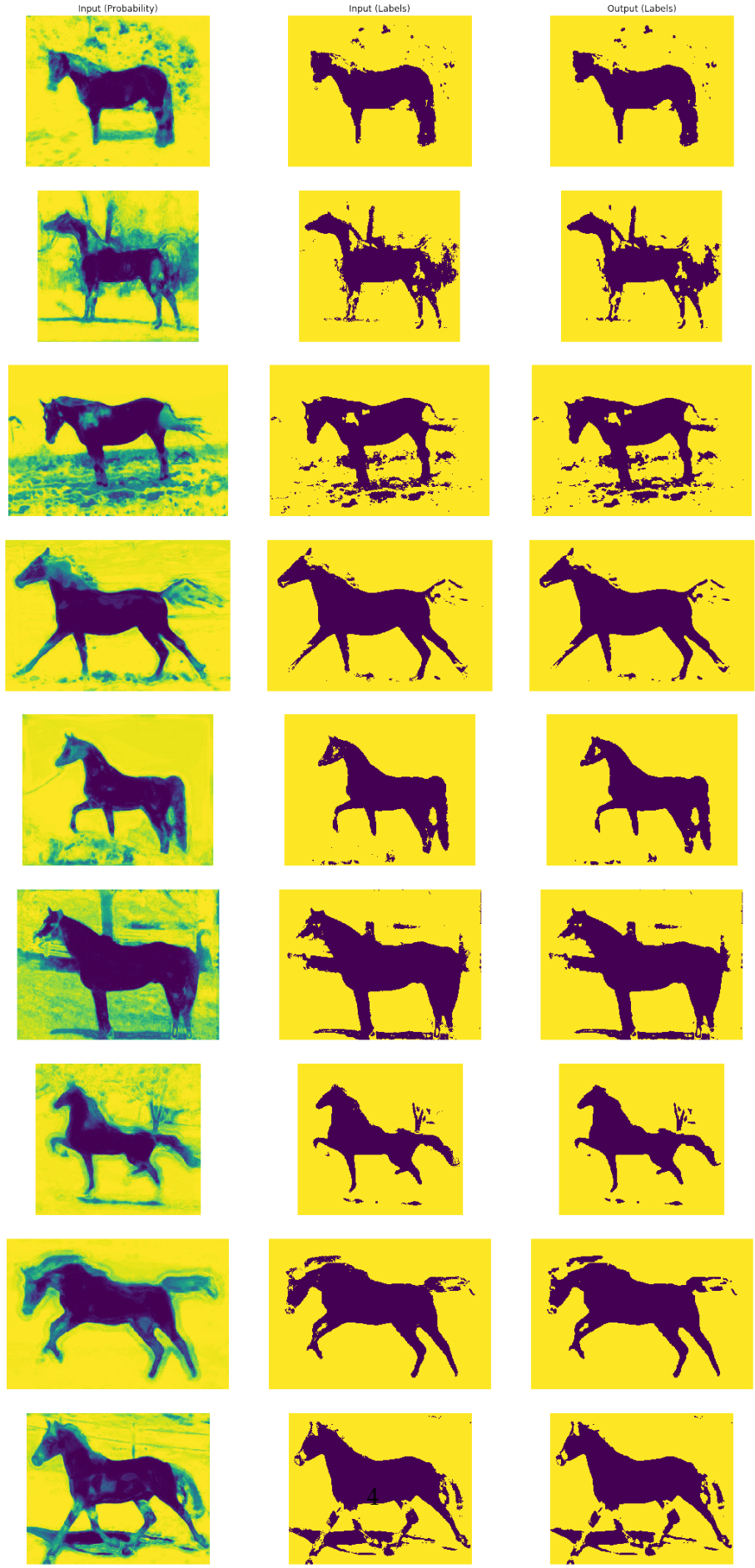
In [88]:
```
def iterated_conditonal_modes(unaries, beta, labels=None):
    shape = unaries.shape[0:2]
    n_labels = unaries.shape[2]

    if labels is None:
        labels = numpy.argmin(unaries, axis=2)

    continue_search = True
    while continue_search:
        continue_search = False

        for x0 in range(1, shape[0]-1):
            for x1 in range(1, shape[1]-1):
                current_label = labels[x0, x1]
                min_energy = float('inf')
                best_label = None

                for l in range(n_labels):
                    # unary terms
                    energy = unaries[x0, x1, l]

                    # pairwise terms
                    if l != labels[x0-1, x1]:
                        energy += beta
                    if l != labels[x0+1, x1]:
                        energy += beta
                    if l != labels[x0, x1-1]:
                        energy += beta
                    if l != labels[x0, x1+1]:
                        energy += beta
```

```
                            if energy < min_energy:
                                min_energy = energy
                                best_label = l

                        if best_label != current_label:
                            labels[x0, x1] = best_label
                            continue_search = True


            return labels
```

