C11 – GA

Geometrie afină euclidiană. Geometrie analitică

Def $(\mathcal{A}, V/_{\mathbb{R}}, \varphi)$ s.n. spațiu afin $\Leftrightarrow$

1) $\mathcal{A} \neq \phi$ (mulțime de puncte)
2) $V/_{\mathbb{R}}$ spațiu vectorial (director)
3) $\varphi : \mathcal{A}\times\mathcal{A} \to V$ structură afină care verifică
   a) $\varphi(A,B) + \varphi(B,C) = \varphi(A,C)$, $\forall A,B,C \in \mathcal{A}$
   b) $\exists\, O \in \mathcal{A}$ a.î. $\varphi_O : \mathcal{A} \to V$, $\varphi_O(A) = \varphi(O,A)$, $\forall A \in \mathcal{A}$ bijecție (de fapt $\forall O$ este adev b))

OBS
a) Not $\varphi(A,B) = \overrightarrow{AB}$
b) $\dim \mathcal{A} = \dim_{\mathbb{R}} V$

Caz particular $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$, $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$
str. afină canonică $\varphi(u,v) = v - u$

Def $M \subseteq \mathbb{R}^n$ subm.

$$\mathcal{A}f(M) = \left\{ \sum_{i=1}^{m} a_i P_i \,,\ \sum_{i=1}^{m} a_i = 1 \,,\ a_i \in \mathbb{R},\ P_i \in M,\ i=\overline{1,m} \right\}\quad m \in \mathbb{N}^*$$

comb. afine de puncte din M

Def $\mathcal{A}' \subseteq \mathbb{R}^n$ subspațiu afin $\Leftrightarrow$ $[\forall P_1, P_2 \in \mathcal{A}' \Rightarrow \mathcal{A}f(\{P_1,P_2\}) \subseteq \mathcal{A}']$

Prop $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ str. can.

a) $\mathcal{A}' \subseteq \mathbb{R}^n$ subsp. afin $\Rightarrow \exists\, \underset{\text{subsp. vect}}{V'} \subseteq V = \mathbb{R}^n$ a.î. $\forall P' \in \mathcal{A}'$
$V' = \{\overrightarrow{P'P},\ \forall P \in \mathcal{A}'\}$

b) $P \in \mathbb{R}^n$, $\underset{\text{subsp vect}}{V'} \subseteq \mathbb{R}^n = V \Rightarrow \exists!\ \underset{\text{sp afin}}{\mathcal{A}'} \subseteq \mathbb{R}^n$, $P \in \mathcal{A}'$
a.î. V' subsp. director.

Ex. $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ $\mathcal{A}' = \{x \in \mathbb{R}^n \mid AX = B\} \subseteq \mathbb{R}^n$ subsp. afin.
$V' = \{x \in \mathbb{R}^n \mid AX = 0\} \subseteq \mathbb{R}^n$ subsp. director

Caz particular $(\mathbb{R}^3, \mathbb{R}^3/_{\mathbb{R}}, \varphi)$

$$\mathcal{A}' = \left\{ x \in \mathbb{R}^3 \,\middle|\, \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 - x_3 = 1 \end{cases} \right\} \subseteq \mathbb{R}^3 \text{ subsp. afin.}$$

$V' = \left\{ x \in \mathbb{R}^3 \mid \begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases} \right\}$ subsp. director

Def $\mathcal{A}', \mathcal{A}'' \subset \mathbb{R}^n$ subsp. afine

$\mathcal{A}' \parallel \mathcal{A}'' \iff V' \subseteq V''$ sau $V'' \subseteq V'$

(subsp. directoare)

Ex $(\mathbb{R}^3, \mathbb{R}^3/_{\mathbb{R}}, \varphi)$

$\mathcal{A}' = \{ x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 2 \}$

$\mathcal{A}'' = \{ x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 1 \}$

$\mathcal{A}' \parallel \mathcal{A}''$, $V' = V'' = \{ x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 0 \}$

Def $(\mathcal{E}, (E, \langle \cdot, \cdot \rangle), \varphi)$ spațiu afin euclidian (spațiu punctual euclidian).

