Distributies

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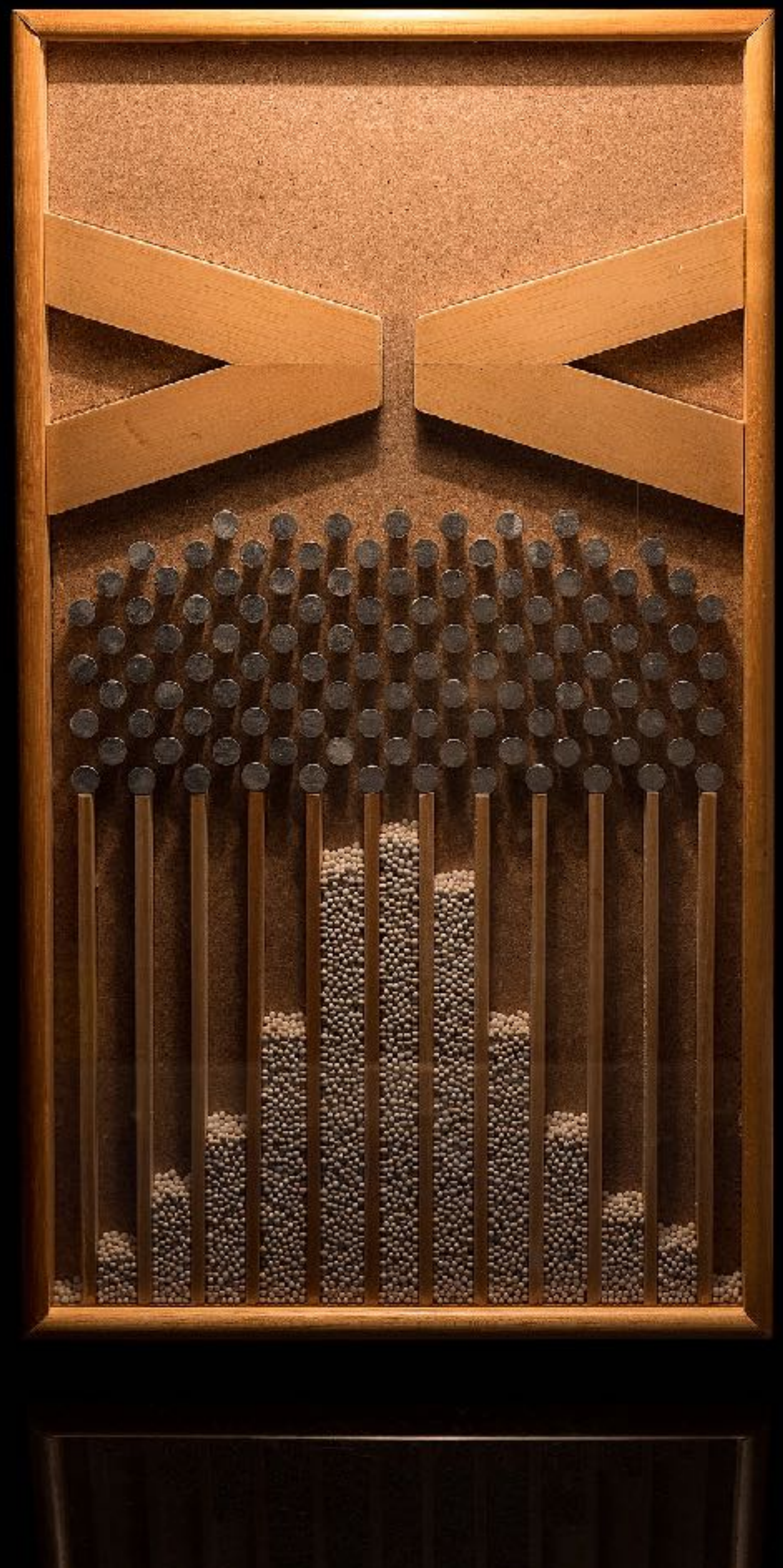
Distributions

Definition

A probability distribution is:

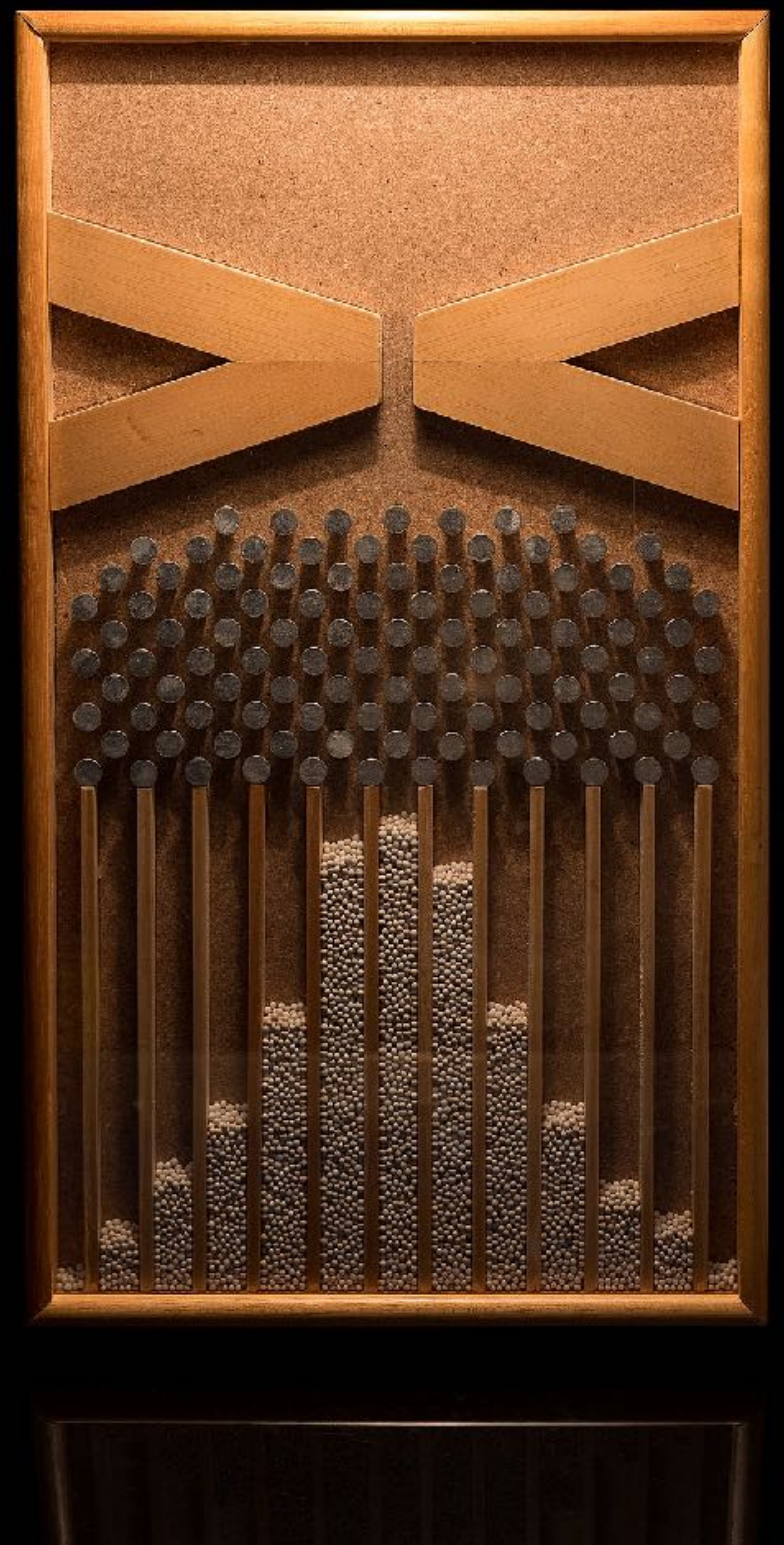
- A mathematical description



Distributions
Definition

A probability distribution is:

