27.05.2021

SEMINAR 14

Aducerea la formă canonică a conicelor
Recapitulare

EX1 Fie $(\mathcal{E}_2, (E_2, \langle\cdot,\cdot\rangle), \varphi)$ sp. punctual euclidian

Fie conica:

$$\Gamma: f(x) = \underline{5x_1^2 + 8x_1x_2 + 5x_2^2} - \underline{18x_1 - 18x_2} + 9 = 0$$

Să se aducă la o formă canonică, utilizând izometrii.

SOL

$$A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{pmatrix}$$

$\delta = \det A = 25 - 16 = 9 \neq 0$ ($\exists!$ centrul conicei)

$$\Delta = \det\tilde{A} = \begin{vmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ 0 & 0 & -9 \end{vmatrix} = -9 \cdot 9 \neq 0$$

(conică nedegenerată)

Determinăm centrul

$$\begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases} \Rightarrow \begin{cases} 10x_1 + 8x_2 - 18 = 0 \\ 8x_1 + 10x_2 - 18 = 0 \end{cases} \Rightarrow \begin{cases} 5x_1 + 4x_2 = 9 & \cdot(-5) \\ 4x_1 + 5x_2 = 9 & \cdot 4 \end{cases} \oplus$$

$$-9x_1 \quad / \quad -9$$

$x_1 = 1 \Rightarrow x_2 = 1 \Rightarrow P_0(1,1)$ centrul unic.

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translație}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{rotație}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}$$

1) $\theta: X = X' + X_0, \quad X_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2) $\tau: X' = RX'', \quad R \in SO(2)$

$\theta(\Gamma)$: $X'^T A X' + \frac{\Delta}{\delta} = 0 \Rightarrow \underline{5x_1'^2 + 8x_1'x_2' + 5x_2'^2} + (-9) = 0$

Fie $Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = 5x_1'^2 + 8x_1'x_2' + 5x_2'^2$

Aducem la o formă canonică, utilizând met. valorilor proprii!

$P(\lambda) = \det(A - \lambda I_2) = 0 \Rightarrow \lambda^2 - Tr(A)\lambda + \det A = 0$

$\lambda^2 - 10\lambda + 9 = 0 \Rightarrow (\lambda - 9)(\lambda - 1) = 0$

$\lambda_1 = 9,\ m_1 = 1$

$\lambda_2 = 1,\ m_2 = 1$ $\qquad \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}$

- $V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = 9X\} = \{(x_1, x_1) = x_1(\underline{\underline{1,1}}),\ x_1 \in \mathbb{R}\}$

$(A - 9I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -4 & 4 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = x_2$

$e_1' = \frac{1}{\sqrt{2}}(1,1)$ versor propriu coresp. val. proprii $\lambda_1 = 9$

- $V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid AX = X\} = \{(-x_2, x_2),\ x_2 \in \mathbb{R}\}$

$(A - I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = -x_2$

$e_2' = \frac{1}{\sqrt{2}}(-1,1)$ versor propriu coresp. val pr $\lambda_2 = 1$

$$R = \begin{array}{cc} e_1' & e_2' \end{array} \atop \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \in SO(2)$$

