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Grupa 143

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EXAMEN - GAL.

1. $(\mathbb{R}_2[X], +, \cdot)$

$S = \{1-x, x+x^2, -3+ax^2\}$

$a = ?$ a.i. S e SLD

$1 - x = 1\cdot 1 + (-1)\cdot x + 0\cdot x^2$

$x+x^2 = 0\cdot 1 + 1\cdot x + 1\cdot x^2$

$-3+ax^2 = 1\cdot(-3) + 0\cdot x + a\cdot x^2$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -3 & 0 & a \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -3 & 0 & a \end{vmatrix}$$

$$= 1\cdot\begin{vmatrix} 1 & 1 \\ 0 & a \end{vmatrix} + (-3)(-1)^4 \cdot \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= (a - 0) + (-3)(-1)$$

$$= a + 3$$

$S(A)$ e SLD $\Leftrightarrow \det A = 0 \Rightarrow a+3=0 \Rightarrow a=-3$

2. $(\mathbb{R}^3, +, \cdot)$ sp. vect.

$V = \langle \{(1,2,1), (1,1,-1), (-1,1,5)\} \rangle$

$\dim V = ?$

Determinăm rangul lui A, unde $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 5 \end{pmatrix}$

$$\det A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -1 \\ 3 & 2 & 1 \\ 6 & 4 & 5 \end{vmatrix} = (-1) \cdot \begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix}$$

$= (-1)(12-12)$

$= (-1) \cdot 0$

$= 0 \Rightarrow \operatorname{rang} A \lneq 2$

$\Delta_1 = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1 \neq 0 \Rightarrow \operatorname{rg} A \geq 2$

$\Big\} \Rightarrow \operatorname{rg} A = 2$

$\Rightarrow \dim V = \operatorname{rg} A = 2.$

3. $f: \mathbb{R}^2 \to \mathbb{R}^4$

$f(x_1, x_2) = (x_1 - x_2, 2x_1 - x_2, x_1, -x_2)$

$\ker f$, $\operatorname{Im} f = ?$

$\ker f = \{ x \in \mathbb{R}^3 \mid f(x) = (0,0,0,0) \}$

$$\Rightarrow \begin{cases} x_1 - x_2 = 0 \\ 2x_1 - x_2 = 0 \\ x_1 = 0 \\ -x_2 = 0 \end{cases} \Rightarrow x_1 = x_2 = 0$$

$\Rightarrow \ker f = \{ (0,0) \}$.

$\operatorname{Im} f = \{ y \in \mathbb{R}^4 \mid f(x) = y \}$

$$\Rightarrow \begin{cases} x_1 - x_2 = y_1 & \Rightarrow \quad y_3 + y_4 = y_1 \\ 2x_1 - x_2 = y_2 & \Rightarrow \quad 2y_3 + y_4 = y_2 \\ x_1 = y_3 \\ -x_2 = y_4 \end{cases}$$

$\operatorname{Im} f = \{ (y_3 + y_4, 2y_3 + y_4, y_3, y_4) \mid y_3, y_4 \in \mathbb{R} \}$

$= \{ y_3(1,2,1,0) + y_4(1,1,0,1) \mid y_3, y_4 \in \mathbb{R} \}$

$= \langle \{ (1,2,1,0), (1,1,0,1) \} \rangle$

4. $f \in End(\mathbb{R}_2[x]) \Rightarrow f(x)+f(y) = f(x+y), \forall x, \forall y \in \mathbb{R}_2[x]$

