8.04.2021

SEMINAR 8

Forme biliniare. Forme pătratice. Formă canonică

OBS

$L(V,V;\mathbb{K}) = \{ g: V\times V \longrightarrow \mathbb{K} \ / \ g$ biliniară $\}$.
(liniară în fiecare argument)

$L^s(V,V;\mathbb{K})$ = forme biliniare simetrice
$(g(x,y)=g(y,x))$

$L^a(V,V;\mathbb{K})$ = forme biliniare antisimetrice
$(g(x,y)=-g(y,x))$

- $\mathcal{R}=\{e_1,..,e_n\}$ reper în V

$g(e_i,e_j)=g_{ij}$ $\quad G=(g_{ij})_{i,j=\overline{1,n}}$

g biliniară $\Leftrightarrow g(x,y)=X^TGY$

$X=\begin{pmatrix} x_1 \\ \vdots \\ x_n\end{pmatrix}, \ Y=\begin{pmatrix} y_1 \\ \vdots \\ y_n\end{pmatrix}, \ x=\sum_{i=1}^{n} x_ie_i$

$y=\sum_{j=1}^{n} y_je_j$

g simetrică $\Leftrightarrow G=G^T$

g antisimetrică $\Leftrightarrow G=-G^T$

- $\mathcal{R}=\{e_1,..,e_n\} \xrightarrow{C} \mathcal{R}'=\{e_1',..,e_n'\}$

$\mathcal{R} \downarrow G \qquad \mathcal{R}' \downarrow G'=C^TGC$

- $g\in L^s(V,V;\mathbb{K})$

$\text{Ker}\, g=\{x\in V \ | \ g(x,y)=0, \forall y\in V\}$

g nedegenerată $\Leftrightarrow \text{Ker}\, g=\{0_V\} \Leftrightarrow \det G\neq 0$.

- $Q: V \to \mathbb{K}$ pătratică $\Leftrightarrow \exists g \in L^s(V, V; \mathbb{K})$ a.î. $Q(x) = g(x,x)$ $\forall x \in V$

$g(x,y) = 2^{-1}(Q(x+y) - Q(x) - Q(y))$, $\operatorname{ch}\mathbb{K} \neq 2$

forma polară

- $Q: V \to \mathbb{R}$ formă pătratică reală

Q poz def $\Leftrightarrow \begin{cases} 1) \ Q(x) > 0, \ \forall x \in V \setminus \{0_V\} \\ 2) \ Q(x) = 0 \Leftrightarrow x = 0_V \end{cases}$

$Q(x) = X^T G X$

$Q(x) = a_1 x_1^2 + \ldots + a_r x_r^2$, $r = \operatorname{rg} Q = \operatorname{rg} g = \operatorname{rg} G$

formă canonică

$$G = \begin{pmatrix} a_1 & & & & & \\ & \ddots & & & 0 & \\ & & a_r & & & \\ & & & 0 & & \\ & 0 & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

<u>Th Gauss</u> $\exists$ un reper în V a.î. $Q: V \to \mathbb{K}$ are o formă canonică

<u>Th</u> $Q: V \to \mathbb{R}$

$\exists$ un reper în V a.î. Q are formă normală

$Q(x) = x_1^2 + \ldots + x_p^2 - x_{p+1}^2 - \ldots - x_r^2$, $r = \operatorname{rg} Q$

$(p, r-p)$ = signatura (invariant la sch de reper)

- Q este p. def $\Leftrightarrow (n, 0)$ = signatura , $n = \dim V$

<u>Ex1</u>

$g: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ formă biliniară antisimetrică

$\mathcal{R}_0 = \{e_1, e_2\}$ reperul canonic și $g_{12} = g(e_1, e_2) = 5$

Să se det matricea G asoc. lui g în raport cu $\mathcal{R}_0$.

SOL

$$G = -G^T \iff g_{ij} = -g_{ji}, \ \forall i,j = \overline{1,2}$$

$$i=j \Rightarrow g_{ii} = -g_{ii} \Rightarrow 2g_{ii} = 0, \ \forall i = \overline{1,2}$$

$$G = \begin{pmatrix} 0 & g_{12} \\ -g_{12} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$$

$x = x_1 e_1 + x_2 e_2$

$y = y_1 e_1 + y_2 e_2$

$$g : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}, \ g(x,y) = \sum_{i,j=1}^{2} g_{ij} x_i y_j =$$

$$= 5x_1 y_2 - 5x_2 y_1$$

Ex2

$$g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}, \ g(x,y) = x_1y_1 - x_2y_2 - x_1y_3 - x_3y_1 + 2x_2y_3 + 2x_3y_2$$

a) $g \in L^s(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$

b) $G = ?$ asoc. în raport cu $\mathcal{R}_0 = \{e_1, e_2, e_3\}$

c) $\operatorname{Ker} g = ?$ Este g nedegenerată?

d) $G' = ?$ asoc. lui g în raport cu reperul

$\mathcal{R}' = \{ e_1' = (1, 1, 1), e_2' = (1, 2, 1), e_3' = (0, 0, 1) \}$.

