25.02.2021

Seminar 2

Rang. Sisteme liniare

Ex (continuare seminar 1)

Fie $A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & -2 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$

a) $P_A = ?$; b) $A^{100} = ?$ utilizând Th H-C

SOL

a) $P_A(X) = \det(A - X I_4) = X^4 - \sigma_1 X^3 + \sigma_2 X^2 - \sigma_3 X + \sigma_4$

$\sigma_1 = Tr(A) = 0$

$\sigma_2 = \begin{vmatrix} -2 & 1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix}$

$= 1 + 1 - 4 + 0 + 1 + 1 = 0$

$\sigma_3 = \begin{vmatrix} 2 & 0 & 1 \\ 0 & -2 & 1 \\ -1 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \\ -1 & 0 & -2 \end{vmatrix} =$

$= \begin{vmatrix} 2 & -2 & 1 \\ 0 & -2 & 1 \\ -1 & 0 & 0 \end{vmatrix} + 0 + 0 + \begin{vmatrix} 0 & 0 & 1 \\ -1 & 2 & 0 \\ -1 & 2 & -2 \end{vmatrix} = 0 + 0 + 0 + 0 = 0$

$\sigma_4 = \begin{vmatrix} 0 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & -2 & 1 \\ 0 & -1 & -1 & 0 \end{vmatrix} = 0$

$P_A = X^4$

b) $A^4 = O_4$ (nilpotentă) $\Rightarrow A^{100} = O_4$

Ex1 $A = \begin{pmatrix} \boxed{1} & 2 & 3 & 1 \\ 2 & 0 & a & 1 \\ 0 & 1 & 3 & b \end{pmatrix}$ $a, b \in \mathbb{R}$

$a, b = ?$ a.î. $rg\, A = 2$.

SOL

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & a \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & a-6 \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} -4 & a-6 \\ 1 & 3 \end{vmatrix} = -12 - a + 6$$

$$= -6 - a = 0 \Rightarrow a = -6.$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & b \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -4 & -1 \\ 0 & 1 & b \end{vmatrix} = -4b + 1 = 0 \Rightarrow b = \frac{1}{4}$$

Ex2

$A = \begin{pmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{pmatrix}$ $rg\, A = ?$ Discuție.

$a \in \mathbb{R}$

SOL

$$\Delta = \begin{vmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{vmatrix} = \begin{vmatrix} a-1 & 1 & 1 \\ 0 & 1 & 0 \\ -2 & 1 & -a \end{vmatrix} =$$

$$= \begin{vmatrix} a-1 & 1 \\ -2 & -a \end{vmatrix} = -a^2 + a + 2 = -(a^2 - a - 2) =$$

$$= -(a+1)(a-2)$$

I. $\Delta \neq 0 \Rightarrow a \in \mathbb{R} \setminus \{-1, 2\} \Rightarrow rg\, A = 3$

II. $\Delta = 0$

a) $a = 2$ $\quad A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$ $rg\, A = 2$

b) $a = -1$ $\quad A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ $rg\, A = 2$

Ex3 $A=\begin{pmatrix} 1 & 0 & 1 & m \\ m & 1 & 2 & -1 \\ m & -2 & -1 & 1 \end{pmatrix}$, $m\in\mathbb{R}$ $rg\,A=?$ Discutie.

SoL

$$\Delta_1=\begin{vmatrix} 1 & 0 & 1 \\ m & 1 & 2 \\ m & -2 & -1 \end{vmatrix}=\begin{vmatrix} 0 & 0 & 1 \\ m-2 & 1 & 2 \\ m+1 & -2 & -1 \end{vmatrix}=\begin{vmatrix} m-2 & 1 \\ m+1 & -2 \end{vmatrix}=$$

$$=-2m+4-m-1=-3m+3=3(1-m)$$

$$\Delta_2=\begin{vmatrix} 0 & 1 & m \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix}=\begin{vmatrix} 0 & 1 & m \\ 1 & 2 & -1 \\ 0 & 3 & -1 \end{vmatrix}=|-|\begin{vmatrix} 1 & m \\ 3 & -1 \end{vmatrix}$$

$$=-(-1-3m)=3m+1.$$

1) $m\neq 1 \Rightarrow \Delta_1\neq 0 \Rightarrow rg\,A=3$

2) $m=1 \Rightarrow \Delta_2\neq 0 \Rightarrow rg\,A=3.$

Ex4. $A\in\mathcal{M}_3(\mathbb{R})$, $A^{2021}-2021A-I_3=O_3$

a) $rg\,A$;