$P_1 = 1+x$

$P_2 = 1-x^2$ vect proprii

$P_3 = x+2x^2$

$\lambda_1 = 1$

$\lambda_2 = 2$ val proprii

$\lambda_3 = -2$

$f(P_1) = \lambda_1 \cdot P_1 = 1+x$

$f(P_2) = \lambda_2 \cdot P_2 = 2(1-x^2) = 2-2x^2$

$f(P_3) = \lambda_3 P_3 = (-2)(x+2x^2) = -2x-4x^2$

$\Rightarrow f(1,1,0) = (1,1,0)$

$f(1,0,-1) = (2,0,-2)$

$f(0,1,2) = (0,-2,-4)$

$1+x+x^2 = 1-x^2+x+2x^2 = P_2+P_3$

$f(1+x+x^2) = f(1-x^2+x+2x^2) = f(P_2+P_3) = f(P_2)+f(P_3)$

$= 2-2x^2-2x-4x^2$

$= 2-2x-6x^2$

$= (2,-2,-6)$

$= 2(1,-1,-3)$

5. $Q:\mathbb{R}^3 \to \mathbb{R}$ formă pătratică

$$G = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$$

a) Q la o formă canonică

$\Delta_1 = 3 > 0$

$\Delta_2 = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6-1 = 5$

$$\Delta_3 = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = 12-1-1-2-3-2$$
$$= 12 - 4 - 5$$
$$= 12 - 9$$
$$= 3$$

$$Q(x) = \frac{1}{\Delta_1} x_1'^2 + \frac{\Delta_1}{\Delta_2} x_2'^2 + \frac{\Delta_2}{\Delta_3} x_3'^2$$

$$Q(x) = \frac{1}{3} x_1'^2 + \frac{3}{5} x_2'^2 + \frac{5}{3} x_3'^2 \Rightarrow (3,0) \text{ signatura}$$

$Q(x) = x_1''^2 + x_2''^2 + x_3''^2$, $x_1'' = \frac{1}{\sqrt{3}} x_1'$, $x_2'' = \frac{\sqrt{3}}{\sqrt{5}} x_2'$, $x_3'' = \frac{\sqrt{5}}{\sqrt{3}} x_3'$

b) $g(x,y) = 3x_1y_1 + 2x_2y_2 + 2x_3y_3 + x_1y_2 - x_1y_3 + x_2y_1 + x_2y_3 - x_3y_1 + x_3y_2$

$$X^T \cdot G \cdot Y = (x_1, x_2, x_3) \cdot \begin{pmatrix} 3 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \begin{pmatrix} 3x_1 + x_2 - x_3 \\ x_1 + 2x_2 + x_3 \\ -x_1 + x_2 + 2x_3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} =$$

$$= 3x_1y_1 + 2x_2y_2 + 2x_3y_3 + x_1y_2 - x_1y_3 + x_2y_1 + x_2y_3 - x_3y_1 + x_3y_2$$

$\Rightarrow$ g e formă biliniară simetrică
Q e poz. def., $G = G^T$ $\Big\} \Rightarrow (\mathbb{R}^3, g)$ e sp. euclidian canonic.

6. $(\mathbb{R}^3, g_0)$

$U = \{x \in \mathbb{R}^3 \mid x_1 + 2x_2 - x_3 = 0\}$

a) $U^\perp = \langle \{(1,2,-1)\} \rangle$

$x_1 + 2x_2 - x_3 = 0 \Rightarrow x_1 + 2x_2 = x_3$

$U = \{(x_1, x_2, x_1 + 2x_2) \mid x_1, x_2 \in \mathbb{R}\}$

$= \{x_1(1,0,1) + x_2(0,1,2) \mid x_1, x_2 \in \mathbb{R}\}$

$= \langle \{(1,0,1), (0,1,2)\} \rangle$

$R_1' = \{(1,0,1), (0,1,2)\}$

$R_2' = \{(1,2,-1)\}$

$e_1 = f_1 = (1,0,1)$

$e_2 = f_2 - \frac{\langle f_2, e_1 \rangle}{\langle e_1, e_1 \rangle} e_1 = (0,1,2) - \frac{2}{2}(1,0,1)$

