## 2) Geometrie analitică euclidiană

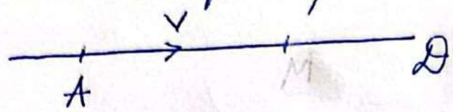
$(\mathbb{R}^3, (\mathbb{R}^3/\mathbb{R}, g_0), \varphi)$ sp. afin euclidian canonic

$\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$, $\varphi(u,v) = v - u$, $\forall u, v \in \mathbb{R}^3$

$\mathcal{R}_0 = \{O; e_1, e_2, e_3\}$ reper cartezian ortonormat

(*) Ec. unei drepte afine

a)



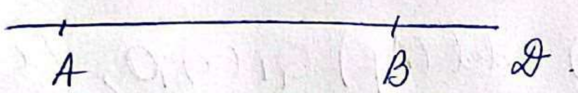
$V_{\mathcal{D}} = \langle\{v\}\rangle$

$\overrightarrow{OA} = \sum_{i=1}^{3} a_i e_i$, $v = \sum_{i=1}^{3} v_i e_i$

$\overrightarrow{OM} = \sum_{i=1}^{3} x_i e_i$

$$\mathcal{D}: \frac{x_1 - a_1}{v_1} = \frac{x_2 - a_2}{v_2} = \frac{x_3 - a_3}{v_3} = t \iff x_i - a_i = t v_i, \ i = \overline{1,3}$$

b)



$V_{\mathcal{D}} = \langle\{\overrightarrow{AB}\}\rangle$

$$\mathcal{D}: \frac{x_1 - a_1}{b_1 - a_1} = \frac{x_2 - a_2}{b_2 - a_2} = \frac{x_3 - a_3}{b_3 - a_3}$$

$\overrightarrow{OA} = \sum_{i=1}^{3} a_i e_i$

$\overrightarrow{OB} = \sum_{i=1}^{3} b_i e_i$

$\boxed{\text{OBS}}$ a) $\mathcal{D}_1 \parallel \mathcal{D}_2 \iff V_{\mathcal{D}_1} = V_{\mathcal{D}_2} \iff \exists \alpha \in \mathbb{R}$ a.î. $v' = \alpha v$

$V_{\mathcal{D}_1} = \langle\{v\}\rangle$, $V_{\mathcal{D}_2} = \langle\{v'\}\rangle$

b) $\mathcal{D}_1 \nparallel \mathcal{D}_2$

$\mathcal{D}_1: x_i - a_i = t v_i$

$\mathcal{D}_2: x_i - b_i = t' v'_i$, $i = \overline{1,3}$

$\mathcal{D}_1 \cap \mathcal{D}_2: \quad t v_i + a_i = t' v'_i + b_i, \ i = \overline{1,3}$

$$\left(\begin{array}{cc|c} v_1 & -v_1' & b_1 - a_1 \\ v_2 & -v_2' & b_2 - a_2 \\ v_3 & -v_3' & b_3 - a_3 \end{array}\right)$$

$$\Delta_c = \begin{vmatrix} v_1 & -v_1' & b_1 - a_1 \\ v_2 & -v_2' & b_2 - a_2 \\ v_3 & -v_3' & b_3 - a_3 \end{vmatrix}$$

1. $\Delta_c = 0$ drepte concurente (coplanare)
2. $\Delta_c \neq 0$ drepte necoplanare.

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**(**) Ec. unui plan afin**

a) $\pi \quad (A \in \pi,\ V_{\pi} = \langle \{u, v\} \rangle),\ \{u, v\}$ SLI

$V_{\pi} = \{\overrightarrow{AM},\ M \in \pi\}.$

$\exists t, s \in \mathbb{R}$ ai $\overrightarrow{AM} = tu + sv \qquad , \overrightarrow{OA} = \sum_{1}^{3} a_i e_i,\ \overrightarrow{OM} = \sum_{1}^{3} x_i e_i.$

$x_i - a_i = t u_i + s v_i,\ i = \overline{1,3}$

$$\pi : \begin{vmatrix} x_1 - a_1 & u_1 & v_1 \\ x_2 - a_2 & u_2 & v_2 \\ x_3 - a_3 & u_3 & v_3 \end{vmatrix} = 0 \qquad u = \sum_{1}^{3} u_i e_i \qquad v = \sum_{1}^{3} v_i e_i$$

$N = u \times v = (A_1, A_2, A_3)$

$\pi : \ A_1(x_1 - a_1) + A_2(x_2 - a_2) + A_3(x_3 - a_3) = 0$

$A_1 x_1 + A_2 x_2 + A_3 x_3 + A_0 = 0$

b) $\pi \quad (A, B, C \in \pi) \quad V_{\pi} = \langle \{\overrightarrow{AB}, \overrightarrow{AC}\} \rangle$

