C_6

Aplicații liniare.
Endomorfisme. Vectori proprii. Valori proprii

$(V_i, +, \cdot)/_{\mathbb{K}}$, $i = \overline{1,2}$ sp. vect

- $f : V_1 \longrightarrow V_2$ apl. lin $\iff f(ax+by) = af(x)+bf(y)$, $\forall x, y \in V_1$, $\forall a, b \in \mathbb{K}$.

$$\iff f\left(\sum_{i=1}^{m} a_i x_i\right) = \sum_{i=1}^{n} a_i f(x_i)$$

$\forall x_1, \dots, x_n \in V_1$, $\forall a_1, \dots, a_n \in \mathbb{K}$.

- $V_1 \simeq V_2 \iff \dim_{\mathbb{K}} V_1 = \dim_{\mathbb{K}} V_2$.

Def $(V, +, \cdot)/_{\mathbb{K}}$ sp. vect

$(V^* = \{ f : V \longrightarrow \mathbb{K} \mid f \text{ liniară} \}, +, \cdot)/_{\mathbb{K}}$

sp. vectorial dual.

$+ : V^* \times V^* \longrightarrow V^*$

$(f+g)(x) := f(x) + g(x)$, $\forall x \in V$

$\cdot : \mathbb{K} \times V^* \longrightarrow V^*$

$(af)(x) := a f(x)$, $\forall x \in V$, $\forall a \in \mathbb{K}$.

Teoremă $V \simeq V^*$

Dem

Fie $\mathcal{R} = \{e_1, \dots, e_n\}$ reper în V, $\dim_{\mathbb{K}} V = n$.

Construim $\mathcal{R}^* = \{e_1^*, \dots, e_n^*\} \subset V^*$ a.î.

$e_i^* : V \longrightarrow \mathbb{K}$ liniară, $\forall i = \overline{1,n}$

$$e_i^*(e_j) := \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Prelungim e_i^* prin liniaritate.

$$e_i^*(x) = e_i^*\left(\sum_{j=1}^{n} x_j e_j\right) = \sum_{j=1}^{n} x_j \underbrace{e_i^*(e_j)}_{\delta_{ij}} = x_i, \ \forall i = \overline{1,n}$$

Dem că $\mathcal{R}^*$ este un reper în V^*

① $\mathcal{R}^*$ este SLI

Fie $a_1, \dots, a_n \in \mathbb{K}$ a.î. $\sum_{i=1}^{n} a_i e_i^* = 0 \quad | \, e_j$

$$\sum_{i=1}^{n} a_i \underbrace{e_i^*(e_j)}_{\delta_{ij}} = 0 \Rightarrow a_j = 0, \ \forall j = \overline{1,n} \Rightarrow \text{SLI}$$

② $\mathcal{R}^*$ este SG.

$\forall f \in V^*$ i.e. $f : V \longrightarrow \mathbb{K}$ liniară

$$f(x) = f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} \underbrace{x_i}_{e_i^*(x)} f(e_i) = \sum_{i=1}^{n} f(e_i) e_i^*(x), \ \forall x \in V$$

$$\Rightarrow f = \sum_{i=1}^{n} f(e_i) e_i^* \Rightarrow \text{SG}.$$

$\Rightarrow \mathcal{R}^* = \{e_1^*, \dots, e_n^*\}$ reper în $V^* \Rightarrow \dim_{\mathbb{K}} V^* = n$

$\Rightarrow \dim_{\mathbb{K}} V = \dim_{\mathbb{K}} V^* \Rightarrow V \simeq V^*$ (izomorfe)

<u>OBS</u> $\varphi : V \longrightarrow V^*$

$$\varphi(e_i) = e_i^*, \ \forall i = \overline{1,n}$$

$$\varphi(x) = \varphi\left(\sum_{i=1}^{n} x_i e_i\right) = \sum x_i \varphi(e_i) = \sum_{i=1}^{n} x_i e_i^*$$

φ izomorfism de spații vectoriale.

