6.05.2020

$\boxed{\text{CURS 11}}$

Endomorfisme simetrice (continuare)
Geometrie analitică euclidiană

- $(E, \langle\cdot,\cdot\rangle)$ s.v.e.r. $f \in End(E)$

$f \in Sim(E) \Leftrightarrow \langle x, f(y)\rangle = \langle f(x), y\rangle, \forall x, y \in E.$

__Prop__. $f \in Sim(E) \Rightarrow$ vectorii proprii corespunzători la valori proprii distincte sunt $\perp$

__Dem__
Fie $\lambda, \mu \in \mathbb{R}$, $\lambda \neq \mu$, valori proprii.
$\exists x, y \in E \setminus \{0_E\}$ aî $f(x) = \lambda x$, $f(y) = \mu y$
$\langle x, f(y)\rangle = \langle f(x), y\rangle \Rightarrow \langle x, \mu y\rangle = \langle \lambda x, y\rangle \Rightarrow$

$$\left.\begin{array}{l}(\mu - \lambda)\langle x, y\rangle = 0 \\ \text{dar } \lambda \neq \mu\end{array}\right\} \Rightarrow \langle x, y\rangle = 0.$$

__Prop__ $f \in End(E)$.
$f \in Sim(E)$, $U \subseteq E$ subsp. invariant $\Rightarrow U^{\perp} \subseteq E$ subspațiu invariant.

__Dem__. Dem că $\forall x \in U^{\perp} \Rightarrow f(x) \in U^{\perp}$.
Fie $y \in U$. $\langle f(x), y\rangle = \langle \underset{U^{\perp}}{x}, \underset{U}{f(y)}\rangle = 0$ ($U \subseteq E$ ssp. inv. al lui f)
$\Rightarrow f(x) \in U^{\perp}$

__Teoremă__ $f \in Sim(E) \Rightarrow$ toate rădăcinile polinomului caracteristic sunt reale.

__Teoremă__
$f \in Sim(E) \Rightarrow \exists R$ un reper ortonormat în E, format din vectori proprii aî $[f]_{R,R}$ este diagonală.

__Dem__ Fie R_0 reper ortonormat și $A = [f]_{R_0, R_0}$.
$P(\lambda) = \det(A - \lambda I_n)$. Toate răd sunt reale.
Fie λ_1 valoare proprie și e_1 versor propriu $f(e_1) = \lambda_1 e_1$
$\Rightarrow \langle\{e_1\}\rangle$ subsp. invariant $\Rightarrow \langle\{e_1\}\rangle^{\perp}$ subsp. invariant.

$\Rightarrow f/_{\langle\{e_1\}\rangle^\perp} \in Sim(\langle\{e_1\}\rangle^\perp)$

Fie λ_2 valoare proprie a restricției și e_2 versor propriu

$$\left.\begin{array}{l}\Rightarrow f(e_2)=\lambda_2 e_2 \\ \text{dar } f(e_1)=\lambda_1 e_1\end{array}\right\} \Rightarrow \begin{array}{l} e_1 \perp e_2 \\ \langle\{e_1, e_2\}\rangle \text{ subsp invar} \Rightarrow\end{array}$$

$\langle\{e_1, e_2\}^\perp\rangle$ subsp. invariant.

Continuăm raționamentul și după n pași construim $\mathcal{R}=\{e_1, .., e_n\}$ sistem de vectori mutual ortog.

$$\left.\begin{array}{l}\Rightarrow \mathcal{R} \text{ este SLI} \\ \text{dar } |\mathcal{R}| = \dim E = n\end{array}\right\} \Rightarrow \mathcal{R} \text{ reper ortonormat în } E$$

$$[f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix}\lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n\end{pmatrix}$$

<u>OBS</u>. a) $f \in Sim(E) \Rightarrow \dim V_{\lambda_i} = m_i$, $i=\overline{1,r}$, unde $\lambda_1, .., \lambda_r$ răd. dist. ale pol. caract, $m_1, .., m_r$ = multiplicitățile corespunzătoare, $m_1 + ... + m_r = n$

