$\mathbb{Z}$ $\quad x \equiv y \iff n \mid x-y \qquad y \equiv x$

1) $x \equiv x \iff n \mid x-x=0$

2) $x \equiv y \Rightarrow n \mid x-y \Rightarrow n \mid y-x \Rightarrow y \equiv x$

3) $x \equiv y$ și $y \equiv z$ lăj.

$n \mid x-y \qquad n \mid y-z \Rightarrow n \mid x-z$

$x \equiv z$

$r \qquad \hat{r} = \{ r + nk \mid k \in \mathbb{Z} \}$

$X \qquad A, B \cap C \qquad X$

$A \sim B \iff f : A \to B$

1) $A \sim A$ reflexivitate

2) $A \sim B \Rightarrow B \sim A$

3) $A \sim B,\ B \sim C \Rightarrow A \sim C$

$X = \bigcup_{i \in I} A_i \qquad A_i \cap A_j = \phi \quad \forall\, i \neq j$

$x \sim y \iff \exists\ i$ a.î. $x, y \in A_i$

1) $x \sim x \qquad$ 2) $x \sim y \Rightarrow y \sim x$

3) $x \sim y \qquad y \sim z \Rightarrow i, j$ a.î.

$x, y \in A_i \qquad y, z \in A_j \Rightarrow x, z \in A_i$

$x \sim z$

$y \in A_i \cap A_j \neq \phi \Rightarrow i = j$

$\hat{x} = A_i$

$X \qquad (P(X), \subset)$

1) $A \subset A$

2) $A \subset B$ și $B \subset A \Rightarrow A = B$

3) $A \subset B \qquad B \subset C \Rightarrow A \subset C$

$X = \{a, b\} \qquad A = \{a\} \qquad B = \{b\}$

supremul $\qquad \sup_{i \in I} (A_i) = \bigcup_{i \in I} A_i$

(1) $A_i \subset A \ \forall i \in I$ $\quad$ A majorant

$B \ \bigcup_{i=1} A_i \subset A$

$\inf_{i \in I} (A_i) = \bigcap_{i \in I} A_i$

$(\mathbb{N}, |)$ $\quad$ 1) $n | n$

2) $n | m$ și $m | n \Rightarrow n = m$

3) $n | m$ și $m | e \Rightarrow n | e$

$(\mathbb{Z}, |)$ $\quad$ $|$ este o rel. de ordine

$\sup \{n, m\}$

K maj $\{n, m\} \Rightarrow n | K$ și $m | K \Rightarrow$
majorant

c.m.m.m.c $\{n, m\} | K$
$\quad \quad \quad \

$\text{maj}\, A = [1, \infty)$

$\sup B = 1 \quad \inf B = 0$

$$C = \left\{ \frac{m}{1+m+n} \;\middle|\; m, n \in \mathbb{N} \;\; m, n \geq 1 \right\}$$

$$0 \leq \frac{m}{1+m+n} \leq 1$$

$$\xrightarrow{m \to \infty} 1$$

$$\lim_{m \to \infty} \frac{m}{1+m+n} = 1 \Rightarrow 1 = \sup C$$

$$\lim_{n \to \infty} \frac{1}{2+n} = 0 \Rightarrow 0 = \inf C$$

$$[a_n, b_n] \supset [a_{n+1}, b_{n+1}]$$

$$a_n \leq a_{n+1} \leq \underset{\sup\{a_n\}}{\underset{\shortparallel}{a}} \qquad \underset{\inf\{b_n\}}{\underset{\shortparallel}{b}} \leq b_{n+1} \leq b_n$$

$$\bigcap_{n \geq 1} \left[1 - \frac{1}{n}, 2 + \frac{1}{n}\right] = [1, 2]$$

$$a_n = 1 - \frac{1}{n} \quad \text{a.i.} \quad a = 1$$

Corpul numerelor reale

0) $(\mathbb{N}, s, 0)$

1) $(\mathbb{N}, +, \leq)$

2) $(\mathbb{Z}, +, \cdot, \leq)$ inel

3) $(\mathbb{Q}, +, \leq)$ corp

4) $(\mathbb{R}, +, \leq)$ corp complet ordonat

$s: \mathbb{N} \to \mathbb{N}$

$s(n) = n+1$

1) $s: \mathbb{N} \to \mathbb{N}$

2) injectivă

3) $s(\mathbb{N}) = \mathbb{N} \setminus \{0\}$

4) $A \subset \mathbb{N}$ și $0 \in A$

$n \in A \Rightarrow s(n) \in A$

$\Rightarrow A = \mathbb{N}$

a) $s(n) = n'$ (succesorul lui n)

