22.04.2021

SEMINAR 10

Spații vectoriale euclidiene
Transformări ortogonale

OBS $(E, \langle \cdot, \cdot \rangle)$ s.v.e.r, $f \in End(E)$

- $f \in O(E)$ (transformare ortogonală) $\Leftrightarrow \langle f(x), f(y) \rangle = \langle x, y \rangle$ $\forall x, y \in E$

$\Leftrightarrow \|f(x)\| = \|x\|, \forall x \in E$.

- $f \in O(E) \Leftrightarrow A = [f]_{R,R} \in O(n)$ $\forall R =$ reper ortonormat $\Leftrightarrow$ schimbare de repere ortonormate

- $f \in O(E) \Rightarrow$ valorile proprii sunt ± 1.

Clasificare

① $\dim E = 1 \Rightarrow O(E) = \{id_E, -id_E\}$

② $\dim E = 2$

a) $\det A = 1$, $\exists R = \{e_1, e_2\}$ reper orton ai $A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$

b) $\det A = -1$ — // — $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

③ $\dim E = 3$

a) $\det A = 1$, $\exists R = \{e_1, e_2, e_3\}$ reper orton ai

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

$\to Tr A = 1 + 2\cos\varphi$

$\to$ Axa: $f(x) = x$

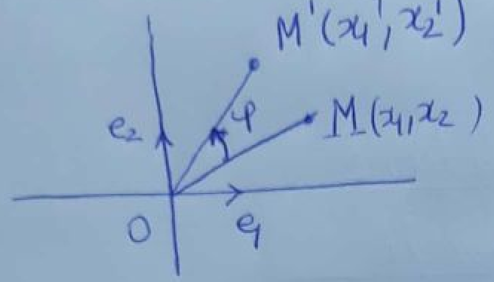
b) $\det A = -1$, $\exists R = \{e_1, e_2, e_3\}$ reper orton ai

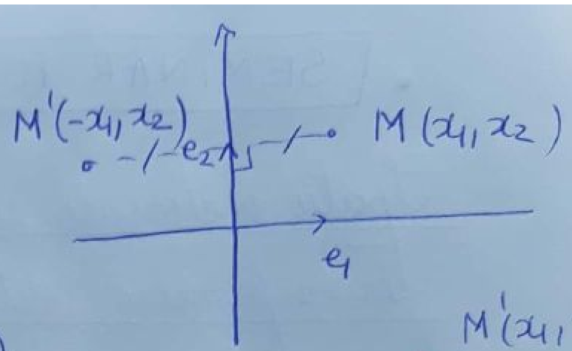
$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

$\to Tr A = -1 + 2\cos\varphi$

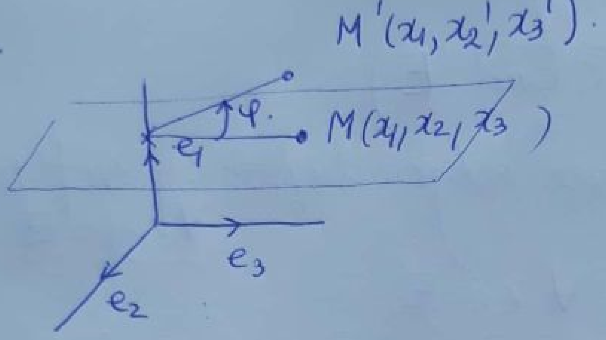
$\to$ Axa: $f(x) = -x$

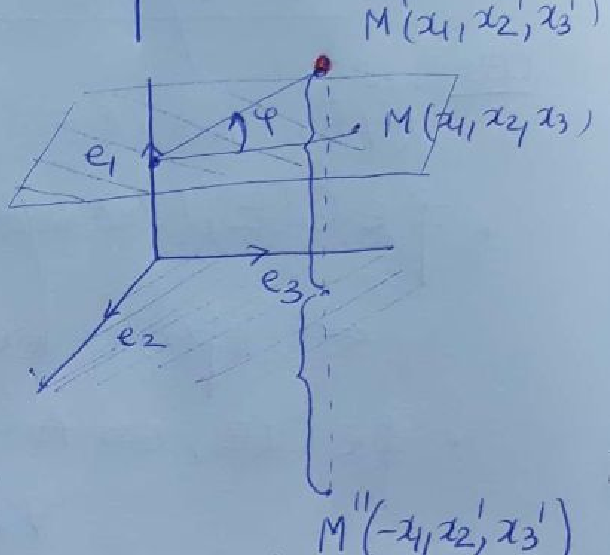
$n=2$





$n=3$.





