BUHAI
DARIUS
G. 135.

EXAMEN. C.D.I.

P1 $\sum_{n=1}^{\infty} tg\left(\sqrt{n^2+n+1} - \sqrt{n^2+n-1}\right)$

Fie $(x_n)_{n\geq 1}$, $x_n = tg\left(\sqrt{n^2+n+1} - \sqrt{n^2+n-1}\right)$

și seria $(y_n)_{n\geq 1} = \sum_{n=1}^{\infty} \frac{1}{n}$
armonică.

$$\lim_{n\to\infty} \frac{x_n}{y_n} = \lim_{n\to\infty} \frac{tg\left(\sqrt{n^2+n+1} - \sqrt{n^2+n-1}\right)}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{tg\left(\frac{2}{\sqrt{n^2+n+1}+\sqrt{n^2+n-1}}\right)}{\frac{2}{\sqrt{n^2+n+1}+\sqrt{n^2+n-1}}} \cdot \frac{2}{\sqrt{n^2+n+1}+\sqrt{n^2+n-1}}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{2n}{\sqrt{n^2+n+1}+\sqrt{n^2+n-1}} = \lim_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1+\frac{1}{n}-\frac{1}{n^2}}\right)}$$

$= 1 = \ell$

~~$\ell = 1$~~ $\xrightarrow[\text{comparare cu limite}]{\text{Criteriul de}}$ ~~serile au aceeași natură~~

~~$\Rightarrow$ seria $\sum_{n=1}^{\infty} tg\left(\sqrt{n^2\right.$~~

$\ell = 1$ (serie divergentă)

Din criteriul de comparare cu limite, seriile au aceeași natură $\Rightarrow$ $\underbrace{\sum_{n=1}^{\infty} tg\left(\sqrt{n^2+n+1} - \sqrt{n^2+n-1}\right)}_{\text{serie divergentă}}$.

$\underline{P_2}$ $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 - 2xy + y^2 + x^3$

I funcție continuă pe $\mathbb{R}^2$

II $\frac{\partial f}{\partial x}(x,y) = 2x - 2y + y^2 + 3x^2$, $\forall (x,y) \in \mathbb{R}^2$

$\frac{\partial f}{\partial y}(x,y) = x^2 - 2x + 2y + x^3$, $\forall (x,y) \in \mathbb{R}^2$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ funcții continue pe $\mathbb{R}^2 \Rightarrow f$. diferențiabilă pe $\mathbb{R}^2$

III $(\mathbb{R}^2)$ $\begin{cases} \frac{\partial f}{\partial x}(x,y) = 0 \\ \frac{\partial f}{\partial y}(x,y) = 0 \end{cases} \Rightarrow \begin{cases} 3x^2 + 2x + y^2 - 2y = 0 \\ x^3 + x^2 - 2x + 2y = 0 \end{cases}$ +

$x^3 + 4x^2 + y^2 = 0$

$x^2(x+4) + y^2 = 0 \Rightarrow$

$\Rightarrow y^2 = -x^2(x+4)$

$\Rightarrow y = \pm x\sqrt{-x-4}$

$x = 0, y = 0$, $(0,0) \in \mathbb{R}^2$

$C = \{(0,0)\}$

IV $\frac{\partial^2 f}{\partial^2 x}(x,y) = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)(x,y) = (2x - 2y + y^2 + 3x^2)'_x =$

$= 2 - 2y + y^2 + 6x$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)(x,y) = (x^2 - 2x + 2y + x^3)'_x$

$= 2x - 2 + 2y + 3x^2$

$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)(x,y) = (2x - 2y + y^2 + 3x^2)'_y =$

$= 2x - 2 + 2y + 3x^2$

$$\frac{\partial^2 f}{\partial^2 y}(x,y) = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial y}\right)(x,y) = (x^2-2x+2y+x^3)'_y =$$

$$= x^2 - 2x + 2 + x^3$$

$\mathbb{R}^2$ mult. deschisă. $\frac{\partial^2 f}{\partial^2 x}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial^2 y}$ fct. continue pe $\mathbb{R}^2$

<u>V</u>

$$Hf(0,0) = \begin{pmatrix} \frac{\partial^2 f}{\partial^2 x}(0,0) & \frac{\partial^2 f}{\partial x \partial y}(0,0) \\ \frac{\partial^2 f}{\partial y \partial x}(0,0) & \frac{\partial^2 f}{\partial^2 y}(0,0) \end{pmatrix} = \begin{vmatrix} \boxed{2} & -2 \\ -2 & 2 \end{vmatrix}$$

$$= 4 - 4 = 0$$

$\Delta_1 = 2$

$\Delta_2 = 0$

Criteriul nu se pronunță.

