Exercise 11 Raphael Michel and Florian Stoertz

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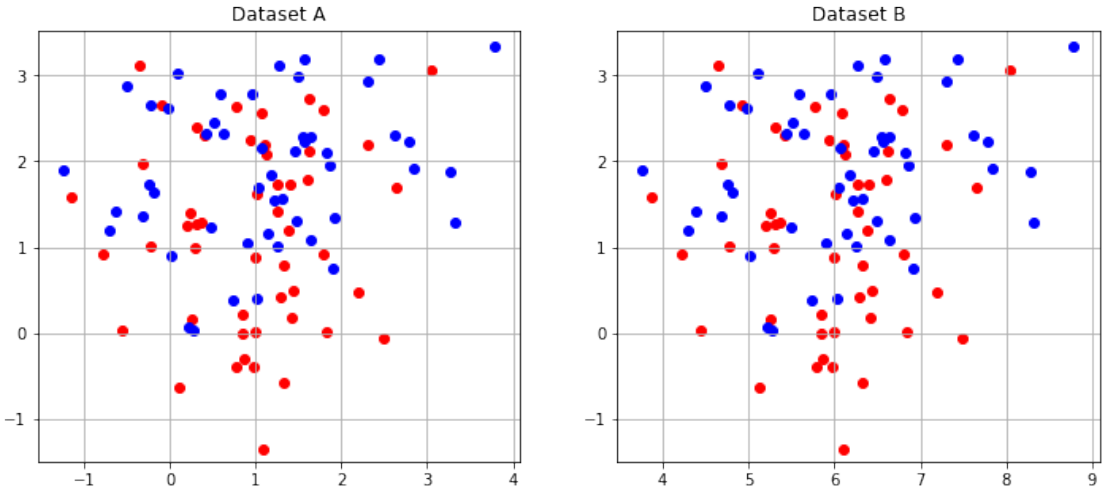
```
In [39]: import matplotlib.pylab as plt
         %matplotlib inline
         import numpy as np
         from sklearn.metrics import log_loss
```

1 Fisher information / Natural gradients

```
In [53]: Nsamples = 100
         dataAclass0 = np.random.multivariate_normal(np.array([1, 1]),
                                                     np.eye(2), Nsamples//2)
         dataAclass1 = np.random.multivariate_normal(np.array([1, 2]),
                                                     np.eye(2), Nsamples//2)
         dataA = np.vstack((dataAclass0, dataAclass1))
         dataBclass0 = np.copy(dataAclass0)
         dataBclass0[:, 0] = dataAclass0[:, 0] + 5
         dataBclass1 = np.copy(dataAclass1)
         dataBclass1[:, 0] = dataAclass1[:, 0] + 5
         dataB = np.vstack((dataBclass0, dataBclass1))
         correct_labels = [0] * 50 + [1] * 50
```

```
In [75]: fig, ax = plt.subplots(1, 2, figsize=(12, 5))
         ax[0].scatter(dataAclass0[:, 0], dataAclass0[:, 1], color='red')
         ax[0].scatter(dataAclass1[:, 0], dataAclass1[:, 1], color='blue')
         ax[0].grid(True)
         ax[0].axis('equal')
         ax[0].set_title('Dataset A')
         ax[1].scatter(dataBclass0[:, 0], dataBclass0[:, 1], color='red')
         ax[1].scatter(dataBclass1[:, 0], dataBclass1[:, 1], color='blue')
         ax[1].grid(True)
         ax[1].axis('equal')
         ax[1].set_title('Dataset B')
         plt.show()
```



