13.05.2021.

Seminar 12

Geometrie analitică euclidiană

EX1 $(\mathbb{R}^3, (\mathbb{R}^3/\mathbb{R}, g_0), \varphi)$ sp. afin euclidian canonic

$A(3,-1,3)$, $B(5,1,-1)$, $u=(-3,5,-6)$

a) Să se scrie ec. dreptei $\mathcal{D}$ ai $A\in\mathcal{D}$, $V_{\mathcal{D}}=\langle\{u\}\rangle$.

b) —//— AB

c) Să se afle punctele de intersecție ale dreptei $\mathcal{D}$ cu planele de coordonate

SOL

a) $\mathcal{D}: \dfrac{x_1-3}{-3}=\dfrac{x_2+1}{5}=\dfrac{x_3-3}{-6}=t$, Ec. carteziană

$$\mathcal{D}: \begin{cases} x_1 = 3-3t \\ x_2 = -1+5t \\ x_3 = 3-6t, \ t\in\mathbb{R} \end{cases}$$ ec. parametrice.

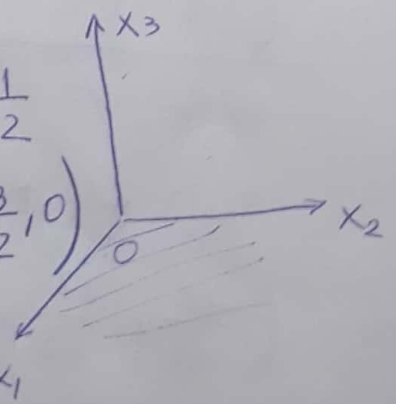
b) $\overrightarrow{AB}=(5-3,\ 1+1,\ -1-3)=(2,2,-4)=2\underbrace{(1,1,-2)}$

$$AB: \frac{x_1-3}{1}=\frac{x_2+1}{1}=\frac{x_3-3}{-2}=t \Rightarrow \begin{cases} x_1=3+t \\ x_2=-1+t \\ x_3=3-2t, t\in R \end{cases}$$

c) 1) $Ox_1x_2: x_3=0$

$Ox_1x_2\cap\mathcal{D}=\{P_3\}$ $\quad 3-6t=0\Rightarrow t=\frac{1}{2}$

$P_3\left(3-\frac{3}{2}, -1+\frac{5}{2}, 3-\frac{6}{2}\right)\Rightarrow P_3\left(\frac{3}{2},\frac{3}{2},0\right)$



2) $Ox_1x_3: x_2=0$

$Ox_1x_3\cap\mathcal{D}=\{P_2\}$ $\quad -1+5t=0\Rightarrow t=\frac{1}{5}$

$P_2\left(3-\frac{3}{5}, -1+\frac{5}{5}, 3-\frac{6}{5}\right)\Rightarrow P_3\left(\frac{12}{5},0,\frac{9}{5}\right)$

3) $Ox_2x_3 \cdot \quad x_1 = 0$

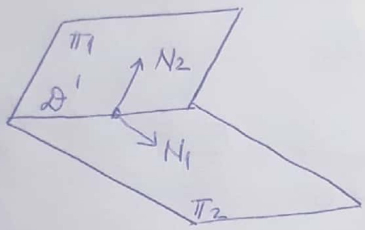
$Ox_2x_3 \cap \mathcal{D} = \{P_1\} \qquad 3 - 3t = 0 \Rightarrow t = 1$

$P_1(3-3, -1+5, 3-6) \Rightarrow P_1(0, 4, -3)$.

<u>Ex 2</u> Să se scrie ec. dreptei $\mathcal{D}$ a.î. $A(2,-5,3) \in \mathcal{D}$

$$\mathcal{D} \parallel \mathcal{D}', \quad \mathcal{D}': \begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 \Rightarrow N_1 = (2,-1,3) \\ 5x_1 + 4x_2 - x_3 + 1 = 0 \Rightarrow N_2 = (5,4,-1) \end{cases}$$

<u>SOL</u>



$$u_{\mathcal{D}'} = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} =$$

$$= (1-12, -(-2-15), 8+5) = (-11, 17, 13)$$

$$\mathcal{D}: \frac{x_1 - 2}{-11} = \frac{x_2+5}{17} = \frac{x_3 - 3}{13} = t \Rightarrow \begin{cases} x_1 = 2 - 11t \\ x_2 = -5 + 17t \\ x_3 = 3 + 13t, \ t \in \mathbb{R} \end{cases}$$

