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CURS 6

Aplicații liniare

OBS
$f: V_1 \to V_2$ s.n. aplicație liniară $\Leftrightarrow$ $f(x+y) = f(x) + f(y)$
$(V_i, +, \cdot)_{/\mathbb{K}}, i = \overline{1,2}$ sp/v. $\quad f(ax) = af(x)$, $\forall x, y \in V_1, \forall a \in \mathbb{K}$

- f liniară $\Leftrightarrow f(ax+by) = af(x) + bf(y)$, $\forall x, y \in V_1, \forall a, b \in \mathbb{K}$.

- $\text{Ker} f = \{x \in V_1 \mid f(x) = 0_{V_2}\} \subset V_1$
  $\text{Im} f = \{y \in V_2 \mid \exists x \in V_1 \text{ a.î. } f(x) = y\} \subset V_2$.

- f inj $\Leftrightarrow \text{Ker} f = \{0_{V_1}\}$
  f surj $\Leftrightarrow \dim_{\mathbb{K}} \text{Im} f = \dim_{\mathbb{K}} V_2$

Teorema dimensiunii $(V_i, +, \cdot)_{/\mathbb{K}}, i = \overline{1,2}$ sp v.
$f: V_1 \to V_2$ aplicație liniară
$\Rightarrow \dim_{\mathbb{K}} V_1 = \dim_{\mathbb{K}} \text{Ker} f + \dim_{\mathbb{K}} \text{Im} f$

Dem
$\dim_{\mathbb{K}} \text{Ker} f = k$, $\dim_{\mathbb{K}} V_1 = n$, $k \leq n$.
Fie $R_0 = \{e_1, \dots, e_k\}$ reper în $\text{Ker} f \subseteq V_1$
Extindem la $R_1 = \{\underline{e_1, \dots, e_k}, \underbrace{e_{k+1}, \dots, e_n}\}$ reper în V_1.
Dem că $R = \{f(e_{k+1}), \dots, f(e_n)\}$ reper în $\text{Im} f$
1) R este SLI
Fie $a_{k+1}, \dots, a_n \in \mathbb{K}$ a.î. $\sum_{i=k+1}^{n} a_i f(e_i) = 0_{V_2} \Rightarrow$

$$f\Big(\underbrace{\sum_{i=k+1}^{n} a_i e_i}_{\in\, \text{Ker} f = \langle R_0 \rangle}\Big) = 0_{V_2} \Rightarrow \sum_{i=k+1}^{n} a_i e_i = \sum_{j=1}^{k} a_j e_j \Rightarrow$$

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$$\sum_{j=1}^{k} a_j e_k - \sum_{i=k+1}^{n} a_i e_i = 0 \overset{R_1\ SLI}{\Longrightarrow} a_j = 0,\ \forall j=\overline{1,k}$$
$$\boxed{a_i = 0,\ \forall i = \overline{k+1,n}}$$

2) R este SG pt $\operatorname{Im} f$ i.e. $\operatorname{Im} f = \langle R \rangle \Leftrightarrow$

$$\forall y \in \operatorname{Im} f \Rightarrow \exists x \in \underset{\langle R_1 \rangle}{\overset{\shortparallel}{V_1}} \text{ ai } y = f(x) = f\Big(\sum_{j=1}^{k} a_j e_j + \sum_{i=k+1}^{n} a_i e_i\Big)$$

$$= \underbrace{f\Big(\sum_{j=1}^{k} a_j e_j\Big)}_{\in \ker f} + f\Big(\sum_{i=k+1}^{n} a_i e_i\Big) = 0_{V_2} = \sum_{i=k+1}^{n} a_i f(e_i)$$

$$\dim V_1 = n = k + n - k$$
$$= \dim \ker f + \dim \operatorname{Im} f.$$

<u>OBS</u>.

$f : V_1 \to V_2$ liniară

a) f inj $\Leftrightarrow \ker f = \{0_{V_1}\} \overset{Th}{\Longleftrightarrow} \dim V_1 = \dim \operatorname{Im} f$

b) f surj $\Leftrightarrow \dim \operatorname{Im} f = \dim V_2 \overset{Th}{\Leftrightarrow} \dim V_1 = \dim \ker f + \dim V_2$

c) f bij $\Leftrightarrow \dim V_1 = \dim V_2$.