- A mathematical description
- Of a random phenomenon



Distributions

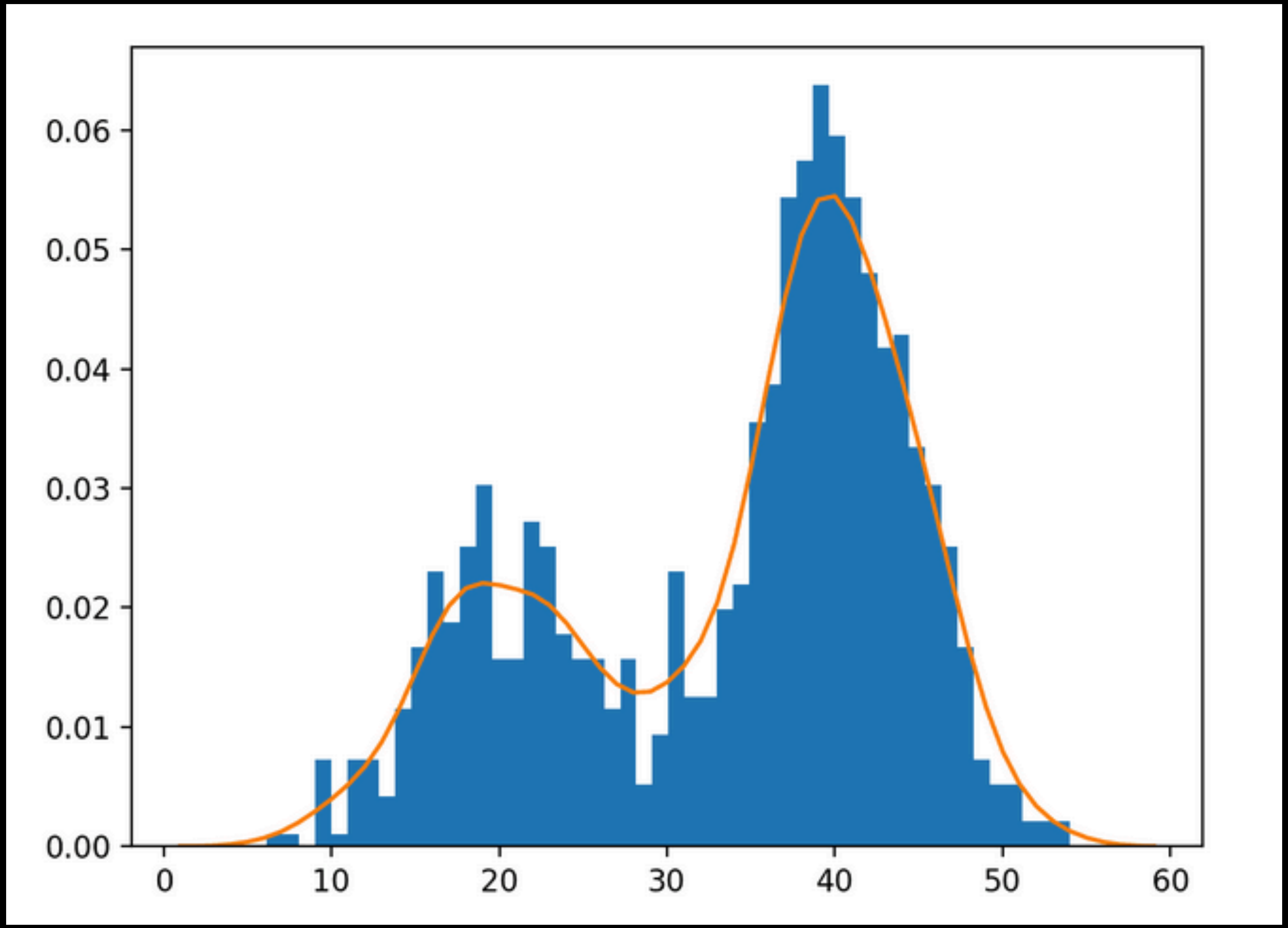
Definition

A probability distribution is:

- A mathematical description
- Of a random phenomenon
- In terms of all its possible outcomes and their associated probabilities

Distributions

- In this image, what are:
  - All the possible outcomes?
  - The associated probabilities?



Distributions

Types

The main types of distributions are:

- **Discrete** : when an outcome can only take discrete values (e.g. number of birds)

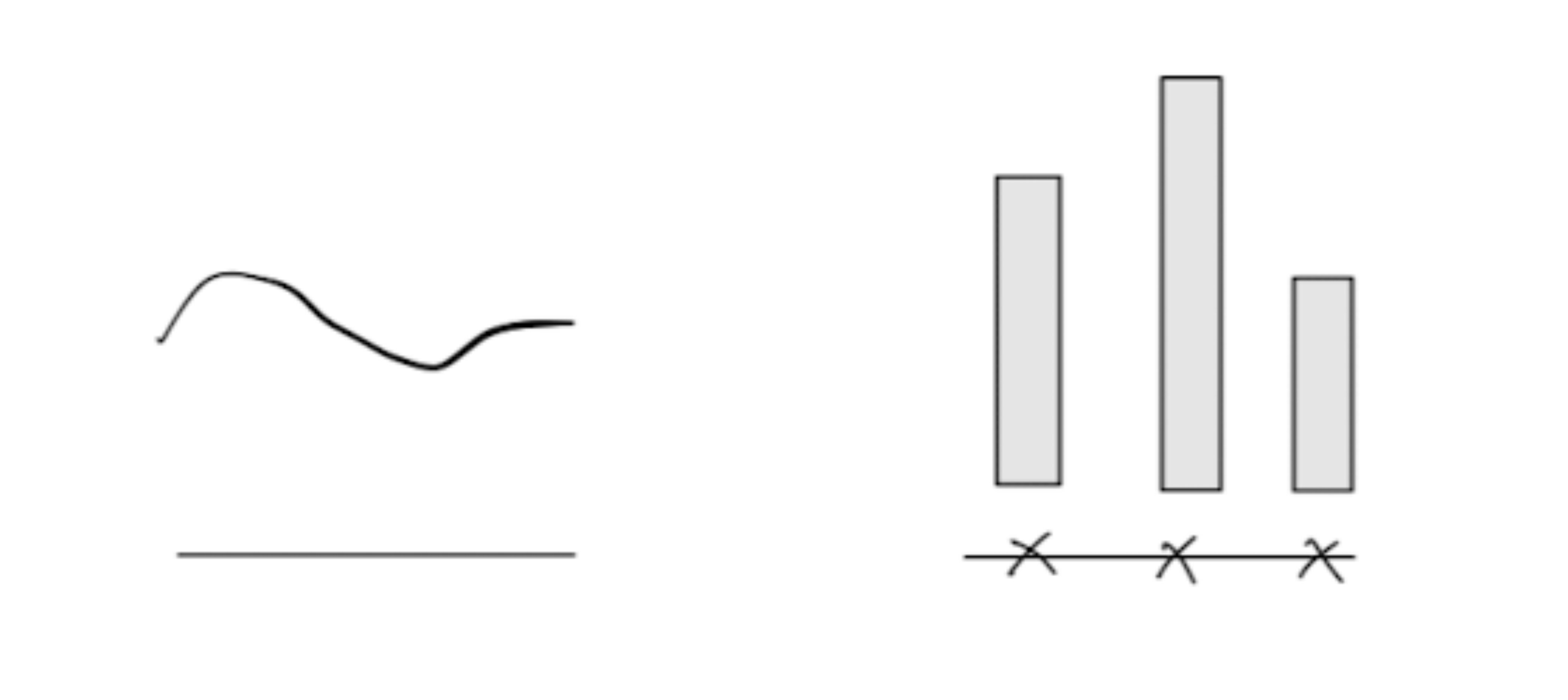
- **Continuous** : when outcomes take continuous values (e.g. blood pressure)

Distributions

Basic visualisation type

<u>Every</u> horizontal line you draw can be interpreted as a continuous distribution. **<u>Every</u>** barplot as a discrete distribution.

<u>All</u> the distributions we are going to discuss are variations of these two basic types!



Distributions

Basic visualisation type

For *parametric distributions*, we have a formula that describes the line / bars. You just put in the parameters, and the output is the line / bars.

Discrete distributions

PMF

A **probability mass function** (pmf) describes the probability distribution of discrete variables.

Consider a toin coss:

$$f(x) = \begin{cases} 0.5 & x \text{ is head} \\ 0.5 & x \text{ is tails} \end{cases}$$

This is the pmf of the *Bernoulli distribution*

Conditions for a PMF

Plain English

1. An event cannot have a negative probability

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Conditions for a PMF

Plain English

1. An event cannot have a negative probability
2. The sum of probabilities of all events must be 1
3. The probability of a subset X of outcomes T is the same as adding the probabilities of the individual elements.

Conditions for a PMF

Mathematical

The probability is a function f over the sample space $\mathcal{S}$ of a discrete random variable X, which gives the probability that X is equal to a certain value.

$$f(x) = P(X = x)$$

Each pmf satisfies these conditions:

1. $f(x) \geq 0, \forall x \in X$
2. $\Sigma_{x \in \mathcal{S}} f(x) = 1$
3. For a collection $\mathcal{A}$, $P(\mathcal{A} \in \mathcal{S}) = \Sigma_{x_i \in \mathcal{A}} f(x_i)$

Continuous Distributions

PDF

For continuous distributions, we use a probability density function (pdf)

Continuous Distributions

PDF

For continuous distributions, we use a probability density function (pdf)

1. $f(x) > 0, \forall x \in X$

2. The integral of the probabilities of all possible events must be 1 (area under the curve)

3. The probability X of values in the interval $[a, b]$ is the integral from a to b

Continuous Distributions

PDF

This might look like unnecessary mathematical details. But it is actually important to understand the difference.

Example: can you answer the question “What is the probability your body temperature is 37.0 C?”

Continuous Distributions

PDF

The answer might be unexpected: 0!

Let’s say your answer is 25%. But what if your temperature is 37.1? does that count? Or 37.01?

Continuous Distributions

PDF

- Because the distribution is *continuous* you can only say something about the *range*
- “What is the probability your temperature is between 36.5 and 37.2 C?”