<u>OBS</u> $x_3 = t'$

$$\begin{cases} 2x_1 - x_2 = -1 - 3t' \\ 5x_1 + 4x_2 = -1 + t' \end{cases} \Bigg| \cdot 4 \quad \oplus$$

$$13x_1 \quad / \quad = -5 - 11t' \Rightarrow x_1 = -\frac{5}{13} - \frac{11}{13}t'$$

$$x_2 = 2x_1 + 1 + 3t' = \frac{-10}{13} - \frac{22}{13}t' + 1 + 3t' = \frac{3}{13} + \frac{17t'}{13}$$

$$\left(-\frac{11}{13}, \frac{17}{13}, 1\right) = \frac{1}{13}\underbrace{(-11, 17, 13)}_{= u_{\mathcal{D}'}}$$

$$\mathcal{D} \parallel \mathcal{D}' \Rightarrow V_{\mathcal{D}} = V_{\mathcal{D}'}$$

Ex3 Fie $\pi: x_1+x_2+x_3=+1$, $M(1,2,-1)$

$$D: \frac{x_1-1}{2}=\frac{x_2-1}{-1}=\frac{x_3}{3}=t \Rightarrow \begin{cases} x_1=1+2t \\ x_2=1-t \\ x_3=3t, t\in\mathbb{R} \end{cases}$$

a) Să se scrie ec dreptei D' ai $M\in D'$ și $D'\perp\pi$.

b) —"— planului π' ai $M\in\pi'$ și $\pi'\perp D$.

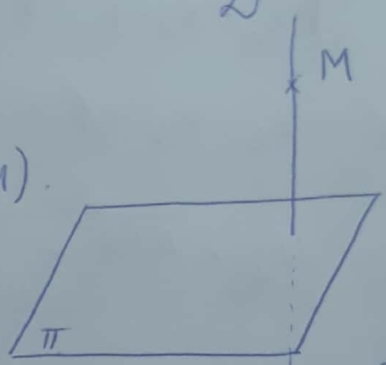
c) —"— planului π'' ai $M\in\pi''$ și $D\subset\pi''$.

d) $pr_D M=?$

e) $pr_\pi M=?$

Sol

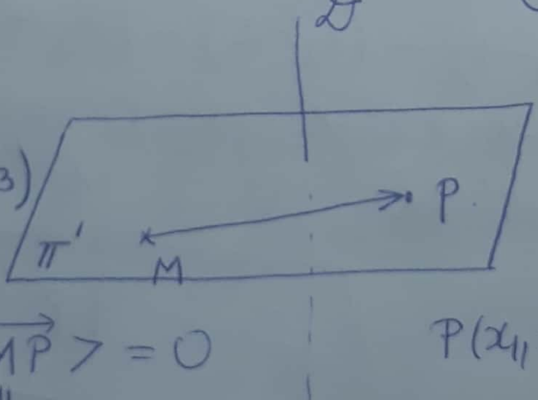
a) $\pi: x_1+x_2+x_3=1$, $N=(1,1,1)$.



$D'\perp\pi \Rightarrow u_{D'}=N$

$$D': \frac{x_1-1}{1}=\frac{x_2-2}{1}=\frac{x_3+1}{1}=t \Rightarrow \begin{cases} x_1=1+t \\ x_2=2+t \\ x_3=-1+t, t\in\mathbb{R} \end{cases}$$

b) $D\perp\pi'$



$u_D=N_{\pi'}=(2,-1,3)$

$\pi': \langle N_{\pi'}, \overrightarrow{MP}\rangle=0$ $\quad P(x_1,x_2,x_3)$

$\overrightarrow{MP}=(x_1-1,x_2-2,x_3+1)$

$\pi': 2(x_1-1)-(x_2-2)+3(x_3+1)=0$

$\pi': 2x_1-x_2+3x_3+3=0$ ec. generală.

$\boxed{\text{OBS}}$ $\pi=Ox_1x_2$ $\quad O(0,0,0)\in\pi$

$N_\pi=(0,0,1)$ $\quad \pi: 0(x_1-0)+0(x_2-0)+1(x_3-0)=0$

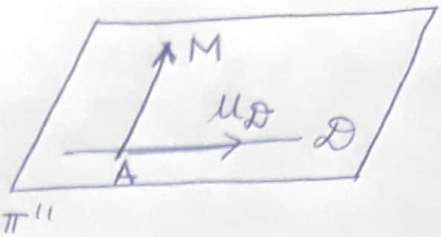
$\Rightarrow x_3=0$

c) $A(1,1,0) \in \mathcal{D}$

$\bullet\ M(1,2,-1)$

$\overrightarrow{AM} = (0,1,-1)$

$u_{\mathcal{D}} = (2,-1,3)$



$$\pi'': \begin{vmatrix} x_1-1 & 2 & 0 \\ x_2-2 & -1 & 1 \\ x_3+1 & 3 & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1-1 & 2 & 0 \\ x_2+x_3-1 & 2 & 0 \\ x_3+1 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \pi'': -2\begin{vmatrix} x_1-1 & 1 \\ x_2+x_3-1 & 1 \end{vmatrix} = 0 \Rightarrow$$

$$\pi''\ x_1 - \cancel{1} - x_2 - x_3 + \cancel{1} = 0 \Rightarrow x_1 - x_2 - x_3 = 0$$