<u>Teoremă</u> $(V_i, +, \cdot)/\mathbb{K}$, $i=\overline{1,2}$ sp vect

$V_1 \simeq V_2$ (sp. vect. izomorfe) $\Leftrightarrow \dim V_1 = \dim V_2$.

<u>Dem</u>

„$\Rightarrow$" $\exists f : V_1 \to V_2$ izomorfism $\overset{OBS}{\Longrightarrow} \dim V_1 = \dim V_2$

„$\Leftarrow$" $\dim V_1 = \dim V_2 = n$.

$R_1 = \{e_1, \dots, e_n\}$ reper în V_1

$R_2 = \{e_1', \dots, e_n'\}$ reper în V_2

Construim $f : V_1 \to V_2$, $f(e_i) = e_i'$, $i = \overline{1,n}$

Prelungim prin liniaritate.

Fie $x \in V_1 = \langle R_1 \rangle$, $x = \sum_{i=1}^{n} x_i e_i$

$f(x) = f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} x_i \underbrace{f(e_i)}_{e_i'} = \sum_{i=1}^{n} x_i e_i' = x'$

$\Rightarrow f$ liniară

· f bij : $\forall x' \in V_2$, $\exists ! \, x = \sum_{i=1}^{n} x_i e_i \in V_1$ a.î. $f(x) = x'$

$\overset{\parallel}{\sum_{i=1}^{n} x_i e_i'}$

$\Rightarrow f$ izomorfism de sp vect. $\Rightarrow V_1 \simeq V_2$.

<u>OBS</u> $(V, +, \cdot)_{/\mathbb{K}}$ sp. vect n-dim, $(\mathbb{K}^n, +, \cdot)_{/\mathbb{K}}$ sp vect n-dim

$V \simeq \mathbb{K}^n$ izomorfe.

$R = \{e_1, \dots, e_n\}$ reper în V

$f : V \longrightarrow \mathbb{K}^n$, $f(x) = (x_1, \dots, x_n)$

izomorfism de sp vect

($\exists$ o infinitate de astfel de izomorfisme; $\exists$ o inf. de repere în V)

$f(e_i) = e_i^0$, $\forall i = \overline{1,n}$ $\quad R_0 = \{e_1^0, \dots, e_n^0\}$ reperul canonic din $\mathbb{K}^n$

<u>Teoremă</u> $f : V_1 \longrightarrow V_2$ liniară

a) f inj $\Leftrightarrow$ f transformă $\forall$ SLI din V_1 într-un SLI din V_2

b) f surj $\Leftrightarrow$ f transformă $\forall$ SG din V_1 într-un SG din V_2

c) f bij $\Longleftrightarrow$ f transformă $\forall$ reper din V_1 într-un reper din V_2

<u>Dem</u>

a) „$\Rightarrow$" Ip: f injectivă

Dem că $\forall S = \{v_1, \dots, v_k\} \subset V_1$ un SLI $\Rightarrow f(S) = \{f(v_1), \dots, f(v_k)\}$ este SLI în V_2

Fie $\sum_{i=1}^{k} a_i f(v_i) = 0_{V_2} \Longrightarrow f\left(\sum_{i=1}^{k} a_i v_i\right) = 0_{V_2} \Rightarrow$

$\sum_{i=1}^{k} a_i v_i \in \operatorname{Ker} f = \{0_{V_1}\} \Rightarrow \sum_{i=1}^{k} a_i v_i = 0_{V_1} \overset{S \text{ e}}{\underset{SLI}{\Longrightarrow}} a_i = 0,$

$\Rightarrow f(S)$ este SLI

"$\Leftarrow$" Fie $x \in \operatorname{Ker} f$.

Sp. prin absurd că $x \neq 0_{V_1} \Rightarrow \{x\}$ e SLI în $V_1 \overset{ip.}{\Rightarrow}$

$\{f(x)\}$ e SLI în V_2

$\Rightarrow f(x) \neq 0_{V_2}$ $\cancel{\varnothing}$

dar $f(x) = 0_{V_2}$

Sp. este falsă $\Rightarrow \operatorname{Ker} f = \{0_{V_1}\}$.

b) "$\Rightarrow$" f surj.

Dem $\forall S = \{v_1, \dots, v_k\}$ SG în $V_1 \Leftrightarrow V_1 = \langle S \rangle \Rightarrow V_2 = \langle f(S) \rangle$

$\forall y \in V_2, \ \exists x \in \underset{\langle S \rangle}{\overset{\shortparallel}{V_1}}, \ x = \sum_{i=1}^{k} a_i v_i$ aî $y = f(x) =$

$= f\left(\sum_{i=1}^{k} a_i v_i\right) = \sum_{i=1}^{k} a_i f(v_i) \Rightarrow V_2 = \langle f(S) \rangle$

$f(S) = \{f(v_1), \dots, f(v_k)\}$.

"$\Leftarrow$" Dem că f e surj.

Ip: Dc $V_1 = \langle S \rangle \Rightarrow V_2 = \langle f(S) \rangle$

$S = \{v_1, \dots, v_k\}$

$\forall y \in V_2, \ \exists x \in V_1$ aî $f(x) = y$

$\overset{\shortparallel}{\sum_{i=1}^{k}} a_i f(v_i) = \sum_{i=1}^{k} f(a_i v_i) = f\left(\sum_{i=1}^{k} a_i v_i\right)$

Considerăm $x = \sum_{i=1}^{k} a_i v_i \in V_1$.

c) f bij $\overset{a), b)}{\Longleftrightarrow}$ [$\mathcal{R}$ reper în $V_1 \Rightarrow f(\mathcal{R})$ reper în V_2]

Matricea asociată unei aplicații liniare

$f : V_1 \longrightarrow V_2$.

$R_1 = \{e_1, \dots, e_n\} \xrightarrow{\ A\ } R_2 = \{\bar{e}_1, \dots, \bar{e}_m\}$

reper în V_1 reper în V_2

$$f(e_i) = \sum_{j=1}^{m} a_{ji}\,\bar{e}_j, \quad \forall i = \overline{1,n}, \quad A = (a_{ji})_{\substack{j=\overline{1,m}\\ i=\overline{1,n}}} \in \mathcal{M}_{m,n}(\mathbb{K})$$

$[f]_{R_1,R_2} = A$; $[f]_{R_1',R_2'} = A$

Modificarea matricei la schimbarea reperelor.

$$\begin{tikzcd}
R_1 = \{e_1, \dots, e_n\} \arrow[r, "A"] \arrow[d, "C"'] & R_2 = \{\bar{e}_1, \dots, \bar{e}_m\} \arrow[d, "D"] \\
R_1' = \{e_1', \dots, e_n'\} \arrow[r, "A'"] & R_2' = \{\bar{e}_1', \dots, \bar{e}_m'\}
\end{tikzcd}$$

$$f(e_k') = \sum_{l=1}^{m} a_{lk}'\,\bar{e}_l', \quad \forall k = \overline{1,n}$$

$$f(e_k') = \sum_{l=1}^{m} a_{lk}' \Big(\sum_{j=1}^{m} d_{jl}\,\bar{e}_j \Big)$$

$$f(e_k') = f\Big(\sum_{i=1}^{n} c_{ik} e_i \Big) = \sum_{i=1}^{n} c_{ik} f(e_i) = \sum_{i=1}^{n} c_{ik} \Big(\sum_{j=1}^{m} a_{ji}\,\bar{e}_j \Big)$$

$$\sum_{j=1}^{m} \Big(\sum_{i=1}^{n} a_{ji} c_{ik} \Big) \bar{e}_j = \sum_{j=1}^{m} \Big(\sum_{l=1}^{m} d_{jl} a_{lk}' \Big) \bar{e}_j$$

$$\sum_{i=1}^{n} a_{ji} c_{ik} = \sum_{l=1}^{m} d_{jl} a_{lk}' \Rightarrow AC = DA'$$

$$\boxed{A' = D^{-1} A C}$$

Prop Rangul matricei asociate lui f nu depinde de reperele alese

Dem $rg(A') = rg(D^{-1}AC) = rg A$.