Ex1. $(\mathbb{R}^3, g_0)$ s.v.e.r, cu str. canonică

$f \in End(\mathbb{R}^3)$, $A = [f]_{R_0,R_0} = \frac{1}{9}\begin{pmatrix} 8 & 1 & -4 \\ 1 & 8 & 4 \\ -4 & 4 & -7 \end{pmatrix}$

R_0 = reperul canonic

a) Să se arate că $f \in O(\mathbb{R}^3)$ de speță 2 $\Rightarrow f = s \circ R_\varphi$

b) Să se det ∢ de rot φ și axa de simetrie

c) Să se det un reper $R \overset{?}{=} \{e_1, e_2, e_3\}$ ortonormat

a.î. $[f]_{R,R} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$

SOL

a) Dem că $A \in O(3)$ i.e $AA^T = I_3$ și $\det A = -1$.

(M1) $A \cdot A^T = \frac{1}{9^2}\begin{pmatrix} 8 & 1 & -4 \\ 1 & 8 & 4 \\ -4 & 4 & -7 \end{pmatrix}\begin{pmatrix} 8 & 1 & -4 \\ 1 & 8 & 4 \\ -4 & 4 & -7 \end{pmatrix}$

$= \frac{1}{9^2}\begin{pmatrix} 9^2 & 0 & 0 \\ 0 & 9^2 & 0 \\ 0 & 0 & 9^2 \end{pmatrix} = I_3 \Rightarrow A \in O(3)$

Scanned with CamScanner

(M2) $\mathcal{R}_0 = \{e_1^0, e_2^0, e_3^0\} \xrightarrow{A} \mathcal{R}' = \{e_1', e_2', e_3'\}$.

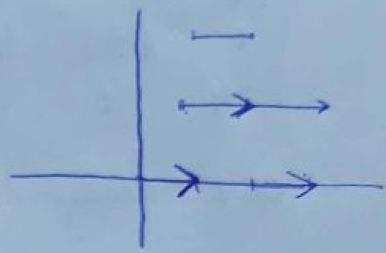
$f(e_i^0) = e_i'$

$e_1' = \frac{1}{9}(8,1,-4)$; $e_2' = \frac{1}{9}(1,8,4)$, $e_3' = \frac{1}{9}(-4,4,-7)$

$\|e_1'\| = \frac{1}{9}\sqrt{64+1+16} = \frac{1}{9}\cdot 9 = 1$

$\|e_2'\| = \frac{1}{9}\sqrt{1+64+16} = 1$

$\|e_3'\| = \frac{1}{9}\sqrt{16+16+49} = \frac{1}{9}\cdot 9 = 1$.



$\langle e_1', e_2' \rangle = \frac{1}{9^2}(8+8-16) = 0$

$\langle e_1', e_3' \rangle = \frac{1}{9^2}(-32+4+28) = 0$

$\langle e_2', e_3' \rangle = \frac{1}{9^2}(-4+32-28) = 0$

$\mathcal{R}'$ reper ortonormat $\Leftrightarrow f \in O(\mathbb{R}^3)$

$$\det A = \frac{1}{9^3}\begin{vmatrix} 8 & 1 & -4 \\ 1 & 8 & 4 \\ -4 & 4 & -7 \end{vmatrix} = \frac{1}{9^3}\begin{vmatrix} 0 & 1 & 0 \\ -63 & 8 & 36 \\ -36 & 4 & 9 \end{vmatrix}$$

$$= +\frac{1}{9^3}\cdot 9^2 \begin{vmatrix} 7 & 4 \\ 4 & 1 \end{vmatrix} = \frac{1}{9}(7-16) = -\frac{9}{9} = -1$$

$f \in O(\mathbb{R}^3)$ de speţa 2.

b) $Tr\, A = -1 + 2\cos\varphi = \frac{1}{9}(8+8-7) = 1$

$$\left.\begin{array}{l} 2\cos\varphi = 2 \Rightarrow \cos\varphi = 1 \\ \text{dar } \varphi \in (-\pi, \pi] \end{array}\right\} \Rightarrow \varphi = 0$$

