C14 27.05.2020.

Aducerea la formă canonică a conicelor ($\delta = 0$)
Cuadrice pe ecuații reduse.

$$\Gamma: \ f(x_1, x_2) = X^T A X + 2BX + c = 0. \qquad \boxed{e = \frac{c}{a}}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = A^T, \quad \tilde{A} = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$$

$$B = (b_1 \ \ b_2) \qquad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\delta = \det A, \ \Delta = \det \tilde{A}, \ r = rg A \geq 1, \ r' = rg \tilde{A}$$

$$r \leq r' \leq r + 2$$

I. $\delta \neq 0$ (conică cu centru unic).

II. $\delta = 0$.

- $(\mathbb{R}^2, \mathbb{R}^2/\mathbb{R}, \varphi)$ sp. afin.

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{transformare afină}]{} \mathcal{R}' = \{O; e_1', e_2'\} \xrightarrow[\text{translație}]{} \mathcal{R}'' = \{P; e_1', e_2'\}$$

$$Q: \mathbb{R}^2 \to \mathbb{R}, \ Q(x) = X^T A X.$$

$$\theta: X = CX', \ C \in GL(2, \mathbb{R})$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \Rightarrow \begin{cases} x_1 = c_{11} x_1' + c_{12} x_2' \\ x_2 = c_{21} x_1' + c_{22} x_2' \end{cases}$$

$$Q(x) = \lambda_1 x_1'^2, \ \lambda_1 \neq 0 \qquad \begin{pmatrix} \lambda_1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \lambda_2 = 0$$

$$\theta(\Gamma): \ \lambda_1 x_1'^2 + 2b_1(c_{11} \underline{x_1'} + c_{12} \underline{\underline{x_2'}}) + 2b_2(c_{21} \underline{x_1'} + c_{22} \underline{\underline{x_2'}}) + c = 0$$

$$\theta(\Gamma): \ \lambda_1 x_1'^2 + 2\underbrace{(b_1 c_{11} + b_2 c_{21})}_{b_1'} x_1' + 2\underbrace{(b_1 c_{12} + b_2 c_{22})}_{b_2'} x_2' + c = 0$$

$$\lambda_1 x_1'^2 + 2b_1' x_1' + 2b_2' x_2' + c = 0.$$

$$\Delta = \det \begin{pmatrix} \lambda_1 & 0 & b_1' \\ 0 & 0 & b_2' \\ b_1' & b_2' & c \end{pmatrix} = -\lambda_1 b_2'^2$$

a) $\Delta \neq 0 \Rightarrow \Gamma$ conică nedegenerată

$b_2' \neq 0$.

$$\theta(\Gamma): \lambda_1 x_1'^2 + 2b_1' x_1' + 2b_2' x_2' + c = 0.$$

$$\lambda_1 \left(x_1'^2 + 2\frac{b_1'}{\lambda_1} x_1' + \frac{b_1'^2}{\lambda_1^2} \right) + 2b_2' x_2' + \underbrace{c - \frac{b_1'^2}{\lambda_1}}_{c'} = 0$$

$$\lambda_1 \left(x_1' + \frac{b_1'}{\lambda_1} \right)^2 + 2b_2' \left(x_2' + \frac{c'}{2b_2'} \right) = 0 \Rightarrow \lambda_1 x_1''^2 + 2b_2' x_2'' = 0$$

(parabolă).

$$\begin{cases} x_1'' = x_1' + \dfrac{b_1'}{\lambda_1} \\ x_2'' = x_2' + \dfrac{c'}{2b_2'} \end{cases} \quad (\text{translație}).$$

$$\tau: X' = X'' + X_0, \quad X_0 = \begin{pmatrix} -\dfrac{b_1'}{\lambda_1} \\ -\dfrac{c'}{2b_2'} \end{pmatrix}$$

$$\tau \circ \theta: X = CX' = C(X'' + X_0) = CX'' + \underbrace{CX_0}_{Y_0}, \quad Y_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$P(\alpha, \beta)$ (α, β) coordonatele în raport cu $\mathcal{R}$

b) $\Delta = 0 \Rightarrow \Gamma$ conică degenerată

$b_2' = 0$.

$$\theta(\Gamma): \lambda_1 \left(x_1' + \frac{b_1'}{\lambda_1} \right)^2 + c' = 0$$

$$\begin{cases} x_1'' = x_1' + \dfrac{b_1'}{\lambda_1} \\ x_2'' = x_2' \end{cases} \quad (\text{translație})$$

$$\tau': X' = X'' + X_0, \quad X_0 = \begin{pmatrix} -\dfrac{b_1'}{\lambda_1} \\ 0 \end{pmatrix}$$

$$\tau \circ \theta: X = CX' = CX'' + \underbrace{CX_0}_{Y_0}, \quad Y_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix},$$

$P(\alpha, \beta)$ în raport cu $\mathcal{R}$.

$$\lambda_1 x_1''^2 + c' = 0.$$

- $c' = 0 \Rightarrow \lambda_1 x_1''^2 = 0 \Rightarrow x_1''^2 = 0$ (dreaptă dublă)
- $c' \neq 0 \Rightarrow \phi$; $x_1''^2 = -\dfrac{c'}{\lambda_1} \Rightarrow x_1'' = \pm\sqrt{\dfrac{-c'}{\lambda_1}}$ (drepte paralele)

• $(\mathcal{E}_2, (E_2, \langle \cdot, \cdot \rangle), \varphi)$ spațiu afin euclidian

$Q: \mathbb{R}^2 \longrightarrow \mathbb{R}$, $Q(x) = X^T A X$.