1.1 Forward pass

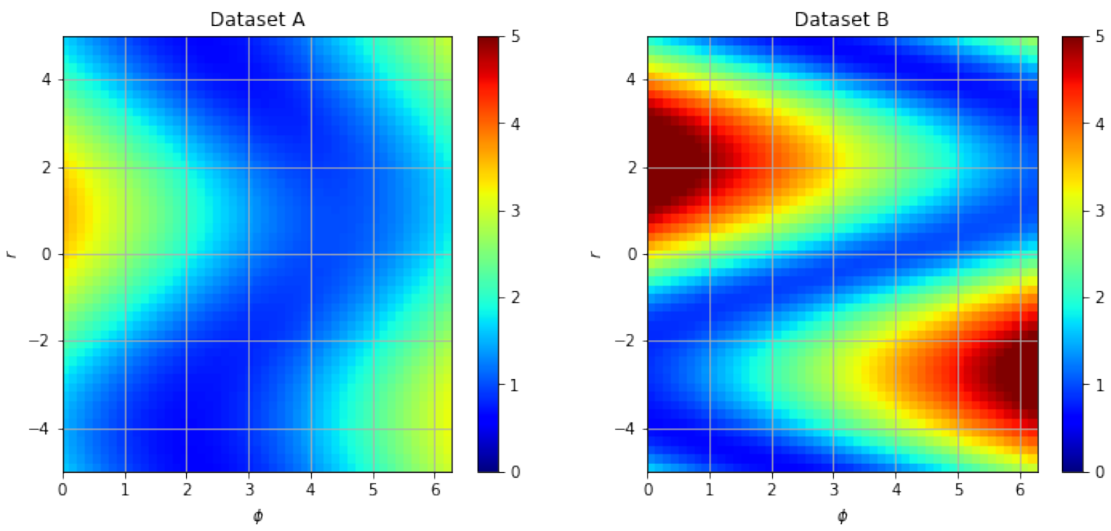
```
In [43]: def z(phi, r, x):
             return 1 / (1 + np.exp(
                 -np.sin(phi) * x[:, 0] - np.cos(phi) * x[:, 1] - r
             ))
```

```
In [56]: N = 50
         M = 50
         phispace = np.linspace(0, 2 * np.pi, N)
         rspace = np.linspace(-5, 5, M)
```

```
In [59]: heatA = np.zeros((M, N))
         heatB = np.zeros((M, N))
         for i, phi in enumerate(phispace):
             for j, r in enumerate(rspace):
                 heatA[i, j] = log_loss(correct_labels, z(phi, r, dataA))
                 heatB[i, j] = log_loss(correct_labels, z(phi, r, dataB))
```

```
In [82]: fig, ax = plt.subplots(1, 2, figsize=(12, 5))
         pcm = ax[0].pcolor(phispace, rspace, heatA, cmap='jet', vmin=0, vmax=5)
         ax[0].grid(True)
         ax[0].set_xlabel('$\phi$')
         ax[0].set_ylabel('$r$')
         ax[0].set_title('Dataset A')
         fig.colorbar(pcm, ax=ax[0])
         pcm = ax[1].pcolor(phispace, rspace, heatB, cmap='jet', vmin=0, vmax=5)
         ax[1].grid(True)
         ax[1].set_xlabel('$\phi$')
         ax[1].set_ylabel('$r$')
         ax[1].set_title('Dataset B')
```

```
fig.colorbar(pcm, ax=ax[1])
plt.show()
```



Observation: Works way better for dataset A in general. Also, the choice of ϕ and R is much more relevant to achieve a good result on B.

