

Temă 3

$$1) q: \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}; q(x, y) = x_2 y_1 + x_1 y_2 + 2x_3 y_1 + 2x_1 y_3$$

a) q FORMĂ BILINI., SIM.b) $G = ?$, ÎN RAP. CU \mathbb{R}_0 c) $\text{Ker } q = ?$ d) $Q: \mathbb{R}^3 \rightarrow \mathbb{R}; Q = ?$ e) SĂ SE ADUCĂ q LA O FORMĂ CANONICĂ. $Q = ?$ + DEF.

$$a) \cdot q \text{ BILINI.} \Leftrightarrow \begin{cases} 1) q(\alpha x + \beta y, z) = \alpha q(x, z) + \beta q(y, z) \\ 2) q(x, \alpha y + \beta z) = \alpha q(x, y) + \beta q(x, z) \end{cases}; (\forall) x, y, z \in \mathbb{R}^3$$

$$\begin{aligned} 1) q(\alpha x + \beta y, z) &= (\alpha x_2 + \beta y_2)z_1 + (\alpha x_1 + \beta y_1)z_2 + 2(\alpha x_3 + \beta y_3)z_1 + 2(\alpha x_1 + \beta y_1)z_3 \\ &= \alpha x_2 z_1 + \beta y_2 z_1 + \alpha x_1 z_2 + \beta y_1 z_2 + \alpha 2x_3 z_1 + \beta 2y_3 z_1 + \alpha 2x_1 z_3 + \beta 2y_1 z_3 \\ &= \alpha (x_2 z_1 + x_1 z_2 + 2x_3 z_1 + 2x_1 z_3) + \beta (y_2 z_1 + y_1 z_2 + 2y_3 z_1 + 2y_1 z_3) = \\ &= \alpha q(x, z) + \beta q(y, z) \checkmark \end{aligned}$$

Idem & pt. 2)

$$\cdot q \text{ SIMETRICĂ} \Leftrightarrow q(x, y) = q(y, x); (\forall) x, y \in \mathbb{R}^3$$

$$\underline{x_2 y_1} + \underline{x_1 y_2} + \underline{2x_3 y_1} + \underline{2x_1 y_3} = \underline{y_2 x_1} + \underline{y_1 x_2} + \underline{2y_3 x_1} + \underline{2y_1 x_3} \checkmark$$

$$b) G = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} \xrightarrow{\text{②} \leftrightarrow \text{①}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix} \xrightarrow{\text{③} - 2 \cdot \text{①}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$c) \text{Ker } q = \{x \in \mathbb{R}^3 \mid q(x, y) = 0, (\forall) y \in \mathbb{R}^3\}$$

$$\begin{cases} q(x, e_1) = 0 \\ q(x, e_2) = 0 \\ q(x, e_3) = 0 \end{cases} \Rightarrow \begin{cases} x_2 + 2x_3 = 0 \Rightarrow x_2 = -2x_3 \\ x_1 = 0 \\ 2x_1 = 0 \end{cases}; \det(G) = 0 \Rightarrow q \text{ NU E NEDEGENERATĂ}$$

$$\text{Ker } q = \{(0, -2x_3, x_3)\}$$

$$d) Q(x) = q(x, x) = x_2 x_1 + x_1 x_2 + 2x_3 x_1 + 2x_1 x_3 = 2x_1 x_2 + 4x_1 x_3 = 2(x_1 x_2 + 2x_1 x_3)$$

$$e) y_1 = \frac{1}{2}(x_1 + x_3); y_2 = \frac{1}{2}(x_1 - x_3); y_3 = x_2 \text{ (PT. CĂ NICIUN } G_{ii} \neq 0)$$

$$Q(x) = 2y_1^2 - 2y_2^2 + 2y_1y_3 + 2y_2y_3 = 2(y_1 + \frac{1}{2}y_3)^2 - 2y_2^2 + 2y_2y_3 - \frac{1}{2}y_3^2 =$$

$$= 2(y_1 + \frac{1}{2}y_3)^2 - 2(y_2 - \frac{1}{2}y_3)^2 + \frac{1}{2}y_3^2 - \frac{1}{2}y_3^2 =$$