Aducem Q la o formă canonică, utilizând metoda valorilor proprii

$$\lambda^2 - Tr(A)\lambda + \underbrace{\det A}_{\delta = 0} = 0 \Rightarrow \begin{array}{l} \lambda_1 \neq 0 \\ \lambda_2 = 0. \end{array}$$

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{rotatie}]{} \mathcal{R}' = \{O; e_1', e_2'\} \xrightarrow[\text{translatie}]{} \mathcal{R}'' = \{P; e_1', e_2'\}$$

e_k' versor propriu coresp. valorii proprii λ_k, $k = \overline{1,2}$

$\theta: X = RX'$ $\quad R = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$, $\quad e_1' = (l_1, m_1)$, $e_2' = (l_2, m_2)$

Alegem $R \in SO(2) \Rightarrow \theta =$ rotatie

$$\theta \begin{cases} x_1 = l_1 x_1' + l_2 x_2' \\ x_2 = m_1 x_1' + m_2 x_2'. \end{cases}$$

$$\theta(\Gamma): \lambda_1 x_1'^2 + 2b_1(l_1 x_1' + l_2 x_2') + 2b_2(m_1 x_1' + m_2 x_2') + c = 0$$

$$\lambda_1 x_1'^2 + 2b_1' x_1' + 2b_2' x_2' + c = 0.$$

Discutia este analoagă cazului afin

a) $\Delta \neq 0$, $b_2' \neq 0$ (Γ nedegenerată)

$$\lambda_1 \underbrace{\left(x_1' + \frac{b_1'}{\lambda_1}\right)}_{x_1''}^2 + 2b_2' \underbrace{\left(x_2' + \frac{c'}{2b_2'}\right)}_{x_2''} = 0$$

$\tau: X' = X'' + X_0$ (translatie), $\quad X_0 = \begin{pmatrix} -\frac{b_1'}{\lambda_1} \\ -\frac{c'}{2b_2'} \end{pmatrix}$

$\tau \circ \theta: X = RX' = RX'' + RX_0$, $\quad RX_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

($P(\alpha, \beta)$ în raport cu $\mathcal{R}$)

$\tau \circ \theta =$ izometrie

b) $\Delta = 0$, $b_2' = 0$

$$\lambda_1 \underbrace{\left(x_1' + \frac{b_1'}{\lambda_1}\right)}_{x_1''}^2 + c' = 0 \; ; \; x_2'' = x_2'$$

$\tau: X' = X'' + X_0$, $\quad X_0 = \begin{pmatrix} -\frac{b_1'}{\lambda_1} \\ 0 \end{pmatrix}$

$\tau \circ \theta: X = RX' = RX'' + RX_0$

$RX_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $P(\alpha, \beta)$ în raport cu $\mathcal{R}$.

Concluzii

Δ (natura)	δ (genul)	Tipul conicei
$\Delta \neq 0$ conică nedeg.	$\delta > 0$	Elipsă sau ϕ
	$\delta < 0$	Hiperbolă
	$\delta = 0$	Parabolă
$\Delta = 0$ conică degen.	$\delta > 0$	Punct dublu
	$\delta < 0$	Drepte concurente
	$\delta = 0$	Drepte //, Drepte conf sau ϕ

Aplicații

Ex1 ($\delta \neq 0$) În spațiul euclidian $\mathcal{E}_2$ se consideră conica:

$\Gamma: f(x) = 7x_1^2 - 8x_1x_2 + x_2^2 - 6x_1 - 12x_2 - 9 = 0.$

Să se aducă la forma canonică, utilizând izometrii.

SOL

$$A = \begin{pmatrix} 7 & -4 \\ -4 & 1 \end{pmatrix}; \quad \tilde{A} = \begin{pmatrix} 7 & -4 & -3 \\ -4 & 1 & -6 \\ -3 & -6 & -9 \end{pmatrix}$$

$\delta = \det A = 7 - 16 = -9 \neq 0.$ (Γ conică cu centru unic)

$$\begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases} \Rightarrow \begin{cases} 14x_1 - 8x_2 - 6 = 0 \\ -8x_1 + 2x_2 - 12 = 0 \end{cases} \Rightarrow \begin{cases} 7x_1 - 4x_2 = 3 \\ -4x_1 + x_2 = 6 \end{cases} \Big| \cdot 4 \quad (+)$$

$$-9x_1 \quad / = 27$$

$x_1 = -3 \Rightarrow x_2 = 6 + 4(-3) = -6$

$P_0(-3, -6)$ centrul conicei

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translație}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{rotație}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}.$$

$\Delta = \det \tilde{A} = -9 \cdot 36 \neq 0 \Rightarrow \Gamma$ conică nedeg.