SOL

a), b) $g(x,y) = \sum_{i,j=1}^{3} g_{ij} x_i y_j$

$$= X^T G Y \Rightarrow g \in L(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$$

$$G = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix}$$

$G = G^T \Rightarrow g$ simetrică

$\Rightarrow g \in L^s(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$

$$g(x,y) = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

c) $\operatorname{Ker} g = \{ x \in \mathbb{R}^3 \mid g(x,y) = 0, \ \forall y \in \mathbb{R}^3 \}$

(M1) $\det G = \begin{vmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ +1 & 2 & -1 \end{vmatrix} \neq 0$

$\Rightarrow G \in GL(3,\mathbb{R}) \Rightarrow g$ nedegenerată i.e. $\mathrm{Ker}\, g = \{0_{\mathbb{R}^3}\}$

(M2) $\begin{cases} g(x, e_1) = 0 \\ g(x, e_2) = 0 \\ g(x, e_3) = 0 \end{cases} \Rightarrow (*) \begin{cases} x_1 - x_3 = 0 \\ -x_2 + 2x_3 = 0 \\ -x_1 + 2x_2 = 0 \end{cases}$ $\quad G = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix}$

$g(x, e_1) = g(x_1 e_1 + x_2 e_2 + x_3 e_3, e_1) = x_1 g_{11} + x_2 g_{21} + x_3 g_{31}$

$\det G \neq 0 \Rightarrow (*)$ SLO cu sol unică nulă $\Rightarrow \mathrm{Ker}\, g = \{0_{\mathbb{R}^3}\}$

$(x_1, x_2, x_3) = (0,0,0)$

d) $R_0 = \{e_1, e_2, e_3\} \xrightarrow{C} R' = \{e_1' = (1,1,1),\ e_2' = (1,2,1),\ e_3' = (0,0,1)\}$

$R_0 \rightsquigarrow G$ $\qquad R' \rightsquigarrow G' = C^T G C$

$e_1' = (1,1,1) = \underline{e_1 + e_2 + e_3}$

$e_2' = (1,2,1) = e_1 + 2e_2 + e_3$

$e_3' = (0,0,1) = e_3$

$C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

$G' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}}_{\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 2 \\ 1 & 3 & 0 \end{pmatrix}}$

$G' = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ $\qquad x = \sum_{i=1}^{3} x_i' e_i'$ $\qquad y = \sum_{j=1}^{3} y_j' e_j'$

$g(x,y) = \sum_{i,j=1}^{3} g_{ij}' x_i' y_j' = 2x_1'y_1' + 3x_1'y_2' + x_1'y_3'$
$+ 3x_2'y_1' + 3x_2'y_2' + 3x_2'y_3' + x_3'y_1' + 3x_3'y_2'.$

OBS

$$g'_{ij} = g(e'_i, e'_j), \ \forall i,j = \overline{1,3}$$

$$g'_{11} = g(e'_1, e'_1) = g((1,1,1),(1,1,1)) = 1 - 1 - \cancel{1} - \cancel{1} + \cancel{2} + 2 = 2$$

$$g(x,y) = x_1y_1 - x_2y_2 - x_1y_3 - x_3y_1 + 2x_2y_3 + 2x_3y_2$$

Ex 3 Fie $f \in End(\mathbb{R}^3)$, $g \in L(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$

Fie $g_f : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, $g_f(x,y) = g(f(x), y)$, $\forall x, y \in \mathbb{R}^3$.

a) $g_f \in L(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$

b) $G = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ -2 & -1 & -1 \end{pmatrix}$, $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

sunt matricele asoc. lui g, resp f în rap cu reperul canonic $\mathcal{R}_0$ $\Rightarrow \tilde{G} = ?$ matricea asociată lui g_f în raport cu reperul canonic $\mathcal{R}_0$.