$\iff$ spațiu afin și sp. vectorial director = sp. vect. euclidian

Def $(\mathcal{E}, E, \varphi)$, $\mathcal{E}_1, \mathcal{E}_2 \subset \mathcal{E}$ subsp. afine.

a) $\mathcal{E}_1, \mathcal{E}_2$ sunt perpendiculare $\iff E_1 \perp E_2$

b) $\mathcal{E}_1, \mathcal{E}_2$ sunt normale $\iff E = E_1 \oplus E_1^{\perp}$

$E_2 = E_1^{\perp}$

- $(\underset{\mathcal{E}}{\mathbb{R}^n}, \underset{E}{(\mathbb{R}^n, g_0)}, \varphi)$ sp. afin euclidian canonic

Ecuații ale varietăților liniare

$\mathcal{R} = \{ O; e_1, \dots, e_n \}$ reper cartezian ortonormat

$O \in \mathcal{E} = \mathbb{R}^n$, $\{ e_1, \dots, e_n \}$ reper ortonormat în $E = \mathbb{R}^n$.

① Ecuația unei drepte

$V_{\mathcal{D}} = \langle \{ v \} \rangle$

a) $\mathcal{D}$. $\underset{A}{\cdot}\xrightarrow{v}$ v vector nenul $(v_1^2 + \dots + v_n^2 > 0)$ $\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$

$\forall M \in \mathcal{D} \Rightarrow \exists t \in \mathbb{R}$ aî $\overrightarrow{AM} = tv \Rightarrow$

$\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$, $v = \sum_{i=1}^{n} v_i e_i$ $\quad \overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}$

$(\overrightarrow{AM} \in V_{\mathcal{D}})$

$\mathcal{D}: \sum_{i=1}^{n} (x_i - a_i) e_i = \sum_{i=1}^{n} t v_i e_i \iff (x_1 - a_1, \dots, x_n - a_n) = t(v_1, \dots, v_n)$

$\iff \frac{x_1 - a_1}{v_1} = \dots = \frac{x_n - a_n}{v_n} = t$ Convenție: Dc $\exists i = \overline{1,n}$ aî $v_i = 0 \Rightarrow x_i - a_i = 0$

b) $\mathcal{D}$

$v = \overrightarrow{AB} = \sum_{i=1}^{n} \underbrace{(b_i - a_i)}_{v_i} e_i$

$A \neq B$

$\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i \ , \ \overrightarrow{OB} = \sum_{i=1}^{n} b_i e_i$

$\mathcal{D}: \ (x_1 - a_1, .., x_n - a_n) = t(b_1 - a_1, .., b_n - a_n), \ t \in \mathbb{R}$

$\frac{x_1 - a_1}{b_1 - a_1} = .. = \frac{x_n - a_n}{b_n - a_n} = t$

Convenție: Dc $\exists i = \overline{1,n}$ ai $b_i - a_i = 0$, at $x_i - a_i = 0$.

Exemplu $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

a) $A(1,2,-3)$, $v = (1,4,3)$

$\mathcal{D} \ni A$, $V_{\mathcal{D}} = \langle \{v\} \rangle$

$\mathcal{D}: \ \frac{x_1 - 1}{1} = \frac{x_2 - 2}{4} = \frac{x_3 + 3}{3} = t \Leftrightarrow \begin{cases} x_1 = t + 1 \\ x_2 = 4t + 2 \\ x_3 = 3t - 3, \ t \in \mathbb{R}. \end{cases}$

b) $A(1,2,-3)$, $B(-1,0,1)$

$A, B \in \mathcal{D}'$

$\mathcal{D}': \ \frac{x_1 - 1}{-1-1} = \frac{x_2 - 2}{0 - 2} = \frac{x_3 + 3}{1 + 3} \Leftrightarrow \frac{x_1 - 1}{\cancel{-2}} = \frac{x_2 - 2}{\cancel{-2}} = \frac{x_3 + 3}{\cancel{4}_{-2}} = t$