$\Delta \neq 0,\ \delta < 0 \Rightarrow \Gamma =$ hiperbolă.

$\theta: X = X' + X_0 \qquad X_0 = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

$\theta(\Gamma): X'^T A X' + \underbrace{\frac{\Delta}{\delta}}_{36.} = 0$

$Q: \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad Q(x) = X'^T A X'$

Aplicăm metoda valorilor proprii.

$\lambda^2 - 8\lambda - 9 = 0 \Rightarrow (\lambda+1)(\lambda-9) = 0$

$\lambda_1 = -1, \quad \lambda_2 = 9.$

$V_{\lambda_1} = \{ x \in \mathbb{R}^2 \mid AX = -X \} = \{ (x_1, 2x_1), x_1 \in \mathbb{R} \}$

$(A + I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-4x_1 + 2x_2 = 0 \Rightarrow x_2 = 2x_1$

$e_1' = \frac{1}{\sqrt{5}}(1,2)$

$V_{\lambda_2} = \{ x \in \mathbb{R}^2 \mid AX = 9X \} = \{ (-2x_2, x_2) \mid x_2 \in \mathbb{R} \}$

$(A - 9I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 & -4 \\ -4 & -8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2x_1 - 4x_2 = 0 \Rightarrow x_1 = -2x_2$

$e_2' = \frac{1}{\sqrt{5}}(-2,1)$

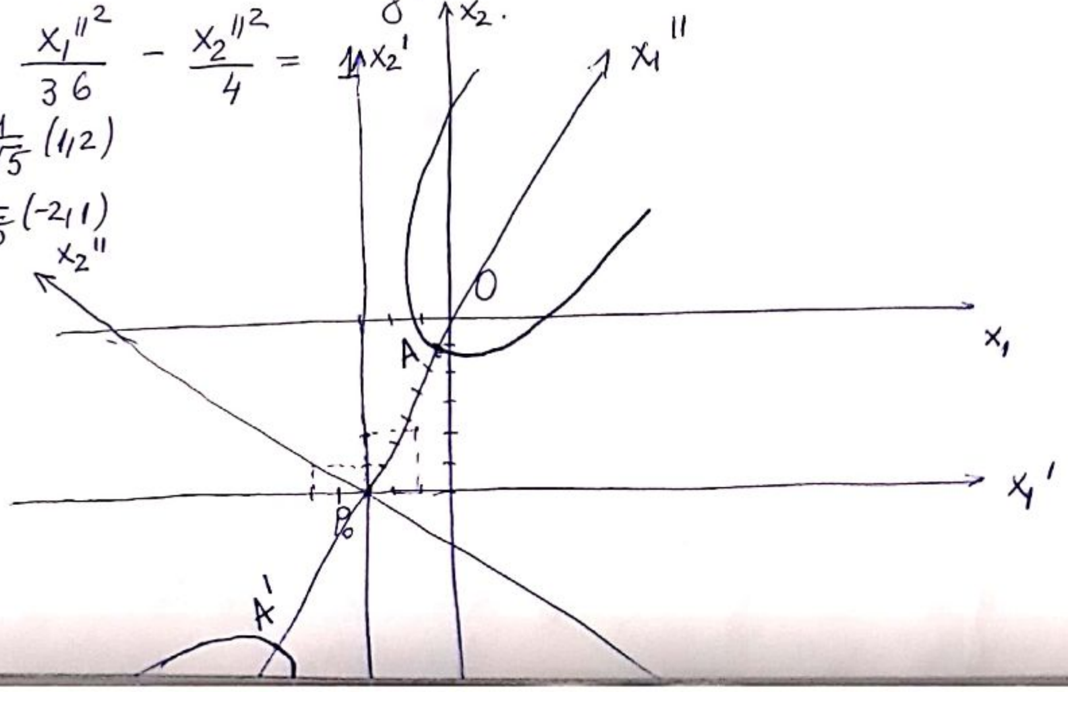
$\tau: X' = RX'', \quad R = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \in SO(2)$

$\tau \circ \theta: X = RX'' + X_0$ (izometrie)

$\lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0 \Rightarrow \tau \circ \theta(\Gamma): -x_1''^2 + 9x_2''^2 + 36 = 0$

$\mathcal{H}: \frac{x_1''^2}{36} - \frac{x_2''^2}{4} = 1$

$\begin{cases} e_1' = \frac{1}{\sqrt{5}}(1,2) \\ e_2' = \frac{1}{\sqrt{5}}(-2,1) \end{cases}$



Ex2 ($\delta=0$) În planul euclidian $\mathcal{E}_2$ se consideră cu[...]

