CURS 13

20.05.2020

Clasificarea izometriilor în $\mathcal{E}_3$.
Hipercuadrice afin echivalente, congruente metric.
Conice ca LG. Formă canonică.

$(\mathcal{E}_3, (E_3, <\cdot,\cdot>), \varphi)$, $\mathcal{R} = \{O; e_1, e_2, e_3\}$ reper cartezian ortonormat, $\tau \in Izo(\mathcal{E}_3)$: $X' = AX + B$, $A \in O(3)$.

(I) τ are speța 1 $(A \in SO(3))$

(1) $\tau = T_u$ (translație de vector u), $u = (b_1, b_2, b_3)$

$X' = X + B$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

a) Dacă $u \neq 0_{\mathbb{R}^3}$, atunci $\nexists$ pcte fixe.

b) Dacă $u = 0_{\mathbb{R}^3}$, atunci $\tau = id_{\mathcal{E}_3}$, $\mathcal{E}_3$ = mult. puncte fixe.

(2) $X' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} X + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Puncte fixe: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + b_1 \\ x_2\cos\varphi - x_3\sin\varphi + b_2 \\ x_2\sin\varphi + x_3\cos\varphi + b_3 \end{pmatrix}$

a) $b_1 = 0 \Rightarrow d$ = dreaptă de puncte.

$\tau = \mathcal{R}_{d,\varphi}$, $V_d = <\{e_1\}>$ rotație de $\measuredangle\varphi$ și axă d.

$\mathcal{R}_{d,\varphi}$: $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \\ b_3 \end{pmatrix}$

Caz particular $\varphi = \pi$. $\Rightarrow \tau = S_d$ simetrie axială

S_d : $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ -x_2 + b_2 \\ -x_3 + b_3 \end{pmatrix}$

b) $b_1 \neq 0$ ($\nexists$ pcte fixe) (mișcare elicoidală, rototranslație)

$\tau = T_w \circ \mathcal{R}_{d,\varphi}$, $w \in V_d$.

$\mathcal{R}_{d,\varphi}$: $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \\ b_3 \end{pmatrix}$; T_w : $\begin{pmatrix} x_1'' \\ x_2'' \\ x_3'' \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} + \begin{pmatrix} b_1 \\ 0 \\ 0 \end{pmatrix}$

II τ izometrie de speța 2 $\left(A \in O(3),\ \det A = -1 \right)$.

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

Dacă $\varphi = 0 \Rightarrow \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -x_1 + b_1 \\ x_2 + b_2 \\ x_3 + b_3 \end{pmatrix}$$

Puncte fixe. $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_1 + b_1 \\ x_2 + b_2 \\ x_3 + b_3 \end{pmatrix} \Rightarrow \pi : x_1 = \frac{b_1}{2}$ (plan), $b_2 = 0$, $b_3 = 0$

① Dacă $b_2 = b_3 = 0 \Rightarrow \pi =$ plan de puncte fixe.

$\tau = S_\pi$ (simetrie ortogonală față de planul π)

$S_\pi : \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -x_1 + b_1 \\ x_2 \\ x_3 \end{pmatrix}$

② Dacă $b_2 \neq 0,\ b_3 \neq 0$ ($\nexists$ pcte fixe)

$\tau = T_w \circ S_\pi$, $w \in V_\pi$, $w \perp e_1$.

(glide reflection.

$S_\pi : \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -x_1 + b_1 \\ x_2 \\ x_3 \end{pmatrix}$; $T_w : \begin{pmatrix} x_1'' \\ x_2'' \\ x_3'' \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' + b_2 \\ x_3' + b_3 \end{pmatrix}$

③ $\tau = R_{d,\varphi} \circ S_\pi$, $V_\pi = \langle \{e_1\} \rangle^\perp$, $d \perp \pi$.

$d \cap \pi = 1$ pct fix. (rotosimetrie).