$= (0,1,2) - (1,0,1)$

$= (-1,1,1)$

$R_1 = \{\frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{3}}(-1,1,1)\}$

$e_3 = \frac{1}{\sqrt{6}}(1,2,-1)$

$R_2 = \{\frac{1}{\sqrt{6}}(1,2,-1)\}$

$R = R_1 \cup R_2 = \{\frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{3}}(-1,1,1), \frac{1}{\sqrt{6}}(1,2,-1)\}$

b)

$$p_1(x'+x'') = x''$$

$$s_1(x'+x'') = 2p_1 - id = 2x'' - x' - x'' = x'' - x'$$

$$p_1(1,2,3) = ?$$

$$(1,2,3) = a(1,0,1) + b(0,1,2) + c(1,2,-1)$$

$$(1,2,3) = (a+c,\ b+2c,\ a+2b-c)$$

$$\begin{cases} a+c = 1 & \Rightarrow a = 1-c \\ b+2c = 2 & \Rightarrow b = 2-2c \\ a+2b-c = 3 & \Rightarrow 1-c+4-4c-c = 3 \Rightarrow 5-6c = 3 \end{cases}$$

$$-6c = -2$$

$$6c = 2$$

$$c = \frac{1}{3} \Rightarrow a = 1 - \frac{1}{3} = \frac{2}{3}$$

$$b = 2 - \frac{2}{3} = \frac{4}{3}$$

$$(a,b,c) = \left(\frac{2}{3}, \frac{4}{3}, \frac{1}{3}\right)$$

$$p_1(1,2,3) = \frac{1}{3}(1,2,-1) = \left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}\right)$$

$$s_1(1,2,3) = \left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}\right) - \frac{2}{3}(1,0,1) - \frac{4}{3}(0,1,2)$$

$$= \left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}\right) - \left(\frac{2}{3}, 0, \frac{2}{3}\right) - \left(0, \frac{4}{3}, \frac{8}{3}\right)$$

$$= \left(-\frac{1}{3}, \frac{2}{3}, -1\right) - \left(0, \frac{4}{3}, \frac{8}{3}\right)$$

$$= \left(-\frac{1}{3}, -\frac{2}{3}, -\frac{11}{3}\right)$$

6. b) M2 $\quad u = (1,2,-1)$

$$s(x) = x - 2 \cdot \frac{\langle x,u\rangle}{\langle u,u\rangle} \cdot u$$

$$= (x_1,x_2,x_3) - \cancel{2}^{1} \cdot \frac{x_1+2x_2-x_3}{\cancel{6}\,3} (1,2,-1)$$

$$= \frac{1}{3}(3x_1,3x_2,3x_3) - \frac{1}{3}(x_1+2x_2-x_3,\ 2x_1+4x_2-2x_3,\ -x_1-2x_2+x_3)$$

$$= \frac{1}{3}\left(2x_1-2x_2+x_3,\ -2x_1-x_2+2x_3,\ x_1+2x_2+2x_3\right)$$

$$s(1,2,3) = \frac{1}{3}(2-4+3, -2-2+6, 1+4+6)$$

$$= \frac{1}{3}(1,2,11)$$

$$p(x) = x - \frac{\langle x,u\rangle}{\langle u,u\rangle} u = (x_1,x_2,x_3) - \frac{x_1+2x_2-x_3}{6}(1,2,-1)$$

$$= \frac{1}{6}(6x_1,6x_2,6x_3) - \frac{1}{6}(x_1+2x_2-x_3,\ 2x_1+4x_2-2x_3,\ -x_1-2x_2+x_3)$$

$$= \frac{1}{6}(5x_1-2x_2+x_3,\ -2x_1+2x_2+2x_3,\ +x_1+2x_2+5x_3)$$

$$p(1,2,3) = \frac{1}{6}(5-4+3, -2+4+6, 1+4+15)$$

$$= \frac{1}{6}(4,8,20)$$

$$= \frac{1}{3}(2,4,10)$$ – am greșit la calcule cred

7.