Γ $f(x_1,x_2) = x_1^2 - 4x_1x_2 + 4x_2^2 - 6x_1 + 2x_2 + 1 = 0$

Să se aducă la o formă canonică, utilizând izometrii.

SOL

$$A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 1 \\ -3 & 1 & 1 \end{pmatrix}$$

$$\left.\begin{array}{l} \delta = 0 \text{ (conică fără centru unic)} \\ \Delta = -25 \neq 0 \text{ (conică nedeg.)} \end{array}\right\} \Rightarrow \Gamma: \text{ parabolă}$$

$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = x_1^2 - 4x_1x_2 + 4x_2^2$

$\lambda^2 - 5\lambda = 0 \Rightarrow \lambda(\lambda-5) = 0.$

$\lambda_1 = 5$, $\lambda_2 = 0$.

$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = 5X\} = \{(x_1, -2x_1) \mid x_1 \in \mathbb{R}\}$

$$(A - 5I_2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-2x_1 - x_2 = 0 \Rightarrow x_2 = -2x_1$

$e_1' = \frac{1}{\sqrt5}(1,-2)$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid AX = 0\} = \{(2x_2, x_2) \mid x_2 \in \mathbb{R}\}$

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$

$e_2' = \frac{1}{\sqrt5}(2,1)$

$\theta: \underset{\text{rotație}}{X = RX'}$, $R = \frac{1}{\sqrt5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \in SO(2)$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \end{pmatrix}, \quad x_1 = \frac{1}{\sqrt5}(x_1' + 2x_2')$$

$$x_2 = \frac{1}{\sqrt5}(-2x_1' + x_2')$$

$$\theta(\Gamma): \ 5x_1'^2 - \frac{6}{\sqrt5}(x_1' + 2x_2') + \frac{2}{\sqrt5}(-2x_1' + x_2') + 1 = 0$$

$$5x_1'^2 - \frac{10}{\sqrt5}x_1' - \frac{10}{\sqrt5}x_2' + 1 = 0. \quad \Big| :5$$

$$x_1'^2 - 2\cdot\frac{1}{\sqrt5}x_1' + \frac15 - 2\cdot\frac{1}{\sqrt5}x_2' = 0$$

$$\left(x_1' - \frac{1}{\sqrt5}\right)^2 = \frac{2}{\sqrt5}x_2'$$

$$\begin{cases} x_1'' = x_1' - \frac{1}{\sqrt5} \\ x_2'' = x_2' \end{cases}; \quad \tau: \underset{\text{translație}}{X' = X'' + X_0}, \quad X_0 = \begin{pmatrix} \frac{1}{\sqrt5} \\ 0 \end{pmatrix}$$

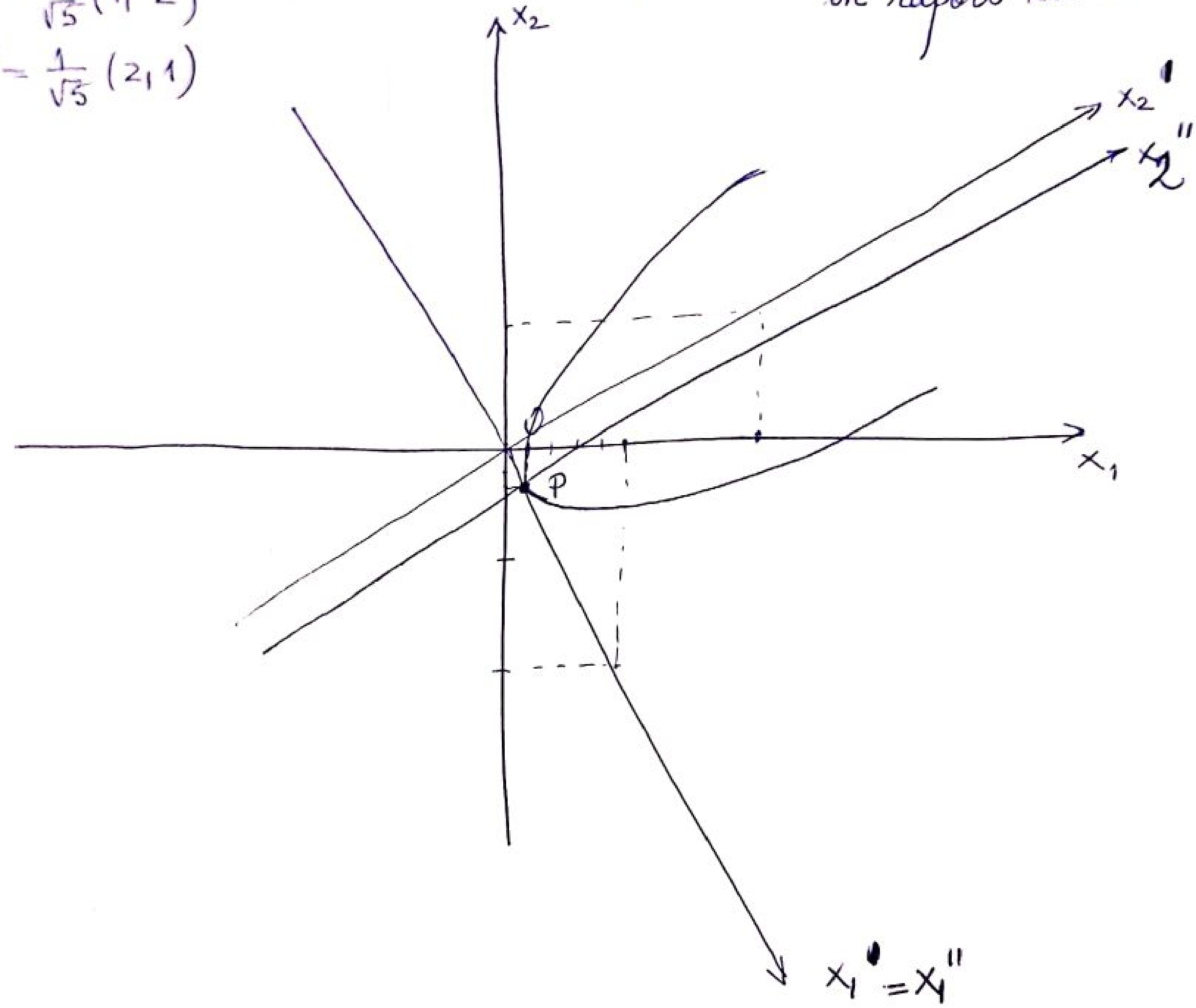
$\tau_0 \theta : X = RX' = R(X'' + X_0) = RX'' + RX_0$.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} X'' + \underbrace{\frac{1}{\sqrt5}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt5} \\ 0 \end{pmatrix}}_{\frac{1}{5}\begin{pmatrix} 1 \\ -2 \end{pmatrix}}$$

