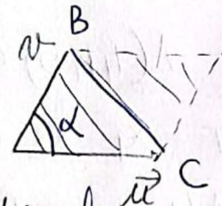
Arii. Volume. Distanțe $\boxed{\text{CURS 12}}$ [redacted]GA



- $\{u, v\}$ SLI, $w = u \times v$

$\|w\| = \|u\| \cdot \|v\| \cdot \sin\alpha$ = Aria paralelogramului

1) Aria unui triunghi

$$A_{\Delta ABC} = \frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}}{2}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ b_1 - a_1 & b_2 - a_2 & b_3 - a_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{vmatrix} = (\Delta_1, \Delta_2, \Delta_3)$$

Ex $A(1,0,1)$, $B(0,-1,0)$, $C(0,1,1)$

$A_{\Delta ABC} = ?$

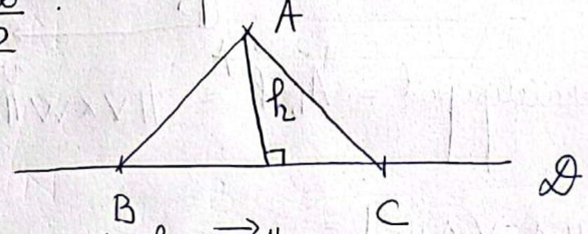
$\vec{AB} = (-1,-1,-1)$

$\vec{AC} = (-1,1,0)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix} = (1, +1, -2)$$

$$A_\Delta = \frac{1}{2}\sqrt{1+1+4} = \frac{\sqrt{6}}{2}$$

2) dist $(A, \mathcal{D})$ $\boxed{\text{CURS 12}}$

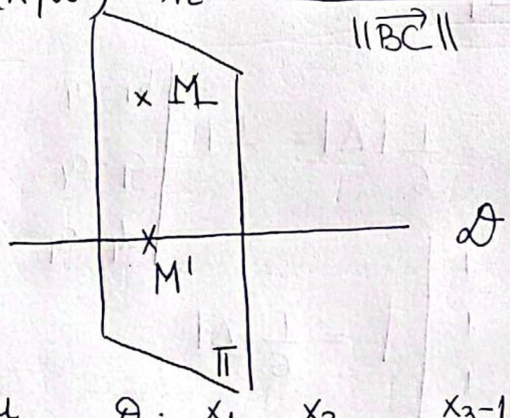


M1

$$A_{\Delta ABC} = \frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{1}{2} h \|\vec{BC}\|$$

$$\text{dist}(A, \mathcal{D}) = h = \frac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{BC}\|} = \frac{\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}}{\sqrt{(c_1 - b_1)^2 + (c_2 - b_2)^2 + (c_3 - b_3)^2}}$$

M2



$\pi \ni M$

$\mathcal{D} \perp \pi$

$\mathcal{D} \cap \pi = \{M'\}$

$\text{dist}(M, \mathcal{D}) = \text{dist}(M, M')$

Exemplu $\mathcal{D}: \frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3 - 1}{2} = t$, $A(1,1,0)$

dist $(A, \mathcal{D})$

SOL M1 $\mathcal{D}: \begin{cases} x_1 = t \\ x_2 = -t \\ x_3 = 1 + 2t \end{cases}$

$t = 0 \Rightarrow B(0,0,1)$ $\vec{AB} = (-1,-1,1)$

$t = 1 \Rightarrow C(1,-1,3)$ $\vec{AC} = (0,-2,3)$

$\vec{BC} = (1,-1,2)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & 1 \\ 0 & -2 & 3 \end{vmatrix} = (-1, 3, 2), \quad \|\vec{AB} \times \vec{AC}\| = \sqrt{1+9+4} = \sqrt{14}$$

$$\|\vec{BC}\| = \sqrt{6}, \quad \text{dist} = \sqrt{\frac{14}{6}} = \sqrt{\frac{7}{3}} = \frac{\sqrt{21}}{3}$$

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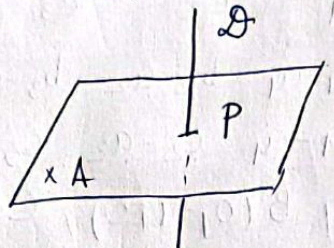
(M2) $A(1,1,0) \in \Pi$, $N_{\Pi} = u_{\mathcal{D}} = (1,-1,2)$

$\Pi : x_1 - x_2 + 2x_3 + d = 0$

$A(1,1,0) \in \Pi \Rightarrow 1 - 1 + 0 + d = 0 \Rightarrow d = 0. \Rightarrow \Pi : x_1 - x_2 + 2x_3 = 0.$

