Ecuația unui (hiper)plan

① 1 Punct, 1 Dreaptă $\perp$ $A(a_1, a_2, \dots, a_n) \in \Pi$

$$d: \frac{x_1 - x_1^0}{\mu_1} = \frac{x_2 - x_2^0}{\mu_2} = \dots = \frac{x_n - x_n^0}{\mu_n} = t$$

$$\Pi: \mu_1(x_1 - a_1) + \mu_2(x_2 - a_2) + \dots + \mu_n(x_n - a_n) = 0$$

② 1 Punct, 2 Vectori: $A(a_1, \dots, a_n) \in \Pi$

$$V_\Pi = \langle \{u, v\} \rangle, \ \{u, v\} \text{ SLI}, \ \mu, v \in \Pi$$

$$N = u \times v \Rightarrow \text{Cazul ① } (N \perp \Pi, A \in \Pi)$$

$\circledast \mathbb{R}^3$

$$\Pi: \begin{vmatrix} x_1 - a_1 & u_1 & v_1 \\ x_2 - a_2 & u_2 & v_2 \\ x_3 - a_3 & u_3 & v_3 \end{vmatrix} = 0$$

③ 3 Puncte $A(a_1, \dots, a_n) \in \Pi$

$B(b_1, \dots, b_n) \in \Pi$

$C(c_1, \dots, c_n) \in \Pi$

$$V_\Pi = \langle \{\overrightarrow{AB}, \overrightarrow{AC}\} \rangle \Rightarrow \text{Cazul ② } (V_\Pi, A \in \Pi)$$

$\circledast \mathbb{R}^3$

$$\Pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0$$

Poziția relativă a 2 (hiper)plane

Fie $\mathcal{H}_1 : A_1x_1 + A_2x_2 + \dots + A_nx_n + A_0 = 0$

$\mathcal{H}_2 : A'_1x_1 + A'_2x_2 + \dots + A'_nx_n + A'_0 = 0$

① $\mathcal{H}_1 \parallel \mathcal{H}_2 \iff \frac{A_1}{A'_1} = \frac{A_2}{A'_2} = \dots = \frac{A_n}{A'_n} \neq \frac{A_0}{A'_0}$

② $\mathcal{H}_1 = \mathcal{H}_2 \iff \frac{A_1}{A'_1} = \frac{A_2}{A'_2} = \dots = \frac{A_n}{A'_n} = \frac{A_0}{A'_0}$

③ $\mathcal{H}_1 \cap \mathcal{H}_2 \neq \emptyset \Rightarrow$ Subspațiu $(n-2)$ af in

ex $\pi_1 : x_1 + x_2 + x_3 - 1 = 0$

$\pi_2 : 2x_1 - x_3 = 0$

$\pi_1 \cap \pi_2 = ?$

$\frac{1}{2} \neq \frac{1}{0} \neq \frac{1}{-1} \neq \frac{-1}{0} \Rightarrow \pi_1 \cap \pi_2 = D$

$N_1 = (1, 1, 1)$

$N_2 = (2, 0, -1)$

$\boxed{\text{Coef. lui } x_1, x_2, x_3}$

$N_D = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{vmatrix} = -e_1 + 2e_2 - 2e_3 + e_3 = -e_1 + 3e_2 - e_3 = (-1, 3, -1)$

Alte formule

① $A_{\text{triunghi}} = \frac{1}{2} \cdot \|\vec{AB} \times \vec{AC}\|$

② $V = \frac{1}{6}|\Delta|$, $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix}$

, $A = 0 \Rightarrow ABCD$ coplanare

③ $\text{dist}(A, D) = \frac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{BC}\|}$, $B, C \in D$

④ $\text{dist}(A, \pi) = \frac{|a a_1 + b a_2 + c a_3 + d|}{\sqrt{a^2 + b^2 + c^2}}$

$A(a_1, a_2, a_3)$ | $\pi : ax + by + cz + d = 0$

⑤ $dist(\mathcal{D}_1, \mathcal{D}_2) = \dfrac{|\langle \vec{AB}, N \rangle|}{\|N\|}$