$\tau_0 \theta(\Gamma) : {x_1''}^2 = \frac{2}{\sqrt5} x_2''$ $\qquad P\left(\frac{1}{5}, -\frac{2}{5}\right)$ în raport cu R.

$e_1' = \frac{1}{\sqrt5}(1,-2)$

$e_2' = \frac{1}{\sqrt5}(2,1)$



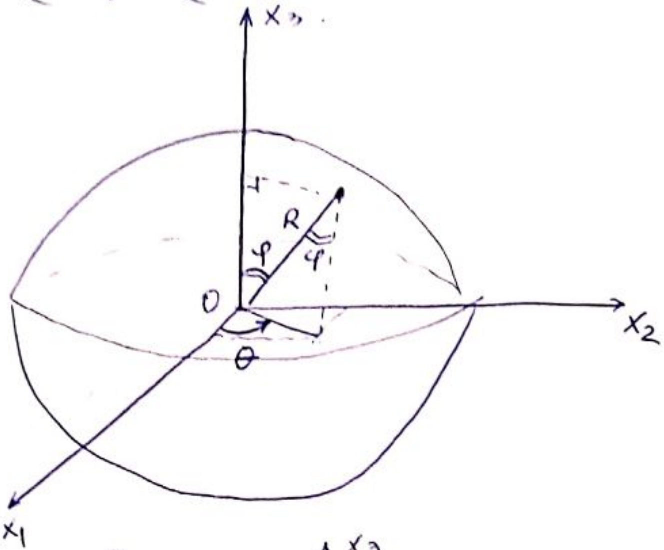
Cuadrice studiate pe ecuații reduse

① $\mathcal{S}(A(a,b,c),R)$: $(x_1-a)^2+(x_2-b)^2+(x_3-c)^2=R^2$
(sfera)

$\mathcal{S}(O(0,0,0),R)$

$$\begin{cases} x_1 = R\sin\varphi\cos\theta \\ x_2 = R\sin\varphi\sin\theta \\ x_3 = R\cos\varphi \end{cases}$$

