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Seminar 6

Aplicații liniare

PROP $f: V_1 \to V_2$ liniară

f bijectivă $\Leftrightarrow$ f transf. $\forall$ reper din V_1 într-un reper în V_2.

① $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1, x_2, x_3) = (2x_1 + 2x_2, x_1 + x_3, x_1 + 3x_2 - 2x_3)$

a) f nu este izomorfism de sp. vect.

b) $f|_{V'}: V' \to V''$ izomorfism, unde

$V' = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0\}$

$V'' = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 3x_1 - 4x_2 - 2x_3 = 0\}$

c) Să se afle $f(V' \cap V'')$.

d) $\mathbb{R}^3 = V' \oplus W$. Dați un exemplu de W.

Fie $p: \mathbb{R}^3 \to \mathbb{R}^3$ proiecția pe V'

$s: \mathbb{R}^3 \to \mathbb{R}^3$ simetria față de V'

Să se calculeze $p(1,3,6)$, $s(1,3,6)$

OBS

a) $p: V_1 \oplus V_2 \to V_1 \oplus V_2$ liniară

$p(v) = p(\underset{V_1}{\underset{\cap}{v_1}} + \underset{V_2}{\underset{\cap}{v_2}}) = v_1$ proiecția pe V_1

b) $s: V_1 \oplus V_2 \to V_1 \oplus V_2$.

$s = 2p - id_V$ simetria față de V_1.

$s(\underset{V_1}{\underset{\cap}{v_1}} + \underset{V_2}{\underset{\cap}{v_2}}) = v_1 - v_2$

② $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 + x_2 - x_3, -x_1 - x_2 + x_3, x_1 + x_2 + x_3)$

a) $[f]_{R,R}$, $R = \{e_1' = e_1 + e_2 + e_3, e_2' = e_1 + e_3, e_3' = e_1 + e_2\}$

b) $\mathbb{R}^3 = \operatorname{Im} f \oplus W$

$s: \mathbb{R}^3 \to \mathbb{R}^3$ simetria față de W

$s(0,1,1) = ?$

c) $\mathbb{R}^3 = f(V') \oplus U$

$V' = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + 2x_2 + x_3 = 0 \\ -x_1 + x_2 + 2x_3 = 0 \end{cases} \right\}$

$p: \mathbb{R}^3 \to \mathbb{R}^3$ proiecția pe $f(V')$

$p(2,-1,3) = ?$

③ $f: \mathbb{R}_2[X] \to \mathbb{R}_1[X]$, $f(P) = P'$

a) $[f]_{R,R'} = ?$ $R = \{X^2, 1+X, 2-X\}$ reper în $\mathbb{R}_2[X]$

$R' = \{X, 1+3X\}$ —״— $\mathbb{R}_1[X]$.

b) $\mathbb{R}_2[X] = \operatorname{Ker} f \oplus W$

$p_1: \mathbb{R}_2[X] \to \mathbb{R}_2[X]$ proiecția pe $\operatorname{Ker} f$

$p_2: \mathbb{R}_2[X] \to \mathbb{R}_2[X]$ —״— W.

$p_1(1 - X + 3X^2)$, $p_2(2X + X^2) = ?$

④ $f: \mathcal{M}_2(\mathbb{R}) \to \mathcal{M}_2^s(\mathbb{R})$, $f(A) = A + A^T$

a) $[f]_{R_0, R_0'}$ $R_0 = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ reper în $\mathcal{M}_2(\mathbb{R})$

$R_0' = \{E_{11}, E_{12} + E_{21}, E_{22}\}$ reper în $\mathcal{M}_2^s(\mathbb{R})$

b) $\ker f$, $\operatorname{Im} f$.

c) $f(V) = ?$, $V = \left\{ \begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix}, c, d \in \mathbb{R} \right\}$

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$(\mathbb{R}^3,+,\cdot)/\mathbb{R}$ $\quad \mathcal{R}=\{e_1=(1,0,0), e_2=(0,1,0), e_3=(0,0,1)\} \xrightarrow{C}$
$\mathcal{R}'=\{e_1'=e_1+e_2+e_3,\ e_2'=e_1+e_2,\ e_3'=e_1\}$.

$((\mathbb{R}^3)^*=\{f:\mathbb{R}^3\to\mathbb{R} \mid f \text{ lin}\},+,\cdot)/\mathbb{R}$ sp. vect dual.

$\mathcal{R}^*=\{e_1^*, e_2^*, e_3^*\} \xrightarrow{D} (\mathcal{R}')^*=\{e_1'^*, e_2'^*, e_3'^*\}$ repere duale în sp. dual.

$e_i^*(e_j)=\delta_{ij}$, $e_i'^*(e_j')=\delta_{ij}$, $\forall i,j=\overline{1,3}$

$C, D=?$

Ex. $f:\mathbb{R}^3\to\mathbb{R}_2[X]$ liniară

$f(1,1,-1)=X^2+3X+3$, $f(-1,1,0)=-X+1$, $f(0,1,-1)=2X^2$

a) $f=?$

b) $[f]_{\mathcal{R}_0,\mathcal{R}_0'}=A=?$ $\quad \mathcal{R}_0=\{e_1,e_2,e_3\}$, $\mathcal{R}_0'=\{1,X,X^2\}$ repere canonice în $\mathbb{R}^3$, $\mathbb{R}_2[X]$

Ex $f:\mathbb{R}_1[X]\to\mathcal{M}_{2,1}(\mathbb{R})$

$[f]_{\mathcal{R}_1,\mathcal{R}_2}=A=\begin{pmatrix}1&1\\0&-1\end{pmatrix}$ $\quad \mathcal{R}_1=\{1+X,-X\}$, $\mathcal{R}_2=\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}-1\\0\end{pmatrix}\right\}$

$f=?$

Ex. $f:\mathbb{R}^n\to\mathbb{R}^n$ endomorfism a.î. $f^2=f+id_{\mathbb{R}^n} \Rightarrow f\in Aut(\mathbb{R}^n)$

Ex $f\in End(\mathbb{R}^4)$

$f(e_1+e_2)=(1,1,-1,-1)$, $f(e_1-e_2)=(-1,-1,1,1)$,
$f(e_3+e_4)=(-1,1,-1,1)$, $f(e_3-e_4)=(1,-1,1,-1)$, $\mathcal{R}_0=\{e_1,e_2,e_3,e_4\}$.

a) $[f]_{\mathcal{R}_0,\mathcal{R}_0}=A=?$

b) Precizați câte o bază în $Ker f$, $Im f$

Ex $f:\mathbb{R}^3\to\mathbb{R}^3$, $f(x)=(x_1+x_2-2x_3,\ x_2,\ x_1-x_2)$

$f\in Aut(\mathbb{R}^3)$

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Ex $f: \mathbb{R}_2[X] \to \mathbb{R}_2[X]$, $f(P) = P + P' + P''$, $P = \tilde{P}$

a) Este apl. f bijectivă?

b) $[f]_{R_0, R_0} = ?$ $R_0 = \{1, X, X^2\}$

Ex $U \xrightarrow{f} V \xrightarrow{g} W$ f, g liniare. a.î. $g \circ f = 0$

a) f surj $\Rightarrow g = 0$

b) g inj $\Rightarrow f = 0$

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