$E = V_{\lambda_1} \oplus ... \oplus V_{\lambda_r}$, $\mathcal{R} = \mathcal{R}_1 \cup ... \cup \mathcal{R}_r$,

$\mathcal{R}_i$ reper orton. în V_{λ_i}, $i=\overline{1,r}$

$$A = [f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix}\overbrace{\lambda_1 \dots \lambda_1}^{m_1 \text{ ori}} & & 0 \\ & \ddots & \\ 0 & & \overbrace{\lambda_r \dots \lambda_r}^{m_r \text{ ori}}\end{pmatrix}$$

b) Matricea asociată lui $f \in Sim(E)$ se poate diagonaliza printr-o schimbare de repere ortonormate $\Leftrightarrow$ o transf. ortogonală.

$\mathcal{R}_0 = \{e_1^0, .., e_n^0\} \xrightarrow{C} \mathcal{R} = \{e_1, .., e_n\}$ repere ortonormate

$C \in O(n)$. , $h \in O(E)$, $h(e_i^0) = e_i$, $i=\overline{1,n}$

$C = [h]_{\mathcal{R}_0, \mathcal{R}_0}$, $h(e_i^0) = \sum_{k=1}^{n} c_{ki} e_k^0$, $\forall i = \overline{1,n}$

c) $A = A^T$ $\rightarrow$ 1) $f \in Sim(E)$

$\searrow$ 2) $Q : E \to \mathbb{R}$, $Q(x) = X^T A X$

formă pătratică asociată

$\langle x, Q(x)\rangle = f(x)$, $\forall x \in E$.

Def $f \in Sim(E)$ s.n. <u>pozitiv definit</u> $\Leftrightarrow$ φ este pozitiv definită

Prop. $f \in Sim(E)$, pozitiv definit $\Rightarrow \exists\, h \in Sim(E)$, pozitiv definit a.î. $f = h^2$.

Teorema (de descompunere polară)

$$\forall f \in Aut(E) \Rightarrow \begin{array}{l} \exists\, h \in Sim(E) \\ \exists\, t \in O(E) \end{array} \text{ a.î. } f = h \circ t.$$

Geometrie analitică euclidiană

Spații afine. Spații punctual euclidiene

Def. $(\mathcal{A} = \mathbb{R}^n, V = \mathbb{R}^n_{/\mathbb{R}}, \varphi)$ spațiu afin $\Leftrightarrow$

- $\mathcal{A} = \mathbb{R}^n$ mulțime de puncte
- $V = \mathbb{R}^n_{/\mathbb{R}}$ spațiu vectorial director
- $\varphi : \mathcal{A} \times \mathcal{A} \longrightarrow V$, $\varphi(A,B) \overset{not}{=} \overrightarrow{AB} = B - A$ <u>structură afină canonică</u>

care verifică:
1) $\varphi(A,B) + \varphi(B,C) = \varphi(A,C)$
2) $\exists\, O \in \mathbb{R}^n$ a.î. $\varphi_0 : \mathcal{A} = \mathbb{R}^n \longrightarrow \mathbb{R}^n = V$ bij.

$\varphi_0(A) = \varphi(O, A)$, $\forall A, B, C \in \mathcal{A} = \mathbb{R}^n$.

Def • $\{P_1, \dots, P_k\} \subset \mathbb{R}^n$ s.n. SA.I (sistem afin independent) (SA.D sistem afin dependent) $\Leftrightarrow$ $\{\overrightarrow{P_1P_2}, \dots, \overrightarrow{P_1P_k}\} \subset \mathbb{R}^n$ este SLI (SLD).

• $\{P_1, \dots, P_k\} \subset \mathbb{R}^n$, $P = \sum_{i=1}^{k} a_i P_i$ s.n. combinație afină $\Leftrightarrow \sum_{i=1}^{k} a_i = 1$.

• $M \subset \mathbb{R}^n$ subm. de pcte.

$$Af(M) = \left\{ \sum_{i=1}^{k} a_i P_i \;\middle|\; a_i \in \mathbb{R}, P_i \in M, i = \overline{1,k}, \sum_{i=1}^{k} a_i = 1 \right\}$$
(comb. afine de pcte din M)

Def. $\mathcal{A}' \subseteq \mathbb{R}^n$ s.n. <u>varietate liniară</u> sau <u>subspațiu afin</u> $\Leftrightarrow \left[\forall P_1, P_2 \in \mathcal{A}' \Rightarrow Af\{P_1, P_2\} \subset \mathcal{A}'\right]$.