Exemple de endomorfisme

<u>Def</u> $p : V_1 \oplus V_2 \longrightarrow \underline{\underline{V_1}} \oplus V_2$ apl. lin.

p s.n. proiecție pe V_1, de-a lungul lui $V_2 \Leftrightarrow$

$p(v_1 + v_2) = v_1$

Prop Fie $p \in End(V)$

p proiecție $\iff p \circ p = p$.

Dem "$\Rightarrow$" Ip: p = proiecție pe V_1 de-a lungul lui V_2

$V = V_1 \oplus V_2$, $p : V \to V$ liniară

$p(\underbrace{v_1 + v_2}_{v}) = v_1$, $v_1 \in V_1$, $v_2 \in V_2$.

$p^2(v) = p(p(v)) = p(v_1) = p(\underset{V_1}{v_1} + \underset{V_2}{0_V}) = v_1 = p(v)$, $\forall v \in V$

$\Rightarrow p^2 = p$

"$\Leftarrow$" Ip: $p \in End(V)$, $p^2 = p$.

Construim $V_1 = Im\, p$, $V_2 = Ker\, p$.

$V = V_1 \oplus V_2$.

"$\supseteq$" din constr

"$\subseteq$" dem.

$\forall v \in V$, $v = \underset{Im f}{p(v)} + \underset{Ker ?}{\boxed{v - p(v)}}$

$p(v - p(v)) \overset{lin}{=} p(v) - p(p(v)) = p(v) - p(v) = 0$

$\Rightarrow v - p(v) \in Ker$.

$V = V_1 + V_2$

"$\oplus$": $V_1 \cap V_2 = \{0_V\}$.

Fie $v \in Im\, p \cap Ker\, p$.

$\exists w \in V$ a.î. $v = p(w)$ $\mid \circ p \Rightarrow \underset{\overset{\|}{0_V}}{p(v)} = p(\underset{\overset{\|}{p(w)} \overset{\|}{v}}{p(w)})$

$p : V = Im\, p \oplus Ker\, p \to V$

$p(v_1 + \underset{Ker\, p}{v_2}) = p(v_1) = v_1$.

p = proiecția pe $V_1 = Im\, p$, de-a lungul lui $V_2 = Ker\, p$.

OBS Fie $\mathcal{R} = \{e_1, \dots, e_k, e_{k+1}, \dots, e_n\}$ reper în V a.î.
$\mathcal{R}_1 = \{e_1, \dots, e_k\}$ reper în $V_1 = \operatorname{Im} p$
$\mathcal{R}_2 = \{e_{k+1}, \dots, e_n\}$ —"— în $V_2 = \operatorname{Ker} p$.

$p(e_i) = e_i, \ \forall i = \overline{1,k}$
$p(e_j) = 0_V, \ \forall j = \overline{k+1,n}$

$$A_p = [p]_{\mathcal{R},\mathcal{R}} = \left(\begin{array}{c|c} I_k & O_{k,n-k} \\ \hline O_{n-k,k} & O_{n-k,n-k} \end{array}\right) \qquad A_p \notin O(n)$$

$p : V \longrightarrow V$

Def. $s \in \operatorname{End}(V)$
s s.n. <u>simetrie</u> sau <u>involuție</u> $\Leftrightarrow s \circ s = id_V$.

Prop $(V, +, \cdot)/_{\mathbb{K}}$ sp. vect, <u>$\operatorname{ch} \mathbb{K} \neq 2$</u> $(1+1 \neq 0)$
$p \in \operatorname{End}(V)$
p proiecție $\Leftrightarrow s = 2p - id_V$ este simetrie

Dem
"$\Rightarrow$" Ip: p = proiecție $\Leftrightarrow p^2 = p$.
Dem că $s^2 = id_V$.
$$s \circ s = (2p - id_V) \circ (2p - id_V) = 4p^2 - 2p - 2p + id_V$$
$$= 4p^2 - 4p + id_V = id_V$$

"$\Leftarrow$" Ip: $s = 2p - id_V$ simetrie
Dem că $p^2 = p$.
$$s^2 = id \Rightarrow 4p^2 - 4p + id_V = id_V \Rightarrow p^2 = p.$$

