Exercise 05 Raphael Michel and Florian Stoertz

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1 Structured Learning:

In this exercise we will implement a structured learning system for foreground background segmentation. We will learn the weights of a CRF Potts model.

The first step is to import all needed modules

```
In [10]: # misc
         import numpy
         import sys

         # visualization
         import matplotlib.pyplot as plt
         import pylab

         # features
         import skimage.filters
         import skimage.feature

         # discrete graphical model package
         from dgm.models import *
         from dgm.solvers import *
         from dgm.value_tables import *
         # misc. tools
         from tools import make_toy_dataset, norm01
         import matplotlib.pyplot as plt

         from tools import make_toy_dataset, norm01
```

2 The Dataset

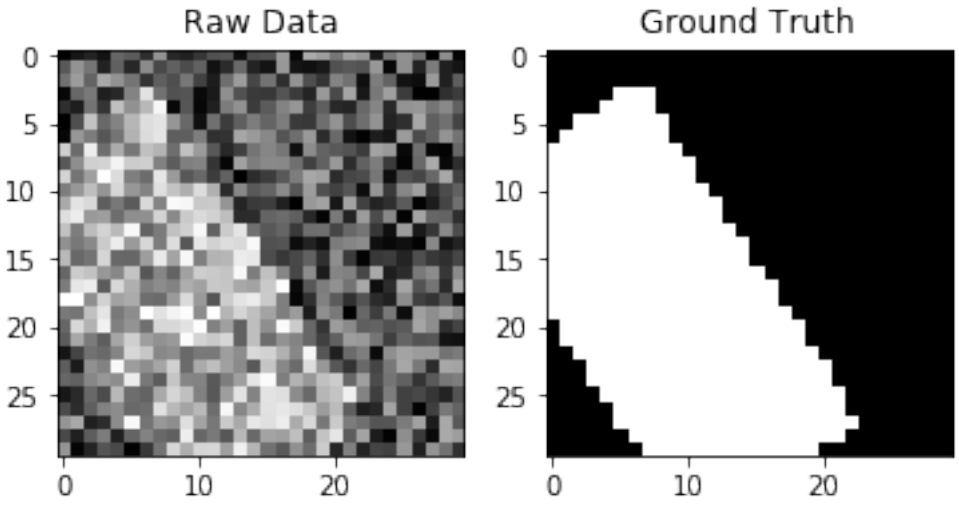
Here we use a toy dataset where we have white squares on black background with some noise. The noise level is given by the variable `noise`.

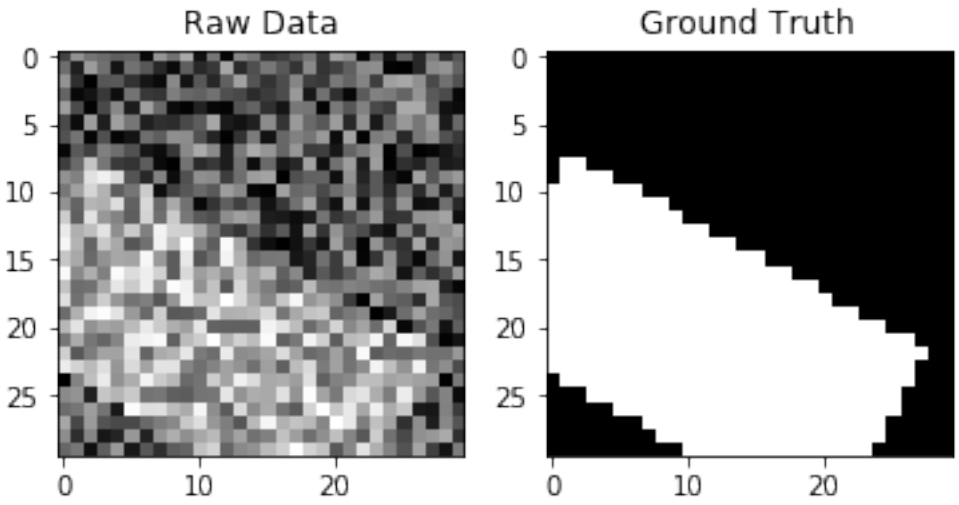
```
In [6]: noise = 2.0
        shape = (30,30)
```

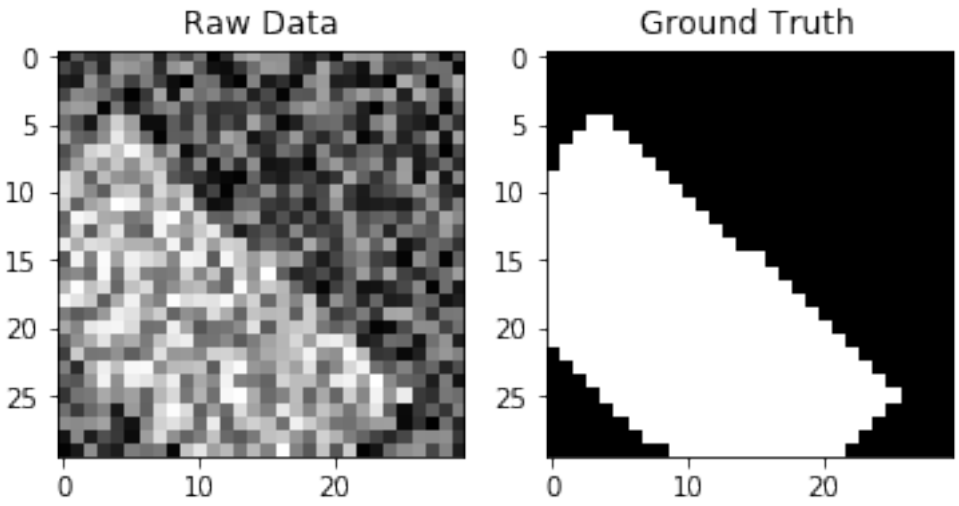
```
x_train, y_train =  make_toy_dataset(shape=shape, n_images=5, noise=noise)
x_test , y_test  =  make_toy_dataset(shape=shape, n_images=5, noise=noise)

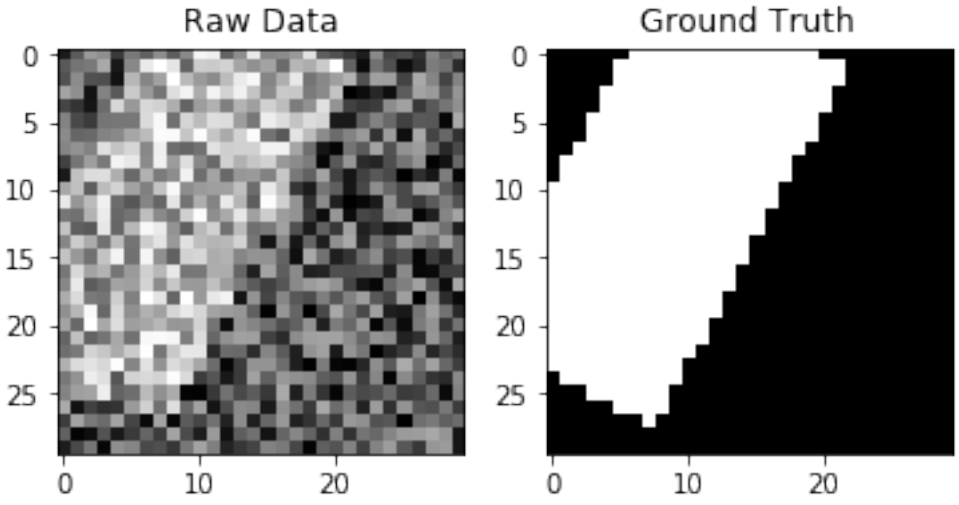
# show a bit from the dataset
for x in range(4):
    f= pylab.figure()
    ax1 = f.add_subplot(1,2,1)
    pylab.imshow(x_train[x],cmap='gray')
    ax1.set_title('Raw Data')

    ax2 = f.add_subplot(1,2,2)
    pylab.imshow(y_train[x],cmap='gray')
    ax2.set_title('Ground Truth')
    plt.show()
```









3 The Unary Features (5Pt):

Here you need to implement a function which should return pixel wise unary features. You can use features like gaussian smooting with differnet sigmas. About 5 features should be enough.

The features should be normalized to be in [0,1]

```
In [9]: from skimage.morphology import disk
```

```
def get_unary_features(raw):
    features = []

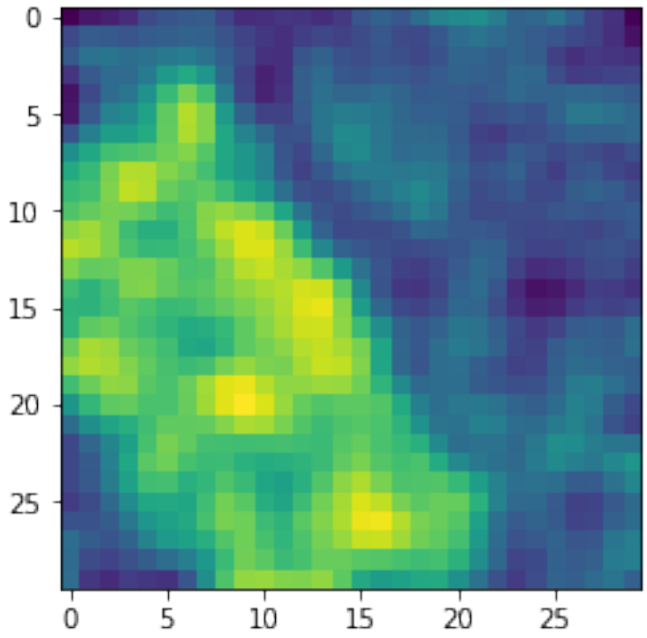
    # Normalize picuteres
    raw -= raw.min()
    raw /= raw.max()

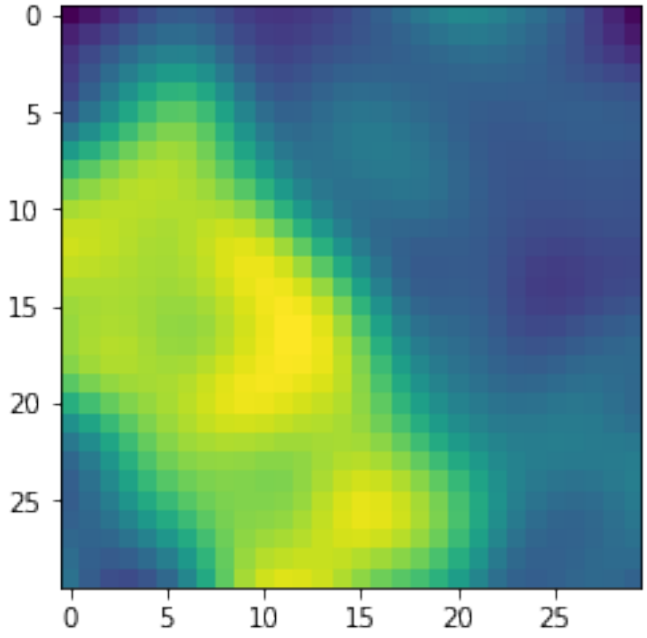
    features.append(skimage.filters.gaussian(raw, sigma=1)[:,:,None])
    features.append(skimage.filters.gaussian(raw, sigma=2)[:,:,None])
    features.append(skimage.filters.gaussian(raw, sigma=5)[:,:,None])
    features.append(skimage.filters.gaussian(raw, sigma=10)[:,:,None])

    # Constant feature, keep, because of linearity
    features.append(numpy.ones(raw.shape)[:,:,None])
    return  numpy.concatenate(features, axis=2)



# visualize the features for a raw image
unary_features = get_unary_features(x_train[0])
n_unary_features = unary_features.shape[2]
for i in range(unary_features.shape[2]):
    pylab.imshow(unary_features[:,:,i])
    pylab.show()
```

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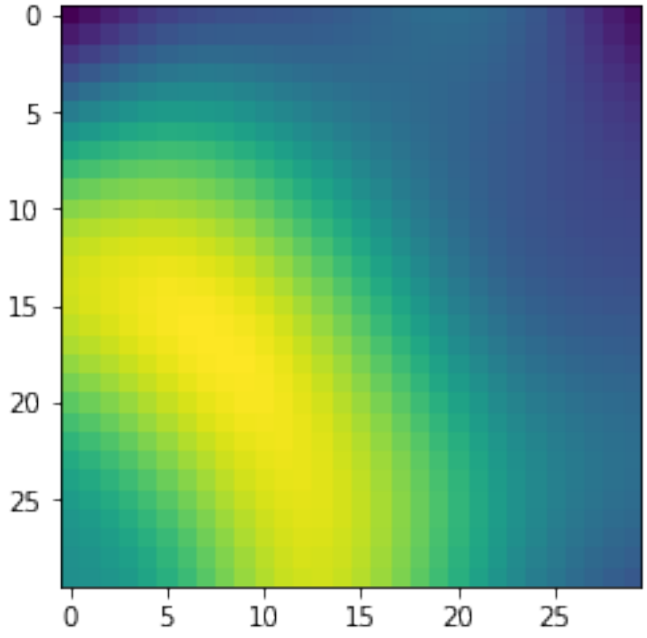
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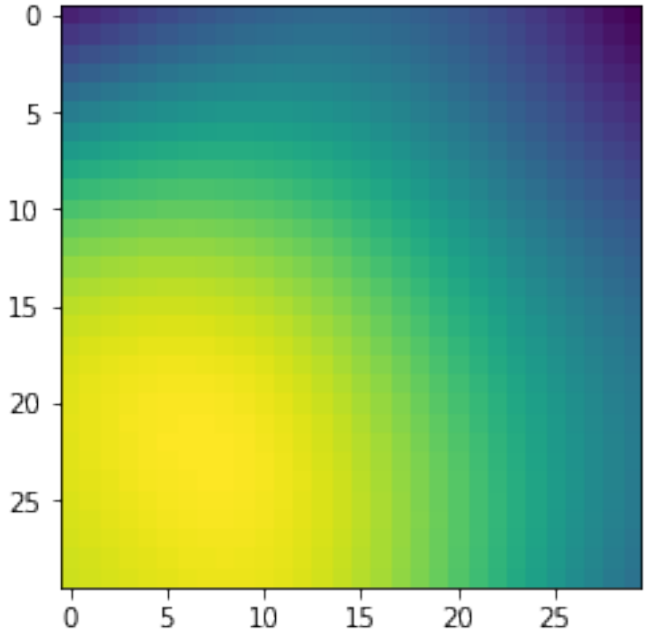
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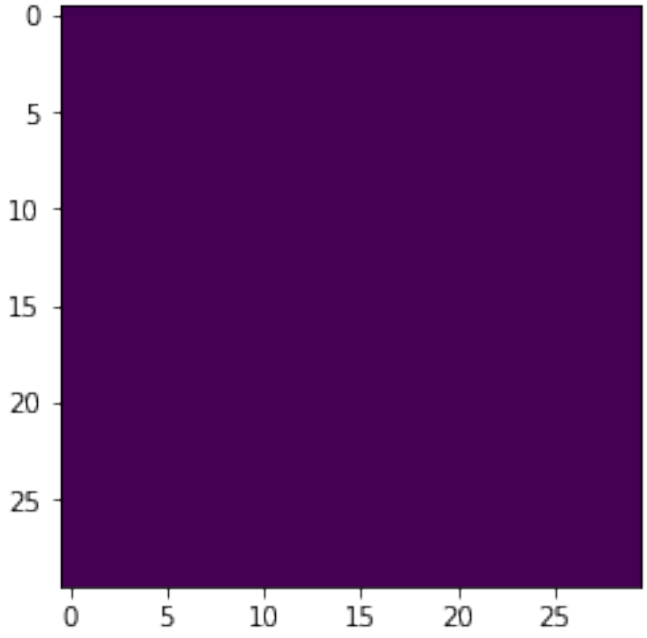
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4 The Potts Features (5 P):

Here youn need to implement a function returing pixel wise features which are used within the potts term. The features should be something like edge detectors.

e^{-1.0 |edge_strength|}$ should work well.

You can compute edge filters on on different sigmas by presmoothing the raw data with a gaussian. About five features should be enough.

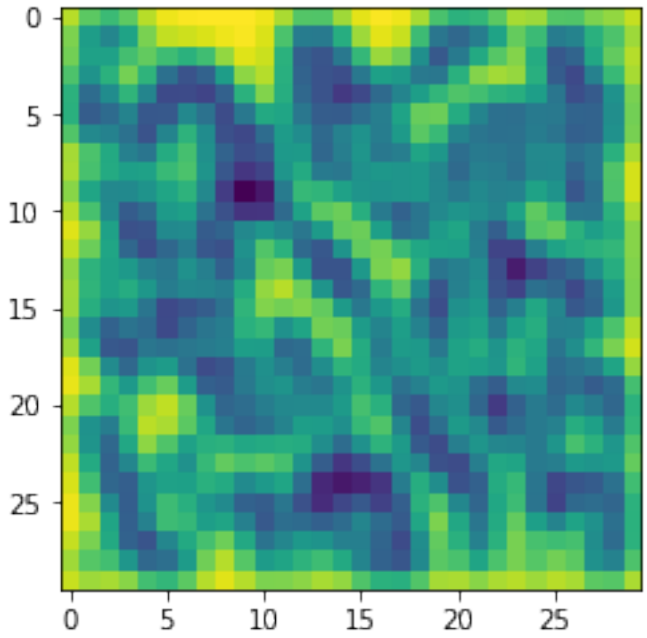
The features should be normalized to be in [0,1]

```
In [23]: def get_potts_features(raw):
             features = []

             for sigma in (1, 2, 3, 4, 5):
                 edges = skimage.feature.canny(raw, sigma=sigma)
                 features.append(
                     skimage.filters.gaussian(
                         numpy.exp(-1.0 * numpy.abs(edges))
                     )[:,:,None]
                 )

             # a constant feature is needed
             features.append(numpy.ones(raw.shape)[:,:,None])
             return  numpy.concatenate(features, axis=2)


         # visualize the features for a raw image
         potts_features = get_potts_features(x_train[0])
         n_potts_features = potts_features.shape[2]
         for i in range(potts_features.shape[2]):
             pylab.imshow(potts_features[:,:,i])
             pylab.show()
```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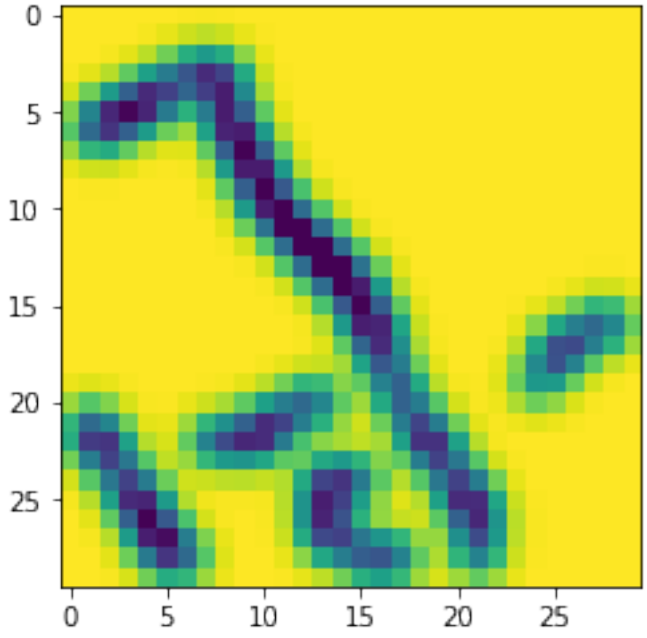
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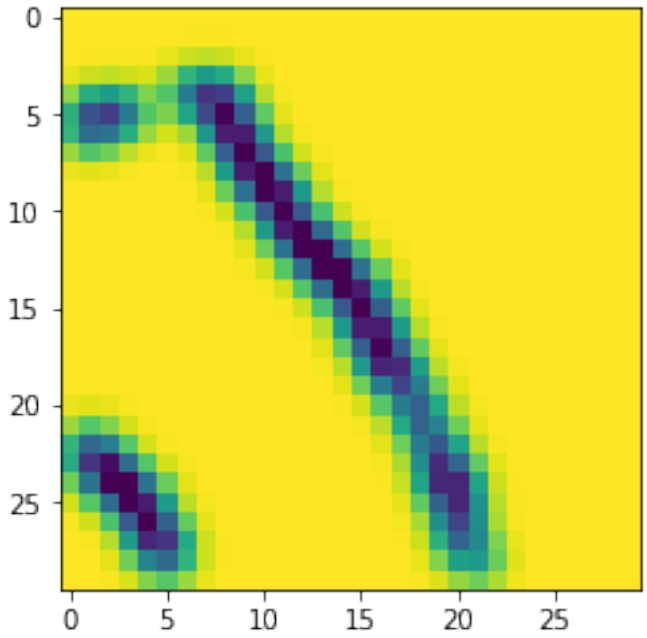
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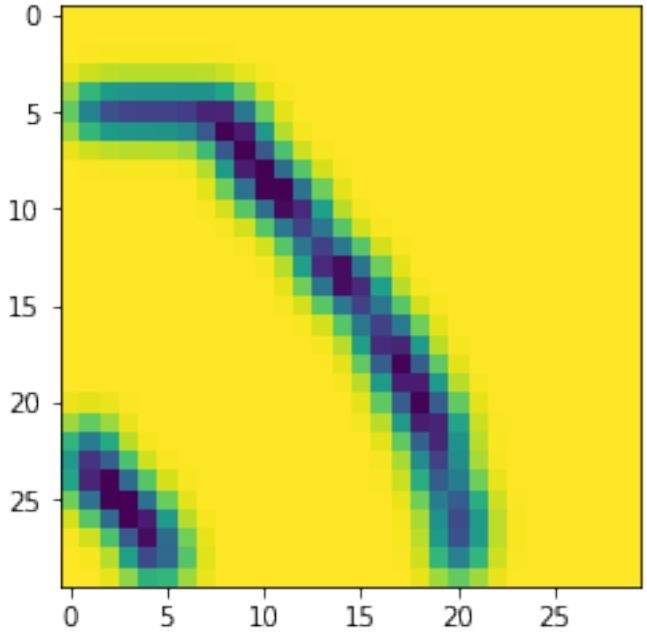
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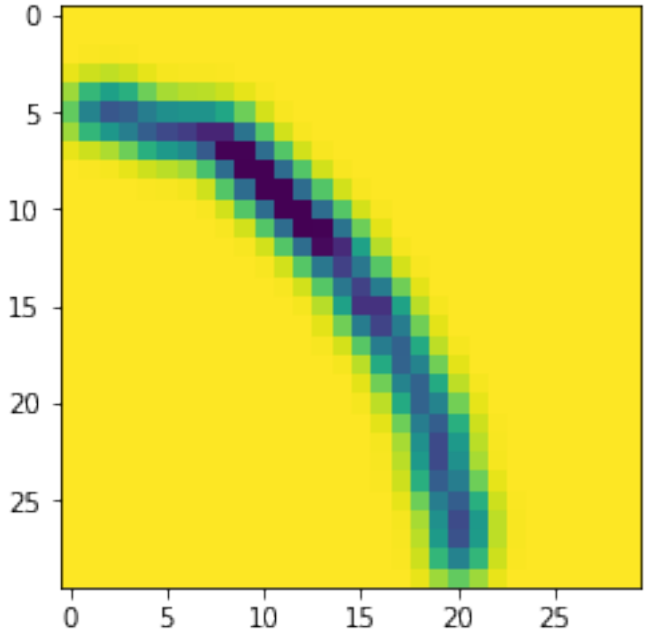
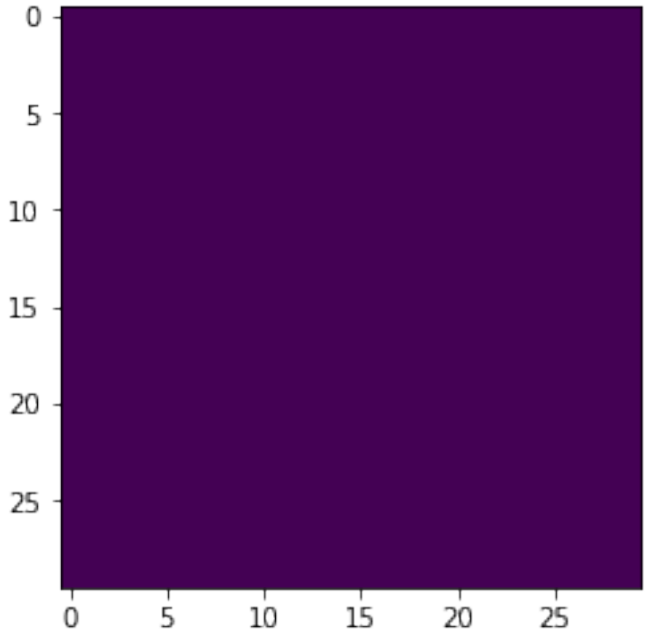




5 Loss Function:

In this example we will use a very simple `Hamming Loss`.

```
In [24]: class HammingLoss(object):
             def __init__(self, y_true):
                 self.y_true = y_true.copy()

             def __call__(self, y_pred):
                 """total loss"""
                 return numpy.sum(self.y_true!=y_pred)
```

