Seminarul 13

Aducerea unei conice la o formă canonică pt. $\Delta = 0$. Cuadrice studiate de ec. reduse.

1) $\Gamma: f(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2 + 2x_1 + 2x_2 - 2 = 0$

Să se aducă la o formă canonică, efectând izometrii. Reprezentare grafică

$$A = \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}, \quad \delta = \det A = 0$$

$$\tilde{A} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 3 & 1 \\ 1 & 1 & -2 \end{pmatrix} \quad \Delta = \det \tilde{A} = -12 \neq 0$$

$\mathcal{R} = \{O; e_1, e_2\} \longrightarrow \mathcal{R}' = \{O; e_1', e_2'\}$.

$Q: \mathbb{R}^2 \to \mathbb{R}, \quad Q(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2$

$$P(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 3-\lambda & -3 \\ -3 & 3-\lambda \end{vmatrix} = 0$$

$P(\lambda) = \lambda^2 - 6\lambda + 0 = 0$

$\lambda(\lambda - 6) = 0$

$\lambda_1 = 6; \; m_1 = 1$

$\lambda_2 = 0, \; m_2 = 1$

$V_{\lambda_1} = \{ x \in \mathbb{R}^2 \mid AX = 6X \}$.

$AX - 6X = 0$

$(A - 6I_2) \cdot X = 0$

$$\begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_1 = -x_2 \Rightarrow V_{\lambda_1} = \langle \{ \underbrace{(+1, -1)}_{f_1} \} \rangle$$

$$e_1' = \frac{1}{\sqrt{2}}(+1, -1)$$

$V_{\lambda_2} = \{ X \in \mathbb{R}^2 \mid AX = 0 \}$

$$\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow V_{\lambda_2} = \langle \{ \underbrace{(1, 1)}_{f_2} \} \rangle \Rightarrow e_2' = \frac{1}{\sqrt{2}}(1, 1)$$

OBS:

$\cancel{R = }$

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} +1 & 1 \\ -1 & 1 \end{pmatrix}$$ (trebuie să fie det = 1)

$\theta: X = RX'$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \quad \begin{cases} x_1 = \frac{1}{\sqrt{2}}(x_1' + x_2') \\ x_2 = \frac{1}{\sqrt{2}}(-x_1' + x_2') \end{cases}$$

$Q(x) = 6x_1'^2$

$$\theta(\Gamma): 6x_1'^2 + \frac{2}{\sqrt{2}}(x_1' + x_2') + \frac{2}{\sqrt{2}}(-x_1' + x_2') - 2 = 0$$

$$6x_1'^2 + \frac{4}{\sqrt{2}}x_2' - 2 = 0 \quad /:6$$

$$x_1'^2 + \frac{2}{3\sqrt{2}}x_2' - \frac{1}{3} = 0$$

$$x_1'^2 + \frac{2}{3\sqrt{2}}\left(x_2' - \frac{\sqrt{2}}{2}\right) = 0$$

$$\begin{cases} x_1'' = x_1' \\ x_2' = x_2' - \frac{\sqrt{2}}{2} \end{cases}$$

$\sigma : X' = X'' + X_0$

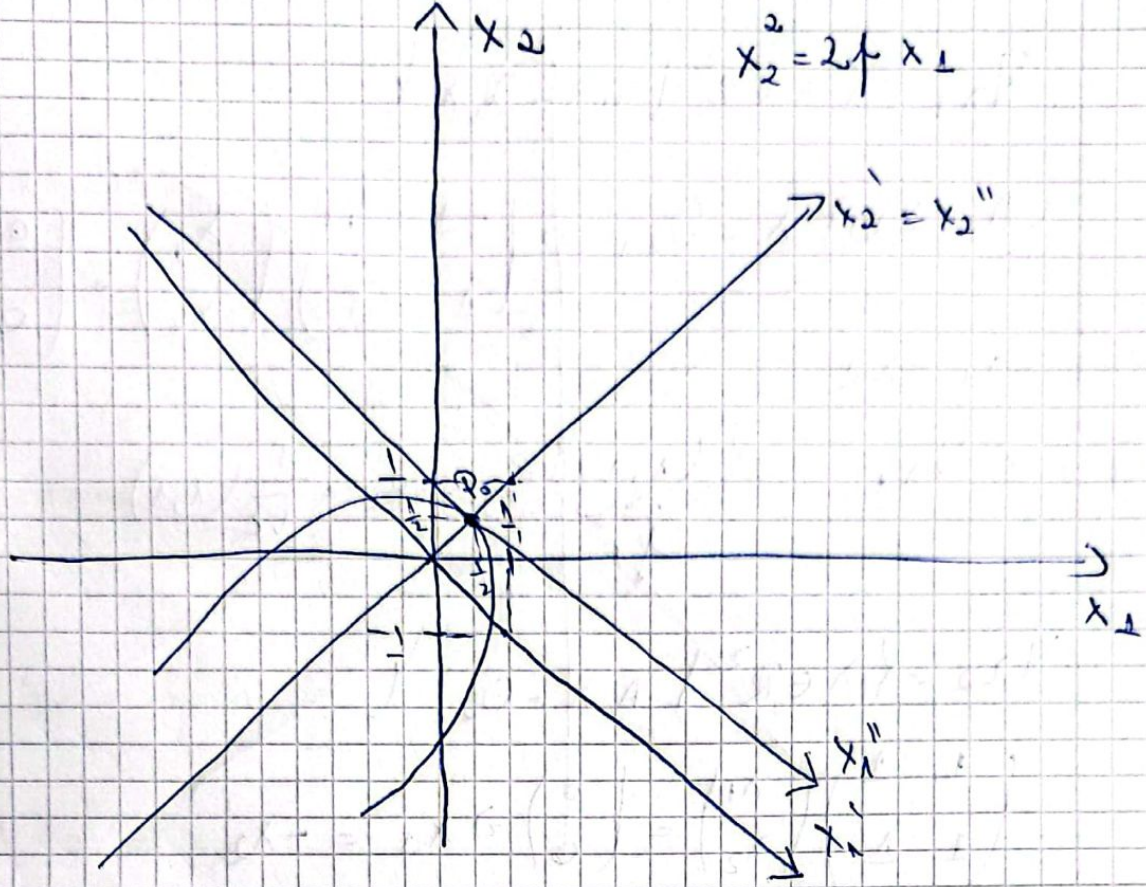
$$X_0 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sigma(\theta(\Pi)) : x_1''^2 \cancel{4} = -\frac{2}{3\sqrt{2}}x_2''$$