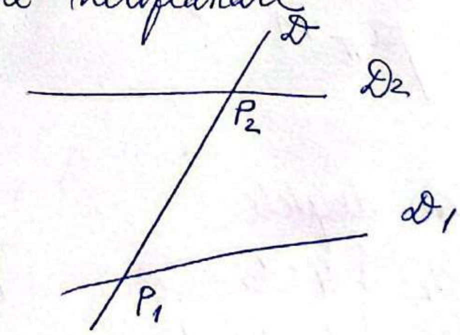
$\pi : \ x_i - a_i = t(b_i - a_i) + s(c_i - a_i) \qquad \overrightarrow{OA} = \sum_{1}^{3} a_i e_i,\ \overrightarrow{OB} = \sum_{1}^{3} b_i e_i,\ \overrightarrow{OC} = \sum_{1}^{3} c_i e_i$

$$\pi : \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0.$$

**(***) $\perp$ comună a 2 drepte necoplanare**

$\mathcal{D}_1 : \ x_i - a_i = t v_i$

$\mathcal{D}_2 : \ x_i - b_i = t' v'_i \qquad i = \overline{1,3}$




$P_1 (a_1 + t v_1,\ a_2 + t v_2,\ a_3 + t v_3)$

$P_2 (b_1 + t' v'_1,\ b_2 + t' v'_2,\ b_3 + t' v'_3)$

$$\begin{cases} \langle \overrightarrow{P_1 P_2}, v \rangle = 0 \\ \langle \overrightarrow{P_1 P_2}, v' \rangle = 0 \end{cases} \Rightarrow t, t' \Rightarrow P_1, P_2.$$

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Ex $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

$A(3,-1,3)$, $B(5,1,-1)$, $u=(-3,5,-6)$

a) Să se scrie ec. dreptei $\mathcal{D}$ a.î. $A \in \mathcal{D}$, $V_{\mathcal{D}} = \langle\{u\}\rangle$.

b) — // — AB.

c) Să se afle punctele de intersecție ale dreptei $\mathcal{D}$ cu planele de coordonate.

Ex Să se scrie ec dreptei $\mathcal{D}$ a.î. $A(2,-5,3) \in \mathcal{D}$ și $\mathcal{D} \parallel \mathcal{D}'$, unde $\mathcal{D}': \begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 \\ 5x_1 + 4x_2 - x_3 + 1 = 0 \end{cases}$

Ex Fie planul $\pi: x_1+x_2+x_3 = 1$, punctul $M(1,2,-1)$ și dreapta $\mathcal{D}: \frac{x_1-1}{2} = \frac{x_2-1}{-1} = \frac{x_3}{3}$

a) Să se scrie ec. dreptei $\mathcal{D}'$ a.î. $M \in \mathcal{D}'$ și $\mathcal{D}' \perp \pi$

b) — // — planului $\pi'$ a.î. $M \in \pi'$ și $\pi' \perp \mathcal{D}$

c) — // — planului $\pi''$ a.î. $M \in \pi''$ și $\mathcal{D} \subset \pi''$.

d) $pr_{\mathcal{D}} M = ?$, unde $M(1,2$

e) $pr_{\pi} M = ?$

Ex. Fie dreptele

$\mathcal{D}_1: \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases}$, $\mathcal{D}_2: \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

a) Să se arate că $\mathcal{D}_1, \mathcal{D}_2$ sunt necoplanare

b) Să se afle ec $\perp$ comune a dreptelor $\mathcal{D}_1, \mathcal{D}_2$

c) Să se determine dist $(\mathcal{D}_1, \mathcal{D}_2)$.

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Ex. Fie dreptele: $D_1: \frac{x_1-1}{1} = \frac{x_2-2}{-1} = \frac{x_3+2}{2}$

$$D_2: \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0 \end{cases}$$

a) Să se arate că $D_1, D_2$ coplanare

b) Să se scrie ec. planului det. de $D_1, D_2$

c) Să se afle dist $(D_1, D_2)$

Ex. Fie $D_1: \frac{x_1-1}{2} = \frac{x_2-1}{-1} = \frac{x_3}{3}$

$\pi_1: x_1 + x_2 + x_3 - 1 = 0$

$\pi_2: x_1 - x_2 + x_3 = 0$, $M(1,2,-1)$

a) Să se det ec. dreptei $D_2 = \pi_1 \cap \pi_2$

b) $\sphericalangle(D_1, D_2)$ ($D_1, D_2$ drepte orientate)

c) $\sphericalangle(\pi_1, \pi_2)$ ($\pi_1, \pi_2$ plane orientate)

d) Să se afle coord. simetricului lui $M$ față de $\pi_1$

Ex. $A(1,3,0)$, $B(3,-2,1)$, $C(\alpha,1,-3)$, $\Delta(7,-2,3)$

$\alpha = ?$ a.î. $A, B, C, \Delta$ = puncte coplanare.

Ex. Fie dreptele

$D_1: \frac{x_1-1}{-1} = \frac{x_2+2}{4} = \frac{x_3}{1}$, $D_2: \frac{x_1}{3} = \frac{x_2}{1} = \frac{x_3-1}{2}$

a) Să se arate că $D_1, D_2$ = necoplanare

b) Aflați ec $\perp$ comune a dreptelor $D_1, D_2$.

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