SOL

a) $g_f(ax + bz, y) = g(f(ax+bz), y) =$

$= g(af(x) + bf(z), y) = a\,\underline{g(f(x), y)} + b\,g(f(z), y) =$

$= a\,g_f(x,y) + b\,g_f(z,y)$

$g_f(x, ay + bz) = g(f(x), ay + bz) = a\,g(f(x), y) + b\,g(f(x), z) =$

$= a\,g_f(x,y) + b\,g_f(x,z)$, $\forall x, y, z \in \mathbb{R}^3$, $\forall a, b \in \mathbb{R}$

$\Rightarrow g_f$ biliniară

b) $\underset{\mathcal{R}_0 = \{e_1, e_2, e_3\}}{\tilde{g}_{ij}} = g_f(e_i, e_j) = g(f(e_i), e_j) = g\left(\sum_{k=1}^{3} a_{ki} e_k, e_j\right) = \underbrace{\sum_{k=1}^{3} a_{ki}\, g(e_k, e_j)}_{\overset{\shortparallel}{\ }}$

$= \underline{\sum_{k=1}^{3} a_{ki}\, g_{kj}} \Rightarrow \boxed{\tilde{G} = A^T G}$

g_{kj}

$$\tilde{G} = A^T G = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ -2 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$g_f : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

$g_f(x,y) = -x_1 y_3 - 2x_2 y_1 - 2x_2 y_2 + x_3 y_2 - x_3 y_3$

$f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 - x_2 + x_3,\ x_2 - x_3,\ x_1 + x_3)$

$f(x) = y \Leftrightarrow Y = AX = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

Ex 4

$Q : \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + x_2x_3$

a) $G = ?$ matricea asoc. lui Q în rap. cu R_0

b) Să se det forma polară $g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

c) Să se aducă Q la o formă canonică.

Este Q poz def?

SOL

a) $Q(x) = \sum_{i,j=1}^{3} g_{ij} x_i x_j = g_{11}x_1^2 + g_{22}x_2^2 + g_{33}x_3^2$

$+ 2g_{12}x_1x_2 + 2g_{13}x_1x_3 + 2g_{23}x_2x_3$.

$Q(x) = g(x,x)$

$$G = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

b) $g(x,y) = \frac{1}{2}\left[Q(x+y) - Q(x) - Q(y)\right]$

$g(x,y) = \sum_{i,j=1}^{3} g_{ij} x_i y_j =$

$= x_1y_1 + \frac{1}{2}x_1y_2 + \frac{1}{2}x_1y_3 + \frac{1}{2}x_2y_1 + x_2y_2 + \frac{1}{2}x_2y_3$

$+ \frac{1}{2}x_3y_1 + \frac{1}{2}x_3y_2 + x_3y_3$

c) $Q(x) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + x_2x_3$

$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 - \frac{1}{4}x_2^2 - \frac{1}{4}x_3^2 - \frac{1}{2}x_2x_3 + x_2^2 + x_3^2 + x_2x_3 =$

$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}x_2^2 + \frac{1}{2}x_2x_3 + \frac{3}{4}x_3^2$

$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2^2 + \frac{2}{3}x_2x_3\right) + \frac{3}{4}x_3^2$

$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2 + \frac{1}{3}x_3\right)^2 - \frac{1}{12}x_3^2 + \frac{\cancel{3}^{3/}}{4}x_3^2$

$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2 + \frac{1}{3}x_3\right)^2 + \frac{2}{3}x_3^2$

Fie sch. de reper:

$$\mathcal{R}': \begin{cases} x_1' = x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ x_2' = x_2 + \frac{1}{3}x_3 \\ x_3' = x_3 \end{cases} \Rightarrow Q(x) = x_1'^2 + \frac{3}{4}x_2'^2 + \frac{2}{3}x_3'^2$$

formă canonică

(3,0) signatura

Q poz definită.

$$\begin{array}{ccccc} \mathcal{R}_0 & \longrightarrow & \mathcal{R}' & \longrightarrow & \mathcal{R}'' \\ \downarrow & & \downarrow & & \downarrow \\ G & & G' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix} & & G'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$\mathcal{R}'': \begin{cases} x_1'' = x_1' \\ x_2'' = \frac{\sqrt{3}}{2}x_2' \\ x_3'' = \sqrt{\frac{2}{3}}x_3' \end{cases} \Rightarrow Q(x) = x_1''^2 + x_2''^2 + x_3''^2$$

<u>Ex5</u> Fie $Q: \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = 2x_1x_2 - 6x_1x_3 - 6x_2x_3$

a) $G = ?$ asoc. în rap. cu $\mathcal{R}_0$

b) g forma polară asoc.

c) Să se aducă Q la o formă canonică. Este Q poz def?