b) $rg(2021A+I_3)$

Sol

a) $A^{2021}-2021A=I_3 \Rightarrow A(A^{2020}-2021I_3)=I_3 \mid \det$

$\det(A)\cdot\det(A^{2020}-2021I_3)=1 \Rightarrow \det A\neq 0 \Rightarrow$

$rg\,A=3$

b) $A^{2021}=2021A+I_3 \mid \det$

$\underset{\neq 0}{\det(A)^{2021}}=\det(2021A+I_3) \Rightarrow rg(2021A+I_3)=3$

Ex5 Fie $A\in\mathcal{M}_n(\mathbb{R})$ ai $A^3-6A^2+12A=O_n \Rightarrow rg(2I_n-A)$.

Sol

$(2I_n-A)^3=8I_n-12A+6A^2-A^3=8I_n \mid \det$

$\det(2I_n-A)^3=8^n \Rightarrow \det(2I_n-A)=2^n\neq 0 \Rightarrow rg(2I_n-A)=n$

Ex6 $\begin{cases} x+\alpha y+z=1 \\ \alpha x-y+z=1 \\ x+y-z=2. \end{cases}$ $\alpha\in\mathbb{R}$. Să se rez. Discuție

Sol

$A=\begin{pmatrix} 1 & \alpha & 1 \\ \alpha & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}\left|\begin{matrix} 1 \\ 1 \\ 2 \end{matrix}\right.$

$$\Delta=\det A=\begin{vmatrix} 1 & \alpha & 1 \\ \alpha & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}=\begin{vmatrix} 1 & \alpha & 1 \\ \alpha & -1 & 1 \\ \alpha+1 & 0 & 0 \end{vmatrix}=(\alpha+1)\begin{vmatrix} \alpha & 1 \\ -1 & 1 \end{vmatrix}=(\alpha+1)^2.$$

I. $\Delta\neq 0\Rightarrow \alpha\in\mathbb{R}\setminus\{-1\}\Rightarrow rg A=rg\bar{A}=3$ SCD

Aplicăm met. Cramer

$$\Delta_x=\begin{vmatrix} 1 & \alpha & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}=\begin{vmatrix} 0 & \alpha & 1 \\ 0 & -1 & 1 \\ 3 & 1 & -1 \end{vmatrix}=3\begin{vmatrix} \alpha & 1 \\ -1 & 1 \end{vmatrix}=3(\alpha+1)$$

$$x=\frac{\Delta x}{\Delta}=\frac{3}{\alpha+1}$$

$$\Delta_y=\begin{vmatrix} 1 & 1 & 1 \\ \alpha & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 0 \\ \alpha & 1 & 0 \\ 1 & 2 & -3 \end{vmatrix}=-3\begin{vmatrix} 1 & 1 \\ \alpha & 1 \end{vmatrix}=-3(1-\alpha)=3(\alpha-1)$$

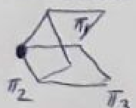
$$y=\frac{\Delta y}{\Delta}=\frac{3(\alpha-1)}{(\alpha+1)^2}$$

$$\Delta_z=\begin{vmatrix} 1 & \alpha & 1 \\ \alpha & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix}=-2+\alpha+\alpha+1-1-2\alpha^2=$$

$$=-2\alpha^2+2\alpha-2=-2(\alpha^2-\alpha+1)$$

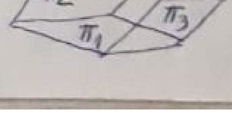
$$z=\frac{\Delta z}{\Delta}=\frac{-2(\alpha^2-\alpha+1)}{(\alpha+1)^2}$$

Soluția unică este $(x,y,z)=\left(\frac{3}{\alpha+1},\frac{3(\alpha-1)}{(\alpha+1)^2},\frac{-2(\alpha^2-\alpha+1)}{(\alpha+1)^2}\right)$.