$\begin{cases} x_1 = t + 1 \\ x_2 = t + 2 \\ x_3 = -2t - 3, \ t \in \mathbb{R} \end{cases}$

Poziția relativă a 2 drepte în $(\mathbb{R}^n, (\mathbb{R}^n, g_0), \varphi)$

$\mathcal{D}_1: \ x_i - a_i = t v_i, \ i = \overline{1,n}$

$\mathcal{D}_2: \ x_i - b_i = t' v'_i, \ i = \overline{1,n}$

$\mathcal{D}_1 \cap \mathcal{D}_2: \ t v_i + a_i = t' v'_i + b_i \rightarrow t v_i - t' v'_i = b_i - a_i, \ i = \overline{1,n}$

$$C = \left(\begin{array}{cc|c} v_1 & -v'_1 & b_1 - a_1 \\ \vdots & \vdots & \vdots \\ v_n & -v'_n & b_n - a_n \end{array}\right)$$

a) $rg\, C = rg\, \overline{C} = 2 \Rightarrow \mathcal{D}_1, \mathcal{D}_2$ concurente

b) $rg\, C = rg\, \overline{C} = 1 \Rightarrow \mathcal{D}_1 = \mathcal{D}_2$

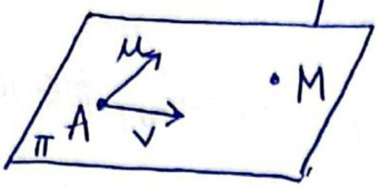
c) $rg\, C = 2$, $rg\, \overline{C} = 3 \Rightarrow \mathcal{D}_1, \mathcal{D}_2$ necoplanare

d) $rg\, C = 1$, $rg\, \overline{C} = 2 \Rightarrow \mathcal{D}_1 \parallel \mathcal{D}_2$!

② Ecuația unui plan

$\{u, v\}$ SLI

a)



$A(a_1, \dots, a_n) \in \Pi$, $V_\Pi = \langle \{u, v\} \rangle$

$\forall M \in \Pi \Rightarrow \vec{AM} \in V_\Pi \Rightarrow$
$(x_1, \dots, x_n)$
$\exists\, t, s \in \mathbb{R}$ a.î. $\vec{AM} = tu + sv$

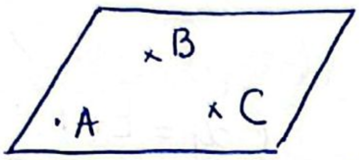
$u = \sum_{i=1}^{n} u_i e_i$

$v = \sum_{i=1}^{n} v_i e_i$

$\pi: x_i - a_i = t u_i + s v_i \,, i = \overline{1,n}$

ec. parametrice

b)



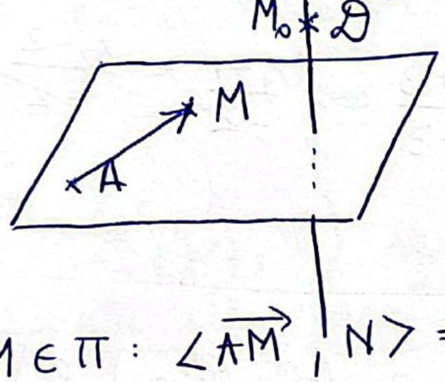
$A(a_1, \dots, a_n), B(b_1, \dots, b_n), C(c_1, \dots, c_n) \in \Pi$

(pcte necolin $\Leftrightarrow \{\vec{AB}, \vec{AC}\}$ SLI)

$V_\Pi = \langle \{ \underset{u}{\vec{AB}}, \underset{v}{\vec{AC}} \} \rangle$

$\Pi: x_i - a_i = t(b_i - a_i) + s(c_i - a_i), \forall i = \overline{1,n}, \; t, s \in \mathbb{R}.$

c)



$A(a_1, \dots, a_n) \in \Pi$, $\mathcal{D} \perp \Pi$

$\mathcal{D}: \dfrac{x_1 - x_1^0}{u_1} = \dots = \dfrac{x_n - x_n^0}{u_n} = t$

$u = (u_1, \dots, u_n) = N$ (normala)