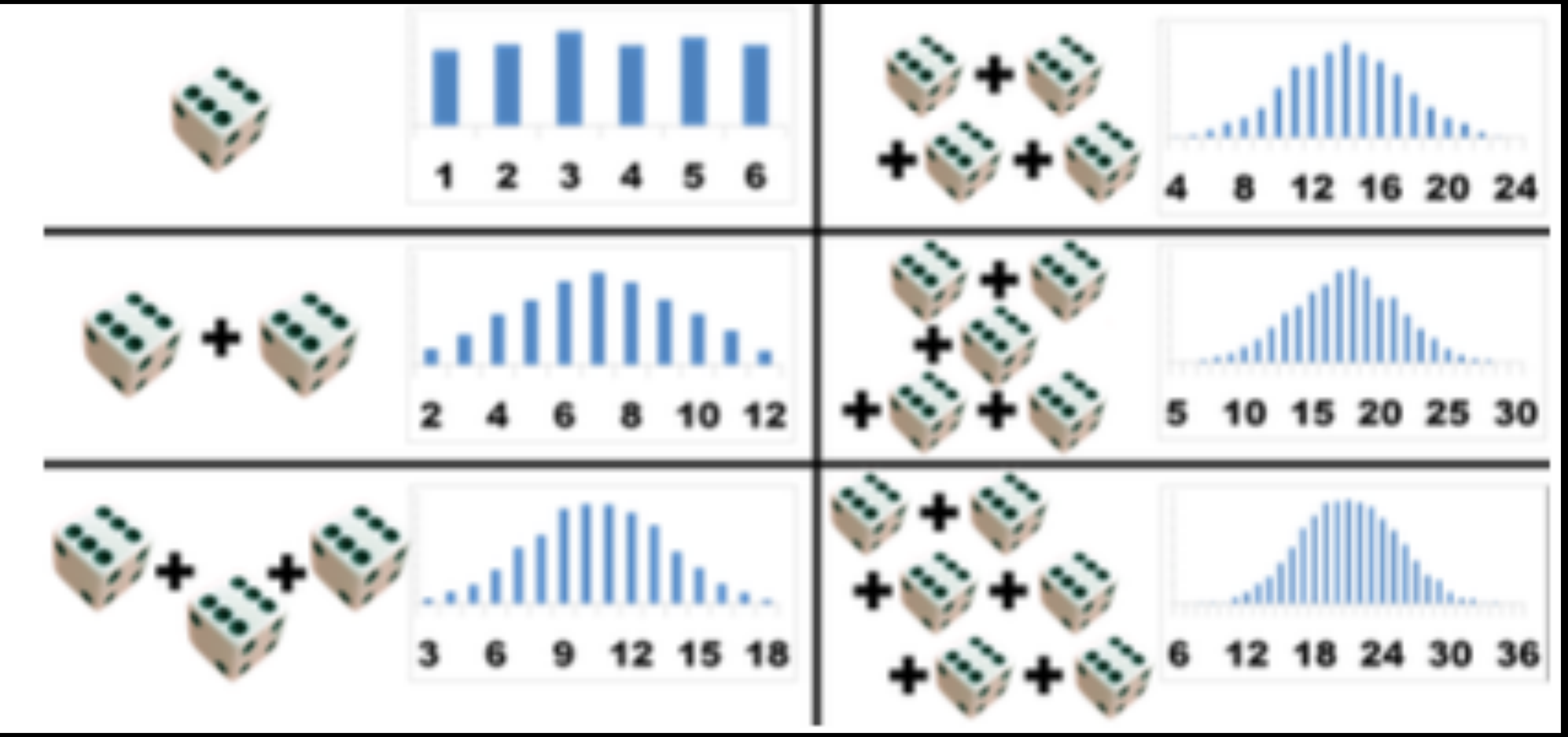
Quiz time

Normal distributions

Central limit theorem

The central limit theorem states that:

- the distribution of a normalized version of the sample mean converges to a standard normal distribution.
- This holds even if the original variables themselves are not normally distributed.

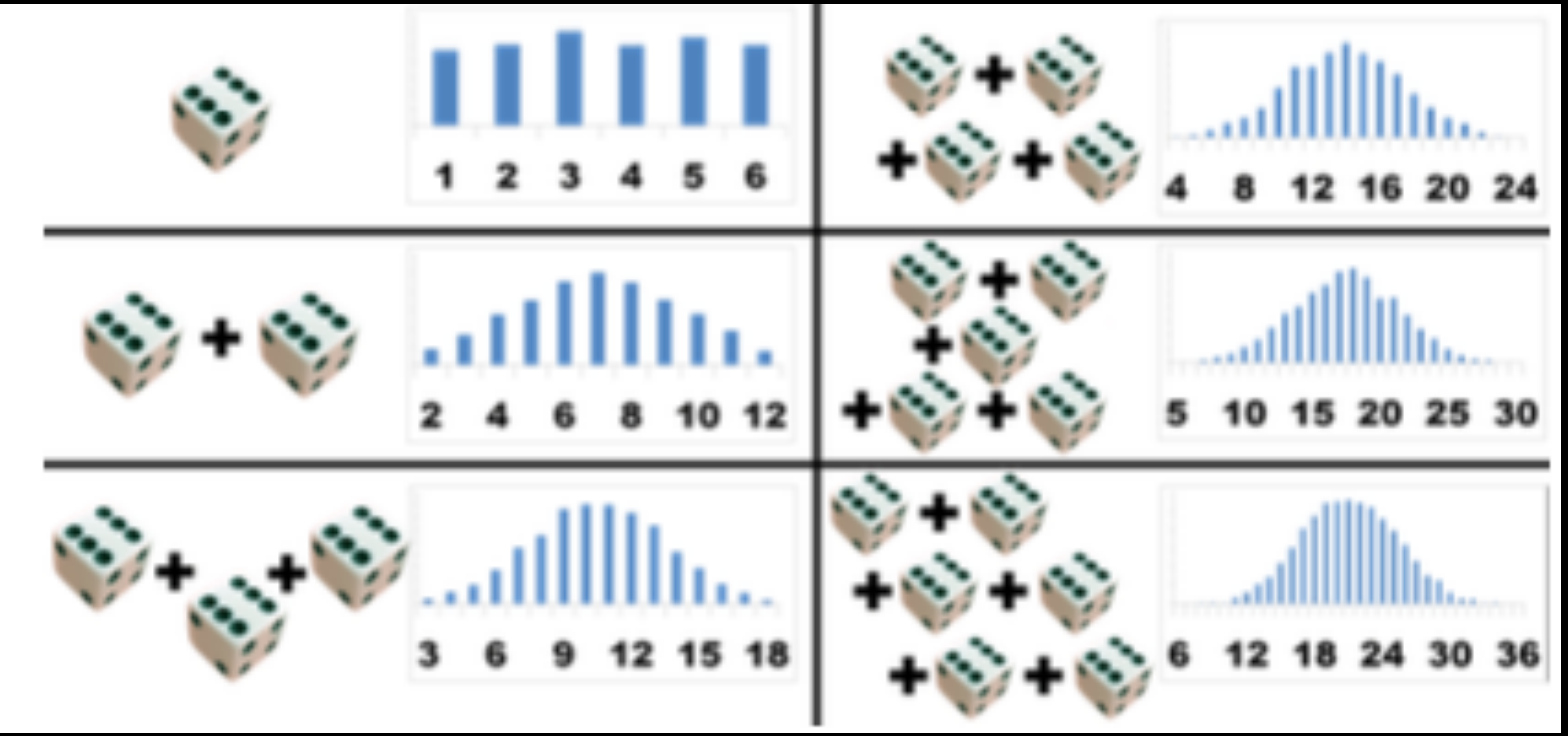


Normal distributions

Central limit theorem

The Normal distribution is one of the distributions that is used most often.

A major reason for this is, that if you keep sampling and **adding** from a population you *always* end up with a normal distribution.

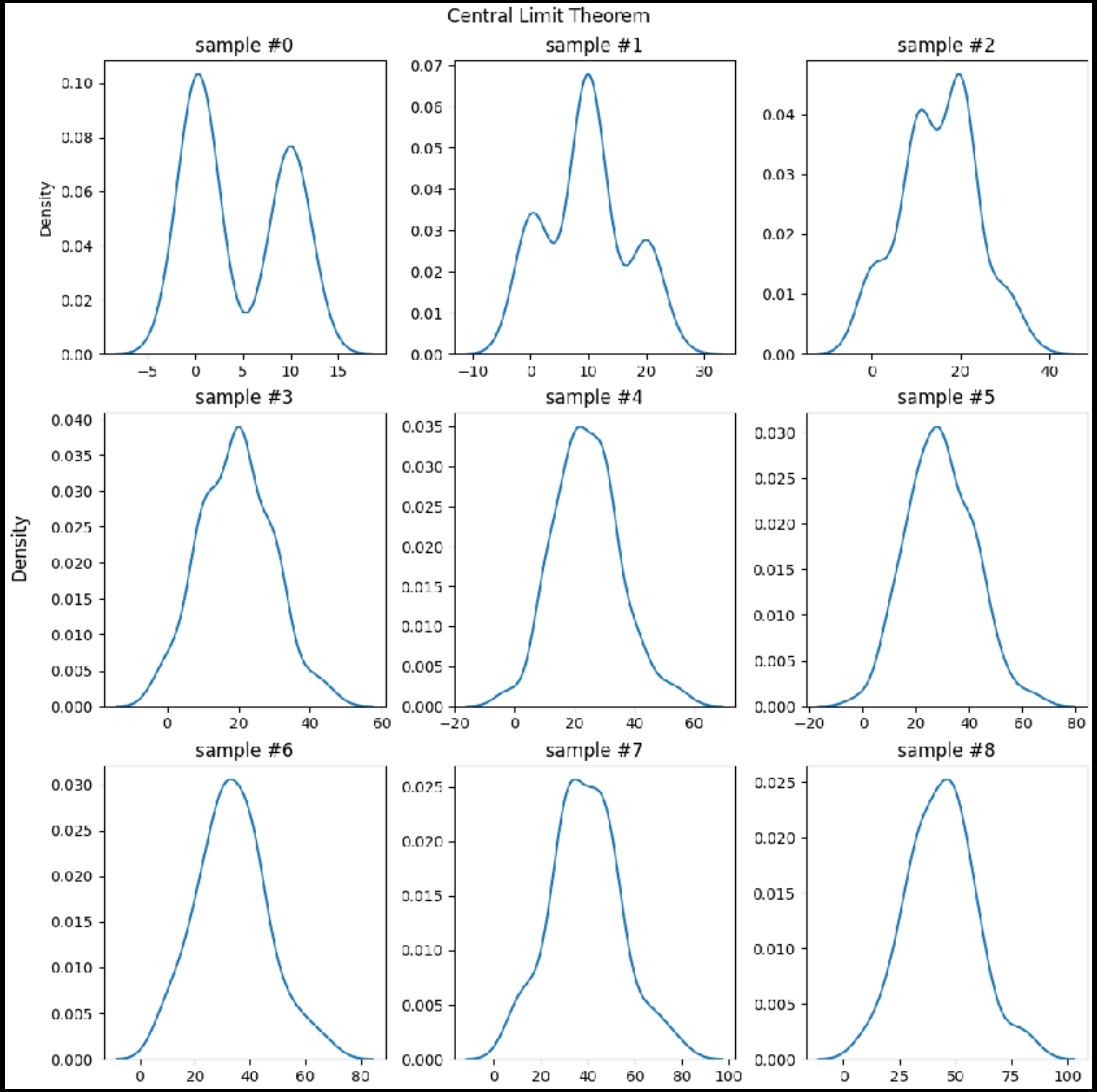


Normal distributions
Central limit theorem

Take a persons height.

