Ex1 Calculați $\det A$

a) $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$; b) $A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$; $\Delta_A = c(a, b, c)$

Ex2 Fie $A = \begin{pmatrix} 2 & -1 & 3m+4 \\ 1 & m & 1 \\ -1 & -1 & 0 \end{pmatrix} \in \mathcal{M}_3(\mathbb{Z})$

a) $m = ?$ a.î. $A^{-1} \in \mathcal{M}_3(\mathbb{Z})$, b) $m = -1$ $A^{-1} = ?$

Ex3 Fie $A = \begin{pmatrix} 1+a^2 & ba & ca \\ ba & 1+b^2 & cb \\ ca & bc & 1+c^2 \end{pmatrix}$

Calculați $\det(A^*)$

Ex4 Fie $A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 4 \\ 2 & 5 & 1 & -1 \\ -1 & -2 & 2 & 4 \end{pmatrix}$

Calculați Δ_A, utilizând Th. Laplace pentru

a) $p = 2$, l_1, l_2 fixate ; b) $p = 2$, l_2, l_3 fixate.

• Ex5 Fie $a_k, b_k, c_k, d_k \in \mathbb{C}$, $k = \overline{1,4}$

$$\Delta = \begin{vmatrix} a_1c_1 & a_2d_1 & a_1c_2 & a_2d_2 \\ a_3c_1 & a_4d_1 & a_3c_2 & a_4d_2 \\ b_1c_3 & b_2d_3 & b_1c_4 & b_2d_4 \\ b_3c_3 & b_4d_3 & b_3c_4 & b_4d_4 \end{vmatrix}$$

Utilizând Th Laplace pt $p = 2$ și l_1, l_2 fixate, să se ar

$$\Delta = \begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} \cdot \begin{vmatrix} b_1 & b_2 \\ b_3 & b_4 \end{vmatrix} \cdot \begin{vmatrix} c_1 & c_2 \\ c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} d_1 & d_2 \\ d_3 & d_4 \end{vmatrix}.$$

• **Ex 6**

$$\Delta = \begin{vmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ x & 0 & 1 & 0 & 4 \\ x & x & 0 & 1 & 5 \\ x & x & x & 0 & 6 \end{vmatrix}$$

Calculați cu Th. Laplace pt $p=3$ și c_1, c_2, c_3 fixate.

Ex 7 Fie $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

Să se calculeze A^{-1}, utilizând Th. Hamilton-Cayley.

Ex 8 Fie $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ și $B = A^4 - 3A^3 + 3A^2 + 2A + 8I_2$

Să se afle $a, b \in \mathbb{R}$ a.î. $B = aA + bI_2$, utilizând Th H-C.

Ex 9 Fie $A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & -2 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$

a) Să se scrie polinomul caracteristic

b) Calculați A^{100}, utilizând th H-C.

Ex 10 Fie $A \in \mathcal{M}_2(\mathbb{C})$.

Dacă $\exists\, k \geq 2$ a.î. $A^k = O_2$, atunci $A^2 = O_2$

• **Ex 11** $f: \mathcal{M}_2(\mathbb{C}) \to \mathcal{M}_2(\mathbb{C})$, $f(X) = X^n$ nu e inj, nu e surj pentru niciun $n \geq 2$

Ex 12 Rez în $\mathcal{M}_2(\mathbb{C})$ ec a) $X^2 = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix} = A$

b) $X^n = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix} = A$

$\boxed{13}$ $X = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ $X^n = x_n X + y_n I_2$.

$x_n = ?$ $y_n = ?$

• $\boxed{EX14}$ $X = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, $X^n = x_n X^2 + y_n X + z_n I_2$

$x_n, y_n, z_n = ?$

$\boxed{EX15}$

a) $A \in \mathcal{M}_n(\mathbb{R})$, $A^2 = O_n \Rightarrow I_n - A, I_n + A$ inversabile

b) —"— $A^3 = O_n \Rightarrow I_n - A, I_n + A$ inversabile

$\boxed{EX16}$

$$\Delta = \begin{vmatrix} a^3 & 3a^2 & 3a & 1 \\ a^2 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{vmatrix} = (a-1)^6$$

$\boxed{EX17}$

$$\Delta = \begin{vmatrix} 1-n & 1 & \cdots & 1 \\ 1 & 1-n & & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & & 1-n \end{vmatrix}$$

$\boxed{EX18}$ $\Delta_n = \det(a_{ij})_{ij} = 1$, unde $a_{ij} = \min\{i, j\}, 1 \le i, j \le n$

$\boxed{EX19}$ $A = \begin{pmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{pmatrix} \in \mathcal{M}_3(\mathbb{R})$ $rg(A) = ?$ Discuție.

$\boxed{EX20}$ $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & a & 1 \\ 0 & 1 & 3 & b \end{pmatrix} \in \mathcal{M}_{3,4}(\mathbb{R})$ $a, b = ?$ ai $rg A = 2$.

$\boxed{EX21}$ $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ $A^{-1} = ?$ (H-C)

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