Axa: $f(x) = -x$

$f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1,x_2,x_3) = \frac{1}{9}(8x_1 + x_2 - 4x_3,\ x_1 + 8x_2 + 4x_3,\ -4x_1 + 4x_2 - 7x_3)$

$$\begin{cases} 8x_1 + x_2 - 4x_3 = -9x_1 \\ x_1 + 8x_2 + 4x_3 = -9x_2 \\ -4x_1 + 4x_2 - 7x_3 = -9x_3 \end{cases} \Rightarrow \begin{cases} 17x_1 + x_2 - 4x_3 = 0 \\ x_1 + 17x_2 + 4x_3 = 0 \\ -4x_1 + 4x_2 + 2x_3 = 0 \end{cases}$$

Scanned with CamScanner

$$M = \left(\begin{array}{ccc|c} 17 & 1 & -4 & 0 \\ 1 & 17 & 4 & 0 \\ -4 & 4 & 2 & 0 \end{array}\right)$$

$\det M = 0$, $rg\, M = 2$.

$$\begin{vmatrix} 17 & 1 & -4 \\ 1 & 17 & 4 \\ -4 & 4 & 2 \end{vmatrix} = 2 \begin{vmatrix} 17 & 1 & -2 \\ 1 & 17 & 2 \\ -4 & 4 & 1 \end{vmatrix} = 2 \begin{vmatrix} 9 & 9 & -2 \\ 9 & 9 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

ec1 + ec2 : $18(x_1+x_2) = 0 \Rightarrow x_2 = -x_1$.

ec 3 : $-4x_1 - 4x_1 + 2x_3 = 0 \Rightarrow 8x_1 = 2x_3 \Rightarrow x_1 = \frac{x_3}{4}$

$x_2 = -\frac{x_3}{4}$

$(x_1, x_2, x_3) = \left(\frac{x_3}{4}, -\frac{x_3}{4}, x_3\right) = \frac{x_3}{4} \underbrace{(1,-1,4)}_{u}$

Axa $= \langle\{u\}\rangle$

$e_1 = \frac{1}{3\sqrt{2}}(1,-1,4)$ versorul axei

c) $\langle\{u\}\rangle^{\perp} = \{x \in \mathbb{R}^3 \mid \underbrace{g_0(x,u)}_{x_1 - x_2 + 4x_3} = 0\} = \{(x_2 - 4x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R}\}$

$x = (x_1, x_2, x_3)$

$u = (1,-1,4)$

$(x_2, x_2, 0) + (-4x_3, 0, x_3)$

$= \{x_2 \underbrace{(1,1,0)}_{f_2} + x_3 \underbrace{(-4,0,1)}_{f_3} \mid x_2, x_3 \in \mathbb{R}\}$

$\{f_2, f_3\}$ SG pt $\langle\{u\}\rangle^{\perp} \Rightarrow \{f_2, f_3\}$ reper în $\langle\{u\}\rangle^{\perp}$

$\mathbb{R}^3 = \underbrace{\langle\{u\}\rangle}_{1\text{-dim}} \oplus \underbrace{\langle\{u\}\rangle^{\perp}}_{2\text{-dim}}$

Aplicăm Gram - Schmidt

$\overline{e_2} = f_2$

$\overline{e_3} = f_3 - \frac{\langle f_3, \overline{e_2}\rangle}{\langle \overline{e_2}, \overline{e_2}\rangle} \cdot \overline{e_2}$

$\bar{e}_2 = (1,1,0)$

$\bar{e}_3 = (-4,0,1) - \frac{-4}{2}(1,1,0) = (-4,0,1) + (2,2,0) = (-2,2,1)$

$\{\bar{e}_2, \bar{e}_3\}$ reper ortog în $<\{u\}>^\perp$

$\left\{ e_2 = \frac{\bar{e}_2}{\|\bar{e}_2\|} = \frac{1}{\sqrt{2}}(1,1,0),\ e_3 = \frac{\bar{e}_3}{\|\bar{e}_3\|} = \frac{1}{3}(-2,2,1) \right\}$

reper ortonormat în $<\{u\}>^\perp$

$R = \{e_1, e_2, e_3\}$ reper orton în $\mathbb{R}^3$ a.î.

$$[f]_{R,R} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Ex2 $(\mathbb{R}^3, g_0)$, $u = (1,1,0)$

a) $<\{u\}>^\perp = ?$. Să se precizeze un reper ortonormat

b) Să se det transf. ortogonală de speță 1, care este rotație de $\measuredangle$ orientat $\varphi = \frac{\pi}{2}$ și axă $<\{u\}>$.