$\varphi = \frac{\pi}{4}$

$\tau \circ \theta: X \longrightarrow X' + X_0 \longrightarrow RX'' + X_0.$

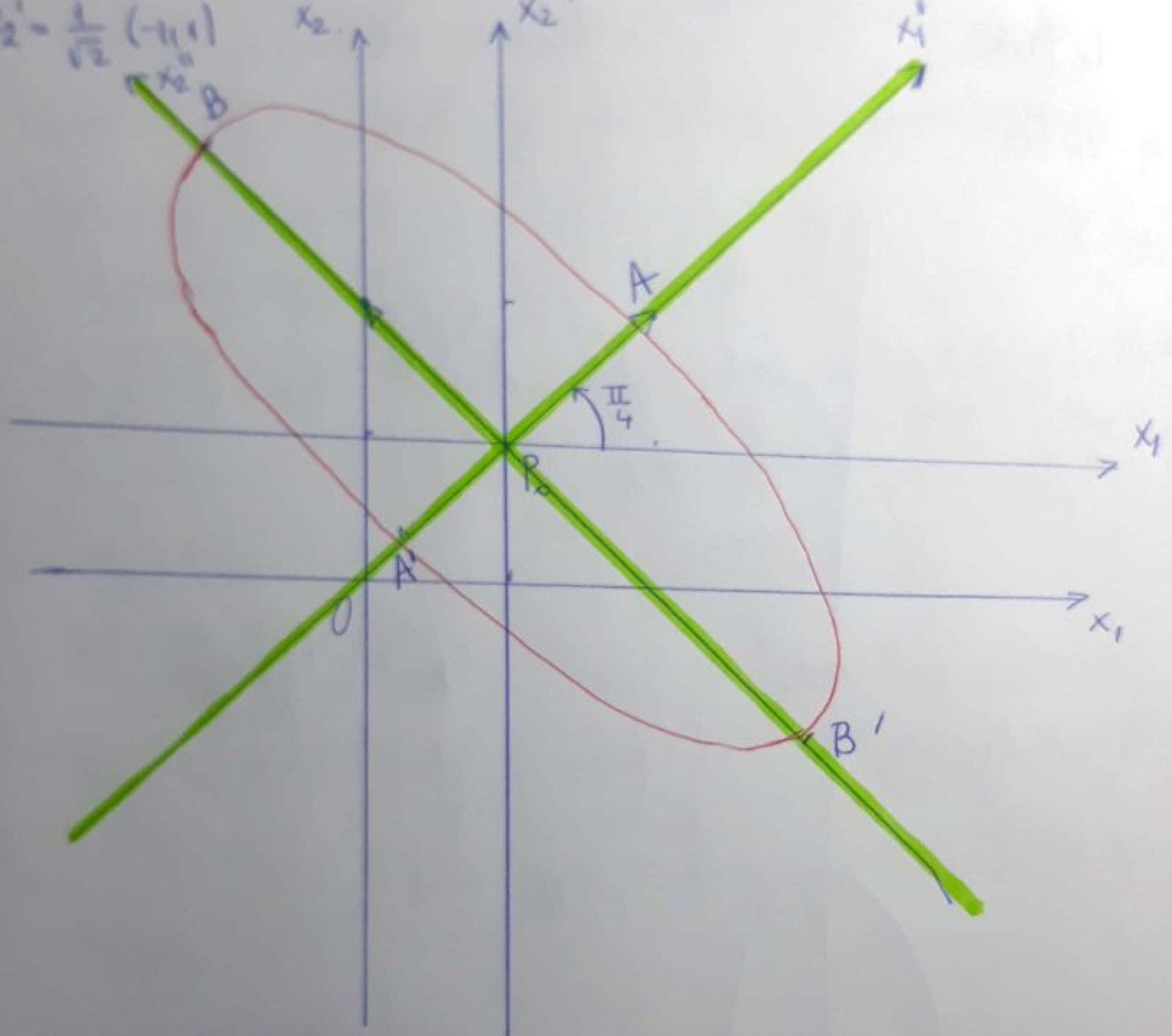
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$T\circ\Theta(\Gamma)$: $9x_1''^2 + x_2''^2 - 9 = 0$ $\qquad$ $\mathcal{E}: \dfrac{x_1''^2}{a^2} + \dfrac{x_2''^2}{b^2} = 1$

$$\frac{x_1''^2}{1} + \frac{x_2''^2}{9} = 1 \qquad a = 1,\ b = 3$$

$e_1' = \frac{1}{\sqrt{2}}(1,1)$

$e_2' = \frac{1}{\sqrt{2}}(-1,1)$



<u>Ex2</u>. În planul punctual euclidian $\mathcal{E}_2$ se consideră

Γ: $f(x) = \underline{3x_1^2 - 6x_1x_2 + 3x_2^2} + \underline{2x_1 + 2x_2} - 2 = 0$

Să se aducă la o formă canonică, efectuând izometrii.

<u>SOL</u>

$$A = \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} \boxed{\begin{matrix} 3 & -3 \\ -3 & 3 \end{matrix}} & \begin{matrix} 1 \\ 1 \end{matrix} \\ \begin{matrix} 1 & 1 \end{matrix} & -2 \end{pmatrix}$$

$\delta = \det A = 0$ ($\nexists$ centru unic)

$$\Delta = \det \tilde{A} = \begin{vmatrix} 3 & -3 & 1 \\ -6 & 6 & 0 \\ 7 & -5 & 0 \end{vmatrix} = 6 \begin{vmatrix} -1 & 1 \\ 7 & -5 \end{vmatrix} = -12 \neq 0$$

(conică nedegenerată)

$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{rotatie}]{\theta} \mathcal{R}' = \{O; e_1', e_2'\} \xrightarrow[\text{translatie}]{\tau} \mathcal{R}'' = \{P; e_1', e_2'\}$