$\theta\in[0,2\pi)$, $\varphi\in[0,\pi]$



② Elipsoid $\dfrac{x_1^2}{a^2}+\dfrac{x_2^2}{b^2}+\dfrac{x_3^2}{c^2}=1$

$a>0, b>0, c>0$

$A(a,0,0), A'(-a,0,0)$

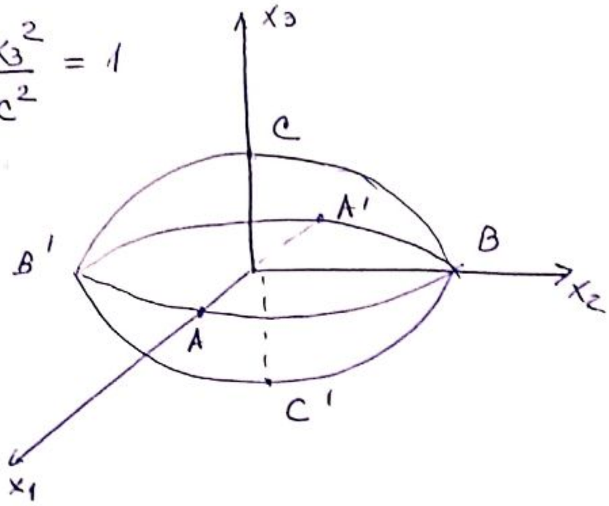
$B(0,b,0), B'(0,-b,0)$

$C(0,0,c), C'(0,0,-c)$

$x_3=\delta\in(-c,c)$

$\dfrac{x_1^2}{a^2}+\dfrac{x_2^2}{b^2}=1-\dfrac{\delta^2}{c^2}>0$

$\Rightarrow$ Elipsă



Analog (secț. cu plane // cu planele de coord)
ϕ, pct, elipse.

③ Hiperboloid cu o pânză.

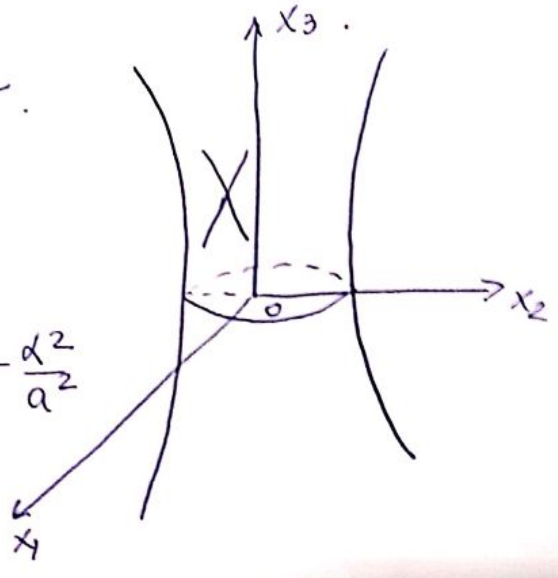
$\mathcal{H}_{1p}$: $\dfrac{x_1^2}{a^2}+\dfrac{x_2^2}{b^2}-\dfrac{x_3^2}{c^2}=1$

$x_3=\delta\in\mathbb{R}\Rightarrow$ elipse.

$x_1=\alpha$, $\alpha\neq\pm a \Rightarrow \dfrac{x_2^2}{b^2}-\dfrac{x_3^2}{c^2}=1-\dfrac{\alpha^2}{a^2}$

Hiperbole.

$x_2=\beta$, $\beta=\pm a \Rightarrow$ hiperbole.



$$\frac{x_1^2}{a^2} - \frac{x_3^2}{c^2} = 1 - \frac{x_2^2}{b^2} \Rightarrow \left(\frac{x_1}{a} - \frac{x_3}{c}\right)\left(\frac{x_1}{a} + \frac{x_3}{c}\right) = \left(1 - \frac{x_2}{b}\right)\left(1 + \frac{x_2}{c}\right)$$

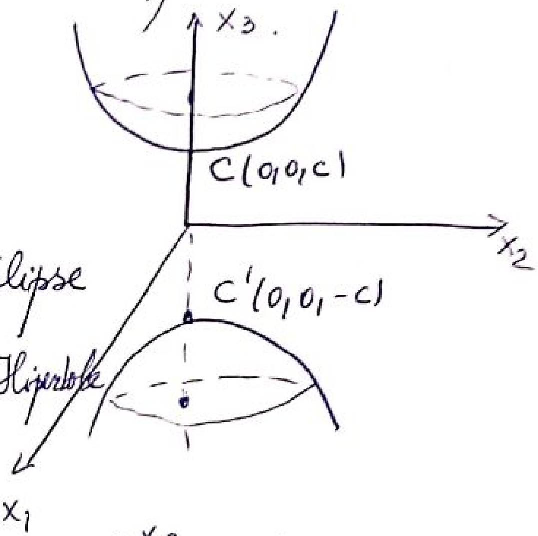
$$d_\lambda \begin{cases} \frac{x_1}{a} - \frac{x_3}{c} = \lambda\left(1 - \frac{x_2}{b}\right) \\ \lambda\left(\frac{x_1}{a} + \frac{x_3}{c}\right) = 1 + \frac{x_2}{b} \end{cases} ; \quad d_\mu : \begin{cases} \frac{x_1}{a} - \frac{x_3}{c} = \mu\left(1 + \frac{x_2}{b}\right) \\ \mu\left(\frac{x_1}{a} + \frac{x_3}{c}\right) = 1 - \frac{x_2}{b} \end{cases}$$

Cuadrică dublu riglată
(2 familii de generatoare)

$\lambda, \mu \in \overline{\mathbb{R}}$

4) Hiperboloidul cu 2 pânze.