OBS. $\mathcal{R} = \{e_1, \dots, e_k, e_{k+1}, \dots, e_n\}$ reper în V, $\mathcal{R}_1 = \{e_1, \dots, e_k\}$, $\mathcal{R}_2 = \{e_{k+1}, \dots, e_n\}$
repere în $V_1 = \operatorname{Im} p$, $V_2 = \operatorname{Ker} p$.
$$s = 2p - id_V \Rightarrow A_s = 2A_p - I_n = \left(\begin{array}{c|c} 2I_k & 0 \\ \hline 0 & 0 \end{array}\right) - \left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & I_{n-k} \end{array}\right)$$

$$A_s = [s]_{\mathcal{R},\mathcal{R}} = \left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & -I_{n-k} \end{array}\right)$$

$$A_s \cdot A_s^T = I_n \implies A_s \in O(n)$$

<u>Aplicatie</u>

$f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = \underline{(x_1 + x_2 - x_3,\ x_1 + x_2,\ x_1 + x_2 + x_3)}$

a) $\mathrm{Im} f = ?$

b) Să se precizeze un reper în $\mathrm{Im} f$

<u>SOL</u>

$\mathcal{R}_0 = \{e_1, e_2, e_3\}$ reper canonic în $\mathbb{R}^3$ $\qquad f(x) = y \Leftrightarrow Y = AX$

$$A_f = [f]_{\mathcal{R}_0, \mathcal{R}_0} = \begin{pmatrix} 1 & \boxed{\begin{matrix} 1 & -1 \\ 1 & 0 \end{matrix}} \\ 1 & \\ 1 & \begin{matrix} 1 & \ \ 1 \end{matrix} \end{pmatrix}$$

$\mathrm{rg}\, A = 2$.

$\mathrm{Ker} f = \{x \in \mathbb{R}^3 \mid AX = 0\}$, $\dim \mathrm{Ker} f = 3 - 2 = 1$.

$$\begin{cases} x_2 - x_3 = -x_1 \\ x_2 \qquad = -x_1 \end{cases} \implies x_3 = 0.$$

$\mathrm{Ker} f = \{(x_1, -x_1, 0) \mid x_1 \in \mathbb{R}\} = \langle\{(1,-1,0)\}\rangle$.

$\boxed{\text{Met 1}}$

Prelungim la un reper în $\mathbb{R}^3$

$\mathcal{R} = \{(1,-1,0), \underline{(0,1,0)}, \underline{(0,0,1)}\}$. reper în $\mathbb{R}^3$

$$\det\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \neq 0$$

$\mathcal{R}' = \{f(0,1,0),\ f(0,0,1)\}$ reper în $\mathrm{Im} f$.

$f(0,1,0) = (1,1,1)$, $\quad f(0,0,1) = (-1,0,1)$

Th. dim: $\underset{3}{\dim \mathbb{R}^3} = \dim \mathrm{Ker} f + \dim \mathrm{Im} f \implies \dim \mathrm{Im} f = 2$

$\mathrm{Im} f = \{a(1,1,1) + b(-1,0,1) \mid \underline{a, b \in \mathbb{R}}\}$

$a(1,1,1) + b(-1,0,1) = (a-b, a, a+b)$

$$\begin{cases} a - b = y_1 \\ a \qquad = y_2 \\ a + b = y_3 \end{cases} \qquad \begin{pmatrix} \boxed{\begin{matrix} 1 & -1 \\ 1 & 0 \end{matrix}} \\ \begin{matrix} 1 & 1 \end{matrix} \end{pmatrix}$$

$\Delta_c = 0$

$$\Delta_c = \begin{vmatrix} 1 & -1 & y_1 \\ 1 & 0 & y_2 \\ 1 & 1 & y_3 \end{vmatrix} = 0 \implies \boxed{y_1 - y_2 \cdot 2 + y_3 = 0}$$

