Exercise 11

Deadline: 17.07.2017, 2:15 pm
Regulations: You should hand in the exercises in groups of two or three persons. Please send a *compressed* (!) directory or file containing your solutions including all graphics, descriptions and source code to *thorsten.beier@iwr.uni-heidelberg.de*. The subject line of this email should start with [MLCV17][EX11] followed by the full names of all group members. Please cross-reference your code files in your writeup, such that it is clear which file has to be run for each exercise.

1 Fischer Information / Natural Gradients

In this exercise we will compute the Fischer Information Matrix for a certain neural network and two datasets. The neural network is given by:

$$z = sigmoid(\sin(\phi) \cdot x_0 + \cos(\phi) \cdot x_1 + r) \tag{1}$$

Where x_0 and x_1 are the input features and ϕ and r are the weighs of the neural network. The output z is a single scalar. We want to use this network to do binary classification. Therefore the targets y are in $\{0, 1\}$. y is Bernoulli distributed and the probability for $y = 0$ is given by:

$$p(y = 0) = z \tag{2}$$

and

$$p(y = 1) = 1 - z \tag{3}$$

As loss function we use cross entropy loss.

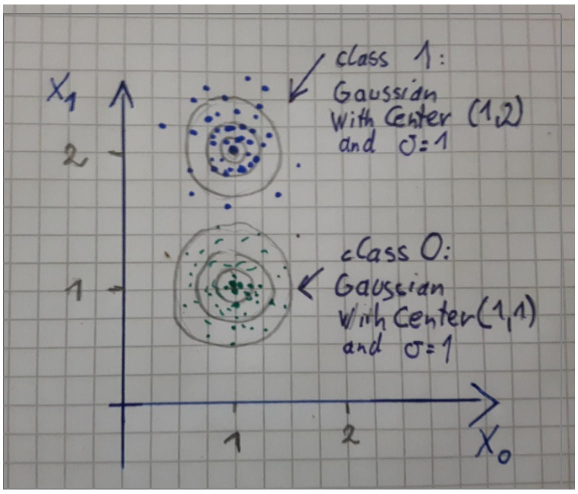

Figure 1: We will use toy datasets which can be generated by sampling from two gaussians.

The dataset we are going to use can be generated by sampling from gaussian distributions as depicted in fig. 1.

All the following experiments shall be done for two different dataset. **Dataset A** is exactly shown in 1. **Dataset B** the same as A, but we shift the whole dataset to the right by 5 units. Therefore the center of class 0 and 1 is given by $(6, 1)$ and $(6, 2)$.

1.1 Forward Pass (10P)

We can easily compute the loss for a given choice of the parameters ϕ and r.

Generated N evenly spaces values for ϕ in the range of 0 to 2π, and M evenly spaces values for r in the range of -5 to $+5$ and compute the loss for any combination of ϕ and r. This should give an image/heat map with the shape $N \times M$. Show this heat map for dataset A and B. Comment on the results.

1.2 Fischer Matrix (10P)

Use the same evenly spaces values for ϕ and r as above. Compute the entries of the fischer matrix $F_{\phi,\phi}$, $F_{r,r}$ and $F_{r,\phi}$.

Use:

$$F_{a,b} = \sum_{x \in X} \sum_{y \in \{0,1\}} \left(\frac{\partial}{\partial a} \log p(y|x)\right)\left(\frac{\partial}{\partial b} \log p(y|x)\right) \cdot p(y|x) \tag{4}$$

Show $F_{\phi,\phi}$, $F_{r,r}$ and $F_{r,\phi}$ as 3 heat maps. Do this for dataset A and dataset B. Comment on the results.

1.3 Bonus: Gradients and Natural Gradients (10P)

To do a gradient step we need to derive the loss w.r.t. ϕ and r.

Compute $\frac{\partial loss}{\partial \phi}$ and $\frac{\partial loss}{\partial r}$.

Use the same evenly spaces values for ϕ and r as above and compute the gradient g for each of these values. Visualize the gradients as a vector field.

Compute the natural gradient by multiplying the inverse fischer matrix $F^{-}1$ with the gradient g. Visualize the natural gradient as a vector field.

Do all of the above for dataset A and B. Comment on the results.

2 Additional Material

- `https://hips.seas.harvard.edu/blog/2013/01/25/the-natural-gradient/`
- `http://kvfrans.com/a-intuitive-explanation-of-natural-gradient-descent/` (do not take this one too seriously)