SOL

a) $<\{u\}>^\perp = \{x \in \mathbb{R}^3 \mid \underbrace{g_0(u,x)}_{x_1+x_2} = 0\} = \{\underbrace{(-x_2, x_2, x_3)}_{x_2(-1,1,0)+x_3(0,0,1)} \mid x_2, x_3 \in \mathbb{R}\}$

$f_2 = (-1,1,0)$, $f_3 = (0,0,1)$

$\left.\begin{array}{l} \{f_2, f_3\} \text{ reper în } <\{u\}>^\perp \\ g_0(f_2, f_3) = 0 \end{array}\right\} \Rightarrow$ reper ortogonal

$\left\{ e_2 = \frac{1}{\sqrt{2}}(-1,1,0),\ e_3 = (0,0,1) \right\}$ reper ortonormat în $<\{u\}>^\perp$.

b) $e_1 = \frac{u}{\|u\|} = \frac{1}{\sqrt{2}}(1,1,0)$ versorul axei

$R = \{e_1, e_2, e_3\}$ reper ortonormat în $\mathbb{R}^3$

$$[f]_{R,R} = A' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$R_0 = \{e_1^0, e_2^0, e_3^0\} \xrightarrow{C} R = \{e_1, e_2, e_3\}$ repere orton

reperul canonic

$C \in O(3)$

$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$e_1 = \frac{1}{\sqrt{2}}(1,1,0) = \frac{1}{\sqrt{2}}\underbrace{(1,0,0)}_{e_1^0} + \frac{1}{2}\underbrace{(0,1,0)}_{e_2^0} + 0 \cdot e_3^0$$

$$\boxed{A' = C^{-1}AC} \Rightarrow A = CA'C^{-1} = CA'C^T$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & -\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} =$$

$$A = \frac{1}{2}\begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ -\sqrt{2} & \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = \frac{1}{2}\left(x_1 + x_2 + \sqrt{2}x_3,\ x_1 + x_2 - \sqrt{2}x_3,\ -\sqrt{2}x_1 + \sqrt{2}x_2\right)$

$f \in O(\mathbb{R}^3)$ de speță 1

Ex3 $(E, \langle \cdot, \cdot \rangle)$ s.v.e.r, $u, v \in E$

UAE

1) $u \perp v$

2) $\|u+v\| = \|u-v\|$

3) $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

SOL.

1) $\Leftrightarrow$ 2

$u \perp v \Leftrightarrow \langle u, v\rangle = 0$

$\circledast \ \|u+v\|^2 = \langle u+v, u+v\rangle = \|u\|^2 + \|v\|^2 + \underline{2\langle u, v\rangle}$

$\|u-v\|^2 = \langle u-v, u-v\rangle = \|u\|^2 + \|v\|^2 - 2\langle u, v\rangle$

$\|u+v\| = \|u-v\| \Leftrightarrow \langle u, v\rangle = 0 \Leftrightarrow u \perp v$

1) $\Leftrightarrow$ 3)

$\|u+v\|^2 = \|u\|^2 + \|v\|^2 \Leftrightarrow \langle u, v\rangle = 0 \Leftrightarrow u \perp v$

Ex 4 $(\mathcal{M}_2(\mathbb{R}), +, \cdot)/\mathbb{R}$

$g: \mathcal{M}_2(\mathbb{R}) \times \mathcal{M}_2(\mathbb{R}) \longrightarrow \mathbb{R},$

$g(X, Y) = 2\,Tr(X \cdot Y) - Tr(X) \cdot Tr(Y), \ \forall X, Y \in \mathcal{M}_2(\mathbb{R})$

Este g produs scalar?

SOL

g produs scalar $\Leftrightarrow$ g formă biliniară, simetrică poz. definită.