$\forall M \in \Pi: \langle \vec{AM}, N \rangle = 0 \Rightarrow (x_1 - a_1) u_1 + \dots + (x_n - a_n) u_n = 0$

$M(x_1, \dots, x_n)$

$x_1 u_1 + \dots + x_n u_n + \alpha = 0$

$\alpha = -a_1 u_1 - \dots - a_n u_n.$

Caz particular $n = 3$

a) $A(a_1, a_2, a_3) \in \Pi$, $u = (u_1, u_2, u_3), V = (v_1, v_2, v_3)$

$\{u, v\}$ SLI

$$\Pi: \begin{vmatrix} x_1 - a_1 & u_1 & v_1 \\ x_2 - a_2 & u_2 & v_2 \\ x_3 - a_3 & u_3 & v_3 \end{vmatrix} = 0$$

$N = u \times v = (N_1, \dots, N_3)$

$\langle \vec{AM}, N \rangle = 0$

$M(x_1, x_2, x_3) \in \Pi \Leftrightarrow (x_1 - a_1) N_1 + \dots + (x_3 - a_3) N_3 = 0$

Exemplu $A(1, -1, 2) \in \Pi$, $V_\Pi = \langle \{ u = (2, 3, 1), v = (4, 1, 3) \} \rangle$

$$\Pi: \begin{vmatrix} x_1 - 1 & 2 & 4 \\ x_2 + 1 & 3 & 1 \\ x_3 - 2 & 1 & 3 \end{vmatrix} = 0$$

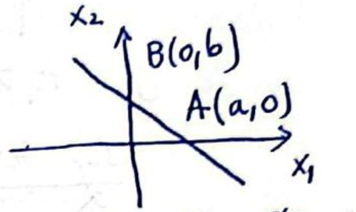
nti $N = u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix} = (8, -2, -10) = 2(4, -1, -5)$

$\langle \overrightarrow{AM}, N \rangle = 0$

$(x_1 - 1)4 + (x_2 + 1)(-1) + (x_3 - 2)(-5) = 0$

$\pi: 4x_1 - x_2 - 5x_3 \underbrace{-4-1+10}_{5} = 0$ ec. generală a planului

b) $A(a_1, a_2, a_3), B(b_1, b_2, b_3), C(c_1, c_2, c_3) \in \pi$

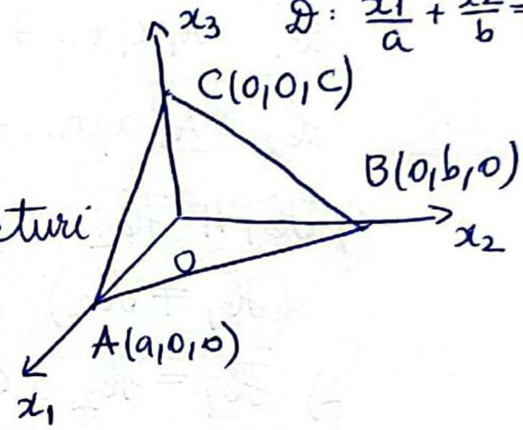


$\mathcal{D}: \frac{x_1}{a} + \frac{x_2}{b} = 1$

$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & b_1 & c_1 & 1 \\ a_2 & b_2 & c_2 & 1 \\ a_3 & b_3 & c_3 & 1 \end{vmatrix} = 0$

Dacă $A(a,0,0), B(0,b,0), C(0,0,c) \in \pi$

$\pi: \frac{x_1}{a} + \frac{x_2}{b} + \frac{x_3}{c} = 1$ (ec. prin tăieturi a planului)