D, C matrice inversabile

Teorema de caracterizare a aplicatiilor liniare

$f: V_1 \to V_2$ functie.

f liniară $\Leftrightarrow$ $\exists A \in \mathcal{M}_{m,n}(\mathbb{K})$ aî coordonatele lui $x \in V_1$ in raport cu reperul $R_1 = \{e_1, .., e_n\}$ din V_1 si coordonatele lui $f(x) = y \in V_2$ in raport cu reperul $R_2 = \{\bar{e}_1, .., \bar{e}_m\}$ din V_2 verifică

$$Y = AX, \quad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad A = (a_{ji})_{\substack{j=\overline{1,m} \\ i=\overline{1,n}}}$$

$$x = \sum_{i=1}^{n} x_i e_i, \quad y = \sum_{j=1}^{m} y_j \bar{e}_j \qquad [f]_{R_1,R_2} = A$$

Dem

"$\Rightarrow$" f lin (ip)

$$f(x) = f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} x_i f(e_i) = \sum_{i=1}^{n} x_i \left(\sum_{j=1}^{m} a_{ji} \bar{e}_j\right)$$

$$= \sum_{j=1}^{m} \underline{\left(\sum_{i=1}^{n} a_{ji} x_i\right)} \bar{e}_j = y = \sum_{j=1}^{m} \underline{y_j} \bar{e}_j$$

$$y_j = \sum_{i=1}^{n} a_{ji} x_i, \ \forall j = \overline{1,m} \qquad Y = AX$$

"$\Leftarrow$" $Y_1 = AX_1$, $Y_2 = AX_2$ $\Rightarrow aY_1 + bY_2 = A(aX_1 + bX_2)$

$f(ax+by) = af(x) + bf(y), \ \forall x, y \in V_1, \ \forall a, b \in \mathbb{K}$.

Prop $f: V_1 \to V_2$ liniară $\quad [f]_{R_1,R_2} = A$

a) f inj $\Leftrightarrow \dim V_1 = \operatorname{rg} A$

b) f surj $\Leftrightarrow \dim V_2 = \operatorname{rg} A$

c) f bij $\Leftrightarrow \dim V_1 = \dim V_2 = \operatorname{rg} A \Leftrightarrow A \in GL(n, K)$

Dem

a) f inj $\Leftrightarrow \operatorname{Ker} f = \{0_{V_1}\}$

$$\operatorname{Ker} f = \{x \in V_1 \mid AX = 0\} \Rightarrow \underbrace{\dim \operatorname{Ker} f}_{=0} = \dim V_1 - \operatorname{rg} A$$

$$= S(A)$$

f inj $\Leftrightarrow \dim V_1 = \operatorname{rg} A$

b) f surj $\Leftrightarrow \dim \operatorname{Im} f = \dim V_2$

T.dim $\quad \dim V_1 = \underbrace{\dim \operatorname{Ker} f}_{\dim V_1 - \operatorname{rg} A} + \underbrace{\dim \operatorname{Im} f}_{\dim V_2}$.

f surj $\Leftrightarrow \dim V_2 = \operatorname{rg} A$

c) f bij $\Leftrightarrow \dim V_1 = \dim V_2 = \operatorname{rg} A = \max$

OBS

a)
$$\begin{tikzcd} V_1 \arrow[r, "f"] \arrow[rr, bend right, "h = g \circ f"'] & V_2 \arrow[r, "g"] & V_3 \end{tikzcd}$$

$h: Z = A_h X$

$g: Z = A_g Y$

$f: Y = A_f X$

$$\left.\begin{array}{l} Z = A_g A_f X \\ \text{dar } Z = A_h X \end{array}\right\} \Rightarrow \boxed{A_{g \circ f} = A_g A_f}$$

b)
$$\begin{tikzcd} V \arrow[r, "f"] \arrow[rr, bend right, "id_V"'] & V \arrow[r, "f^{-1}"] & V \end{tikzcd}$$

$f \in \operatorname{Aut}(V)$

$I_n = A_{f^{-1} \circ f} = A_{f^{-1}} \cdot A_f$, Analog $I_n = A_{f \circ f^{-1}} = A_f A_{f^{-1}}$

$\Rightarrow A_f^{-1} = A_{f^{-1}}$

$GL(V) = \{ f: V \longrightarrow V \mid f \in Aut(V) \}$

$\varphi : (GL(V), \circ) \longrightarrow (GL(n, \mathbb{K}), \cdot)$

$f \longrightarrow A_f$ izomorfism de grupuri

$A_f = [f]_{R,R}$, $R = \{e_1, \dots, e_n\}$ reper în V

1) $\varphi(f \circ g) = \varphi(f) \cdot \varphi(g)$

2) φ bijecție.