Met2

$$\operatorname{Im} f = \{ y \in \mathbb{R}^3 \mid \underline{\exists x \in \mathbb{R}^3} \text{ ai } f(x) = y \}$$

$$\begin{cases} x_1 + x_2 - x_3 = y_1 \\ x_1 + x_2 = y_2 \\ x_1 + x_2 + x_3 = y_3 \end{cases} \qquad \text{SCN} \qquad \left(\begin{array}{ccc|c} 1 & 1 & -1 & y_1 \\ 1 & 1 & 0 & y_2 \\ 1 & 1 & 1 & y_3 \end{array}\right)$$

$$\Delta_p = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} \neq 0$$

$$\Delta_c = \begin{vmatrix} 1 & -1 & y_1 \\ 1 & 0 & y_2 \\ 1 & 1 & y_3 \end{vmatrix} = 0 \Rightarrow y_1 - 2y_2 + y_3 = 0$$

$$\operatorname{Im} f = \{ y \in \mathbb{R}^3 \mid y_1 - 2y_2 + y_3 = 0 \}$$

$$y_1 = 2y_2 - y_3 .$$

$$\operatorname{Im} f = \{ (\underbrace{2y_2 - y_3}_{y_1}, y_2, y_3) \mid y_2, y_3 \in \mathbb{R} \}$$

$$y_2(2,1,0) + y_3(-1,0,1)$$

$$= \langle \{ (2,1,0), (-1,0,1) \} \rangle$$

$\dim \operatorname{Im} f = 2$.

$\{ (2,1,0), (-1,0,1) \}$ reper în $\operatorname{Im} f$.

Aplicatie 2. $V = \langle \{ (1,1,0), (1,0,0) \} \rangle$, $V \subset \mathbb{R}^3$.

$p : \mathbb{R}^3 \to \mathbb{R}^3$ proiectia pe V, de-a lungul lui W, unde $\mathbb{R}^3 = V \oplus W$

a) $p(\underline{1,2,1}) = ?$ b) $s(1,2,1) = ?$ s = simetria fata de V.

SOL.

$$\operatorname{rg}\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} = 2 \qquad W = \langle \{ (0,0,1) \} \rangle .$$

$$\mathbb{R}^3 = V \oplus W$$

$$\mathcal{R} = \{ (1,1,0), (1,0,0), (0,0,1) \} \text{ reper în } \mathbb{R}^3$$

$$(1,2,1) = \boxed{a(1,1,0) + b(1,0,0)} + \underline{c(0,0,1)} = (a+b, a, c)$$

$a = 2 \Rightarrow b = -1$

$c = 1$

$$(1,2,1) = \underset{V}{\underset{\cap}{v}} + \underset{W}{\underset{\cap}{w}}, \quad v = 2(1,1,0) - (1,0,0) = (1,2,0)$$

$p(1,2,1) = (1,2,0)$

b) $s = 2p - id_{\mathbb{R}^3}$

$s(1,2,1) = 2(1,2,0) - (1,2,1) = (1,2,-1)$

<u>Problemă</u>

$f \in End(V)$.

Determinarea unui reper $\mathcal{R} = \{e_1, .., e_n\}$ în V a.î. $A_f = [f]_{\mathcal{R},\mathcal{R}}$ să fie diagonală

$$A_f = \begin{pmatrix} \alpha_1 & & 0 \\ & \ddots & \\ 0 & & \alpha_n \end{pmatrix}$$

$f(e_1) = \alpha_1 e_1$
$f(e_2) = \alpha_2 e_2$
$f(e_n) = \alpha_n e_n$

$\alpha_i \in \mathbb{K}$
$i = \overline{1,n}$

<u>Def</u> $f \in End(V)$

$x \neq 0_V$ s.n. <u>vector propriu</u> $\Leftrightarrow$ <u>$\exists \lambda \in \mathbb{K}$</u> a.î. $f(x) = \lambda x$.

λ s.n. valoare proprie asociată vectorului propriu x.

Not $V_\lambda = \{0_V\} \cup \{$vectorilor proprii asoc. lui $\lambda\}$.