$S_\pi \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} -x_1 + b_1 \\ x_2 \\ x_3 \end{pmatrix}$; $R_{d,\varphi} : \begin{pmatrix} x_1'' \\ x_2'' \\ x_3'' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \\ b_3 \end{pmatrix}$

Concluzie

I (speța 1)
- T_u $\emptyset$, E_3 (pcte fixe)
- $R_{d,\varphi}$, $d =$ mulțime de pcte fixe.
  (Dacă $\varphi = \pi$, at $S_d =$ simetrie axială)
- $T_w \circ R_{d,\varphi}$ $\emptyset$ (m. p. fixe) ($w \in V_d$)
  (rototranslație)

II (specia 2) $\left\langle \begin{array}{l} S_\pi, \ \pi = \text{plan pcte fixe} \\ T_w \circ S_\pi, \ w \in V_\pi, \ w \perp e_1 \ (\not\exists \text{ pcte fixe}) \\ R_{d,\varphi} \circ S_\pi \quad 1 \text{ pct fix.} \end{array} \right.$

Hipercuadrice

Def $(\mathbb{R}^n, \mathbb{R}^n_{/\mathbb{R}}, \varphi)$ $(\mathcal{E}_n, (E_n, \langle \cdot, \cdot \rangle), \varphi)$

$\mathcal{R} = \{O; e_1, \dots, e_n\}$ reper cartezian.

S.n. hipercuadrică în $\mathbb{R}^n$ L.G al punctelor $P(x_1, \dots, x_n)$ a.î.

$\Gamma: f(x) = a_{11}x_1^2 + \dots + a_{nn}x_n^2 + 2a_{12}x_1x_2 + \dots + 2a_{n-1n}x_{n-1}x_n + 2b_1x_1 + \dots + 2b_nx_n + c = 0$

$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$

$\Gamma: X^TAX + 2BX + c = 0, \ A = A^T, \ rg A \geq 1.$

$\tilde{A} = \begin{pmatrix} A & B^T \\ B & c \end{pmatrix}$ $\quad r = rg A, \ r' = rg \tilde{A}$

$r \leq r' \leq r + 2.$

$\delta = \det A, \ \Delta = \det \tilde{A}.$

Dacă $\Delta = 0$, atunci Γ s.n. hipercuadrică degenerată.
Dacă $\Delta \neq 0$ — // — nedegenerată.

OBS a) $(\mathbb{R}^n, \mathbb{R}^n_{/\mathbb{R}}, \varphi)$ spațiu afin.

$\Gamma_1 \sim \Gamma_2$ afin echivalente $\Leftrightarrow \exists \tau: \mathbb{R}^n \to \mathbb{R}^n$ transformare afină a.î. $\Gamma_2 = \tau(\Gamma_1)$

$\tau: X' = CX + D, \ C \in GL(n, \mathbb{R})$; Invarianți afini: $\frac{\Delta}{\delta}, r, r'$.

b) $(\mathcal{E}_n, (E_n, \langle \cdot, \cdot \rangle), \varphi)$ spațiu punctual euclidian.

$\Gamma_1 \sim \Gamma_2$ congruente metric $\Leftrightarrow \exists \tau \in Izo(\mathcal{E}^n)$ a.î. $\Gamma_2 = \tau(\Gamma_1)$, $\tau: X' = CX + D, \ C \in O(n)$.

Invarianți metrici $\frac{\Delta}{\delta}, r, r', \Delta, \delta$.

c) $n=2 \Rightarrow \Gamma$ conică

$n=3 \Rightarrow \Gamma$ cuadrică.

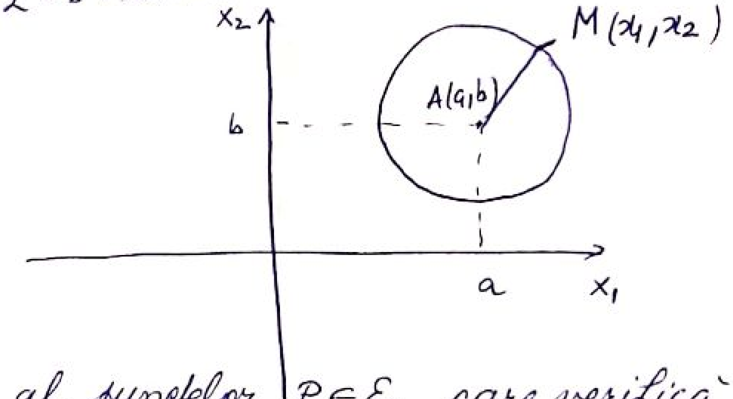
Conice ca locuri geometrice

① Cercul $\mathcal{C}(A(a,b),r)$ = LG al punctelor egal depărtate de punctul fix A.