In [89]:
```
n_imgs = 9
n_labels = 2

# regularizer strength
beta = 10

fig, axs = plt.subplots(n_imgs, 3, figsize=(15, 30))
axs[0, 0].set_title('Input (Probability)')
axs[0, 1].set_title('Input (Labels)')
axs[0, 2].set_title('Output (Labels)')
for imgnr in range(n_imgs):
    in_file = ndimage.imread('out%d.png' % imgnr)
    shape = in_file.shape

    # Load output from exercise 01
    prob_horse = img_as_float(in_file)

    # Normalize
    prob_horse = prob_horse / np.max(prob_horse)

    # Avoid zero probabilities
    prob_horse = np.clip(prob_horse, 0.00001, 0.99999)

    axs[imgnr, 0].imshow(prob_horse)
    axs[imgnr, 0].axis('off')
    axs[imgnr, 1].imshow(prob_horse > 0.5)
    axs[imgnr, 1].axis('off')

    # Unary factors: Negative logarithm of probability
    unaries = np.zeros(shape + (2,))
    unaries[:, :, 0] = - np.log(prob_horse)
    unaries[:, :, 1] = - np.log(1 - prob_horse)

    labels = iterated_conditonal_modes(unaries, beta=beta)
    axs[imgnr, 2].imshow(1 - labels)
    axs[imgnr, 2].axis('off')
```

```
        fig.tight_layout()
        fig.show()
```

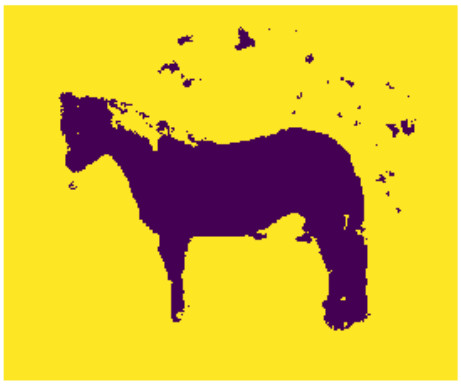
```
/home/ubuntu/.local/lib/python3.5/site-packages/matplotlib/figure.py:403: UserWarning: matplotl
  "matplotlib is currently using a non-GUI backend, "
```


Input (Probability)
Input (Labels)
Output (Labels)

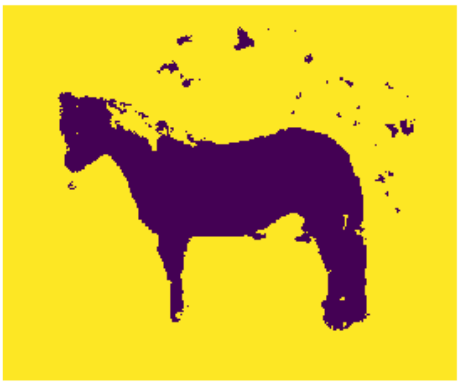
In [90]:
```
n_imgs = 9
n_labels = 2

in_file = ndimage.imread('out0.png')
shape = in_file.shape

# Load output from exercise 01
prob_horse = img_as_float(in_file)

# Normalize
prob_horse = prob_horse / np.max(prob_horse)

# Avoid zero probabilities
prob_horse = np.clip(prob_horse, 0.00001, 0.99999)

# Unary factors: Negative logarithm of probability
unaries = np.zeros(shape + (2,))
unaries[:, :, 0] = - np.log(prob_horse)
unaries[:, :, 1] = - np.log(1 - prob_horse)

betas = (0.0001, 0.001, 0.01, 0.1, 1, 10, 100, 1000)
fig, axs = plt.subplots(int(np.ceil(len(betas) / 2)), 2, figsize=(10, 15))

for i, beta in enumerate(betas):
    labels = iterated_conditonal_modes(unaries, beta=beta)
    axs[i//2, i % 2].set_title('beta = %f' % beta)
    axs[i//2, i % 2].imshow(1 - labels)
    axs[i//2, i % 2].axis('off')

axs[i//2, 1].axis('off')
fig.tight_layout()
fig.show()
```

```
/home/ubuntu/.local/lib/python3.5/site-packages/matplotlib/figure.py:403: UserWarning: matplotl
  "matplotlib is currently using a non-GUI backend, "
```

