Exercise 07 Raphael Michel and Florian Stoertz

June 11, 2017

1 Gaussian Graphical Model

The non-zero off-diagonal values of Q should be of negative sign, such that the cost ist positive/high if c_i and c_j are of different sign and low if they are of the same sign.

With the non-constant values, α should be negative.

The diagonal values should be the absolute value of the sum of all neighbored off-diagonal elements. In the case of the constant values of -1, that is 4.

1.1 Implementation

```
In [1]: import skimage.data
        import skimage.filters
        import matplotlib.pylab as plt
        %matplotlib inline
        import numpy as np
        import scipy.sparse
```

```
In [2]: def norm_rgb(data):
            for C in range(data.shape[2]):
                data[:, :, C] /= np.max(data[:, :, C])
                data[:, :, C] -= np.min(data[:, :, C])
```

```
In [7]: data = skimage.data.astronaut()
        data = skimage.transform.resize(data, (250, 250)).astype('float32')
        norm_rgb(data)

        def make_noise(data, s=.1):
            noise = np.random.normal(0, scale=s, size=data.size)
            data_noisy = np.clip(data + noise.reshape(data.shape), 0, 1)
            return data_noisy
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/skimage/transform/_warps.py:84: Us
  warn("The default mode, 'constant', will be changed to 'reflect' in "
```

```
In [23]: def make_q_matrix_simple(data, gamma, alpha):
             img_shape = data.shape[0:2]
```

```
    n_pixel = data.shape[0] * data.shape[1]
    Q = scipy.sparse.lil_matrix((n_pixel, n_pixel), dtype='float32')

    def var_index(x0, x1):
        return x0 * img_shape[1] + x1

    for x0 in range(img_shape[0]):
        for x1 in range(img_shape[1]):
            vi0 = var_index(x0, x1)
            # Diagonal element
            Q[vi0, vi0] = 4

            # Right neigbor
            if x0 + 1 < img_shape[0]:
                vi1 = var_index(x0 + 1, x1)
                Q[vi0, vi1] = -1.0
                Q[vi1, vi0] = -1.0

            # Bottom neigbor
            if x1 + 1 < img_shape[1]:
                vi1 = var_index(x0, x1 + 1)
                Q[vi0, vi1] = -1.0
                Q[vi1, vi0] = -1.0

            # Top neigbor
            if x0 > 0:
                vi1 = var_index(x0 - 1, x1)
                Q[vi0, vi1] = -1.0
                Q[vi1, vi0] = -1.0

            # Left neigbor
            if x1 > 0:
                vi1 = var_index(x0, x1 - 1)
                Q[vi0, vi1] = -1.0
                Q[vi1, vi0] = -1.0

    return Q

def make_q_matrix_advanced(data, gamma, alpha):
    img_shape = data.shape[0:2]
    n_pixel = data.shape[0] * data.shape[1]
    Q = scipy.sparse.lil_matrix((n_pixel, n_pixel), dtype='float32')
    def var_index(x0, x1):
        return x0 * img_shape[1] + x1

    def col2val(c0, c1):
        diff_norm = np.linalg.norm(c0-c1)
        val = np.exp(-gamma * diff_norm)
```

```
    return alpha * val

for x0 in range(img_shape[0]):
    for x1 in range(img_shape[1]):
        vi0 = var_index(x0, x1)
        c0 = data[x0, x1]

        # right neighbor
        if x0 + 1 < img_shape[0]:
            vi1 = var_index(x0 + 1, x1)
            c1 = data[x0 + 1, x1]
            Q[vi0, vi1] = Q[vi0, vi1] = col2val(c0, c1)

        # Bottom neigbor
        if x1 + 1 < img_shape[1]:
            vi1 = var_index(x0, x1 + 1)
            c1 = data[x0, x1 + 1]
            Q[vi0, vi1] = Q[vi0, vi1] = col2val(c0, c1)

        # Top neigbor
        if x0 > 0:
            vi1 = var_index(x0 - 1, x1)
            Q[vi0, vi1] = -1.0
            Q[vi1, vi0] = -1.0
            c1 = data[x0 - 1, x1]
            Q[vi0, vi1] = Q[vi0, vi1] = col2val(c0, c1)

        # Left neigbor
        if x1 > 0:
            vi1 = var_index(x0, x1 - 1)
            Q[vi0, vi1] = -1.0
            Q[vi1, vi0] = -1.0
            c1 = data[x0, x1 - 1]
            Q[vi0, vi1] = Q[vi0, vi1] = col2val(c0, c1)

for x0 in range(img_shape[0]):
    for x1 in range(img_shape[1]):
        vi0 = var_index(x0, x1)
        s = 0
        if x0 + 1 < img_shape[0]:
            s += Q[vi0, var_index(x0+1, x1)]
        if x1 + 1 < img_shape[1]:
            s += Q[vi0, var_index(x0, x1+1)]
        if x0 > 0:
            s += Q[vi0, var_index(x0-1, x1)]
        if x1 > 0:
            s += Q[vi0, var_index(x0, x1-1)]
```