Sol a) $Q(x) = \sum_{i,j=1}^{3} g_{ij} x_i x_j$

$$G = \begin{pmatrix} 0 & \boxed{1} & -3 \\ 1 & 0 & -3 \\ -3 & -3 & 0 \end{pmatrix}$$

b) $g : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$$g(x,y) = x_1 y_2 - 3x_1 y_3 + x_2 y_1 - 3x_2 y_3 - 3x_3 y_1 - 3x_3 y_2$$

c) $Q(x) = \underline{2x_1 x_2} - 6x_1 x_3 - 6x_2 x_3$

Sch. de reper

$$\begin{cases} x_1' = x_1 + x_2 \\ x_2' = x_1 - x_2 \\ x_3' = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}(x_1' + x_2') \\ x_2 = \frac{1}{2}(x_1' - x_2') \\ x_3 = x_3' \end{cases}$$

$$Q(x) = \frac{1}{2}(x_1'^2 - x_2'^2) - 6x_3' x_1' = \frac{1}{2}x_1'^2 - 6x_1' x_3' - \frac{1}{2}x_2'^2 =$$

$$= 2\left(\frac{1}{4}x_1'^2 - 3x_1' x_3'\right) - \frac{1}{2}x_2'^2 =$$

$$= 2\left(\frac{1}{2}x_1' - 3x_3'\right)^2 - \frac{1}{2}x_2'^2 - 18x_3'^2$$

Sch. de reper:

$$\begin{cases} x_1'' = \sqrt{2}\left(\frac{1}{2}x_1' - 3x_3'\right) \\ x_2'' = \frac{1}{\sqrt{2}}x_2' \\ x_3'' = 3\sqrt{2}\,x_3' \end{cases} \Rightarrow Q(x) = x_1''^2 - x_2''^2 - x_3''^2$$

formă normală

$(1,2)$ = signatura $\Rightarrow Q$ nu e poz def

$$\begin{array}{ccccc} R_0 & \longrightarrow & R' & \longrightarrow & R'' \\ \downarrow & & \downarrow & & \downarrow \\ G & & G' & & G'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{array}$$

Ex6

Fie $g, g_s, g_a : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ forme biliniare.

$G = \begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$, $G_s = \frac{1}{2}(G + G^T)$, $G_a = \frac{1}{2}(G - G^T)$

matricele asoc. lui g, g_s, resp g_a, în rap. cu R_0

a) $g, g_s, g_a = ?$

b) Fie $Q : \mathbb{R}^3 \to \mathbb{R}$ forma pătratică asoc. lui g_s

Să se aducă la o formă canonică.

Precizați reperul în care se realizează.

Este Q poz. def?

a) $G_s = \frac{1}{2}\left(\begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}\right) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$

$G_a = \frac{1}{2}\begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$g(x, y) = 2x_1y_1 + x_1y_2 - x_2y_1 - x_2y_2 - x_2y_3 - x_3y_2 - 2x_3y_3$

$g_a(x, y) = x_1y_2 - x_2y_1$ forma biliniară antisimetrică asociată lui g

$g_s(x, y) = 2x_1y_1 - x_2y_2 - x_2y_3 - x_3y_2 - 2x_3y_3$.

forma biliniară simetrică asoc. lui g

b) $Q : \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = g_s(x, x) = \underline{2x_1^2} - \underline{\underline{x_2^2}} - 2x_3^2 - \underline{\underline{2x_2x_3}}$

$Q(x) = 2x_1^2 - (x_2 + x_3)^2 - x_3^2$

Sch reper: $\begin{cases} x_1' = x_1 \\ x_2' = x_2 + x_3 \\ x_3' = x_3 \end{cases}$ $\quad Q(x) = 2x_1'^2 - x_2'^2 - x_3'^2$

(1, 2) signatura

Nu e poz def

$\mathcal{R}_0 = \{e_1, e_2, e_3\} \xrightarrow{C} \mathcal{R}' = \{e_1', e_2', e_3'\}$

$G \qquad\qquad G' = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$X = CX'$

$$\begin{cases} x_1 = x_1' \\ x_2 = x_2' - x_3' \\ x_3 = x_3' \end{cases} \Leftrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} & \begin{matrix} 0 \\ -1 \\ 1 \end{matrix} \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

$e_1' = 1 \cdot e_1 + 0 \cdot e_2 + 0 e_3 = e_1 = (1, 0, 0)$

$e_2' = e_2 = (0, 1, 0)$

$e_3' = -e_2 + e_3 = (0, -1, 1)$

T_4 (sem)

1) $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 - 2x_2, -2x_1 + 2x_2 - 2x_3, -2x_2 + 3x_3)$

Este f endomorfism diagonalizabil?

2) $G = \begin{pmatrix} 3 & -2 & -4 \\ -2 & 6 & 2 \\ -4 & 2 & 3 \end{pmatrix}$

matricea asoc. f. pătratice $Q : \mathbb{R}^3 \to \mathbb{R}$ în rap. cu $\mathcal{R}_0$.

a) $Q = ?$; $g = ?$ (forma polară); $\ker g = ?$

b) Să se aducă Q la o f. canonică. Precizați reperul coresp.