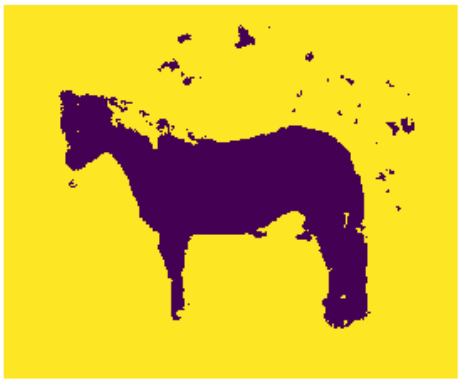
beta = 0.000100



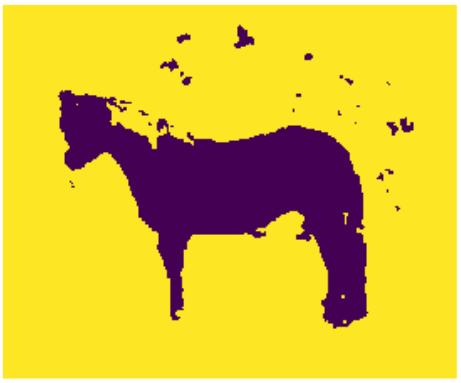
beta = 0.001000



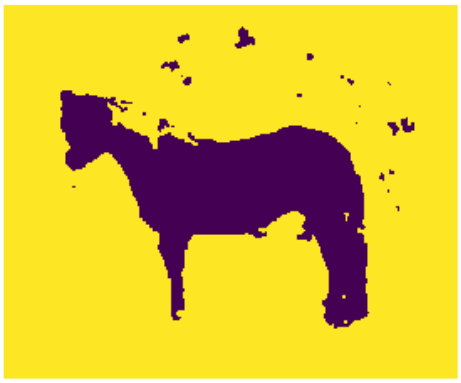
beta = 0.010000



beta = 0.100000



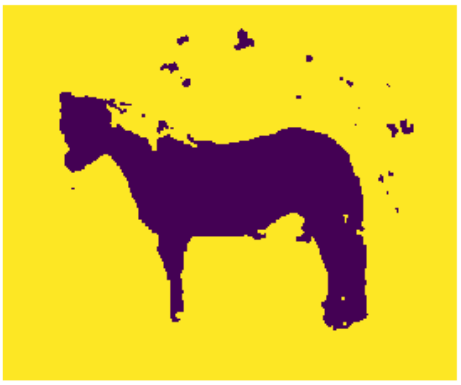
beta = 1.000000



beta = 10.000000

beta = 100.000000

beta = 1000.000000



Different values of beta make the smoothing stronger, i.e. small objects are more strongly penalized. However, the overall performance is still a bit disappointing. We believe that the reason for this is that the penalty only takes the very next neighbors into account which is a way to small radius to work well on a picture of this size.

0.2 Exercise 2

1. The new random variable x_z can take 8 different values, one for each combination of x_1, x_2, x_3. We will name them 1-8 in the order they are presented in the table of the exercise sheet.

2. The energies of the pairwise factors $\phi_{iz}(x_i, x_z)$ are then given by $E = E_0 \times E_1 \times E_2$ using the E_k values from the following table:

$x_i, i \in \{0,1,2\}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_z	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
E_0	$a^{1/3}$	$b^{1/3}$	$c^{1/3}$	$d^{1/3}$	∞	∞	∞	∞	∞	∞	∞	∞	$e^{1/3}$	$f^{1/3}$	$g^{1/3}$	$h^{1,3}$
E_1	$a^{1/3}$	$b^{1/3}$	∞	∞	$e^{1/3}$	$f^{1/3}$	∞	∞	∞	∞	$c^{1/3}$	$d^{1/3}$	∞	∞	$g^{1/3}$	$h^{1,3}$
E_2	$a^{1/3}$	∞	$c^{1/3}$	∞	$e^{1/3}$	∞	$g^{1/3}$	∞	∞	$b^{1/3}$	∞	$d^{1/3}$	∞	$f^{1/3}$	∞	$h^{1,3}$