$h \in O(\mathbb{R}^3)$

$h(e_i^0) = e_i \quad (\forall)\ i = \overline{1,3}$

Seminarul 11

Geometrie analitică euclidiană

1) $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

$A(3,-1,3),\ B(5,1,-1),\ u = (-3,5,-6)$

a) Să se scrie ec. dreptei D a.î. $A \in D$, $V_D = \langle\{u\}\rangle$

b) Să se scrie ec. dreptei AB

c) Să se afle punctele de intersecție ale dreptei D cu planele de coordonate

Sol

a) $D: \dfrac{x_1-3}{-3} = \dfrac{x_2+1}{5} = \dfrac{x_3-3}{-6} = t \Rightarrow \begin{cases} x_1 = -3t+3 \\ x_2 = 5t-1 \\ x_3 = -6t+3 \end{cases}$

b) $AB: \dfrac{x_1-3}{\underbrace{b_1-a_1}_{2}} = \dfrac{x_2+1}{\underbrace{b_2-a_2}_{2}} = \dfrac{x_3-3}{\underbrace{b_3-a_3}_{-4}} = t \Rightarrow \begin{cases} x_1 = 2t+3 \\ x_2 = 2t-1 \\ x_3 = -4t+3 \end{cases}$

c) $O(x_1, x_2): x_3 = 0$

$D \cap O_{x_1x_2} \Rightarrow -6t+3=0 \Rightarrow t = \frac{1}{2}$

$$\left(\frac{-3}{2}+3, \frac{5}{2}-1, 0\right)$$

$$\left(\frac{3}{2}, \frac{3}{2}, 0\right)$$

$O_{x_1x_3} : x_2 = 0 \quad \mathcal{D} \cap O_{x_1,x_3} \Rightarrow 5t - 1 = 0 \Rightarrow t = \frac{1}{5}$

$$\left(\frac{-3}{5}+3, 0, \frac{-6}{5}+3\right) = \left(\frac{12}{5}, 0, \frac{9}{5}\right)$$

$\cancel{O_{x_1x_3}}\ O_{x_2x_3} : x_1 = 0 \quad \mathcal{D} \cap O_{x_2x_3} \Rightarrow -3t + 3 = 0$

$t = \frac{3}{3} = 1$

$(0, 4, -3)$

2) Fie dreptele

$$\mathcal{D}_1 : \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases}, \quad \mathcal{D}_2 : \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$$

a) $\mathcal{D}_1, \mathcal{D}_2$ necoplanare.

b) Să se afle ec $\perp$ comune a dreptelor $\mathcal{D}_1, \mathcal{D}_2$.

c) Să se det. $dist(\mathcal{D}_1, \mathcal{D}_2)$

Sol.

$$\mathcal{D}_1 : \begin{cases} \cancel{x_3 = t} \\ \cancel{x_1 = -t} \\ \cancel{x_2 = t+1} \end{cases} \qquad \begin{cases} x_1 = -t \\ x_2 = t+1 \\ x_3 = t \end{cases}$$

$u = (-1, 1, 1)$

Fie $t = 0 \Rightarrow A(0, 1, 0)$.

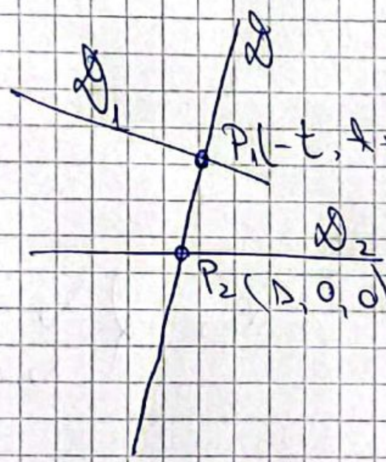
$$\mathcal{D}_2 : \begin{cases} x_1 = \lambda \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

Fie $\lambda = 0$; $\vec{v} = (1, 0, 0)$, $B(0,0,0)$

$f(B-A) = (0, -1, 0)$

$$\det = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow \text{drepte necoplanare}$$

b)



$P_1(-t, t+1, t) = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

$P_2(\lambda, 0, 0) = \left(\frac{1}{2}, 0, 0\right)$

$$\overrightarrow{P_1P_2} = (\lambda + t, -t-1, -t) = \left(0, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}(0, -1, 1)$$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, \vec{u} \rangle = 0 \Rightarrow -\lambda - t - t - 1 - t = 0 \\ \langle \overrightarrow{P_1P_2}, \vec{v} \rangle = 0 \Rightarrow \lambda + t = 0 \end{cases} \Rightarrow 2t = -1, \quad t = -\frac{1}{2}$$

$\Rightarrow \lambda = \frac{1}{2}$

$$\mathcal{D} : \frac{x_1 - \frac{1}{2}}{0} = \frac{x_2 - 0}{-1} = \frac{x_3 - 0}{1}$$

$$\mathcal{D} \begin{cases} x_1 - \frac{1}{2} = 0 \\ x_2 + x_3 = 0 \end{cases}$$

c) $dist\, \mathcal{D}_1, \mathcal{D}_2 = dist\, P_1, P_2 = \|\overrightarrow{P_1P_2}\| = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$