6 Function to set up the weighted Model:

This function will set up the weighted graphical model and also the loss augmented Model:

$$\mathbf{argmin}_y \quad w \cdot \phi(x, y)$$

Loss Augmented Model:

$$\mathbf{argmin}_y \quad w \cdot \phi(x, y) - \Delta(\hat{y}, y)$$

```
In [25]: def build_model(raw_data, gt_image, weights):
             shape = raw_data.shape
             n_var = shape[0] * shape[1]
             n_labels = 2
             variable_space = numpy.ones(n_var)*n_labels


             # lets compute some filters for the uanry features
             unary_features = get_unary_features(raw_data)

             # lets compute some filters for the potts features
             potts_features = get_potts_features(raw_data)

             n_weights =potts_features.shape[2] + unary_features.shape[2]

             #print("n_weights",n_weights)
             assert n_weights == len(weights)


             # both graphical models
             gm = WeightedDiscreteGraphicalModel(variable_space=variable_space, weights=weights
             loss_augmented_gm = WeightedDiscreteGraphicalModel(variable_space=variable_space,

             # convert coordinates to scalar
             def vi(x0,x1):  # vi = variable index
                 return x1 + x0*shape[1]


             # weight ids for the unaries
             # (just plain numbers to remeber which weights
```

```
# are associated with the unary features)
weight_ids = numpy.arange(unary_features.shape[2])
for x0 in range(shape[0]):
    for x1 in range(shape[1]):

        pixel_val = raw_data[x0, x1]
        gt_label = gt_image[x0, x1]
        features = unary_features[x0, x1, :]

        unary_function = WeightedTwoClassUnary(features=features, weight_ids=weigh
                                               weights=weights)

        if gt_label == 0:
            loss = numpy.array([0,1])
        else:
            loss = numpy.array([1,0])

        loss_augmented_unary_function = WeightedTwoClassUnary(features=features,
                                                weights=weights, const_terms=-1.0*

        variables = vi(x0,x1)
        gm.add_factor(variables=variables, value_table=unary_function)
        loss_augmented_gm.add_factor(variables=variables, value_table=loss_augmen

# add pairwise factors
# the weight id's for the pairwise factors

# average over 2 coordinates to extract
# extract feature vectors for potts functins
def get_potts_feature_vec(coord_a, coord_b):

    fa = potts_features[coord_a[0],coord_a[1],:]
    fb = potts_features[coord_b[0],coord_b[1],:]
    return (fa+fb)/2.0

# weight ids for the potts functions
# (just plain numbers to remeber which weights
# are associated with the potts features)
weight_ids = numpy.arange(potts_features.shape[2]) + unary_features.shape[2]

for x0 in range(shape[0]):
    for x1 in range(shape[1]):

        # horizontal edge
        if x0 + 1 < shape[0]:
            variables = [vi(x0,x1),vi(x0+1,x1)]
            features = get_potts_feature_vec((x0,x1), (x0+1,x1))
            # the weighted potts function
```

```
                potts_function = WeightedPottsFunction(shape=[2,2],
                                                       features=features,
                                                       weight_ids=weight_ids,
                                                       weights=weights)
                # add factors to both models
                gm.add_factor(variables=variables, value_table=potts_function)
                loss_augmented_gm.add_factor(variables=variables, value_table=potts_fu

            # vertical edge
            if x1 + 1 < shape[1]:
                variables = [vi(x0,x1),vi(x0, x1+1)]
                features = get_potts_feature_vec((x0,x1), (x0,x1+1))
                # the weighted potts function
                potts_function = WeightedPottsFunction(shape=[2,2],
                                                       features=features,
                                                       weight_ids=weight_ids,
                                                       weights=weights)
                # add factors to both models
                gm.add_factor(variables=variables, value_table=potts_function)
                loss_augmented_gm.add_factor(variables=variables, value_table=potts_fu

        # gm, loss augmented and the loss
        return gm,loss_augmented_gm, HammingLoss(gt_image.ravel())
```

7 Build the weighted models:

```
In [27]: #
         n_weights = n_unary_features + n_potts_features
         weights = numpy.zeros(n_weights)

          # build the graphical models
         models_train  = [build_model(x,y, weights) for x,y in zip(x_train, y_train)]
         models_test   = [build_model(x,y, weights) for x,y in zip(x_test, y_test)]


         # very simple helper class to combine things
         class Dataset(object):
             def __init__(self, models_train, models_test, weights):
                 self.models_train = models_train
                 self.models_test = models_test
                 self.weights = weights
         dset = Dataset(models_train, models_test, weights)
```

8 Subgradient SSVM

Instead of a cutting plane approach, we use a subgradient decent to find the optimal weights
Learn more about subgradient ssvm

```
In [68]: def subgradient_ssvm(dataset, n_iter=20, learning_rate=1.0, c=0.5, lower_bounds=None,

             weights = dataset.weights
             n = len(dataset.models_train)

             if lower_bounds is None:
                 lower_bounds = numpy.ones(len(weights))*-1.0*float('inf')

             if upper_bounds is None:
                 upper_bounds = numpy.ones(len(weights))*float('inf')

             do_opt = True
             for iteration in range(n_iter):

                 effective_learning_rate = learning_rate*float(learning_rate)/(1.0+iteration)

                 # compute gradient
                 diff = numpy.zeros(weights.shape)
                 for gm, gm_loss_augmented, loss_function in dataset.models_train:

                     # update the weights to the current weight vector
                     gm.change_weights(weights)
                     gm_loss_augmented.change_weights(weights)

                     # the gt vector
                     y_true = loss_function.y_true



                     # optimize loss augmented /
                     # find most violated constraint
                     graphcut = algorithm(model=gm_loss_augmented)
                     y_hat = graphcut.optimize()

                     # compute joint feature vector
                     phi_y_hat  = gm.phi(y_hat)
                     phi_y_true = gm.phi(y_true)

                     diff +=  phi_y_true - phi_y_hat



                 new_weights = weights - effective_learning_rate*(c/n)*diff

                 # project new weights
                 where_to_large = numpy.where(new_weights>upper_bounds)
                 new_weights[where_to_large]  = upper_bounds[where_to_large]
                 where_to_small = numpy.where(new_weights<lower_bounds)
```

```
            new_weights[where_to_small]  = lower_bounds[where_to_small]


            delta = numpy.abs(new_weights-weights).sum()
            if(delta<convergence):
                print("converged")
                break
            print('iter',iteration, 'delta',delta,"  ",numpy.round(new_weights,3))

            weights = new_weights


        return weights
```

9 Learn The Weights:

We call use graphcut to find the argmin of the gm and the loss augmented gm. Graphcut will find optimal values iff the potts regularizer is positive. To ensure a positive regularizer is learned, we can constraint the weights of the potts function to be positive.

```
In [29]: lower_bounds = numpy.ones(len(weights))*(-1.0*float('inf'))

         # we want the regularizer 'beta' to be positive
         lower_bounds[n_unary_features:n_unary_features+n_potts_features] = 0

         weights = subgradient_ssvm(dset,c=0.5,learning_rate=1.0, lower_bounds=lower_bounds, n
```

```
iter 0 delta 148.301948269    [-32.05  -25.162  -7.94   11.149  72.      0.      0.      0.
   0.   ]
iter 1 delta 315.881335136    [-93.278 -84.535 -62.031 -35.541 -22.5     0.      0.      0.
   0.   ]
iter 2 delta 216.586094929    [-63.143 -53.341 -28.618  -0.698  64.5     0.      0.      0.
   0.   ]
iter 3 delta 15.1156490128    [-61.085 -51.222 -26.47    1.362  68.925   0.4     0.572   0.475
   0.37    0.125]
iter 4 delta 5.8495989785    [-61.669 -51.795 -27.047   0.812  67.665   0.      0.      0.
   0.   ]
iter 5 delta 0.770754366329    [-61.523 -51.65  -26.91    0.938  67.882   0.      0.      0.
   0.   ]
iter 6 delta 0.131381588897    [-61.531 -51.658 -26.923   0.923  67.796   0.      0.      0.
   0.   ]
iter 7 delta 0.069690615693    [-61.52  -51.647 -26.917   0.926  67.758   0.      0.      0.
   0.   ]
iter 8 delta 0.0619472139493    [-61.511 -51.636 -26.911   0.928  67.725   0.      0.      0.
   0.   ]
iter 9 delta 0.0835031517545    [-61.488 -51.612 -26.891   0.945  67.725   0.      0.      0.
   0.   ]
iter 10 delta 0.0506840841403    [-61.48  -51.603 -26.886   0.947  67.698   0.      0.      0.
```

```
   0.   ]
iter 11 delta 0.046460410462    [-61.473 -51.596 -26.881   0.949  67.673   0.      0.      0.
   0.   ]
iter 12 delta 0.0642331936573    [-61.455 -51.577 -26.866   0.961  67.673   0.      0.      0.
   0.   ]
iter 13 delta 0.0398232089674    [-61.449 -51.57  -26.862   0.963  67.651   0.      0.      0.
   0.   ]
iter 14 delta 0.0371683283696    [-61.443 -51.564 -26.859   0.965  67.631   0.      0.      0.
   0.   ]
iter 15 delta 0.0461726220169    [-61.432 -51.552 -26.849   0.972  67.625   0.      0.      0.
   0.   ]
iter 16 delta 0.0327955838555    [-61.427 -51.547 -26.846   0.974  67.608   0.      0.      0.
   0.   ]
iter 17 delta 0.0359898123466    [-61.419 -51.539 -26.841   0.977  67.596   0.      0.      0.
   0.   ]
iter 18 delta 0.0340956116968    [-61.412 -51.531 -26.835   0.981  67.586   0.      0.      0.
   0.   ]
iter 19 delta 0.0278762462772    [-61.408 -51.526 -26.832   0.982  67.571   0.      0.      0.
   0.   ]
iter 20 delta 0.0397634055974    [-61.397 -51.515 -26.823   0.99   67.571   0.      0.      0.
   0.   ]
iter 21 delta 0.0253420420702    [-61.393 -51.511 -26.821   0.991  67.557   0.      0.      0.
   0.   ]
iter 22 delta 0.0242402141541    [-61.39  -51.507 -26.818   0.992  67.544   0.      0.      0.
   0.   ]
iter 23 delta 0.0307817480113    [-61.382 -51.499 -26.812   0.997  67.54    0.      0.      0.
   0.   ]
iter 24 delta 0.0223009970218    [-61.379 -51.495 -26.81    0.998  67.528   0.      0.      0.
   0.   ]
iter 25 delta 0.0284139212412    [-61.371 -51.488 -26.804   1.002  67.524   0.      0.      0.
   0.   ]
iter 26 delta 0.0206490713164    [-61.368 -51.484 -26.802   1.003  67.513   0.      0.      0.
   0.   ]
iter 27 delta 0.0231363079371    [-61.364 -51.479 -26.798   1.006  67.506   0.      0.      0.
   0.   ]
iter 28 delta 0.0223385042151    [-61.359 -51.474 -26.795   1.008  67.499   0.      0.      0.
   0.   ]
iter 29 delta 0.0215938874079    [-61.355 -51.469 -26.791   1.011  67.492   0.      0.      0.
   0.   ]
iter 30 delta 0.0208973103948    [-61.35  -51.465 -26.788   1.013  67.486   0.      0.      0.
   0.   ]
iter 31 delta 0.0202442694449    [-61.346 -51.46  -26.785   1.015  67.48    0.      0.      0.
   0.   ]
iter 32 delta 0.0196308067345    [-61.342 -51.456 -26.782   1.017  67.474   0.      0.      0.
   0.   ]
iter 33 delta 0.0163977919278    [-61.34  -51.453 -26.78    1.018  67.465   0.      0.      0.
   0.   ]
iter 34 delta 0.0211074843506    [-61.335 -51.448 -26.776   1.021  67.462   0.      0.      0.
```

```
   0.   ]
iter 35 delta 0.0154868034873    [-61.332 -51.445 -26.774   1.022  67.454   0.      0.      0.
   0.   ]
iter 36 delta 0.0175085573578    [-61.329 -51.441 -26.772   1.024  67.448   0.      0.      0.
   0.   ]
iter 37 delta 0.0170478058484    [-61.325 -51.437 -26.769   1.026  67.443   0.      0.      0.
   0.   ]
iter 38 delta 0.0166106826215    [-61.322 -51.434 -26.766   1.027  67.438   0.      0.      0.
   0.   ]
iter 39 delta 0.016195415556    [-61.319 -51.43  -26.764   1.029  67.433   0.      0.      0.
   0.   ]
iter 40 delta 0.0158004054204    [-61.315 -51.427 -26.761   1.031  67.428   0.      0.      0.
   0.   ]
iter 41 delta 0.0154242052914    [-61.312 -51.423 -26.759   1.033  67.423   0.      0.      0.
   0.   ]
iter 42 delta 0.0150655028427    [-61.309 -51.42  -26.756   1.034  67.419   0.      0.      0.
   0.   ]
iter 43 delta 0.0147231050509    [-61.306 -51.417 -26.754   1.036  67.414   0.      0.      0.
   0.   ]
iter 44 delta 0.0143959249386    [-61.303 -51.414 -26.752   1.037  67.41    0.      0.      0.
   0.   ]
iter 45 delta 0.0140829700487    [-61.3   -51.41  -26.749   1.039  67.405   0.      0.      0.
   0.   ]
iter 46 delta 0.0137833323881    [-61.298 -51.407 -26.747   1.04   67.401   0.      0.      0.
   0.   ]
iter 47 delta 0.01349617963    [-61.295 -51.404 -26.745   1.042  67.397   0.      0.      0.
   0.   ]
iter 48 delta 0.011378059705    [-61.293 -51.402 -26.744   1.042  67.391   0.      0.      0.
   0.   ]
iter 49 delta 0.0129563324448    [-61.29  -51.4   -26.742   1.044  67.387   0.      0.      0.
   0.   ]
iter 50 delta 0.0127022867106    [-61.288 -51.397 -26.74    1.045  67.383   0.      0.      0.
   0.   ]
iter 51 delta 0.0124580119661    [-61.285 -51.394 -26.738   1.046  67.379   0.      0.      0.
   0.   ]
iter 52 delta 0.0122229551366    [-61.283 -51.391 -26.736   1.048  67.375   0.      0.      0.
   0.   ]
iter 53 delta 0.0119966041155    [-61.28  -51.389 -26.734   1.049  67.371   0.      0.      0.
   0.   ]
iter 54 delta 0.0117784840407    [-61.278 -51.386 -26.732   1.05   67.368   0.      0.      0.
   0.   ]
iter 55 delta 0.0115681539685    [-61.276 -51.384 -26.73    1.052  67.364   0.      0.      0.
   0.   ]
iter 56 delta 0.0113652038989    [-61.273 -51.381 -26.729   1.053  67.361   0.      0.      0.
   0.   ]
iter 57 delta 0.0111692521076    [-61.271 -51.379 -26.727   1.054  67.357   0.      0.      0.
   0.   ]
iter 58 delta 0.0109799427498    [-61.269 -51.376 -26.725   1.055  67.354   0.      0.      0.
```

```
   0.   ]
iter 59 delta 0.010796943704    [-61.267 -51.374 -26.723   1.056  67.35    0.      0.      0.
   0.   ]
iter 60 delta 0.0106199446269    [-61.264 -51.371 -26.722   1.058  67.347   0.      0.      0.
   0.   ]
iter 61 delta 0.0104486551974    [-61.262 -51.369 -26.72    1.059  67.344   0.      0.      0.
   0.   ]
iter 62 delta 0.0102828035276    [-61.26  -51.367 -26.718   1.06   67.341   0.      0.      0.
   0.   ]
iter 63 delta 0.0101221347225    [-61.258 -51.365 -26.717   1.061  67.338   0.      0.      0.
   0.   ]
iter 64 delta 0.00857730654683    [-61.257 -51.363 -26.716   1.061  67.333   0.      0.      0
   0.   ]
iter 65 delta 0.00981540336725    [-61.255 -51.361 -26.714   1.062  67.33    0.      0.      0
   0.   ]
iter 66 delta 0.00966890480953    [-61.253 -51.359 -26.713   1.063  67.327   0.      0.      0
   0.   ]
iter 67 delta 0.00952671503291    [-61.251 -51.357 -26.711   1.064  67.324   0.      0.      0
   0.   ]
iter 68 delta 0.00938864669911    [-61.249 -51.355 -26.71    1.065  67.321   0.      0.      0
   0.   ]
iter 69 delta 0.00925452317483    [-61.247 -51.353 -26.708   1.066  67.318   0.      0.      0
   0.   ]
iter 70 delta 0.00912417777801    [-61.245 -51.351 -26.707   1.067  67.316   0.      0.      0
   0.   ]
iter 71 delta 0.00899745308665    [-61.244 -51.349 -26.705   1.068  67.313   0.      0.      0
   0.   ]
iter 72 delta 0.00887420030464    [-61.242 -51.347 -26.704   1.069  67.31    0.      0.      0
   0.   ]
iter 73 delta 0.00875427867891    [-61.24  -51.345 -26.703   1.07   67.307   0.      0.      0
   0.   ]
iter 74 delta 0.00863755496318    [-61.238 -51.343 -26.701   1.071  67.305   0.      0.      0
   0.   ]
iter 75 delta 0.0085239029242    [-61.236 -51.341 -26.7     1.072  67.302   0.      0.      0.
   0.   ]
iter 76 delta 0.00841320288621    [-61.235 -51.339 -26.698   1.073  67.299   0.      0.      0
   0.   ]
iter 77 delta 0.00830534131075    [-61.233 -51.337 -26.697   1.074  67.297   0.      0.      0
   0.   ]
iter 78 delta 0.00820021040808    [-61.231 -51.335 -26.696   1.075  67.294   0.      0.      0
   0.   ]
iter 79 delta 0.0069690615693    [-61.23  -51.334 -26.695   1.075  67.291   0.      0.      0.
   0.   ]
iter 80 delta 0.00799773607702    [-61.229 -51.333 -26.694   1.076  67.288   0.      0.      0
   0.   ]
iter 81 delta 0.00790020271022    [-61.227 -51.331 -26.693   1.077  67.286   0.      0.      0
   0.   ]
iter 82 delta 0.00780501954504    [-61.226 -51.329 -26.691   1.078  67.283   0.      0.      0
```

```
   0.   ]
iter 83 delta 0.00771210264569    [-61.224 -51.327 -26.69    1.079  67.281   0.      0.      0
   0.   ]
iter 84 delta 0.00762137202633    [-61.222 -51.326 -26.689   1.079  67.279   0.      0.      0
   0.   ]
iter 85 delta 0.00753275142138    [-61.221 -51.324 -26.688   1.08   67.276   0.      0.      0
   0.   ]
iter 86 delta 0.00744616807172    [-61.219 -51.322 -26.687   1.081  67.274   0.      0.      0
   0.   ]
iter 87 delta 0.00736155252543    [-61.218 -51.321 -26.685   1.082  67.272   0.      0.      0
   0.   ]
iter 88 delta 0.00727883845212    [-61.216 -51.319 -26.684   1.083  67.269   0.      0.      0
   0.   ]
iter 89 delta 0.00719796246931    [-61.215 -51.318 -26.683   1.083  67.267   0.      0.      0
   0.   ]
iter 90 delta 0.00711886398064    [-61.213 -51.316 -26.682   1.084  67.265   0.      0.      0
   0.   ]
iter 91 delta 0.00704148502433    [-61.212 -51.314 -26.681   1.085  67.263   0.      0.      0
   0.   ]
iter 92 delta 0.00696577013159    [-61.211 -51.313 -26.68    1.086  67.261   0.      0.      0
   0.   ]
iter 93 delta 0.00593111622918    [-61.21  -51.312 -26.679   1.086  67.257   0.      0.      0
   0.   ]
iter 94 delta 0.00681912233936    [-61.208 -51.31  -26.678   1.087  67.255   0.      0.      0
   0.   ]
iter 95 delta 0.00674808981498    [-61.207 -51.309 -26.677   1.087  67.253   0.      0.      0
   0.   ]
iter 96 delta 0.00667852187875    [-61.206 -51.307 -26.676   1.088  67.251   0.      0.      0
   0.   ]
iter 97 delta 0.00661037369631    [-61.204 -51.306 -26.675   1.089  67.249   0.      0.      0
   0.   ]
iter 98 delta 0.00654360224483    [-61.203 -51.305 -26.674   1.089  67.247   0.      0.      0
   0.   ]
iter 99 delta 0.00647816622238    [-61.202 -51.303 -26.673   1.09   67.245   0.      0.      0
   0.   ]
```

10 Training Set Performance:

```
In [30]: print("learnedn weights",weights)
         for i,(gm,_,loss_function) in enumerate(models_train):
             gm.change_weights(weights)


             graphcut = GraphCut(model=gm)
             y_pred = graphcut.optimize()
```

```
prediction_image = y_pred.reshape(shape)

# show a bit from the dataset

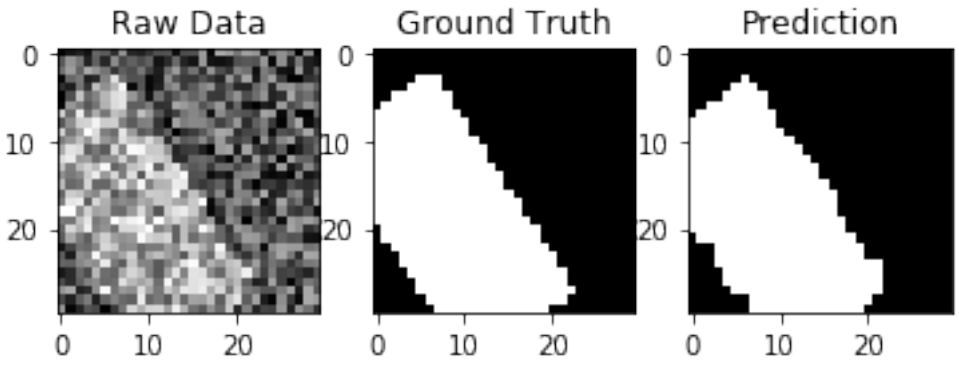
f= pylab.figure()
ax1 = f.add_subplot(1,3,1)
pylab.imshow(x_train[i],cmap='gray')
ax1.set_title('Raw Data')

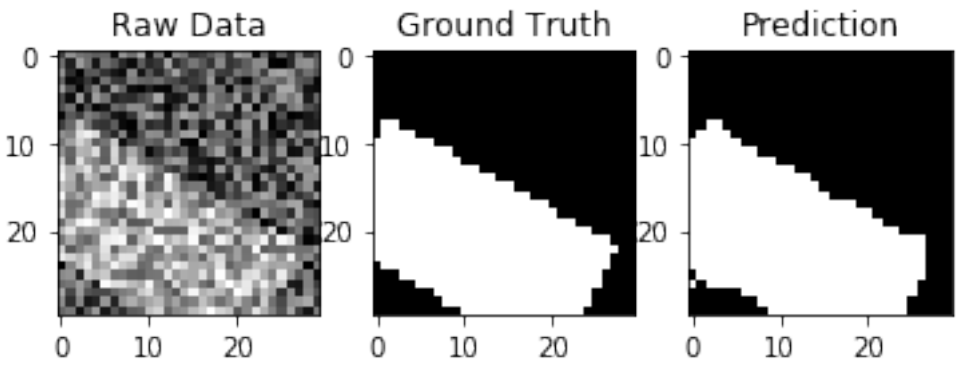
ax2 = f.add_subplot(1,3,2)
pylab.imshow(y_train[i],cmap='gray')
ax2.set_title('Ground Truth')

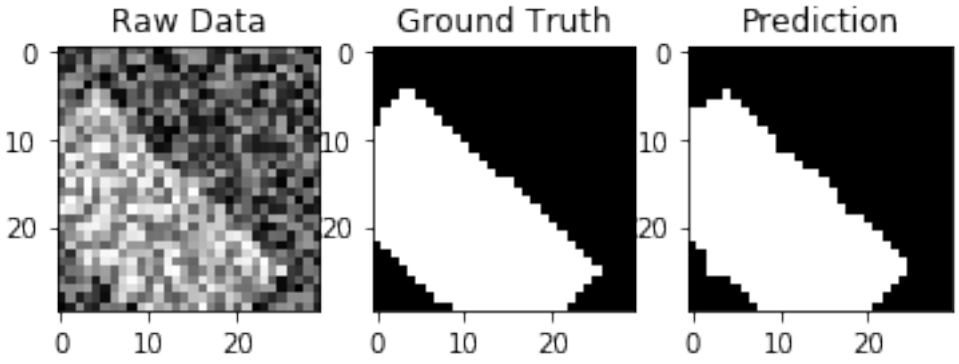
ax3 = f.add_subplot(1,3,3)
pylab.imshow(prediction_image,cmap='gray')
ax3.set_title('Prediction')

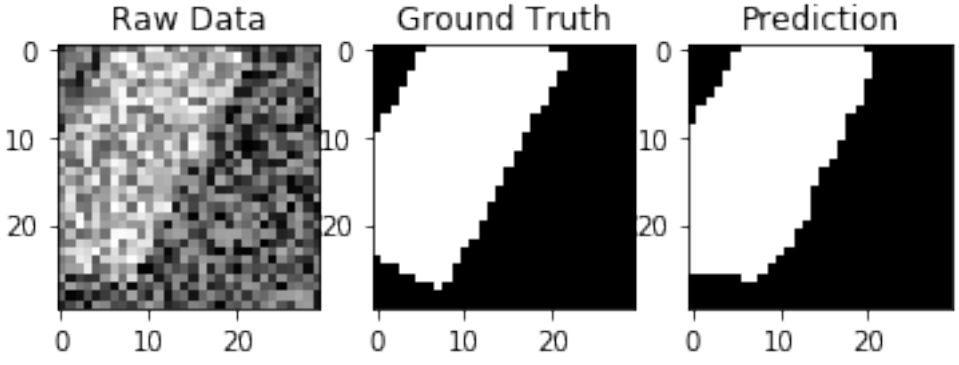
plt.show()
```