$Tr(X \cdot Y) = Tr(Y \cdot X) \Rightarrow g(X, Y) = g(Y, X) \Rightarrow g$ simetrică

$g(aX + bZ, Y) \overset{?}{=} a\,g(X, Z) + b\,g(Z, Y)$
$\forall X, Y, Z \in \mathcal{M}_2(\mathbb{R})$
$\forall a, b \in \mathbb{R}$

$g(aX + bZ, Y) = 2\,Tr\underbrace{((aX + bZ)Y)}_{aXY + bZY} - Tr(aX + bZ)\,Tr\,Y$

$= \underline{2a\,Tr(XY)} + \underline{2b\,Tr(ZY)} - \underline{a\,Tr(X)\,Tr\,Y} - b\,\underline{Tr(Z)}\,Tr(Y)$

$= a\,g(X, Y) + b\,g(Z, Y)$

$\left.\begin{array}{l} g \text{ simetrică} \\ g \text{ liniară în primul arg} \end{array}\right\} \Rightarrow g$ biliniară

$Q: \mathcal{M}_2(\mathbb{R}) \longrightarrow \mathbb{R}, \quad Q(X) = g(X, X) = 2Tr(X^2) - (Tr X)^2$

$$X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \equiv (x_1, x_2, x_3, x_4)$$

$$X^2 = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} x_1^2 + x_2 x_3 & x_1 x_2 + x_2 x_4 \\ x_1 x_3 + x_3 x_4 & x_4^2 + x_2 x_3 \end{pmatrix}$$

$$Q(X) = 2(x_1^2 + x_4^2 + 2x_2 x_3) - (x_1 + x_4)^2$$
$$= \underline{x_1^2} + x_4^2 + 4x_2 x_3 - \underline{2x_1 x_4}.$$

$$G = \begin{pmatrix} \boxed{1} & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

$$Q(X) = (x_1 - x_4)^2 + 4x_2 x_3$$

$$\begin{cases} x_1' = x_1 - x_4 \\ x_k' = x_k, \quad k \in \{2,3,4\} \end{cases} \Rightarrow Q(X) = x_1'^2 + \underline{4x_2' x_3'}$$

$$\begin{cases} x_2'' = x_2' + x_3' \\ x_3'' = x_2' - x_3' \\ x_k'' = x_k', \quad k \in \{1,4\} \end{cases} \Rightarrow \begin{cases} x_2' = \frac{1}{2}(x_2'' + x_3'') \\ x_3' = \frac{1}{2}(x_2'' - x_3'') \\ x_k' = x_k'', \quad k \in \{1,4\} \end{cases}$$

$$Q(X) = x_1''^2 + x_2''^2 - x_3''^2$$

Signatura: $(2, -1) \Rightarrow Q$ nu e poz def
$\Rightarrow g$ -"-

Deci g nu e produs scalar.

<u>Ex5</u> $(\mathbb{R}^3, g_0)$, $U = \langle \{ \underset{f_1}{(1,0,1)}, \underset{f_2}{(1,1,2)} \} \rangle$

a) $U^\perp = ?$

b) Să se determine $R = R_1 \cup R_2$ reper ortonormat în $\mathbb{R}^3$

$-g-$

a.î. $\mathcal{R}_1$ reper ortonormat în U

$\mathcal{R}_2$ $\quad$ — $\,$ — $\quad$ $U^\perp$

Sol

a) M1 $\{f_1, f_2\}$ SLI $\quad rg\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{pmatrix} = 2$

$\dim U = 2 \Rightarrow \dim U^\perp = 1 \qquad \mathbb{R}^3 = U \oplus U^\perp$

$f_1 \times f_2 \perp f_k, \ k = \overline{1,2}$

$$f_1 \times f_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix} = e_1 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} - e_2 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + e_3 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$= (-1, -1, 1) \Rightarrow U^\perp = \langle \{(-1,-1,1)\} \rangle.$

M2 $U^\perp = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} g_0(x, f_1) = 0 \\ g_0(x, f_2) = 0 \end{cases} \right\}$

$= \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases} \right\} = \{(-x_3, -x_3, x_3) \mid x_3 \in \mathbb{R}\}$

$(-x_3, -x_3, x_3) = x_3(-1,-1,1)$

$$\begin{cases} x_1 = -x_3 \\ x_2 = x_3 - 2x_3 = -x_3 \end{cases}$$

b) $\{f_1, f_2\}$ reper $\forall$ în U. Aplicăm G-S.

$$\begin{cases} e_1 = f_1 = (1,0,1) \\ e_2 = f_2 - \dfrac{\langle f_2, e_1 \rangle}{\langle e_1, e_1 \rangle} e_1 = (1,1,2) - \dfrac{3}{2}(1,0,1) = \left(-\dfrac{1}{2}, 1, \dfrac{1}{2}\right) \end{cases}$$

$= \frac{1}{2}(-1, 2, 1)$

$\{e_1, e_2\}$ reper ortogonal în U.

