Seminar 10.

Geometrie analitică euclidiană

1

$(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$ $A(3,-1,3)$, $B(5,1,-1)$, $v=(-3,5,-6)$

a) Ec dreptei D a.î. $A \in D$, $V_D = \langle \{v\} \rangle$

b) Ec AB

c) D intersecția cu ~~planele~~ planele de coordonate

a) $D: \frac{x_1-3}{-3} = \frac{x_2+1}{5} = \frac{x_3-3}{-6} = t \Leftrightarrow \begin{cases} x_1 = -3t+3 \\ x_2 = 5t-1 \\ x_3 = -6t+3 \end{cases}$

b) $\overrightarrow{AB} = v_{AB} = (5-3, 1+1, -1-3) = (2,2,-4) = 2(1,1,-2)$

$AB: \frac{x_1-3}{1} = \frac{x_2+1}{1} = \frac{x_3+3}{-2}$

c) $D \cap O_{x_1x_2}: x_3 = 0$

$x_3 = -6t+3 = 0 \Rightarrow t = \frac{1}{2} \Rightarrow \begin{cases} x_1 = -\frac{3}{2}+3 = \frac{3}{2} \\ x_2 = \frac{5}{2}-1 = \frac{3}{2} \\ x_3 = 0 \end{cases}$

$D \cap O_{x_2x_3}: x_1 = 0$

$x_1 = -3t+3 = 0 \Rightarrow t = 1 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 4 \\ x_3 = -3 \end{cases}$

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$D \cap O_{x_3 x_1} : x_2 = 0$

$x_2 = 5t - 1 = 0 \Leftrightarrow t = \frac{1}{5} \Rightarrow \begin{cases} x_1 = -\frac{3}{5} + 3 = \frac{12}{5} \\ x_2 = 0 \\ x_3 = -\frac{6}{5} + 3 = \frac{9}{5} \end{cases}$

④ Fie dreptele

$D_1 : \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases}$ $\quad D_2 : \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

a) D_1, D_2 necoplanare

b) Ec dr $\perp$ comune a dreptelor D_1, D_2

c) Să se det dist (D_1, D_2)

a)

$D_1 : \begin{cases} x_1 = -t \\ x_2 = t + 1 \\ x_3 = t. \end{cases}$ $\quad v = (-1, 1, 1)$ - direcția lui D_1

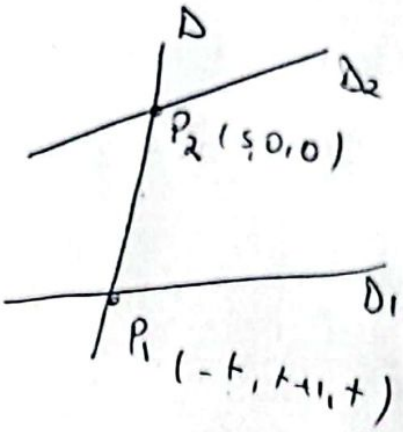
$D_1 \ni A(0, 1, 0)$ pt $t = 0$

$D_2 : \begin{cases} x_1 = 5 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$ $\quad v = (1, 0, 0)$ - direcția lui D_2

$O(0, 0, 0) \in D_2$

$$\begin{vmatrix} u & v & \overrightarrow{OA} \\ -1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \neq 0 \Rightarrow D_1, D_2 \text{ necoplanare}$$

b) Fie $D \perp D_1$ și $D \perp D_2$



$\overrightarrow{P_1P_2} = (s+t, -t-1, -t)$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Leftrightarrow \begin{cases} -s-t-t-1-t = 0 \\ s+t = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} t = -\frac{1}{2} \\ s = \frac{1}{2} \end{cases}$$

Deci $P_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ și $P_2\left(\frac{1}{2}, 0, 0\right)$

$\overrightarrow{P_1P_2} = \left(0, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}(0, -1, 1) \Rightarrow D: \frac{x_1 - \frac{1}{2}}{0} = \boxed{\frac{x_2 - \frac{1}{2}}{-1} = \frac{x_3 + \frac{1}{2}}{1}}$

$$\begin{cases} x_1 - \frac{1}{2} = 0 \\ x_2 + x_3 = 0 \end{cases}$$

c) $dist(D_1, D_2) = dist(P_1, P_2) = \|\overrightarrow{P_1P_2}\| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$

(*) $D_1: \frac{x_1 - 1}{1} = \frac{x_2 - 2}{-1} = \frac{x_3 + 2}{2}$

$$D_2: \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0 \end{cases}$$

a) Să se arate că D_1, D_2 coplanare

b) Ec planului det de D_1 și D_2

a) $u_1 = (1, -1, 2)$

$$\begin{cases} x_3 = t \\ x_1 = \frac{1}{2} + \frac{1}{2}t \\ x_2 = -\frac{3}{2} - \frac{1}{2}t \end{cases} \qquad v = \left(\frac{1}{2}, -\frac{1}{2}, 1\right) = \frac{1}{2}(1, -1, 2) \Rightarrow$$

$\Rightarrow D_1 \parallel D_2 \Rightarrow D_1, D_2$ coplanare.

b)

Fie $P_1(1, 2, -2) \in D_1$

$P_2(1, -2, 1) \in D_2$ ($p + t = 1$)

$P_1 \in \pi$

$v = (1, -1, 2)$

$\overrightarrow{P_1P_2} = (0, -4, 3)$

$$\pi: \begin{vmatrix} x_1 - 1 & 1 & 0 \\ x_2 - 2 & -1 & -4 \\ x_3 + 2 & 2 & 3 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow (x_1 - 1) \cdot 5 - (x_2 - 2) \cdot 3 + (x_3 + 2) \cdot (-4) = 0$$

$$5x_1 - 5 - 3x_2 + 6 - 4x_3 - 8 = 0$$

$$5x_1 - 3x_2 - 4x_3 - 7 = 0$$