$\sigma(\theta) : X = R(X'' + X_0) = RX'' + RX_0$

$$R \cdot X_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & +1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$P_0\left(\frac{1}{2}, \frac{1}{2}\right)$



2) $\Gamma: f(x) = x_1^2 + 2x_1x_2 + x_2^2 + 2x_1 + 2x_2 - 3 = 0$

Să se aducă la o f. canonică, utilizând izometrii. Reprezentare grafică

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

$\delta = 0 \qquad \Delta = 0$

$Q(x) = x_1^2 + x_2^2 + 2x_1x_2$

$$P(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1 =$$
$$= \lambda^2 - 2\lambda = \lambda(\lambda - 2)$$

$$\begin{cases} \lambda_1 = 2;\ m_{\lambda_1} = 1 \\ \lambda_2 = 0,\ m_{\lambda_2} = 1 \end{cases}$$

$V_{\lambda_1} = \{ x \in \mathbb{R}^2 \mid Ax = \lambda_1 x \}$

$(A - 2 I_2) \cdot x = O_{21} \Leftrightarrow \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = x_2$

$\Rightarrow V_{\lambda_1} = \langle \{ \underbrace{(1,1)}_{v_1} \} \rangle \Rightarrow e_1' = \frac{1}{\sqrt{2}}(1,1)$

$V_{\lambda_2} = \{ x \in \mathbb{R}^2 \mid Ax = O_{21} \}$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_1 = -x_2$

$V_{\lambda_2} = \langle \{ (-1,1) \} \rangle \Rightarrow e_2' = \frac{1}{\sqrt{2}}(-1,1)$

$\Theta: \ X = R \cdot X', \ R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$

$x_1 = \frac{1}{\sqrt{2}}(x_1' - x_2')$

$x_2 = \frac{1}{\sqrt{2}}(x_1' + x_2')$

$Q(x) = 2x_1'^2$

$\Theta(\Gamma): 2x_1'^2 + \frac{2}{\sqrt{2}}(x_1' - x_2') + \frac{2}{\sqrt{2}}(x_1' + x_2')$

$-3 = 0$

$: 2x_1'^2 + \frac{4}{\sqrt{2}} x_1' - 3 = 0.$

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$$x_1'^2 + \sqrt{2}\, x_1' - \frac{3}{2} = 0$$

$$x_1'^2 + 2 \cdot \frac{\sqrt{2}}{2}\, x_1' + \frac{1}{2} - 2 = 0$$

$$\Big(\underbrace{x_1' + \frac{1}{\sqrt{2}}}_{x_1''}\Big)^2 - 2 = 0$$

$$\begin{cases} x_1'' = x_1' + \dfrac{1}{\sqrt{2}} \\ x_2'' = x_2' \end{cases}$$

$\sigma : x' = x'' + x_0$

$$x_0 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$\sigma(\Theta(\Gamma)) : x_1''^2 - 2 = 0 \Rightarrow x_1'' = \pm\sqrt{2}$

$\sigma(\Theta) : x = R(x'' + x_0) = R x'' + R x_0$

$$R x_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} =$$

$$= \frac{1}{2} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$P_0\left(-\frac{1}{2}, -\frac{1}{2}\right)$$

OBS: $f(x) = (x_1+x_2)^2 + 2(x_1+x_2) + 1 - 4 = 0$

$= (x_1+x_2+1)^2 - 2^2 = 0$

$= (x_1+x_2+3)(x_1+x_2-1) = 0$

$d_1 : x_1 + x_2 = 1$

$d_2 : x_1 + x_2 = -3$