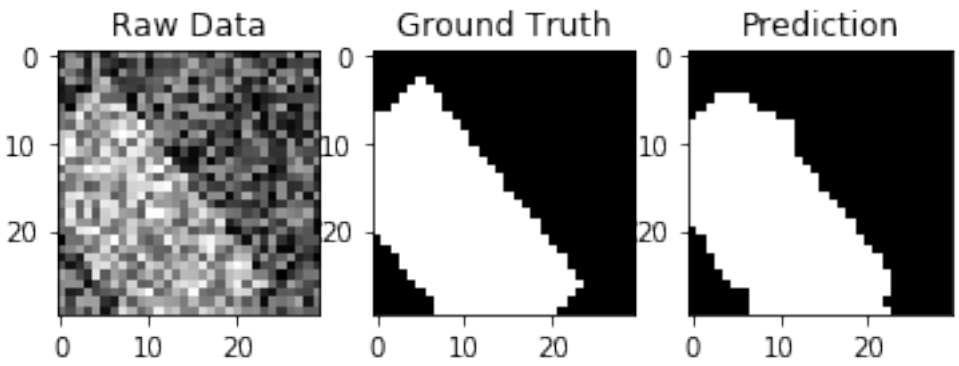
```
learnedn weights [-61.20162564 -51.30311871 -26.67284331   1.09018464  67.24514042   0.
   0.           0.           0.           0.        ]
```




Raw Data
Ground Truth
Prediction
0
10
20


Raw Data
Ground Truth
Prediction
0
10
20


Raw Data
Ground Truth
Prediction
0
10
20



11 Test set performance:

```
In [31]: for i,(gm,_,loss_function) in enumerate(models_test):
             gm.change_weights(weights)


             graphcut = GraphCut(model=gm)
             y_pred = graphcut.optimize()

             prediction_image = y_pred.reshape(shape)

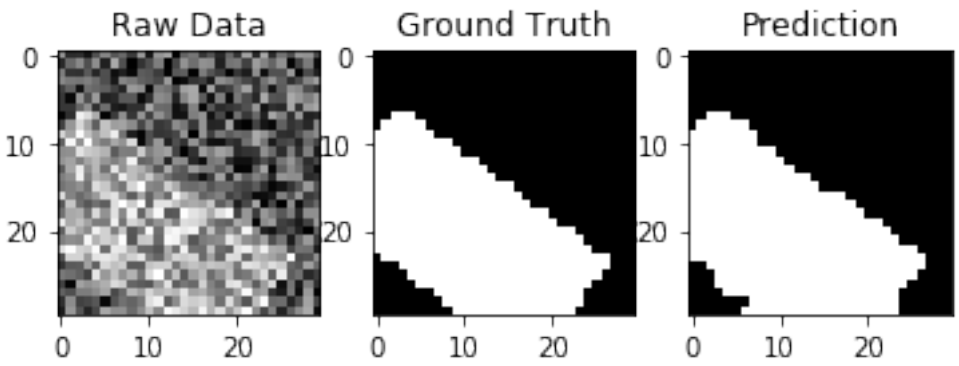
             # show a bit from the dataset

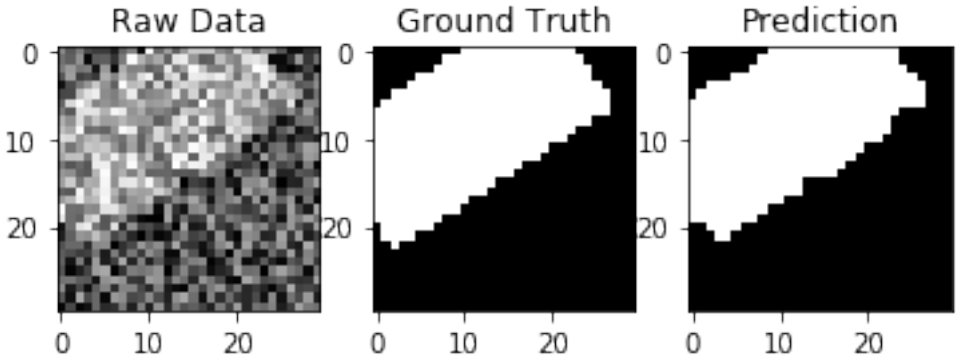
             f= pylab.figure()
             ax1 = f.add_subplot(1,3,1)
             pylab.imshow(x_test[i],cmap='gray')
             ax1.set_title('Raw Data')

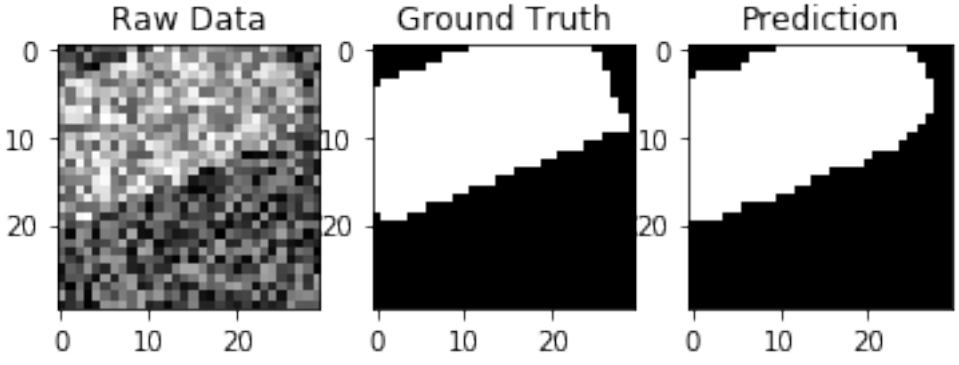
             ax2 = f.add_subplot(1,3,2)
             pylab.imshow(y_test[i],cmap='gray')
             ax2.set_title('Ground Truth')

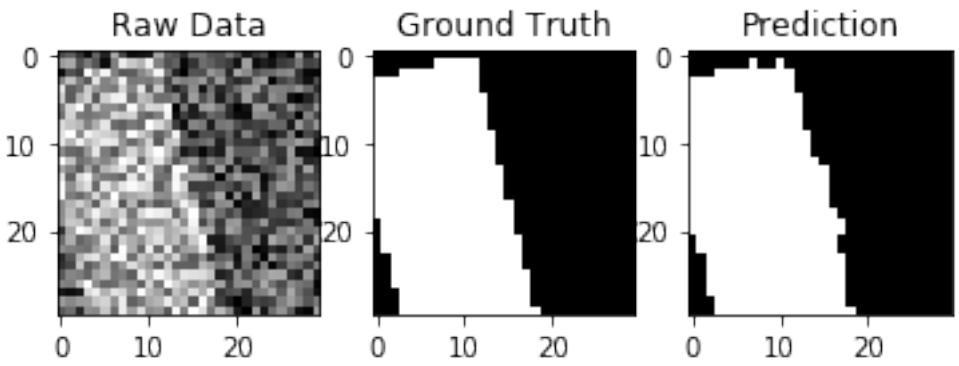
             ax3 = f.add_subplot(1,3,3)
             pylab.imshow(prediction_image,cmap='gray')
             ax3.set_title('Prediction')

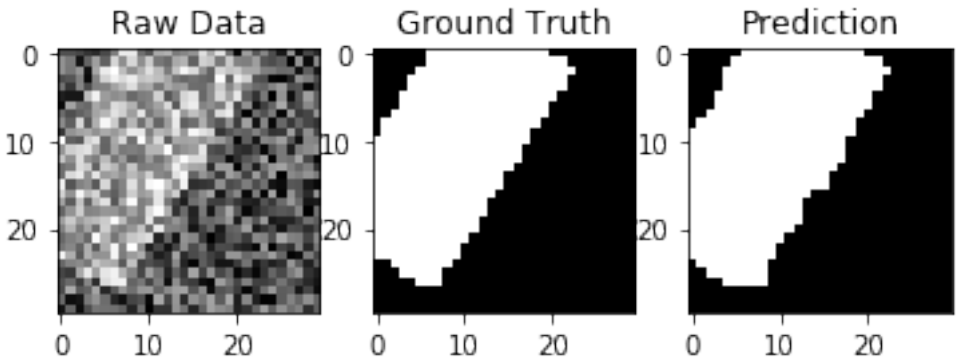
             plt.show()
```


Raw Data
Ground Truth
Prediction
0
10
20


Raw Data
Ground Truth
Prediction
0
10
20


Raw Data
Ground Truth
Prediction
0
10
20





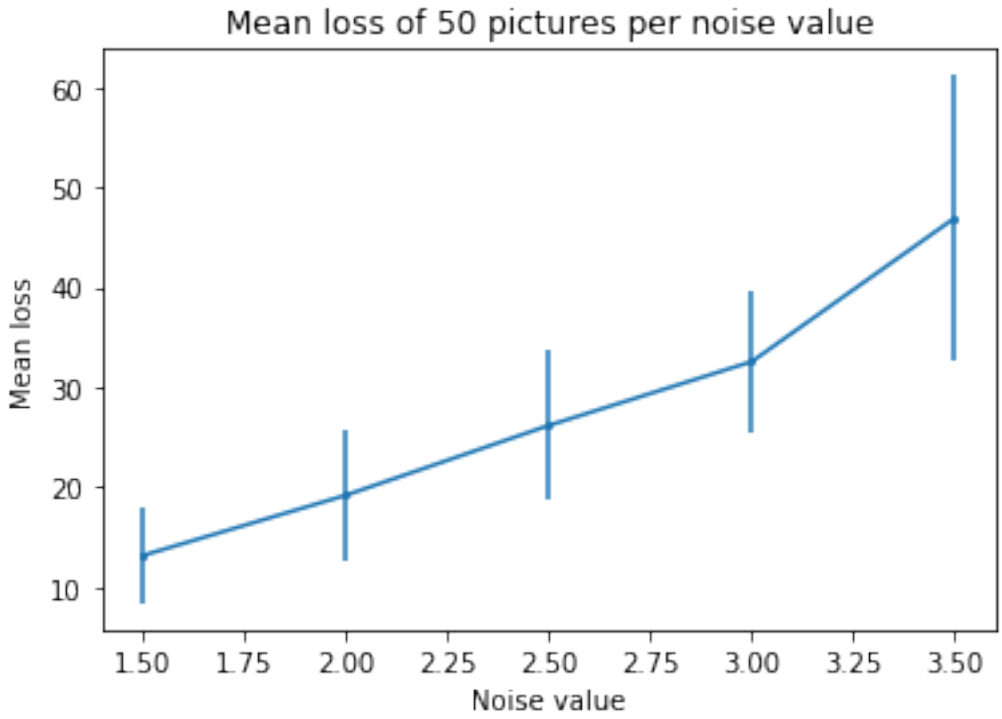
12 Experiments (5 P):

Compute The test set performance (loss on test test) for different noise values (1.5, 2.0, 2.5, 3.0, 3.5) and different regularizers C (0.1 0.5 0.9, 5, 10).

```
In [70]: noises = [1.5, 2., 2.5, 3., 3.5]
         dataset = numpy.zeros((len(noises), 2))
         for j, noise in enumerate(noises):
             x_test , y_test  =  make_toy_dataset(shape=shape, n_images=50, noise=noise)
             models_test   = [build_model(x,y, weights) for x,y in zip(x_test, y_test)]
             losses = []
             for i, (gm, _, loss_function) in enumerate(models_test):
                 lf = HammingLoss(y_test[i])

                 gm.change_weights(weights)
                 graphcut = GraphCut(model=gm)
                 y_pred = graphcut.optimize()
                 prediction_image = y_pred.reshape(shape)
```

```
            losses.append(lf(prediction_image))
        dataset[j, 0] = numpy.mean(losses)
        dataset[j, 1] = numpy.std(losses)
    plt.cla()
    plt.errorbar(noises, dataset[:, 0], marker='o', markersize=2, yerr=dataset[:, 1])
    plt.title('Mean loss of 50 pictures per noise value')
    plt.xlabel('Noise value')
    plt.ylabel('Mean loss')
    plt.show()
```




```
In [72]: Cs = [.1, .5, .9, 5, 10]
         dataset = numpy.zeros((len(Cs), 2))
         for j, C in enumerate(Cs):
             weights = subgradient_ssvm(dset,c=C,learning_rate=1.0, lower_bounds=lower_bounds,

             x_test , y_test  =  make_toy_dataset(shape=shape, n_images=50, noise=noise)
             models_test   = [build_model(x,y, weights) for x,y in zip(x_test, y_test)]
             losses = []
             for i, (gm, _, loss_function) in enumerate(models_test):
                 lf = HammingLoss(y_test[i])

                 gm.change_weights(weights)
                 graphcut = GraphCut(model=gm)
```

```
                y_pred = graphcut.optimize()
                prediction_image = y_pred.reshape(shape)

                losses.append(lf(prediction_image))
            dataset[j, 0] = numpy.mean(losses)
            dataset[j, 1] = numpy.std(losses)
```

```
iter 0 delta 29.6603896538    [ -6.41   -5.032  -1.588   2.23   14.4     0.      0.      0.
   0.   ]
iter 1 delta 63.1762670272    [-18.656 -16.907 -12.406  -7.108  -4.5     0.      0.      0.
   0.   ]
iter 2 delta 43.3172189859    [-12.629 -10.668  -5.724  -0.14   12.9     0.      0.      0.
   0.   ]
iter 3 delta 10.9121293799    [-11.773  -9.799  -4.833   0.734  14.86    0.769   0.977   0.975
   0.937   0.865]
iter 4 delta 16.8335039699    [-14.236 -12.168  -6.992  -1.163  10.936   0.164   0.375   0.346
   0.242   0.037]
iter 5 delta 23.0058643946    [-11.232  -9.059  -3.663   2.307  19.589   0.      0.      0.
   0.   ]
iter 6 delta 18.0503620078    [-14.731 -12.452  -6.754  -0.361  14.189   0.      0.      0.
   0.   ]
iter 7 delta 8.52923815112    [-13.988 -11.7    -5.979   0.41   15.964   0.504   0.672   0.675
   0.641   0.57 ]
iter 8 delta 1.50564230749    [-14.105 -11.808  -6.081   0.315  15.744   0.385   0.54    0.53
   0.484   0.412]
iter 9 delta 0.964085611698    [-14.151 -11.848  -6.117   0.28   15.658   0.286   0.428   0.408
   0.354   0.28 ]
iter 10 delta 0.83674041079    [-14.19  -11.882  -6.147   0.251  15.586   0.199   0.329   0.302
   0.241   0.164]
iter 11 delta 0.567399451723    [-14.201 -11.889  -6.153   0.246  15.566   0.127   0.247   0.2
   0.15    0.067]
iter 12 delta 0.476585850915    [-14.212 -11.896  -6.159   0.24   15.546   0.066   0.178   0.13
   0.072   0.   ]
iter 13 delta 0.295545932678    [-14.219 -11.901  -6.163   0.236  15.531   0.022   0.126   0.08
   0.016   0.   ]
iter 14 delta 0.167357767938    [-14.22  -11.899  -6.161   0.238  15.53    0.      0.088   0.04
   0.   ]
iter 15 delta 0.0452748587265    [-14.216 -11.894  -6.155   0.245  15.539   0.      0.081   0.0
   0.   ]
iter 16 delta 0.0292397413746    [-14.215 -11.891  -6.151   0.249  15.542   0.      0.075   0.0
   0.   ]
iter 17 delta 0.0276153112983    [-14.214 -11.888  -6.148   0.253  15.546   0.      0.069   0.0
   0.   ]
iter 18 delta 0.0146926642719    [ -1.42140000e+01  -1.18860000e+01  -6.14600000e+00   2.55000
   1.55470000e+01   1.00000000e-03   6.70000000e-02   1.60000000e-02
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 19 delta 0.0122530650533    [ -1.42150000e+01  -1.18860000e+01  -6.14500000e+00   2.57000
   1.55460000e+01   3.00000000e-03   6.50000000e-02   1.20000000e-02
```

```
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 20 delta 0.011669585765    [ -1.42160000e+01  -1.18850000e+01  -6.14400000e+00   2.580000
   1.55450000e+01   4.00000000e-03   6.30000000e-02   9.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 21 delta 0.00949742466846    [ -1.42170000e+01  -1.18850000e+01  -6.14300000e+00   2.59000
   1.55430000e+01   8.00000000e-03   6.30000000e-02   8.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 22 delta 0.0089936066466    [ -1.42180000e+01  -1.18850000e+01  -6.14200000e+00   2.60000
   1.55420000e+01   1.10000000e-02   6.40000000e-02   8.00000000e-03
   0.00000000e+00   1.00000000e-03   0.00000000e+00]
iter 23 delta 0.0107526020727    [ -1.42180000e+01  -1.18840000e+01  -6.14100000e+00   2.62000
   1.55420000e+01   1.20000000e-02   6.20000000e-02   5.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 24 delta 0.00827411811487    [ -1.42190000e+01  -1.18840000e+01  -6.14000000e+00   2.63000
   1.55410000e+01   1.50000000e-02   6.30000000e-02   4.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 25 delta 0.00968023849551    [ -1.42200000e+01  -1.18830000e+01  -6.13900000e+00   2.64000
   1.55400000e+01   1.60000000e-02   6.10000000e-02   1.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 26 delta 0.00766122047673    [ -1.42200000e+01  -1.18830000e+01  -6.13900000e+00   2.65000
   1.55400000e+01   1.90000000e-02   6.10000000e-02   1.00000000e-03
   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 27 delta 0.00738760545971    [-14.221 -11.883  -6.138   0.266  15.539   0.022   0.062   0
   0.   ]
iter 28 delta 0.00642775629849    [-14.221 -11.882  -6.137   0.267  15.539   0.023   0.06    0
   0.   ]
iter 29 delta 0.00568432708756    [-14.222 -11.882  -6.136   0.268  15.538   0.024   0.059   0
   0.   ]
iter 30 delta 0.00624280635919    [-14.223 -11.882  -6.136   0.269  15.537   0.026   0.059   0
   0.   ]
iter 31 delta 0.00556620492612    [-14.223 -11.881  -6.135   0.27   15.537   0.027   0.058   0
   0.   ]
iter 32 delta 0.0051675700796    [-14.224 -11.881  -6.134   0.271  15.536   0.028   0.056   0.
   0.   ]
iter 33 delta 0.00501558272432    [-14.224 -11.88   -6.133   0.272  15.536   0.029   0.055   0
   0.   ]
iter 34 delta 0.00487228036077    [-14.225 -11.88   -6.133   0.273  15.535   0.03    0.054   0
   0.   ]
iter 35 delta 0.00473693923964    [-14.226 -11.88   -6.132   0.273  15.535   0.03    0.053   0
   0.   ]
iter 36 delta 0.00523045938203    [-14.226 -11.88   -6.132   0.274  15.534   0.032   0.053   0
   0.   ]
iter 37 delta 0.00567333128625    [-14.226 -11.879  -6.131   0.275  15.534   0.033   0.052   0
   0.   ]
iter 38 delta 0.00438917473592    [-14.227 -11.879  -6.131   0.276  15.533   0.034   0.051   0
   0.   ]
iter 39 delta 0.00426324531567    [-14.228 -11.879  -6.13    0.276  15.533   0.034   0.049   0
   0.   ]
```

```
iter 40 delta 0.00415926372261    [-14.228 -11.879  -6.13    0.277  15.532   0.035   0.048   0
   0.   ]
iter 41 delta 0.00460778564607    [-14.229 -11.879  -6.129   0.277  15.531   0.037   0.049   0
   0.   ]
iter 42 delta 0.00501299714133    [-14.229 -11.878  -6.128   0.279  15.532   0.037   0.048   0
   0.   ]
iter 43 delta 0.00389040487957    [-14.229 -11.878  -6.128   0.279  15.531   0.038   0.047   0
   0.   ]
iter 44 delta 0.00378955139171    [-14.23  -11.878  -6.128   0.28   15.53    0.039   0.046   0
   0.   ]
iter 45 delta 0.00370716983971    [-14.23  -11.877  -6.127   0.28   15.53    0.039   0.045   0
   0.   ]
iter 46 delta 0.00411759568372    [-14.231 -11.878  -6.127   0.281  15.529   0.041   0.045   0
   0.   ]
iter 47 delta 0.00449035544593    [-14.231 -11.877  -6.126   0.282  15.529   0.041   0.044   0
   0.   ]
iter 48 delta 0.00348020025769    [-14.231 -11.877  -6.126   0.282  15.529   0.042   0.043   0
   0.   ]
iter 49 delta 0.00341059625254    [-14.232 -11.876  -6.125   0.283  15.529   0.043   0.042   0
   0.   ]
iter 50 delta 0.00334372181621    [-14.232 -11.876  -6.125   0.283  15.528   0.043   0.041   0
   0.   ]
iter 51 delta 0.00372167302183    [-14.233 -11.876  -6.125   0.284  15.527   0.045   0.041   0
   0.   ]
iter 52 delta 0.00333625201309    [-14.233 -11.876  -6.124   0.284  15.527   0.045   0.041   0
   0.   ]
iter 53 delta 0.00315795949309    [-14.233 -11.876  -6.124   0.285  15.527   0.045   0.04    0
   0.   ]
iter 54 delta 0.00310054204776    [-14.233 -11.876  -6.123   0.286  15.527   0.046   0.039   0
   0.   ]
iter 55 delta 0.00304517522548    [-14.234 -11.875  -6.123   0.286  15.526   0.046   0.038   0
   0.   ]
iter 56 delta 0.00339521047605    [-14.234 -11.875  -6.123   0.286  15.526   0.048   0.038   0
   0.   ]
iter 57 delta 0.00371560193091    [-14.234 -11.875  -6.122   0.287  15.526   0.048   0.038   0
   0.   ]
iter 58 delta 0.00290131889324    [-14.235 -11.875  -6.122   0.288  15.525   0.049   0.037   0
   0.   ]
iter 59 delta 0.00284216354378    [-14.235 -11.875  -6.121   0.288  15.525   0.049   0.036   0
   0.   ]
iter 60 delta 0.00275480160479    [-14.235 -11.874  -6.121   0.289  15.525   0.05    0.035   0
   0.   ]
iter 61 delta 0.00329139513302    [-14.236 -11.875  -6.121   0.289  15.524   0.051   0.036   0
   0.   ]
iter 62 delta 0.00335627886686    [-14.236 -11.874  -6.12    0.29   15.524   0.051   0.035   0
   0.   ]
iter 63 delta 0.0026746533547     [-14.236 -11.874  -6.12    0.29   15.524   0.052   0.034   0.
   0.   ]
```

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iter 64 delta 0.00258527535219    [-14.237 -11.874  -6.119   0.29   15.524   0.052   0.033   0
   0.   ]
iter 65 delta 0.00309191664011    [-14.237 -11.874  -6.119   0.291  15.523   0.053   0.034   0
   0.   ]
iter 66 delta 0.00315583645964    [-14.237 -11.874  -6.119   0.291  15.523   0.053   0.033   0
   0.   ]
iter 67 delta 0.00251732080443    [-14.238 -11.873  -6.119   0.292  15.523   0.054   0.032   0
   0.   ]
iter 68 delta 0.00243540431728    [-14.238 -11.873  -6.118   0.292  15.523   0.054   0.032   0
   0.   ]
iter 69 delta 0.00244539735287    [-14.238 -11.873  -6.118   0.292  15.522   0.054   0.031   0
   0.   ]
iter 70 delta 0.00268715988036    [-14.238 -11.873  -6.118   0.293  15.522   0.056   0.031   0
   0.   ]
iter 71 delta 0.00246423659582    [-14.239 -11.873  -6.117   0.293  15.521   0.056   0.03    0
   0.   ]
iter 72 delta 0.00230195750537    [-14.239 -11.873  -6.117   0.294  15.521   0.056   0.03    0
   0.   ]
iter 73 delta 0.0022796067563    [-14.239 -11.873  -6.117   0.294  15.521   0.056   0.029   0.
   0.   ]
iter 74 delta 0.00246245334026    [-14.239 -11.873  -6.117   0.294  15.521   0.057   0.029   0
   0.   ]
iter 75 delta 0.0022813492433    [-14.24  -11.872  -6.116   0.295  15.52    0.058   0.029   0.
   0.   ]
iter 76 delta 0.00219079090865    [-14.24  -11.872  -6.116   0.295  15.52    0.058   0.028   0
   0.   ]
iter 77 delta 0.00215439612682    [-14.24  -11.872  -6.116   0.296  15.52    0.058   0.028   0
   0.   ]
iter 78 delta 0.00233777215847    [-14.24  -11.872  -6.115   0.296  15.52    0.059   0.028   0
   0.   ]
iter 79 delta 0.00216724271016    [-14.241 -11.872  -6.115   0.296  15.519   0.06    0.027   0
   0.   ]
iter 80 delta 0.0020665306707    [-14.241 -11.872  -6.115   0.297  15.519   0.06    0.027   0.
   0.   ]
iter 81 delta 0.00205720609715    [-14.241 -11.872  -6.115   0.297  15.519   0.06    0.026   0
   0.   ]
iter 82 delta 0.00222510843999    [-14.241 -11.872  -6.114   0.297  15.519   0.061   0.026   0
   0.   ]
iter 83 delta 0.00204850805673    [-14.241 -11.871  -6.114   0.298  15.519   0.061   0.026   0
   0.   ]
iter 84 delta 0.00198459882313    [-14.242 -11.871  -6.114   0.298  15.518   0.061   0.025   0
   0.   ]
iter 85 delta 0.00196152209263    [-14.242 -11.871  -6.114   0.299  15.518   0.062   0.024   0
   0.   ]
iter 86 delta 0.00212280460367    [-14.242 -11.871  -6.113   0.299  15.518   0.063   0.025   0
   0.   ]
iter 87 delta 0.00195536490992    [-14.242 -11.871  -6.113   0.299  15.518   0.063   0.024   0
   0.   ]
```

```
iter 88 delta 0.00189540337041    [-14.243 -11.871  -6.113   0.3    15.517   0.063   0.024   0
   0.   ]
iter 89 delta 0.00187434333296    [-14.243 -11.871  -6.113   0.3    15.517   0.063   0.023   0
   0.   ]
iter 90 delta 0.0020294945112    [-14.243 -11.871  -6.112   0.3    15.517   0.064   0.023   0.
   0.   ]
iter 91 delta 0.00187032361792    [-14.243 -11.87   -6.112   0.301  15.517   0.064   0.023   0
   0.   ]
iter 92 delta 0.0018138806448    [-14.243 -11.87   -6.112   0.301  15.517   0.065   0.022   0.
   0.   ]
iter 93 delta 0.00179458404219    [-14.244 -11.87   -6.112   0.301  15.516   0.065   0.022   0
   0.   ]
iter 94 delta 0.00194404211073    [-14.244 -11.87   -6.111   0.301  15.516   0.066   0.022   0
   0.   ]
iter 95 delta 0.0017923711522    [-14.244 -11.87   -6.111   0.302  15.516   0.066   0.021   0.
   0.   ]
iter 96 delta 0.00173908144295    [-14.244 -11.87   -6.111   0.302  15.516   0.066   0.021   0
   0.   ]
iter 97 delta 0.00172133571394    [-14.244 -11.87   -6.111   0.302  15.516   0.066   0.02    0
   0.   ]
iter 98 delta 0.00186549495474    [-14.245 -11.87   -6.111   0.303  15.515   0.067   0.02    0
   0.   ]
iter 99 delta 0.00172065661484    [-14.245 -11.87   -6.11    0.303  15.515   0.067   0.02    0
   0.   ]
iter 0 delta 148.301948269    [-32.05  -25.162  -7.94   11.149  72.      0.      0.      0.
   0.   ]
iter 1 delta 315.881335136    [-93.278 -84.535 -62.031 -35.541 -22.5     0.      0.      0.
   0.   ]
iter 2 delta 216.586094929    [-63.143 -53.341 -28.618  -0.698  64.5     0.      0.      0.
   0.   ]
iter 3 delta 15.1156490128    [-61.085 -51.222 -26.47    1.362  68.925   0.4     0.572   0.475
   0.37    0.125]
iter 4 delta 5.8495989785    [-61.669 -51.795 -27.047   0.812  67.665   0.      0.      0.
   0.   ]
iter 5 delta 0.770754366329    [-61.523 -51.65  -26.91    0.938  67.882   0.      0.      0.
   0.   ]
iter 6 delta 0.131381588897    [-61.531 -51.658 -26.923   0.923  67.796   0.      0.      0.
   0.   ]
iter 7 delta 0.069690615693    [-61.52  -51.647 -26.917   0.926  67.758   0.      0.      0.
   0.   ]
iter 8 delta 0.0619472139493    [-61.511 -51.636 -26.911   0.928  67.725   0.      0.      0.
   0.   ]
iter 9 delta 0.0835031517545    [-61.488 -51.612 -26.891   0.945  67.725   0.      0.      0.
   0.   ]
iter 10 delta 0.0506840841403    [-61.48  -51.603 -26.886   0.947  67.698   0.      0.      0.
   0.   ]
iter 11 delta 0.046460410462    [-61.473 -51.596 -26.881   0.949  67.673   0.      0.      0.
   0.   ]
```

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iter 12 delta 0.0642331936573    [-61.455 -51.577 -26.866   0.961  67.673   0.      0.      0.
   0.   ]
iter 13 delta 0.0398232089674    [-61.449 -51.57  -26.862   0.963  67.651   0.      0.      0.
   0.   ]
iter 14 delta 0.0371683283696    [-61.443 -51.564 -26.859   0.965  67.631   0.      0.      0.
   0.   ]
iter 15 delta 0.0461726220169    [-61.432 -51.552 -26.849   0.972  67.625   0.      0.      0.
   0.   ]
iter 16 delta 0.0327955838555    [-61.427 -51.547 -26.846   0.974  67.608   0.      0.      0.
   0.   ]
iter 17 delta 0.0359898123466    [-61.419 -51.539 -26.841   0.977  67.596   0.      0.      0.
   0.   ]
iter 18 delta 0.0340956116968    [-61.412 -51.531 -26.835   0.981  67.586   0.      0.      0.
   0.   ]
iter 19 delta 0.0278762462772    [-61.408 -51.526 -26.832   0.982  67.571   0.      0.      0.
   0.   ]
iter 20 delta 0.0397634055974    [-61.397 -51.515 -26.823   0.99   67.571   0.      0.      0.
   0.   ]
iter 21 delta 0.0253420420702    [-61.393 -51.511 -26.821   0.991  67.557   0.      0.      0.
   0.   ]
iter 22 delta 0.0242402141541    [-61.39  -51.507 -26.818   0.992  67.544   0.      0.      0.
   0.   ]
iter 23 delta 0.0307817480113    [-61.382 -51.499 -26.812   0.997  67.54    0.      0.      0.
   0.   ]
iter 24 delta 0.0223009970218    [-61.379 -51.495 -26.81    0.998  67.528   0.      0.      0.
   0.   ]
iter 25 delta 0.0284139212412    [-61.371 -51.488 -26.804   1.002  67.524   0.      0.      0.
   0.   ]
iter 26 delta 0.0206490713164    [-61.368 -51.484 -26.802   1.003  67.513   0.      0.      0.
   0.   ]
iter 27 delta 0.0231363079371    [-61.364 -51.479 -26.798   1.006  67.506   0.      0.      0.
   0.   ]
iter 28 delta 0.0223385042151    [-61.359 -51.474 -26.795   1.008  67.499   0.      0.      0.
   0.   ]
iter 29 delta 0.0215938874079    [-61.355 -51.469 -26.791   1.011  67.492   0.      0.      0.
   0.   ]
iter 30 delta 0.0208973103948    [-61.35  -51.465 -26.788   1.013  67.486   0.      0.      0.
   0.   ]
iter 31 delta 0.0202442694449    [-61.346 -51.46  -26.785   1.015  67.48    0.      0.      0.
   0.   ]
iter 32 delta 0.0196308067345    [-61.342 -51.456 -26.782   1.017  67.474   0.      0.      0.
   0.   ]
iter 33 delta 0.0163977919278    [-61.34  -51.453 -26.78    1.018  67.465   0.      0.      0.
   0.   ]
iter 34 delta 0.0211074843506    [-61.335 -51.448 -26.776   1.021  67.462   0.      0.      0.
   0.   ]
iter 35 delta 0.0154868034873    [-61.332 -51.445 -26.774   1.022  67.454   0.      0.      0.
   0.   ]
```

```
iter 36 delta 0.0175085573578    [-61.329 -51.441 -26.772   1.024  67.448   0.      0.      0.
   0.   ]
iter 37 delta 0.0170478058484    [-61.325 -51.437 -26.769   1.026  67.443   0.      0.      0.
   0.   ]
iter 38 delta 0.0166106826215    [-61.322 -51.434 -26.766   1.027  67.438   0.      0.      0.
   0.   ]
iter 39 delta 0.016195415556    [-61.319 -51.43  -26.764   1.029  67.433   0.      0.      0.
   0.   ]
iter 40 delta 0.0158004054204    [-61.315 -51.427 -26.761   1.031  67.428   0.      0.      0.
   0.   ]
iter 41 delta 0.0154242052914    [-61.312 -51.423 -26.759   1.033  67.423   0.      0.      0.
   0.   ]
iter 42 delta 0.0150655028427    [-61.309 -51.42  -26.756   1.034  67.419   0.      0.      0.
   0.   ]
iter 43 delta 0.0147231050509    [-61.306 -51.417 -26.754   1.036  67.414   0.      0.      0.
   0.   ]
iter 44 delta 0.0143959249386    [-61.303 -51.414 -26.752   1.037  67.41    0.      0.      0.
   0.   ]
iter 45 delta 0.0140829700487    [-61.3   -51.41  -26.749   1.039  67.405   0.      0.      0.
   0.   ]
iter 46 delta 0.0137833323881    [-61.298 -51.407 -26.747   1.04   67.401   0.      0.      0.
   0.   ]
iter 47 delta 0.01349617963    [-61.295 -51.404 -26.745   1.042  67.397   0.      0.      0.
   0.   ]
iter 48 delta 0.011378059705    [-61.293 -51.402 -26.744   1.042  67.391   0.      0.      0.
   0.   ]
iter 49 delta 0.0129563324448    [-61.29  -51.4   -26.742   1.044  67.387   0.      0.      0.
   0.   ]
iter 50 delta 0.0127022867106    [-61.288 -51.397 -26.74    1.045  67.383   0.      0.      0.
   0.   ]
iter 51 delta 0.0124580119661    [-61.285 -51.394 -26.738   1.046  67.379   0.      0.      0.
   0.   ]
iter 52 delta 0.0122229551366    [-61.283 -51.391 -26.736   1.048  67.375   0.      0.      0.
   0.   ]
iter 53 delta 0.0119966041155    [-61.28  -51.389 -26.734   1.049  67.371   0.      0.      0.
   0.   ]
iter 54 delta 0.0117784840407    [-61.278 -51.386 -26.732   1.05   67.368   0.      0.      0.
   0.   ]
iter 55 delta 0.0115681539685    [-61.276 -51.384 -26.73    1.052  67.364   0.      0.      0.
   0.   ]
iter 56 delta 0.0113652038989    [-61.273 -51.381 -26.729   1.053  67.361   0.      0.      0.
   0.   ]
iter 57 delta 0.0111692521076    [-61.271 -51.379 -26.727   1.054  67.357   0.      0.      0.
   0.   ]
iter 58 delta 0.0109799427498    [-61.269 -51.376 -26.725   1.055  67.354   0.      0.      0.
   0.   ]
iter 59 delta 0.010796943704    [-61.267 -51.374 -26.723   1.056  67.35    0.      0.      0.
   0.   ]
```

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iter 60 delta 0.0106199446269    [-61.264 -51.371 -26.722   1.058  67.347   0.      0.      0.
   0.   ]
iter 61 delta 0.0104486551974    [-61.262 -51.369 -26.72    1.059  67.344   0.      0.      0.
   0.   ]
iter 62 delta 0.0102828035276    [-61.26  -51.367 -26.718   1.06   67.341   0.      0.      0.
   0.   ]
iter 63 delta 0.0101221347225    [-61.258 -51.365 -26.717   1.061  67.338   0.      0.      0.
   0.   ]
iter 64 delta 0.00857730654683    [-61.257 -51.363 -26.716   1.061  67.333   0.      0.      0
   0.   ]
iter 65 delta 0.00981540336725    [-61.255 -51.361 -26.714   1.062  67.33    0.      0.      0
   0.   ]
iter 66 delta 0.00966890480953    [-61.253 -51.359 -26.713   1.063  67.327   0.      0.      0
   0.   ]
iter 67 delta 0.00952671503291    [-61.251 -51.357 -26.711   1.064  67.324   0.      0.      0
   0.   ]
iter 68 delta 0.00938864669911    [-61.249 -51.355 -26.71    1.065  67.321   0.      0.      0
   0.   ]
iter 69 delta 0.00925452317483    [-61.247 -51.353 -26.708   1.066  67.318   0.      0.      0
   0.   ]
iter 70 delta 0.00912417777801    [-61.245 -51.351 -26.707   1.067  67.316   0.      0.      0
   0.   ]
iter 71 delta 0.00899745308665    [-61.244 -51.349 -26.705   1.068  67.313   0.      0.      0
   0.   ]
iter 72 delta 0.00887420030464    [-61.242 -51.347 -26.704   1.069  67.31    0.      0.      0
   0.   ]
iter 73 delta 0.00875427867891    [-61.24  -51.345 -26.703   1.07   67.307   0.      0.      0
   0.   ]
iter 74 delta 0.00863755496318    [-61.238 -51.343 -26.701   1.071  67.305   0.      0.      0
   0.   ]
iter 75 delta 0.0085239029242    [-61.236 -51.341 -26.7     1.072  67.302   0.      0.      0.
   0.   ]
iter 76 delta 0.00841320288621    [-61.235 -51.339 -26.698   1.073  67.299   0.      0.      0
   0.   ]
iter 77 delta 0.00830534131075    [-61.233 -51.337 -26.697   1.074  67.297   0.      0.      0
   0.   ]
iter 78 delta 0.00820021040808    [-61.231 -51.335 -26.696   1.075  67.294   0.      0.      0
   0.   ]
iter 79 delta 0.0069690615693    [-61.23  -51.334 -26.695   1.075  67.291   0.      0.      0.
   0.   ]
iter 80 delta 0.00799773607702    [-61.229 -51.333 -26.694   1.076  67.288   0.      0.      0
   0.   ]
iter 81 delta 0.00790020271022    [-61.227 -51.331 -26.693   1.077  67.286   0.      0.      0
   0.   ]
iter 82 delta 0.00780501954504    [-61.226 -51.329 -26.691   1.078  67.283   0.      0.      0
   0.   ]
iter 83 delta 0.00771210264569    [-61.224 -51.327 -26.69    1.079  67.281   0.      0.      0
   0.   ]
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iter 84 delta 0.00762137202633    [-61.222 -51.326 -26.689   1.079  67.279   0.      0.      0
   0.   ]
iter 85 delta 0.00753275142138    [-61.221 -51.324 -26.688   1.08   67.276   0.      0.      0
   0.   ]
iter 86 delta 0.00744616807172    [-61.219 -51.322 -26.687   1.081  67.274   0.      0.      0
   0.   ]
iter 87 delta 0.00736155252543    [-61.218 -51.321 -26.685   1.082  67.272   0.      0.      0
   0.   ]
iter 88 delta 0.00727883845212    [-61.216 -51.319 -26.684   1.083  67.269   0.      0.      0
   0.   ]
iter 89 delta 0.00719796246931    [-61.215 -51.318 -26.683   1.083  67.267   0.      0.      0
   0.   ]
iter 90 delta 0.00711886398064    [-61.213 -51.316 -26.682   1.084  67.265   0.      0.      0
   0.   ]
iter 91 delta 0.00704148502433    [-61.212 -51.314 -26.681   1.085  67.263   0.      0.      0
   0.   ]
iter 92 delta 0.00696577013159    [-61.211 -51.313 -26.68    1.086  67.261   0.      0.      0
   0.   ]
iter 93 delta 0.00593111622918    [-61.21  -51.312 -26.679   1.086  67.257   0.      0.      0
   0.   ]
iter 94 delta 0.00681912233936    [-61.208 -51.31  -26.678   1.087  67.255   0.      0.      0
   0.   ]
iter 95 delta 0.00674808981498    [-61.207 -51.309 -26.677   1.087  67.253   0.      0.      0
   0.   ]
iter 96 delta 0.00667852187875    [-61.206 -51.307 -26.676   1.088  67.251   0.      0.      0
   0.   ]
iter 97 delta 0.00661037369631    [-61.204 -51.306 -26.675   1.089  67.249   0.      0.      0
   0.   ]
iter 98 delta 0.00654360224483    [-61.203 -51.305 -26.674   1.089  67.247   0.      0.      0
   0.   ]
iter 99 delta 0.00647816622238    [-61.202 -51.303 -26.673   1.09   67.245   0.      0.      0
   0.   ]
iter 0 delta 266.943506884    [ -57.691  -45.292  -14.293   20.068  129.6      0.       0.
    0.       0.   ]
iter 1 delta 568.586403245    [-167.9   -152.163 -111.656  -63.974  -40.5      0.       0.
    0.       0.   ]
iter 2 delta 389.854970873    [-113.657  -96.013  -51.512   -1.257  116.1      0.       0.
    0.       0.   ]
iter 3 delta 24.6526479912    [-110.105  -92.357  -47.812    2.292  123.705    0.495    0.739
    0.388    0.392    0.   ]
iter 4 delta 7.5790676712    [-110.902  -93.151  -48.627    1.509  121.905    0.       0.
    0.       0.   ]
iter 5 delta 0.969161533781    [-110.714  -92.958  -48.445    1.675  122.145    0.       0.
    0.       0.   ]
iter 6 delta 0.167681809831    [-110.709  -92.958  -48.459    1.655  122.016    0.       0.
    0.       0.   ]
iter 7 delta 0.146721583602    [-110.705  -92.957  -48.472    1.638  121.904    0.       0.
    0.       0.   ]
```

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iter 8 delta 0.115294996725    [-110.683  -92.937  -48.463    1.641  121.844    0.       0.
    0.       0.   ]
iter 9 delta 0.103765497052    [-110.663  -92.918  -48.455    1.644  121.79     0.       0.
    0.       0.   ]
iter 10 delta 0.109811240516    [-110.637  -92.893  -48.439    1.654  121.757    0.       0.
    0.       0.   ]
iter 11 delta 0.100660303806    [-110.614  -92.869  -48.424    1.664  121.727    0.       0.
    0.       0.   ]
iter 12 delta 0.0929172035134    [-110.592  -92.847  -48.411    1.672  121.7      0.       0.
    0.       0.   ]
iter 13 delta 0.0741182121802    [-110.578  -92.834  -48.405    1.674  121.661    0.       0.
    0.       0.   ]
iter 14 delta 0.080528243045    [-110.559  -92.815  -48.393    1.682  121.637    0.       0.
    0.       0.   ]
iter 15 delta 0.0754952278546    [-110.541  -92.798  -48.383    1.689  121.614    0.       0.
    0.       0.   ]
iter 16 delta 0.0710543320985    [-110.525  -92.781  -48.372    1.695  121.593    0.       0.
    0.       0.   ]
iter 17 delta 0.0671068692041    [-110.509  -92.766  -48.363    1.701  121.573    0.       0.
    0.       0.   ]
iter 18 delta 0.0635749287197    [-110.494  -92.751  -48.354    1.707  121.554    0.       0.
    0.       0.   ]
iter 19 delta 0.0603961822837    [-110.48   -92.737  -48.345    1.713  121.536    0.       0.
    0.       0.   ]
iter 20 delta 0.0494121414535    [-110.471  -92.728  -48.341    1.714  121.511    0.       0.
    0.       0.   ]
iter 21 delta 0.0549056202579    [-110.458  -92.715  -48.333    1.719  121.494    0.       0.
    0.       0.   ]
iter 22 delta 0.0525184193771    [-110.446  -92.703  -48.325    1.724  121.479    0.       0.
    0.       0.   ]
iter 23 delta 0.0503301519031    [-110.434  -92.691  -48.318    1.729  121.464    0.       0.
    0.       0.   ]
iter 24 delta 0.048316945827    [-110.423  -92.68   -48.311    1.733  121.449    0.       0.
    0.       0.   ]
iter 25 delta 0.0464586017567    [-110.412  -92.669  -48.305    1.737  121.435    0.       0.
    0.       0.   ]
iter 26 delta 0.0447379128028    [-110.401  -92.659  -48.298    1.741  121.422    0.       0.
    0.       0.   ]
iter 27 delta 0.0431401302027    [-110.391  -92.649  -48.292    1.745  121.409    0.       0.
    0.       0.   ]
iter 28 delta 0.0357812058801    [-110.384  -92.642  -48.289    1.746  121.391    0.       0.
    0.       0.   ]
iter 29 delta 0.0402641215225    [-110.375  -92.633  -48.283    1.75   121.379    0.       0.
    0.       0.   ]
iter 30 delta 0.0389652788927    [-110.366  -92.624  -48.278    1.754  121.367    0.       0.
    0.       0.   ]
iter 31 delta 0.0377476139273    [-110.357  -92.615  -48.272    1.757  121.356    0.       0.
    0.       0.   ]
```

```
iter 32 delta 0.0366037468386    [-110.349  -92.606  -48.267    1.761  121.345    0.       0.
    0.       0.   ]
iter 33 delta 0.0355271660493    [-110.34   -92.598  -48.262    1.764  121.334    0.       0.
    0.       0.   ]
iter 34 delta 0.0345121041621    [-110.332  -92.59   -48.257    1.767  121.324    0.       0.
    0.       0.   ]
iter 35 delta 0.0288237491812    [-110.327  -92.585  -48.255    1.768  121.309    0.       0.
    0.       0.   ]
iter 36 delta 0.0326465850182    [-110.319  -92.577  -48.25     1.771  121.299    0.       0.
    0.       0.   ]
iter 37 delta 0.0317874643599    [-110.312  -92.57   -48.245    1.774  121.29     0.       0.
    0.       0.   ]
iter 38 delta 0.0309724011712    [-110.305  -92.563  -48.241    1.777  121.28     0.       0.
    0.       0.   ]
iter 39 delta 0.0301980911419    [-110.298  -92.556  -48.237    1.779  121.271    0.       0.
    0.       0.   ]
iter 40 delta 0.0294615523335    [-110.291  -92.549  -48.232    1.782  121.263    0.       0.
    0.       0.   ]
iter 41 delta 0.0247060707267    [-110.286  -92.544  -48.23     1.783  121.25     0.       0.
    0.       0.   ]
iter 42 delta 0.0280912475738    [-110.279  -92.538  -48.226    1.785  121.241    0.       0.
    0.       0.   ]
iter 43 delta 0.027452810129    [-110.273  -92.531  -48.222    1.788  121.233    0.       0.
    0.       0.   ]
iter 44 delta 0.0268427476816    [-110.267  -92.525  -48.219    1.79   121.225    0.       0.
    0.       0.   ]
iter 45 delta 0.0262592096886    [-110.261  -92.519  -48.215    1.793  121.217    0.       0.
    0.       0.   ]
iter 46 delta 0.0257005030995    [-110.255  -92.513  -48.211    1.795  121.21     0.       0.
    0.       0.   ]
iter 47 delta 0.0251650759515    [-110.249  -92.507  -48.207    1.798  121.202    0.       0.
    0.       0.   ]
iter 48 delta 0.0211766320515    [-110.245  -92.503  -48.206    1.798  121.191    0.       0.
    0.       0.   ]
iter 49 delta 0.0241584729135    [-110.239  -92.498  -48.202    1.8    121.184    0.       0.
    0.       0.   ]
iter 50 delta 0.0236847773662    [-110.234  -92.492  -48.199    1.803  121.177    0.       0.
    0.       0.   ]
iter 51 delta 0.0232293008783    [-110.228  -92.487  -48.196    1.805  121.17     0.       0.
    0.       0.   ]
iter 52 delta 0.0227910121825    [-110.223  -92.482  -48.192    1.807  121.163    0.       0.
    0.       0.   ]
iter 53 delta 0.0223689564014    [-110.218  -92.476  -48.189    1.809  121.157    0.       0.
    0.       0.   ]
iter 54 delta 0.0219622481032    [-110.213  -92.471  -48.186    1.811  121.15     0.       0.
    0.       0.   ]
iter 55 delta 0.0185295530451    [-110.209  -92.468  -48.184    1.811  121.14     0.       0.
    0.       0.   ]
```