II. $\Delta=0\Rightarrow\alpha=-1$ $A=\begin{pmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}\left|\begin{matrix} 1 \\ 1 \\ 2 \end{matrix}\right.$

$$\left.\begin{aligned} &\Delta_p=\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix}\neq 0\Rightarrow rg A=2 \\ &\Delta_c=\begin{vmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix}=\begin{vmatrix} 1 & -1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 3 \end{vmatrix}=3\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix}\neq 0\Rightarrow rg\bar{A}=3 \end{aligned}\right\}\text{SI}$$



$\pi_2 // \pi_3$

Ex7 $\begin{cases} x+2y+3z=0 \\ 4x+5y+6z=0 \\ x+\lambda^2 z=0, \lambda\in\mathbb{R} \end{cases}$ Să se rez.

$$A=\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 1 & 0 & \lambda^2 & 0 \end{array}\right)$$

SOL

$\det A = 1(12-15)+\lambda^2(5-8) = -3(\lambda^2+1) \neq 0$

$rg A = rg \bar{A} = 3$ SCD

$\Delta_x = \Delta_y = \Delta_z = 0 \Rightarrow$ sol unică nulă $(x,y,z)=(0,0,0)$

Ex8 $\begin{cases} x+y+z=0 \\ ax+by+cz=0 \\ (b+c)x+(a+c)y+(a+b)z=0 \end{cases}$ $a,b,c\in\mathbb{R}$, $a\neq b$.

Să se rezolve.

SOL

$$A=\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ a & b & c & 0 \\ b+c & a+c & a+b & 0 \end{array}\right)$$

$$\det A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a+b+c & a+b+c & a+b+c \end{vmatrix} = 0$$

$l_3' = l_3 + l_2$

$\left.\begin{array}{l} \Delta_p = \begin{vmatrix} 1 & 1 \\ a & b \end{vmatrix} \neq 0 \Rightarrow rg A = 2 \\ \Delta_c = \begin{vmatrix} 1 & 1 & 0 \\ a & b & 0 \\ b+c & a+c & 0 \end{vmatrix} = 0 \Rightarrow rg\bar{A}=2 \end{array}\right\} \Rightarrow$ SC simplu N

x, y = var. principale, $z=\alpha$ var. secundară

$\begin{cases} x+y=-\alpha \\ ax+by=-c\alpha \end{cases} \Big| -b$

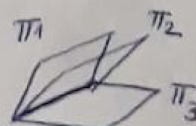
1) $b=0 \Rightarrow ax=-c\alpha \Rightarrow x=-\frac{c\alpha}{a}$, $y=-\alpha+\frac{c\alpha}{a}=\frac{\alpha(c-a)}{a}$

$(x,y,z)\in\left\{\left(-\frac{c\alpha}{a}, \frac{\alpha(c-a)}{a}, \alpha\right), \alpha\in\mathbb{R}\right\}$

2) $b\neq 0 \Rightarrow x(a-b)=\alpha(b-c) \Rightarrow x=\frac{\alpha(b-c)}{a-b}$

$y=-\alpha-\frac{\alpha(b-c)}{a-b}=\frac{-\alpha(a-b+b-c)}{a-b}=\frac{\alpha(c-a)}{a-b}$

$(x,y,z)\in\left\{\left(\frac{\alpha(b-c)}{a-b}, \frac{\alpha(c-a)}{a-b}, \alpha\right), \alpha\in\mathbb{R}\right\}$



Ex 9
$$\begin{cases} x+y+mz-t=0 \\ 2x+y-z+t=0 \\ 3x-y-z-t=0 \\ mx-2y-2t=0 \end{cases}$$

$m=?$ a.î. sistemul are și soluții nenule. $(m\in\mathbb{R})$

$$A=\begin{pmatrix} 1 & 1 & m & -1 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -1 \\ m & -2 & 0 & -2 \end{pmatrix}\Bigg|\begin{matrix}0\\0\\0\\0\end{matrix}$$

Sol

$$\Delta=\det A=\begin{vmatrix} 1 & 1 & m & -1 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -1 \\ m & -2 & 0 & -2 \end{vmatrix} \overset{C_4'=C_4-C_2}{=} \begin{vmatrix} 1 & 1 & m & -2 \\ 2 & 1 & -1 & 0 \\ 3 & -1 & -1 & 0 \\ m & -2 & 0 & 0 \end{vmatrix}$$

$$=(-2)(-1)^{1+4}\begin{vmatrix} 2 & 1 & -1 \\ 3 & -1 & -1 \\ m & -2 & 0 \end{vmatrix}=2\begin{vmatrix} 2 & 1 & -1 \\ 1 & -2 & 0 \\ m & -2 & 0 \end{vmatrix}=$$

$$=(-2)(-2)\begin{vmatrix} 1 & 1 \\ m & 1 \end{vmatrix}=4(1-m)$$

SCN $\Leftrightarrow \Delta=0 \Leftrightarrow m=1$.