$$= 2(y_1 + \frac{1}{2}y_3)^2 - 2(y_2 - \frac{1}{2}y_3)^2$$

$$\begin{cases} x_1 = y_1 + \frac{1}{2}y_3 \\ x_2 = y_2 - \frac{1}{2}y_3 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = \frac{1}{2}(x_1 + x_3) + \frac{1}{2}x_2 = \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ x_2 = \frac{1}{2}(x_1 - x_3) - \frac{1}{2}x_2 = \frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 \\ x_3 = x_3 \end{cases}$$

$$Q(x) = 2x_1^2 - 2x_2^2$$

$(1, 1) \rightarrow$ SIGNATURA $\Rightarrow Q$ NU E "DEF." $(+2, 0)$

2) $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ AP. LINI.

$$A = \begin{pmatrix} 5 & 1 & 3 \\ 0 & 7 & 6 \\ 0 & 1 & 8 \end{pmatrix}; [f]_{\mathcal{L}_0, \mathcal{L}_0} = A; \mathcal{L}_0 = \text{REPER CANONIC}$$

a) VALORI PROPRII = ?

b) UN REPER ÎN CARE MATRICEA ASOCIATĂ LUI f E DIAGONALĂ

$$a) P(\lambda) = \begin{vmatrix} 5-\lambda & 1 & 3 \\ 0 & 7-\lambda & 6 \\ 0 & 1 & 8-\lambda \end{vmatrix} = (5-\lambda) \begin{vmatrix} 7-\lambda & 6 \\ 1 & 8-\lambda \end{vmatrix} = 0$$

$$\begin{cases} \lambda_1 = 5; m_1 = 2 \\ \lambda_2 = 10; m_2 = 1 \end{cases}$$

$$\begin{aligned} \lambda_1 = 5 \in \mathbb{R} \\ (7-2)(8-2) - 6 &= 0 \\ 56 - 72 - 82 + 2^2 - 6 &= 0 \\ \lambda^2 - 15\lambda + 50 &= 0 \\ \Delta = 15^2 - 4 \cdot 50 = 25 \Rightarrow \sqrt{\Delta} = 5 \\ \lambda_2 = \frac{15+5}{2} = 10 \in \mathbb{R} \\ \lambda_3 = \frac{15-5}{2} = 5 \in \mathbb{R} \end{aligned}$$

$$b) V_{\lambda_1} = \{x \in \mathbb{R}^3 / f(x) = 5x\} \Rightarrow \begin{cases} x_2 + 3x_3 = 0 \\ 2x_2 + 6x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} 0 & 1 & 3 \\ 0 & 2 & 6 \\ 0 & 1 & 3 \end{vmatrix} = 0; \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \dim V_{\lambda_1} = 3 - 1 = 2 = m_{\lambda_1}, \begin{cases} x_2 + 3x_3 = 0 \Rightarrow 3x_3 = -x_2 \Rightarrow x_2 = -3x_3 \\ 2x_2 + 6x_3 = 0 \Rightarrow 2x_2 - 2x_2 = 0 \end{cases}$$

$$V_{\lambda_1} = \{(0, -3x_3, x_3) / x_3 \in \mathbb{R}\} = \{x_3(0, -3, 1) / x_3 \in \mathbb{R}\} \Rightarrow \mathcal{R}_1 = \{(0, -3, 1)\}$$

$$V_{\lambda_2} = \{x \in \mathbb{R}^3 / f(x) = 10x\} \Rightarrow \begin{cases} -5x_1 + x_2 + 3x_3 = 0 \\ -3x_2 + 6x_3 = 0 \\ x_2 - 2x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} -5 & 1 & 3 \\ 0 & -3 & 6 \\ 0 & 1 & -2 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \dim V_{\lambda_2} = 3 - 2 = 1 = m_{\lambda_2} \begin{cases} -5x_1 + x_2 = -3x_3 \Rightarrow -5x_1 = -5x_3 \Rightarrow x_1 = x_3 \\ x_2 = 2x_3 \end{cases} \Rightarrow V_{\lambda_2} = \{x_3(1, 2, 1) / x_3 \in \mathbb{R}\}$$

$$\Rightarrow \mathcal{R}_2 = \{(1, 2, 1)\} \Rightarrow \mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$$