Aplicație

Fie $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 - x_2 + x_3, x_1 - x_2, x_1 - x_2 + x_3)$

a) $[f]_{R_0, R_0} = A = ?$

b) dim Ker f, dim Im f

c) Precizați o bază în Im f

SOL

a) $R_0 = \{ e_1 = (1,0,0), e_2 = (0,1,0), e_3 (0,0,1) \}$.

$f(e_1) = f(1,0,0) = (1,1,1) = e_1 + e_2 + e_3$

$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$

$f(e_2) = f(0,1,0) = (-1,-1,-1) = -e_1 - e_2 - e_3$

$f(e_3) = f(0,0,1) = (1,0,1) = e_1 + 0 e_2 + e_3$

$$f(x) = y \Leftrightarrow Y = AX = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + x_3 \\ x_1 - x_2 \\ x_1 - x_2 + x_3 \end{pmatrix}$$

b) $\text{Ker} f = \{ x \in \mathbb{R}^3 \mid AX = 0 \} = S(A)$

$\dim \text{Ker} f = 3 - rg A = 3 - 2 = 1$, $\det A = 0$

T. dim: $\dim \mathbb{R}^3 = \dim \text{Ker} f + \dim \text{Im} f \Rightarrow \dim \text{Im} f = 2$

$$\ker f \quad \begin{cases} x_1 - x_2 + x_3 = 0 \\ x_1 - x_2 \qquad = 0 \\ x_1 - x_2 + x_3 = 0 \end{cases} \qquad A = \left(\begin{array}{ccc} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{array}\right) \left|\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right.$$

$$\begin{cases} -x_2 + x_3 = -x_1 \Rightarrow x_3 = -x_1 + x_1 = 0 \\ x_2 = x_1 \end{cases}$$

$\ker f = \{(x_1, x_1, 0) \mid x_1 \in \mathbb{R}\} = \langle\{(1,1,0)\}\rangle$

$R_1 = \{(1,1,0)\}$ reper în $\ker f$

$\det \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \neq 0 \quad \{(1,1,0), \underline{e_2}, \underline{e_3}\}$ reper în $\mathbb{R}^3$

$\{ f(e_2) = (-1,-1,-1), f(e_3) = (1,0,1)\}$ reper în $\operatorname{Im} f$

<u>Def</u> $(V,+,\cdot)/_{\mathbb{K}}$ sp. vect

$(V^* = \{f : V \to \mathbb{K} \mid f \text{ lin}\}, +, \cdot)/_{\mathbb{K}}$ <u>sp. vectorial dual</u>

$+ : V^* \times V^* \to V^*, \quad (f+g)(x) = f(x) + g(x)$

$\cdot : \mathbb{K} \times V^* \to V^* \quad (af)(x) = a f(x)$

$\forall f, g \in V^*, \forall a \in \mathbb{K}, \forall x \in V$

<u>Teorema</u> $V \simeq V^*$

<u>Dem</u> $R = \{e_1, \dots, e_n\}$ reper în V

Construim $R^* = \{e_1^*, \dots, e_n^*\} \subset V^*$

$e_i^* : V \to \mathbb{K}, \quad e_i^*(e_j) = \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

Prelungim prin liniaritate

$$e_i^*(x) = e_i^*\left(\sum_{j=1}^n x_j e_j\right) = \sum_{j=1}^n x_j \overbrace{\underline{e_i^*(e_j)}}^{\delta_{ij}} = x_i, \quad \forall i = \overline{1,n}$$

① R^* e SLI în V^*

$$\sum_{i=1}^n a_i e_i^* = 0 \Rightarrow \sum_{i=1}^n a_i \overbrace{e_i^*(e_j)}^{\delta_{ij}} = 0 \Rightarrow a_j = 0, \forall j = \overline{1,n}$$

$$a_1 \underbrace{e_1^*(e_j)}_{0} + \dots + a_j \underbrace{e_j^*(e_j)}_{1} + \dots + a_n \underbrace{e_n^*(e_j)}_{0} = 0 \Rightarrow a_j = 0$$

$\forall j = \overline{1,n}$

② R^* e SG

$$\forall f \in V^*, \quad f(x) = f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} x_i f(e_i)$$

$$= \sum_{i=1}^{n} f(e_i) e_i^*(x), \ \forall x \in V$$

$$\Rightarrow f = \sum_{i=1}^{n} f(e_i) e_i^* \Rightarrow \langle R^* \rangle = V^*$$

OBS $\varphi : V \longrightarrow V^*$

$\varphi(e_i) = e_i^*, \ \forall i = \overline{1,n}$ izom sp v.

Proiecții și simetrii

Def $p : V_1 \oplus V_2 \longrightarrow V_1 \oplus V_2$ lin.

p s.n. proiecție pe V_1 de-a lungul lui $V_2 \Leftrightarrow$

$p(v_1 + v_2) = v_1, \quad v_1 \in V_1, v_2 \in V_2.$

Prop $p \in End(V) \qquad V = V_1 \oplus V_2$

p proiecție $\Leftrightarrow p \circ p = p$

Dem "$\Rightarrow$" $p : V_1 \oplus V_2 \longrightarrow V_1 \oplus V_2.$

$p(v) = p(v_1 + v_2) = v_1$

$p \circ p(v) = p(v_1) = p(v_1 + 0) = v_1 = p(v)$

"$\Leftarrow$" $p \in End(V), \ p^2 = p.$

$V_1 = Im\, p, \ V_2 = Ker\, p.$

$V = V_1 \oplus V_2$

"$\subset$" $v = \underbrace{p(v)}_{\in Im\, p} + \underbrace{v - p(v)}_{\in Ker\, p}$

"$\supset$" din constr

$p(v - p(v)) = p(v) - p \circ p(v) = p(v) - p(v) = 0$

Fie $v \in Im\, p \cap Ker\, p$

$$\left.\begin{array}{l} v = p(w) \\ p(v) = 0 \end{array}\right\} \Rightarrow \underset{\overset{\shortparallel}{0}}{p(v)} = p(\underset{\underset{\overset{\shortparallel}{v}}{\overset{\shortparallel}{p(w)}}}{p(w)}) \Rightarrow v = 0_V$$

$$V = Im\, p \oplus Ker\, p$$

$$p(\underbrace{v_1 + v_2}_{v}) = p(v_1) = p(p(v)) = p(v) = v_1$$

<u>OBS</u>

$R = \{e_1, \dots, e_n\}$ reper în V

$R_1 = \{e_1, \dots, e_k\}$ reper în $V_1 = Im\, p$ $\quad p(e_i) = e_i,\ i = \overline{1,k}$

$R_2 = \{e_{k+1}, \dots, e_n\}$ $\quad V_2 = Ker\, p \quad p(e_j) = 0,\ \forall j = \overline{k+1,n}$

$$A_p = [p]_{R,R} = \left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & 0 \end{array}\right) \in \mathcal{M}_n(\mathbb{K})$$

<u>Def</u> $s \in End(V)$

s s.n. simetrie sau involuție $\Leftrightarrow s \circ s = id_V$

$\boxed{\text{Temă 3}}$ (CURS)

(1) $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1, x_1 + x_2 - x_3, 0)$

a) $[f]_{R_0, R_0} = ?$ b) $Ker\, f$, $Im\, f$.

Precizați câte un reper în $Ker\, f$, $Im\, f$

(2) $(V, +, \cdot)/\mathbb{K}$, $(W, +, \cdot)/\mathbb{K}$ sp vect

$S: V \to W$ apl liniară

$S^*: W^* \to V^*$, $S^*(f) = f \circ S$, $\forall f \in W^*$

a) S^* apl. liniară

b) S surj $\Rightarrow S^*$ inj.

(3) a) $\mathbb{R}_3[X] = V_1 \oplus V_2$ $\quad V_1 = \langle\{1, X - X^3\}\rangle$, $V_2 = \langle\{1+X, 1+X^2\}\rangle$

b) $p(1 + 2X + 3X^2 + 4X^3) = ?$ p proiecția pe V_1 de-a lungul lui V_2.