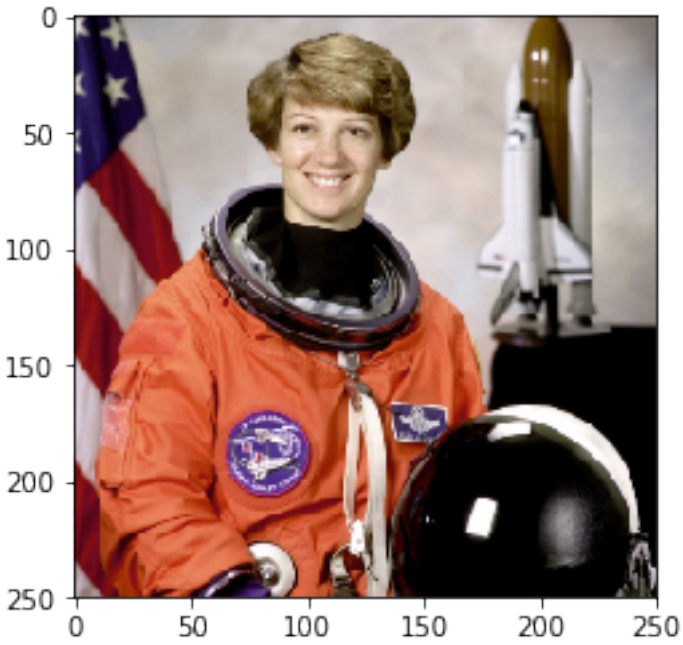
```
                Q[vi0, vi0] = np.abs(s)
        return Q

    def optimize(mu, Q, sigma):
        A = scipy.sparse.identity(Q.shape[0]) + sigma ** 2 * Q
        x = scipy.sparse.linalg.spsolve(A=A, b=mu)
        return x

    def experiment(data_noisy, alpha=-1, gamma=10.1, sigma=1., fun=make_q_matrix_advanced
        # Sigma: How much to trust the observerions (more sigma, less smoothing)
        Q = fun(data_noisy, gamma=gamma, alpha=alpha)
        result = np.copy(data_noisy)
        for c in range(data_noisy.shape[2]):
            result[:, :, c] = optimize(mu=data_noisy[:, :, c].ravel(), Q=Q, sigma=sigma).
        norm_rgb(result)
        return result
```