$\mathcal{R}_1 = \left\{ e_1' = \frac{1}{\sqrt{2}}(1,0,1),\ e_2' = \frac{1}{\sqrt{6}}(-1,2,1) \right\}$ reper ortonormat în U

$\mathcal{R}_2 = \left\{ e_3' = \frac{1}{\sqrt{3}}(-1,-1,1) \right\}$ reper orton în $U^\perp$

$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ reper ortonormat în $\mathbb{R}^3$.

Ex6 $\left(\mathbb{R}_2[X],+,\cdot\right)/\mathbb{R}$, $g: \mathbb{R}_2[X]\times\mathbb{R}_2[X]\longrightarrow\mathbb{R}$

$$g(P,Q)=\sum_{k=0}^{2}a_k b_k, \quad P=a_0+a_1X+a_2X^2\equiv(a_0,a_1,a_2)\in\mathbb{R}^3$$
$$Q=b_0+b_1X+b_2X^2\equiv(b_0,b_1,b_2)\in\mathbb{R}^3$$

$\mathbb{R}_2[X]\simeq\mathbb{R}^3$

g produs scalar canonic.

Să se ortonormeze $\{2,\ 3-2X,\ 1-2X+X^2\}$

Sol

$$\{\underset{f_1}{(2,0,0)},\ \underset{f_2}{(3,-2,0)},\ \underset{f_3}{(1,-2,1)}\} \text{ reper în } \mathbb{R}^3$$

$$\det\begin{pmatrix}2&3&1\\0&-2&-2\\0&0&1\end{pmatrix}\neq 0\Rightarrow SLI$$

$e_1=f_1=(2,0,0)=\underline{2(1,0,0)}$

$$e_2=f_2-\frac{\langle f_2,e_1\rangle}{\langle e_1,e_1\rangle}e_1=(3,-2,0)-\frac{6}{4}(2,0,0)=$$
$$=(3,-2,0)-(3,0,0)=\underline{(0,-2,0)}=2(0,-1,0)$$

$$e_3=f_3-\frac{\langle f_3,e_1\rangle}{\langle e_1,e_1\rangle}e_1-\frac{\langle f_3,e_2\rangle}{\langle e_2,e_2\rangle}\cdot e_2=$$
$$=(1,-2,1)-\frac{2}{4}(2,0,0)-\frac{4}{4}(0,-2,0)$$
$$=(1,-2,1)-(1,0,0)+(0,2,0)=(0,0,1)$$

$\{e_1'=(1,0,0),\ e_2'=(0,-1,0),\ e_3'=(0,0,1)\}$

$\{1,-X,X^2\}$ reper ortonormat în $\mathbb{R}_2[X]$ în raport cu g.

T5 (seminar)

Ex 1 $(\mathbb{R}^3, g_0)$, $u = (0,1,-1)$

Să se determine transf. ortog. de speță 1, care este rotație de $\sphericalangle \varphi = \pi$ și axă $<\{u\}>$.

Ex 2 $(\mathbb{R}_2[X], g_0)$, $g_0(P,Q) = \sum_{i=0}^{2} a_i b_i$, $P = a_0 + a_1 X + a_2 X^2$
$Q = b_0 + b_1 X + b_2 X^2$

Să se ortonormeze $R = \{X, X - X^2, 1 + X + X^2\}$ în rap cu g_0

Ex 3 $(\mathbb{R}^3, g_0)$, $U = \left\{ x \in \mathbb{R}^3 \mid \begin{cases} x_1 + x_3 = 0 \\ 3x_2 + x_3 = 0 \end{cases} \right\}$

a) $U^\perp$

b) Să se determine $R = R_1 \cup R_2$ reper ortonormat în $\mathbb{R}^3$ a.î. R_1 reper ortonormat în U

R_2 — // — $U^\perp$.