$Q: \mathbb{R}^2 \to \mathbb{R}, \quad Q(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2$

Metoda valorilor proprii:

$\lambda^2 - Tr(A)\lambda + \det A = 0 \Rightarrow \lambda^2 - 6\lambda = 0 \Rightarrow (\lambda - 6)\lambda = 0$

$\lambda_1 = 6, \quad m_1 = 1$

$\lambda_2 = 0, \quad m_2 = 1$

$\begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix}$

• $V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = 6X\} = \{(x_1, -x_1) = x_1\underline{(1,-1)}, x_1 \in \mathbb{R}\}$

$(A - 6I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_2 = -x_1$

$e_1' = \frac{1}{\sqrt{2}}(1, -1)$

• $V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid AX = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\} = \{(x_1, x_1) = x_1(1,1), x_1 \in \mathbb{R}\}$

$\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = x_2$

$e_2' = \frac{1}{\sqrt{2}}(1, 1)$

$\theta: X = RX', \quad R = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \in SO(2)$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{1}{\sqrt{2}}(x_1' + x_2') \\ x_2 = \frac{1}{\sqrt{2}}(-x_1' + x_2') \end{cases}$

$\theta(\Gamma): 6x_1'^2 + \frac{2}{\sqrt{2}}(\cancel{x_1'} + x_2') + \frac{2}{\sqrt{2}}(\cancel{-x_1'} + x_2') - 2 = 0$

$6x_1'^2 + \frac{4}{\sqrt{2}}x_2' - 2 = 0 \Rightarrow 3x_1'^2 + \underline{\sqrt{2}x_2' - 1 = 0}$

Scanned with CamScanner

$$\theta(\Gamma):\ 3x_1'^2+\sqrt{2}\left(x_2'-\frac{1}{\sqrt{2}}\right)=0$$

$$\begin{cases} x_1''=x_1' \\ x_2''=x_2'-\frac{1}{\sqrt{2}} \end{cases} \Rightarrow \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}=\begin{pmatrix} x_1'' \\ x_2''+\frac{1}{\sqrt{2}} \end{pmatrix}=\begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix}+\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\tau: X'=X''+X_0\ ,\quad X_0=\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \text{ translatie}$$

$$\tau\circ\theta: X\longrightarrow RX'\longrightarrow R(X''+X_0)=RX''+RX_0$$

$$RX_0=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

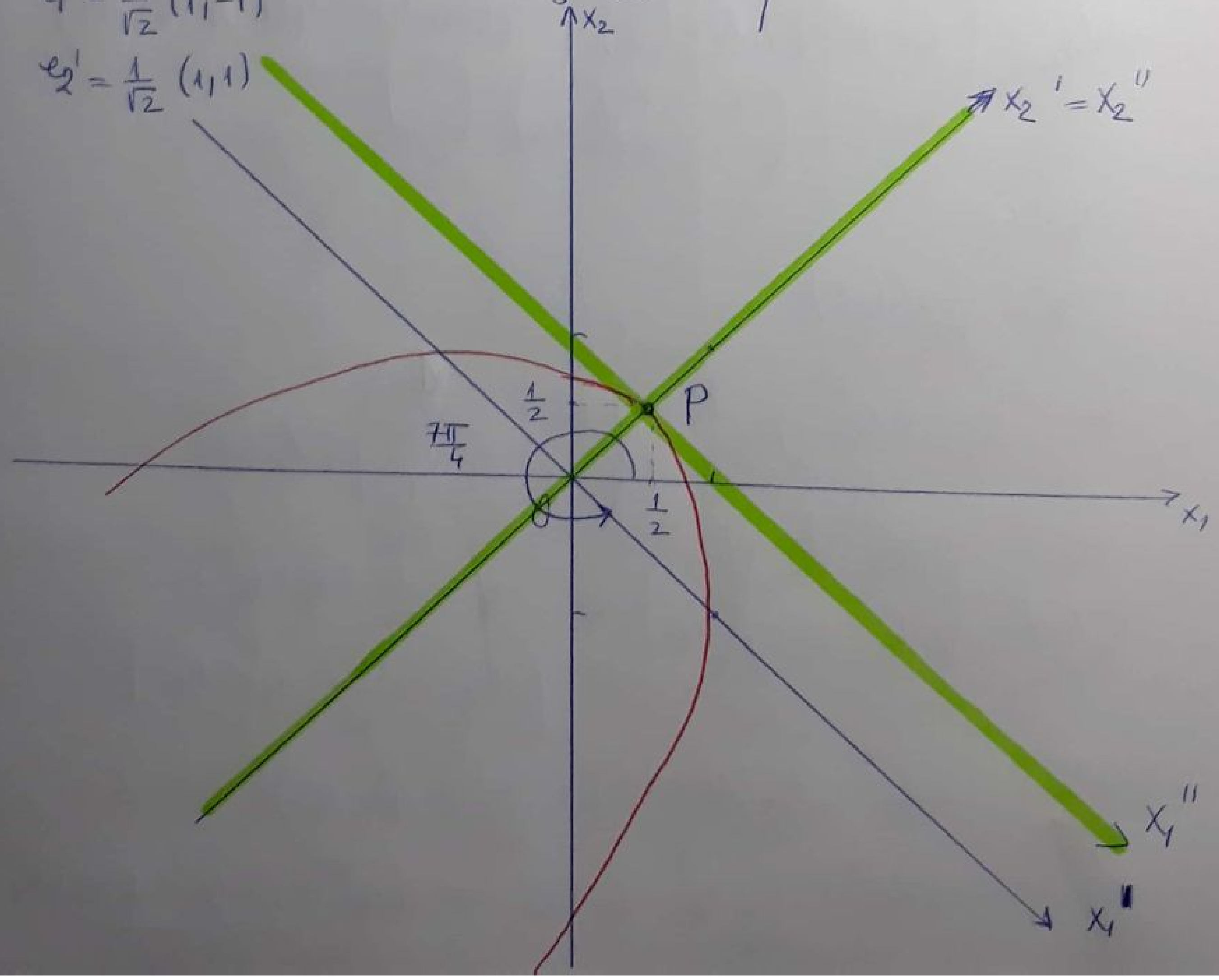
$$=\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \Rightarrow P\left(\frac{1}{2},\frac{1}{2}\right) \text{ (în rap cu } \mathcal{R}\text{)}$$