$\mathcal{D} \cap \Pi$

$\mathcal{D} \begin{cases} x_1 = t \\ x_2 = -t \\ x_3 = 1 + 2t \end{cases} \Rightarrow t + t + 2(1+2t) = 0 \Rightarrow 6t = -2 \Rightarrow t = -\frac{1}{3}.$

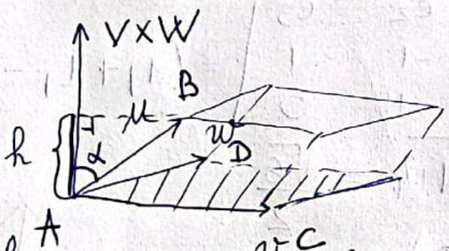


$P\left(-\frac{1}{3}, \frac{1}{3}, \overbrace{1 - \frac{2}{3}}^{\frac{1}{3}}\right)$

$dist(A, \mathcal{D}) = dist(A, P) = \sqrt{\left(-\frac{1}{3} - 1\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{1}{3} - 0\right)^2}$

Volume

$\{u, v, w\}$ SLI



$V_{paralelipiped} = A_b \cdot h = \|v \times w\| \cdot \underset{\|u\| \cdot |\cos\alpha|}{\underset{\|}{h}} = |\langle u, v \times w \rangle|$

$= |u \wedge v \wedge w| =$

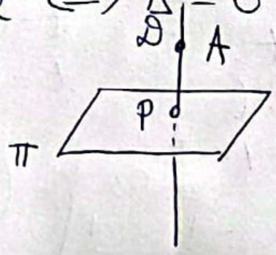
$= \left| \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \right| = |\Delta|$

$V_{ABCD} = \frac{1}{6} V_{paralelip} = \frac{1}{6}|\Delta| = \frac{1}{6} \left| \begin{vmatrix} b_1 - a_1 & b_2 - a_2 & b_3 - a_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \\ d_1 - a_1 & d_2 - a_2 & d_3 - a_3 \end{vmatrix} \right|$

$= \frac{1}{6} \left| \begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix} \right| = \frac{1}{6}|\Delta|$

$\boxed{\text{OBS}}$ A, B, C, D = coplanare $\Leftrightarrow \Delta = 0$.

- $Dist(A, \Pi)$
  $\|$
  $dist(A, P)$



$\mathcal{D} \perp \Pi$

$u_{\mathcal{D}} = N_{\Pi}$

$\mathcal{D} \cap \Pi = \{P\}$

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Ex $\pi : x_1 - 2x_2 + 3x_3 + 1 = 0, \quad A(1,2,3)$

$dist(A, \pi) = ?$

Sol $\mathcal{D} \perp \pi, \quad A \in \mathcal{D} \Rightarrow \mathcal{D}: \frac{x_1 - 1}{1} = \frac{x_2 - 2}{-2} = \frac{x_3 - 3}{3} = t$

$N_\pi = (1, -2, 3) = u_{\mathcal{D}}$

$\mathcal{D}: \begin{cases} x_1 = 1 + t \\ x_2 = 2 - 2t \\ x_3 = 3 + 3t \end{cases}$ $\quad \mathcal{D} \cap \pi : 1 + \underline{t} - 2(2 - \underline{2t}) + 3(3 + \underline{3t}) + 1 = 0$

$t + 4t + 9t + 1 - 4 + 9 + 1 = 0 \Rightarrow 14t = -7 \Rightarrow t = -\frac{1}{2}$

$P\left(\frac{1}{2}, 3, \overbrace{3 - \frac{3}{2}}^{3/2}\right)$

$dist(A, \pi) = dist(A, P) = \sqrt{\left(\frac{1}{2} - 1\right)^2 + (3-2)^2 + \left(\frac{3}{2} - 3\right)^2}$

$= \sqrt{\frac{1}{4} + 1 + \frac{9}{4}} = \frac{\sqrt{14}}{2}$.

Hipercuadrice în spațiul afin $\mathbb{R}^n$

Def $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ (sau $(\mathbb{R}^n, (\mathbb{R}^n/_{\mathbb{R}}, g_0), \varphi)$)

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian

S.n. hipercuadrică în $\mathbb{R}^n$ Δ LG al punctelor $P(x_1, .., x_n)$ a.î.

$\Gamma : f(x_1, .., x_n) = a_{11} x_1^2 + ... + a_{nn} x_n^2 + 2a_{12} x_1 x_2 + ... + 2a_{n-1\,n} x_{n-1} x_n$.

$+ 2b_1 x_1 + ... + 2b_n x_n + c = 0$.

$\Gamma : X^T A X + 2BX + c = 0$

$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \in \mathcal{M}_n(\mathbb{R})$ $\quad B = (b_1 \dots b_n)$ $\quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$A = A^T$

$\tilde{A} = \left(\begin{array}{c|c} A & B^T \\ \hline B & c \end{array}\right)$ $\quad r = rg A, \quad r' = rg \tilde{A}$

$r \le r' \le r + 2$.

$\delta = det(A)$

$\Delta = det(\tilde{A})$

1) $\Delta \ne 0 \Rightarrow \Gamma$ hipercuadrică nedegenerată

2) $\Delta = 0 \Rightarrow \Gamma$ —"— degenerată

OBS a) $(\mathbb{R}^n, \mathbb{R}^n/\mathbb{R}, \varphi)$

$\Gamma_1 \sim \Gamma_2$ afin echivalente $\Leftrightarrow$ $\tau: \mathbb{R}^n \to \mathbb{R}^n$ transf. afină

$\tau: X' = CX + D, \quad C \in GL(n, \mathbb{R})$

a.î. $\Gamma_2 = \tau(\Gamma_1)$; invarianți afini: $\frac{\Delta}{\delta}, r, r'$

b) $(\mathbb{R}^n, (\mathbb{R}^n/\mathbb{R}, g_0), \varphi)$

$\Gamma_1 \equiv \Gamma_2$ congruente metric $\Leftrightarrow$ $\tau: \mathbb{R}^n \to \mathbb{R}^n$ izometrie

$\tau: X' = CX + D, \ C \in O(n)$

a.î. $\Gamma_2 = \tau(\Gamma_1)$; invarianți metrici

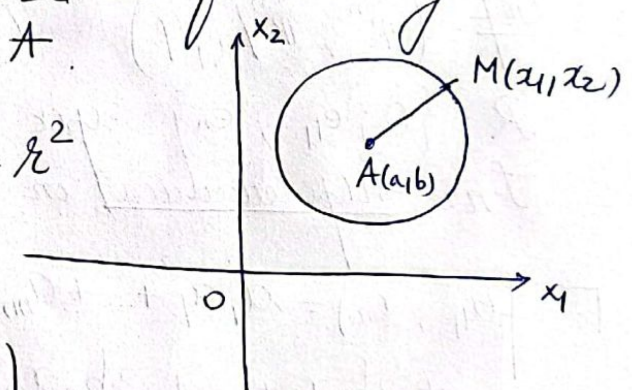
$\frac{\Delta}{\delta}, r, r', \Delta, \delta.$

OBS $n=2$ Γ conică

$n=3$ Γ cuadrică.

Conice ca locuri geometrice

1) Cercul $\mathcal{C}(A(a,b), r)$ = LG al punctelor egal depărtate de un pct fix A.



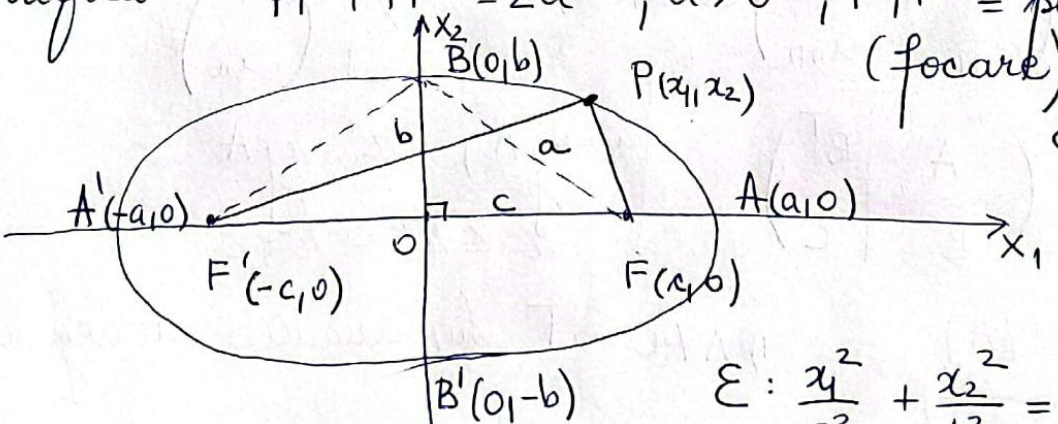
$AM = r \Leftrightarrow (x_1-a)^2 + (x_2-b)^2 = r^2$

Ec. parametrice:

$$\begin{cases} x_1 - a = r\cos t \\ x_2 - b = r\sin t \end{cases}, \quad t \in [0, 2\pi)$$

2) Elipsa este LG al punctelor P din plan care verifică $PF + PF' = 2a$, $a > 0$, F, F' = puncte fixe (focare)

$a^2 = b^2 + c^2$



$$\mathcal{E}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$$

Ec. parametrice:

$$\begin{cases} x_1 = a\cos t \\ x_2 = b\sin t \end{cases}, \quad t \in [0, 2\pi)$$

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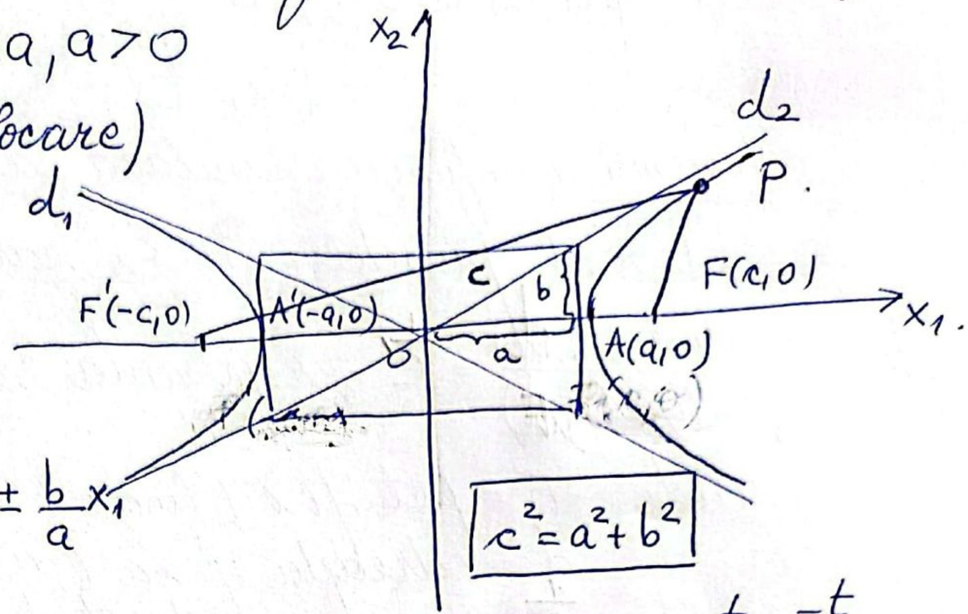
③ *Hiperbola* este LG al punctelor $P \in \mathcal{E}_2$ care verifică

$$|PF - PF'| = 2a,\ a > 0$$

F, F' pcte fixe (focare)



$$\mathcal{H}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1.$$

$$\boxed{c^2 = a^2 + b^2}$$

$d_1 \cup d_2$: $x_2 = \pm \frac{b}{a} x_1$

(asimptotele)

<u>OBS</u> Ec. parametrice: $\begin{cases} x_1 = a\,\mathrm{ch}\,t \\ x_2 = b\,\mathrm{sh}\,t \end{cases}$ $\quad \mathrm{ch}\,t = \frac{e^t + e^{-t}}{2}$

$\mathrm{sh}\,t = \frac{e^t - e^{-t}}{2}$

$$\mathrm{ch}^2 t - \mathrm{sh}^2 t = 1$$

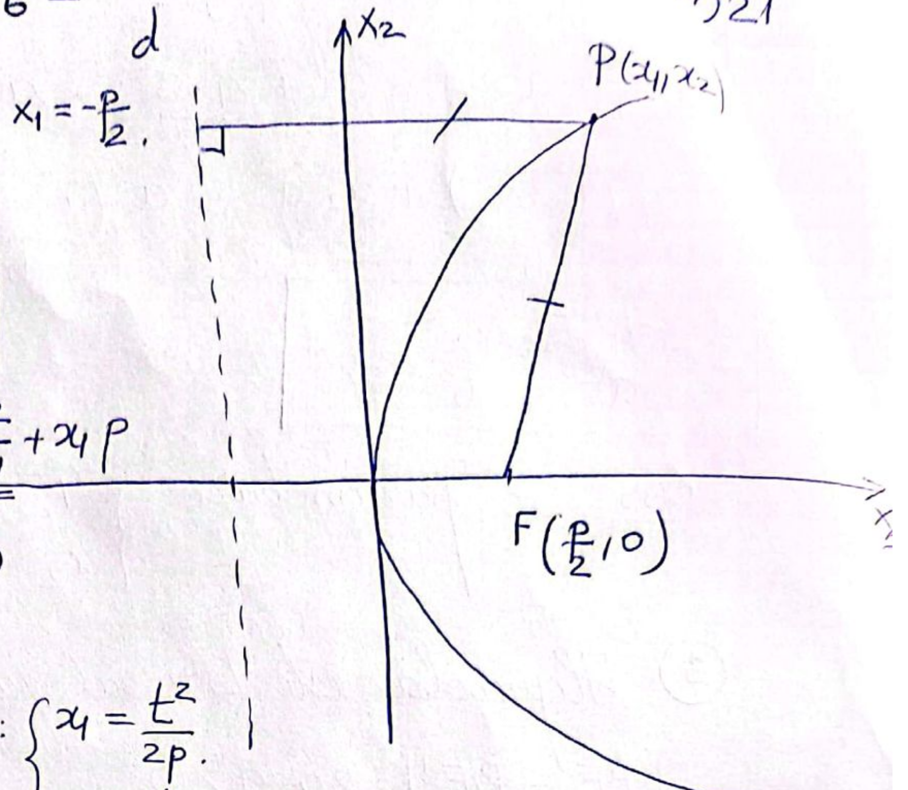
$t \in \mathbb{R}$.

④ *Parabola* = LG al punctelor $P \in \mathcal{E}_2$ care verifică

$\frac{dist(P, F)}{dist(P, d)} = 1$, $\quad F$ = pct fix (focar)

d = dreaptă fixă (directoare)

$F \notin d$.



$$\frac{dist(P,F)}{dist(P,d)}=1$$

$$\sqrt{\left(x_1-\frac{p}{2}\right)^2+x_2^2}=\left|x_1+\frac{p}{2}\right|$$

$$x_1^2+\frac{p^2}{4}-x_1p+x_2^2=x_1^2+\frac{p^2}{4}+x_1p$$

$\boxed{\mathcal{P}:\ x_2^2=2px_1}$, $p>0$

OBS. Ec parametrice: $\begin{cases} x_1=\frac{t^2}{2p} \\ x_2=t \end{cases}$

Teoremă (definirea unitară a conicelor nedegenerate)

LG al punctelor $P\in E_2$ care verifică

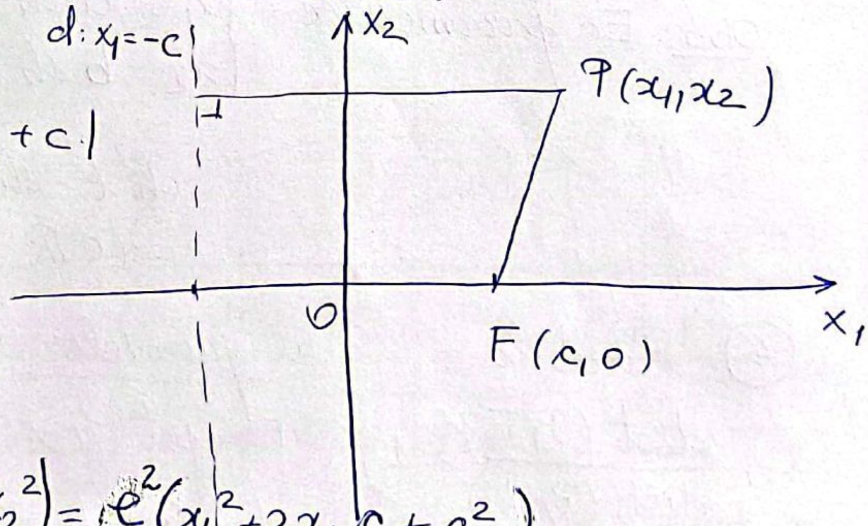
$\frac{dist(P,F)}{dist(P,d)}=e$ și reprezintă o conică nedegenerată,

unde F = pct fix (focar)

d = dreaptă fixă (directoare), $F\notin d$

e = s.n. excentricitate.

Dem.



$$\sqrt{(x_1-c)^2+x_2^2}=e|x_1+c|$$

$$(x_1^2+c^2-2x_1c+x_2^2)=e^2(x_1^2+2x_1c+c^2)$$

$$x_1^2(1-e^2)+x_2^2-2x_1c(1+e^2)+c^2(1-e^2)=0.$$

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OBS

Conica $\Gamma: f(x_1,x_2) = \underline{a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2} +$
$+ 2\underline{b_1}x_1 + 2\underline{b_2}x_2 + c = 0$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = A^T, \quad \tilde{A} = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$$

$\delta = \det A, \quad \Delta = \det \tilde{A}$

$\Gamma: \underline{x_1^2(1+e^2) + x_2^2} - \underline{2x_1 c(1+e^2)} + c^2(1+e^2) = 0$

$$A = \begin{pmatrix} 1+e^2 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 1+e^2 & 0 & -c(1+e^2) \\ 0 & 1 & 0 \\ -c(1+e^2) & 0 & c^2(1+e^2) \end{pmatrix}$$

Γ nedegenerată $\Leftrightarrow \Delta = \det \tilde{A} \neq 0$

$$\Delta = e^2 \begin{vmatrix} 1+e^2 & -c(1+e^2) \\ -c(1+e^2) & c^2(1+e^2) \end{vmatrix} = [c^2(1-e^2)^2 - c^2(1+e^2)^2] e^2 = c^2(-4e^2) = -4c^2e^2 \neq 0$$

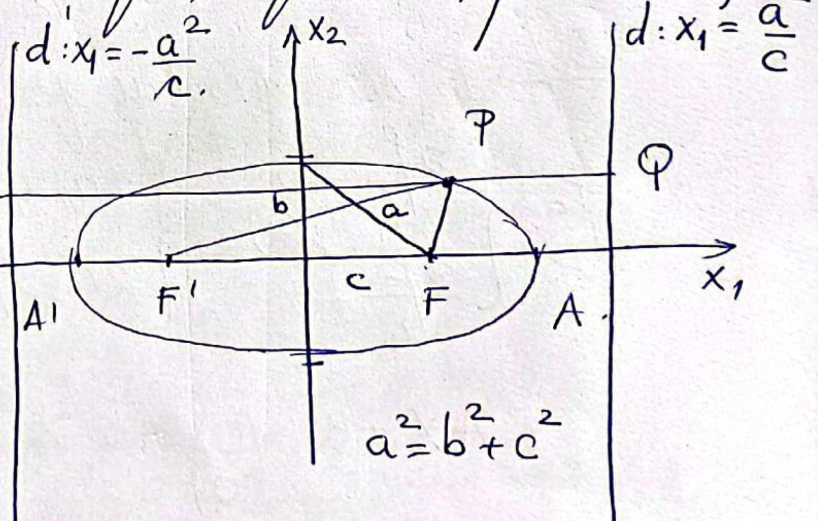
$F \notin d \Rightarrow e \neq 0$. $\Rightarrow \Gamma$ nedegenerată
$c \neq 0$.
(elipsă, hiperbolă, parabolă)

OBS

① $\mathcal{E}: \dfrac{x_1^2}{a^2} + \dfrac{x_2^2}{b^2} = 1$

$e = \dfrac{c}{a} < 1$
(excentricitatea)

$d \cup d': x_1 = \pm \dfrac{a^2}{c}$
directoarele.



$\dfrac{a^2}{c} > a \Leftrightarrow a > c$

Ⓐ $\dfrac{PQ}{PF} = \dfrac{a}{c} \Rightarrow PF = \dfrac{c}{a} PQ$

$\dfrac{PQ'}{PF'} = \dfrac{a}{c} \Rightarrow PF' = \dfrac{c}{a} PQ'$

$\Rightarrow PF + PF' = \dfrac{c}{a} \cdot \dfrac{2a^2}{c} = 2a;$

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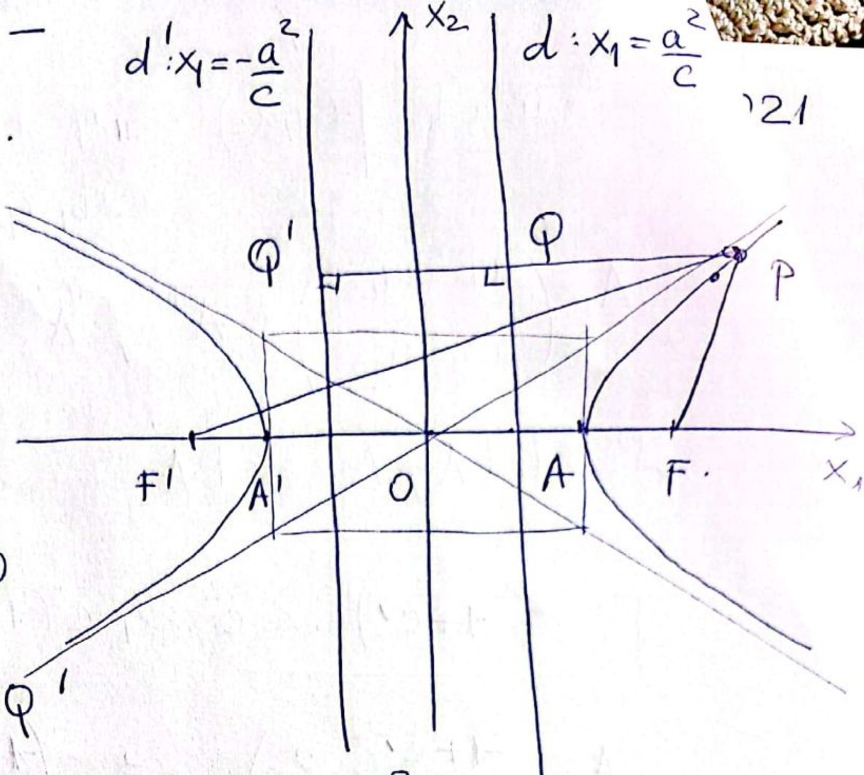
② $\mathcal{H}: \dfrac{x_1^2}{a^2} - \dfrac{x_2^2}{b^2} = 1.$



$c^2 = a^2 + b^2$

$e = \dfrac{c}{a} > 1.$

$\dfrac{a^2}{c} < a \Leftrightarrow a < c$

$\dfrac{PQ}{PF} = \dfrac{a}{c} \Rightarrow PF = \dfrac{c}{a} PQ$

$\dfrac{PQ'}{PF'} = \dfrac{a}{c} \Rightarrow PF' = \dfrac{c}{a} PQ'$

$|PF - PF'| = \dfrac{c}{a}\,|PQ - PQ'| = \dfrac{c}{a} \cdot \dfrac{2a^2}{c} = 2a.$

③ $\mathcal{P}: x_2^2 = 2p x_1.$

$e = 1.$

Aducerea la forma canonică a conicelor cu centru unic

$\Gamma: X^T A X + 2BX + c = 0.$

$\Gamma: a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2 + 2b_1x_1 + 2b_2x_2 + c = 0$

<u>Def</u> P_0 s.n. centru pentru $\Gamma \Leftrightarrow [\forall P \in \Gamma \Leftrightarrow S_{P_0}(P) \in \Gamma]$

$$P_0: \begin{cases} \dfrac{\partial f}{\partial x_1} = 0 \\ \dfrac{\partial f}{\partial x_2} = 0 \end{cases} \overset{(*)}{\Rightarrow} \begin{cases} 2a_{11}x_1 + 2a_{12}x_2 + 2b_1 = 0 \\ 2a_{12}x_1 + 2a_{22}x_2 + 2b_2 = 0 \end{cases}$$

$$AX + B^T = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow X^T A + B = (0\ \ 0)$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$\delta = \det A \neq 0 \Rightarrow (*)$ are sol unică.

Prop Dacă $\delta \neq 0$, atunci $f(x_1^0, x_2^0) = \frac{\Delta}{\delta}$, unde $P_0(x_1^0, x_2^0)$ este centrul conicei Γ.

Dem

$$f(x_1^0, x_2^0) = X_0^T A X_0 + 2BX_0 + C =$$
$$= \underbrace{(X_0^T A + B)}\, X_0 + BX_0 + C = BX_0 + c = \frac{\Delta}{\delta} \text{ (dem)}.$$

$$b_1 x_1^0 + b_2 x_2^0 + c = \frac{\Delta}{\delta} \text{ (dem)}$$

$$x_1^0 = \frac{\begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix}}{\delta}, \quad x_2^0 = \frac{\begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix}}{\delta} \quad (1)$$

$$\frac{\Delta}{\delta} = \frac{1}{\delta} \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{vmatrix} =$$

$$= \frac{1}{\delta} \cdot b_1 \begin{vmatrix} a_{12} & a_{22} \\ b_1 & b_2 \end{vmatrix} - \frac{b_2}{\delta} \begin{vmatrix} a_{11} & a_{12} \\ b_1 & b_2 \end{vmatrix} + \frac{c}{\delta}\,\delta \quad (2)$$

$$b_1 x_1^0 + b_2 x_2^0 + c \overset{(1)}{=} \frac{b_1}{\delta} \begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix} + \frac{b_2}{\delta} \begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix} + c \quad (3)$$

$$(2), (3) \Rightarrow f(x_1^0, x_2^0) = \frac{\Delta}{\delta}.$$

I. $\delta \neq 0$

- Dacă $(\mathbb{R}^2, \mathbb{R}^2/_{\mathbb{R}}, \varphi)$ sp. afin.

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translație}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{transf. afină}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}.$$

$\theta: X = X' + X_0$.

$$\theta(\Gamma): (X' + X_0)^T A (X' + X_0) + 2B(X' + X_0) + c = 0.$$

$$X'^T A X' + X_0^T A X' + X'^T A X_0 + X_0^T A X_0 + 2BX' + 2BX_0 + C = 0$$

$$\theta(\Gamma): X'^T A X' + \frac{\Delta}{\delta} = 0$$

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$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = X'^T A X'$ formă pătratică

Aducem Q la formă canonică (met Gauss)

$$Q(x) = \lambda_1 x_1''^2 + \lambda_2 x_2''^2$$

$$\tau: X' = CX'', \quad C \in GL(2, \mathbb{R})$$

$$\tau \circ \theta(\Gamma): \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0.$$

$$\overset{\shortparallel}{\Gamma'}$$

Γ, Γ' conice afin echivalente

$X \to X' + X_0 \to CX'' + X_0$ (transf. afină)

• $(\mathcal{E}_2, (E_2, \langle \cdot, \cdot \rangle), \varphi)$ sp. afin euclidian.

$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = X'^T A X'$

$\exists$ un reper ortonormat format din vectori proprii ai A = diagonală.

a) $P(\lambda) = \det(A - \lambda I_2) = 0$

$\lambda^2 - Tr(A)\lambda + \det(A) = 0$.

$\lambda_1 \neq \lambda_2$, $m_1 = m_2 = 1$

$V_{\lambda_i} = \langle \{e_i'\} \rangle$ $i = \overline{1,2}$ $\quad \langle e_i', e_j' \rangle = \delta_{ij}$

$e_1' = (l_1, m_1)$, $e_2' = (l_2, m_2)$ $\quad \forall i, j = \overline{1,2}$

$R = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$ ($\det R = 1$, reper pozitiv orientat)

$\tau: X' = RX''$ izometrie (de speța 1).

$$\tau \circ \theta(\Gamma): \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0$$

$$\overset{\shortparallel}{\Gamma'}$$

Γ, Γ' conice congruente metric.

$X = X' + X_0$, $X' = RX'' \Rightarrow \tau \circ \theta: X = RX'' + X_0$.

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b) $\lambda_1 = \lambda_2$, $m_1 = 2$

$V_{\lambda_1} = \langle \{ f_1, f_2 \} \rangle$ Aplicăm Gram-Schmidt

$\Rightarrow \{ e_1', e_2' \}$ reper ortonormat.

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Aplicatii

Ex1 $(\delta \neq 0)$

În sp. euclidian $\mathcal{E}_2$ se consideră conica:

$$\Gamma: f(x) = 7x_1^2 - 8x_1x_2 + x_2^2 - 6x_1 - 12x_2 - 9 = 0$$

Să se aducă la o formă canonică, utilizând izometrii. Reprez. grafică

SOL

$$A = \begin{pmatrix} 7 & -4 \\ -4 & 1 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 7 & -4 & -3 \\ -4 & 1 & -6 \\ -3 & -6 & -9 \end{pmatrix}$$

$\delta = \det A = 7 - 16 = -9 \neq 0$ (Γ are centru unic)

$\Delta = \det \tilde{A} = -9 \cdot 36 \neq 0$ (Γ nedegenerată)

Det. centrul conicei

$$P_0: \begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases} \Rightarrow \begin{cases} 14x_1 - 8x_2 - 6 = 0 \\ -8x_1 + 2x_2 - 12 = 0 \end{cases}$$

$$\begin{cases} 7x_1 - 4x_2 = 3 \\ -4x_1 + x_2 = 6 \quad | \cdot 4 \end{cases}$$

$-9x_1 \quad / = 27$

$x_1 = -3$

$x_2 = 6 - 12 = -6$

$P_0(-3, -6)$.

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translatie}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{rotatie}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}$$

$\theta: X = X' + X_0, \quad X_0 = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

$$\theta(\Gamma): X'^T A X' + \frac{\Delta}{\delta_{11}} = 0$$

36

$\theta(\Gamma): \underbrace{7x_1'^2 - 8x_1'x_2' + x_2'^2} + 36 = 0.$

$Q: \mathbb{R}^2 \longrightarrow \mathbb{R}$, $Q(x) = 7x_1'^2 - 8x_1'x_2' + x_2'$

Aplicăm met val. proprii

$\lambda^2 - Tr(A)\lambda + det(A) = 0 \Rightarrow \lambda^2 - 8\lambda - 9 = 0$

$(\lambda+1)(\lambda-9) = 0$

1. $\lambda_1 = -1$, 2. $\lambda_2 = 9$.

$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = -X\}$

$(A+I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-4x_1 + 2x_2 = 0 \Rightarrow x_2 = 2x_1$

$V_{\lambda_1} = \{(x_1, 2x_1) = x_1\underline{(1,2)}, x_1 \in \mathbb{R}\}$

$e_1' = \frac{1}{\sqrt{5}}(1,2)$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid AX = 9X\}$

$(A - 9I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 & -4 \\ -4 & -8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2x_1 - 4x_2 = 0 \Rightarrow x_1 = -2x_2$

$V_{\lambda_2} = \{(-2x_2, x_2) = x_2(-2,1), x_2 \in \mathbb{R}\}$

$e_2' = \frac{1}{\sqrt{5}}(-2,1).$

$\tau: X' = RX''$, $R = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \in SO(2).$

$\tau \circ \theta: X = RX'' + X_0$ (izometrie).

$\tau \circ \theta(\Gamma): \quad -x_1''^2 + 9x_2''^2 + 36 = 0$

$\frac{x_1''^2}{36} - \frac{x_2''^2}{4} = 1$ (hiperbolă).

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$e_1' = \frac{1}{\sqrt{5}}(1,2)$

$e_2' = \frac{1}{\sqrt{5}}(-2,1)$

$$\frac{x_1''^2}{36} - \frac{x_2''^2}{4} = 1.$$

$a = 6, b = 2$