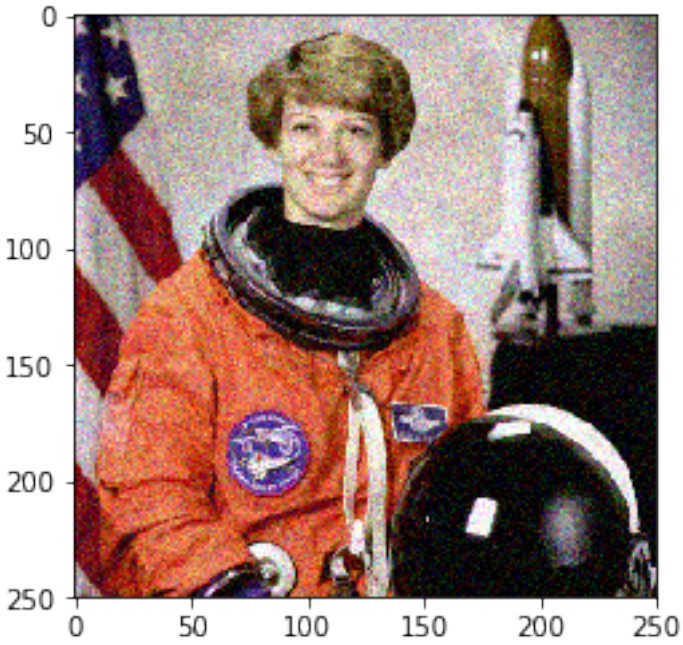
In [9]:
```
plt.imshow(data)
```

Out[9]: <matplotlib.image.AxesImage at 0x7f3dea67bcf8>



In [10]:
```
data_noisy = make_noise(data, .1)
plt.imshow(data_noisy)
```

Out[10]: <matplotlib.image.AxesImage at 0x7f3de8dcd978>

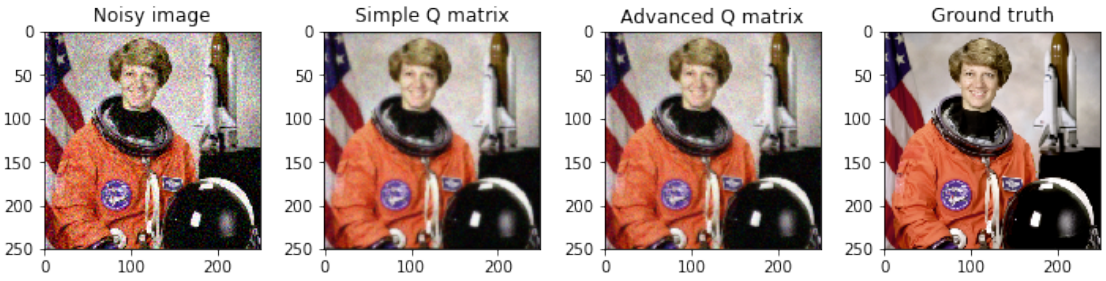


Simple vs. exponential off-diagonal values

In [13]:
```
result_simple = experiment(data_noisy, fun=make_q_matrix_simple)
result_advanced = experiment(data_noisy, fun=make_q_matrix_advanced)

fig, axs = plt.subplots(1, 4, figsize=(10, 5))
axs[0].imshow(data_noisy)
axs[0].set_title('Noisy image')
axs[1].imshow(result_simple)
axs[1].set_title('Simple Q matrix')
axs[2].imshow(result_advanced)
axs[2].set_title('Advanced Q matrix')
axs[3].imshow(data)
axs[3].set_title('Ground truth')
fig.tight_layout()
fig.show()
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/matplotlib/figure.py:403: UserWarn
  "matplotlib is currently using a non-GUI backend, "
```



The simple Q matrix leads to a result that basically looks like Gaussian blurring, while with the advanced Q matrix, edges are better retained.

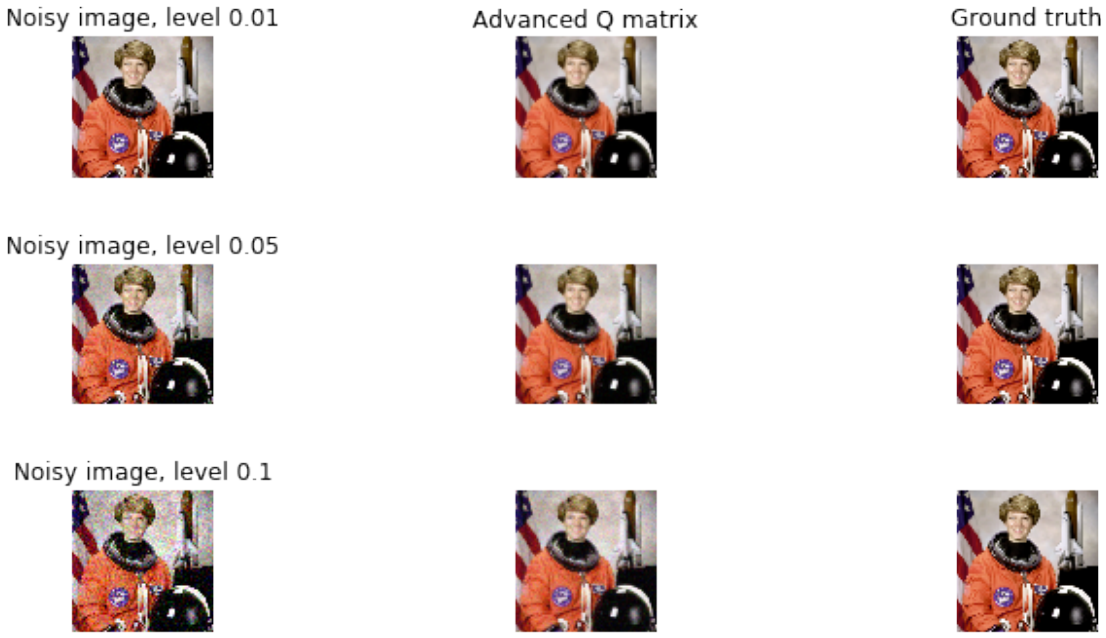
1.2 Different noise levels s

```
In [31]: s_ = (.01, .05, .1)
         fig, axs = plt.subplots(len(s_), 3, figsize=(10, 5))
         axs[0, 0].set_title('Noisy image')
         axs[0, 1].set_title('Advanced Q matrix')
         axs[0, 2].set_title('Ground truth')
         for i, s in enumerate(s_):
             data_noisy = make_noise(data, s)
             result_advanced = experiment(data_noisy, fun=make_q_matrix_advanced)
             print("Noise level: {}, Sum of squared error: {}".format(
                 s,
                 np.sum((result_advanced - data) ** 2)
             ))
             axs[i, 0].set_title("Noisy image, level {}".format(s))
             axs[i, 0].imshow(data_noisy)
             axs[i, 1].imshow(result_advanced)
             axs[i, 2].imshow(data)
             for x in axs[i]:
                 x.axis('off')

         fig.tight_layout()
         fig.show()
```

```
Noise level: 0.01, Sum of squared error: 616.6914402561705
Noise level: 0.05, Sum of squared error: 745.1831479280739
Noise level: 0.1, Sum of squared error: 1018.6534486060536
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/matplotlib/figure.py:403: UserWarn
  "matplotlib is currently using a non-GUI backend, "
```



1.3 Different α values

```
In [27]: alphas = (-.01, -.1, -1, -10)
         data_noisy = make_noise(data, s=.1)
         fig, axs = plt.subplots(1, len(alphas) + 2, figsize=(10, 5))
         for i, alpha in enumerate(alphas):
             result_advanced = experiment(data_noisy, fun=make_q_matrix_advanced,
                                          alpha=alpha)
             print("Alpha: {}, Sum of squared error: {}".format(
                 alpha,
                 np.sum((result_advanced - data) ** 2)
             ))
             axs[i + 1].imshow(result_advanced)
             axs[i + 1].set_title(r"$\alpha = {}$".format(alpha))

         for x in axs:
             x.axis('off')
         axs[0].imshow(data_noisy)
         axs[-1].imshow(data)
         axs[0].set_title('Noisy image')
         axs[-1].set_title('Ground truth')
         fig.tight_layout()
         fig.show()
```

```
Alpha: -0.01, Sum of squared error: 1104.208429853545
Alpha: -0.1, Sum of squared error: 1095.7630379111424
```

```
Alpha: -1, Sum of squared error: 1019.1974658642641
Alpha: -10, Sum of squared error: 654.9968510776486
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/matplotlib/figure.py:403: UserWarn
  "matplotlib is currently using a non-GUI backend, "
```