<u>OBS</u>

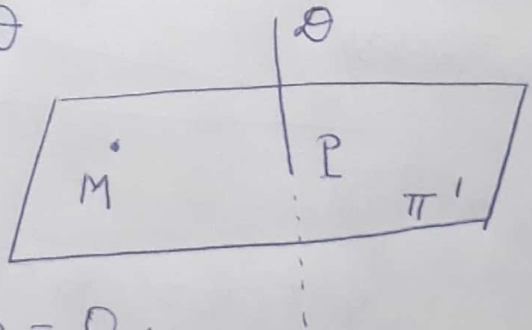
$$N_{\pi''} = u_{\mathcal{D}} \times \overrightarrow{AM} = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & -1 \end{vmatrix} =$$

$$= (1-3, -(-2), 2) = (-2,2,2) = -2\underline{(1,-1,-1)}$$

$$\pi'': 1(x_1-1) - (x_2-2) - (x_3+1) = 0$$

$$x_1 - 1 - x_2 + 2 - x_3 - 1 = 0 \Rightarrow x_1 - x_2 - x_3 = 0$$

d) $pr_{\mathcal{D}}(M) = P = \pi' \cap \mathcal{D}$



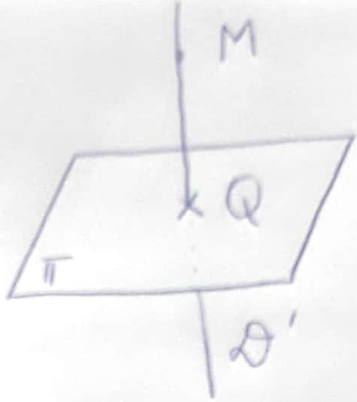
$$\mathcal{D}: \begin{cases} x_1 = 1+2t \\ x_2 = 1-t \\ x_3 = 3t \end{cases}$$

$$\pi': 2x_1 - x_2 + 3x_3 + 3 = 0.$$

$$2(1+2t) - (1-t) + 3\cdot 3t + 3 = 0 \Rightarrow t = -\frac{2}{7}$$

$$P\left(1-\frac{4}{7}, 1+\frac{2}{7}, -\frac{6}{7}\right) \Rightarrow P\left(\frac{3}{7}, \frac{9}{7}, -\frac{6}{7}\right)$$

e) $pr_{\pi} M = \{Q\} = \mathcal{D}' \cap \pi$



$$\mathcal{D}': \begin{cases} x_1 = 1+t \\ x_2 = 2+t \\ x_3 = -1+t \end{cases}$$

$\pi: x_1 + x_2 + x_3 - 1 = 0.$

$1+t+\cancel{2}+t-\cancel{1}+t-\cancel{1}=0 \Rightarrow 3t=-1 \Rightarrow t=-\frac{1}{3}$

$Q\left(1-\frac{1}{3}, 2-\frac{1}{3}, -1-\frac{1}{3}\right) \Rightarrow Q\left(\frac{2}{3}, \frac{5}{3}, -\frac{4}{3}\right)$

T_6 (seminar)

1) Fie $\mathcal{D}: \frac{x_1-1}{4} = \frac{x_2-2}{1} = \frac{z}{6}$ și $M(1,1,1)$

a) Să se scrie ec. planului π care conține $\mathcal{D}$ și trece prin M

b) Să se scrie ec. dreptei $\mathcal{D}'$ a.î. $M \in \mathcal{D}'$ și $\mathcal{D} \parallel \mathcal{D}'$

c) dist$(M, \mathcal{D})$

2) Fie conica $\Gamma: f(x) = x_1^2 + x_1x_2 + x_2^2 - 6x_1 - 16 = 0$
Să se aducă la o formă canonică, utilizând izometrii

3) Fie hiperbola $\mathcal{H}: 16x^2 - 25y^2 = 400$.
Să se afle coord. vârfurilor, focarelor; distanța focală; ec. asimptotelor, directoarelor.