Prop a) $\mathcal{A}' \subseteq \mathbb{R}^n$ subsp. afin $\Rightarrow \exists!\, V' \subseteq \mathbb{R}^n$ subspațiu vectorial director a.î. $\forall P' \in \mathcal{A}'$, $V' = \{\overrightarrow{P'P},\ P \in \mathcal{A}'\}$!

- $\dim \mathcal{A}' = \dim V'$

b) Fie $P \in \mathbb{R}^n$, $V' \subseteq \mathbb{R}^n$ subsp. vect $\Rightarrow$ $\exists!\, \mathcal{A}' \subseteq \mathbb{R}^n$ subsp. afin a.î. $P \in \mathcal{A}'$ și V' = sp. director pentru $\mathcal{A}'$.

OBS. $\mathcal{A}_i \subseteq \mathbb{R}^n$, $i = \overline{1,2}$ subsp. afine $\Rightarrow \mathcal{A}_1 \cap \mathcal{A}_2$ subsp. afin.

În general, $\mathcal{A}_1 \cup \mathcal{A}_2$ nu e subsp. afin.

$\mathcal{A}_1 + \mathcal{A}_2 = \mathcal{A}f(\mathcal{A}_1 \cup \mathcal{A}_2)$.

Def. $\mathcal{A}_1, \mathcal{A}_2 \subseteq \mathbb{R}^n$ subspații afine

$\mathcal{A}_1 \parallel \mathcal{A}_2 \Longleftrightarrow V_1 \subseteq V_2$ sau $V_2 \subseteq V_1$.

Exemplu
$$\mathcal{A}' = \{x \in \mathbb{R}^n \mid AX = B\} \subset \mathbb{R}^n$$
$$V' = \{x \in \mathbb{R}^n \mid AX = 0\}$$

$\dim \mathcal{A}' = \dim V' = n - rg A$.

Caz part. $n = 3$.

$$\mathcal{A}' = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 - x_3 = 1 \end{cases} \right\} \subset \mathbb{R}^3$$

$$V' = \left\{ x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases} \right\}.$$

$\mathcal{A}'$ varietate liniară:

$$\begin{matrix} AX_1 = B \\ AX_2 = B \end{matrix} \Longrightarrow \underbrace{A(aX_1 + bX_2)} = B, \quad a + b = 1.$$

$$A(aX_1 + bX_2) \overset{\text{alg}}{=} a\underbrace{AX_1}_{B} + b\underbrace{AX_2}_{B} = (a+b)B = B.$$

Exemplu
$$\mathcal{A}' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 2\}$$
$$\mathcal{A}'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 3\}$$

$\mathcal{A}' \parallel \mathcal{A}''$, deoarece $V' = V'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 0\}$

<u>Def</u> $(\mathcal{E} = \mathbb{R}^n, (E_{/\mathbb{R}}, \langle\cdot,\cdot\rangle), \varphi)$ s.n. spațiu afin euclidian (spațiu punctual euclidian)

(i.e spațiu afin în care sp. director este sp. vectorial euclidian)

<u>Def</u> a) $\mathcal{E}_1, \mathcal{E}_2 \subset \mathcal{E}$ s.n. <u>subspații afine perpendiculare</u> $\Leftrightarrow E_1 \perp E_2$

b) $\mathcal{E}_1, \mathcal{E}_2 \subset \mathcal{E}$ s.n. <u>subspații afine normale</u> $\Leftrightarrow$ $E_2^{\perp} = E_1$ i.e. $E = E_1 \oplus E_1^{\perp}$

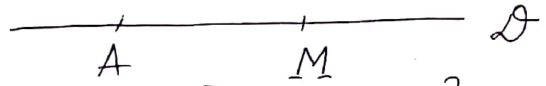
<u>Ecuații varietăți liniare</u>

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian ortonormat în $(\mathcal{E}, E, \varphi)$ unde $O \in \mathcal{E}$, $\{e_1, .., e_n\}$ reper ortonormat în E.