$A \in \mathcal{D}_1, B \in \mathcal{D}_2$

$N = u_{\mathcal{D}_1} \times u_{\mathcal{D}_2}$

⑥ $\sphericalangle(\mathcal{D}_1, \mathcal{D}_2) = \sphericalangle(u_{\mathcal{D}_1}, u_{\mathcal{D}_2}) = \varphi$, $\cos\varphi = \dfrac{\langle u_1, u_2 \rangle}{\|u_1\| \cdot \|u_2\|}$ } PRODUS SCALAR } NORME

⑦ $\sphericalangle(\pi_1, \pi_2) = \sphericalangle(N_1, N_2) = \varphi$, $\cos\varphi = \dfrac{\langle N_1, N_2 \rangle}{\|N_1\| \cdot \|N_2\|}$

⑧ $\sphericalangle(\mathcal{D}, \pi) = \sphericalangle(\mathcal{D}, \mathcal{D}') = \varphi = \dfrac{\pi}{2} - \sphericalangle(\mathcal{D}, N_\pi)$

PROIECȚIA

CLASA A 8-A

$N_1 = (1, 1, 1)$

$N_2 = (2, 0, -1)$

④ $\boxed{\text{coef. lui } x_1, x_2, x_3}$

$u_{\mathcal{D}} = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{vmatrix} = -e_1 + 2e_2 - 2e_3 + e_2 = \ldots = (-1, 3, -2)$

Alte formule

① $A_{\triangle ABC} = \frac{1}{2} \cdot \|\vec{AB} \times \vec{AC}\|$

② $V = \frac{1}{6}|\Delta|$, $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix}$

$, \Delta = 0 \Rightarrow A, B, C, D$ coplanare

③ $dist(A, \mathcal{D}) = \dfrac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{BC}\|}$, $B, C \in \mathcal{D}$

④ $dist(A, \pi) = \dfrac{|a a_1 + b a_2 + c a_3 + d|}{\sqrt{a^2 + b^2 + c^2}}$

$A(a_1, a_2, a_3)$ | $\pi: ax + by + cz + d = 0$

c) $\Pi_{BCD}: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 2 & 1 & 3 & 1 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 1 \end{vmatrix} \xlongequal{C_2 - 2C_4} \begin{vmatrix} x_1 & x_2-2 & x_3 & 1 \\ 2 & -1 & 3 & 1 \\ -2 & -1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \cdot (-1)^{8} \cdot \begin{vmatrix} x_1 & x_2-2 & x_3 \\ 2 & -1 & 3 \\ -2 & -1 & 3 \end{vmatrix}$

$= -3x_1 - 2x_3 - 6x_2 + 12 - 2x_3 + \cancel{3x_1} - 6x_2 + 12 = 0$

$\Rightarrow \Pi_{BCD}: -12x_2 - 4x_3 + 24 = 0 \quad | : (-4)$

$\Pi_{BCD}: 3x_2 + x_3 - 6 = 0$

$dist(A, \Pi_{BCD}) = \dfrac{|0 \cdot 1 + 3 \cdot 2 + 1 \cdot 1 - 6|}{\sqrt{0^2 + 3^2 + 1^2}} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

① $A(1,2,1), B(2,1,3), C(-2,1,3), D(0,2,0)$

a) V_{ABCD}, b) $S_{\Delta BCD}$, c) $dist(A, (BCD))$

a) $V_{ABCD} = \frac{1}{6} \cdot |\Delta|$

$\Delta = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 1 \end{vmatrix} \xlongequal{C_2 - 2C_4} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 2 & -1 & 3 & 1 \\ -2 & -1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \cdot (-1)^{8} \cdot \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ -2 & -1 & 3 \end{vmatrix} = -3 - 2 - 2 + 3 = -4$

$V_{ABCD} = \frac{4}{6} = \frac{2}{3}$

b) $S_{\Delta BCD} = \frac{1}{2} \cdot \| \vec{BC} \times \vec{BD} \|$

$\vec{BC} = (-4, 0, 0), \quad \vec{BD} = (-2, 1, -3), \quad \vec{BC} \times \vec{BD} = (0, -12, -4)$

