Lucrare II (142)

① $f \in End(\mathbb{R}^2)$, $A = [f]_{R_0, R_0} = \begin{pmatrix} 1 & -3 \\ 3 & -6 \end{pmatrix}$, R_0 = reperul canonic

a) f nu se poate diagonaliza; b) f se poate diagonaliza;
c) polin. caract. are răd. $\in \mathbb{C} \setminus \mathbb{R}$; d) valorile proprii sunt egale.

② $(\mathbb{R}^3, g_0)$. Fie reperul $R = \{f_1 = (0,-1,1), f_2 = (0,0,1), f_3 = (1,1,1)\}$.
Reperul ortonormat obținut cu Gram-Schmidt este:

a) $\left\{\frac{1}{\sqrt2}(1,-1,0), \frac{1}{\sqrt2}(1,0,1), \frac{1}{\sqrt3}(-1,-1,1)\right\}$; b) $\left\{\frac{1}{\sqrt2}(0,-1,1), \frac{1}{\sqrt2}(0,1,1), \frac{1}{\sqrt2}(1,0,1)\right\}$
c) $\left\{\frac{1}{\sqrt2}(0,-1,1), \frac{1}{\sqrt2}(0,1,1), (1,0,0)\right\}$; d) $\left\{(1,0,0), (0,1,0), \frac{1}{\sqrt2}(1,1,0)\right\}$.

③ $(\mathbb{R}^3, g_0)$, $u = (0,1,1)$
$s \in End(\mathbb{R}^3)$ simetria ortogonală față de $\langle\{u\}\rangle^\perp$

a) $s(x) = (x_1, -x_3, -x_2)$, b) $s(x) = (x_1, -x_2, -x_3)$
c) $s(x) = (-x_2, x_1, x_3)$, d) $s(x) = (-x_1, -x_2, -x_3)$

④ $(\mathbb{R}^3, g_0)$, $U = \{(\alpha, -\alpha, 2\alpha) \mid \alpha \in \mathbb{R}\}$
Complementul ortogonal $U^\perp$ este

a) $\{x \in \mathbb{R}^3 \mid x_1 + x_2 + 2x_3 = 0\}$; b) $\{x \in \mathbb{R}^3 \mid -x_1 + x_2 + 2x_3 = 0\}$
c) $\{x \in \mathbb{R}^3 \mid x_1 - x_2 + 2x_3 = 0\}$; d) $\{x \in \mathbb{R}^3 \mid 2x_1 + x_2 + x_3 = 0\}$.

⑤ $Q : \mathbb{R}^3 \to \mathbb{R}$ formă pătratică, $Q(x) = x_1^2 + 2x_1x_2 + x_2x_3$
Signatura lui Q este:

a) $(1,1)$; b) $(1,2)$; c) $(3,0)$, d) $(2,1)$

⑥ $(\mathbb{R}^3, g_0)$, $u = (1,2,-1)$, $f \in End(\mathbb{R}^3)$, $f(x) = u \cdot \langle x, u\rangle$
$g_0 = \langle \cdot, \cdot \rangle$ produs scalar canonic

a) $\dim Ker f = 1$, b) $\dim Ker f = 2$; c) $f \in Sim(\mathbb{R}^3)$, d) $f \in Aut(\mathbb{R}^3)$.

⑦ $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0, R_0} = A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$
$\exists$ un reper ortonormat a.î. matricea lui f are forma diagonală

a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$; b) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$; c) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix}$; d) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(8) $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0,R_0} = \frac{1}{9}\begin{pmatrix} 1 & 8 & -4 \\ 8 & 1 & 4 \\ -4 & 4 & 7 \end{pmatrix}$

a) $f = s \circ R_\varphi$, $\cos\varphi = +1$, b) $f = R_\varphi$, $\cos\varphi = -\frac{1}{3}$

c) $f = R_\varphi$, $\cos\varphi = 0$; d) $f = s \circ R_\varphi$, $\cos\varphi = 0$

unde R_φ = rotație de $\sphericalangle\varphi$, axă $\langle\{e_1\}\rangle$,

s = simetrie ortogonală față de $\langle\{e_1\}\rangle^\perp$.

(9) $Q : \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = x_1^2 + 2x_1x_2 + x_2x_3 + x_3^2$

$g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ forma polară asociată

a) $g(x,y) = x_1y_1 + x_1y_2 + x_2y_1 + x_3y_3$; b) $g(x,y) = x_1y_1 + \frac{1}{2}x_2y_3 + \frac{1}{2}x_3y_2$

c) $g(x,y) = x_1y_1 + x_3y_3 + x_1y_2 + x_2y_1 + \frac{1}{2}x_2y_3 + \frac{1}{2}x_3y_2$; d) $g(x,y) = x_1y_1 + x_2y_2$