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iter 56 delta 0.0211916429066    [-110.204  -92.463  -48.181    1.813  121.134    0.       0.
    0.       0.   ]
iter 57 delta 0.020826269753    [-110.199  -92.458  -48.178    1.815  121.128    0.       0.
    0.       0.   ]
iter 58 delta 0.0204732821301    [-110.194  -92.453  -48.175    1.817  121.122    0.       0.
    0.       0.   ]
iter 59 delta 0.0201320607612    [-110.19   -92.449  -48.173    1.819  121.116    0.       0.
    0.       0.   ]
iter 60 delta 0.0198020269783    [-110.185  -92.444  -48.17     1.821  121.11     0.       0.
    0.       0.   ]
iter 61 delta 0.0194826394463    [-110.181  -92.439  -48.167    1.823  121.104    0.       0.
    0.       0.   ]
iter 62 delta 0.0164707138178    [-110.177  -92.437  -48.166    1.823  121.096    0.       0.
    0.       0.   ]
iter 63 delta 0.0188738069637    [-110.173  -92.432  -48.163    1.825  121.09     0.       0.
    0.       0.   ]
iter 64 delta 0.0185834407027    [-110.169  -92.428  -48.16     1.827  121.084    0.       0.
    0.       0.   ]
iter 65 delta 0.0183018734193    [-110.164  -92.424  -48.158    1.828  121.079    0.       0.
    0.       0.   ]
iter 66 delta 0.0180287111295    [-110.16   -92.419  -48.155    1.83   121.074    0.       0.
    0.       0.   ]
iter 67 delta 0.0177635830246    [-110.156  -92.415  -48.152    1.832  121.068    0.       0.
    0.       0.   ]
iter 68 delta 0.0150384778337    [-110.153  -92.412  -48.151    1.832  121.06     0.       0.
    0.       0.   ]
iter 69 delta 0.0172560520811    [-110.149  -92.408  -48.149    1.834  121.055    0.       0.
    0.       0.   ]
iter 70 delta 0.017013009094    [-110.145  -92.405  -48.146    1.835  121.05     0.       0.
    0.       0.   ]
iter 71 delta 0.016776717301    [-110.141  -92.401  -48.144    1.837  121.045    0.       0.
    0.       0.   ]
iter 72 delta 0.0165468992558    [-110.137  -92.397  -48.141    1.838  121.04     0.       0.
    0.       0.   ]
iter 73 delta 0.0163232925091    [-110.134  -92.393  -48.139    1.84   121.035    0.       0.
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iter 74 delta 0.016105648609    [-110.13   -92.389  -48.137    1.841  121.031    0.       0.
    0.       0.   ]
iter 75 delta 0.0136533548753    [-110.127  -92.387  -48.136    1.842  121.023    0.       0.
    0.       0.   ]
iter 76 delta 0.0156873200737    [-110.124  -92.383  -48.133    1.843  121.019    0.       0.
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iter 77 delta 0.0154862005856    [-110.12   -92.379  -48.131    1.844  121.014    0.       0.
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iter 78 delta 0.0152901727301    [-110.116  -92.376  -48.129    1.846  121.01     0.       0.
    0.       0.   ]
iter 79 delta 0.0150990455709    [-110.113  -92.372  -48.127    1.847  121.005    0.       0.
    0.       0.   ]
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iter 80 delta 0.0149126376009    [-110.109  -92.369  -48.125    1.849  121.001    0.       0.
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iter 81 delta 0.0126543289088    [-110.107  -92.367  -48.124    1.849  120.994    0.       0.
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iter 82 delta 0.0145532969358    [-110.104  -92.363  -48.122    1.85   120.99     0.       0.
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iter 83 delta 0.0143800434009    [-110.1    -92.36   -48.12     1.852  120.985    0.       0.
    0.       0.   ]
iter 84 delta 0.0142108664197    [-110.097  -92.357  -48.118    1.853  120.981    0.       0.
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iter 85 delta 0.0140456237869    [-110.094  -92.353  -48.115    1.854  120.977    0.       0.
    0.       0.   ]
iter 86 delta 0.0138841798353    [-110.09   -92.35   -48.113    1.856  120.973    0.       0.
    0.       0.   ]
iter 87 delta 0.0137264050645    [-110.087  -92.347  -48.112    1.857  120.969    0.       0.
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iter 88 delta 0.0116590446126    [-110.085  -92.345  -48.111    1.857  120.963    0.       0.
    0.       0.   ]
iter 89 delta 0.0134213738408    [-110.082  -92.342  -48.109    1.858  120.959    0.       0.
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iter 90 delta 0.0132738862162    [-110.079  -92.339  -48.107    1.86   120.955    0.       0.
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iter 91 delta 0.0131296048443    [-110.076  -92.336  -48.105    1.861  120.951    0.       0.
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iter 92 delta 0.0129884262976    [-110.073  -92.333  -48.103    1.862  120.947    0.       0.
    0.       0.   ]
iter 93 delta 0.0128502515497    [-110.07   -92.33   -48.101    1.863  120.943    0.       0.
    0.       0.   ]
iter 94 delta 0.0109226839002    [-110.067  -92.328  -48.1      1.863  120.938    0.       0.
    0.       0.   ]
iter 95 delta 0.0125825379758    [-110.065  -92.325  -48.098    1.865  120.934    0.       0.
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iter 96 delta 0.0124528210894    [-110.062  -92.322  -48.097    1.866  120.93     0.       0.
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iter 97 delta 0.0123257514865    [-110.059  -92.319  -48.095    1.867  120.926    0.       0.
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iter 98 delta 0.0122012489462    [-110.056  -92.316  -48.093    1.868  120.923    0.       0.
    0.       0.   ]
iter 99 delta 0.0120792364568    [-110.053  -92.313  -48.091    1.869  120.919    0.       0.
    0.       0.   ]
iter 0 delta 1483.01948269    [-320.504 -251.624  -79.404  111.488  720.       0.       0.
    0.       0.   ]
iter 1 delta 3158.81335136    [-932.78  -845.352 -620.314 -355.412 -225.       0.       0.
    0.       0.   ]
iter 2 delta 2165.86094929    [-631.43  -533.408 -286.176   -6.983  645.       0.       0.
    0.       0.   ]
iter 3 delta 116.346263131    [ -6.12971000e+02  -5.14437000e+02  -2.66988000e+02   1.13980000e
   6.84250000e+02   4.17000000e-01   1.28400000e+00   3.98000000e-01
```

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   0.00000000e+00   0.00000000e+00   0.00000000e+00]
iter 4 delta 21.5789024119    [-615.968 -517.47  -270.153    8.312  677.05     0.       0.
    0.       0.   ]
iter 5 delta 3.42522196655    [-615.23  -516.716 -269.482    8.907  677.717    0.       0.
    0.       0.   ]
iter 6 delta 1.1121238556    [-614.939 -516.418 -269.264    9.069  677.574    0.       0.
    0.       0.   ]
iter 7 delta 0.857719497828    [-614.742 -516.221 -269.136    9.154  677.324    0.       0.
    0.       0.   ]
iter 8 delta 0.762417331403    [-614.567 -516.045 -269.021    9.23   677.102    0.       0.
    0.       0.   ]
iter 9 delta 0.686175598262    [-614.409 -515.887 -268.919    9.298  676.902    0.       0.
    0.       0.   ]
iter 10 delta 0.62379599842    [-614.266 -515.744 -268.825    9.36   676.72     0.       0.
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iter 11 delta 0.571812998552    [-614.134 -515.612 -268.74     9.417  676.553    0.       0.
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iter 12 delta 0.527827383279    [-614.013 -515.491 -268.661    9.469  676.399    0.       0.
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iter 13 delta 0.49012542733    [-613.901 -515.378 -268.588    9.517  676.256    0.       0.
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iter 14 delta 0.457450398842    [-613.796 -515.273 -268.519    9.563  676.123    0.       0.
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iter 15 delta 0.428859748914    [-613.697 -515.174 -268.455    9.605  675.998    0.       0.
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iter 16 delta 0.40363270486    [-613.605 -515.081 -268.395    9.645  675.88     0.       0.
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iter 17 delta 0.381208665701    [-613.517 -514.993 -268.337    9.683  675.769    0.       0.
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iter 18 delta 0.361145051717    [-613.434 -514.91  -268.283    9.719  675.664    0.       0.
    0.       0.   ]
iter 19 delta 0.343087799131    [-613.355 -514.831 -268.232    9.753  675.564    0.       0.
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iter 20 delta 0.326750284887    [-613.28  -514.756 -268.183    9.785  675.469    0.       0.
    0.       0.   ]
iter 21 delta 0.31189799921    [-613.209 -514.684 -268.137    9.816  675.378    0.       0.
    0.       0.   ]
iter 22 delta 0.298337216636    [-613.14  -514.615 -268.092    9.846  675.291    0.       0.
    0.       0.   ]
iter 23 delta 0.285906499276    [-613.075 -514.55  -268.049    9.874  675.208    0.       0.
    0.       0.   ]
iter 24 delta 0.274470239305    [-613.011 -514.487 -268.008    9.901  675.128    0.       0.
    0.       0.   ]
iter 25 delta 0.263913691639    [-612.951 -514.426 -267.969    9.928  675.051    0.       0.
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iter 26 delta 0.254139110468    [-612.893 -514.367 -267.93     9.953  674.977    0.       0.
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iter 27 delta 0.245062713665    [-612.836 -514.311 -267.894    9.977  674.905    0.       0.
```

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    0.       0.   ]
iter 28 delta 0.236612275263    [-612.782 -514.256 -267.858   10.     674.836    0.       0.
    0.       0.   ]
iter 29 delta 0.228725199421    [-612.729 -514.204 -267.824   10.023  674.77     0.       0.
    0.       0.   ]
iter 30 delta 0.221346967181    [-612.679 -514.153 -267.791   10.045  674.705    0.       0.
    0.       0.   ]
iter 31 delta 0.214429874457    [-612.629 -514.104 -267.759   10.066  674.643    0.       0.
    0.       0.   ]
iter 32 delta 0.207931999473    [-612.582 -514.056 -267.728   10.087  674.582    0.       0.
    0.       0.   ]
iter 33 delta 0.20181635243    [-612.535 -514.009 -267.698   10.107  674.523    0.       0.
    0.       0.   ]
iter 34 delta 0.196050170932    [-612.49  -513.964 -267.668   10.126  674.466    0.       0.
    0.       0.   ]
iter 35 delta 0.190604332851    [-612.447 -513.92  -267.64    10.145  674.41     0.       0.
    0.       0.   ]
iter 36 delta 0.185452864395    [-612.404 -513.878 -267.612   10.164  674.356    0.       0.
    0.       0.   ]
iter 37 delta 0.180572525859    [-612.362 -513.836 -267.585   10.182  674.304    0.       0.
    0.       0.   ]
iter 38 delta 0.175942461093    [-612.322 -513.796 -267.559   10.199  674.252    0.       0.
    0.       0.   ]
iter 39 delta 0.171543899566    [-612.283 -513.756 -267.533   10.216  674.202    0.       0.
    0.       0.   ]
iter 40 delta 0.167359902015    [-612.244 -513.718 -267.508   10.233  674.154    0.       0.
    0.       0.   ]
iter 41 delta 0.163375142443    [-612.207 -513.68  -267.484   10.249  674.106    0.       0.
    0.       0.   ]
iter 42 delta 0.159575720526    [-612.17  -513.643 -267.46    10.265  674.06     0.       0.
    0.       0.   ]
iter 43 delta 0.155948999605    [-612.134 -513.607 -267.436   10.28   674.014    0.       0.
    0.       0.   ]
iter 44 delta 0.152483466281    [-612.099 -513.572 -267.413   10.295  673.97     0.       0.
    0.       0.   ]
iter 45 delta 0.149168608318    [-612.065 -513.538 -267.391   10.31   673.926    0.       0.
    0.       0.   ]
iter 46 delta 0.145994808141    [-612.032 -513.504 -267.369   10.324  673.884    0.       0.
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iter 47 delta 0.142953249638    [-611.999 -513.471 -267.348   10.339  673.842    0.       0.
    0.       0.   ]
iter 48 delta 0.14003583638    [-611.967 -513.439 -267.327   10.353  673.801    0.       0.
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iter 49 delta 0.137235119652    [-611.935 -513.408 -267.306   10.366  673.761    0.       0.
    0.       0.   ]
iter 50 delta 0.134544234953    [-611.904 -513.377 -267.286   10.379  673.722    0.       0.
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iter 51 delta 0.13195684582    [-611.874 -513.346 -267.267   10.393  673.683    0.       0.
```

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    0.       0.   ]
iter 52 delta 0.129467094012    [-611.844 -513.316 -267.247   10.405  673.646    0.       0.
    0.       0.   ]
iter 53 delta 0.127069555234    [-611.815 -513.287 -267.228   10.418  673.609    0.       0.
    0.       0.   ]
iter 54 delta 0.124759199684    [-611.786 -513.258 -267.209   10.43   673.572    0.       0.
    0.       0.   ]
iter 55 delta 0.122531356833    [-611.758 -513.23  -267.191   10.443  673.537    0.       0.
    0.       0.   ]
iter 56 delta 0.120381683906    [-611.731 -513.203 -267.173   10.454  673.501    0.       0.
    0.       0.   ]
iter 57 delta 0.118306137632    [-611.703 -513.175 -267.155   10.466  673.467    0.       0.
    0.       0.   ]
iter 58 delta 0.116300948858    [-611.677 -513.149 -267.138   10.478  673.433    0.       0.
    0.       0.   ]
iter 59 delta 0.11436259971    [-611.65  -513.122 -267.121   10.489  673.4      0.       0.
    0.       0.   ]
iter 60 delta 0.112487802994    [-611.625 -513.096 -267.104   10.5    673.367    0.       0.
    0.       0.   ]
iter 61 delta 0.110673483591    [-611.599 -513.071 -267.088   10.511  673.335    0.       0.
    0.       0.   ]
iter 62 delta 0.108916761629    [-611.574 -513.046 -267.071   10.522  673.303    0.       0.
    0.       0.   ]
iter 63 delta 0.107214937228    [-611.55  -513.021 -267.055   10.533  673.272    0.       0.
    0.       0.   ]
iter 64 delta 0.105565476656    [-611.525 -512.997 -267.039   10.543  673.241    0.       0.
    0.       0.   ]
iter 65 delta 0.103965999737    [-611.501 -512.973 -267.024   10.553  673.211    0.       0.
    0.       0.   ]
iter 66 delta 0.102414268397    [-611.478 -512.949 -267.009   10.564  673.181    0.       0.
    0.       0.   ]
iter 67 delta 0.100908176215    [-611.455 -512.926 -266.993   10.574  673.151    0.       0.
    0.       0.   ]
iter 68 delta 0.0994457388786    [-611.432 -512.903 -266.979   10.583  673.122    0.       0.
    0.       0.   ]
iter 69 delta 0.0980250854661    [-611.409 -512.881 -266.964   10.593  673.094    0.       0.
    0.       0.   ]
iter 70 delta 0.0966444504595    [-611.387 -512.858 -266.949   10.603  673.066    0.       0.
    0.       0.   ]
iter 71 delta 0.0953021664254    [-611.365 -512.837 -266.935   10.612  673.038    0.       0.
    0.       0.   ]
iter 72 delta 0.0939966572963    [-611.344 -512.815 -266.921   10.621  673.011    0.       0.
    0.       0.   ]
iter 73 delta 0.0927264321976    [-611.322 -512.794 -266.907   10.631  672.983    0.       0.
    0.       0.   ]
iter 74 delta 0.0914900797683    [-611.301 -512.773 -266.894   10.64   672.957    0.       0.
    0.       0.   ]
iter 75 delta 0.0902862629291    [-611.281 -512.752 -266.88    10.649  672.93     0.       0.
```

```
    0.       0.   ]
iter 76 delta 0.0891137140602    [-611.26  -512.731 -266.867   10.657  672.905    0.       0.
    0.       0.   ]
iter 77 delta 0.0879712305463    [-611.24  -512.711 -266.853   10.666  672.879    0.       0.
    0.       0.   ]
iter 78 delta 0.0868576706661    [-611.22  -512.691 -266.84    10.675  672.854    0.       0.
    0.       0.   ]
iter 79 delta 0.0857719497827    [-611.2   -512.671 -266.828   10.683  672.829    0.       0.
    0.       0.   ]
iter 80 delta 0.0847130368225    [-611.181 -512.652 -266.815   10.692  672.804    0.       0.
    0.       0.   ]
iter 81 delta 0.0836799510076    [-611.162 -512.632 -266.802   10.7    672.779    0.       0.
    0.       0.   ]
iter 82 delta 0.0826717588269    [-611.143 -512.613 -266.79    10.708  672.755    0.       0.
    0.       0.   ]
iter 83 delta 0.0816875712217    [-611.124 -512.595 -266.778   10.716  672.732    0.       0.
    0.       0.   ]
iter 84 delta 0.080726540972    [-611.105 -512.576 -266.766   10.724  672.708    0.       0.
    0.       0.   ]
iter 85 delta 0.0797878602631    [-611.087 -512.558 -266.754   10.732  672.685    0.       0.
    0.       0.   ]
iter 86 delta 0.0788707584209    [-611.069 -512.54  -266.742   10.74   672.662    0.       0.
    0.       0.   ]
iter 87 delta 0.0779744998026    [-611.051 -512.522 -266.73    10.748  672.639    0.       0.
    0.       0.   ]
iter 88 delta 0.0770983818273    [-611.033 -512.504 -266.719   10.755  672.617    0.       0.
    0.       0.   ]
iter 89 delta 0.0762417331403    [-611.016 -512.486 -266.707   10.763  672.594    0.       0.
    0.       0.   ]
iter 90 delta 0.075403911897    [-610.999 -512.469 -266.696   10.77   672.572    0.       0.
    0.       0.   ]
iter 91 delta 0.074584304159    [-610.981 -512.452 -266.685   10.778  672.551    0.       0.
    0.       0.   ]
iter 92 delta 0.0737823223937    [-610.965 -512.435 -266.674   10.785  672.529    0.       0.
    0.       0.   ]
iter 93 delta 0.0729974040704    [-610.948 -512.418 -266.663   10.792  672.508    0.       0.
    0.       0.   ]
iter 94 delta 0.0722290103434    [-610.931 -512.401 -266.652   10.8    672.487    0.       0.
    0.       0.   ]
iter 95 delta 0.0714766248191    [-610.915 -512.385 -266.642   10.807  672.466    0.       0.
    0.       0.   ]
iter 96 delta 0.0707397523982    [-610.899 -512.369 -266.631   10.814  672.445    0.       0.
    0.       0.   ]
iter 97 delta 0.0700179181901    [-610.882 -512.353 -266.62    10.821  672.425    0.       0.
    0.       0.   ]
iter 98 delta 0.0693106664911    [-610.867 -512.337 -266.61    10.827  672.405    0.       0.
    0.       0.   ]
iter 99 delta 0.0686175598262    [-610.851 -512.321 -266.6     10.834  672.385    0.       0.
```

```
    0.       0.   ]
iter 0 delta 2966.03896538    [ -641.007  -503.247  -158.809   222.976  1440.        0.
     0.        0.        0.   ]
iter 1 delta 6317.62670272    [-1865.559 -1690.704 -1240.628  -710.824  -450.        0.
     0.        0.        0.   ]
iter 2 delta 4331.72189859    [-1262.86  -1066.815  -572.351   -13.966  1290.        0.
     0.        0.        0.   ]
iter 3 delta 234.624743165    [ -1.22658700e+03  -1.02955100e+03  -5.34653000e+02   2.21590000e
   1.36700000e+03   2.47300000e+00   4.13700000e+00   2.41600000e+00
   5.38000000e-01   7.01000000e-01   0.00000000e+00]
iter 4 delta 46.7956913275    [-1232.215 -1035.21   -540.547    16.409  1353.4       0.
     0.        0.        0.   ]
iter 5 delta 7.8211173194    [-1230.584 -1033.532  -539.043    17.75   1355.067     0.        0
     0.        0.        0.   ]
iter 6 delta 2.50616669892    [-1229.865 -1032.797  -538.46     18.22   1355.067     0.
     0.        0.        0.   ]
iter 7 delta 1.71543899566    [-1229.471 -1032.403  -538.203    18.39   1354.567     0.
     0.        0.        0.   ]
iter 8 delta 1.52483466281    [-1229.121 -1032.052  -537.975    18.541  1354.122     0.
     0.        0.        0.   ]
iter 9 delta 1.37235119652    [-1228.806 -1031.736  -537.77     18.677  1353.722     0.
     0.        0.        0.   ]
iter 10 delta 1.24759199684    [-1228.52  -1031.449  -537.583    18.801  1353.359     0.
     0.        0.        0.   ]
iter 11 delta 1.1436259971    [-1228.257 -1031.186  -537.412    18.914  1353.025     0.
     0.        0.        0.   ]
iter 12 delta 1.05565476656    [-1228.015 -1030.943  -537.254    19.019  1352.718     0.
     0.        0.        0.   ]
iter 13 delta 0.980250854661    [-1227.79  -1030.717  -537.107    19.116  1352.432     0.
     0.        0.        0.   ]
iter 14 delta 0.914900797683    [-1227.58  -1030.506  -536.97     19.207  1352.165     0.
     0.        0.        0.   ]
iter 15 delta 0.857719497828    [-1227.383 -1030.309  -536.842    19.292  1351.915     0.
     0.        0.        0.   ]
iter 16 delta 0.807265409721    [-1227.197 -1030.123  -536.721    19.372  1351.68      0.
     0.        0.        0.   ]
iter 17 delta 0.762417331403    [-1227.022 -1029.948  -536.607    19.447  1351.458     0.
     0.        0.        0.   ]
iter 18 delta 0.722290103434    [-1226.856 -1029.782  -536.499    19.519  1351.247     0.
     0.        0.        0.   ]
iter 19 delta 0.686175598262    [-1226.699 -1029.624  -536.396    19.587  1351.047     0.
     0.        0.        0.   ]
iter 20 delta 0.653500569774    [-1226.549 -1029.473  -536.299    19.652  1350.857     0.
     0.        0.        0.   ]
iter 21 delta 0.62379599842    [-1226.406 -1029.33   -536.205    19.714  1350.675     0.
     0.        0.        0.   ]
iter 22 delta 0.596674433272    [-1226.269 -1029.192  -536.116    19.773  1350.501     0.
     0.        0.        0.   ]
```

