Produse

① Produs scalar. $x \cdot y = \langle x, y \rangle = x_1 y_1 + x_2 y_2 + x_3 y_3$, în $(\mathbb{R}^3, g_0)$

② Produs vectorial $x \times y = \begin{vmatrix} e_1 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$

! Obținem un vector perpendicular pe x și y

③ Produs mixt. $x \wedge y \wedge z = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$

! Obținem un scalar

① $(\mathbb{R}^3, g_0)$, $u=(1,-1,2)$, $v=(0,1,3)$, $w=(1,4,0)$

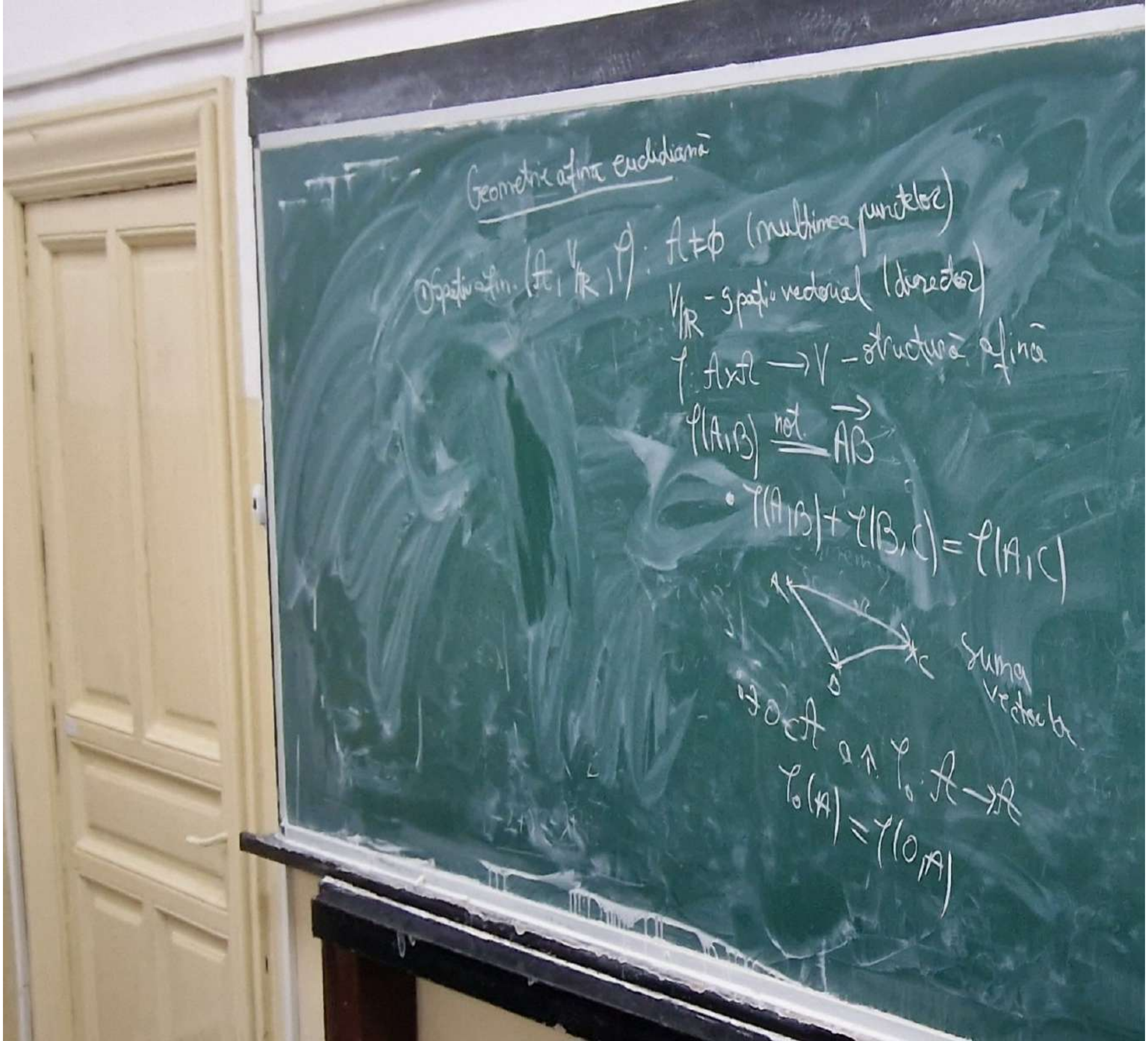
a) $u \times v$

b) $u \wedge v \wedge w$

a) $u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 2 \\ 0 & 1 & 3 \end{vmatrix} = -3e_1 + e_3 - 2e_1 - 3e_2 = -5e_1 - 3e_2 + e_3 = (-5,-3,1)$

$e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

b) $u \wedge v \wedge w = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 1 & 4 & 0 \end{vmatrix} = -3 - 2 - 12 = -17$

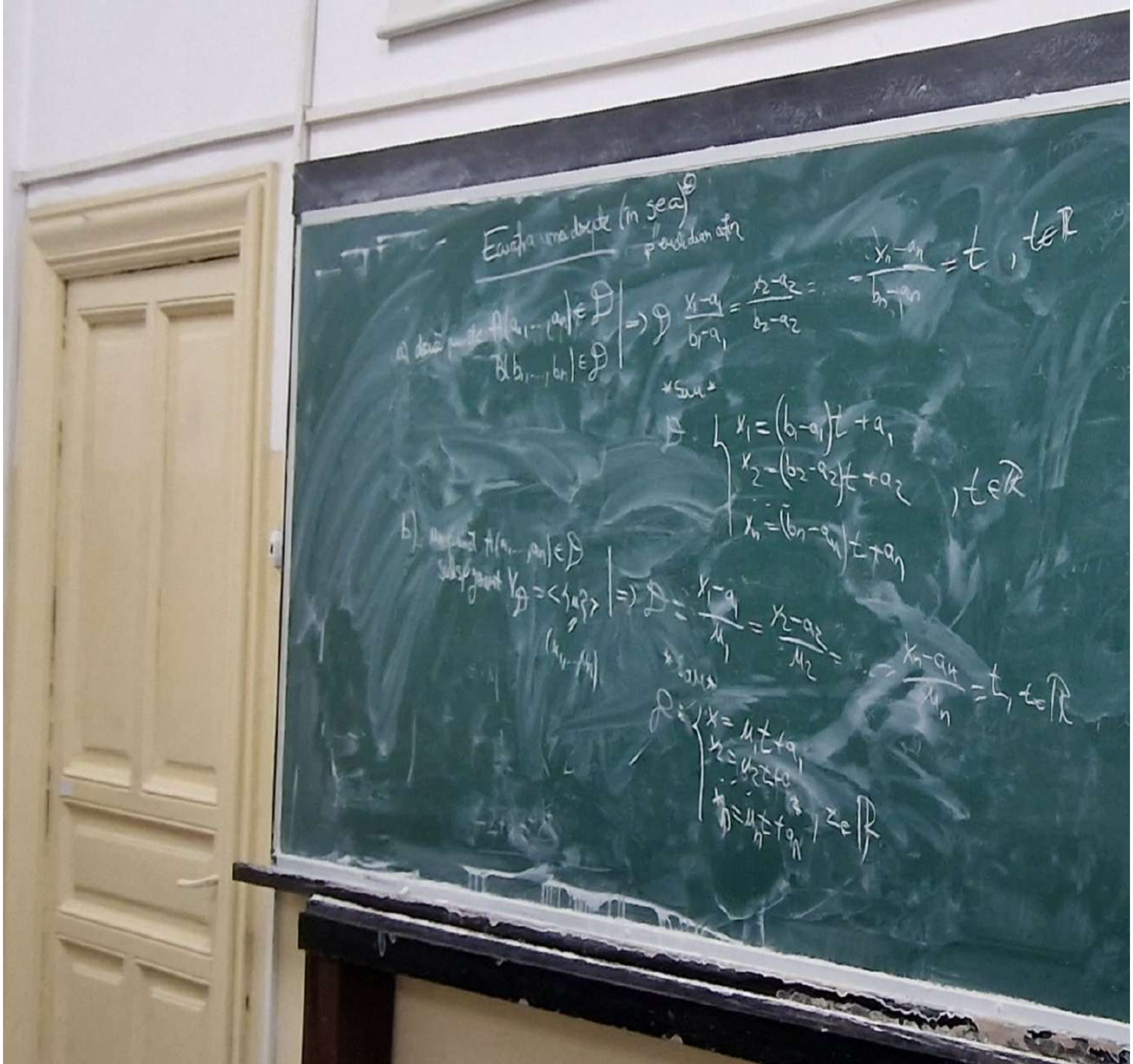

Geometrie afină euclidiană
① Spațiu afin. (A, V/R, φ): A ≠ ∅ (mulțimea punctelor)
V/R - spațiu vectorial (director)
φ: A×A → V - structură afină
φ(A,B) not. AB
• φ(A,B) + φ(B,C) = φ(A,C)
A
B
C
suma vectorilor
∀ O ∈ A a.î. φ_O: A → A
φ_O(A) = φ(O,A)

② Spațiu euclidian afin $(\mathcal{E}, (E, \langle\cdot,\cdot\rangle), \varphi)$

sp. afin + produs scalar

translații, linii, plane, paralelism

distanțe, unghiuri


Ecuația unei drepte (în sea)
$\frac{x_1-a_1}{b_1-a_1} = \frac{x_2-a_2}{b_2-a_2} = \dots = \frac{x_n-a_n}{b_n-a_n} = t, \; t \in \mathbb{R}$
sau
$x_1 = (b_1-a_1)t + a_1$
$x_2 = (b_2-a_2)t + a_2$
$x_n = (b_n-a_n)t + a_n$
$, t \in \mathbb{R}$
$\frac{x_1-a_1}{u_1} = \frac{x_2-a_2}{u_2} = \dots = \frac{x_n-a_n}{u_n} = t, \; t \in \mathbb{R}$
sau

Poziția relativă a două drepte

$D_1 : x_i = u_i t + a_i$

$D_2 : x_i' = u_i' t' + a_i'$

$$C = \left(\begin{array}{cc|c} u_1 & -u_1' & -(a_1 - a_1') \\ u_2 & -u_2' & -(a_2 - a_2') \\ \vdots & \vdots & \vdots \\ u_n & -u_n' & -(a_n - a_n') \end{array}\right)$$

1. $\operatorname{rg} C = \operatorname{rg} \overline{C} = 2 \Rightarrow D_1, D_2$ concurente ($D_1 \cap D_2 = \{P\}$)
2. $\operatorname{rg} C = \operatorname{rg} \overline{C} = 1 \Rightarrow D_1 = D_2$
3. $\operatorname{rg} C = 2, \operatorname{rg} \overline{C} = 3 \Rightarrow D_1, D_2$ necoplanare
4. $\operatorname{rg} C = 1, \operatorname{rg} \overline{C} = 2 \Rightarrow D_1 \parallel D_2$

Distanța dintre două puncte

$$d(P_1, P_2) = \|\underbrace{\overrightarrow{P_1P_2}}_{(u_1, u_2, \ldots, u_n)}\| = \sqrt{\langle \overrightarrow{P_1P_2}, \overrightarrow{P_1P_2} \rangle} = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2}$$

1) $A(3,-1,3)$, $B(5,1,-1)$, $u=(-3,5,-6)$, $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$ s.e.a

a) ecuația dreptei d a.î. $A \in d$, $V_d = \langle \{u\} \rangle$

b) ecuația dreptei AB

c) punctul de intersecție $d \cap$ planele sist. de coord.

a) $d: \dfrac{x_1-3}{-3} = \dfrac{x_2+1}{5} = \dfrac{x_3-3}{-6} = t, \ t \in \mathbb{R}$

$$d: \begin{cases} x_1 = -3t+3 \\ x_2 = 5t-1 \\ x_3 = -6t+3 \end{cases}, \ t \in \mathbb{R}$$

b) $AB: \dfrac{x_1-3}{5-3} = \dfrac{x_2+1}{1+1} = \dfrac{x_3-3}{-1-3} = t, \ t \in \mathbb{R}$

$AB: \dfrac{x_1-3}{2} = \dfrac{x_2+1}{2} = \dfrac{x_3-3}{-4} = t$

c) $\mathcal{D} \cap Ox_2x_3$

$x_1 = 0 \Rightarrow t = 1 \Rightarrow \begin{cases} x_2 = 4 \\ x_3 = -3 \end{cases} \Rightarrow \mathcal{D} \cap Ox_2x_3 = \{P_1(0, 4, -3)\}$

$\mathcal{D} \cap Ox_1x_2$

$x_3 = 0 \Rightarrow t = \frac{1}{2} \Rightarrow \begin{cases} x_1 = \frac{3}{2} \\ x_2 = \frac{3}{2} \end{cases} \Rightarrow \mathcal{D} \cap Ox_1x_2 = \{P_2(\frac{3}{2}, \frac{3}{2}, 0)\}$

$\mathcal{D} \cap Ox_1x_3$

$x_2 = 0 \Rightarrow t = \frac{1}{5} \Rightarrow \begin{cases} x_1 = 12/5 \\ x_3 = 9/5 \end{cases} \Rightarrow \mathcal{D} \cap Ox_1x_3 = \{P_3(\frac{12}{5}, 0, \frac{9}{5})\}$

(2) $D' \begin{cases} 2x_1 - x_2 + 3x_3 = -1 & \} \text{PLAN 1} \\ 5x_1 + 4x_2 - x_3 = -1 & \} \text{PLAN 2} \end{cases} \Big\} \Rightarrow$ DREAPTĂ

$D \parallel D'$, $A(2,-5,3) \in D$

ecuația dreptei D

$\pi_1 : 2x_1 - x_2 + 3x_3 = -1 \Rightarrow N_1 = (2,-1,3)$

$\pi_2 : 5x_1 + 4x_2 - x_3 = -1 \Rightarrow N_2 = (5,4,-1)$

$$N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} = e_1 + 8e_3 + 15e_2 - (-5e_3) - 12e_1 - (-2e_2)$$

$$= -11e_1 + 17e_2 + 13e_3 = (-11, 17, 13) = u_D$$

$D \parallel D' \Rightarrow u_D = u_{D'}$

$D : \dfrac{x_1 - 2}{-11} = \dfrac{x_2 + 5}{17} = \dfrac{x_3 - 3}{13} = t, \; t \in \mathbb{R}$

② $\mathcal{D}_1: \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 = 1 \end{cases}$, $\mathcal{D}_2: \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

a) $\mathcal{D}_1, \mathcal{D}_2$ necoplanare

a) $x_3 = t \Rightarrow x_2 = 1 + t$, $x_1 = -t$ $\Rightarrow \mathcal{D}_1: \begin{cases} x_1 = -t \\ x_2 = t + 1 \\ x_3 = t \end{cases}, \ t \in \mathbb{R}$

$x_1 = s \Rightarrow x_2 = 0, x_3 = 0 \Rightarrow \mathcal{D}_2: \begin{cases} x_1 = s \\ x_2 = 0 \\ x_3 = 0 \end{cases}, \ s \in \mathbb{R}$

$C = \left(\begin{array}{cc|c} -1 & -1 & -(0-0) \\ 1 & 0 & -(1-0) \\ 1 & 0 & -(0-0) \end{array}\right)$

$$C = \left(\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{l} \Delta_2 = \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} = 0-(-1) = 1 \neq 0 \Rightarrow rg\, C = 2 \\ \det \overline{C} = \begin{vmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = 1 - 0 = 1 \Rightarrow rg\, \overline{C} = 3 \end{array}\right| \Rightarrow \mathcal{D}_1, \mathcal{D}_2 \text{ necoplanare.}$$