# Lucrare II (141)

① $f \in End(\mathbb{R}^2)$, $A = [f]_{R_0,R_0} = \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}$, $R_0$ = reperul canonic.

a) f se poate diagonaliza; b) f nu se poate diagonaliza;
c) valorile proprii sunt egale; d) pol. caract. are răd $\in \mathbb{C} \setminus \mathbb{R}$

② $(\mathbb{R}^3, g_0)$. Fie reperul $R = \{f_1 = (1,-1,1), f_2 = (0,1,0), f_3 = (1,1,0)\}$
Reperul ortonormat obtinut cu Gram-Schmidt este

a) $\{\frac{1}{\sqrt{3}}(1,-1,1), \frac{1}{\sqrt{6}}(1,2,1), \frac{1}{\sqrt{2}}(1,0,-1)\}$; b) $\{\frac{1}{\sqrt{2}}(1,0,0), \frac{1}{\sqrt{2}}(1,-1,1), \frac{1}{\sqrt{3}}(1,1,1)\}$;
c) $\{\frac{1}{\sqrt{6}}(2,1,-1), \frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{3}}(1,1,1)\}$; d) $\{\frac{1}{\sqrt{2}}(1,-1,0), \frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{3}}(-1,-1,1)\}$.

③ $(\mathbb{R}^3, g_0)$, $u = (1,0,1)$
$s \in End(\mathbb{R}^3)$ simetria ortogonală față de $\langle\{u\}\rangle^{\perp}$

a) $s(x) = (x_1, -x_3, -x_2)$, b) $s(x) = (-x_3, x_2, -x_1)$
c) $s(x) = (x_1, x_2, -x_3)$, d) $s(x) = (-x_1, x_2, -x_3)$.

④ $(\mathbb{R}^3, g_0)$, $u = (1,1,-1)$. Complementul ortogonal $\langle\{u\}\rangle^{\perp}$ este

a) $\{x \in \mathbb{R}^3 \mid x_1 - x_2 - x_3 = 0\}$; b) $\{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 0\}$;
c) $\{x \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0\}$; d) $\{x \in \mathbb{R} \mid x_1 + x_2 = 0\}$.

⑤ $Q: \mathbb{R}^3 \to \mathbb{R}$ formă pătratică, $A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ matricea asociată. Signatura lui Q este:

a) (3,0); b) (1,2); c) (1,1); d) (2,1)

⑥ $(\mathbb{R}^3, g_0)$, $u = (1,-1,2)$, $f \in End(\mathbb{R}^3)$, $f(x) = \langle x, u\rangle u$
$g_0 = \langle\,;\,\rangle$ produs scalar canonic

a) $\dim Ker f = 2$, b) $\dim Ker f = 1$; c) $f \in Aut(\mathbb{R}^3)$, d) $f \in Sim(\mathbb{R}^3)$

⑦ $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0,R_0} = A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$
$\exists$ un reper ortonormat în $\mathbb{R}^3$ ai matricea lui f are forma diagonală:

a) $\begin{pmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; b) $\begin{pmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{pmatrix}$; d) $\begin{pmatrix} -7 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

(8) $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0,R_0} = \frac{1}{7}\begin{pmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{pmatrix}$

a) $f = R_\varphi$, $\cos\varphi = \frac{5}{7}$ ; b) $f = R_\varphi$, $\cos\varphi = -\frac{5}{7}$ ;

c) $f = s \circ R_\varphi$, $\cos\varphi = -\frac{7}{5}$ , d) $f = s \circ R_\varphi$, $\cos\varphi = \frac{1}{7}$,

unde $R_\varphi$ = rotație de $\measuredangle \varphi$, axă $<\{e_1\}>$,

$s$ = simetrie ortogonală față de $<\{e_1\}>^\perp$.

(9) $Q : \mathbb{R}^3 \longrightarrow \mathbb{R}$, $Q(x) = x_1^2 + 2x_1x_2$

$g : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ forma polară asociată

a) $g(x,y) = x_1y_1 + 2x_1y_2 + 2x_2y_1$ ; b) $g(x,y) = x_1y_1 + x_2y_1 + x_2y_2$

c) $g(x,y) = x_1y_1 + x_2y_2 + x_3y_3$ ; d) $g(x,y) = x_1y_1 + x_1y_2 + x_2y_1$.