① <u>Ecuația unei drepte afine</u>

a) $A(a_1, .., a_n) \in \mathcal{D}$, $V_{\mathcal{D}} = \langle\{v\}\rangle$, $\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$

$v = \sum_{i=1}^{n} v_i e_i$



$V_{\mathcal{D}} = \{\overrightarrow{AM}, \forall M \in \mathcal{D}\} = \langle\{v\}\rangle$

$\exists t \in \mathbb{R}$ a.î. $\overrightarrow{AM} = tv \Rightarrow$

• $(x_1 - a_1, .., x_n - a_n) = t(v_1, .., v_n)$, $M(x_1, .., x_n)$ (ec. parametrice)

$\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$

<u>Ec. carteziană</u>:

$\mathcal{D}: \dfrac{x_1 - a_1}{v_1} = \ldots = \dfrac{x_n - a_n}{v_n} = t$ (Convenție: Dc pt un indice j avem $v_j = 0$, atunci $x_j = a_j$)

<u>Ec. vectorială parametrică</u>

$\vec{r} = \vec{r}_0 + tv$, unde $\vec{r} = \overrightarrow{OM}$, $\vec{r}_0 = \overrightarrow{OA}$

b). $A(a_1, .., a_n), B(b_1, .., b_n) \in \mathcal{D}$, $V_{\mathcal{D}} = \langle\{\overrightarrow{AB}\}\rangle$

<u>Ec. carteziană</u>

$\mathcal{D}: \dfrac{x_1 - a_1}{b_1 - a_1} = \ldots = \dfrac{x_n - a_n}{b_n - a_n} = t$

$\vec{r} = \overrightarrow{OM}, M \in \mathcal{D}$

<u>Ec vectorială</u>: $\vec{r} = \vec{r}_1 + t(\vec{r}_2 - \vec{r}_1)$

$\vec{r}_1 = \overrightarrow{OA}$

$\vec{r}_2 = \overrightarrow{OB}$

$x_i - a_i = t(b_i - a_i)$, $i = \overline{1,n}$ ec. parametrice.

$x_i = t b_i + (1-t) a_i$, $\forall i = \overline{1,n}$

$M = tB + (1-t)A$ baricentru, sau centru de greutate

$M(x_1, \dots, x_n)$

Poziția relativă a 2 drepte

$\mathcal{D}_1 : x_i - a_i = t v_i$, $i = \overline{1,n}$; $\mathcal{D}_2 : x_i - a_i' = t' v_i'$, $i = \overline{1,n}$

$a_i + \underline{\underline{t}} v_i = a_i' + \underline{\underline{t'}} v_i'$, $\forall i = \overline{1,n}$

$t v_i - t' v_i' = a_i' - a_i$, $\forall i = \overline{1,n}$

$$C = \left(\begin{array}{cc|c} v_1 & -v_1' & a_1' - a_1 \\ \vdots & & \vdots \\ v_n & -v_n' & a_n' - a_n \end{array}\right).$$

① $\operatorname{rg} C = \operatorname{rg} \bar{C} = 2 \Rightarrow \mathcal{D}_1, \mathcal{D}_2$ concurente.

② $\operatorname{rg} C = 2$, $\operatorname{rg} \bar{C} = 3 \Rightarrow$ necoplanare

③ $\operatorname{rg} C = 1 = \operatorname{rg} \bar{C} \Rightarrow \mathcal{D}_1 = \mathcal{D}_2$.

④ $\operatorname{rg} C = 1$, $\operatorname{rg} \bar{C} = 2$ $\quad \mathcal{D}_1 \parallel \mathcal{D}_2$, dist.

