Seminar 11 - CONTINUARE [redacted]

Geometrie analitică euclidiană

§1 Arii, volume, distanțe, unghiuri

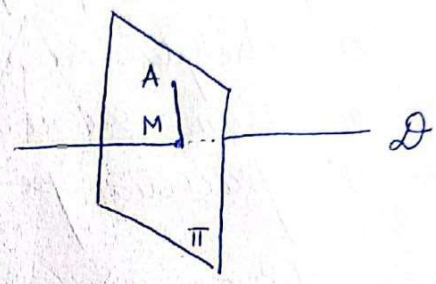
- $S_{\Delta ABC} = \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$

- $dist(A, \mathcal{D}) = \dfrac{\| \overrightarrow{AB} \times \overrightarrow{AC} \|}{\| \overrightarrow{BC} \|}$, $B, C \in \mathcal{D}$ sau

$dist(A, \mathcal{D}) = dist(A, M)$,

unde $\pi \perp \mathcal{D}$, $A \in \pi$

$\mathcal{D} \cap \pi = \{M\}$



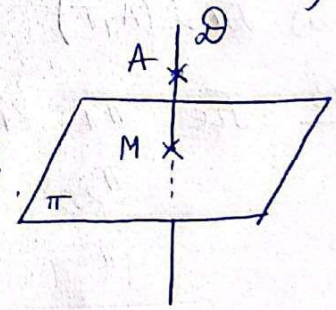
- $V_{ABCD} = \frac{1}{6} |\Delta|$, $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix}$

- A, B, C, D coplanare $\Leftrightarrow \Delta = 0$.

- $dist(A, \pi) = \dfrac{|a a_1 + b a_2 + c a_3 + d|}{\sqrt{a^2 + b^2 + c^2}}$, $\pi : a x_1 + b x_2 + c x_3 + d = 0$, $A(a_1, a_2, a_3)$

sau $dist(A, \pi) = dist(A, M)$,

unde $A \in \mathcal{D}$, $\mathcal{D} \perp \pi$, $\mathcal{D} \cap \pi = \{M\}$.



- $dist(\mathcal{D}_1, \mathcal{D}_2) = \dfrac{| \langle \overrightarrow{AB}, N \rangle |}{\| N \|}$,

unde $\mathcal{D}_1, \mathcal{D}_2$ drepte necoplanare, $A \in \mathcal{D}_1$, $u = u_{\mathcal{D}_1}$, $B \in \mathcal{D}_2$, $v = u_{\mathcal{D}_2}$

$N = u \times v$

- $\sphericalangle(\mathcal{D}_1, \mathcal{D}_2) = \sphericalangle(u_1, u_2) = \sphericalangle\varphi \in [0, \pi]$

  $\cos\varphi = \dfrac{\langle u_1, u_2 \rangle}{\|u_1\| \cdot \|u_2\|}$, unde $\mathcal{D}_K$ = dreaptă orientată de u_K, $K=\overline{1,2}$

- $\sphericalangle(\pi_1, \pi_2) = \sphericalangle(N_1, N_2) = \sphericalangle\varphi$

  $\cos\varphi = \dfrac{\langle N_1, N_2 \rangle}{\|N_1\| \cdot \|N_2\|}$, unde π_k = plan orientat de N_K, $K=\overline{1,2}$

- $\sphericalangle(\mathcal{D}, \pi) = \sphericalangle(\mathcal{D}, \mathcal{D}') = \sphericalangle\varphi$.

  $\mathcal{D}$ = dreaptă orientată de u

  π = plan orientat de N.

  $\mathcal{D}'$ = proiecția pe π a lui $\mathcal{D}$.

$(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

<u>Ex1</u> Fie $A(1,2,1)$, $B(2,1,3)$, $C(-2,1,3)$, $D(0,2,0)$

a) V_{ABCD}; b) $S_{\Delta BCD}$; c) dist $(A, (BCD))$

<u>Ex2</u> Fie $A(1,1,1)$, $\mathcal{D}: \begin{cases} x_1 + x_2 - x_3 + 1 = 0 \\ 2x_1 + x_2 - 3x_3 + 2 = 0 \end{cases}$; $\pi: x_1 + x_2 + x_3 = 0$

a) dist $(A, \mathcal{D}) = ?$

b) Dist (A, π)

<u>Ex3</u> Fie $\pi_1: x_1 - 3x_2 - 1 = 0$ $\quad \mathcal{D}_1: \begin{cases} x_1 - x_2 - 2 = 0 \\ x_1 + x_3 - 3 = 0 \end{cases}$

$\pi_2: 2x_2 + x_3 - 2 = 0$ $\quad \mathcal{D}_2: \dfrac{x_1 - 1}{3} = \dfrac{x_2 + 1}{0} = \dfrac{x_3 - 1}{-1}$

$\pi: x_2 - x_3 - 1 = 0$ $\quad \mathcal{D}: \dfrac{x_1 - 1}{-1} = \dfrac{x_2}{2} = \dfrac{x_3 + 1}{-5}$

a) $\sphericalangle(\mathcal{D}_1, \mathcal{D}_2)$; b) $\sphericalangle(\mathcal{D}, \pi)$; c) $\sphericalangle(\pi_1, \pi_2)$

<u>Ex4</u> Fie $\mathcal{D}_1: \begin{cases} x_1 - x_2 = 2 \\ x_1 + x_3 = 3 \end{cases}$, $\mathcal{D}_2: \dfrac{x_1 - 1}{3} = \dfrac{x_2 + 1}{0} = 1 - x_3$

Dist $(\mathcal{D}_1, \mathcal{D}_2) = ?$