$$A=\begin{pmatrix} 1 & 1 & 1 & -1 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -1 \\ 1 & -2 & 0 & -2 \end{pmatrix}\Bigg|\begin{matrix}0\\0\\0\\0\end{matrix}$$

$$\Delta_p=\begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & -2 \\ 3 & -1 & 0 \end{vmatrix}=(-2)(-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}=2(-1-4)=-8\neq 0$$

$\Delta_c=0$ (are o col. nulă) $\Rightarrow rgA=rg\bar{A}=3$

$t=\alpha$ var secundară, x,y,z = var principale.

$$\begin{cases} x+y+z=\alpha \\ 2x+y-z=-\alpha \\ 3x-y-z=\alpha \end{cases}$$

ec1 + ec3 : $4x=2\alpha \Rightarrow \boxed{x=\frac{\alpha}{2}}$

$$\begin{cases} y+z=\alpha-\frac{\alpha}{2}=\frac{\alpha}{2} \\ y-z=-\alpha-\alpha=-2\alpha \end{cases} \; (+)$$

$$2y=-2\alpha+\frac{\alpha}{2}=-\frac{3\alpha}{2}\Rightarrow \boxed{y=-\frac{3\alpha}{4}}$$

$$z=\frac{\alpha}{2}+\frac{3\alpha}{4}=\frac{5\alpha}{4}$$

$$(x,y,z,t)\in\left\{\left(\frac{\alpha}{2},-\frac{3\alpha}{4},\frac{5\alpha}{4},\alpha\right),\ \alpha\in\mathbb{R}\right\}$$

Ex10 $\begin{cases} x+2y-3z=0 \\ 5x-3y+z=10 \end{cases}$ Să se rezolve.

α

$A=\left(\begin{array}{cc|c|c} \boxed{1} & 2 & -3 & 0 \\ 5 & -3 & 1 & 10 \end{array}\right)$

SOL

$\Delta_p=\begin{vmatrix} 1 & 2 \\ 5 & -3 \end{vmatrix} \neq 0 \Rightarrow rg A = rg \bar{A} = 2$ SC simplu N

x, y = var principale, $z=\alpha$ var. secundară

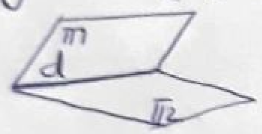
$\begin{cases} x+2y=3\alpha \\ 5x-3y=10-\alpha \end{cases} \Big| -5$

$-13y = 10-16\alpha$

$y=\frac{16}{13}\alpha-\frac{10}{13}$

$x=3\alpha-\frac{32}{13}\alpha+\frac{20}{13}=\frac{7\alpha}{13}+\frac{20}{13}$

$(x,y,z)\in\left\{\left(\frac{7\alpha}{13}+\frac{20}{13}, \frac{16}{13}\alpha-\frac{10}{13}, \alpha\right) \Big| \alpha\in\mathbb{R}\right\}$



Ex11 $\begin{cases} x+2y=m+1 \\ 2x+3y=m-1 \\ mx+y=3 \end{cases}, m\in\mathbb{R}$ m = ? aî SI

$A=\left(\begin{array}{cc|c} \boxed{\begin{matrix} 1 & 2 \\ 2 & 3 \end{matrix}} & & m+1 \\ & & m-1 \\ m & 1 & 3 \end{array}\right)$

SOL

$\Delta_p=\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \neq 0 \Rightarrow rg A = 2.$

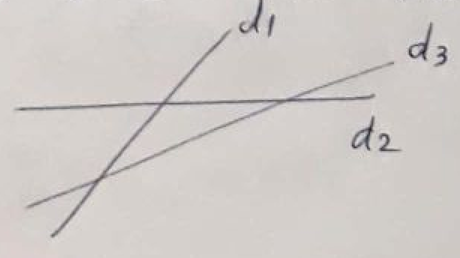
$\Delta_c=\begin{vmatrix} 1 & 2 & m+1 \\ 2 & 3 & m-1 \\ m & 1 & 3 \end{vmatrix} \neq 0 \Leftrightarrow rg \bar{A}=3.$

$\Delta_c = 1(9-m+1)-2(6-m-1)+m(2m-2-3m-3)$

$= 10-m-10+2m-m^2-5m = -m^2-4m$

$\Delta_c = -m(m+4) \neq 0$

$m\in\mathbb{R}\setminus\{-4,0\}$



Ex12 $\sum_{i=1}^{k}(1+i)x_i + \sum_{i=1}^{4-k} i\, x_{i+k} = 0,\ \forall k=\overline{1,3}$