$$\tau\circ\theta(\Gamma):\ 3x_1''^2+\sqrt{2}x_2''=0 \Rightarrow$$

$$x_1''^2=-\frac{\sqrt{2}}{3}x_2'' \quad \text{(parabolă)}$$

$e_1'=\frac{1}{\sqrt{2}}(1,-1)$

$e_2'=\frac{1}{\sqrt{2}}(1,1)$



Recapitulare

Ex1 $(\mathcal{M}_2(\mathbb{R}), g)$, $g(A,B) = \sum_{i,j=1}^{2} a_{ij} b_{ij}$

$U = \langle \left\{ \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \rangle$.

a) $U^\perp$; b) $p\left(\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\right) = ?$, unde p = proiecția ortogonală pe $U^\perp$

SOL

a) $\mathcal{M}_2(\mathbb{R}) \simeq \mathbb{R}^4$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow (a,b,c,d)$

$U = \langle \{ \underbrace{(0,0,1,1)}_{f_1}, \underbrace{(0,1,0,1)}_{f_2}, \underbrace{(1,0,0,1)}_{f_3} \} \rangle$

$rg \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 3 \Rightarrow \{f_1, f_2, f_3\}$ SLI

$\dim U = 3$

$\mathbb{R}^4 = U \oplus U^\perp$, $\dim U^\perp = 1$.

$U^\perp = \left\{ x \in \mathbb{R}^4 \;\middle|\; \begin{cases} g(x, f_1) = 0 \\ g(x, f_2) = 0 \\ g(x, f_3) = 0 \end{cases} \right\} = \left\{ x \in \mathbb{R}^4 \;\middle|\; \begin{cases} x_3 + x_4 = 0 \\ x_2 + x_4 = 0 \\ x_1 + x_4 = 0 \end{cases} \right\}$

$= \{ (-x_4, -x_4, -x_4, x_4) = x_4(-1,-1,-1,1),\ x_4 \in \mathbb{R} \}$

$U^\perp = \langle \{ (-1,-1,-1,1) \} \rangle$; $\langle \left\{ \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \right\} \rangle = U^\perp$

b) $\underset{\mathbb{R}^4}{\underset{\cap}{x}} = \underset{U}{\underset{\cap}{x'}} + \underset{U^\perp}{\underset{\cap}{x''}}$ $\quad p$ = proiecția ortog pe $U^\perp$

