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# Exercise 6

**Deadline: 2.06.2017, 2:15 pm**
**Regulations:** You should hand in the exercises in groups of two or three persons. Please send a *compressed* (!) directory or file containing your solutions including all graphics, descriptions and source code to *thorsten.beier@iwr.uni-heidelberg.de*. The subject line of this email should start with [MLCV17][EX06] followed by the full names of all group members. Please cross-reference your code files in your writeup, such that it is clear which file has to be run for each exercise.

## 1 Potts MRF with chain structure (10 points)

Consider a potts model with binary variables with a chain structure. The argmin of such a model can be found by applying dijkstra's shortest path algorithm on an auxiliary graph.

Implemented a function which finds the argmin for such a graphical model.

Run your function on a chain with 20 variables. Use random unary energies in $[0, 1]$. Try different values for $\beta$ $(0.01, 0.1, 0.2, 0.5, 1.0)$

There are some limitations to dijkstra's algorithm if the graph has negative weights. How can one overcome this limitation? Use random unary energies in $[-1, 1]$. Try different values for $\beta$ $(-1.0, -0.1, -0.01, 0.01, 0.1, 0.2, 0.5, 1.0)$

## 2 Potts MRF with grid sructure (10 points)

Consider a potts model with binary variables with a grid structure. Let $E(X)$ be the energy function of this model. We can split this model into two subproblems $E_h(X)$ and $E_v(X)$. We demand that for any configuration $X$, the sum of the energies of the two subproblems must match the energy of the original energy function $E(X)$.

$$E(X) = E_h(X) + E_v(X) \tag{1}$$

We can do this by the following rules:

- each unary $\phi_i(X_i)$ is split into $\phi_h(X_i)$ and $\phi_v(X_i)$ such that $\phi_i(X_i) = \phi_h(X_i) + \phi_v(X_i)$
- $E_h(X)$ contains only the horizontal pairwise factors
- $E_v(X)$ contains only the vertical pairwise factors

We now find the *argmin* $E_h(X)$ and $E_v(X)$ independent of each other. $E_h(X)$ and $E_v(X)$ can be solved by using the solver from section 1 on all rows/columns of the two problems.

### 2.1 Implementation (5 Points)

Implement code to optimize $E_h(X)$ and $E_v(X)$. Use the same random randomized weighs as in section 1

## 2.2 Interpretation (5 Points)

With $\hat{X}^h = argmin_X E_h(X)$ and $\hat{X}^v = argmin_X E_v(X)$ what do we know about $E(\hat{X})$ with $\hat{X} = argmin_X E(X)$.

Is $E(\hat{X}) \leq E_h(\hat{X}^h) + E_v(\hat{X}^v)$ true?
Is $E(\hat{X}) \geq E_h(\hat{X}^h) + E_v(\hat{X}^v)$ true? If $\hat{X}^h = \hat{X}^v$, what can we conclude from that.

## 2.3 Bonus (10 Points)

Can you find a better strategy to make $\hat{X}^h$ agree $\hat{X}^v$ in as many variables as possible. Are there better rules to distribute the energy between the two subproblems. **Hint:** Dual Decomposition.