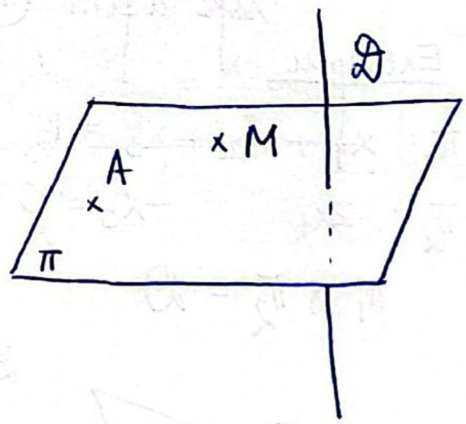


Exemplu

$A(1,1,1), B(-1,1,1), C(2,0,0) \in \pi$

$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{vmatrix} = 0$

c) Exemplu $\mathcal{D} \perp \pi$, $A(1,0,3) \in \pi$



$\mathcal{D}: \frac{x_1 - 1}{2} = \frac{x_2 - 1}{1} = \frac{x_3}{2}$

$N_\pi = v_{\mathcal{D}} = (2, 1, 2)$

$\pi: \langle \overrightarrow{AM}, N \rangle = 0 \Leftrightarrow$

$(x_1 - 1)2 + (x_2 - 0) \cdot 1 + (x_3 - 3)2 = 0 \Rightarrow$

$\pi: 2x_1 + x_2 + 2x_3 - 8 = 0$

3 Ecuația unui hiperplan afin (ssp. afin $(n-1)$-dim)

$A(a_1, .., a_n) \in \mathcal{H}$, $\vec{V}_{\mathcal{H}} = \langle \{u_1, .., u_{n-1}\} \rangle$

$\{u_1, .., u_{n-1}\}$ SLI

$\forall M(x_1,..,x_n) \in \mathcal{H} \Rightarrow \overrightarrow{AM} \in V_{\mathcal{H}} \Rightarrow \exists\, t_1,..,t_{n-1} \in \mathbb{R}$ a.î.

$x_i - a_i = t_1 u_1^i + \dots + t_{n-1} u_{n-1}^i \,,\ i=\overline{1,n}$ $\quad \overrightarrow{AM} = \sum_{j=1}^{n-1} t_j u_j$

$$\mathcal{H}: \begin{vmatrix} x_1-a_1 & u_1^1 & \dots & u_{n-1}^1 \\ \vdots & \vdots & & \vdots \\ x_n-a_n & u_1^n & & u_{n-1}^n \end{vmatrix} = 0 \Longrightarrow$$

$\mathcal{H}: A_1x_1 + \dots + A_nx_n + A_0 = 0,\ N = (A_1,..,A_n)\,,\ A_1^2+..+A_n^2>0$

Poz. relativă a 2 hiperplane.

$\mathcal{H}_1: A_1x_1 + .. + A_nx_n + A_0 = 0$

$\mathcal{H}_2: A_1'x_1 + \dots + A_n'x_n + A_0' = 0$

1) $\mathcal{H}_1 \parallel \mathcal{H}_2$ $(\mathcal{H}_1 \neq \mathcal{H}_2)$ $\iff \dfrac{A_1}{A_1'} = .. = \dfrac{A_n}{A_n'} \neq \dfrac{A_0}{A_0'}$

2) $\mathcal{H}_1 = \mathcal{H}_2 \iff \dfrac{A_1}{A_1'} = .. = \dfrac{A_n}{A_n'} = \dfrac{A_0}{A_0'}$

3) $\underbrace{\mathcal{H}_1 \cap \mathcal{H}_2}_{\text{ssp. afin } (n-2) \text{dimensional}} \neq \phi$ $\quad (N \neq \alpha N')$

Exemplu

$\pi_1: x_1 + x_2 + x_3 = 1$

$\pi_2: 2x_1 \quad\quad - x_3 = 0$

$\left(\begin{array}{cc|c} \boxed{\begin{matrix}1 & 1 \\ 2 & 0\end{matrix}} & \begin{matrix}1 \\ -1\end{matrix} \end{array}\right) \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$x_3 = t$

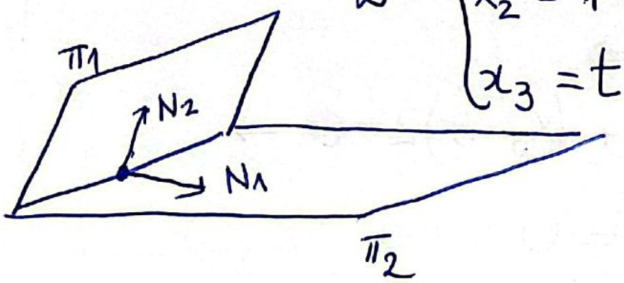
$\begin{cases} x_1 + x_2 = 1 - t \\ 2x_1 = t \end{cases}$