$$-\frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1.$$



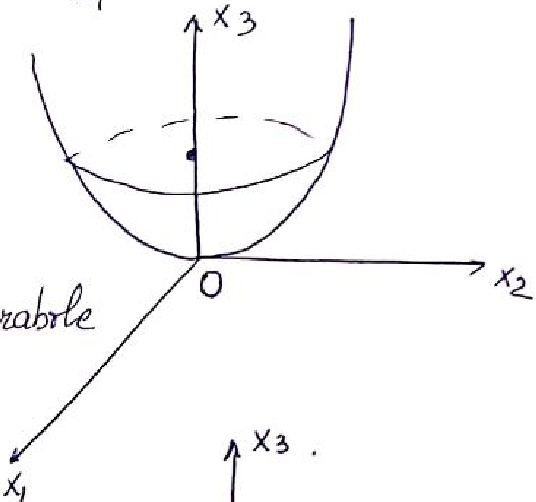
- $x_3 = \gamma \in (-\infty, -c) \cup (c, \infty)$

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = \frac{\gamma^2}{c^2} - 1 > 0 \Rightarrow \text{Elipse}$$

- $x_1 = \alpha \Rightarrow -\frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1 + \frac{\alpha^2}{a^2} \Rightarrow$ Hiperbole
- $x_2 = \gamma \Rightarrow$ Hiperbole.

5) Paraboloid eliptic.

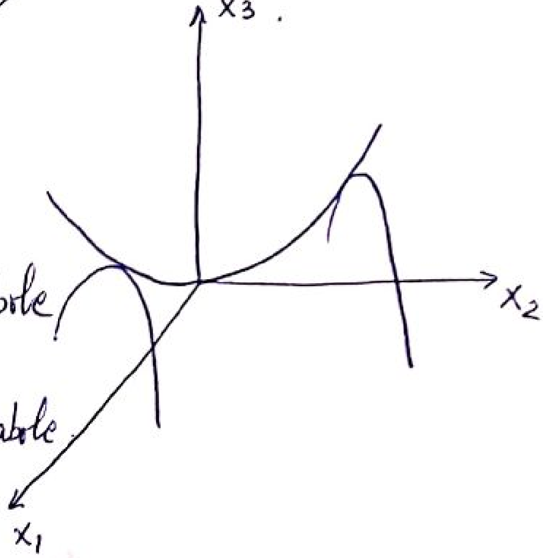
$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 2x_3.$$



- $x_3 = \gamma \in (0, \infty) \Rightarrow$ Elipse
- $x_1 = \alpha \Rightarrow \frac{x_2^2}{b^2} = 2x_3 - \frac{\alpha^2}{a^2} \Rightarrow$ Parabole
- $x_2 = \beta \Rightarrow$ Parabole.

6) Paraboloid hiperbolic.

$$\frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 2x_3$$



- $x_3 = \gamma$, $\gamma \neq 0$ $\Rightarrow \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 2\gamma$ hiperbole
- $x_1 = \alpha \Rightarrow -\frac{x_2^2}{b^2} = 2x_3 - \frac{\alpha^2}{a^2}$ parabole
- $x_2 = \beta \Rightarrow$ parabole.

Cuadrică dublu riglată (2 familii de generatoare)

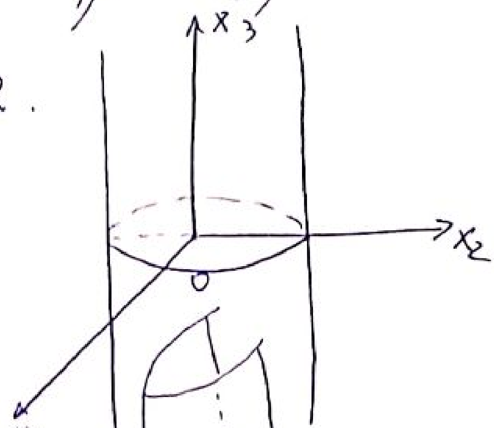
$$\frac{x_1^2}{a^2}-\frac{x_2^2}{b^2}=2x_3 \Rightarrow \left(\frac{x_1}{a}-\frac{x_2}{b}\right)\left(\frac{x_1}{a}+\frac{x_2}{b}\right)=2 \cdot x_3$$

$$d_\lambda : \begin{cases} \frac{x_1}{a}-\frac{x_2}{b}=2\lambda \\ \lambda\left(\frac{x_1}{a}+\frac{x_2}{b}\right)=x_3 \end{cases} ; \quad d_\mu : \begin{cases} \frac{x_1}{a}-\frac{x_2}{b}=\mu x_3 \\ \mu\left(\frac{x_1}{a}+\frac{x_2}{b}\right)=2. \end{cases}$$