- This is determined by a combination of 180 genes.
- One gene will contribute to a longer neck, the other to longer legs
- If we assume the genes contribute independently, height equals the sum of 180 genes.

Thus, height will be normally distributed. So will the weight of wolves or the length of a penguins wing.



Log Normal distribution

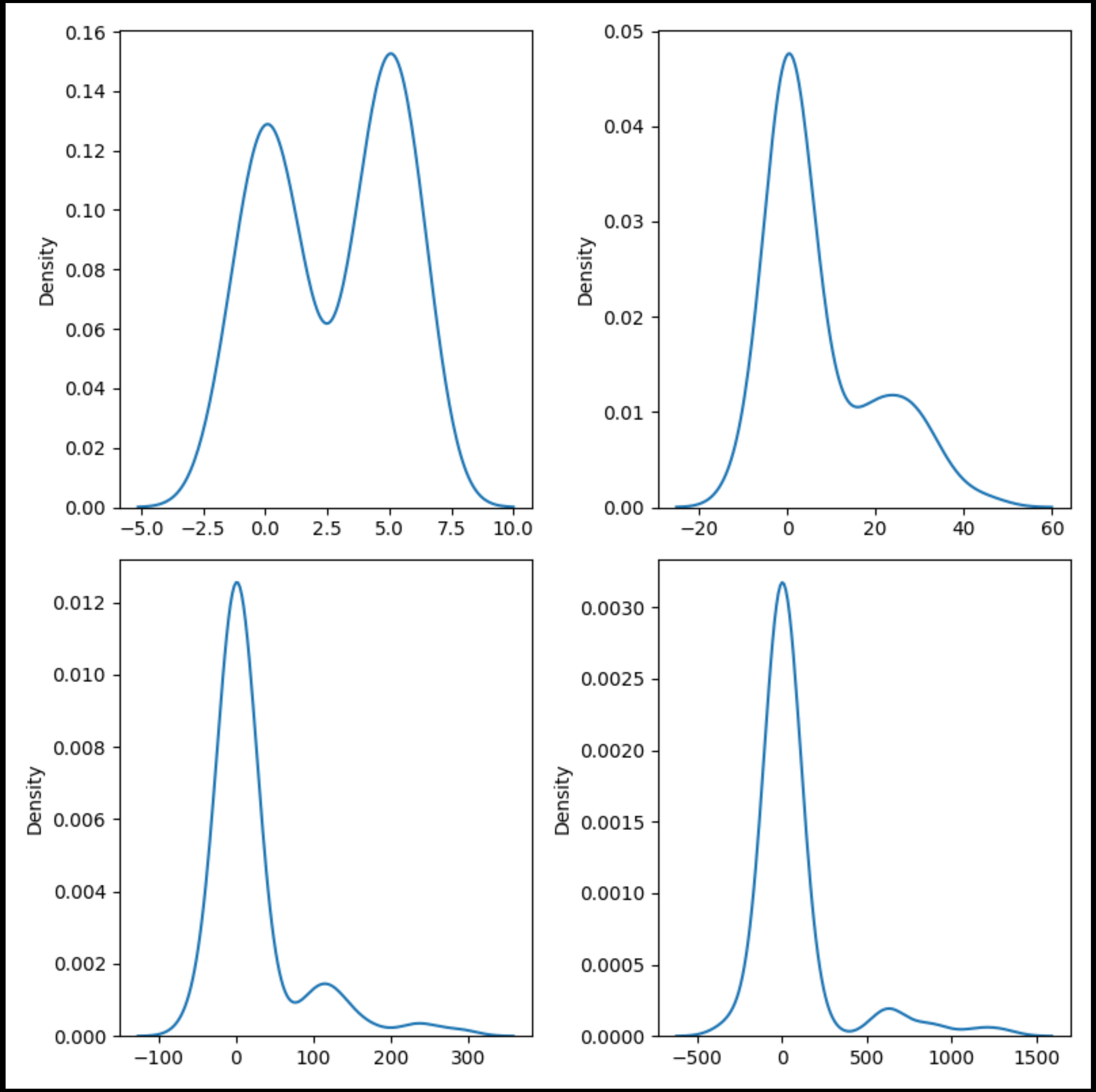
However, **multiplying** values will give you a long tail!

This is the case when variables interact in some way, and are not independent.

$$4 + 4 + 4 + 4 = 16$$

but

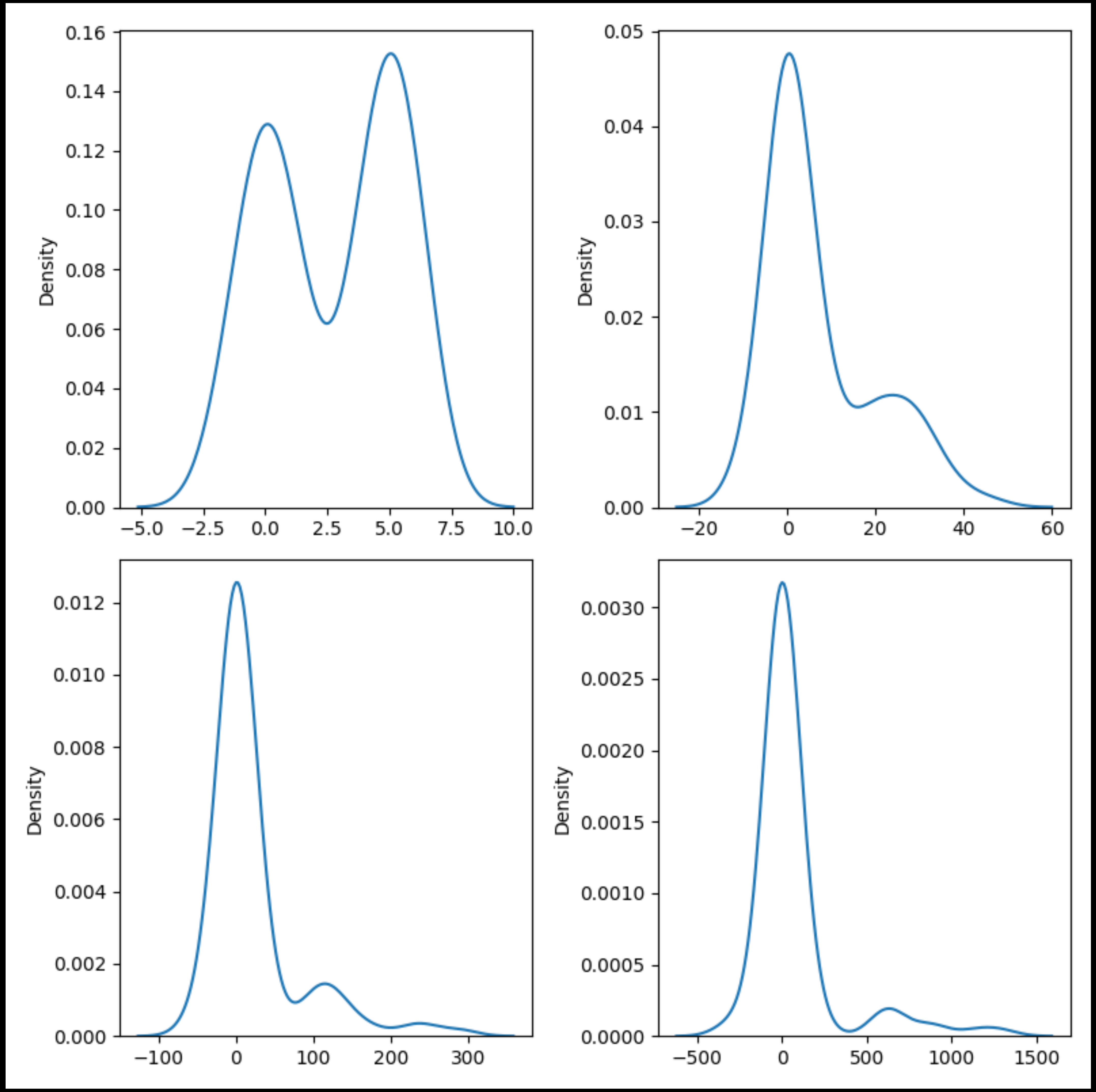
$$4 \times 4 \times 4 \times 4 = 256$$



Log Normal distribution

Multiplying should be expected if variables interact with each other.

Examples are stock prices, failures of machines, ping times on a network, income distribution.



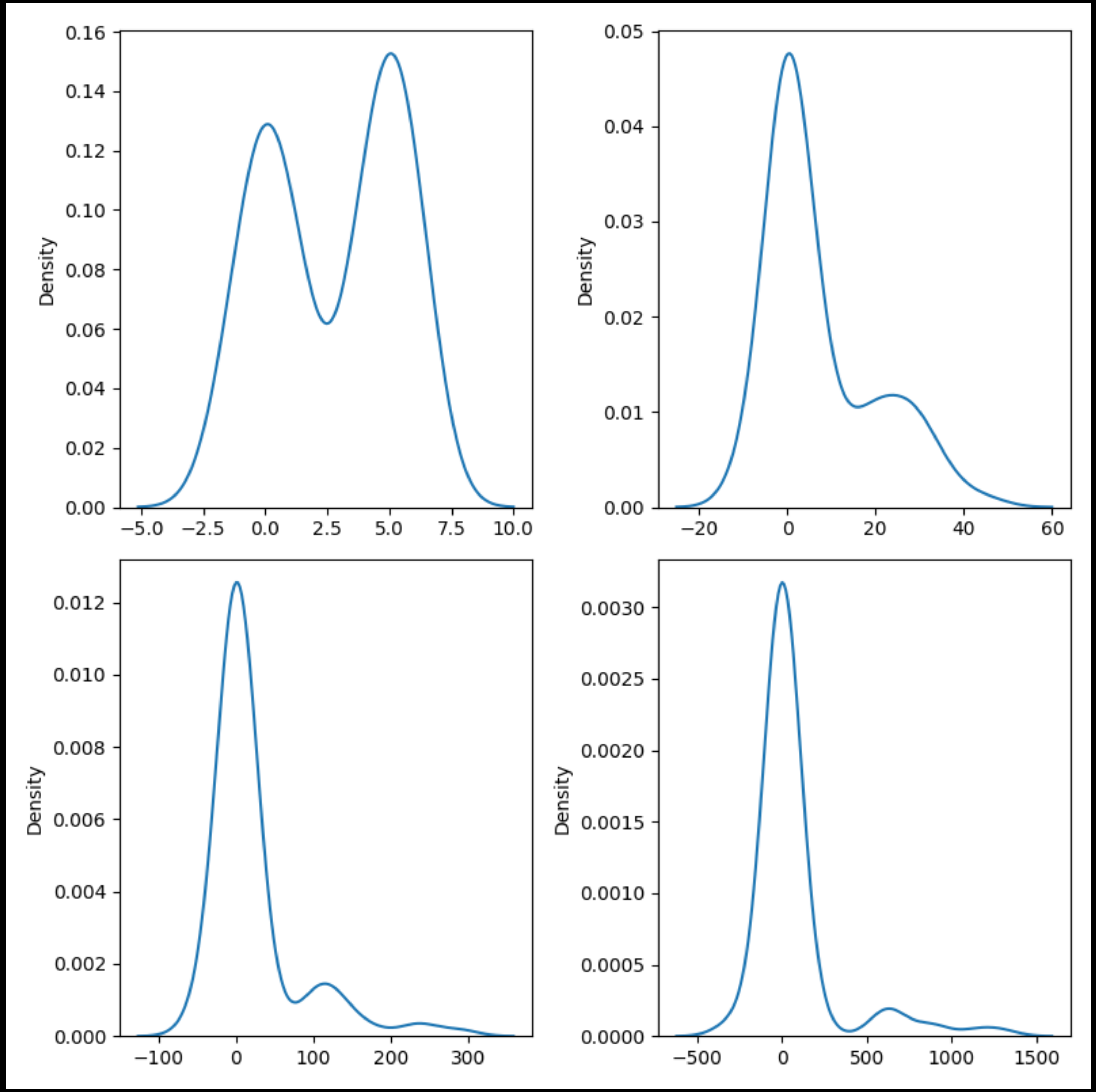
Log Normal distribution

multiplying values will give you a fat-tail distribution! This will typically be a log-normal distribution:

If X is log-normal distibuted, then

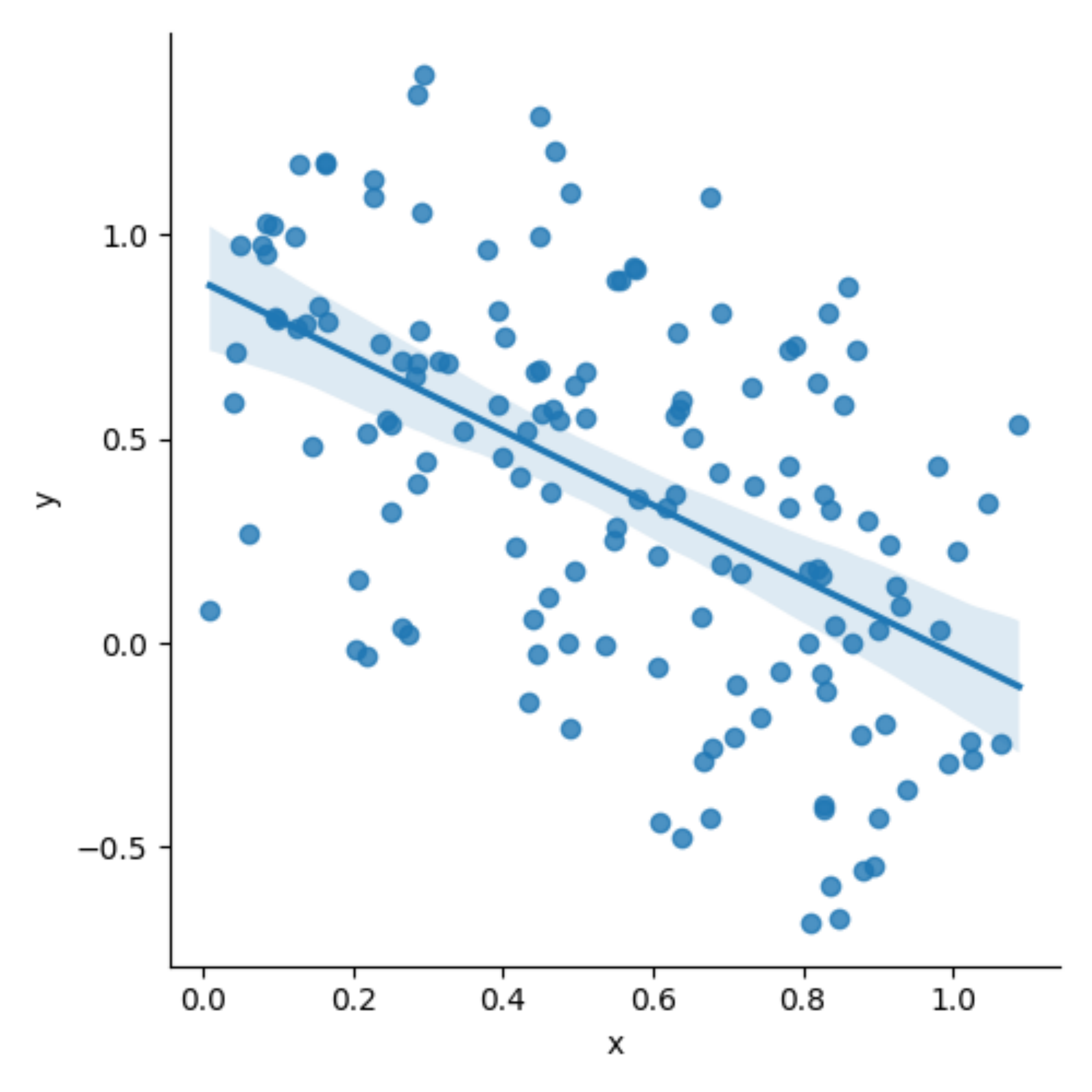
$$y = log(X)$$

will be a normal distribution.



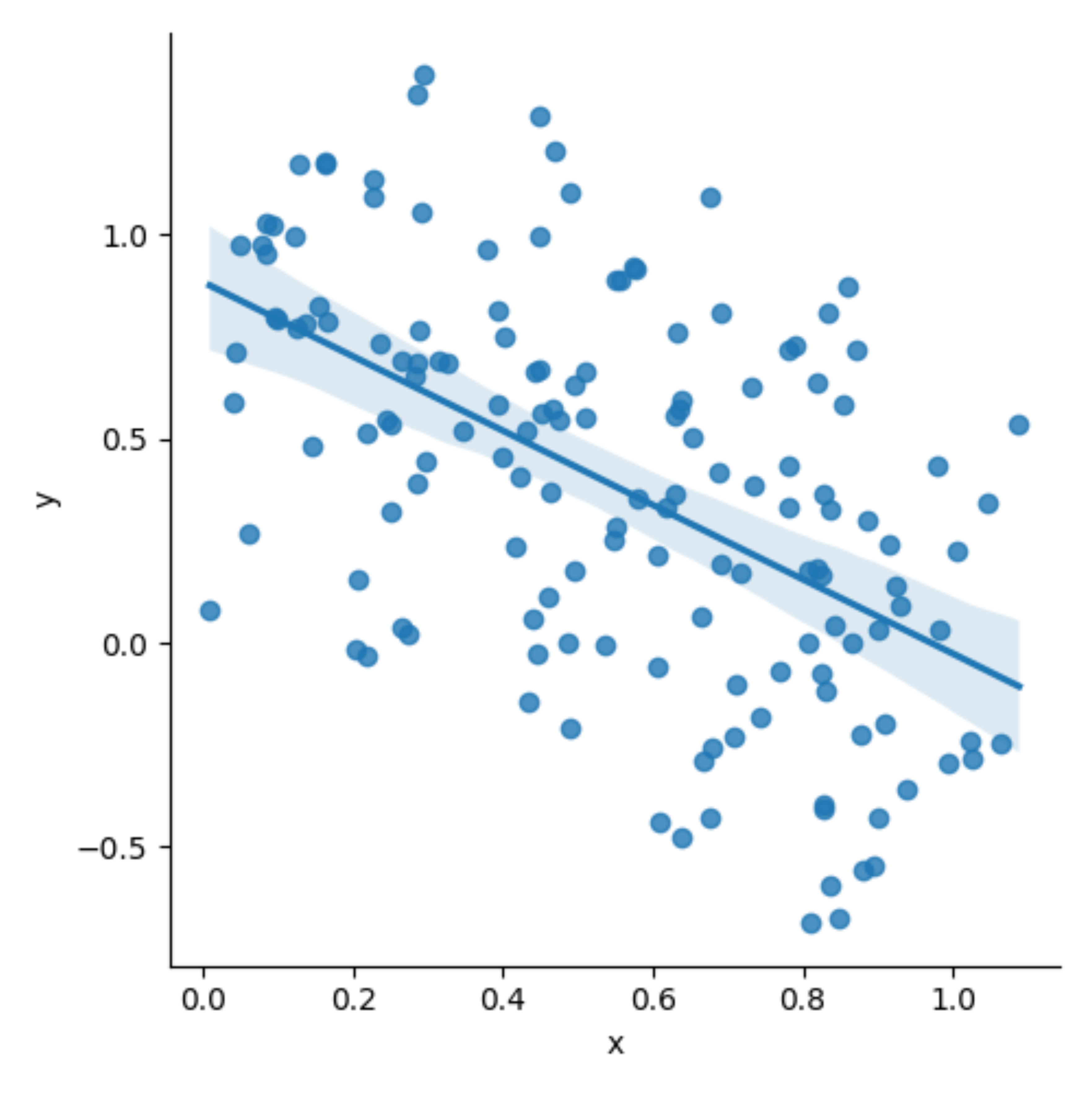
The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- What is your conclusion about the data?



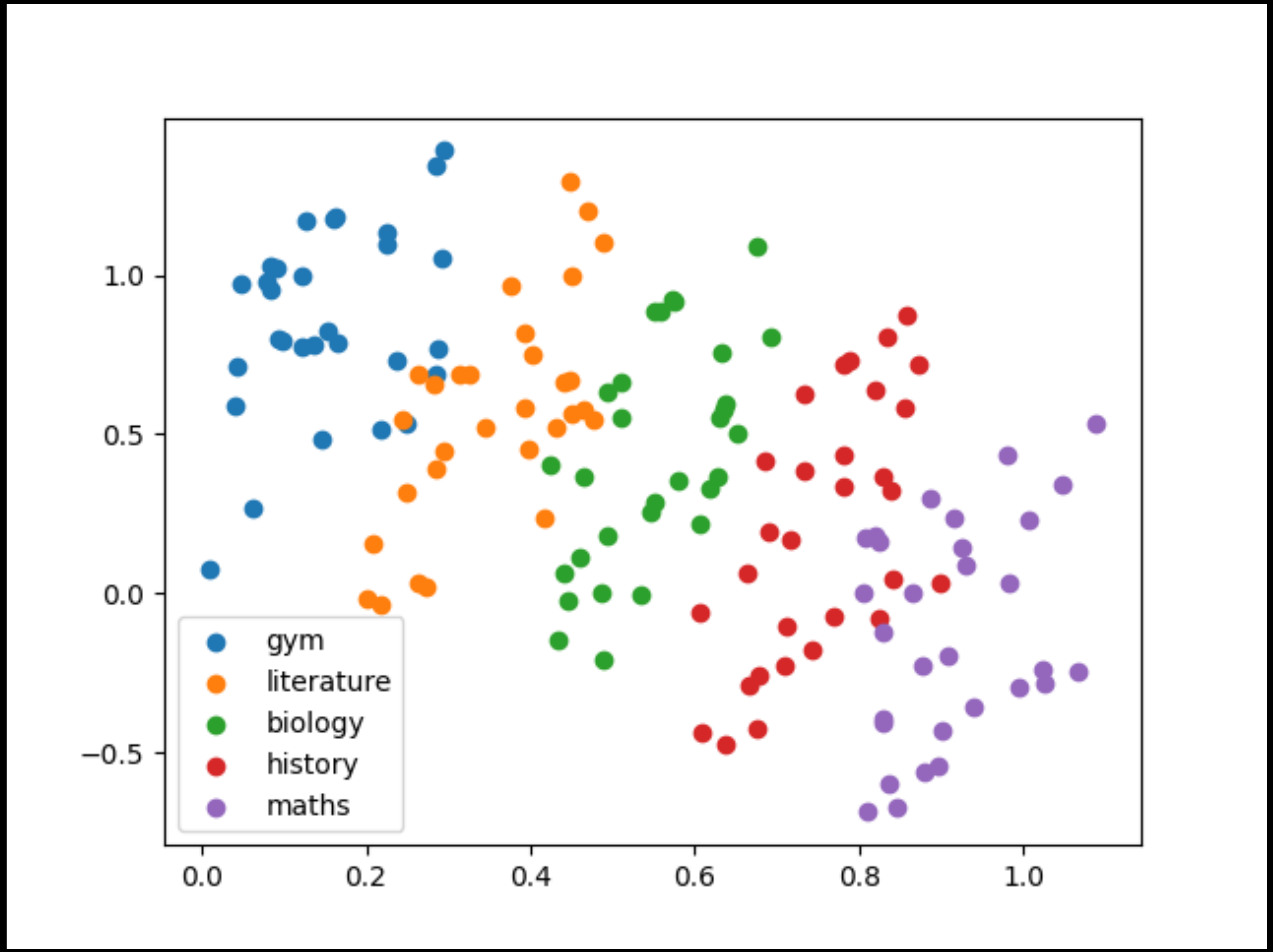
The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- Does your conclusion change if I tell you that the x axis is the amount of hours invested in study, and the y axis is the average grade of a student? Why?



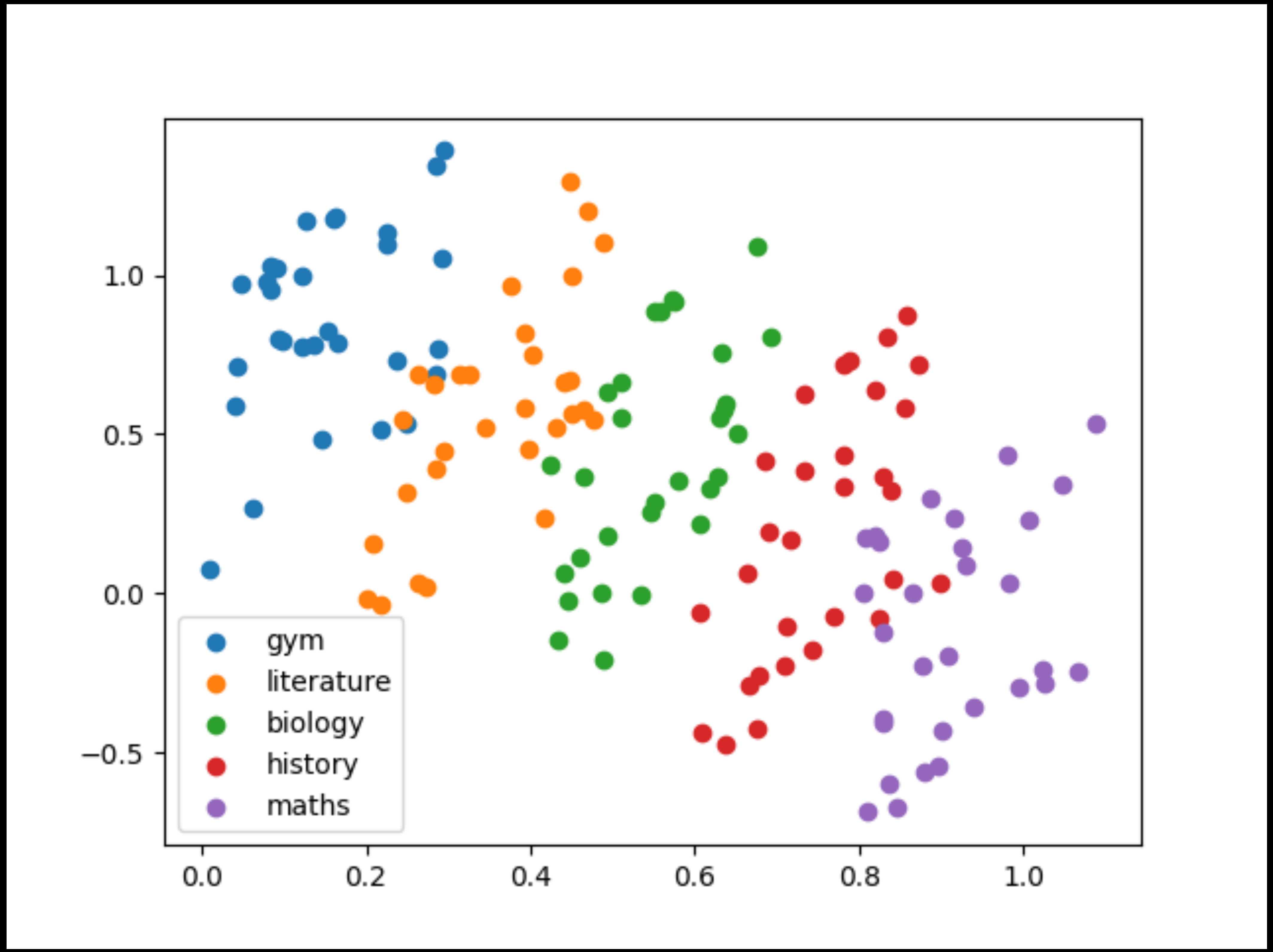
The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- the x axis is the amount of hours invested in study, and the y axis is the average grade of a student
- Does changing the colors change your conclusion?
- If so, should you have changed your inital conclusion, even without this extra information?



The Simpsons paradox

- Simpson's paradox is a phenomenon in which a trend appears in several groups of data but **disappears** or **reverses** with different groups.
- This result is often encountered in social-science and medical-science statistics
- It is particularly problematic when frequency data are undeservedly given ***causal interpretations***



The Simpsons paradox

UC Berkely Gender Bias (admission fall 1973)

	All		Men		Women	
	Applicants	Admitted	Applicants	Admitted	Applicants	Admitted
Total	12,763	41%	8,442	44%	4,321	35%

The Simpsons paradox

UC Berkely Gender Bias (admission fall 1973)

Department	All		Men		Women	
	Applicants	Admitted	Applicants	Admitted	Applicants	Admitted
A	933	64%	**825**	62%	108	82%
B	585	63%	**560**	63%	25	68%
C	918	35%	325	37%	**593**	34%
D	792	34%	417	33%	375	35%
E	584	25%	191	28%	**393**	24%
F	714	6%	373	6%	341	7%
Total	**4526**	**39%**	**2691**	**45%**	**1835**	**30%**

Outliers $\neq$ Errors

What are outliers?

Outliers may represent:

- A signal you picked the wrong parametric distribution for your data
- Genuine unusual observations
- A symptom of your samplebias
- Measurement errors
- Data entry errors

What are outliers?
Assumptions

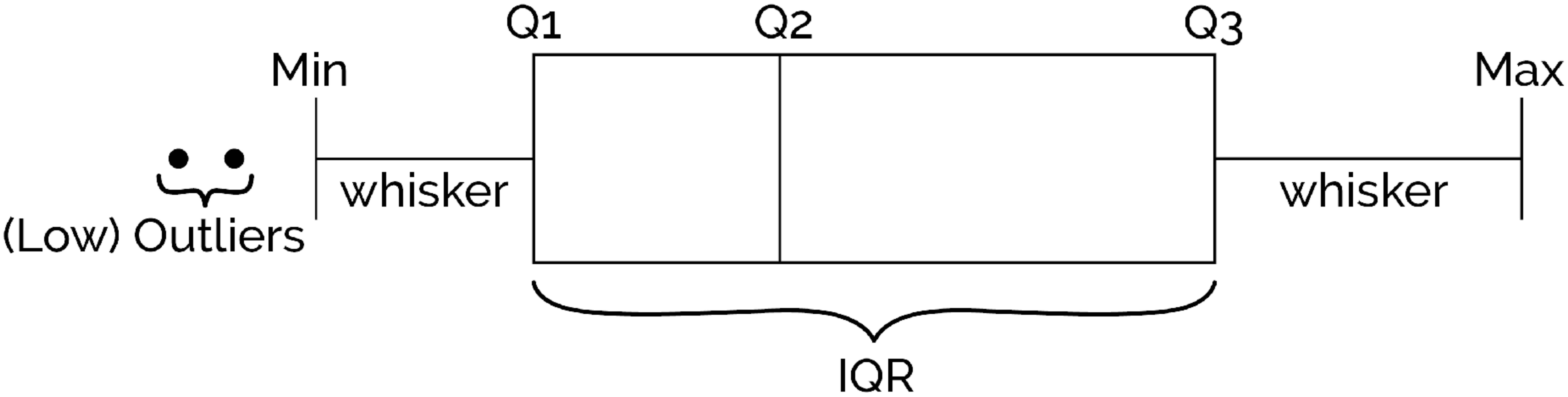
- Assume your data follows a normal distribution, are at least is approximately symmetric and not skewed.
- Assume the data comes from a single population. Multimodal distributions arent handled well (what appears an outlier might be a different subpopulation).
- Assume your data is continous numerical data. Categorical, count or discretised data might not work well.
- Assume observations are indepent of each other. It doesnt account for data with temporal, spatial, or hierarchical dependencies.

What are outliers?

Tukey's method

- Calculate the Interquartile range (IQR): $IQR = Q_3 - Q_1$
- Outliers are defined as
  - Values $< Q_1 - 1.5 \times IQR$
  - Values $> Q_3 + 1.5 \times IQR$

Boxplot



IQR = Q3 - Q1

Low Outliers = Q1 - (1.5 X IQR)

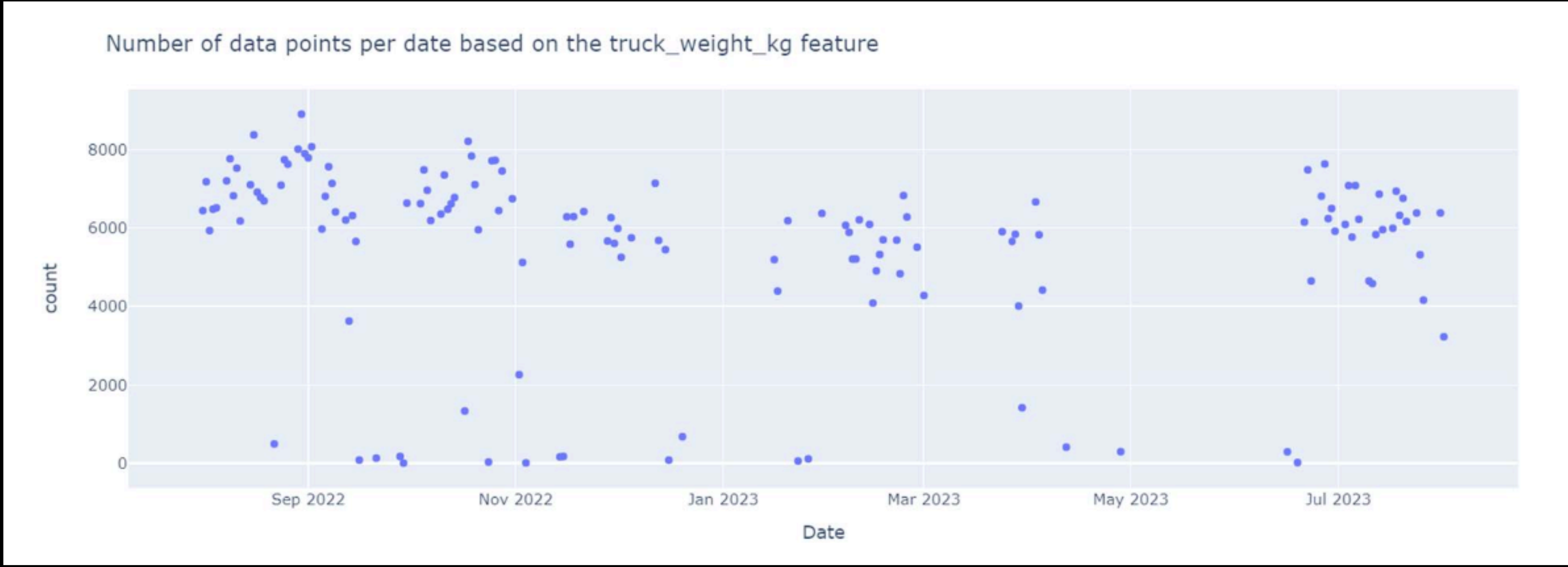
High Outliers = Q3 + (1.5 X IQR)

What are outliers?

Z-scores

- $z = (x - \mu)/\sigma$, where x is the datapoint, μ the average, and σ the standard-deviation
- Flag $|z| > 3$ as outlier
- Sensitive to extreme values; outliers affect μ and σ (bill gates walks into a bar; everyone is millionaire, on average).

Garbage truck analysis



Garbage truck analysis

Student analysis

“The graph suggests inconsistent data collection, which to certain extent is understandable given the fact that garbage collection can be different per date. The inconsistency suggests periods where the data was either not collected or not recorded properly, indicating potenial issues in the data collection process.”

“If we were to draw a line to only take into account dates that have 5500 data points or more in it, assuming this is the minimum amount of data points to consider any date as having good data density, then only 70% of the dates would be considered for any further analysis. In other words, about 30% of the data is unreliable.”

“we already know that 30% of it is not reliable”

Garbage truck analysis

Garbage truck analysis

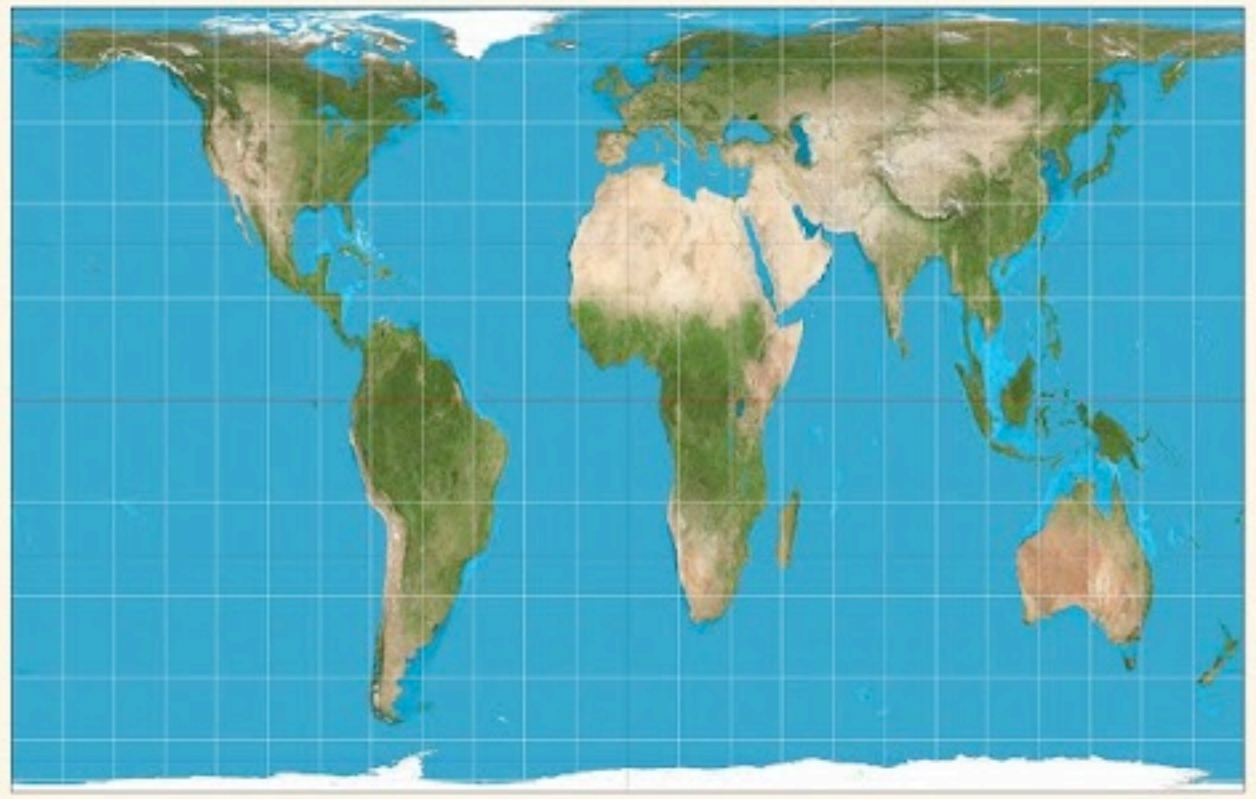
“Further analysis of the data shows that the truck never gets filled to its maximum capacity. The filtering removes outliers above 28,000 Kg.”