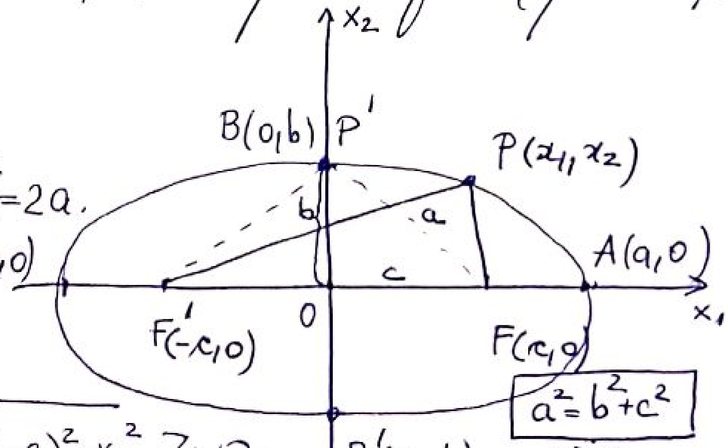
$$\mathcal{C}(A(a,b),r):\ (x_1-a)^2+(x_2-b)^2=r^2$$

$$\Leftrightarrow x_1^2+x_2^2-2ax_1-2bx_2=\underbrace{r^2-a^2-b^2}$$

Ec. parametrice $\begin{cases} x_1=a+r\cos t \\ x_2=b+r\sin t \end{cases},\ t\in[0,2\pi)$



② Elipsa este LG al punctelor $P\in\mathcal{E}_2$ care verifică $PF+PF'=2a$, $a>0$, F,F' = puncte fixe (focare).



$PF+PF'=2a$.

$$\sqrt{(x_1-c)^2+x_2^2}+\sqrt{(x_1+c)^2+x_2^2}=2a.$$

$$\sqrt{(x_1+c)^2+x_2^2}=2a-\sqrt{(x_1-c)^2+x_2^2}\geq 0$$

$$\underline{x_1^2}+2x_1c+\underline{\underline{c^2}}+\underset{\sim}{x_2^2}=4a^2+\underline{x_1^2}-2x_1c+\underline{\underline{c^2}}+\underset{\sim}{x_2^2}-4a\sqrt{(x_1-c)^2+x_2^2}$$

$$4a\sqrt{(x_1-c)^2+x_2^2}=4a^2-4x_1c \Rightarrow$$

$$a^2(x_1^2-\underline{2x_1c}+c^2+x_2^2)=a^4+x_1^2c^2-\underline{2a^2cx_1}$$

$$a^2x_1^2 + a^2x_2^2 = a^4 + x_1^2c^2 - a^2c^2$$

$$x_1^2(\underbrace{a^2-c^2}_{b^2}) + x_2^2a^2 = a^2(\underbrace{a^2-c^2}_{b^2}) \quad | \; a^2b^2$$

$a>0, b>0$.

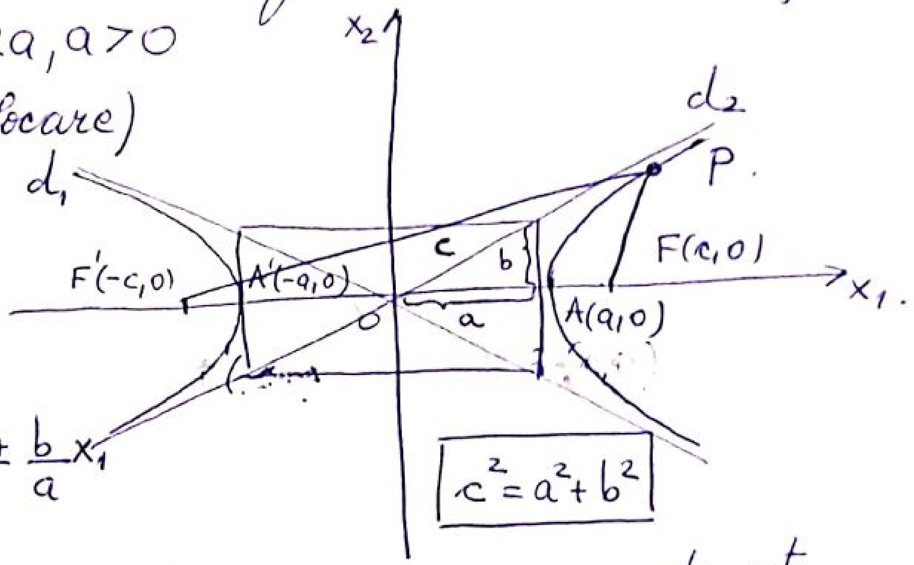
$$\boxed{\mathcal{E}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1}$$