$\pi_1 \cap \pi_2 = \mathcal{D}$

$$\mathcal{D}: \begin{cases} x_1 = \dfrac{t}{2} \\ x_2 = 1 - t - \dfrac{t}{2} = 1 - \dfrac{3}{2}t \\ x_3 = t \end{cases}$$

$\left(\frac{1}{2}, -\frac{3}{2}, 1\right) = -\frac{1}{2}(-1, 3, -2)$

OBS



$$N_1 \times N_2 = u_{\mathcal{D}} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{vmatrix}$$

$= (-1, +3, -2)$

Intersecția unei drepte cu un hiperplan.

$M_0(a_1,\dots,a_n)\in\mathcal{D}$
$u_{\mathcal{D}}=(u_1,\dots,u_n)$

$$\mathcal{D}: \frac{x_1-a_1}{u_1}=\dots=\frac{x_n-a_n}{u_n}=t \Rightarrow \begin{cases} x_1=u_1t+a_1\\ \vdots\\ x_n=u_nt+a_n,\ t\in\mathbb{R}\end{cases} \quad (*)$$

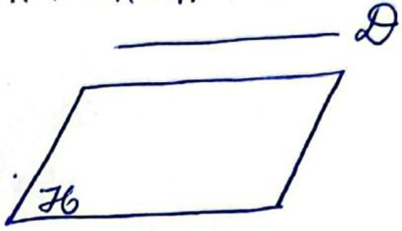
$$\mathcal{H}: A_1x_1+\dots+A_nx_n+A_0=0\ ,\ N=(A_1,\dots,A_n).$$

$$\mathcal{D}\cap\mathcal{H}: t(A_1u_1+\dots+A_nu_n)+A_1a_1+\dots+A_na_n+A_0=0$$

1) $A_1u_1+\dots+A_nu_n=0$

$M_0\notin\mathcal{H}\Rightarrow A_1a_1+\dots+A_na_n+A_0\neq 0.$

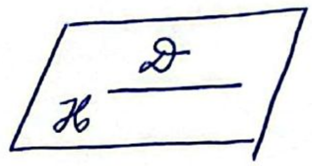
$\mathcal{D}\parallel\mathcal{H}\quad(\mathcal{D}\cap\mathcal{H}=\emptyset)$



2) $A_1u_1+\dots+A_nu_n=0$

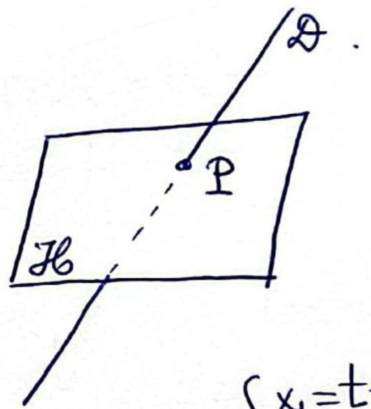
$M_0\in\mathcal{H}\Rightarrow A_1a_1+\dots+A_na_n+A_0=0$

$\mathcal{D}\subset\mathcal{H}$



3) $A_1u_1+\dots+A_nu_n\neq 0\Rightarrow$

$$t=-\frac{A_1a_1+\dots+A_na_n+A_0}{A_1u_1+\dots+A_nu_n}\overset{(*)}{\Rightarrow}\{P\}=\mathcal{D}\cap\mathcal{H}$$



Exemplu

$$\pi: x_1+x_2+x_3=2,\ \mathcal{D}: \frac{x_1-3}{1}=\frac{x_2}{3}=\frac{x_3}{1}=t\to\begin{cases}x_1=t+3\\ x_2=3t\\ x_3=t\end{cases}\ (*)$$

$$\mathcal{D}\cap\pi: t+3+3t+t=2\Rightarrow 5t=-1\Rightarrow t=-\frac{1}{5}$$

$$P: \begin{cases} x_1=-\frac{1}{5}+3=\frac{14}{5}\\ x_2=-\frac{3}{5}\\ x_3=-\frac{1}{5}.\end{cases}$$