Caz particular : $n = 3$.

$$C = \left(\begin{array}{cc|c} v_1 & -v_1' & a_1' - a_1 \\ v_2 & -v_2' & a_2' - a_2 \\ v_3 & -v_3' & a_3' - a_3 \end{array}\right).$$

$\mathcal{D}_1, \mathcal{D}_2$ necoplanare $\Leftrightarrow \Delta = \begin{vmatrix} v_1 & -v_1' & a_1' - a_1 \\ v_2 & -v_2' & a_2' - a_2 \\ v_3 & -v_3' & a_3' - a_3 \end{vmatrix} \neq 0$

$\phantom{\mathcal{D}_1, \mathcal{D}_2 \text{ necoplanare } \Leftrightarrow \Delta = } \; v \quad -v' \quad \overrightarrow{A_1A_2}$

$A_1(a_1, a_2, a_3) \in \mathcal{D}_1$

$A_2(a_1', a_2', a_3') \in \mathcal{D}_2$

OBS. $\mathcal{D}_1, \mathcal{D}_2$ drepte afine perpendiculare $\Leftrightarrow \langle v, v' \rangle = 0$

Exemplu

$\mathcal{D}_1 : \dfrac{x_1}{1} = \dfrac{x_2 - 1}{1} = \dfrac{x_3}{1} = t \Leftrightarrow \begin{cases} x_1 = t \\ x_2 = 1 + t \\ x_3 = t \end{cases}$ $\quad A_1(0,1,0) \in \mathcal{D}_1$, $v = (1,1,1)$

$\mathcal{D}_2 : \dfrac{x_1 - 2}{1} = \dfrac{x_2}{1} = \dfrac{x_3 - 1}{1} = t' \Leftrightarrow \begin{cases} x_1 = 2 + t' \\ x_2 = t' \\ x_3 = 1 + t' \end{cases}$ $\quad A_2(2,0,1) \in \mathcal{D}_2$. $v' = (1,1,1)$

$v_{\mathcal{D}_1} = v_{\mathcal{D}_2} \Rightarrow \mathcal{D}_1 \parallel \mathcal{D}_2$.

$\begin{cases} t = 2 + t' \\ 1 + t = t' \\ t = 1 + t' \end{cases} \Rightarrow \begin{cases} t - t' = 2 \\ t - t' = -1 \\ t - t' = 1 \end{cases}$ $\quad C = \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{array}\right)$ $\quad \operatorname{rg} C = 1$, $\operatorname{rg} \bar{C} = 2$

② Ecuația unui plan afin (varietate liniară 2-dim)

a) π: $A(a_1,\dots,a_n)\in\pi$, $V_\pi=\langle\{u,v\}\rangle$, $\{u,v\}$ SLI

$V_\pi=\{\overrightarrow{AM},\ M\in\pi\}\Rightarrow\exists t,s\in\mathbb{R}$ ai $\overrightarrow{AM}=tu+sv$

π: $x_i-a_i=tu_i+sv_i,\ \forall i=\overline{1,n}$, $M(x_1,\dots,x_n)$

$v=\sum_{i=1}^{n}v_ie_i,\ u=\sum_{i=1}^{n}u_ie_i$

OBS $n=3$

$$\pi\quad\begin{vmatrix}x_1-a_1 & u_1 & v_1\\ x_2-a_2 & u_2 & v_2\\ x_3-a_3 & u_3 & v_3\end{vmatrix}=0$$

$N=u\times v\Rightarrow\pi:\langle\vec{r}-\vec{r_0},N\rangle=0$ ec. vectorială

$\vec{r}=\overrightarrow{OM}$, $\vec{r_0}=\overrightarrow{OA}$, $N=(A_1,A_2,A_3)$.

π: $A_1(x_1-a_1)+A_2(x_2-a_2)+A_3(x_3-a_3)=0$

$A_1x_1+A_2x_2+A_3x_3+A_0=0$, $A_1^2+A_2^2+A_3^2>0$

(ec. generală)

b) π: $\{A(a_1,\dots,a_n),B(b_1,\dots,b_n),C(c_1,\dots,c_n)\}$ SAI $\Leftrightarrow$ $\{\overrightarrow{AB},\overrightarrow{AC}\}$ SLI