OBS Ec. parametrice $\begin{cases} x_1 = a\cos t \\ x_2 = b\sin t, \; t\in[0,2\pi) \end{cases}$

③ Hiperbola este LG al punctelor $P \in \mathcal{E}_2$ care verifică

$|PF - PF'| = 2a, \; a>0$

F, F' pcte fixe (focare)



$$\mathcal{H}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1.$$

$d_1 \cup d_2$ (asimptotele) $\quad x_2 = \pm \frac{b}{a}x_1$

$$\boxed{c^2 = a^2 + b^2}$$

OBS Ec. parametrice: $\begin{cases} x_1 = a\,ch\,t \\ x_2 = b\,sh\,t \end{cases}$ $\quad ch\,t = \frac{e^t + e^{-t}}{2}$

$sh\,t = \frac{e^t - e^{-t}}{2}$

$ch^2t - sh^2t = 1$

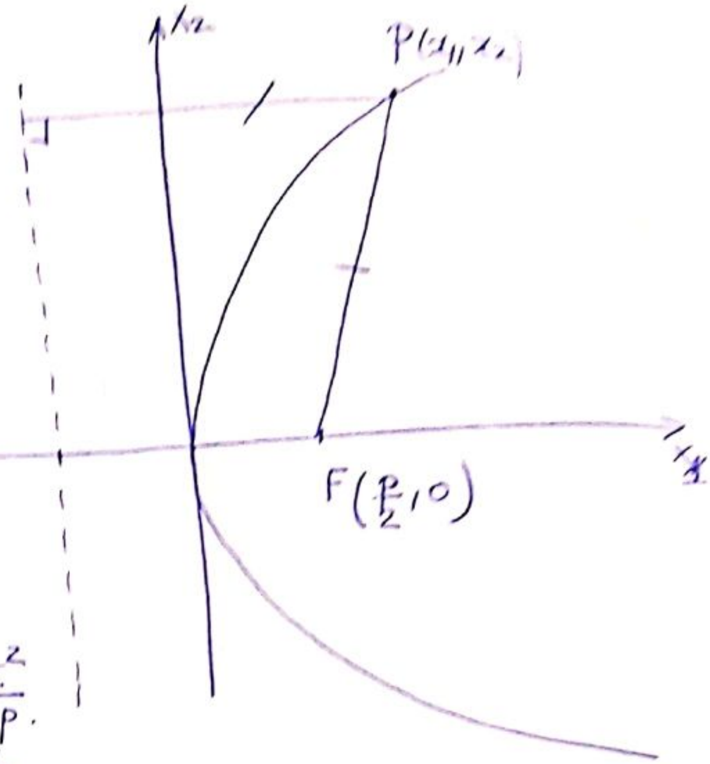
$t \in \mathbb{R}$.

④ Parabola = LG al punctelor $P \in \mathcal{E}_2$ care verifică

$\frac{dist(P,F)}{dist(P,d)} = 1$, $\quad F$ = pct fix (focar)

d = dreaptă fixă (directoare)

$F \notin d$.



$$\frac{dist(P,F)}{dist(P,d)} = 1$$

$$\sqrt{\left(x_1 - \frac{p}{2}\right)^2 + x_2^2} = \left|x_1 + \frac{p}{2}\right|$$

$$x_1^2 + \frac{p^2}{4} - x_1 p + x_2^2 = x_1^2 + \frac{p^2}{4} + x_1 p$$

$$\boxed{\mathcal{P}: \ x_2^2 = 2px_1} \ , \ p > 0$$

OBS. Ec. parametrice: $\begin{cases} x_1 = \frac{t^2}{2p} \\ x_2 = t \end{cases}$

Teoremă (definirea unitară a conicelor nedegenerate)