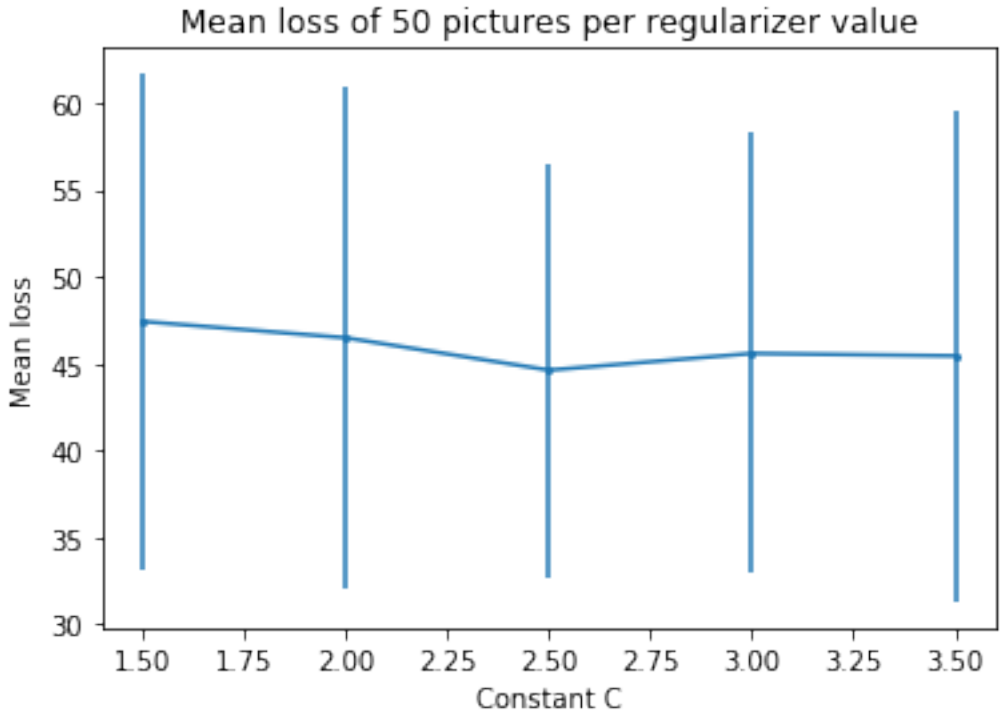
```
iter 23 delta 0.571812998552    [-1226.137 -1029.061  -536.03     19.829  1350.334     0.
     0.         0.         0.   ]
iter 24 delta 0.54894047861    [-1226.011 -1028.935  -535.948    19.884  1350.174     0.
     0.         0.         0.   ]
iter 25 delta 0.527827383279    [-1225.89  -1028.813  -535.869    19.936  1350.02      0.
     0.         0.         0.   ]
iter 26 delta 0.508278220935    [-1225.773 -1028.696  -535.793    19.987  1349.872     0.
     0.         0.         0.   ]
iter 27 delta 0.49012542733    [-1225.661 -1028.583  -535.72     20.035  1349.729     0.
     0.         0.         0.   ]
iter 28 delta 0.473224550526    [-1225.552 -1028.474  -535.649    20.082  1349.591     0.
     0.         0.         0.   ]
iter 29 delta 0.457450398842    [-1225.447 -1028.369  -535.581    20.127  1349.458     0.
     0.         0.         0.   ]
iter 30 delta 0.442693934363    [-1225.345 -1028.267  -535.514    20.171  1349.329     0.
     0.         0.         0.   ]
iter 31 delta 0.428859748914    [-1225.247 -1028.168  -535.45     20.214  1349.204     0.
     0.         0.         0.   ]
iter 32 delta 0.415863998947    [-1225.151 -1028.073  -535.388    20.255  1349.083     0.
     0.         0.         0.   ]
iter 33 delta 0.40363270486    [-1225.059 -1027.98   -535.327    20.295  1348.965     0.
     0.         0.         0.   ]
iter 34 delta 0.392100341864    [-1224.969 -1027.89   -535.269    20.334  1348.851     0.
     0.         0.         0.   ]
iter 35 delta 0.381208665701    [-1224.881 -1027.802  -535.212    20.372  1348.74      0.
     0.         0.         0.   ]
iter 36 delta 0.370905728791    [-1224.796 -1027.717  -535.156    20.408  1348.632     0.
     0.         0.         0.   ]
iter 37 delta 0.361145051717    [-1224.713 -1027.633  -535.102    20.444  1348.526     0.
     0.         0.         0.   ]
iter 38 delta 0.351884922186    [-1224.632 -1027.552  -535.05     20.479  1348.424     0.
     0.         0.         0.   ]
iter 39 delta 0.343087799131    [-1224.554 -1027.474  -534.998    20.513  1348.324     0.
     0.         0.         0.   ]
iter 40 delta 0.33471980403    [-1224.477 -1027.396  -534.948    20.546  1348.226     0.
     0.         0.         0.   ]
iter 41 delta 0.326750284887    [-1224.402 -1027.321  -534.899    20.579  1348.131     0.
     0.         0.         0.   ]
iter 42 delta 0.319151441052    [-1224.328 -1027.248  -534.851    20.61   1348.038     0.
     0.         0.         0.   ]
iter 43 delta 0.31189799921    [-1224.257 -1027.176  -534.805    20.641  1347.947     0.
     0.         0.         0.   ]
iter 44 delta 0.304966932561    [-1224.187 -1027.106  -534.759    20.672  1347.858     0.
     0.         0.         0.   ]
iter 45 delta 0.298337216636    [-1224.118 -1027.037  -534.714    20.701  1347.771     0.
     0.         0.         0.   ]
iter 46 delta 0.291989616282    [-1224.051 -1026.97   -534.671    20.73   1347.686     0.
     0.         0.         0.   ]
```

```
iter 47 delta 0.285906499276    [-1223.986 -1026.904  -534.628    20.758  1347.603     0.
     0.        0.        0.   ]
iter 48 delta 0.28007167276    [-1223.921 -1026.84   -534.586    20.786  1347.521     0.
     0.        0.        0.   ]
iter 49 delta 0.274470239305    [-1223.858 -1026.777  -534.545    20.813  1347.441     0.
     0.        0.        0.   ]
iter 50 delta 0.269088469907    [-1223.796 -1026.715  -534.505    20.84   1347.363     0.
     0.        0.        0.   ]
iter 51 delta 0.263913691639    [-1223.736 -1026.654  -534.465    20.866  1347.286     0.
     0.        0.        0.   ]
iter 52 delta 0.258934188024    [-1223.676 -1026.594  -534.427    20.892  1347.21      0.
     0.        0.        0.   ]
iter 53 delta 0.254139110468    [-1223.618 -1026.536  -534.388    20.917  1347.136     0.
     0.        0.        0.   ]
iter 54 delta 0.249518399368    [-1223.561 -1026.478  -534.351    20.942  1347.064     0.
     0.        0.        0.   ]
iter 55 delta 0.245062713665    [-1223.504 -1026.422  -534.314    20.966  1346.992     0.
     0.        0.        0.   ]
iter 56 delta 0.240763367811    [-1223.449 -1026.367  -534.278    20.99   1346.922     0.
     0.        0.        0.   ]
iter 57 delta 0.236612275263    [-1223.395 -1026.312  -534.243    21.013  1346.853     0.
     0.        0.        0.   ]
iter 58 delta 0.232601897716    [-1223.341 -1026.259  -534.208    21.036  1346.785     0.
     0.        0.        0.   ]
iter 59 delta 0.228725199421    [-1223.289 -1026.206  -534.174    21.059  1346.719     0.
     0.        0.        0.   ]
iter 60 delta 0.224975605987    [-1223.237 -1026.154  -534.14     21.081  1346.653     0.
     0.        0.        0.   ]
iter 61 delta 0.221346967181    [-1223.186 -1026.103  -534.107    21.103  1346.589     0.
     0.        0.        0.   ]
iter 62 delta 0.217833523258    [-1223.136 -1026.053  -534.075    21.125  1346.525     0.
     0.        0.        0.   ]
iter 63 delta 0.214429874457    [-1223.087 -1026.004  -534.043    21.146  1346.463     0.
     0.        0.        0.   ]
iter 64 delta 0.211130953312    [-1223.039 -1025.955  -534.011    21.167  1346.401     0.
     0.        0.        0.   ]
iter 65 delta 0.207931999473    [-1222.991 -1025.907  -533.98     21.188  1346.34      0.
     0.        0.        0.   ]
iter 66 delta 0.204828536795    [-1222.944 -1025.86   -533.949    21.208  1346.281     0.
     0.        0.        0.   ]
iter 67 delta 0.20181635243    [-1222.898 -1025.814  -533.919    21.228  1346.222     0.
     0.        0.        0.   ]
iter 68 delta 0.198891477757    [-1222.852 -1025.768  -533.889    21.248  1346.164     0.
     0.        0.        0.   ]
iter 69 delta 0.196050170932    [-1222.807 -1025.723  -533.86     21.267  1346.107     0.
     0.        0.        0.   ]
iter 70 delta 0.193288900919    [-1222.763 -1025.678  -533.831    21.286  1346.05      0.
     0.        0.        0.   ]
```

```
iter 71 delta 0.190604332851    [-1222.719 -1025.635  -533.802    21.305  1345.995     0.
     0.        0.        0.   ]
iter 72 delta 0.187993314593    [-1222.676 -1025.591  -533.774    21.324  1345.94      0.
     0.        0.        0.   ]
iter 73 delta 0.185452864395    [-1222.633 -1025.549  -533.747    21.342  1345.886     0.
     0.        0.        0.   ]
iter 74 delta 0.182980159537    [-1222.591 -1025.506  -533.719    21.361  1345.833     0.
     0.        0.        0.   ]
iter 75 delta 0.180572525858    [-1222.55  -1025.465  -533.692    21.378  1345.78      0.
     0.        0.        0.   ]
iter 76 delta 0.17822742812    [-1222.509 -1025.424  -533.665    21.396  1345.728     0.
     0.        0.        0.   ]
iter 77 delta 0.175942461093    [-1222.468 -1025.383  -533.639    21.414  1345.677     0.
     0.        0.        0.   ]
iter 78 delta 0.173715341332    [-1222.428 -1025.343  -533.613    21.431  1345.626     0.
     0.        0.        0.   ]
iter 79 delta 0.171543899565    [-1222.389 -1025.304  -533.587    21.448  1345.576     0.
     0.        0.        0.   ]
iter 80 delta 0.169426073645    [-1222.35  -1025.265  -533.562    21.465  1345.527     0.
     0.        0.        0.   ]
iter 81 delta 0.167359902015    [-1222.312 -1025.226  -533.537    21.481  1345.478     0.
     0.        0.        0.   ]
iter 82 delta 0.165343517654    [-1222.274 -1025.188  -533.512    21.498  1345.43      0.
     0.        0.        0.   ]
iter 83 delta 0.163375142443    [-1222.236 -1025.151  -533.488    21.514  1345.382     0.
     0.        0.        0.   ]
iter 84 delta 0.161453081944    [-1222.199 -1025.114  -533.464    21.53   1345.335     0.
     0.        0.        0.   ]
iter 85 delta 0.159575720526    [-1222.162 -1025.077  -533.44     21.546  1345.289     0.
     0.        0.        0.   ]
iter 86 delta 0.157741516842    [-1222.126 -1025.041  -533.416    21.561  1345.243     0.
     0.        0.        0.   ]
iter 87 delta 0.155948999605    [-1222.09  -1025.005  -533.393    21.577  1345.197     0.
     0.        0.        0.   ]
iter 88 delta 0.154196763655    [-1222.055 -1024.969  -533.37     21.592  1345.152     0.
     0.        0.        0.   ]
iter 89 delta 0.152483466281    [-1222.02  -1024.934  -533.347    21.607  1345.108     0.
     0.        0.        0.   ]
iter 90 delta 0.150807823794    [-1221.985 -1024.899  -533.324    21.622  1345.064     0.
     0.        0.        0.   ]
iter 91 delta 0.149168608318    [-1221.951 -1024.865  -533.302    21.637  1345.02      0.
     0.        0.        0.   ]
iter 92 delta 0.147564644787    [-1221.917 -1024.831  -533.28     21.651  1344.977     0.
     0.        0.        0.   ]
iter 93 delta 0.145994808141    [-1221.884 -1024.798  -533.258    21.666  1344.935     0.
     0.        0.        0.   ]
iter 94 delta 0.144458020687    [-1221.851 -1024.764  -533.237    21.68   1344.893     0.
     0.        0.        0.   ]
```

```
iter 95 delta 0.142953249638    [-1221.818 -1024.731  -533.215    21.694  1344.851     0.
     0.         0.         0.   ]
iter 96 delta 0.141479504796    [-1221.785 -1024.699  -533.194    21.708  1344.81      0.
     0.         0.         0.   ]
iter 97 delta 0.14003583638    [-1221.753 -1024.667  -533.173    21.722  1344.769     0.
     0.         0.         0.   ]
iter 98 delta 0.138621332982    [-1221.721 -1024.635  -533.152    21.736  1344.729     0.
     0.         0.         0.   ]
iter 99 delta 0.137235119652    [-1221.69  -1024.603  -533.132    21.75   1344.689     0.
     0.         0.         0.   ]
```

```
In [73]: plt.cla()
         plt.errorbar(noises, dataset[:, 0], marker='o', markersize=2, yerr=dataset[:, 1])
         plt.title('Mean loss of 50 pictures per regularizer value')
         plt.xlabel('Constant C')
         plt.ylabel('Mean loss')
         plt.show()
```



13 Experiments (5 P):

Replace GraphCut with IteratedConditionalModes (ICM) to see if optimality is important.

```
In [74]: lower_bounds = numpy.ones(len(weights))*(-1.0*float('inf'))

         # we want the regularizer 'beta' to be positive
         lower_bounds[n_unary_features:n_unary_features+n_potts_features] = 0

         weights = subgradient_ssvm(dset,c=0.5,learning_rate=1.0, lower_bounds=lower_bounds, n
                                    algorithm=IteratedConditionalModes)

         for i,(gm,_,loss_function) in enumerate(models_test):
             gm.change_weights(weights)


             a = IteratedConditionalModes(model=gm)
             y_pred = a.optimize()

             prediction_image = y_pred.reshape(shape)

             # show a bit from the dataset

             f= pylab.figure()
             ax1 = f.add_subplot(1,3,1)
             pylab.imshow(x_test[i],cmap='gray')
             ax1.set_title('Raw Data')

             ax2 = f.add_subplot(1,3,2)
             pylab.imshow(y_test[i],cmap='gray')
             ax2.set_title('Ground Truth')

             ax3 = f.add_subplot(1,3,3)
             pylab.imshow(prediction_image,cmap='gray')
             ax3.set_title('Prediction')

             plt.show()

iter 0 delta 148.301948269    [-32.05  -25.162  -7.94   11.149  72.      0.      0.      0.
   0.   ]
iter 1 delta 315.881335136    [-93.278 -84.535 -62.031 -35.541 -22.5     0.      0.      0.
   0.   ]
iter 2 delta 216.586094929    [-63.143 -53.341 -28.618  -0.698  64.5     0.      0.      0.
   0.   ]
iter 3 delta 15.1156490128    [-61.085 -51.222 -26.47    1.362  68.925   0.4     0.572   0.475
   0.37    0.125]
iter 4 delta 6.02735133396    [-61.701 -51.825 -27.075   0.785  67.605   0.      0.      0.
   0.   ]
iter 5 delta 0.869048533361    [-61.54  -51.663 -26.921   0.927  67.855   0.      0.      0.
   0.   ]
iter 6 delta 0.0723001606803    [-61.535 -51.657 -26.921   0.924  67.798   0.      0.      0.
   0.   ]
```