1.4 Different γ values

```
In [32]: gammas = (.1, 1, 5, 10, 25)
         data_noisy = make_noise(data, s=.1)
         fig, axs = plt.subplots(1, len(gammas) + 2, figsize=(10, 5))
         for i, gamma in enumerate(gammas):
             result_advanced = experiment(data_noisy, fun=make_q_matrix_advanced,
                                          gamma=gamma)
             print("Gamma: {}, Sum of squared error: {}".format(
                 gamma,
                 np.sum((result_advanced - data) ** 2)
             ))
             axs[i + 1].imshow(result_advanced)
             axs[i + 1].set_title(r"$\gamma = {}$".format(gamma))

         for x in axs:
             x.axis('off')
         axs[0].imshow(data_noisy)
         axs[-1].imshow(data)
         axs[0].set_title('Noisy image')
         axs[-1].set_title('Ground truth')
         fig.tight_layout()
         fig.show()
```

```
Gamma: 0.1, Sum of squared error: 693.5202669296342
Gamma: 1, Sum of squared error: 682.8082897900254
Gamma: 5, Sum of squared error: 903.9490848257216
Gamma: 10, Sum of squared error: 1026.5042784712807
Gamma: 25, Sum of squared error: 1096.6418896404887
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/matplotlib/figure.py:403: UserWarn
  "matplotlib is currently using a non-GUI backend, "
```

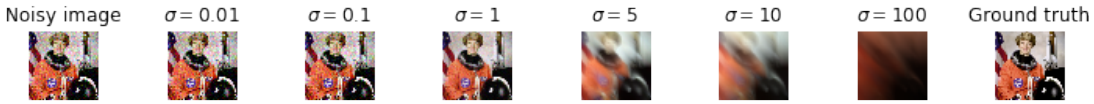
1.5 Different σ values

```
In [33]: sigmas = (.01, .1, 1, 5, 10, 100)
         data_noisy = make_noise(data, s=.1)
         fig, axs = plt.subplots(1, len(sigmas) + 2, figsize=(10, 5))
         for i, sigma in enumerate(sigmas):
             result_advanced = experiment(data_noisy, fun=make_q_matrix_advanced,
                                          sigma=sigma)
             print("Sigma: {}, Sum of squared error: {}".format(
                 sigma,
                 np.sum((result_advanced - data) ** 2)
             ))
             axs[i + 1].imshow(result_advanced)
             axs[i + 1].set_title(r"$\sigma = {}$".format(sigma))

         for x in axs:
             x.axis('off')
         axs[0].imshow(data_noisy)
         axs[-1].imshow(data)
         axs[0].set_title('Noisy image')
         axs[-1].set_title('Ground truth')
         fig.tight_layout()
         fig.show()
```

```
Sigma: 0.01, Sum of squared error: 1599.5491282242813
Sigma: 0.1, Sum of squared error: 1537.463806624807
Sigma: 1, Sum of squared error: 1021.2623225795844
Sigma: 5, Sum of squared error: 9558.177945988185
Sigma: 10, Sum of squared error: 14245.675828624926
Sigma: 100, Sum of squared error: 29891.80930380664
```

```
/home/raphael/proj/uni-ml4cv/env/lib/python3.6/site-packages/matplotlib/figure.py:403: UserWarn
  "matplotlib is currently using a non-GUI backend, "
```


Noisy image
$\sigma = 0.01$
$\sigma = 0.1$
$\sigma = 1$
$\sigma = 5$
$\sigma = 10$
$\sigma = 100$
Ground truth

In [ ]: