11.05.2021

CURS 12

Geometrie analitică euclidiană

Def (sp. afin) $(\mathcal{A}, V/_{\mathbb{R}}, \varphi)$

Fie $\mathcal{A} \neq \phi$ (mult. de puncte),

$V/_{\mathbb{R}}$ sp. vectorial (spațiu director)

$\varphi: \mathcal{A}\times\mathcal{A} \to V$ (structură afină) aplicație

care verif. 1) $\varphi(A,B) + \varphi(B,C) = \varphi(A,C)$, $\forall A,B,C \in \mathcal{A}$

2) $\exists\, O \in \mathcal{A}$ a.î. $\varphi_O : \mathcal{A} \to V$ bijecție

$\varphi_O(A) = \varphi(O,A)$, $\forall A \in \mathcal{A}$

(de fapt $\exists \Rightarrow \forall$)

Not $\varphi(A,B) = \overrightarrow{AB}$

Caz particular $\mathcal{A} = \mathbb{R}^n$, $V = \mathbb{R}^n/_{\mathbb{R}}$, $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$

$\varphi(u,v) = v - u$ (str. afină canonică).

$(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ sp. afin

Def $M \subset \mathbb{R}^n$ m. de puncte.

$\mathcal{A}f(M) = \left\{ \sum_{i=1}^{k} a_i P_i,\ a_i \in \mathbb{R},\ P_i \in M,\ i=\overline{1,k},\ \sum_{i=1}^{k} a_i = 1 \right\}$

combinații afine de puncte din M.

Def $\mathcal{A}' \subseteq \mathbb{R}^n$ s.n. varietate liniară sau subspațiu afin $\Leftrightarrow [\forall P_1, P_2 \in \mathcal{A}' \Rightarrow \mathcal{A}f(\{P_1,P_2\}) \subset \mathcal{A}']$

i.e. $a_1P_1 + a_2P_2 \in \mathcal{A}'$, $a_1 + a_2 = 1$

Prop a) $\mathcal{A}' \subseteq \mathbb{R}^n$ subspațiu afin $\Rightarrow \exists!\, V' \subseteq \mathbb{R}^n$ subspațiu vect. director a.î. $\forall P' \in \mathcal{A}'$, $V' = \{\overrightarrow{P'P},\ P \in \mathcal{A}'\}$

și $\dim \mathcal{A}' = \dim V'$.

b) Fie $P \in \mathbb{R}^n$, $V' \subseteq \mathbb{R}^n$ subsp. vect $\Rightarrow$
$\exists! \ \mathcal{A}' \ni P$ și V' = sp. vect director.

$$\underset{(m,n)}{A}\ \underset{(n,1)}{X} \to \underset{(m,1)}{B}$$

Exemplu $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ $\quad A = (a_{ij})_{\substack{i=\overline{1,m} \\ j=\overline{1,n}}}$, $X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

- $\mathcal{A}' = \{x \in \mathbb{R}^n \mid AX = B\} \subset \mathbb{R}^n$ subsp. afin. $\quad B = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$V' = \{x \in \mathbb{R}^n \mid AX = 0\}$ sp. director

$\dim \mathcal{A}' = \dim V' = n - \operatorname{rg} A$.

$\forall x_1, x_2 \in \mathcal{A}' \Rightarrow a_1 x_1 + a_2 x_2 \in \mathcal{A}'$, unde $a_1 + a_2 = 1$.

$AX_1 = B$, $\quad a_1, a_2 \in \mathbb{R}$, $a_1 + a_2 = 1$

$AX_2 = B$ $\quad A(a_1 X_1 + a_2 X_2) = a_1 AX_1 + a_2 AX_2 = (a_1 + a_2) B = B$

Caz particular

$$\mathcal{A}' = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 - x_3 = 1 \end{cases} \right\}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$(2,3)\ (3,1) \longrightarrow (2,1)$$

$$V' = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases} \right\} \text{ sp. director.}$$

Def $\mathcal{A}_1, \mathcal{A}_2 \subseteq \mathbb{R}^n$ subsp. afine

$\mathcal{A}_1 \parallel \mathcal{A}_2 \Leftrightarrow V_1 \subseteq V_2$ sau $V_2 \subseteq V_1$

Exemplu

$\mathcal{A}' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 2\}$

$\mathcal{A}'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 3\}$.

$V' = V'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 0\} \Rightarrow \mathcal{A}' \parallel \mathcal{A}''$

Def $(\mathcal{E} = \mathbb{R}^n, E/_{\mathbb{R}}, \langle\,,\rangle, \varphi)$ s.n. spațiu afin euclidian sau spațiu punctual euclidian dacă este un spațiu afin în care spațiul vectorial director este spațiu euclidian.

Def a) $\mathcal{E}_1, \mathcal{E}_2 \subset \mathcal{E}$ subsp. afine perpendiculare $\Leftrightarrow E_1 \perp E_2$

b) — // — normale $\Leftrightarrow E_2 = E_1^{\perp}$

$E = E_1 \oplus E_1^{\perp}$

Ecuații ale varietăților liniare în spațiul afin euclidian $(\mathcal{E}, E_{/\mathbb{R}}, \varphi)$

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian ortonormat,

$O \in \mathcal{E}$, $\{e_1, .., e_n\}$ reper ortonormat în E.

① Ecuația unei drepte afine

a) 

$v \in \mathbb{R}^n$.

$A(a_1, .., a_n)$, $\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$

$V_{\mathcal{D}} = \langle \{v\} \rangle$, $v = \sum_{i=1}^{n} v_i e_i$

$\forall M \in \mathcal{D} \Rightarrow V_{\mathcal{D}} = \{\overrightarrow{AM}, \forall M \in \mathcal{D}\}$

$M(x_1, .., x_n)$

$\exists t \in \mathbb{R}$ a.î. $\overrightarrow{AM} = t\vec{v}$

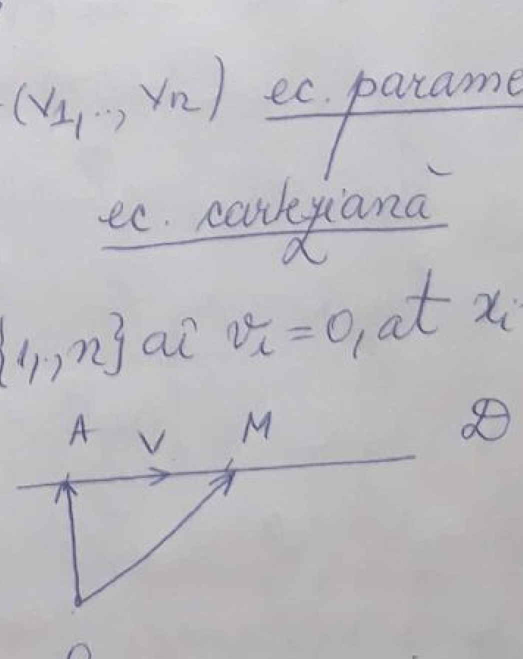
$\mathcal{D}: (x_1 - a_1, .., x_n - a_n) = t(v_1, .., v_n)$ ec. parametrice

$\mathcal{D}: \frac{x_1 - a_1}{v_1} = \dots = \frac{x_n - a_n}{v_n}$ ec. carteziană

Convenție: dacă $\exists i \in \{1, .., n\}$ a.î. $v_i = 0$, at $x_i - a_i = 0$

$\mathcal{D}: r = r_0 + tv$

$r = \overrightarrow{OM}$, $r_0 = \overrightarrow{OA}$



b) 

$\mathcal{D}$ $A(a_1, \dots, a_n)$
$B(b_1, \dots, b_n)$

$V_{\mathcal{D}} = \langle \{\overrightarrow{AB}\} \rangle$ $\quad \overrightarrow{AB} = (b_1 - a_1, \dots, b_n - a_n)$

$\exists\, t \in \mathbb{R}$ a.î.

$\mathcal{D}: (x_1 - a_1, \dots, x_n - a_n) = t(b_1 - a_1, \dots, b_n - a_n)$

$$\mathcal{D}: \frac{x_1 - a_1}{b_1 - a_1} = \dots = \frac{x_n - a_n}{b_n - a_n}$$

$\mathcal{D}: r = r_1 + t(r_2 - r_1)$ $\quad r = \overrightarrow{OM}$, $r_1 = \overrightarrow{OA}$, $r_2 = \overrightarrow{OB}$

Poziția relativă a 2 drepte

$\mathcal{D}_1: x_i - a_i = t v_i, \ \forall i = \overline{1,n}$

$\mathcal{D}_2: x_i - a'_i = t' v'_i, \ \forall i = \overline{1,n}$

$\mathcal{D}_1 \cap \mathcal{D}_2: \ t v_i - t' v'_i = a'_i - a_i, \ i = \overline{1,n}$ $\quad (t, t' = \text{necunoscute})$

$$C = \left(\begin{array}{cc|c} v_1 & -v'_1 & a'_1 - a_1 \\ v_2 & -v'_2 & a'_2 - a_2 \\ \vdots & \vdots & \\ v_n & -v'_n & a'_n - a_n \end{array}\right)$$

1. $\operatorname{rg} C = \operatorname{rg} \overline{C} = 2 \Rightarrow \mathcal{D}_1 \cap \mathcal{D}_2$ concurente
2. $\operatorname{rg} C = 2, \ \operatorname{rg} \overline{C} = 3$ necoplanare
3. $\operatorname{rg} C = \operatorname{rg} \overline{C} = 1$ $\quad \mathcal{D}_1 = \mathcal{D}_2$
4. $\operatorname{rg} C = 1, \ \operatorname{rg} \overline{C} = 2$ $\quad \mathcal{D}_1 \parallel \mathcal{D}_2$.

Exemplu $(\mathbb{R}^3, (\mathbb{R}^3_{/\mathbb{R}}, g_0), \varphi)$

$\mathcal{D} \ni A(1, 2, -1)$, $\ V_{\mathcal{D}} = \langle \{v = (2, 3, 1)\} \rangle$

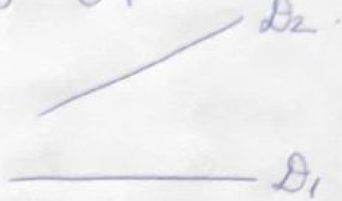
$\mathcal{D}' \ni A'(0, 1, -1)$, $\ V_{\mathcal{D}'} = \langle \{v' = (1, -2, 3)\} \rangle$

$\overrightarrow{AA'} = (-1, -1, 0)$

$$C = \left(\begin{array}{cc|c} 2 & -1 & -1 \\ 3 & -(-2) & -1 \\ 1 & -3 & 0 \end{array}\right)$$
$\qquad v \quad -v'$

$$\det \begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & -1 \\ 1 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 0 \\ 1 & 3 & 0 \end{vmatrix} \neq 0 \Rightarrow$$

$\mathcal{D}_1, \mathcal{D}_2$ necoplanare



OBS $\mathcal{D}, \mathcal{D}'$ drepte afine perpendiculare $\Rightarrow \langle v, v' \rangle = 0$

Exemplu

a) $\mathcal{D} \ni A(0,1,0)$, $v = (1,1,1)$

$\mathcal{D}' \ni A'(2,0,1)$, $v' = (1,-2,1)$

$\langle v, v' \rangle = 1 - 2 + 1 = 0 \Rightarrow \mathcal{D} \perp \mathcal{D}'$

b) $\mathcal{D} \ni A(0,1,0)$, $v = (1,1,1)$

$\mathcal{D}' \ni A'(2,0,1)$, $v' = (1,1,1)$

$v = v' \Rightarrow \mathcal{D} \parallel \mathcal{D}'$

$$C = \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{array}\right) \qquad \begin{array}{l} rg\, C = 1 \\ rg\, \overline{C} = 2 \end{array}$$

(2) Ec. unui plan afin (var. liniară 2-dim)

a) π ; $A \in \pi$, $V_\pi = \langle \{u, v\} \rangle$, $\{u, v\}$ SLI

$V_\pi = \{ \overrightarrow{AM}, M \in \pi \}$

$\exists t, s \in \mathbb{R}$ a.î. $\overrightarrow{AM} = tu + sv$. $\qquad A(a_1, .., a_n) \qquad r = \overrightarrow{OM}$

$M(x_1, .., x_n) \qquad r_0 = \overrightarrow{OA}$

π: $x_i - a_i = tu_i + sv_i, \forall i = \overline{1,n} \qquad v = \sum_{i=1}^{n} v_i e_i$

π: $r - r_0 = tu + sv \qquad u = \sum_{i=1}^{n} u_i e_i$

b) π: $A(a_1, .., a_n)$, $B(b_1, .., b_n)$, $C(c_1, .., c_n) \in \pi$

$V_\pi = \langle \{\overrightarrow{AB}, \overrightarrow{AC}\} \rangle$ $\quad \{\overrightarrow{AB}, \overrightarrow{AC}\}$ SLI

$\exists t, s \in \mathbb{R}$ ai

$\pi: \; x_i - a_i = t(b_i - a_i) + s(c_i - a_i), \; i = \overline{1,n}$

Caz particular $n = 3$.

a) $\pi: \; x_i - a_i = t u_i + s v_i, \; i = \overline{1,3}$

$$\pi: \begin{vmatrix} x_1 - a_1 & u_1 & v_1 \\ x_2 - a_2 & u_2 & v_2 \\ x_3 - a_3 & u_3 & v_3 \end{vmatrix} = 0$$

$N = u \times v \Rightarrow \pi: \langle r - r_0, N \rangle = 0$, $r = \overrightarrow{OM}$, $r_0 = \overrightarrow{OA}$

$N = (A_1, A_2, A_3)$, $A_1^2 + A_2^2 + A_3^2 > 0$

$\pi: \; A_1(x_1 - a_1) + A_2(x_2 - a_2) + A_3(x_3 - a_3) = 0$

$\pi: \; A_1 x_1 + A_2 x_2 + A_3 x_3 + A_0 = 0$ ec. generală a planului

Exemplu

π: $A(1, -1, 2)$, $u = (2, 3, 1)$
$v = (4, 1, 3)$

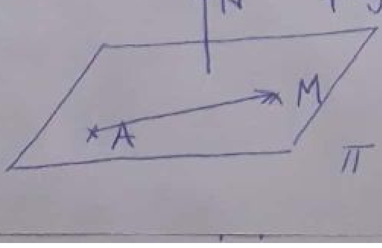
$a = (1, -1, 2)$

$$\pi: \begin{vmatrix} x_1 - 1 & 2 & 4 \\ x_2 + 1 & 3 & 1 \\ x_3 - 2 & 1 & 3 \end{vmatrix} = 0 \qquad \langle N, \underset{\overrightarrow{AM}}{x - a} \rangle = 0$$

SAU
$$N = u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix} = (8, -2, -10) = 2(4, -1, -5)$$

$\pi: \; 4(x_1 - 1) - 1(x_2 + 1) - 5(x_3 - 2) = 0$

$\pi: \; 4x_1 - x_2 - 5x_3 \underbrace{-4 - 1 + 10}_{+5} = 0$ ec. generala



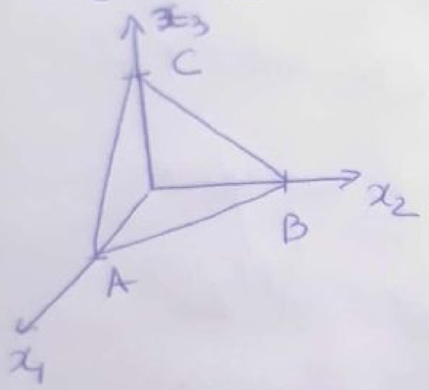
$\{e_1, e_2, e_3\}$ reperul canonic, $e_1 = i$, $e_2 = j$, $e_3 = k$

b) $A(a_1, a_2, a_3), B(b_1, b_2, b_3), C(c_1, c_2, c_3)$

$$\pi \quad \begin{vmatrix} x_1 - a_1 & b_1 - a_1 & c_1 - a_1 \\ x_2 - a_2 & b_2 - a_2 & c_2 - a_2 \\ x_3 - a_3 & b_3 - a_3 & c_3 - a_3 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0$$

Dacă $A(a,0,0), B(0,b,0), C(0,0,c)$, atunci

$\pi: \dfrac{x_1}{a} + \dfrac{x_2}{b} + \dfrac{x_3}{c} = 1$ (ec. prin tăieturi a planului)



Exemplu $A(1,1,1), B(-1,1,1), C(2,0,0)$

$$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = 0$$

③ Ecuația unui hiperplan afin $\mathcal{H}$ ($\dim \mathcal{H} = n-1$)

$A \in \mathcal{H}$, $V_{\mathcal{H}} = \langle \{v_1, \dots, v_{n-1}\} \rangle$, $\{v_1, \dots, v_{n-1}\}$ SLI

$A(a_1, \dots, a_n)$ $\quad \overset{\shortparallel}{\{\overrightarrow{AM}, M \in \mathcal{H}\}}$

$\exists\, t_1, \dots, t_{n-1} \in \mathbb{R}$ a.î. $\overrightarrow{AM} = \sum_{k=1}^{n-1} t_k v_k$, $M(x_1, \dots, x_n)$

$x_i - a_i = \sum_{k=1}^{n-1} t_k v_{ki}$, $i = \overline{1,n}$

$$\begin{cases} t_1 v_{11} + \dots + t_{n-1} v_{n-1\,1} = x_1 - a_1 \\ t_1 v_{1n} + \dots + t_{n-1} v_{n-1\,n} = x_n - a_n \end{cases}$$

sist. de n ecuații cu $n-1$ necunosc. $(t_1, \dots, t_{n-1})$

$$C = \begin{pmatrix} v_{11} & \cdots & v_{n-1\,1} \\ v_{1n} & & v_{n-1\,n} \end{pmatrix} \left| \begin{matrix} x_1 - a_1 \\ x_n - a_n \end{matrix} \right.$$

$$\Delta_c = 0 \Rightarrow \begin{vmatrix} v_{11} & \cdot & v_{n-1\,1} & x_1 - a_1 \\ \vdots & & & \\ v_{1n} & & v_{n-1\,n} & x_n - a_n \end{vmatrix} = 0$$

$\mathcal{H}$ $A_1 x_1 + \ldots + A_n x_n + A_0 = 0$, $\sum_{i=1}^{n} A_i^2 > 0$

$N = (A_1, \ldots, A_n)$ normala.

$\mathcal{H}$: $\langle N, r - r_0 \rangle = 0 \iff \langle N, \overrightarrow{AM} \rangle = 0$

OBS $\forall p$ plan $= \cap$ a $(n-p)$ hiperplane.

OBS $\mathcal{D} \perp \mathcal{H} \iff \langle \{u_{\mathcal{D}}\} \rangle = \langle \{N_{\mathcal{H}}\} \rangle$.

$\mathcal{H}$: $\sum_{i=1}^{n} A_i x_i + A_0 = 0$, $\mathcal{D}$: $\dfrac{x_1 - a_1}{A_1} = \ldots = \dfrac{x_n - a_n}{A_n}$

$A(a_1, \ldots, a_n) \in \mathcal{D}$

Poziția relativă a 2 hiperplane

$\mathcal{H}_1$: $A_1 x_1 + \ldots + A_n x_n + A_0 = 0$, $N = (A_1, \ldots, A_n)$

$\mathcal{H}_2$: $A_1' x_1 + \ldots + A_n' x_n + A_0' = 0$, $N' = (A_1', \ldots, A_n')$

$\mathcal{H}_1 \cap \mathcal{H}_2$ $\quad C = \begin{pmatrix} A_1 & \ldots & A_n \\ A_1' & \ldots & A_n' \end{pmatrix} \left| \begin{matrix} -A_0 \\ -A_0' \end{matrix} \right.$

- $\mathcal{H}_1 \parallel \mathcal{H}_2 \iff \langle \{N\} \rangle = \langle \{N'\} \rangle \iff \dfrac{A_1}{A_1'} = \ldots = \dfrac{A_n}{A_n'} \neq \dfrac{A_0}{A_0'}$
  $\mathcal{H}_1 \neq \mathcal{H}_2$ $\quad (rg\, C = 1,\ rg\, \overline{C} = 2)$
- $\mathcal{H}_1 = \mathcal{H}_2 \iff \dfrac{A_1}{A_1'} = \ldots = \dfrac{A_n}{A_n'} = \dfrac{A_0}{A_0'}$ $\quad (rg\, C = rg\, \overline{C} = 1)$
- $\mathcal{H}_1 \cap \mathcal{H}_2 = \mathcal{A}$ sp afin $(n-2)$
  $rg\, C = rg\, \overline{C} = 2$

<u>Exemplu</u> $n=3$

$\pi_1: \ x_1+x_2+x_3=1$

$\pi_2: \ 2x_1 \quad -x_3=0$

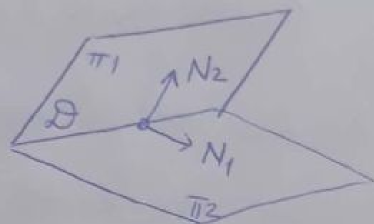
$$C=\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 \end{array}\right)$$

$$rg\, C = rg\, \overline{C} = 2$$

$\pi_1 \cap \pi_2 = \mathcal{D}$, $x_3 = t$.

$$\mathcal{D}: \begin{cases} x_1+x_2 = 1-t \\ 2x_1 = t \end{cases} \Rightarrow \begin{cases} x_1 = \frac{t}{2} \\ x_2 = 1-t-\frac{t}{2} = 1-\frac{3t}{2}. \end{cases}$$

$$\mathcal{D}: \ \frac{x_1}{\frac{1}{2}} = \frac{x_1-1}{-\frac{3}{2}} = \frac{x_3}{1}$$



$$\mathcal{D}: \ \frac{x_1}{1} = \frac{x_1-1}{-3} = \frac{x_3}{2}$$

$u_{\mathcal{D}} = N_1 \times N_2 \qquad N_1 = (1,1,1)$

$\qquad\qquad\qquad\qquad N_2 = (2,0,-1)$

<u>Intersecția unei drepte cu un hiperplan</u>

$$\mathcal{D}: \ \frac{x_1-a_1}{v_1} = \ldots = \frac{x_n-a_n}{v_n} = t \Rightarrow \begin{cases} x_1 = a_1+t v_1 \\ \vdots \\ x_n = a_n + t v_n \end{cases}$$

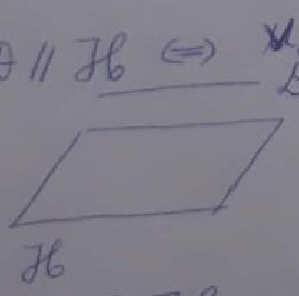
- $\mathcal{H} \quad A_1x_1+\ldots+A_nx_n+A_0=0$.

$\mathcal{D}\cap\mathcal{H}: \ A_1(a_1+\underline{t}v_1)+\ldots+A_n(a_n+\underline{t}v_n)+A_0=0$

$$t\cdot\sum_{i=1}^{n} A_i v_i + \sum_{i=1}^{n} A_i a_i + A_0 = 0$$

1) $\mathcal{D} \parallel \mathcal{H} \Leftrightarrow u_{\mathcal{D}} \perp N \Leftrightarrow A_1v_1+\ldots+A_nv_n=0 \quad (\nexists t)$

$\qquad A \notin \pi$, $A(a_1,\ldots,a_n)$



2) $\mathcal{D} \subset \mathcal{H} \Leftrightarrow A_1v_1+\ldots+A_nv_n = 0 \quad (\forall t)$

$\qquad A \in \pi$

3) ~~[illegible]~~ $\mathcal{D}\cap\mathcal{H}=\{P\}: \ t = -\dfrac{\sum_{i=1}^{n} A_i a_i + A_0}{\sum_{i=1}^{n} A_i v_i} \quad (\exists!\, t)$

Scanned with CamScanner

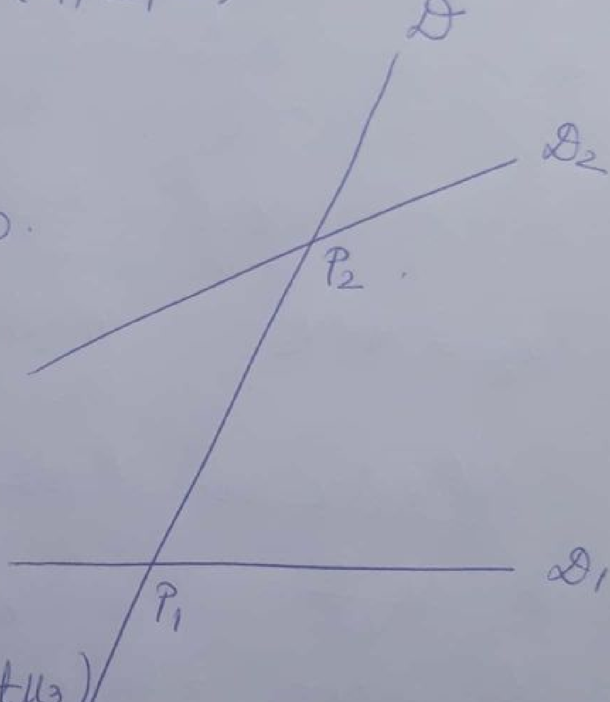
Perpendiculara comună a 2 drepte necoplanare.

$\mathcal{D}_1 \quad \dfrac{x_1-a_1}{u_1} = \dfrac{x_2-a_2}{u_2} = \dfrac{x_3-a_3}{u_3} \iff \begin{cases} x_1 = a_1 + t u_1 \\ x_2 = a_2 + t u_2 \\ x_3 = a_3 + t u_3 \end{cases}$

$A_1(a_1, a_2, a_3), \quad u = (u_1, u_2, u_3)$

$\mathcal{D}_2 : \dfrac{x_1-b_1}{v_1} = \dfrac{x_2-b_2}{v_2} = \dfrac{x_3-b_3}{v_3}$.

$A_2(b_1, b_2, b_3), \quad v = (v_1, v_2, v_3)$.



$\mathcal{D}_1, \mathcal{D}_2$ necoplanare.

$$\begin{array}{c} \begin{vmatrix} u_1 & v_1 & b_1-a_1 \\ u_2 & v_2 & b_2-a_2 \\ u_3 & v_3 & b_3-a_3 \end{vmatrix} \neq 0. \\ \begin{array}{ccc} u & v & \overrightarrow{A_1A_2} \end{array} \end{array}$$

(M_1) $\mathcal{D} = \perp$ comună

$\mathcal{D} \cap \mathcal{D}_k = \{P_k\}, \ k = \overline{1,2}$

$P_1(a_1 + t u_1, a_2 + t u_2, a_3 + t u_3)$

$P_2(b_1 + s v_1, b_2 + s v_2, b_3 + s v_3) \quad \mathcal{D}_2 : x_i = b_i + s v_i, \ i = \overline{1,3}$

$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow t, s \Rightarrow P_1, P_2$.

$\mathcal{D}$ este det. de P_1, P_2

$\text{dist}(\mathcal{D}_1, \mathcal{D}_2) = \text{dist}(P_1, P_2)$.

(M_2) π_1 planul det de $\mathcal{D}$ și $\mathcal{D}_1$

π_2 —"— $\mathcal{D}, \mathcal{D}_2$

$\pi_1 \cap \pi_2 = \mathcal{D}$

$N = u \times v$ (direcția lui $\mathcal{D}$)

$N_1 = N \times u$ normala lui π_1, $A_1(a_1, a_2, a_3) \in \pi_1$

$N_2 = N \times v$ —\/\/— π_2, $A_2(b_1, b_2, b_3) \in \pi_2$.

Temă 6 (curs)

1) Fie $\mathcal{D}_1 : \dfrac{x_1 - 1}{1} = \dfrac{x_2 - 3}{1} = \dfrac{x_3 - 2}{0}$

$\mathcal{D}_2 : \dfrac{x_1 - 1}{3} = \dfrac{x_2}{0} = \dfrac{x_3 - 1}{2}$

a) Să se arate că $\mathcal{D}_1, \mathcal{D}_2$ sunt necoplanare

b) Să se afle ec. $\perp$ comune la $\mathcal{D}_1, \mathcal{D}_2$; $dist(\mathcal{D}_1, \mathcal{D}_2) = ?$

2) Fie conica $\Gamma : f(x) = 3x_1^2 - 10x_1x_2 + 3x_2^2 + 4x_1 + 4x_2 + 4 = 0$

Să se aducă la forma canonică, utilizând izometrii.

3) Să se scrie ec. hiperbolei care trece prin $A(1, 0)$ și are asimptotele $d_1 \cup d_2 : 2x_1 \pm x_2 = 0$

Precizați excentricitatea și ec. directoarelor.