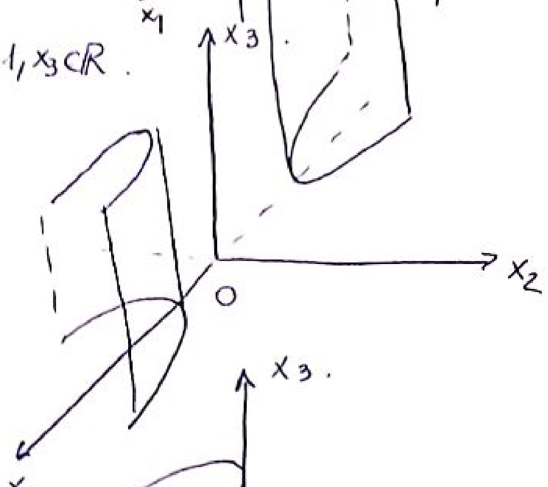
$\lambda, \mu \in \overline{\mathbb{R}}$, $\mu \neq 0$

7) Cilindrul

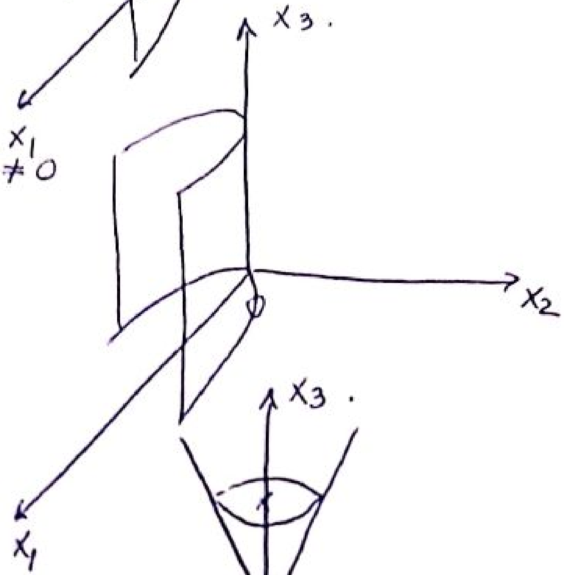
a) eliptic: $\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}=1$, $x_3 \in \mathbb{R}$.



b) hiperbolic: $\frac{x_1^2}{a^2}-\frac{x_2^2}{b^2}=1$, $x_3 \in \mathbb{R}$.

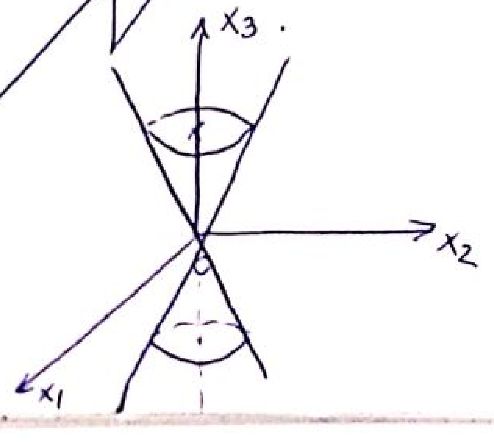


c) parabolic $x_2^2=2px_1$, $p \neq 0$
$x_3 \in \mathbb{R}$.

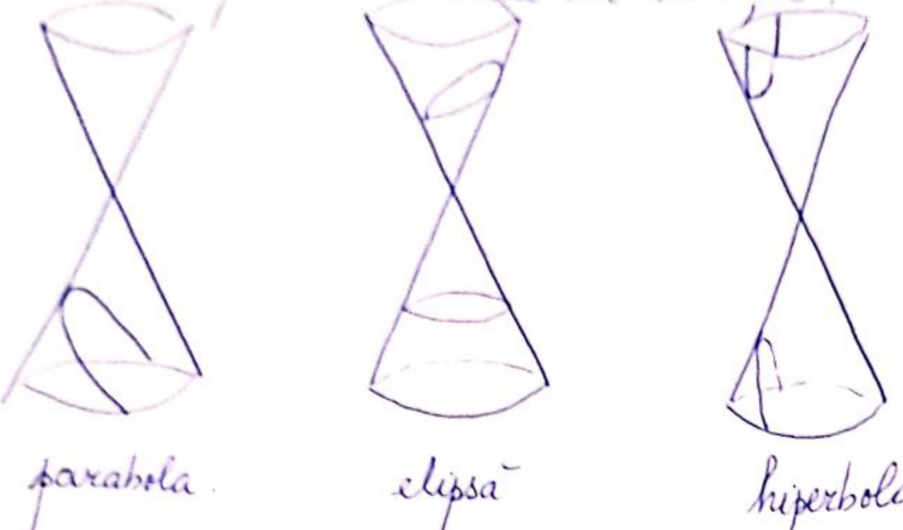


8) Con pătratic.

$$\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}-\frac{x_3^2}{c^2}=0.$$



Conice ca secțiuni în conul de rotație.



OBS Cuadrice degenerate

1. perechi de plane $(ax_1+bx_2+cx_3+d)(a'x_1+b'x_2+c'x_3+d')=0$
$a^2+b^2+c^2>0;\ a'^2+b'^2+c'^2>0.$
2. dreaptă dublă $x_1^2+x_2^2=0$
3. punct dublu $x_1^2+x_2^2+x_3^2=0$
4. cuadrică ϕ $x_1^2+x_2^2+x_3^2+1=0$