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iter 7 delta 0.069690615693    [-61.524 -51.645 -26.914   0.927  67.76    0.      0.      0.
   0.   ]
iter 8 delta 0.0619472139493    [-61.515 -51.635 -26.908   0.93   67.727   0.      0.      0.
   0.   ]
iter 9 delta 0.0835031517545    [-61.492 -51.611 -26.888   0.946  67.727   0.      0.      0.
   0.   ]
iter 10 delta 0.0506840841403    [-61.484 -51.602 -26.883   0.949  67.7     0.      0.      0.
   0.   ]
iter 11 delta 0.046460410462    [-61.477 -51.594 -26.879   0.951  67.675   0.      0.      0.
   0.   ]
iter 12 delta 0.0642331936573    [-61.459 -51.576 -26.864   0.963  67.675   0.      0.      0.
   0.   ]
iter 13 delta 0.0398232089674    [-61.453 -51.569 -26.86    0.965  67.653   0.      0.      0.
   0.   ]
iter 14 delta 0.0371683283696    [-61.447 -51.563 -26.856   0.967  67.633   0.      0.      0.
   0.   ]
iter 15 delta 0.0404885388899    [-61.439 -51.554 -26.85    0.971  67.621   0.      0.      0.
   0.   ]
iter 16 delta 0.0327955838555    [-61.434 -51.548 -26.847   0.972  67.603   0.      0.      0.
   0.   ]
iter 17 delta 0.0463906398636    [-61.421 -51.535 -26.836   0.981  67.603   0.      0.      0.
   0.   ]
iter 18 delta 0.0293434171339    [-61.416 -51.53  -26.833   0.983  67.587   0.      0.      0.
   0.   ]
iter 19 delta 0.0278762462772    [-61.412 -51.525 -26.83    0.984  67.572   0.      0.      0.
   0.   ]
iter 20 delta 0.0351791405843    [-61.403 -51.516 -26.823   0.989  67.568   0.      0.      0.
   0.   ]
iter 21 delta 0.0253420420702    [-61.399 -51.512 -26.821   0.991  67.554   0.      0.      0.
   0.   ]
iter 22 delta 0.0321200848813    [-61.391 -51.504 -26.814   0.996  67.55    0.      0.      0.
   0.   ]
iter 23 delta 0.023230205231    [-61.388 -51.5   -26.812   0.997  67.537   0.      0.      0.
   0.   ]
iter 24 delta 0.0259126648895    [-61.383 -51.494 -26.808   0.999  67.529   0.      0.      0.
   0.   ]
iter 25 delta 0.0249160239322    [-61.378 -51.488 -26.804   1.002  67.521   0.      0.      0.
   0.   ]
iter 26 delta 0.0239932082311    [-61.373 -51.483 -26.8     1.005  67.514   0.      0.      0.
   0.   ]
iter 27 delta 0.0231363079371    [-61.368 -51.478 -26.796   1.007  67.507   0.      0.      0.
   0.   ]
iter 28 delta 0.0223385042151    [-61.363 -51.473 -26.793   1.01   67.5     0.      0.      0.
   0.   ]
iter 29 delta 0.0215938874079    [-61.359 -51.468 -26.789   1.012  67.493   0.      0.      0.
   0.   ]
iter 30 delta 0.0179846750175    [-61.356 -51.465 -26.787   1.013  67.484   0.      0.      0.
   0.   ]
```

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iter 31 delta 0.0230863110084    [-61.351 -51.459 -26.783   1.016  67.481   0.      0.      0.
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iter 32 delta 0.0168946947135    [-61.348 -51.456 -26.781   1.017  67.471   0.      0.      0.
   0.   ]
iter 33 delta 0.0217282927138    [-61.343 -51.451 -26.777   1.02   67.468   0.      0.      0.
   0.   ]
iter 34 delta 0.015929283587    [-61.34  -51.448 -26.775   1.021  67.46    0.      0.      0.
   0.   ]
iter 35 delta 0.0179949061733    [-61.337 -51.444 -26.772   1.023  67.454   0.      0.      0.
   0.   ]
iter 36 delta 0.0175085573578    [-61.333 -51.44  -26.77    1.025  67.449   0.      0.      0.
   0.   ]
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iter 38 delta 0.0166106826215    [-61.326 -51.433 -26.764   1.029  67.439   0.      0.      0.
   0.   ]
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   0.   ]
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   0.   ]
iter 41 delta 0.0154242052914    [-61.317 -51.422 -26.757   1.034  67.424   0.      0.      0.
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iter 42 delta 0.0150655028427    [-61.313 -51.419 -26.754   1.035  67.419   0.      0.      0.
   0.   ]
iter 43 delta 0.0147231050509    [-61.31  -51.416 -26.752   1.037  67.415   0.      0.      0.
   0.   ]
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   0.   ]
iter 45 delta 0.0140829700487    [-61.305 -51.41  -26.747   1.04   67.406   0.      0.      0.
   0.   ]
iter 46 delta 0.0118622324584    [-61.303 -51.408 -26.746   1.041  67.4     0.      0.      0.
   0.   ]
iter 47 delta 0.01349617963    [-61.3   -51.405 -26.744   1.042  67.395   0.      0.      0.
   0.   ]
iter 48 delta 0.0132207473926    [-61.297 -51.402 -26.742   1.043  67.391   0.      0.      0.
   0.   ]
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   0.   ]
iter 51 delta 0.0124580119661    [-61.29  -51.393 -26.736   1.048  67.38    0.      0.      0.
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iter 52 delta 0.0122229551366    [-61.287 -51.391 -26.734   1.049  67.376   0.      0.      0.
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iter 55 delta 0.0115681539685    [-61.28  -51.383 -26.728   1.053  67.365   0.      0.      0.
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iter 69 delta 0.00925452317483    [-61.252 -51.352 -26.706   1.068  67.319   0.      0.      0
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iter 73 delta 0.00875427867891    [-61.244 -51.344 -26.701   1.071  67.308   0.      0.      0