$S_{\Delta BCD} = \frac{1}{2} \cdot \sqrt{160} = \sqrt{40} = 2\sqrt{10}$

② $\Pi_1 : x_1 - 2x_2 - 1 = 0$

$\Pi_2 : 2x_2 + x_3 - 2 = 0$

$\Pi : x_2 - x_3 - 1 = 0$

$D_1 : \begin{cases} x_1 - x_2 - 2 = 0 \\ x_1 + x_3 - 3 = 0 \end{cases}$

$D_2 : \frac{x_1 - 1}{3} = \frac{x_2 + 1}{0} = \frac{x_3 - 1}{-1} = t$

$D : \frac{x_1 - 1}{-1} = \frac{x_2}{2} = \frac{x_3 + 1}{-5} = \lambda, \quad t, \lambda \in \mathbb{R}$

a) $\sphericalangle(D_1, D_2)$, b) $\sphericalangle(D, \Pi)$, c) $\sphericalangle(\Pi_1, \Pi_2)$

$N_1 = (1, -1, 0), \; N_2 = (1, 0, 1)$

$$u_{D_1} = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = (-1, -1, 1)$$

$u_{D_2} = (3, 0, -1)$

$\sphericalangle(D_1, D_2) = \sphericalangle(u_{D_1}, u_{D_2}) = \varphi$

$$\cos\varphi = \frac{\langle u_{D_1}, u_{D_2} \rangle}{\|u_{D_1}\| \cdot \|u_{D_2}\|} = \frac{-4}{\sqrt{3} \cdot \sqrt{10}} = \frac{-4}{\sqrt{30}} = \frac{-2\sqrt{30}}{15}$$

$$\Rightarrow \varphi = \arccos \frac{-2\sqrt{30}}{15} = \pi - \arccos \frac{2\sqrt{30}}{15}$$

b) $\sphericalangle(D, \Pi) = \frac{\pi}{2} - \sphericalangle(D, N_\Pi)$

$u_D = (-1, 2, -5)$

$N_\Pi = (0, 1, -1)$

$$\Rightarrow \cos\sphericalangle(D, N) = \frac{\langle u_D, N \rangle}{\|u_D\| \|N\|} = \frac{7}{\sqrt{30} \cdot \sqrt{2}} = \frac{7\sqrt{60}}{60}$$

$\Rightarrow \sphericalangle(D, N) = \arccos \frac{7\sqrt{60}}{60}$

$\Rightarrow \sphericalangle(D, \Pi) = \frac{\pi}{2} - \arccos \frac{7\sqrt{60}}{60}$

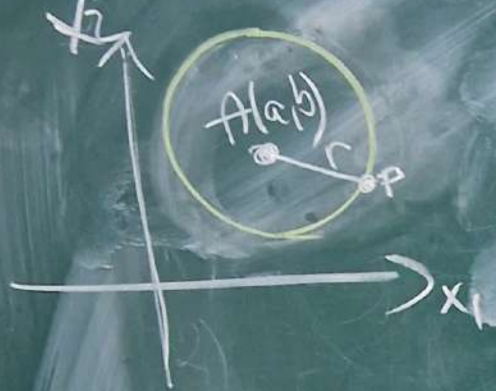
c) $N_1 = (1, -2, 0)$

$N_2 = (0, 2, 1)$

$$\Rightarrow \cos \sphericalangle(N_1, N_2) = \frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|} = \frac{-4}{\sqrt{5} \cdot \sqrt{5}} = -\frac{4}{5}$$

$\sphericalangle(\Pi_1, \Pi_2) = \arccos\left(-\frac{4}{5}\right) = \pi - \arccos \frac{4}{5}$

Cercul



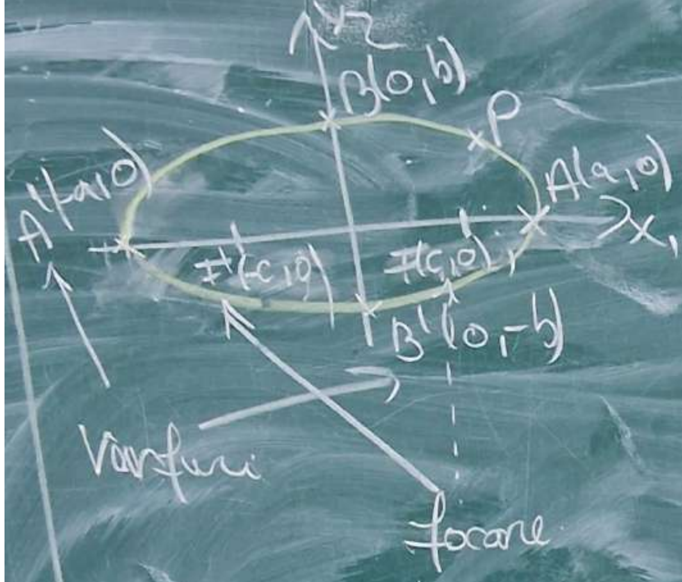
$$\mathcal{C}: (x_1-a)^2+(x_2-b)^2=R^2$$

$$PA=R$$

$$e=0$$

excentricitatea

Elipsa



$$\mathcal{E}: \frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}=1,$$

$$a, b>0$$

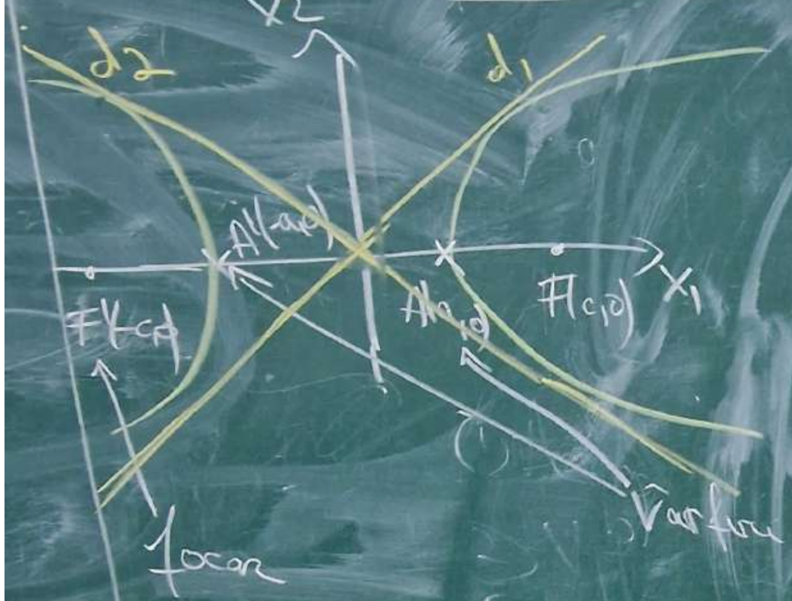
$$a^2=b^2+c^2$$

$$PF+PF'=2a$$

$$e=\frac{c}{a}<1$$

ICE

Hiperbola



$$\mathcal{H}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1$$

$$a, b > 1$$

$$c^2 = a^2 + b^2$$

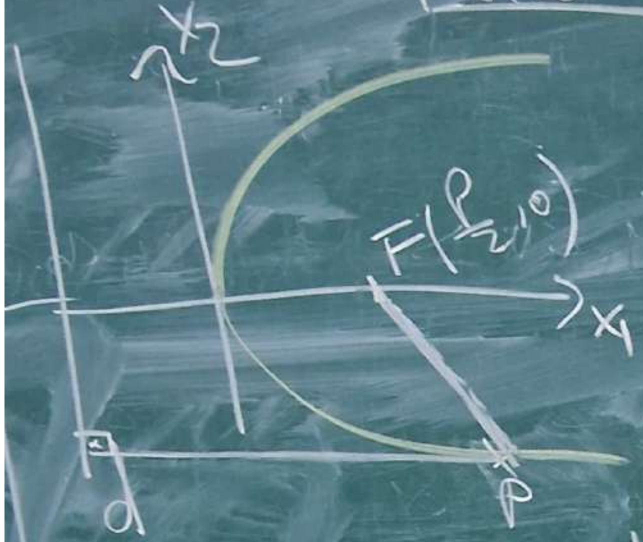
$$|PF - PF'| = 2a$$

$$\underbrace{d_1 \cup d_2}_{\text{asimptote}} : x_2 = \pm \frac{b}{a} x_1$$

$$e = \frac{c}{a} > 1$$

drepte directoare $d' \cup d'' : x_1 = \pm \frac{a}{c}$

Parabola



$\mathcal{P}: x_2^2 = 2 \cdot p x_1$

$d: x_1 = -\frac{p}{2}$

dreapta directoare

$PF = \text{dist}(P, d)$

$e = 1$