$(\mathbb{R}^3, g_0)$ sp. vect. euclidian canonic

$f : \mathbb{R}^3 \to \mathbb{R}^3$

$f(x) = u \cdot g_0(x, u)$, $u = (1, 2, 2)$

a) f endomorfism simetric

$$f(x) = (1,2,2) \cdot g_0((x_1, x_2, x_3), (1,2,2))$$

$$f(x) = (1,2,2) \cdot (x_1 + 2x_2 + 2x_3)$$

$$f(x) = (x_1 + 2x_2 + 2x_3,\ 2x_1 + 4x_2 + 4x_3,\ 2x_1 + 4x_2 + 4x_3)$$

$$Q(x) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 8x_2x_3$$

$$\left.\begin{array}{l} A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix} \\ \\ A = A^T \\ f(x) = y \Leftrightarrow Y = A \cdot X \Leftrightarrow f \in End(\mathbb{R}^3) \end{array}\right\} \Rightarrow$$

$\Rightarrow f \in Sim(\mathbb{R}^3)$

b)

$$Q(x) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 8x_2x_3$$

$$P(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 4-\lambda & 4 \\ 2 & 4 & 4-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \sigma_1\lambda^2 + \sigma_2\lambda - \sigma_3 = 0$$

$$\sigma_1 = \operatorname{Tr} A = 9$$

$$\sigma_2 = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 4 & 4 \\ 4 & 4 \end{vmatrix} = 4-4+4-4+16-16 = 0$$

$$\sigma_3 = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 4 \end{vmatrix} = 0$$

$$\lambda^3 - 9\lambda^2 = 0$$

$$\lambda^2(\lambda - 9) = 0$$

$$\lambda_1 = 0, \quad m_1 = 2$$

$$\lambda_2 = 9, \quad m_2 = 1$$

$$V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid f(x) = 0\} =$$

$$\begin{cases} x_1 + 2x_2 + 2x_3 = 0 \\ 2x_1 + 4x_2 + 4x_3 = 0 \end{cases} \Rightarrow \begin{array}{l} x_1 + 2x_2 + 2x_3 = 0 \\ x_1 = -2x_2 - 2x_3 \end{array}$$

$$V_{\lambda_1} = \{(-2x_2 - 2x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R}\}$$

$$= \{x_2(-2,1,0) + x_3(-2,0,1) \mid x_2, x_3 \in \mathbb{R}\}$$

$$= \langle \{(-2,1,0), (-2,0,1)\} \rangle \qquad \dim V_{\lambda_1} = 2 = m_1$$

$\Rightarrow \{f_1, f_2\}$ repert in V_λ

$e_1 = f_1 = (-2, 1, 0)$

$e_2 = f_2 - \dfrac{\langle f_2, e_1\rangle}{\langle e_1, e_1\rangle} e_1$

$e_2 = (-2, 0, 1) - \dfrac{4}{5}(-2, 1, 0)$

$e_2 = (-2, 0, 1) - \left(\dfrac{-8}{5}, \dfrac{4}{5}, 0\right)$

$e_2 = (-2, 0, 1) + \left(\dfrac{8}{5}, -\dfrac{4}{5}, 0\right)$

$e_2 = \left(\dfrac{8-10}{5}, -\dfrac{4}{5}, 1\right)$

$e_2 = \left(-\dfrac{2}{5}, -\dfrac{4}{5}, 1\right) = \dfrac{1}{5}(-2, -4, 5)$

$\{e_1, e_2\}$ reper ortogonal in V_{λ_1}

$R_1 = \left\{ e_1' = \dfrac{1}{\sqrt{5}}(-2, 1, 0),\ e_2' = \dfrac{1}{\sqrt{45}}(-2, -4, 5) \right\}$

$= \left\{ e_1' = \dfrac{1}{\sqrt{5}}(-2, 1, 0),\ e_2' = \dfrac{1}{3\sqrt{5}}(-2, -4, 5) \right\}$

$V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid f(x) = 9x\} = \left\{ x \in \mathbb{R}^3 \,\middle|\, \begin{cases} x_1 + 2x_2 + 2x_3 = 9x_1 \\ 2x_1 + 4x_2 + 4x_3 = 9x_2 \\ 2x_1 + 4x_2 + 4x_3 = 9x_3 \end{cases} \right\}$

$$\begin{cases} -8x_1 + 2x_2 + 2x_3 = 0 \\ 2x_1 - 5x_2 + 4x_3 = 0 \\ 2x_1 + 4x_2 - 5x_3 = 0 \end{cases}$$

$$\det\begin{pmatrix} -8 & 2 & 2 \\ 2 & -5 & 4 \\ 2 & 4 & -5 \end{pmatrix} = \begin{vmatrix} -8 & 2 & 2 \\ 2 & -5 & 4 \\ 2 & 4 & -5 \end{vmatrix} =$$

$$= -200 + 16 + 16 + 20 + 128 + 20$$

$$= 0$$

$$\begin{cases} -8x_1 + 2x_2 + 2x_3 = 0 \\ 2x_1 - 5x_2 + 4x_3 = 0 \quad | \cdot 4 \end{cases} \Rightarrow \begin{array}{r} -8x_1 + 2x_2 + 2x_3 = 0 \\ 8x_1 - 20x_2 + 16x_3 = 0 \\ \hline -18x_2 + 18x_3 = 0 \\ x_2 = x_3 \end{array}$$

$$-8x_1 + 4x_2 = 0$$

$$4x_2 = 8x_1$$

$$\frac{1}{2}x_2 = x_1$$

$$V_{\lambda_2} = \left\{\left(\frac{1}{2}x_2, x_2, x_2\right) \mid x_2 \in \mathbb{R}\right\} = \left\langle\left\{\left(\frac{1}{2}, 1, 1\right)\right\}\right\rangle = \left\langle\left\{(1,2,2)\right\}\right\rangle$$

$$R_2 = \left\{\frac{1}{\sqrt{9}}(1,2,2)\right\} = \left\{\frac{1}{3}(1,2,2)\right\}$$

$$R_0 = \{e_1^0, e_2^0, e_3^0\} \xrightarrow{C} R = \{e_1', e_2', e_3'\}$$

$$C = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 & -\frac{2}{3} & \frac{1}{3} \\ 1 & -\frac{4}{3} & \frac{2}{3} \\ 0 & \frac{5}{3} & \frac{2}{3} \end{pmatrix} = \frac{1}{3\sqrt{5}}\begin{pmatrix} -6 & -2 & 1 \\ 3 & -4 & 2 \\ 0 & 5 & 2 \end{pmatrix}$$

$$h(x) \in O(\mathbb{R}^3)$$

$$h(e_i^0) = e_i', \quad \forall i = \overline{1,3}$$

$$h(x) = \frac{1}{3\sqrt{5}}(-6x_1 - 2x_2 + x_3,\ 3x_1 - 4x_2 + 2x_3,\ 5x_2 + 2x_3)$$

8.

$\Gamma: f(x_1, x_2) = 3x_1^2 - 4x_1x_2 - 4x_1 + 8x_2 - 3 = 0$

forma canonică, efectuând izometrii

$$A = \begin{pmatrix} 3 & -2 \\ -2 & 0 \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} 3 & -2 & -2 \\ -2 & 0 & 4 \\ -2 & 4 & -3 \end{pmatrix}$$

$$\delta = \begin{vmatrix} 3 & -2 \\ -2 & 0 \end{vmatrix} = 0 - 4 = -4 \neq 0$$ ($\exists !$ centrul conicei).

$$\Delta = \det \tilde{A} = \begin{vmatrix} 3 & -2 & -2 \\ -2 & 0 & 4 \\ -2 & 4 & 3 \end{vmatrix}$$

$$= 0 + 16 + 16 - 0 - 48 + 12$$

$$= 32 - 60$$

$$= -28$$ (conică nedegenerată)

$$\begin{cases} \frac{df}{dx_1} = 0 \\ \frac{df}{dx_2} = 0 \end{cases} \Rightarrow \begin{cases} 6x_1 - 4x_2 - 4 = 0 \\ -4x_1 + 8 = 0 \end{cases} \Rightarrow x_1 = 2$$

$$6 \cdot 2 - 4x_2 - 4 = 0$$

$$8 = 4x_2$$

$$x_2 = 2$$

$\Rightarrow P(2, 2)$ centru unic

$$R=\{O, e_1, e_2\} \xrightarrow{\theta} R'=\{P_0, e_1, e_2\} \xrightarrow{\tau} R'=\{P_0, e_1', e_2'\}$$

$$\theta: 3x_1'^2 - 4x_1'x_2' + \frac{\Delta}{\delta} = 0$$

$$\theta: 3x_1'^2 - 4x_1'x_2' + \frac{-28}{-4} = 0$$

$$\theta: 3x_1'^2 - 4x_1'x_2' + 7 = 0$$

$$Q: \mathbb{R}^2 \to \mathbb{R}$$

$$Q(x) = 3x_1^2 - 4x_1x_2 \qquad A = \begin{pmatrix} 3 & -2 \\ -2 & 0 \end{pmatrix}$$

$$\operatorname{Tr} A = 3$$

$$\det A = 0 - 4 = -4$$

$$P(\lambda) = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\Delta = 9 + 16 = 25$$

$$\lambda_{1,2} = \frac{3 \pm 5}{2} \quad \lambda_1 = \frac{8}{2} = 4, \; m_1 = 1 \qquad \lambda_2 = \frac{-2}{2} = -1, \; m_2 = 1$$

$$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid Ax = 4x\}$$

$$(A - 4I_3)x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -x_1 - 2x_2 = 0 \Rightarrow x_1 = -2x_2$$

$$V_{\lambda_1} = \{(-2x_2, x_2) \mid x_2 \in \mathbb{R}\} = \langle \{(-2,1)\} \rangle$$

$$e_1 = \frac{1}{\sqrt{5}}(-2, 1)$$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid Ax = -x\}$

$(A + I_3)x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -2x_1 + x_2 = 0 \Rightarrow x_2 = 2x_1$

$V_{\lambda_2} = \{ (x_1, 2x_1) \mid x_1 \in \mathbb{R} \} = \langle \{(1,2)\} \rangle$

$e_2' = \frac{1}{\sqrt{5}}(1,2)$

$\tau : x' = R \cdot x''$

$R = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

$\tau \circ \theta(\Gamma)$

$4x_1''^2 - x_2''^2 + 7 = 0$

$x = x' + x_0 = Rx'' + x_0$

$$\begin{pmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \cdot \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}}(-2x_1'' + x_2'') + 2 \\ \frac{1}{\sqrt{5}}(x_1'' + 2x_2'') + 2 \end{pmatrix}$$

9.

$$D_1: \frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{2}$$

$$D_2: \frac{x_1-1}{1} = \frac{x_2+1}{2} = \frac{x_3-2}{3}$$

a) D_1, D_2 coplanare.

$u_1 = (1,-1,2)$

$A_1 = (0,0,0) \in D_1$

$u_2 = (1,2,3)$

$A_2 = (1,-1,2) \in D_2$

$\overrightarrow{A_1A_2} = (1,-1,2)$

$$C = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & 2 & -1 \\ 2 & 3 & 2 \end{array}\right)$$

$$\det \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & -1 \\ 2 & 3 & 2 \end{vmatrix} = \cancel{4} - 3 - \cancel{2} - \cancel{4} + 3 + \cancel{2} = 0$$

$\Rightarrow rg\,\bar{C} \leq 2$

$\Delta_1 = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 2+1 = 3 > 0 \Rightarrow rg\,C = 2$

$\Big\} \Rightarrow rg\,\bar{C} = 2$

$rg\,\bar{C} = 2 = rg\,C \Rightarrow$

$\Rightarrow$ concurente, deci coplanare

b) π ?

π - planul determinat de cele doua drepte

$A(0,0,0)$

$u_1 = (1,-1,2)$

$u_2 = (1,2,3)$

$$\pi : \begin{vmatrix} x_1 & 1 & 1 \\ x_2 & -1 & 2 \\ x_3 & 2 & 3 \end{vmatrix} = 0$$

$\pi : \ x_1(-3-4) + x_2(-1)(3-2) + x_3(2+1) = 0$

$\pi : \ x_1(-7) + x_2(-1) + x_3 \cdot 3 = 0$

$\pi : \ -7x_1 - x_2 + 3x_3 = 0$