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iter 74 delta 0.00863755496318    [-61.243 -51.342 -26.699   1.072  67.305   0.      0.      0
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iter 79 delta 0.00809770777798    [-61.235 -51.334 -26.693   1.076  67.291   0.      0.      0
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   0.   ]
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   0.   ]
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   0.   ]
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   0.   ]
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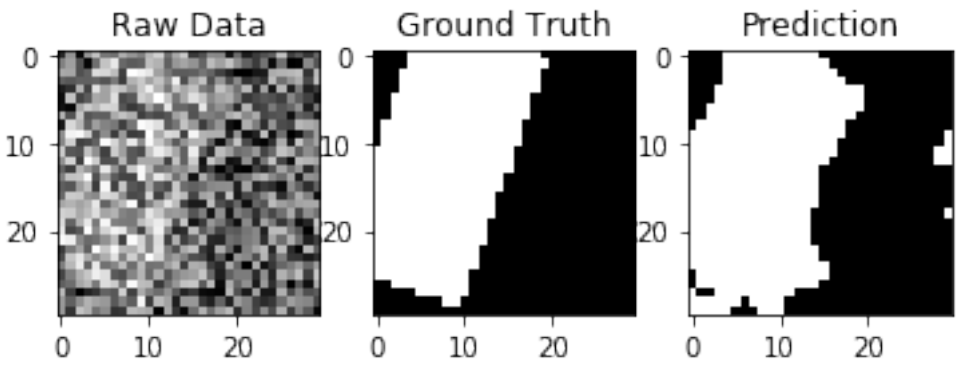
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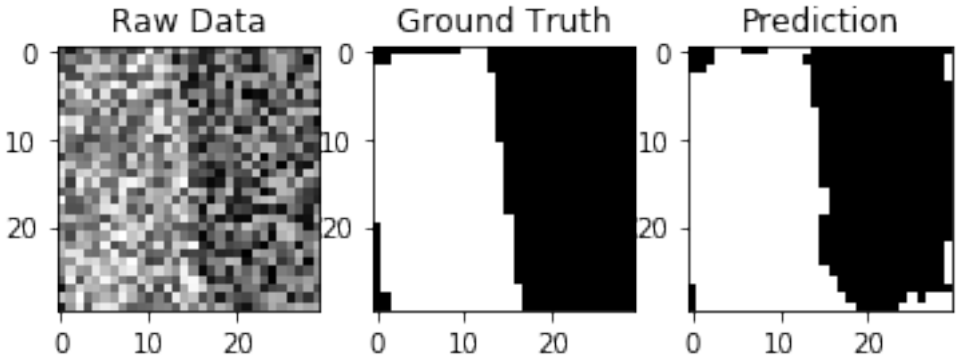
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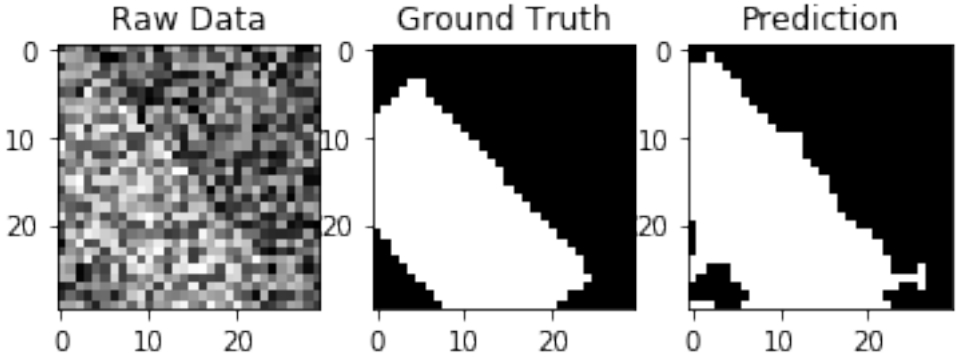
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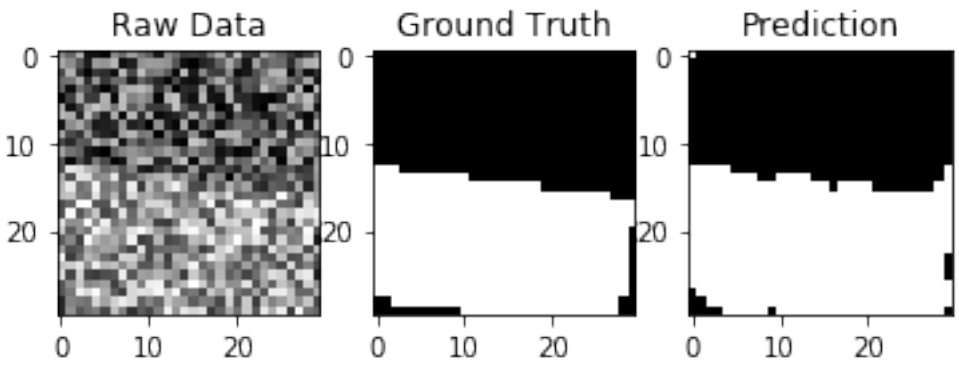
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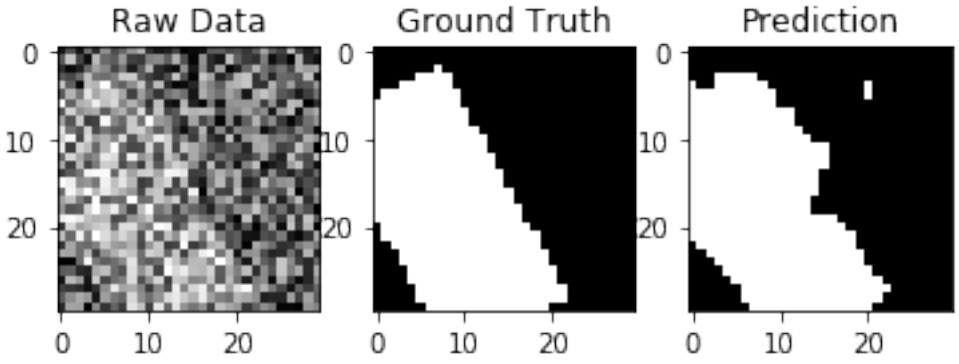
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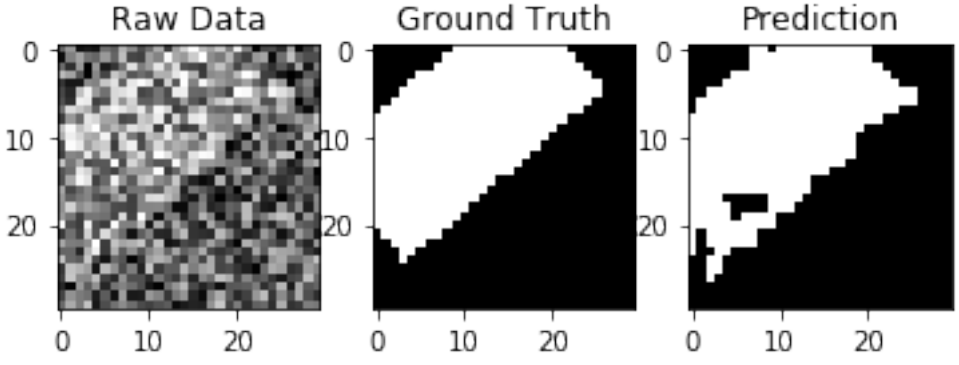
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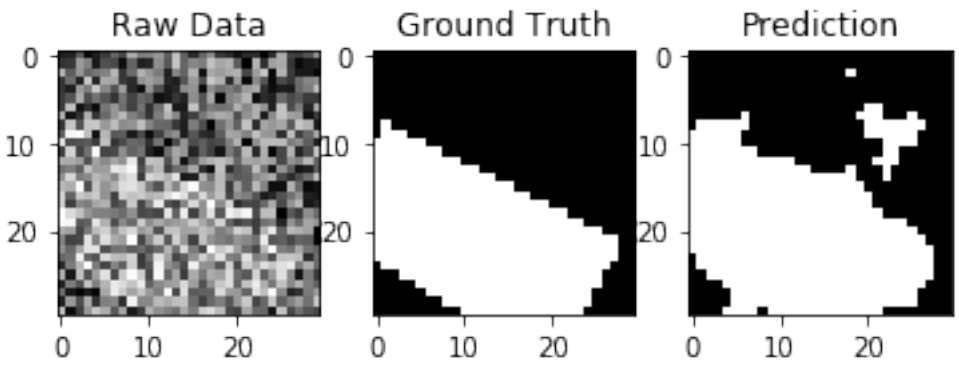
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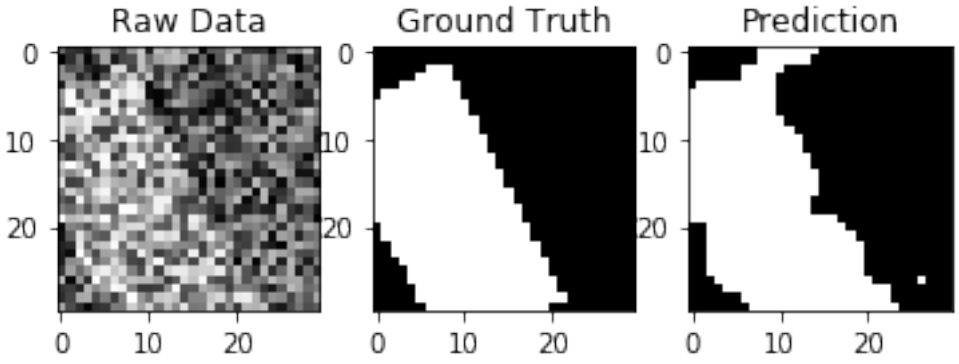
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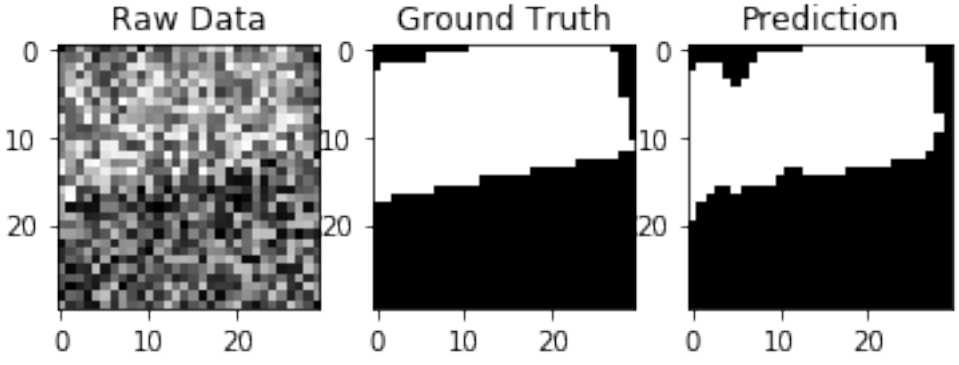
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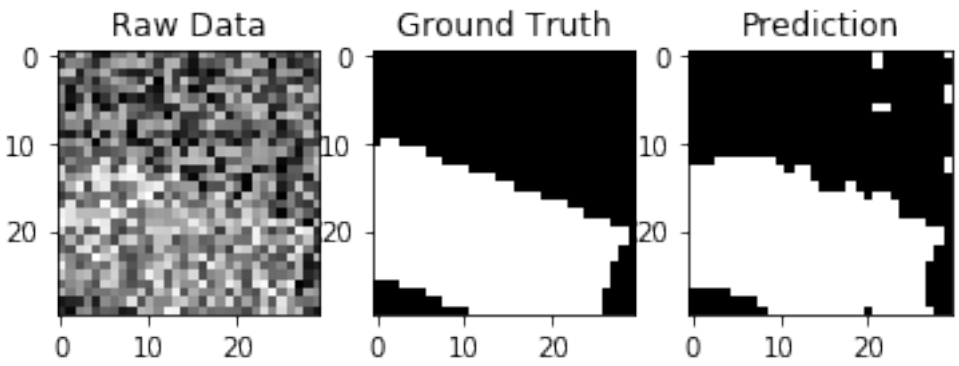
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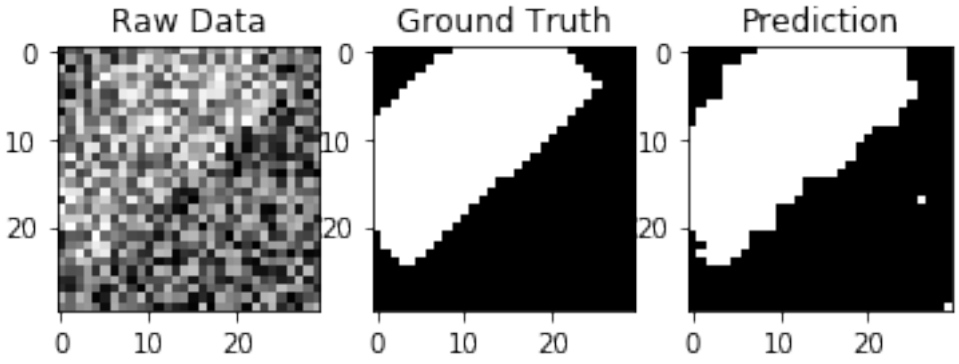
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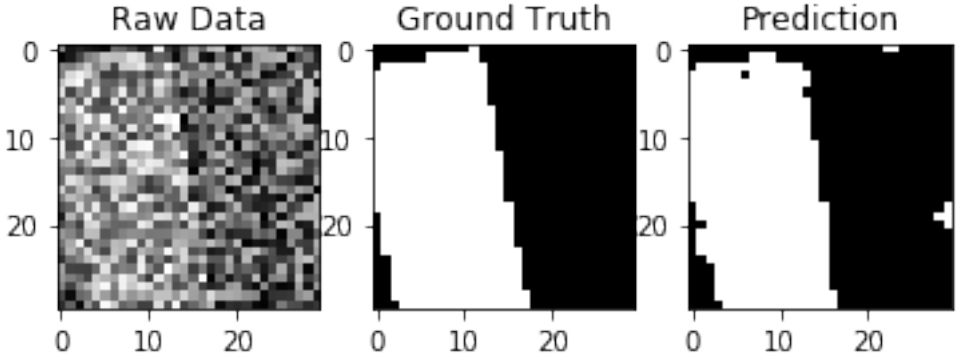
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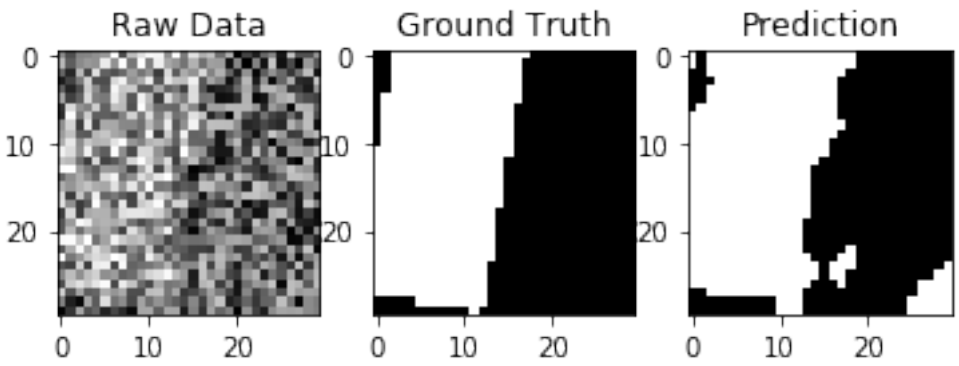
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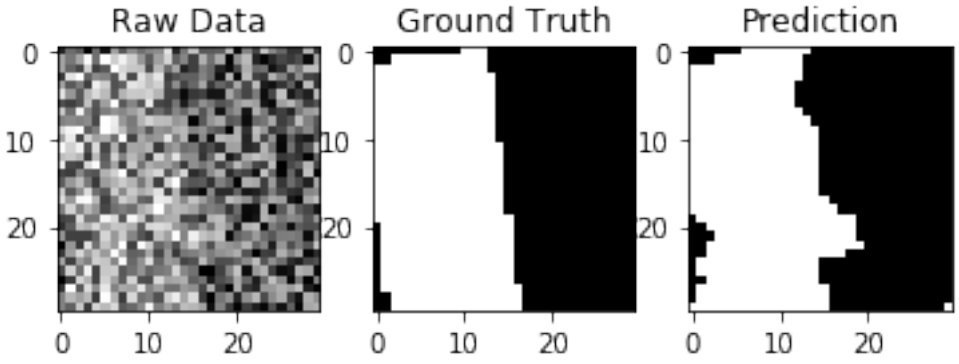
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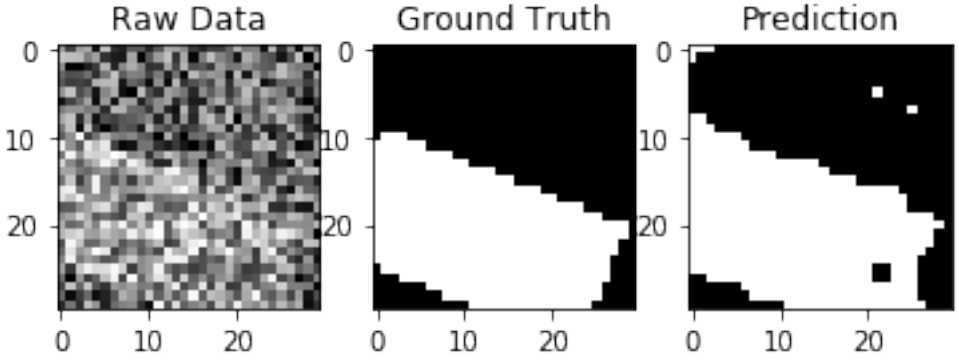
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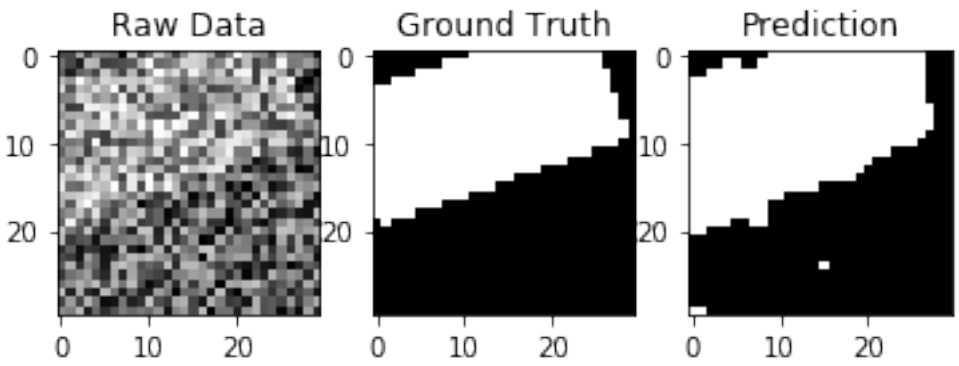
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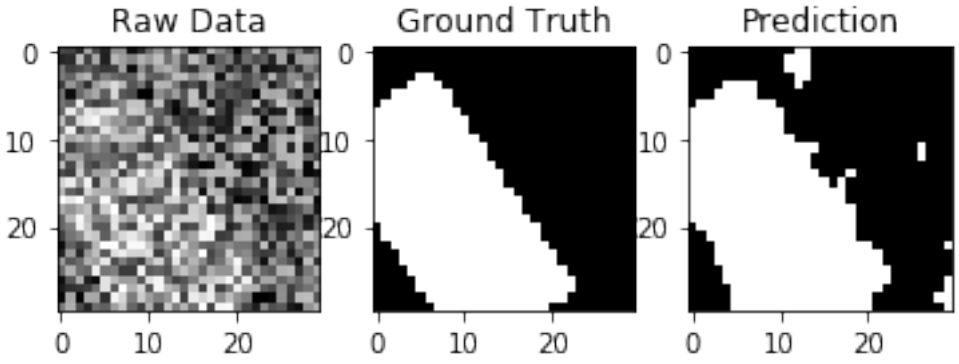
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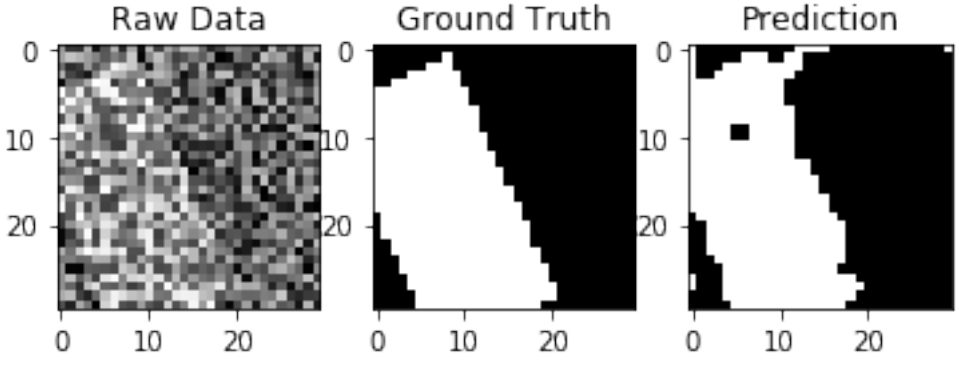
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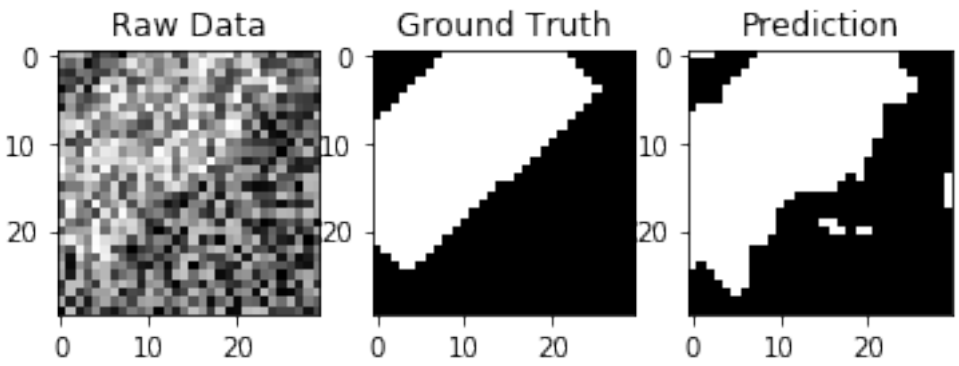
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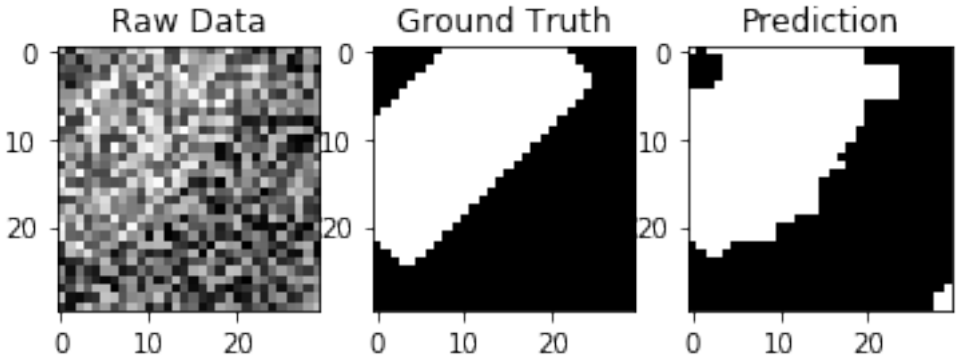
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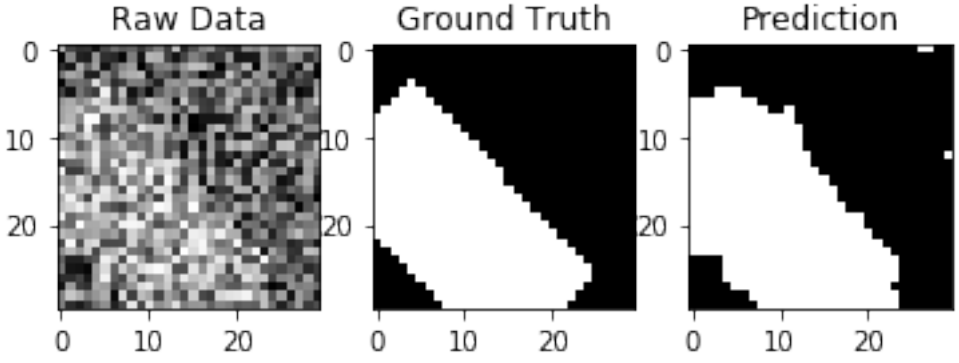
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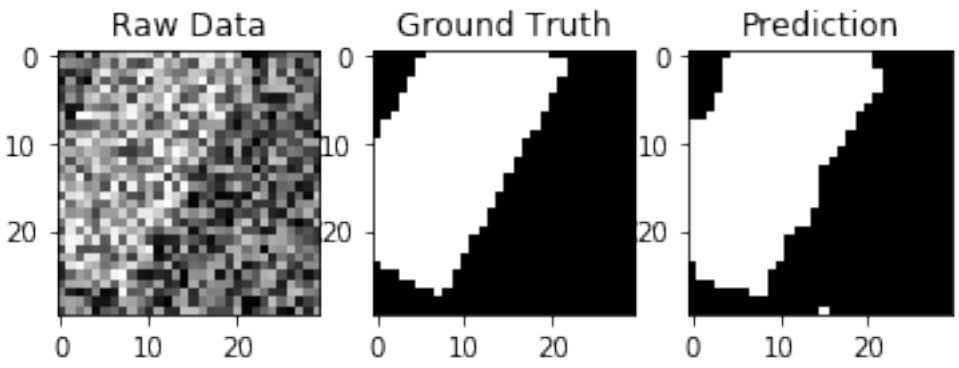
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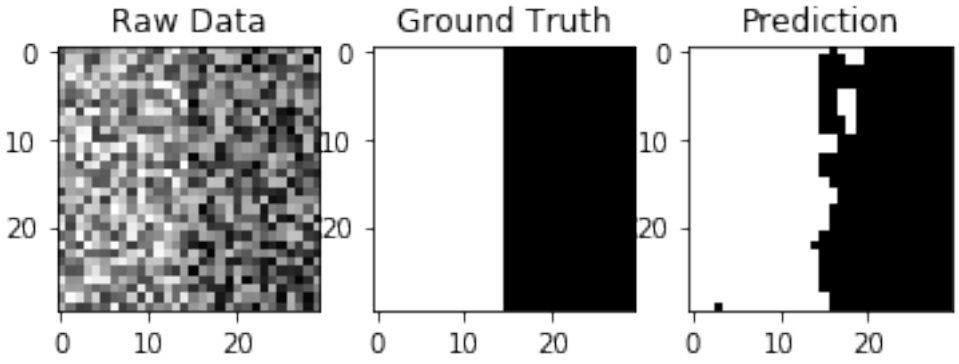
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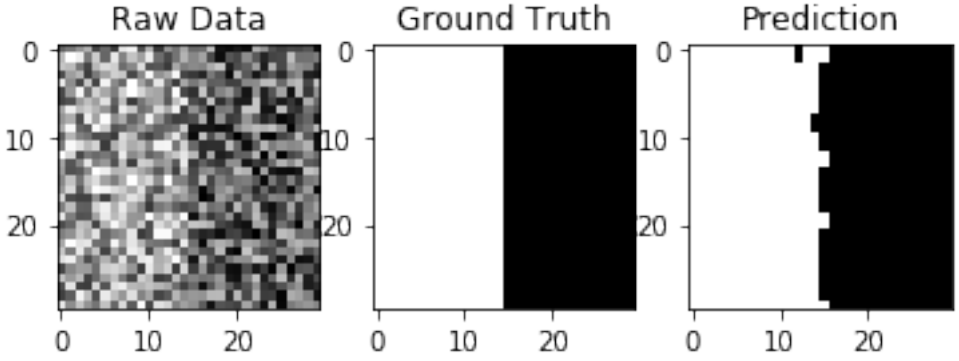
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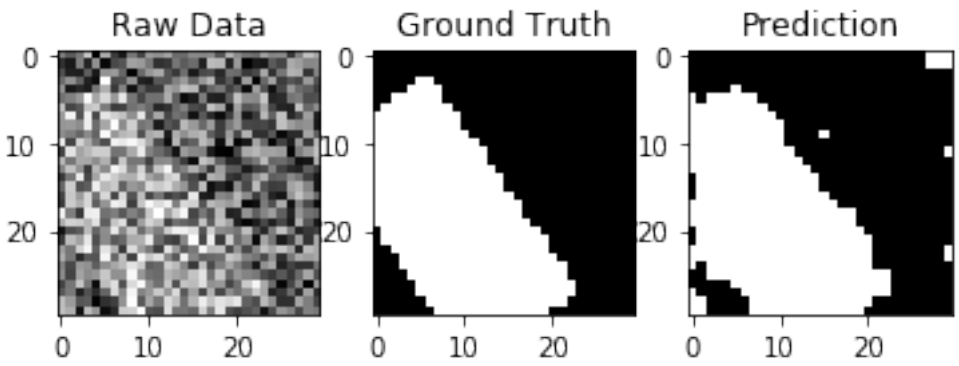
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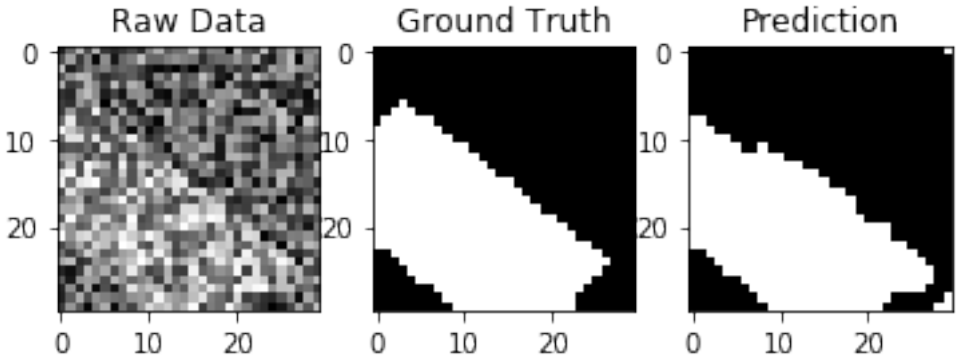
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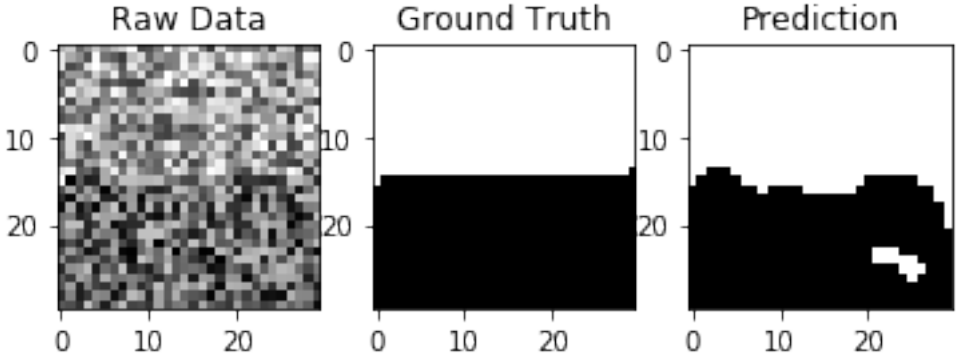
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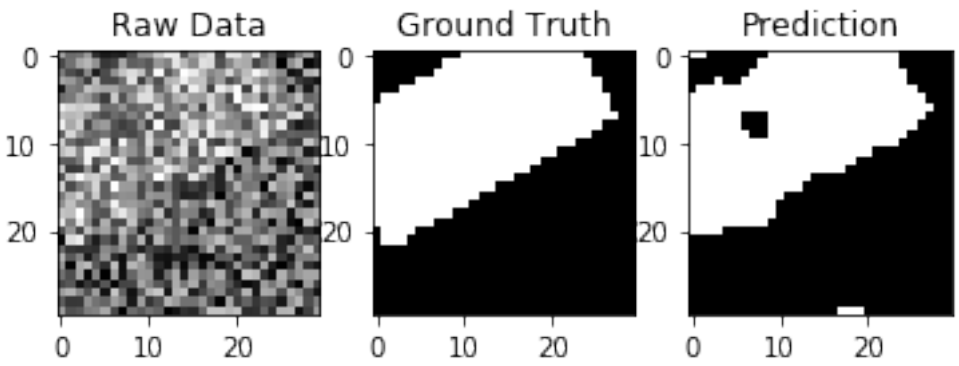
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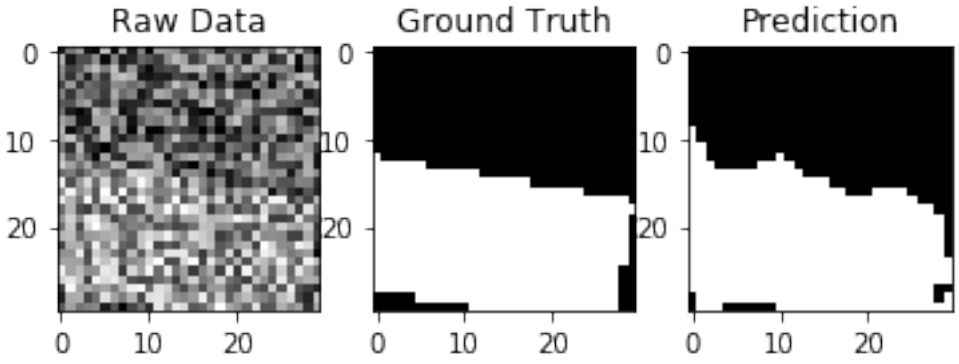
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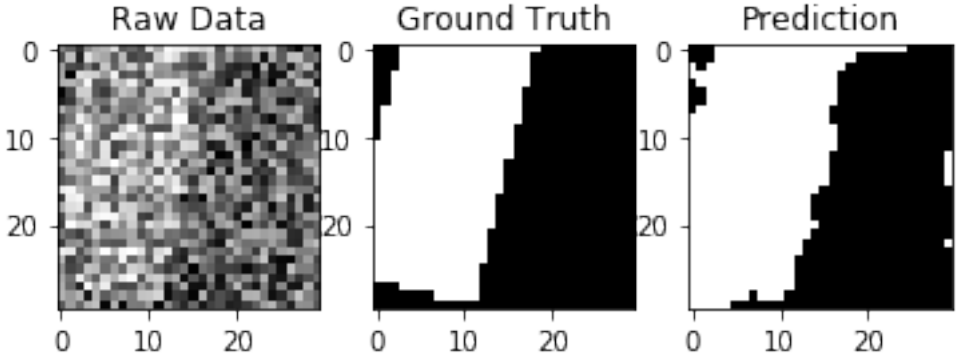
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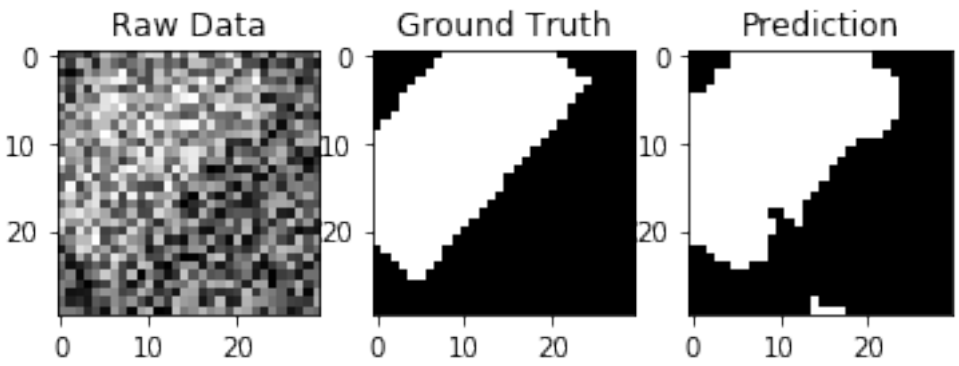
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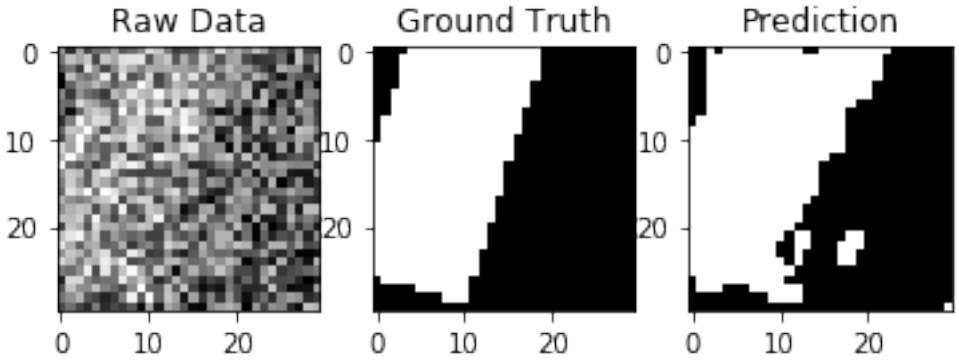
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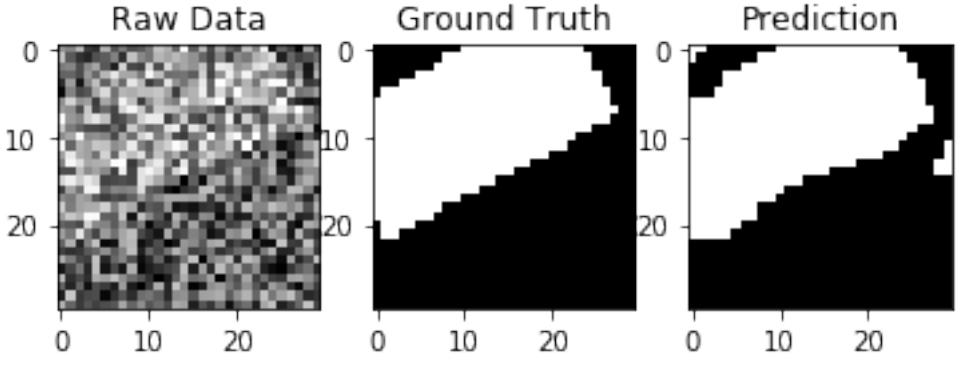
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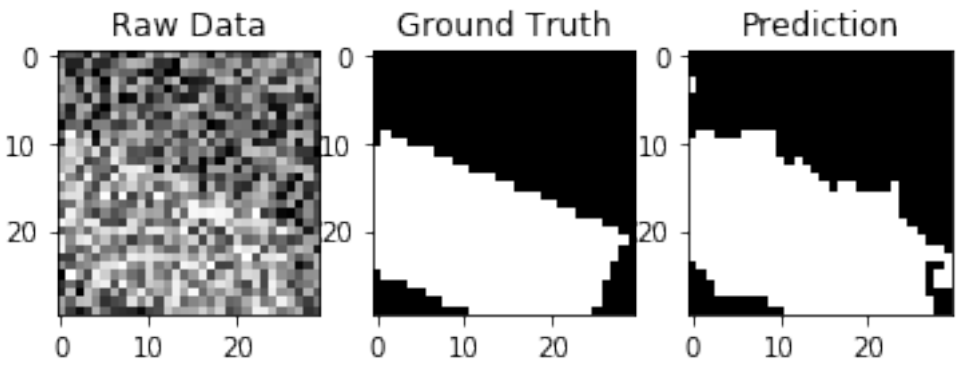
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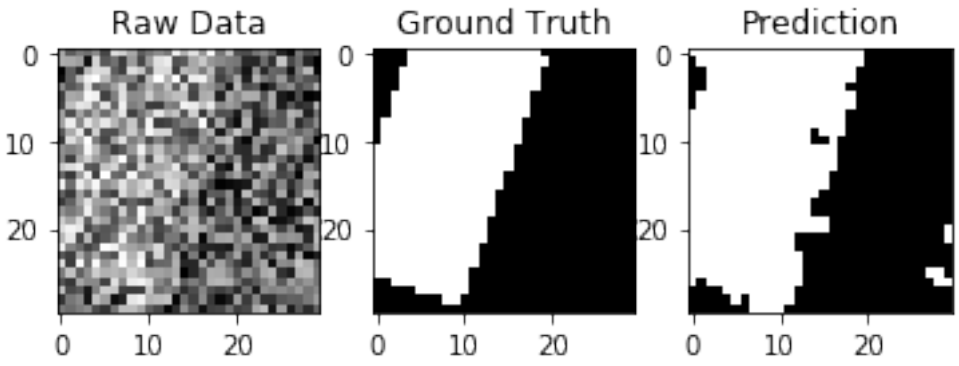
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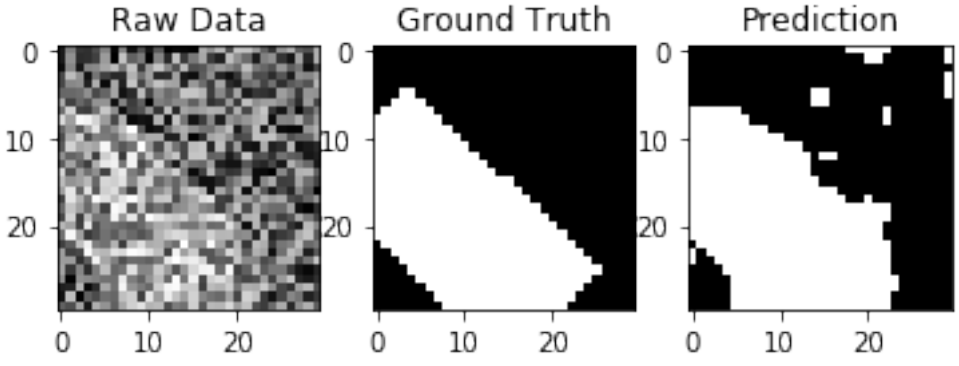
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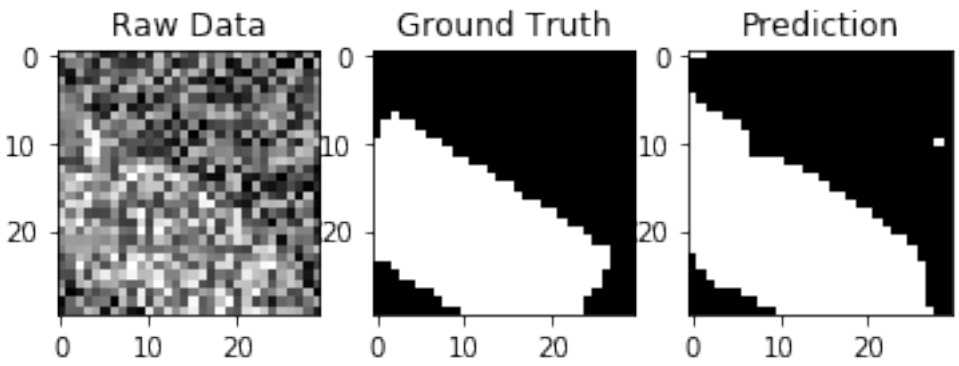
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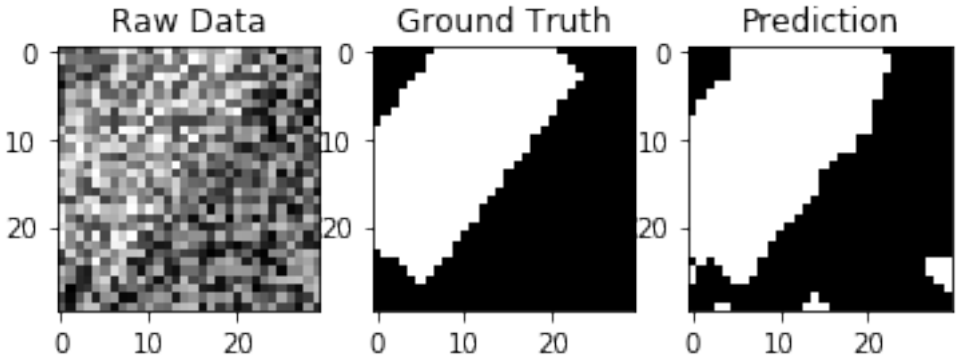
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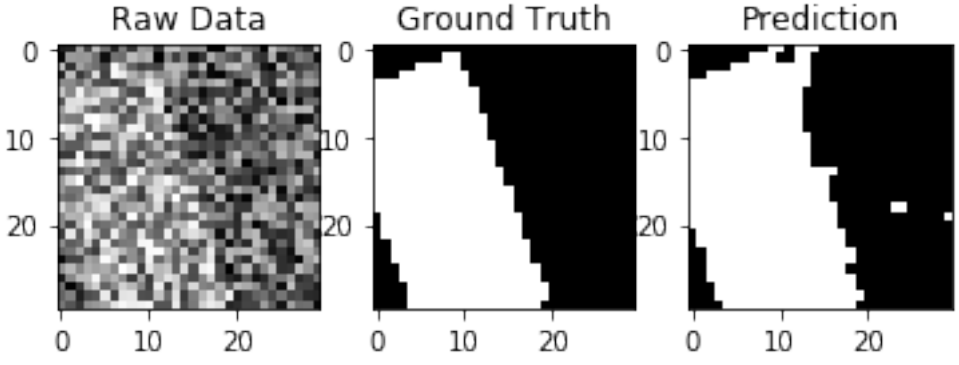
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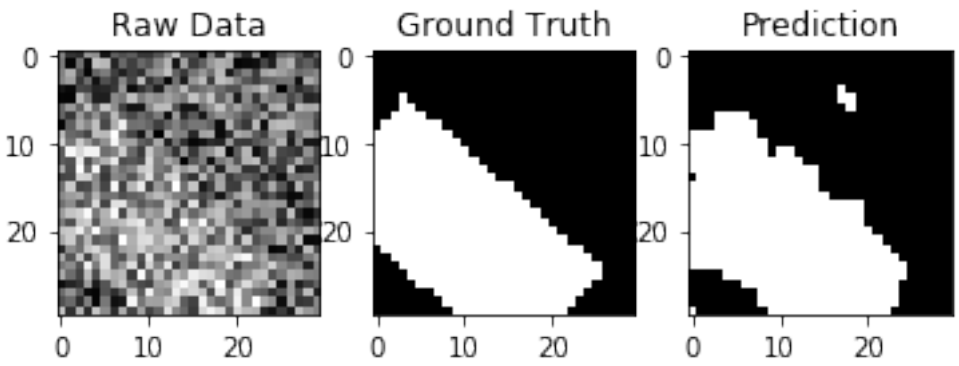
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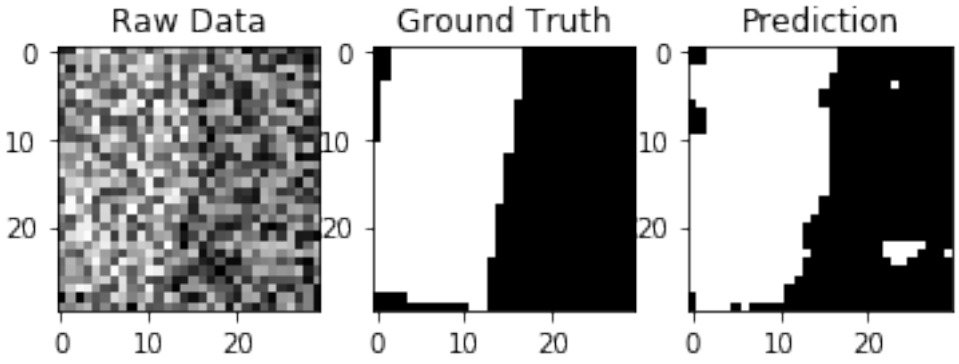
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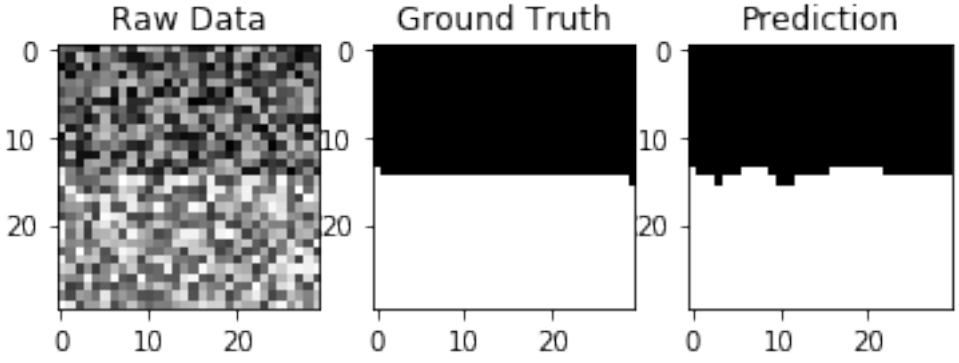
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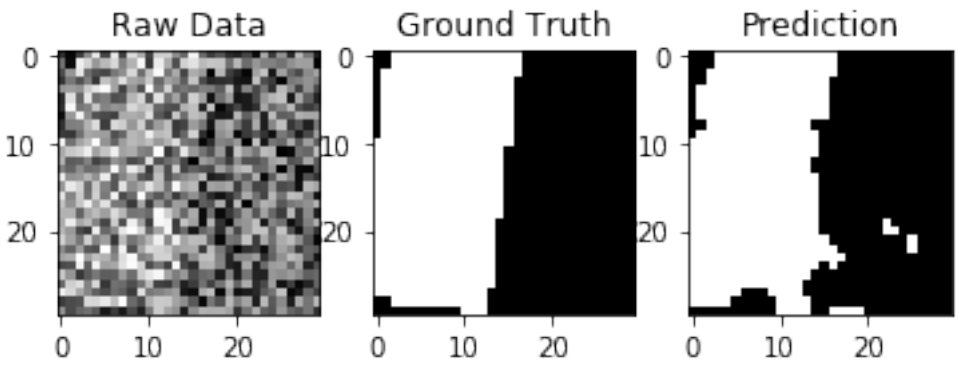
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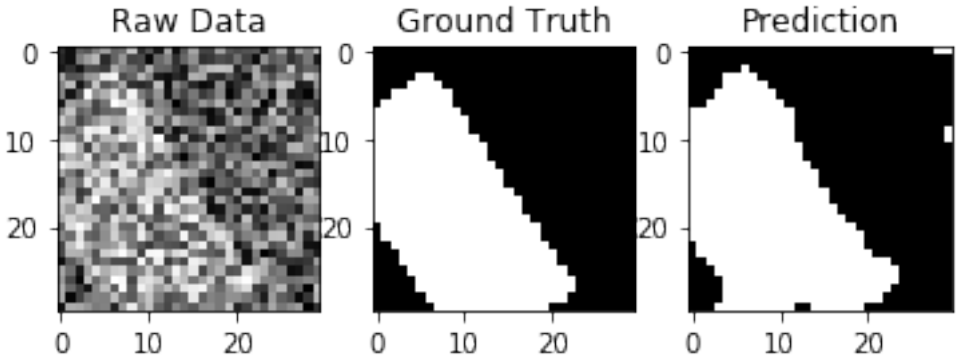
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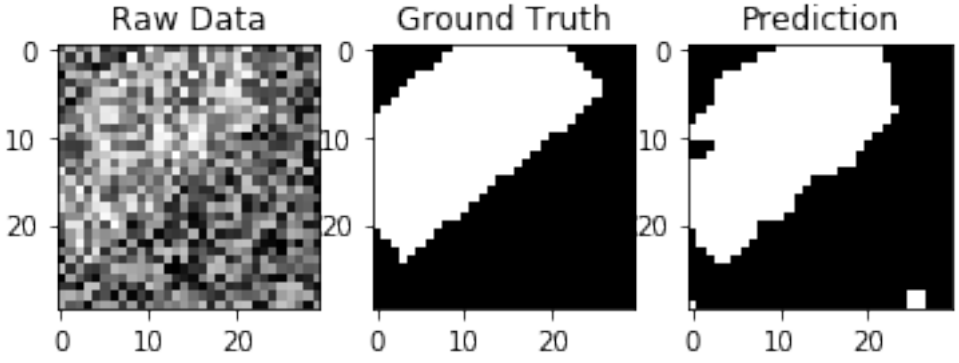
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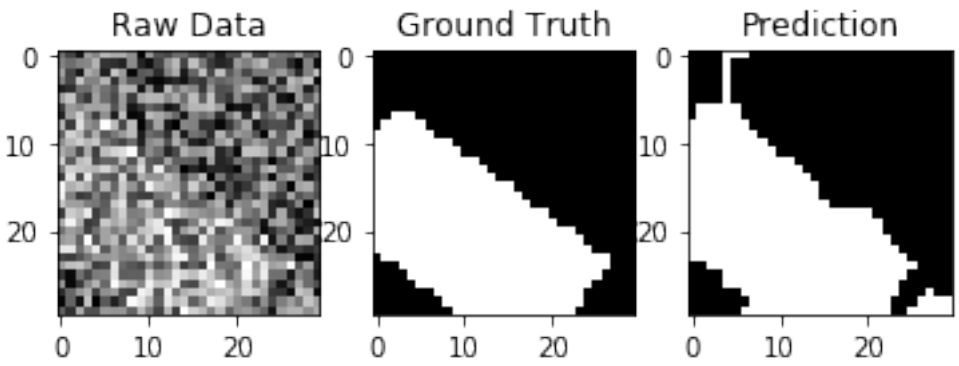
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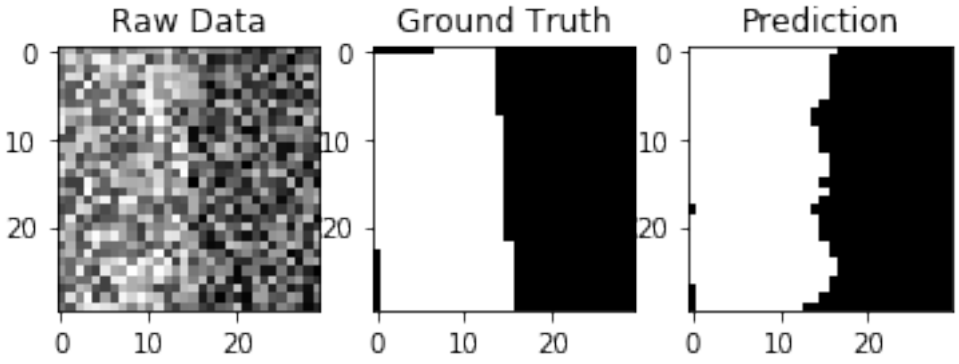
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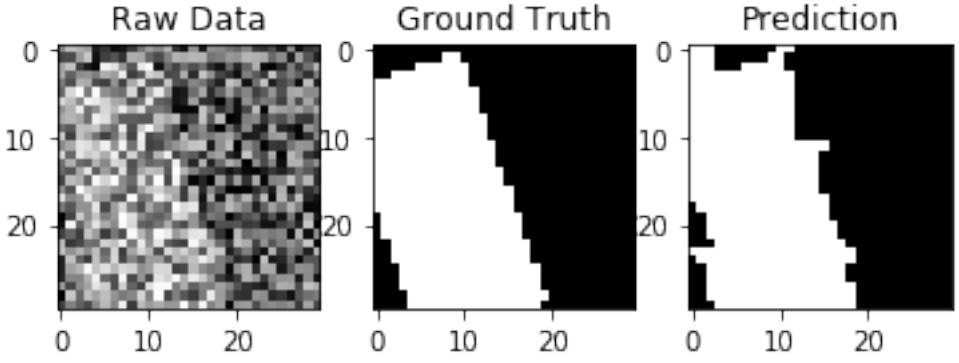
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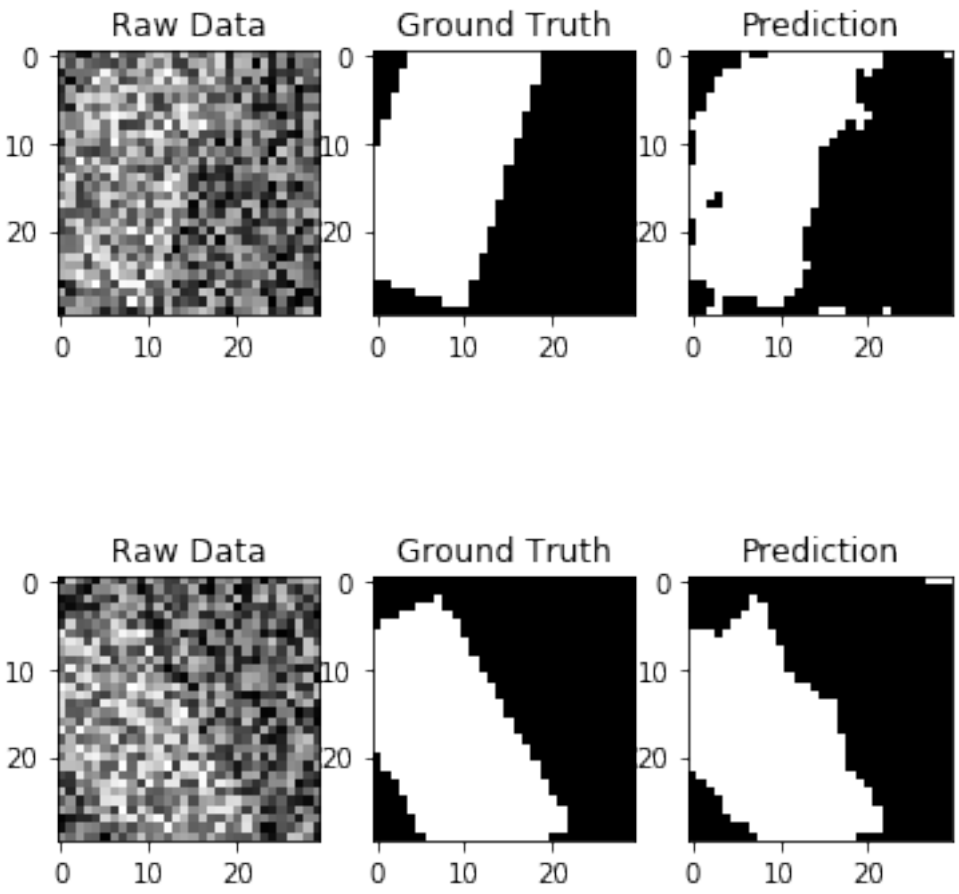

Yep, Graph cut works way better than ICM.

In [ ]: