Exercise 6

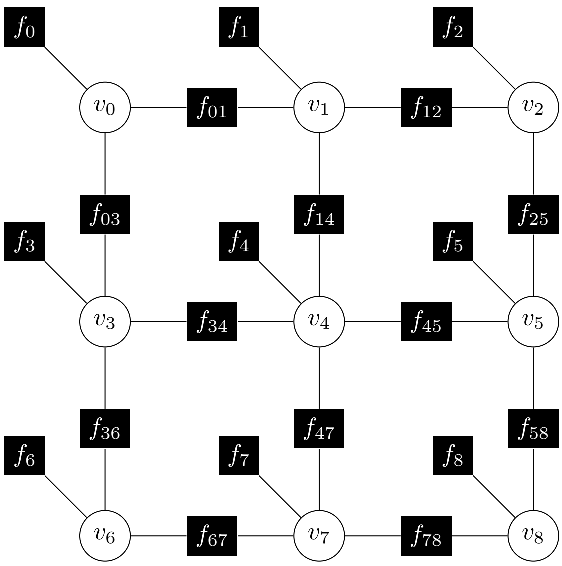
Deadline: 5.05.2017, 2:15 pm
Regulations: You should hand in the exercises in groups of two or three persons. Please send a *compressed* (!) directory or file containing your solutions including all graphics, descriptions and source code to *thorsten.beier@iwr.uni-heidelberg.de*. The subject line of this email should start with [MLCV17][EX02] followed by the full names of all group members. Please cross-reference your code files in your writeup, such that it is clear which file has to be run for each exercise.

1 Iterated Conditional Models (ICM) (10 points)

The ICM algorithm is one of the easiest algorithms to find the approximate argmin/min of a graphical model. We initialize all variables by some starting value. Next we iterate over the variables in order. When we are at some variable x_i all other variables are fixed and we consider all labels for x_i and replace the current label with the label that minimizes the energy. We keep iterating over all variables until we complete a full iteration over all variables without changing a single variable. At this point, we cannot improve the energy any more by flipping only a single variable at once. We reached a local minimum.

1.1 Implementation (15 points)

In this exercise we implement an ICM solver for a graphical model defined over an image. For a $3x3$ image, the factor graph will look like the picture below.



The pairwise factors are a potts functions:

$$\phi_p(x_i, x_j) = \begin{cases} 0 & \text{if } x_i = x_j \\ 1 & \text{if } x_i \neq x_j \end{cases} \tag{1}$$

The source code with missing part is given below. Implement the missing parts (marked with a "TODO"). Can you thing of ways how to improve the speed?

Use your code from exercise 1 to predict pixel wise probabilities for a few “horse-images”(use `predict_proba` instead of `predict`). Generate unaries by converting the per-class-probabilities into energies by using the negative logarithm. Select different values for `beta` and use the ICM code from above to optimize the graphical model. Comment on the results. How do different values of `beta` change the results?

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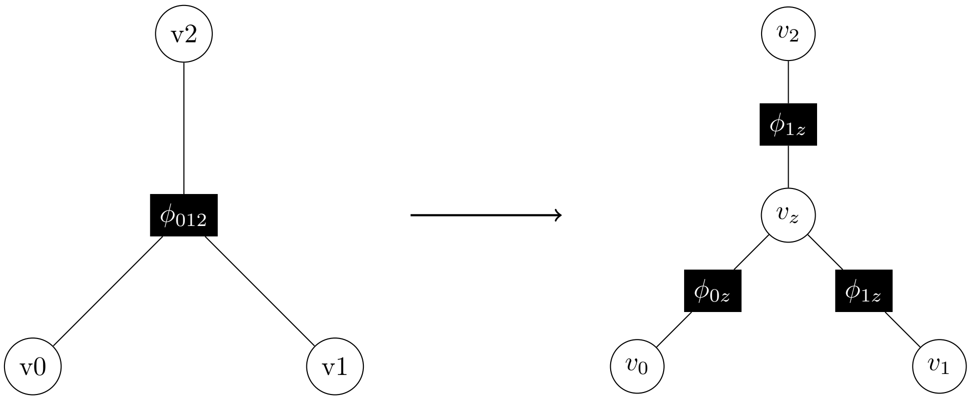
```python
import numpy

def iterated_conditonal_modes(unaries, beta, labels=None):
    shape = unaries.shape[0:2]
    n_labels = unaries.shape[2]

    if labels is None:
        labels = numpy.argmin(unaries, axis=2)

    continue_search  = True
    while(continue_search):
        continue_search  = False
        for x0 in range(1, shape[0]-1):
            for x1 in range(1, shape[1]-1):

                current_label = labels[x0, x1]
                min_energy = float('inf')
                best_label = None

                for l in range(n_labels):

                    # evaluate cost
                    energy = 0.0

                    # unary terms
                    # energy += TODO

                    # pairwise terms
                    # energy += TODO

                    if energy < min_energy:
                        min_energy  = energy
                        best_label = l

                if best_label != current_label:
                    labels[x0, x1] = best_label
                    continue_search = True
    return labels

if __name__ == "__main__":

    import matplotlib.pyplot as plt

    shape = [100, 100]
    n_labels = 2

    # unaries
    unaries = numpy.random.rand(shape[0],shape[1], n_labels)

    # regularizer strength
    beta = 0.01

    labels = iterated_conditonal_modes(unaries, beta=beta)
```

```
plt.imshow(labels)
plt.show()
```

2 Higher order factors (5 points)

A third order factor $\varphi_{012}(x_0, x_1, x_2)$ can be translated to a pairwise MRF by adding a new node x_z and pairwise factors $\varphi_{0z}(x_0, x_z)$, $\varphi_{1z}(x_1, x_z)$ and $\varphi_{2z}(x_2, x_z)$.



Without loss of generality, let x_0, x_1, x_2 be binary variables $x_i \in \{0, 1\}$ and the energies of $\varphi_0(x_0, x_1, x_2)$ are given as follows:

x_0	0	0	0	0	1	1	1	1
x_1	0	0	1	1	0	0	1	1
x_2	0	1	0	1	0	1	0	1
E	a	b	c	d	e	f	g	h

1. What is the domain of the additional random variable x_z?
2. Please give the energies of the pairwise factors $\varphi_{0z}(x_0, x_z)$, $\varphi_{1z}(x_1, x_z)$ and $\varphi_{2z}(x_2, x_z)$.
   Hint: some of the energies in the pairwise factors can be ∞

Bonus 2 (5 points): A third order factor $\varphi_{012}(x_0, x_1, x_2)$ can be also translated to a pairwise MRF by adding **multiple binary** variables $x_{z0}, x_{z1}, \ldots, x_{zn-1}$ and multiple pairwise factors. Give the energies of all pairwise factors.