1.2 Fisher matrix

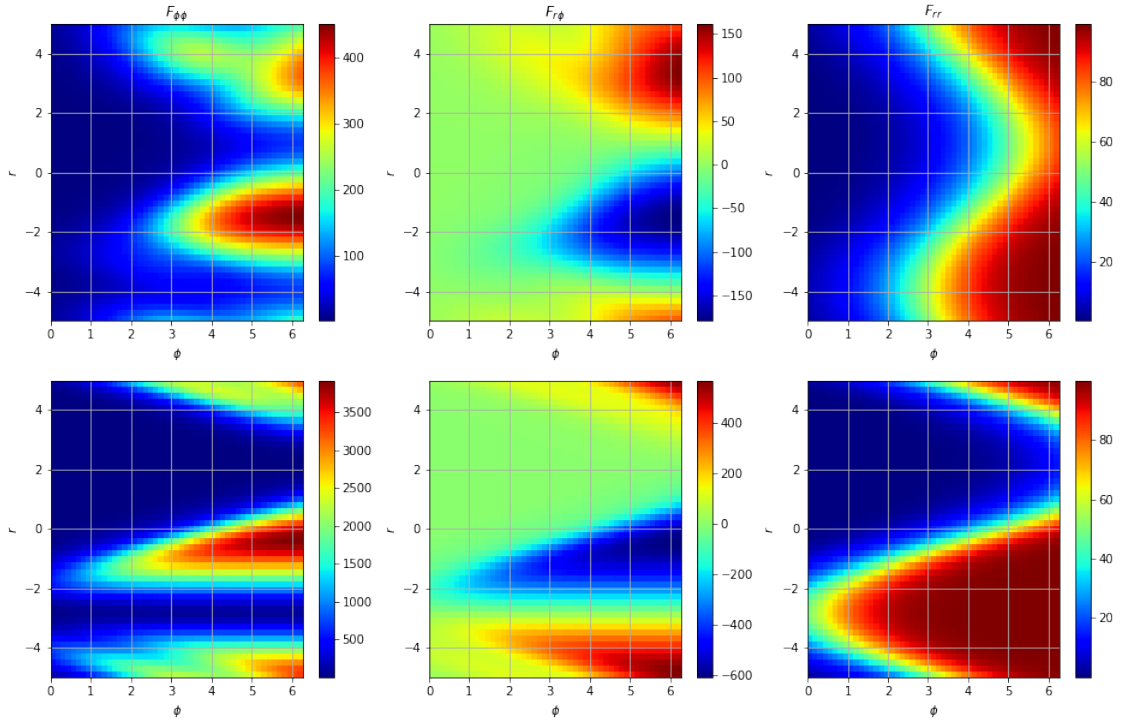
```
In [83]: def make_fisher(phi, r, x):
             mat = np.zeros((2, 2))  # [[F_phiphi, F_phir], [F_phir, F_rr]]
             term_dphi_y0 = (
                 (np.cos(phi) * x[:, 0] - np.sin(phi) * x[:, 1])
                 / (1 + np.exp(np.sin(phi) * x[:, 0] - np.cos(phi) * x[:, 1] + r))
             )
             term_dphi_y1 = (
                 (np.cos(phi) * x[:, 0] - np.sin(phi) * x[:, 1])
                 / (1 + np.exp(np.sin(phi) * x[:, 0] - np.cos(phi) * x[:, 1] + r))
                 - np.cos(phi) * x[:, 0]
                 + np.sin(phi) * x[:, 1]
             )
             term_dr_y0 = 1 / (1 + np.exp(
                 np.sin(phi) * x[:, 0] + np.cos(phi) * x[:, 1] + r
             ))
             term_dr_y1 = 1 / (1 + np.exp(
                 np.sin(phi) * x[:, 0] + np.cos(phi) * x[:, 1] + r
             )) - 1
             mat[0, 0] = np.sum(
                 term_dphi_y0 * term_dphi_y0 * z(phi, r, x)
                 + term_dphi_y1 * term_dphi_y1 * z(phi, r, x)
             )
```

```
            mat[1, 0] = np.sum(
                term_dphi_y0 * term_dr_y0 * z(phi, r, x)
                + term_dphi_y1 * term_dr_y1 * z(phi, r, x)
            )
            mat[1, 1] = np.sum(
                term_dr_y0 * term_dr_y0 * z(phi, r, x)
                + term_dr_y1 * term_dr_y1 * z(phi, r, x)
            )

            mat[0, 1] = mat[1, 0]
            return mat

        fisherA = np.zeros((M, N, 2, 2))
        fisherB = np.zeros((M, N, 2, 2))
        for i, phi in enumerate(phispace):
            for j, r in enumerate(rspace):
                fisherA[i, j, :, :] = make_fisher(phi, r, dataA)
                fisherB[i, j, :, :] = make_fisher(phi, r, dataB)
```

In [95]:
```
        fig, ax = plt.subplots(2, 3, figsize=(16, 10))
        for i, fisher in enumerate([fisherA, fisherB]):
            pcm = ax[i, 0].pcolor(phispace, rspace, fisher[:, :, 0, 0], cmap='jet')
            ax[i, 0].grid(True)
            ax[i, 0].set_xlabel('$\phi$')
            ax[i, 0].set_ylabel('$r$')
            fig.colorbar(pcm, ax=ax[i, 0])
            pcm = ax[i, 1].pcolor(phispace, rspace, fisher[:, :, 1, 0], cmap='jet')
            ax[i, 1].grid(True)
            ax[i, 1].set_xlabel('$\phi$')
            ax[i, 1].set_ylabel('$r$')
            fig.colorbar(pcm, ax=ax[i, 1])
            pcm = ax[i, 2].pcolor(phispace, rspace, fisher[:, :, 1, 1], cmap='jet')
            ax[i, 2].grid(True)
            ax[i, 2].set_xlabel('$\phi$')
            ax[i, 2].set_ylabel('$r$')
            fig.colorbar(pcm, ax=ax[i, 2])
        ax[0, 0].set_title(r'$F_{\phi\phi}$')
        ax[0, 1].set_title(r'$F_{r\phi}$')
        ax[0, 2].set_title(r'$F_{rr}$')
        plt.show()
```



In [ ]: