Module 8, Discussion, Part 1

Link to SOA Probability exercises from which this problem was taken:

https://people.math.osu.edu/ban.1/as/ExamP/P-09-05ques.pdf
(https://people.math.osu.edu/ban.1/as/ExamP/P-09-05ques.pdf)

1. The loss due to a fire in a commercial building is modeled by a random variable X with density function

f(x) = 0.005(20 - x) for 0 < x < 20 f(x) = 0 otherwise

Given that a fire loss exceeds 8, what is the probability that it exceeds 16 ?

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
import scipy.integrate as integrate
```

In [2]:
```
# set up list from zero to 20
x = np.linspace(0, 20, 21)
```

In [3]:
```
# create function for f(x)
def f(x):
    y = 0.005*(20 - x)
    return y
```

In [4]:
```
"""In this case we know that this is a valid probability density funct
ion but for verification let's integrate from
zero to 20 to make sure that the area under the curve is precisely 1.0
00."""

integral_f = integrate.quad(f, x[0], x[-1])
print(round(integral_f[0],7))
```

```
1.0
```

In [5]:
```
"""Solution: This is a conditional probability so we will need to calc
ulate the probability of the
intersection of the two events, which is the loss being greater than 1
6, and divide that by the probability of
the loss being greater than 8.    P(x > 16 | x > 8) = P(x > 16 ∩ x > 8
) / P(x > 8)"""

loss_grtr_8 = integrate.quad(f, x[8], x[-1])      # integrate from 8 t
o 20
loss_grtr_16 = integrate.quad(f, x[16], x[-1])    # integrate from 16
to 20

prob_grtr16_given_grtr8 = loss_grtr_16[0] / loss_grtr_8[0]    # prob.
of loss greater than 16 given greater than 8
print('P(x > 16 | x > 8) =',round(prob_grtr16_given_grtr8,5))
```

```
P(x > 16 | x > 8) = 0.11111
```

Things are much clearer if we graph them...

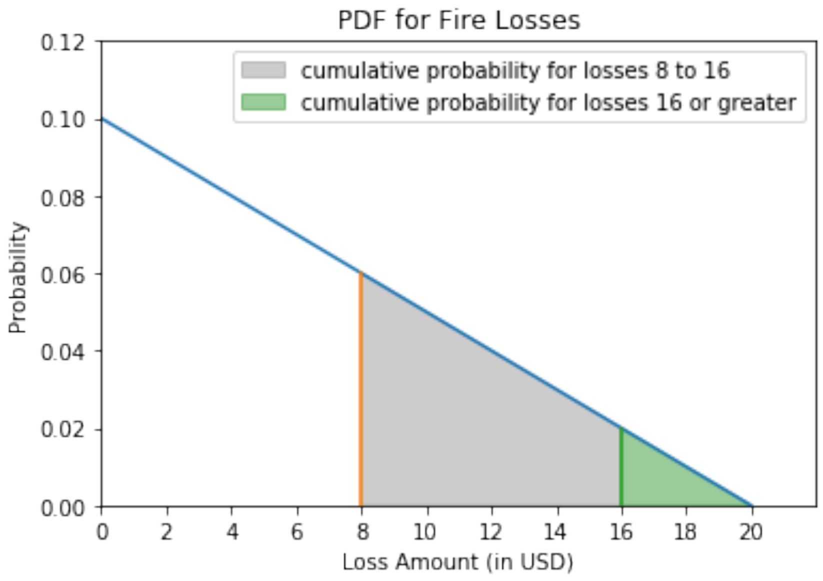
In [6]:
```
# this is the set up that I used for the vertical lines at y = 8 and y
= 16
y1 = np.arange(0, f(8) + .005, .005)
x1 = [8 for i in y1]

y2 = np.arange(0, f(16) + .005, .005)
x2 = [16 for i in y2]
```

```
In [7]: plt.xlim(0, 22)
        plt.ylim(0, .12)
        plt.xticks(np.arange(0, 22, 2))
        plt.plot(x, f(x))
        plt.plot(x1, y1)
        plt.plot(x2, y2)
        plt.title('PDF for Fire Losses')
        plt.xlabel('Loss Amount (in USD)')
        plt.ylabel('Probability')

        x3 = [8, 8, 16, 16]
        y3 = [0, f(8), f(16), 0]
        x4 = [16, 16, 20]
        y4 = [0, .02, 0]

        plt.fill(x3, y3, color='grey', alpha=0.4, label='cumulative probabilit
        y for losses 8 to 16')
        plt.fill(x4, y4, color='green', alpha=0.4, label='cumulative probabili
        ty for losses 16 or greater')
        plt.legend();
```



Note that we can verify our answer by using basic geometry... The area of the green triangle is 0.04 and the area of the sum of the green and grey triangles is 0.36.

0.04 / 0.36 = 1/9 = 0.11111

When we have similar problems that have curves rather then straight lines we need to use the integration option rather than geometry.

Module 8, Discussion, Part 2

In [8]:
```
y1 = np.arange(0, f(8) + .005, .005)
x1 = [8 for i in y1]

y2 = np.arange(0, f(16) + .005, .005)
x2 = [16 for i in y2]
```

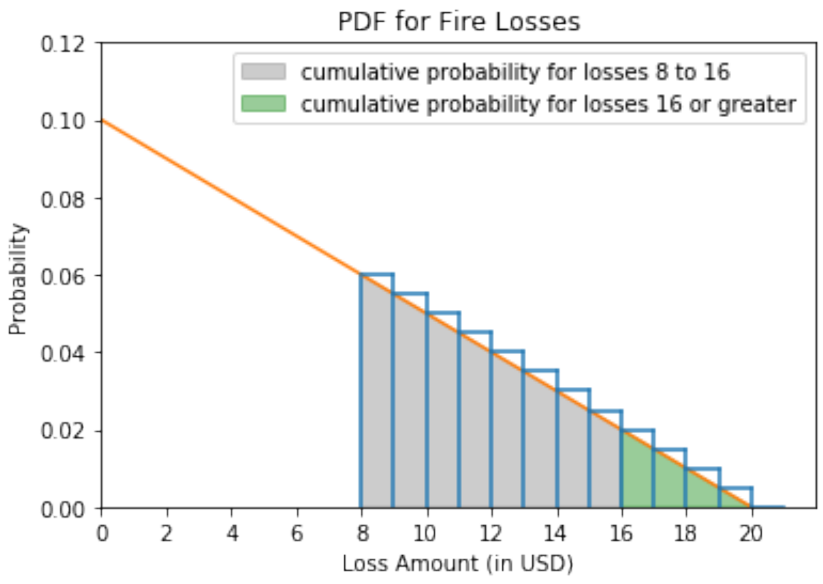
In [9]:
```
plt.xlim(0, 22)
plt.ylim(0, .12)
plt.xticks(np.arange(0, 22, 2))
plt.plot(x1, y1)
plt.plot(x, f(x))
plt.title('PDF for Fire Losses')
plt.xlabel('Loss Amount (in USD)')
plt.ylabel('Probability')

x3 = [8, 8, 16, 16]
y3 = [0, f(8), f(16), 0]
x4 = [16, 16, 20]
y4 = [0, .02, 0]

x_values = np.arange(8, 21, 1)
y_values = [f(i) for i in x_values]
for i, j in zip(x_values[1:], y_values[1:]):
    plt.axvline(x=i, ymax=(j+0.005)/.12)

for i, j in zip(x_values, y_values):
    plt.axhline(y=j, xmin=i/22, xmax=(i+1)/22)

plt.fill(x3, y3, color='grey', alpha=0.4, label='cumulative probabilit
y for losses 8 to 16')
plt.fill(x4, y4, color='green', alpha=0.4, label='cumulative probabili
ty for losses 16 or greater')
plt.legend();
```



Helpful article:

https://stackoverflow.com/questions/24988448/how-to-draw-vertical-lines-on-a-given-plot-in-matplotlib (https://stackoverflow.com/questions/24988448/how-to-draw-vertical-lines-on-a-given-plot-in-matplotlib)

In [10]:
```
area_rectangles = sum(y_values)
print(area_rectangles)
```
```
0.39
```

In [11]:
```
area_green_rect = sum(y_values[-5:])
print(area_green_rect)
```
```
0.05
```

In [12]:
```
print('Probability using rectagles =', round(0.05 / 0.39,4))
```
```
Probability using rectagles = 0.1282
```

Module 8, Discussion, Part 3

In [13]:
```
# define function
def parabola(x):
    y = -(1/4)*((x-2)**2) + 8
    return y
```

```
In [15]: # set up array of x-values
         x = np.arange(-4, 10.2, 0.2)
```

```
In [16]: # determine y-intercepts
         y_int1 = math.sqrt(-8/(-1/4)) + 2
         y_int2 = 2 - (y_int1 - 2)
```

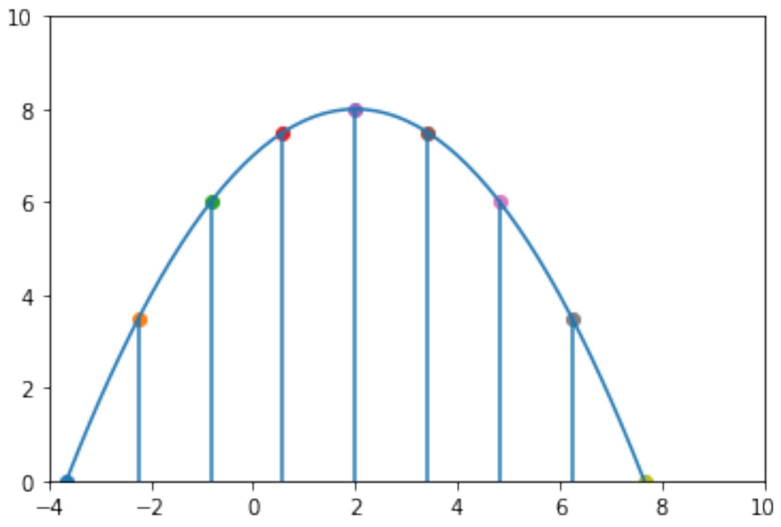
```
In [17]: print(y_int1, y_int2)

         7.656854249492381 -3.6568542494923806
```

```
In [18]: # set up intervals for trapezoid areas
         n = 8
         interval = (y_int1 - y_int2) / n

         # x values at each interval
         intervals_x = np.arange(y_int2, y_int1 + interval, interval)
```

```
In [19]: plt.xlim(-4, 10)
         plt.ylim(0, 10)
         plt.plot(x, parabola(x))
         for i, j in zip(intervals_x, parabola(intervals_x)):
             plt.axvline(x=i, ymax=j/10)
         for i, j in zip(intervals_x, parabola(intervals_x)):
             plt.scatter(i, j);
```



Now that we can see the graph, the next step will be to take the average height between each of the points on the curve and multiply that figure by the length of the x-axis intervals since that will provide us with the area of the trapezoids.

In [20]:
```
# set up empty list then feed the average heights for each pair of adj
acent points into the list
avg_heights = []
for i in range(1, len(intervals_x)):
    avg_height = np.average([parabola(intervals_x[i-1]), parabola(inte
rvals_x[i])])
    avg_heights.append(avg_height)

# multiply each value by the interval width to obtain areas
areas = [i*interval for i in avg_heights]
```

In [21]:
```
print('area using trapezoidal method:', sum(areas))
```

```
area using trapezoidal method: 59.39696961967001
```

In [22]:
```
# verification using standard formula:
print('trapezoidal area =', ((intervals_x[-1] - intervals_x[0]) / n)*(
0.5*parabola(intervals_x[0]) + parabola(intervals_x[1]) + parabola(int
ervals_x[2]) +
      parabola(intervals_x[3]) + parabola(intervals_x[4]) + parabola(i
ntervals_x[5]) + parabola(intervals_x[6]) +
      parabola(intervals_x[7]) + 0.5*parabola(intervals_x[8])))
```

```
trapezoidal area = 59.396969619669996
```

In [23]:
```
integ_parabola = integrate.quad(parabola, y_int2, y_int1)
print(integ_parabola[0])
```

```
60.33977866125207
```

Note: need to input a verification here using the actual trapezoidal formula

In [24]:
```
print('So the trapezoidal areas are only short by about', round(((inte
g_parabola[0]-sum(areas))/integ_parabola[0])*100,2),\
      '%.')
```

```
So the trapezoidal areas are only short by about 1.56 %.
```