Să se rezolve sistemul

Sol

$k=1 \Rightarrow 2x_1 + \sum_{i=1}^{3} i\, x_{i+1} = 0 \Rightarrow 2x_1 + x_2 + 2x_3 + 3x_4 = 0$

$k=2 \Rightarrow \sum_{i=1}^{2}(1+i)x_i + \sum_{i=1}^{2} i\, x_{i+2} = 0 \Rightarrow 2x_1 + 3x_2 + x_3 + 2x_4 = 0$

$k=3 \Rightarrow \sum_{i=1}^{3}(1+i)x_i + x_4 = 0 \Rightarrow 2x_1 + 3x_2 + 4x_3 + x_4 = 0.$

$$\begin{cases} 2x_1 + x_2 + 2x_3 + 3x_4 = 0 \\ 2x_1 + 3x_2 + x_3 + 2x_4 = 0 \\ 2x_1 + 3x_2 + 4x_3 + x_4 = 0 \end{cases} \qquad A = \left(\begin{array}{cccc|c} 2 & 1 & 2 & 3 & 0 \\ 2 & 3 & 1 & 2 & 0 \\ 2 & 3 & 4 & 1 & 0 \end{array}\right)$$

$$\Delta_p = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 2 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} \neq 0 \Rightarrow rg A = 3 = rg \bar{A}$$

SCsN $\quad x_1, x_2, x_3 =$ var. principale, $x_4 = \alpha$ var secundară

$$\begin{cases} 2x_1 + x_2 + 2x_3 = -3\alpha \\ 2x_1 + 3x_2 + x_3 = -2\alpha \\ 2x_1 + 3x_2 + 4x_3 = -\alpha. \end{cases} \qquad ec3 - ec2:\ 3x_3 = \alpha \Rightarrow x_3 = \frac{\alpha}{3}$$

$$\begin{cases} 2x_1 + x_2 = -3\alpha - \frac{2\alpha}{3} = -\frac{11\alpha}{3} \\ 2x_1 + 3x_2 = -2\alpha - \frac{\alpha}{3} = -\frac{7\alpha}{3} \end{cases} \ominus$$

$/ \quad 2x_2 = \frac{4\alpha}{3} \Rightarrow x_2 = \frac{2\alpha}{3}$

$x_1 = \frac{1}{2}\left(-\frac{11\alpha}{3} - \frac{2\alpha}{3}\right) = -\frac{13\alpha}{6}$

$(x_1, x_2, x_3, x_4) \in \left\{\left(-\frac{13\alpha}{6}, \frac{2\alpha}{3}, \frac{\alpha}{3}, \alpha\right),\ \alpha \in \mathbb{R}\right\}.$

Ex13 $\sum_{j=1}^{4} a_{ij} x_j = 4^{i-1}, \forall i=\overline{1,4}; \quad a_{ij} = j^{i-1}, \forall i,j=\overline{1,4}$

Să se rez. sistemul

SOL

$$A = \left(\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \end{matrix}\right)\left|\begin{matrix} 1 \\ 4 \\ 4^2 \\ 4^3 \end{matrix}\right.$$

$$\Delta = \det A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \end{vmatrix} = \underbrace{(4-3)(4-2)(4-1)}_{3!}\underbrace{(3-2)(3-1)}_{2!}\underbrace{(2-1)}_{1!}$$

(det. Vandermonde) $= 1! \cdot 2! \cdot 3! \neq 0$ SCD

$\Delta_{x_1} = \Delta_{x_2} = \Delta_{x_3} = 0 \quad ; \Delta_{x_4} = \Delta$

(2 col. egale)

$(x_1, x_2, x_3, x_4) = (0, 0, 0, 1)$

Tema 1 (seminar)

① $\begin{cases} x + y + az - t = 0 \\ 2x + y - z + t = 0 \\ 3x - y - z - t = 0 \\ ax - 2y - 2z - 2t = 0, a \in \mathbb{R}. \end{cases}$ Să se rez. Discuție.

② $A = \begin{pmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{pmatrix}$

Să se arate că $\det A = 4(a^2+b^2)(c^2+d^2)$, utilizând Th. Laplace

③ Fie $A = \mathcal{M}_2(\mathbb{R})$ a.î. $A^2 = O_2$

Fie P_A pol. caract.

Calculați $P_A(1) + \ldots + P_A(n)$, $n \in \mathbb{N}^*$.