<u>Prop</u> $f \in End(V)$, <u>$f(x) = \lambda x$</u>, $\lambda \in \mathbb{K}$ val proprie.

a) $V_\lambda \subseteq V$ subsp. vect

b) V_λ subsp. invariant al lui f i.e. $f(V_\lambda) \subseteq V_\lambda$.

<u>Dem</u>

a) $\forall x, y \in V_\lambda$, $\forall a, b \in \mathbb{K}$ $\overset{?}{\Longrightarrow}$ $ax + by \in V_\lambda$

$f(ax + by) = a f(x) + b f(y) = a\lambda x + b\lambda y = \lambda(ax + by)$

$\Rightarrow V_\lambda \subseteq V$ subsp. vect.

b) Fie $x \in V_\lambda \Rightarrow f(x) = \lambda x \in V_\lambda \Rightarrow$ subsp. invar.

<u>OBS</u> $f(0_V) = 0_V = \lambda \cdot 0_V$

Polinom caracteristic

Fie $f \in End(V)$, $f(x) = \lambda x$, $x \neq 0_V$.

$$f(x) = f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} x_i f(e_i) =$$

$$\| \quad = \sum_{i=1}^{n} x_i \left(\sum_{j=1}^{n} a_{ji} e_j\right) = \underbrace{\sum_{j=1}^{n}\left(\sum_{i=1}^{n} a_{ji} x_i\right) e_j}$$

$$\lambda x = \lambda \underbrace{\sum_{j=1}^{n} x_j e_j}$$

$$\sum_{i=1}^{n} a_{ji} x_i = \lambda x_j \quad , \forall j = \overline{1,n}$$

$$(*) \quad \sum_{i=1}^{n} (a_{ji} - \lambda \delta_{ji}) x_i = 0 \quad , \forall j = \overline{1,n}$$

$(*)$ este SLO care are și sol. nenule $\Rightarrow$

$P(\lambda) = \det(A - \lambda I_n) = 0$ (polinomul caracteristic)

<u>Prop</u> polinomul caracteristic este invariant la schimbarea de reper.

<u>Dem</u>

$$\begin{array}{ccc} R = \{e_1, .., e_n\} & \xrightarrow{A_f} & R = \{e_1, .., e_n\} \\ C \downarrow & & \downarrow C \\ R' = \{e_1', .., e_n'\} & \xrightarrow{A_f'} & R' = \{e_1', .., e_n'\} \end{array} \qquad A' = C^{-1} A C$$

$$\det(A' - \lambda I_n) = \det(C^{-1} A C - \lambda C^{-1} I_n C)$$

$$= \det\left[C^{-1}(A - \lambda I_n) C\right] = \det C^{-1} \det(A - \lambda I_n) \det C$$

$$= \det(A - \lambda I_n)$$

<u>OBS</u>. valorile proprii sunt <u>rădăcinile din $\mathbb{K}$</u> ale polinomului caracteristic.

OBS

$P(\lambda) = \det(A - \lambda I_n) = 0 \Rightarrow$

$\lambda^n - \sigma_1 \lambda^{n-1} + \dots + (-1)^n \sigma_n = 0$;

σ_k = suma minorilor diagonali de ord k.

$\sigma_1 = Tr A, \dots, \sigma_n = \det A$

Ex. $J : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $J(x) = (-x_2, x_1)$

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\qquad \lambda^2 - Tr(A)\lambda + \det(A) = 0.$

$\lambda^2 + 1 = 0 \Rightarrow \lambda_{1,2} = \pm i \notin \mathbb{R}.$

$(\mathbb{R}^2, +, \cdot)/\mathbb{R}$

Aplicatie

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x) = (x_1 + 2x_2, 2x_1 + x_2)$.

a) val. proprii

b) Subsp. proprii și repere în fiecare.

SOL. $R_0 = \{(1,0), (0,1)\}$.