<u>VI</u>

$$f(x,y) - f(0,0) = x^2 - 2xy + y^2 + x^3$$

$$f\left(\frac{1}{n}, 0\right) - f(0,0) = \frac{1}{n^2} + \frac{1}{n^3} > 0$$

$$f\left(-\frac{1}{n}, 0\right) - f(0,0) = \frac{1}{n^2} - \frac{1}{n^3} = \frac{n-1}{n^3} > 0, \forall n \geq 1$$

$\Rightarrow (0,0)$ nu este pct. de extrem al funcției f.

$\underline{P_3}$ a) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\sin^5 x \cos^3 x}\, dx = \int_{0+0}^{\frac{\pi}{2}-0} \sin^{\frac{1}{5}} x \cos^{\frac{1}{3}} x \, dx$

$$B(p,q) = 2\int (\sin x)^{2p-1} (\cos x)^{2q-1} \qquad \begin{cases} 2p-1 = \frac{5}{2} \\ 2q-1 = \frac{3}{2} \end{cases} \Rightarrow \begin{cases} p = \frac{7}{4} \\ q = \frac{5}{4} \end{cases}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin^5 \cdot \cos^3 x}\, dx = \frac{B\left(\frac{7}{4}, \frac{5}{4}\right)}{2} = \frac{\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{5}{4}\right)}{2\Gamma\left(\frac{7}{4} \cdot \frac{5}{4}\right)}$$

$$= \frac{\Gamma\left(1+\frac{3}{4}\right)\Gamma\left(1+\frac{1}{4}\right)}{2\Gamma(3)} = \frac{\frac{3}{4}\Gamma\left(\frac{3}{4}\right) \cdot \frac{1}{4}\Gamma\left(\frac{1}{4}\right)}{4} = \frac{\frac{3}{16} \frac{\pi}{\sin\frac{\pi}{4}}}{4}$$

$$= \frac{3 \cdot 2 \cdot \pi}{16 \cdot 4 \cdot \sqrt{2}} = \frac{3\pi}{32\sqrt{2}}$$

b) $\displaystyle\iint_D (x-y)\, dx\, dy$, $\forall D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le 4, y \ge 0\}$

$f : \mathbb{R}^2 \to \mathbb{R}^2$

$f(R,\alpha) = (\overbrace{R\cos\alpha}^{x}, \overbrace{R\sin\alpha}^{y})$

$$D: \begin{cases} x^2+y^2 \le 4 \\ 0 \le y \end{cases}$$

$$D': \begin{cases} R^2\cos^2\alpha + R^2\sin^2\alpha \le 4 \\ 0 \le R\sin\alpha \\ R \ge 0 \\ \alpha \in [0, 2\pi] \end{cases}$$

$$D': \begin{cases} R^2 \le 4 \\ 0 \le \sin\alpha \\ R \ge 0 \\ \alpha \in [0, 2\pi] \end{cases} \Rightarrow \begin{cases} 0 \le R \le \sqrt{4} \\ \alpha \in [0, \pi] \\ 0 \le \sin\alpha \end{cases} \Rightarrow \begin{cases} 0 \le R \le 2 \\ 0 \le \alpha \le \pi \end{cases}$$

4/5.

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial R} & \frac{\partial x}{\partial \alpha} \\ \frac{\partial y}{\partial R} & \frac{\partial y}{\partial \alpha} \end{vmatrix} = \begin{vmatrix} \cos\alpha & -R\sin\alpha \\ \sin\alpha & R\cos\alpha \end{vmatrix} = R\cos^2\alpha + R\sin^2\alpha = R.$$

$$dx\,dy = |R|\,dR\,d\alpha = R\,dR\,d\alpha$$

$$\iint_{D^2} (x-y)\,dx\,dy = \iint_{D'} (R\cos\alpha - R\sin\alpha)\,R\,dR\,d\alpha$$

$$= \int_0^{\frac{\pi}{4}} \left(\int_0^2 R^2\cos\alpha - R^2\sin\alpha\,dR \right) d\alpha$$

$$= \int_0^{\frac{\pi}{4}} \frac{R^3}{3}\Big|_0^2 (\cos\alpha - \sin\alpha)\,d\alpha = \frac{8}{3}\int_0^{\frac{\pi}{4}} \cos\alpha - \sin\alpha\,d\alpha$$

$$= \frac{8}{3}\cdot \int_0^{\frac{\pi}{2}} (\sin\alpha + \cos\alpha)\,d\alpha = \frac{8}{3}\cdot(-2) = -\frac{16}{3}$$