$n + 0 = 0$

$1 = s(0) = 0'$

$n + 1 = s(n)$

$n + s(m) = s(n+m)$

b) $n \cdot 0 = 0$ $\quad n \cdot 1 = 1$

$n\, s(m) = nm + n$

c) $n, m \in \mathbb{N} \Rightarrow$ I) $\exists p \in \mathbb{N}$ $n = m + p$

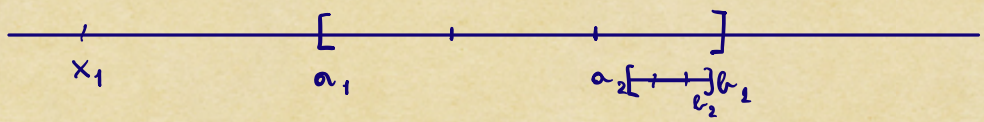
sau

II) $\exists p \in \mathbb{N}$ $m = n + p$

$n = m + p \Rightarrow n \geq m$

$\mathbb{R} \overset{\text{în bijecție}}{\sim} \mathbb{N}$ $\qquad \mathbb{R} = \{x_1, x_2, \dots, x_n, \dots\}$

$I_1 \not\ni x_1$



$x_1 \notin I_1$
$\cup$
$x_2 \notin I_2$
$\cup$
$\vdots$
$x_n \notin I_n$

$\bigcap_{n \geq 1} I_n \ni y \in \mathbb{R}$

$y \neq x_n \ \forall n \in \mathbb{N}$

Ex $f: \mathbb{R} \to \mathbb{R}$

1) f inj.
2) $f(x+y) = f(x) + f(y)$
3) $f(x \cdot y) = f(x) \cdot f(y)$
4) f cont

$x = y = 0 \quad \cancel{f(0)} = f(0+0) = f(0) + \cancel{f(0)} \Rightarrow f(0) = 0$

$x = y = 1 \quad f(1) = f(1^2) = (f(1))^2 \Rightarrow f(1) \in \{0, 1\}$

I. $f(1) = 0 \Rightarrow f(x) = f(x \cdot 1) = f(x) \cdot f(1) = 0$
dar f inj. $\times$

II. $f(1) = 1 \quad f(2) = f(1+1) = f(1) + f(1) = 2$

ind. $f(n) = n \ \forall n \in \mathbb{N}$

$f(2x) = 2f(x) \qquad f(nx) = nf(x)$

$0 = f(0) = f(x + (-x)) = f(x) + f(-x) \Rightarrow$

$\Rightarrow f(-x) = -f(x)$

$f(-n) = -f(n) = -n \ \forall n \in \mathbb{N}$

$f(m) = m \ \forall m \in \mathbb{Z}$

$f(nx) = nf(x) \qquad x = \frac{1}{n}$

$$1 = f\left(n \cdot \frac{1}{n}\right) = n\, f\left(\frac{1}{n}\right) \Rightarrow f\left(\frac{1}{n}\right) = \frac{1}{n}$$

$r \in \mathbb{Q} \quad r = \frac{m}{n}, \; m, n \in \mathbb{Z}$

$$f(r) = f\left(\frac{m}{n}\right) = m \cdot f\left(\frac{1}{n}\right) = \frac{m}{n}$$

$\forall r \in \mathbb{Q} \quad f(r) = r$

$\forall x \in \mathbb{R} \Rightarrow \exists\, r_n \to x \quad r_n \in \mathbb{R}$

f cont. $f(r_n) \to f(x)$

$\quad\quad\quad\quad \overset{\shortparallel}{r_n} \to x \quad \Rightarrow \boxed{f(x) = x}$

f cont in $a \quad b \in \mathbb{R}$

$$\lim_{y \to b} f(y) - f(b) = \lim_{y \to b} f(\underbrace{y - b + a}_{x \to a} - a) =$$

$$= \lim_{x \to a} (f(x) - f(a)) = 0$$

$f \nearrow$ (crescatoare)

$x \leq y \Rightarrow f(x) \leq f(y) \Leftrightarrow 0 \leq f(y - x)$

$z \geq 0 \Rightarrow f(z) \geq 0$

$z \geq 0 \Leftrightarrow \exists\, y \in \mathbb{R}$ a.î. $y^2 = z$

$$f(z) = f(y^2) = (f(y))^2 \geq 0$$

$x \in \mathbb{R} \quad (r_n)_n, (g_n)_n \subset \mathbb{Q}$

$r_n \nearrow$ și $\swarrow g_n$

$f(r_n) \leq f(x) \leq f(g_n)$

$\overset{\shortparallel}{r_n} \longrightarrow x \longleftarrow g_n$