π: $x_i-a_i=t(b_i-a_i)+s(c_i-a_i),\ \forall i=\overline{1,n}$

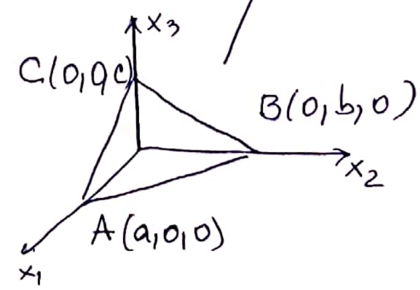
OBS $n=3$

$$\pi:\begin{vmatrix}x_1-a_1 & b_1-a_1 & c_1-a_1\\ x_2-a_2 & b_2-a_2 & c_2-a_2\\ x_3-a_3 & b_3-a_3 & c_3-a_3\end{vmatrix}=0\Leftrightarrow\pi:\begin{vmatrix}1 & 1 & 1 & 1\\ x_1 & a_1 & b_1 & c_1\\ x_2 & a_2 & b_2 & c_2\\ x_3 & a_3 & b_3 & c_3\end{vmatrix}=0$$

• Dacă $A(a,0,0)$, $B(0,b,0)$, $C(0,0,c)$

$$\pi:\begin{vmatrix}x_1-a & -a & -a\\ x_2 & b & 0\\ x_3 & 0 & c\end{vmatrix}=0\Leftrightarrow\frac{x_1}{a}+\frac{x_2}{b}+\frac{x_3}{c}=1$$

Ec prin tăieturi



Exemplu Planul π trece prin $\underline{A(2,1,4)}$ și $V_\pi = \langle\{u=(1,-1,1),$
$v=(2,1,-4)\}\rangle$.

$$N = u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & -4 \end{vmatrix} = (4-1,-(-4-2),1+2) = (3,6,3) = 3\underline{(1,2,1)}$$

π: $1(x_1-2)+2(x_2-1)+1(x_3-4)=0$

π: $x_1+2x_2+x_3-8=0$.

③ <u>Ecuația unui hiperplan</u> $\mathcal{H}$.

$A(a_1,\dots,a_n)\in\mathcal{H}$, $V_{\mathcal{H}} = \langle\{v_1,\dots,v_{n-1}\}\rangle$. $\{v_1,\dots,v_{n-1}\}$ SLI

$V_{\mathcal{H}} = \{\overrightarrow{AM},\ M\in\mathcal{H}\} \Rightarrow \exists t_1,\dots,t_{n-1}\in\mathbb{R}$ ai

$\overrightarrow{AM} = \sum_{k=1}^{n-1} t_k v_k \Rightarrow x_i - a_i = \sum_{k=1}^{n-1} t_k v_{ki}$, $\forall i=\overline{1,n}$

$$\begin{cases} t_1 v_{11} + \dots + t_{n-1} v_{n-1\,1} = x_1 - a_1 \\ \vdots \\ t_1 v_{1n} + \dots + t_{n-1} v_{n-1\,n} = x_n - a_n \end{cases}$$

sist. de n ecuații cu n-1 necunosc.

$$C = \left(\begin{matrix} v_{11} & \dots & v_{n-1\,1} \\ \vdots & & \vdots \\ v_{1n} & & v_{n-1\,n} \end{matrix}\right|\left.\begin{matrix} x_1-a_1 \\ \vdots \\ x_n-a_n \end{matrix}\right).$$

$$\Delta_c = 0 \Rightarrow \begin{vmatrix} v_{11} & \dots & v_{n-1\,1} & x_1-a_1 \\ \vdots & & \vdots & \vdots \\ v_{1n} & & v_{n-1\,n} & x_n-a_n \end{vmatrix} = 0$$

$\mathcal{H}$: $A_1x_1+\dots+A_nx_n+A_0=0$, $\sum_{i=1}^n A_i^2 > 0$

$N=(A_1,\dots,A_n)$ normala la hiperplan.

$\mathcal{H}$: $\langle N, x\rangle = 0$, $x=(x_1,\dots,x_n)$

<u>OBS</u> $\forall p$-plan $= \cap$ a $(n-p)$ hiperplane.

<u>OBS</u> $\mathcal{D}\perp\mathcal{H} \Leftrightarrow \langle\{N_{\mathcal{H}}\}\rangle = \langle\{u_{\mathcal{D}}\}\rangle$

$\mathcal{H}$: $\sum_{i=1}^n A_i x_i + A_0 = 0$; $\mathcal{D}: \frac{x_1-a_1}{A_1} = \dots = \frac{x_n-a_n}{A_n}$.