$p(x) = x''$

$(1,2,0,1) = \overbrace{a(0,0,1,1) + b(0,1,0,1) + c(1,0,0,1)}^{x'} + \overbrace{\underline{\underline{d}}(-1,-1,-1,1)}^{x''}$

$= (c-d, b-d, a-d, a+b+c+d)$

$$\begin{cases} c-d=1 \\ b-d=2 \\ a-d=0 \\ a+b+c+d=1 \end{cases} \Rightarrow \begin{cases} c=d+1 \\ b=d+2 \\ a=d \\ d+d+\cancel{1}+d+2+d=\cancel{1} \Rightarrow 4d=-2 \end{cases}$$

$$d=-\frac{1}{2}$$

$$p((1,2,0,1))=-\frac{1}{2}(-1,-1,-1,1)=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)$$

$$p\left(\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\right)=\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

<u>Ex</u> $f:\mathbb{R}^2 \to \mathbb{R}^2$, $f(x)=(x_1+x_2, 3x_1+x_2)$

$R'=\{e_1'=e_1+2e_2, e_2'=e_1+e_2\}$, $R_0=\{e_1,e_2\}$ reperul canonic

$[f]_{R',R'}=?$

<u>SOL</u>

(M1) $\begin{cases} f(e_1')=ae_1'+be_2' \\ f(e_2')=ce_1'+de_2' \end{cases}$ $A'=[f]_{R',R'}=\begin{pmatrix} a & c \\ b & d \end{pmatrix}=\begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix}$

$f(e_1')=f(e_1+2e_2)=f(1,2)=(3,5)=a(1,2)+b(1,1)=(a+b,2a+b)$

$\begin{cases} a+b=3 \\ 2a+b=5 \end{cases}$ $\quad a=2$, $b=1$

$a \;/=2$

$f(e_2')=f(e_1+e_2)=f(1,1)=(2,4)=c(1,2)+d(1,1)=(c+d,2c+d)$

$\begin{cases} c+d=2 \\ 2c+d=4 \end{cases}$ $\quad \begin{cases} c=2 \\ d=0 \end{cases}$

$c \;/=2$

(M2) $R_0=\{e_1,e_2\} \xrightarrow{C} R'=\{e_1'=e_1+2e_2, e_2'=e_1+e_2\}$

$[f]_{R_0,R_0}=A$, $[f]_{R',R'}=A'$ $\quad C=\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

$A'=C^{-1}AC$ $\quad f(x)=y \Leftrightarrow Y=\underbrace{\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}}_{A}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Ex $(\mathbb{R}^3, g_0)$, $U = \{x \in \mathbb{R}^3 \mid 2x_1 + x_2 - x_3 = 0\}$

a) $U^\perp$; b) $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ reper orton. în $\mathbb{R}^3$, unde

$\mathcal{R}_1$ — " — U,

$\mathcal{R}_2$ " — $U^\perp$

c) s_1, s_2 = simetrii ortogonale față de U, și $U^\perp$,

$\Rightarrow s_1 + s_2$

Sol

a) $U = \{x \in \mathbb{R}^3 \mid g_0((x_1, x_2, x_3), (2, 1, -1)) = 0\}$

$U^\perp = \langle\{(2, 1, -1)\}\rangle$

b) $U = \{x \in \mathbb{R}^3 \mid x_3 = 2x_1 + x_2\} = \{(x_1, x_2, 2x_1 + x_2) \mid x_1, x_2 \in \mathbb{R}\}$

$x_1 \underbrace{(1, 0, 2)}_{f_1} + x_2 \underbrace{(0, 1, 1)}_{f_2}$.

$\dim U = 3 - 1 = 2$

$\{f_1, f_2\}$ reper în U.

OBS a) $U^\perp = \left\{x \in \mathbb{R}^3 \,\middle|\, \begin{cases} g_0(x, f_1) = 0 \\ g_0(x, f_2) = 0 \end{cases}\right\} \Rightarrow \left\{x \in \mathbb{R}^3 \,\middle|\, \begin{cases} x_1 + 2x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}\right\}$

b) $U^\perp = \langle\{f_1 \times f_2\}\rangle$

$$f_1 \times f_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = (-2, -1, 1) = -1(2, 1, -1)$$

Aplicăm Gram-Schmidt pt reperul $\{f_1, f_2\}$.

$$\begin{cases} e_1 = f_1 = (1, 0, 2) \\ e_2 = f_2 - \dfrac{\langle f_2, e_1\rangle}{\langle e_1, e_1\rangle} e_1 = (0, 1, 1) - \dfrac{2}{5}(1, 0, 2) = \left(-\dfrac{2}{5}, 1, \dfrac{1}{5}\right) = \\ \quad = \dfrac{1}{5}(-2, 5, 1) \end{cases}$$

$\{e_1, e_2\}$ reper ortogonal în U

$\mathcal{R}_1 = \left\{e_1' = \dfrac{1}{\sqrt{5}}(1, 0, 2),\ e_2' = \dfrac{1}{\sqrt{30}}(-2, 5, 1)\right\}$ reper ortonormat în U

OBS

$u = \alpha v, \ \alpha > 0.$

$$\frac{u}{\|u\|} = \frac{\alpha v}{\|\alpha v\|} = \frac{\alpha v}{|\alpha|\,\|v\|} = \frac{v}{\|v\|}$$

$\mathcal{R}_2 = \left\{ \frac{1}{\sqrt{6}}(2,1,-1) \right\}$ reper orton în $U^\perp$

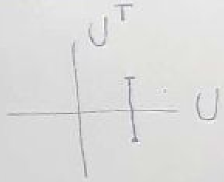
$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ reper orton. în $\mathbb{R}^3 = U \oplus U^\perp$; $\underset{\mathbb{R}^3}{x} = \underset{U}{x'} + \underset{U^\perp}{x''}$

c) • p_1 proiecție ortog pe U

$$p_1(x) = p_1(\underset{U}{x'} + \underset{U^\perp}{x''}) = x'$$

s_1 = simetria ortog față de U : $s_1 = 2p_1 - id_{R^3}$

$$s_1(x) = 2p_1(x) - x = 2x' - (x' + x'') = x' - x''$$



• p_2 proiecție ortog pe $U^\perp$

$$p_2(x) = p_2(x' + x'') = x''$$

s_2 = simetria ortog față de $U^\perp$: $s_2 = 2p_2 - id_{R^3}$

$$s_2(x) = 2p_2(x) - x = 2x'' - (x' + x'') = x'' - x'$$

$$(s_1 + s_2)(x) = 0.$$

Ex $f \in End(\mathbb{R}^2)$, $u_1 = (1,2)$, $u_2 = (1,1)$ vect pr. coresp val pr $\lambda_1 = 1$, $\lambda_2 = 3$

$\Rightarrow f(4,5) = ?$

SOL

$f(u_1) = \lambda_1 u_1 = (1,2)$

$f(u_2) = \lambda_2 u_2 = 3(1,1) = (3,3)$

$\{u_1, u_2\}$ reper în $\mathbb{R}^2$

$(4,5) = a u_1 + b u_2 = a(1,2) + b(1,1) = (a+b, 2a+b)$

$\begin{cases} a+b = 4 \\ 2a+b = 5 \end{cases}$ $\quad a = 1$, $b = 3$

$$f(4,5) = f(u_1 + 3u_2) = f(u_1) + 3f(u_2) =$$
$$= (1,2) + 3(3,3) = (10,11)$$

OBS. $R_0 = \{e_1, e_2\} \xrightarrow{C} R' = \{u_1 = e_1 + 2e_2,\ u_2 = e_1 + e_2\}$

$$C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$A' = C^{-1}AC, \quad A' = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$A = \dots \Rightarrow f(x) = \dots \Rightarrow f(4,5)$$

OBS. 1) $f \in End(\mathbb{R}^3)$, $(\mathbb{R}^3, g_0)$, $u = (1,1,1)$.

$$f(x) = g_0(x,u)\,u = (x_1+x_2+x_3)(1,1,1) =$$
$$= (x_1+x_2+x_3,\ x_1+x_2+x_3,\ x_1+x_2+x_3)$$

2) $f \in End(\mathbb{R}^3)$, $(\mathbb{R}^3, g_0)$, $u = (-1,0,1)$

$$f(x) = u \times x = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & 0 & 1 \\ x_1 & x_2 & x_3 \end{vmatrix} =$$

$$f(x) = (-x_2,\ x_1 + x_3,\ -x_2) \Rightarrow [f]_{R_0 R_0} = A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

Se poate diagonaliza?

$$\det(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 1 \\ 0 & -1 & -\lambda \end{vmatrix} = 0 \quad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$-\lambda(\lambda^2+1) - \lambda = 0 \Rightarrow -\lambda(\lambda^2+2) = 0, \quad \lambda_2, \lambda_3 \in \mathbb{C} \setminus \mathbb{R}$$

Nu se poate diag