$A = [f]_{R_0, R_0} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

$P(\lambda) = 0 = \det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0$

$\lambda^2 - Tr(A)\lambda + \det A = 0 \Rightarrow \lambda^2 - 2\lambda - 3 = 0 \Rightarrow$

$(\lambda+1)(\lambda-3) = 0 \quad \lambda_1 = -1, \quad \lambda_2 = 3.$

$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid f(x) = \lambda_1 x\}$, $AX = \lambda_1 X$ $\qquad \begin{cases} x_1 + 2x_2 = -x_1 \\ 2x_1 + x_2 = -x_2 \end{cases} \Rightarrow \begin{cases} 2x_1 + 2x_2 = 0 \\ 2x_1 + 2x_2 = 0 \end{cases} \Rightarrow x_2 = -x_1.$

$= \{(x_1, -x_1) / x_1 \in \mathbb{R}\} = \langle\{(1,-1)\}\rangle.$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid f(x) = \lambda_2 x\}$ $\qquad \begin{cases} x_1 + 2x_2 = 3x_1 \\ 2x_1 + x_2 = 3x_2 \end{cases} \Rightarrow \begin{cases} -2x_1 + 2x_2 = 0 \\ 2x_1 - 2x_2 = 0 \end{cases}$

$x_1 = x_2$ $\qquad = \{(x_1, x_1) \mid x_1 \in \mathbb{R}\} = \langle\{(1,1)\}\rangle.$

$\mathbb{R}^2 = V_{\lambda_1} \oplus V_{\lambda_2}$. $\qquad R = \{(1,-1), (1,1)\}$ reper în $\mathbb{R}^2$

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$A' = [f]_{R,R}$

$A' = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$

- $f(1,-1) = -(1,-1)$
  $f(1,1) = 3(1,1)$

OBS $P(\lambda) = 0 \Rightarrow (\lambda-\lambda_1)^{m_1} \cdot \ldots \cdot (\lambda-\lambda_r)^{m_r} = 0$

$\lambda_1, \ldots, \lambda_r$ = valori proprii dist.

$m_1, \ldots, m_r$ = multiplicitățile.

$\sigma(f) = \{\lambda_1, \ldots, \lambda_r\}$ spectrul

$Spec(f) = \{\underbrace{\lambda_1 = \ldots = \lambda_1}_{m_1}, \ldots, \underbrace{\lambda_r = \ldots = \lambda_r}_{m_r}\}$.

OBS a) $f \in Aut(V)$

$f(x) = \lambda x \Rightarrow f^{-1} \circ f(x) = f^{-1}(\lambda x)$

$x = \lambda f^{-1}(x)$.

- $\lambda \in \sigma(f) \Leftrightarrow \lambda^{-1} \in \sigma(f^{-1})$

b) $f \in End(V)$

$f(x) = \lambda x \Rightarrow f \circ f(x) = f(\lambda x) = \lambda f(x) = \lambda^2 x$.

$f^m(x) = \lambda^m x$

$\lambda \in \sigma(f) \Rightarrow \lambda^m \in \sigma(f)$

$\boxed{\text{Temă 3}}$

1) $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 - x_2 + x_3, x_1 - x_2 + x_3, x_3)$

a) $A = [f]_{R_0, R_0}$

b) valorile proprii; subsp. proprii și repere în fiecare

c) $Ker f$, $Im f$, reper în fiecare

d) $\mathbb{R}^3 = Ker f \oplus W$, $W = ?$

$p: Ker f \oplus W \longrightarrow Ker f$, $p(1,0,3)$

s = simetria față de $Ker f$. $s(1,0,3)$

Ex2 $S : V \longrightarrow W$ liniară

$S^* : W^* \longrightarrow V^*$, $S^*(f) = f \circ S$, $\forall f \in W^*$.

(pull-back)

a) S^* liniară

b) S surj $\Rightarrow$ S^* inj

Ex3 $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ $A = \begin{pmatrix} \alpha & 1-\alpha \\ 1 & 2 \end{pmatrix} = [f]_{R_0, R_0}$.

$\alpha = ?$ aî 1) $\lambda = 1$ valoare proprie.

2) $\lambda = -1$ —||—

3) $0 \notin \sigma(f)$