Poziția relativă a 2 hiperplane $\mathcal{H}_1, \mathcal{H}_2$.

$\mathcal{H}_1$: $A_1x_1+\dots+A_nx_n+A_0=0$ $\quad V_1 = \{x\in\mathbb{R}^n \mid A_1x_1+\dots+A_nx_n=0\}$

$N_1 = (A_1,\dots,A_n)$

$\mathcal{H}_2: A_1'x_1+\dots+A_n'x_n+A_0'=0$, $V_2=\{x\in\mathbb{R}^n \mid A_1'x_1+\dots+A_n'x_n=0\}$
$N_2=(A_1',\dots,A_n')$

- $\mathcal{H}_1 \parallel \mathcal{H}_2 \iff \langle\{N_1\}\rangle=\langle\{N_2\}\rangle$ $(rg\, C=1, rg\, \bar{C}=2)$

$$\frac{A_1}{A_1'}=\dots=\frac{A_n}{A_n'}\neq\frac{A_0}{A_0'},$$

- $\mathcal{H}_1=\mathcal{H}_2 \iff \frac{A_1}{A_1'}=\dots=\frac{A_n}{A_n'}=\frac{A_0}{A_0'}$, $(rg\, C=rg\, \bar{C}=1)$

- $\mathcal{H}_1\cap\mathcal{H}_2=\mathcal{A}$ (ssp. afin $(n-2)$-dim) $(rg\, C=rg\, \bar{C}=2)$

$$C=\begin{pmatrix}A_1 & \dots & A_n\\ A_1' & \dots & A_n'\end{pmatrix}\left|\begin{matrix}-A_0\\ -A_0'\end{matrix}\right.$$

<u>Exemplu</u> $n=3$.

$\pi_1: x_1+x_2+x_3-1=0$

$\pi_2: 2x_1 - x_3 = 0.$

$$\begin{pmatrix}1 & 1 & 1\\ 2 & 0 & -1\end{pmatrix}\left|\begin{matrix}1\\ 0\end{matrix}\right.$$

$\pi_1\cap\pi_2=\mathcal{D}$. , $x_3=t$

$$\begin{cases}x_1+x_2=1-t & \Rightarrow x_2=1-t-\frac{t}{2}=1-\frac{3}{2}t\\ 2x_1 = t \Rightarrow x_1=\frac{t}{2}\end{cases}$$

$$\mathcal{D}: \frac{x_1}{\frac{1}{2}}=\frac{x_2-1}{-\frac{3}{2}}=\frac{x_3}{1}=t \iff \frac{x_1}{1}=\frac{x_2-1}{-3}=\frac{x_3}{2}=t$$

<u>OBS</u> $$u_{\mathcal{D}}=N_1\times N_2=\begin{vmatrix} i & j & k\\ 1 & 1 & 1\\ 2 & 0 & -1\end{vmatrix}=(-1,3,-2)=-(1,-3,2)$$

- *Intersecția unei drepte cu un hiperplan.*

$\mathcal{D}: x_i=a_i+tv_i$, $i=\overline{1,n}$

$\mathcal{H}: A_1x_1+\dots+A_nx_n+A_0=0.$

$\mathcal{D}\cap\mathcal{H}: A_1(a_1+tv_1)+\dots+A_n(a_n+tv_n)+A_0=0$

$t(A_1v_1+\dots+A_nv_n)=-(A_1a_1+\dots+A_na_n)$.

Dacă $\mathcal{D}\parallel\mathcal{H} \iff u_{\mathcal{D}}\perp N \iff A_1v_1+\dots+A_nv_n=0.$

$$\mathcal{D}\nparallel\mathcal{H} \Rightarrow t=-\frac{\sum_{i=1}^{n}A_ia_i}{\sum_{i=1}^{n}A_iv_i}$$

$$\mathcal{D}\cap\mathcal{H}=\{M_0\} \quad M_0: x_i=a_i-\frac{\sum_{i=1}^{n}A_ia_i}{\sum_{i=1}^{n}A_iv_i}v_i, \forall i=\overline{1,n}$$