Perpendiculara comună a 2 drepte necoplanare

$$\mathcal{D}_1: \frac{x_1-a_1}{u_1}=\frac{x_2-a_2}{u_2}=\frac{x_3-a_3}{u_3}=t \qquad u=(u_1,u_2,u_3),\ A(a_1,a_2,a_3)$$

$$\mathcal{D}_2: \frac{x_1-b_1}{v_1}=\frac{x_2-b_2}{v_2}=\frac{x_3-b_3}{v_3}=s \qquad v=(v_1,v_2,v_3),\ B(b_1,b_2,b_3)$$

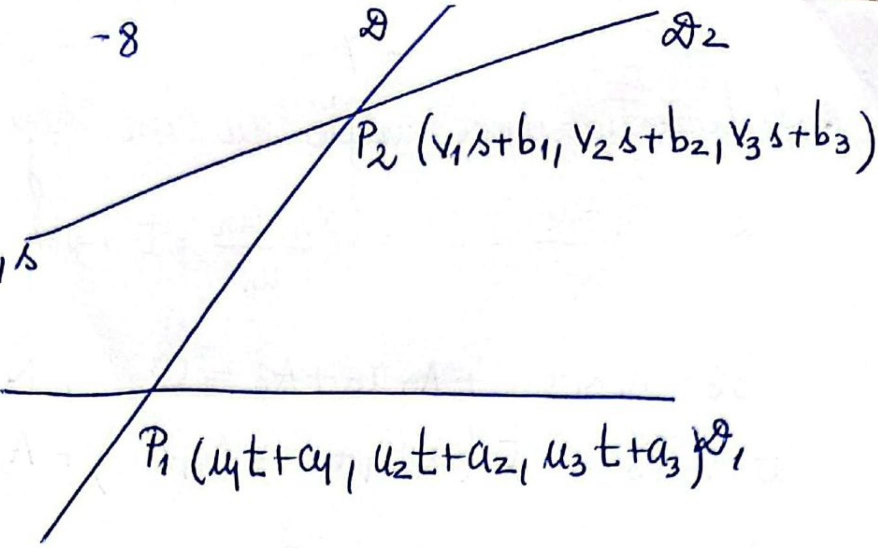
$$\begin{vmatrix} u_1 & v_1 & b_1-a_1\\ u_2 & v_2 & b_2-a_2\\ u_3 & v_3 & b_3-a_3\end{vmatrix}\neq 0 \iff \mathcal{D}_1,\mathcal{D}_2 \text{ necoplanare}$$

$\overrightarrow{AB}$

$\mathcal{D} \perp \mathcal{D}_i,\ i=\overline{1,2}$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow t, s$$

$\Rightarrow P_1, P_2$.



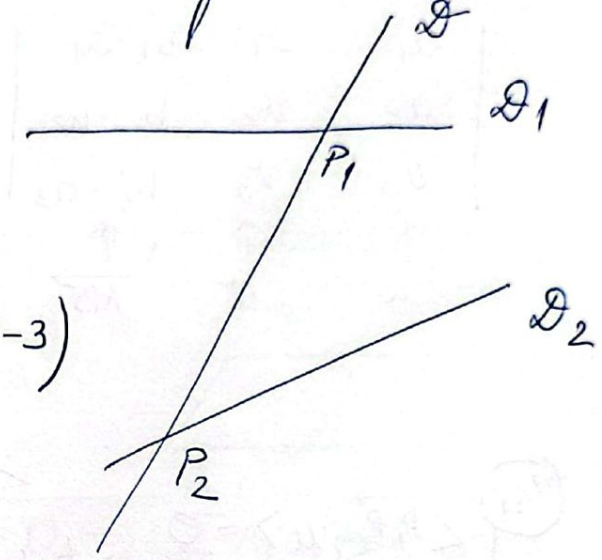
EXEMPLU

$\mathcal{D}_1: \dfrac{x_1-2}{1} = \dfrac{x_2}{2} = \dfrac{x_3-3}{1} = t \quad u=(1,2,1),\ A(2,0,3)$

$\mathcal{D}_2: \dfrac{x_1-1}{2} = \dfrac{x_2-3}{1} = \dfrac{x_3}{1} = s \quad v=(2,1,1),\ B(1,3,0)$

$\overrightarrow{AB}=(-1,3,-3)$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & -3 \end{vmatrix} \neq 0 \Rightarrow \mathcal{D}_1, \mathcal{D}_2 \text{ necoplanare.}$$



(M1) $P_1(t+2, 2t, t+3)$

$P_2(2s+1, s+3, s)$

$\overrightarrow{P_1P_2} = (2s-t-1, s-2t+3, s-t-3)$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 2s-t-1+2s-4t+6+s-t-3=0 \\ 4s-2t-2+s-2t+\cancel{3}+s-t-\cancel{3}=0 \end{cases} \Rightarrow \begin{cases} -6t+5s=-2 \\ -5t+6s=2 \end{cases}$$

$$11(-t+s)=0$$

$t=s=2.$

$P_1(4,4,5),\ P_2(5,5,2),\ \overrightarrow{P_1P_2}=(1,1,-3)$

$\mathcal{D}: \dfrac{x_1-4}{1} = \dfrac{x_2-4}{1} = \dfrac{x_3-5}{-3}$

$\text{dist}(\mathcal{D}_1, \mathcal{D}_2) = \text{dist}(P_1, P_2) = \|\overrightarrow{P_1P_2}\| = \sqrt{11}.$

(M2) π_1 plan det de $\mathcal{D}$ și $\mathcal{D}_1$

$A(2,0,3) \in \pi_1$

$$N = u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} =$$

$N = (1,1,-3) = u_{\mathcal{D}}$

$$N_1 = N \times u = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & -3 \\ 1 & 2 & 1 \end{vmatrix} = (7,-4,1)$$

$\pi_1: 7(x_1-2) - 4(x_2-0) + 1\cdot(x_3-3) = 0 \Rightarrow \boxed{7x_1-4x_2+x_3-17=0}$

π_2 plan det de $\mathcal{D}_2$ și $\mathcal{D}$, $B(1,3,0) \in \pi_2$.

$N = (1,1,-3) = u_{\mathcal{D}}$

$$N_2 = N \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix} = (4,-7,-1)$$

b3)

$\pi_2 : 4(x_1-1) - 7(x_2-3) - 1(x_3-0) = 0 \Rightarrow 4x_1 - 7x_2 - x_3 + 17 = 0$

$$\mathcal{D} = \pi_1 \cap \pi_2 : \begin{cases} 7x_1 - 4x_2 + x_3 - 17 = 0 \\ 4x_1 - 7x_2 - x_3 + 17 = 0. \end{cases}$$

$x_3 = t$

$$\begin{cases} 7x_1 - 4x_2 = -t + 17 \\ 4x_1 - 7x_2 = t - 17 \end{cases} \quad \begin{vmatrix} 7 \\ -4 \end{vmatrix}$$

$$x_1 \underbrace{(49-16)}_{33} = t(-11) + 17 \cdot 11 \Rightarrow x_1 = -\frac{1}{3}t + \frac{17}{3}.$$

$$x_2 = \frac{1}{4}\left[7x_1 + t - 17\right] = \frac{1}{4}\left[-\frac{7}{3}t + \frac{7 \cdot 17}{3} + t - 17\right]$$

$$= \frac{1}{4}\left(-\frac{4}{3}t + 17 \cdot \frac{4}{3}\right) = -\frac{1}{3}t + \frac{17}{3}.$$

$$\mathcal{D} : \begin{cases} x_1 = -\frac{1}{3}t + \frac{17}{3} \\ x_2 = -\frac{1}{3}t + \frac{17}{3} \\ x_3 = t \end{cases}$$

$P_1, P_2 \in \mathcal{D}$.