LG al punctelor $P \in \mathcal{E}_2$ care verifică

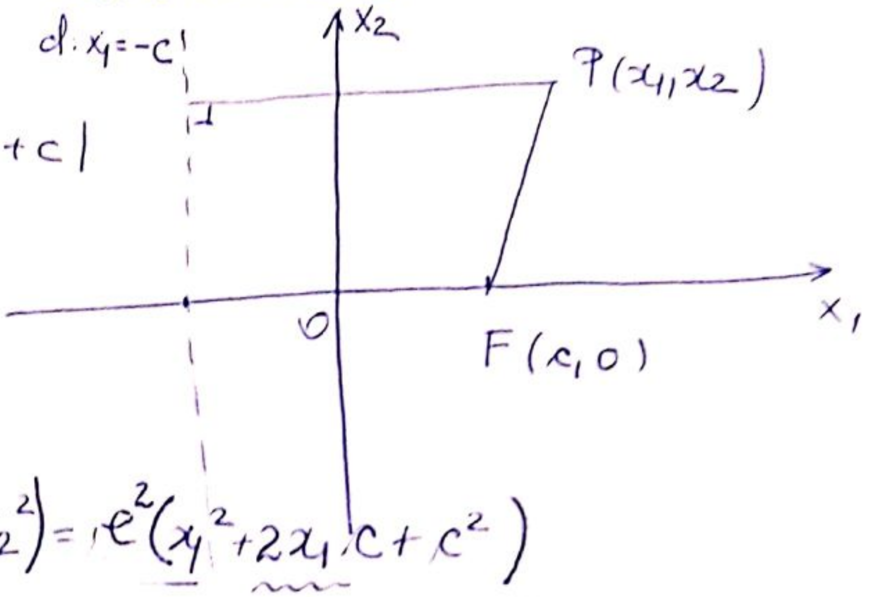
$\frac{dist(P, F)}{dist(P, d)} = e$, reprezintă o conică nedegenerată,

unde F = pct fix (focar)

d = dreapta fixă (directoare), $F \notin d$

e s.n. excentricitate.

Dem.



$$\sqrt{(x_1 - c)^2 + x_2^2} = e\,|x_1 + c|$$

$$(x_1^2 + c^2 - 2x_1 c + x_2^2) = e^2(x_1^2 + 2x_1 c + c^2)$$

OBS Conica $\Gamma: f(x_1,x_2) = \underline{a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2} +$
$+ \underline{2b_1x_1 + 2b_2x_2} + c = 0$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = A^T, \quad \tilde{A} = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$$

$\delta = \det A, \ \Delta = \det \tilde{A}$

$\Gamma: \underline{x_1^2(1-e^2) + x_2^2} - \underline{2x_1 c(1+e^2) + c^2(1-e^2)} = 0$

$$A = \begin{pmatrix} 1-e^2 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 1-e^2 & 0 & -c(1+e^2) \\ 0 & 1 & 0 \\ -c(1+e^2) & 0 & c^2(1-e^2) \end{pmatrix}$$

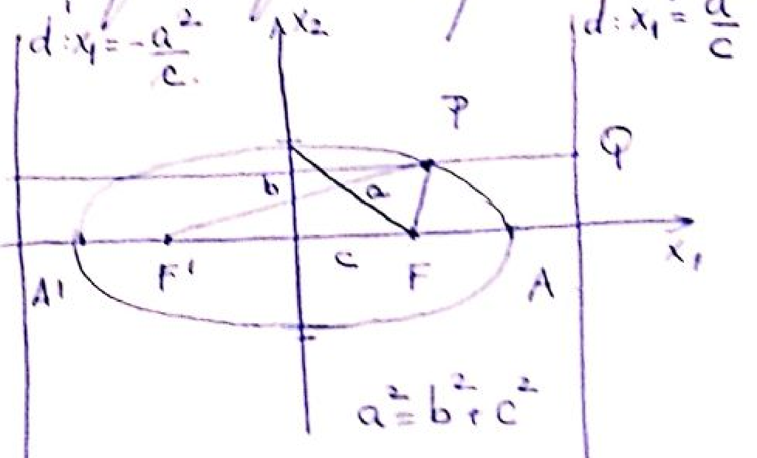
Γ nedegenerată $\Leftrightarrow \Delta = \det \tilde{A} \neq 0$

$$\Delta = \begin{vmatrix} 1-e^2 & -c(1+e^2) \\ -c(1+e^2) & c^2(1-e^2) \end{vmatrix} = \left[c^2(1-e^2)^2 - c^2(1+e^2)^2\right] = c^2(-4e^2) = -4c^2e^2 \neq 0$$

$F \notin d \Rightarrow e \neq 0$. $\Rightarrow \Gamma$ nedegenerată
$c \neq 0$.
(elipsă, hiperbolă, parabolă)

OBS

① $\mathcal{E}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$



$a^2 = b^2 + c^2$

$e = \frac{c}{a} < 1$
(excentricitatea)

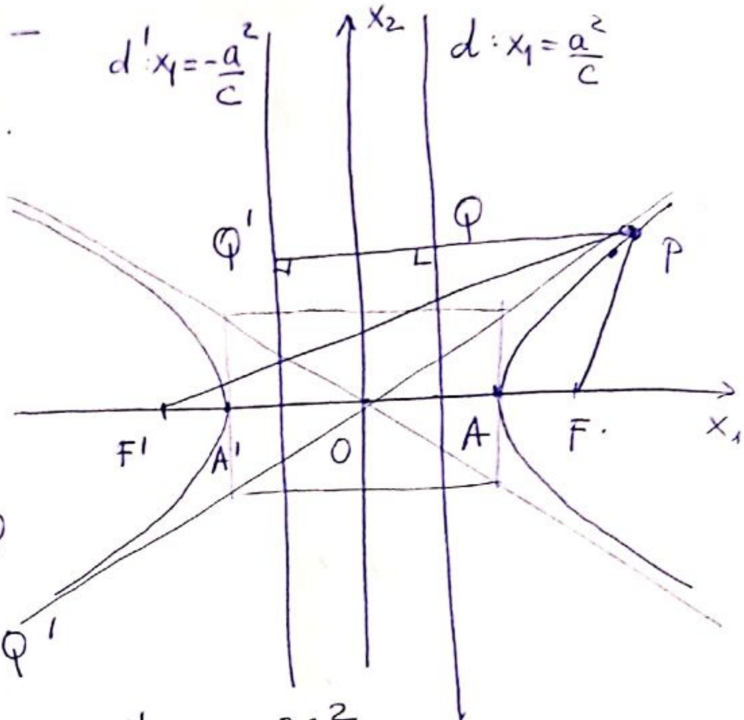
$d \cup d': x_1 = \pm \frac{a^2}{c}$
directoarele.

$\frac{a^2}{c} > a \Leftrightarrow a > c$ Ⓐ $\frac{PQ}{PF} = \frac{a}{c} \Rightarrow PF = \frac{c}{a} PQ$

$\frac{PQ'}{PF'} = \frac{a}{c} \Rightarrow PF' = \frac{c}{a} PQ'$

$\Rightarrow PF + PF' = \frac{c}{a} \cdot \frac{2a^2}{c} = 2a$;

② $\mathcal{H}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1.$



$c^2 = a^2 + b^2$

$e = \frac{c}{a} > 1.$

$\frac{a^2}{c} < a \Leftrightarrow a < c$

$\frac{PQ}{PF} = \frac{a}{c} \Rightarrow PF = \frac{c}{a} PQ$

$\frac{PQ'}{PF'} = \frac{a}{c} \Rightarrow PF' = \frac{c}{a} PQ'$

$$|PF - PF'| = \frac{c}{a} |PQ - PQ'| = \frac{c}{a} \cdot \frac{2a^2}{c} = 2a.$$

③ $\mathcal{P}: x_1^2 = 2p x_1.$

$e = 1.$

Aducerea la forma canonică a conicelor cu centru unic

$$\Gamma: X^T A X + 2BX + c = 0.$$

$$\Gamma: a_{11} x_1^2 + a_{22} x_2^2 + 2a_{12} x_1 x_2 + 2b_1 x_1 + 2b_2 x_2 + c = 0$$

Def P_0 s.n. centru pentru $\Gamma \Leftrightarrow [\forall P \in \Gamma \Leftrightarrow \mathcal{S}_{P_0}(P) \in \Gamma]$

$$P_0: \begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases} (*) \Rightarrow \begin{cases} 2a_{11} x_1 + 2a_{12} x_2 + 2b_1 = 0 \\ 2a_{12} x_1 + 2a_{22} x_2 + 2b_2 = 0 \end{cases}$$

$$AX + B^T = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow X^T A + B = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$\delta = \det A \neq 0 \Rightarrow (*)$ are sol unică.

Prop Dacă $\delta \neq 0$, atunci $f(x_1^0, x_2^0) = \frac{\Delta}{\delta}$, unde $P_0(x_1^0, x_2^0)$ este centrul conicei Γ.

Dem

$$f(x_1^0, x_2^0) = X_0^T A X_0 + 2BX_0 + C =$$
$$= \underbrace{(X_0^T A + B)} X_0 + BX_0 + C = BX_0 + c = \frac{\Delta}{\delta} \text{(dem)}.$$

$$b_1 x_1^0 + b_2 x_2^0 + c = \frac{\Delta}{\delta} \text{ (dem)}$$

$$x_1^0 = \frac{\begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix}}{\delta}, \quad x_2^0 = \frac{\begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix}}{\delta} \quad (1)$$

$$\frac{\Delta}{\delta} = \frac{1}{\delta} \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{vmatrix} =$$

$$= \frac{1}{\delta} \cdot b_1 \begin{vmatrix} a_{12} & a_{22} \\ b_1 & b_2 \end{vmatrix} - \frac{b_2}{\delta} \begin{vmatrix} a_{11} & a_{12} \\ b_1 & b_2 \end{vmatrix} + \frac{c}{\delta}\delta \quad (2)$$

$$b_1 x_1^0 + b_2 x_2^0 + c \overset{(1)}{=} \frac{b_1}{\delta} \begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix} + \frac{b_2}{\delta} \begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix} + c \quad (3)$$

$$\overset{(2),(3)}{\Rightarrow} f(x_1^0, x_2^0) = \frac{\Delta}{\delta}.$$

I) $\delta \neq 0$

- Dacă $(\mathbb{R}^2, \mathbb{R}^2_{/\mathbb{R}}, \varphi)$ sp. afin.

$$\mathcal{R} = \{O; e_1, e_2\} \underset{\text{translatie}}{\xrightarrow{\theta}} \mathcal{R}' = \{P_0; e_1, e_2\} \underset{\text{transf. afină}}{\xrightarrow{\tau}} \mathcal{R}'' = \{P_0; e_1', e_2'\}.$$

$$\theta: X = X' + X_0.$$

$$\theta(\Gamma): (X' + X_0)^T A (X' + X_0) + 2B(X' + X_0) + c = 0.$$

$$\underline{X'^T A X'} + X_0^T A X' + X'^T A X_0 + \underline{\underline{X_0^T A X_0}} + 2BX' + \underline{\underline{2BX_0}} + \underline{\underline{C}} = 0$$

$$\theta(\Gamma): X'^T A X' + \frac{\Delta}{\delta} = 0$$

$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = X'^T A X'$ formă pătratică

Aducem Q la formă canonică (met Gauss)

$Q(x) = \lambda_1 x_1''^2 + \lambda_2 x_2''^2$

$\tau : X' = C X''$, $C \in GL(2, \mathbb{R})$

$\tau \circ \theta(\Gamma) : \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \dfrac{\Delta}{\delta} = 0$.

$\overset{\shortparallel}{\Gamma'}$ $\quad\Gamma, \Gamma'$ conice afin echivalente

- $(\mathcal{E}_2, (E_2, \langle \cdot, \cdot \rangle), \varphi)$ sp. afin euclidian.

$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = X'^T A X'$

$\exists$ un reper ortonormat format din vectori proprii ai A = diagonală.

a) $P(\lambda) = \det(A - \lambda I_2) = 0$

$\lambda^2 - Tr(A)\lambda + \det(A) = 0$.

$\lambda_1 \neq \lambda_2$, $m_1 = m_2 = 1$

$V_{\lambda_i} = \langle \{e_i'\} \rangle \quad i = \overline{1,2} \qquad \langle e_i', e_j' \rangle = \delta_{ij}$

$e_1' = (l_1, m_1)$, $e_2' = (l_2, m_2)$ $\quad \forall i, j = \overline{1,2}$

$R = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$ ($\det R = 1$., reper pozitiv orientat)

$\tau : X' = R X''$ izometrie (de speța 1).

~~$\theta \circ \tau$~~ $\tau \circ \theta(\Gamma) : \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \dfrac{\Delta}{\delta} = 0$

$\overset{\shortparallel}{\Gamma'}$

Γ, Γ' conice congruente metric.

$X = X' + X_0$, $X' = R X'' \implies \tau \circ \theta : X = R X'' + X_0$.

b) $\lambda_1 = \lambda_2$, $m_1 = 2$

$V_{\lambda_1} = \langle \{ f_1, f_2 \} \rangle$ Aplicăm Gram-Schmidt

$\Rightarrow \{ e_1', e_2' \}$ reper ortonormat.