The map is not the territory

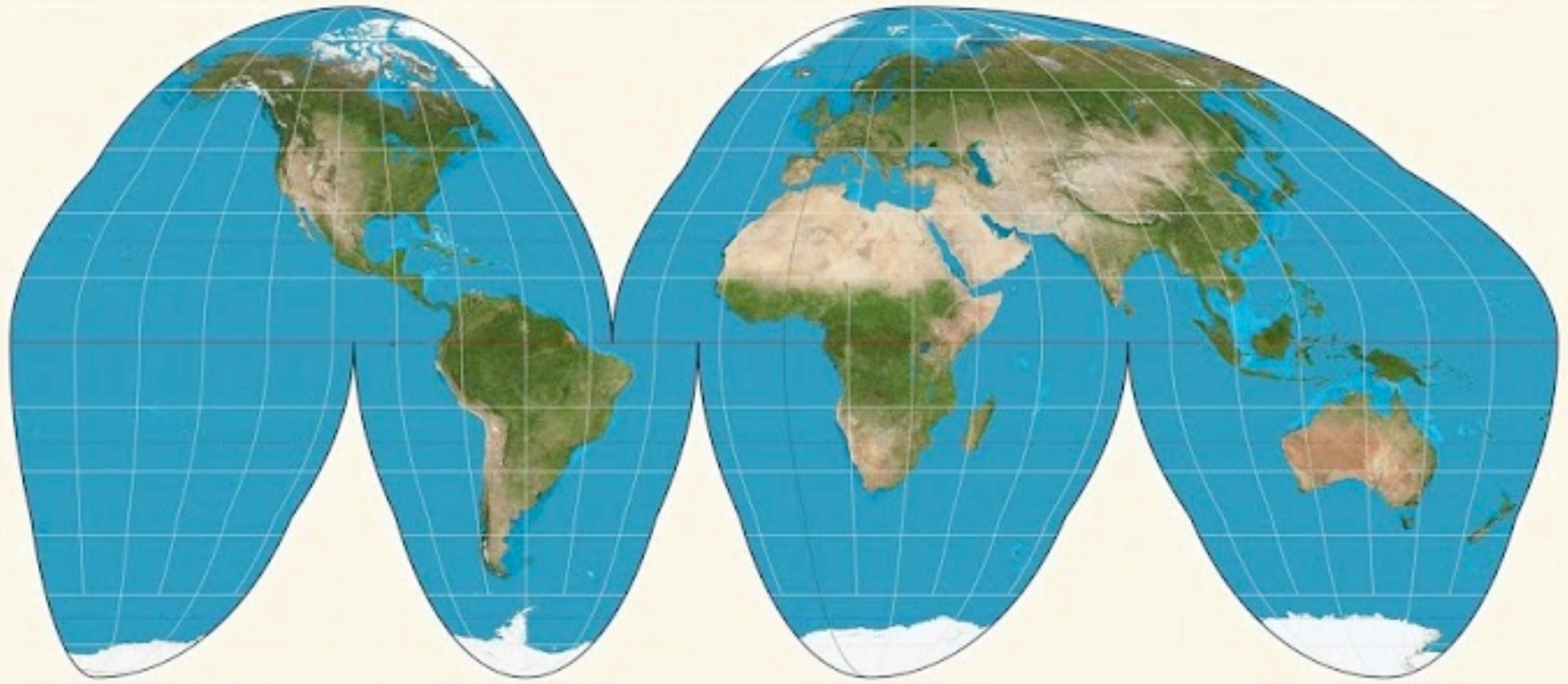
MAP PROJECTIONS

MERCATOR

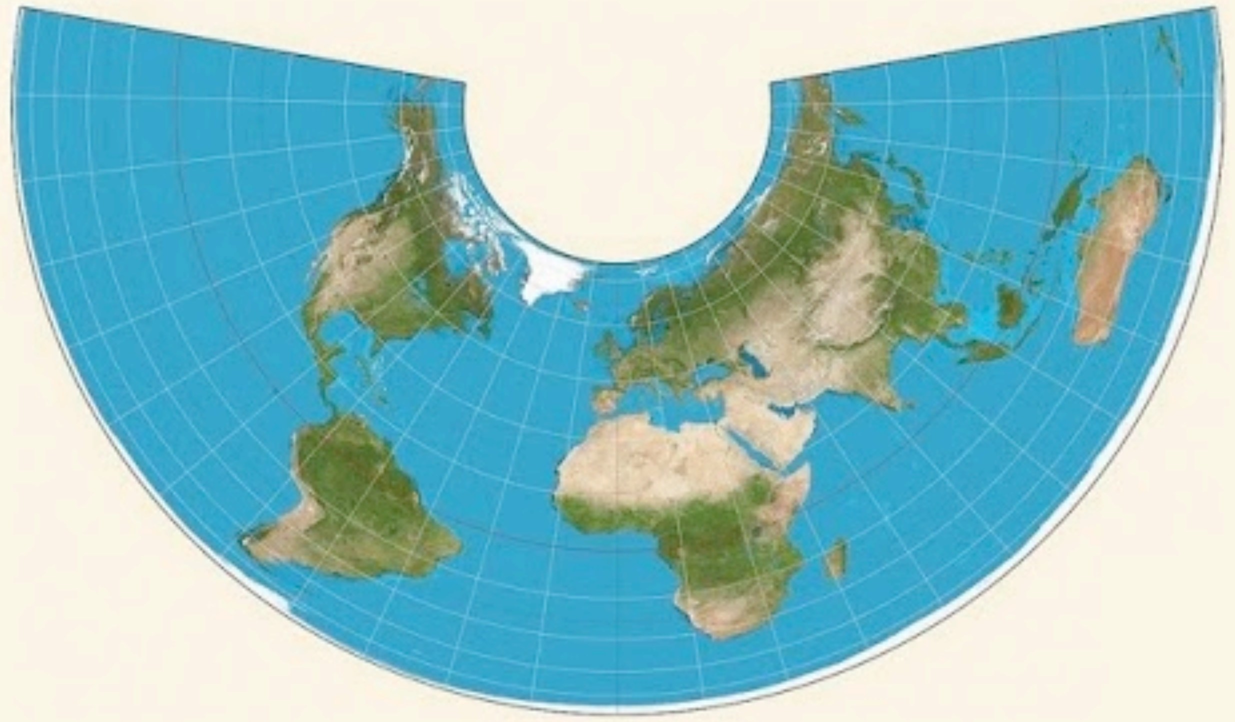
GALL-PETERS



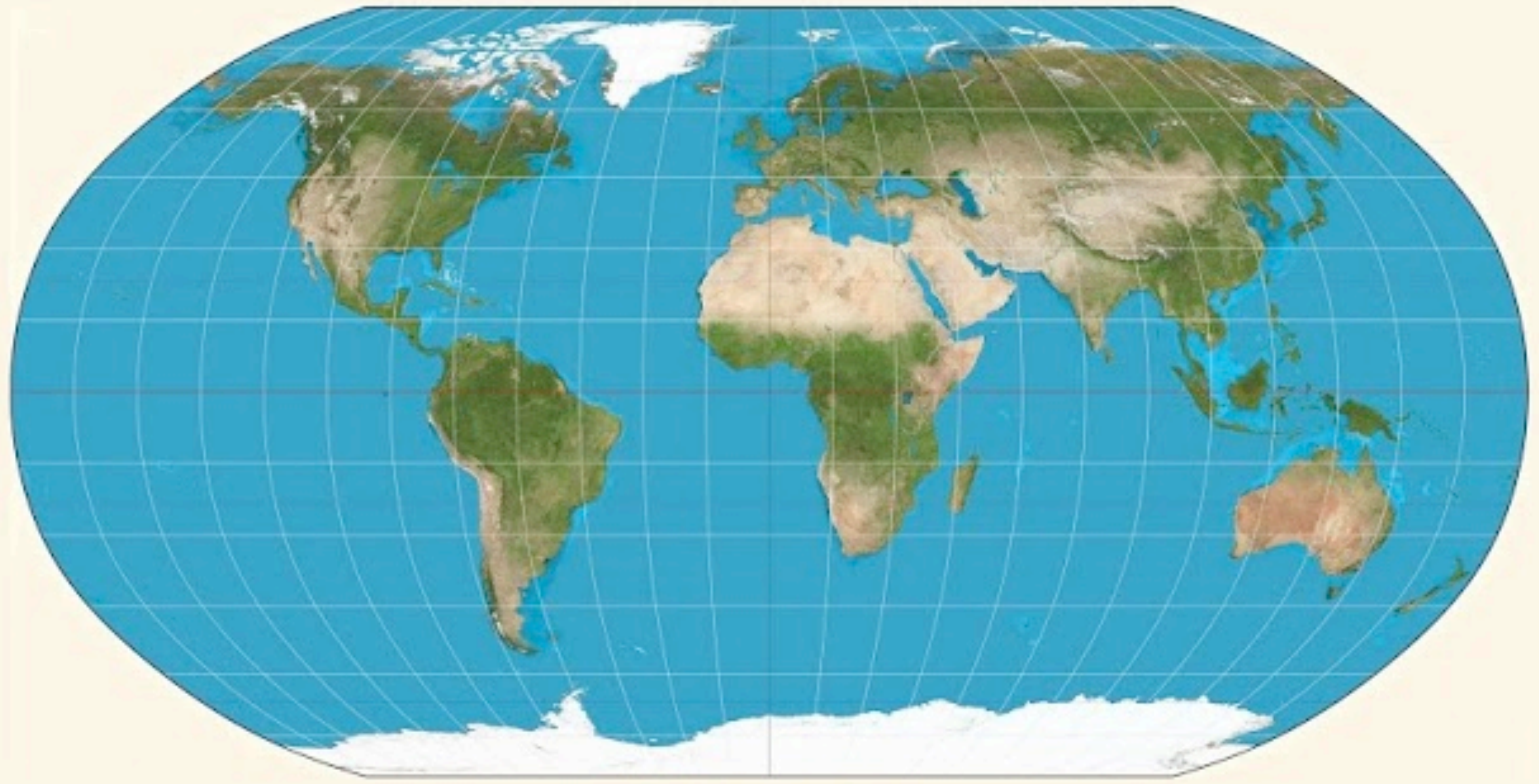
GOODE-HOMOLOSINE



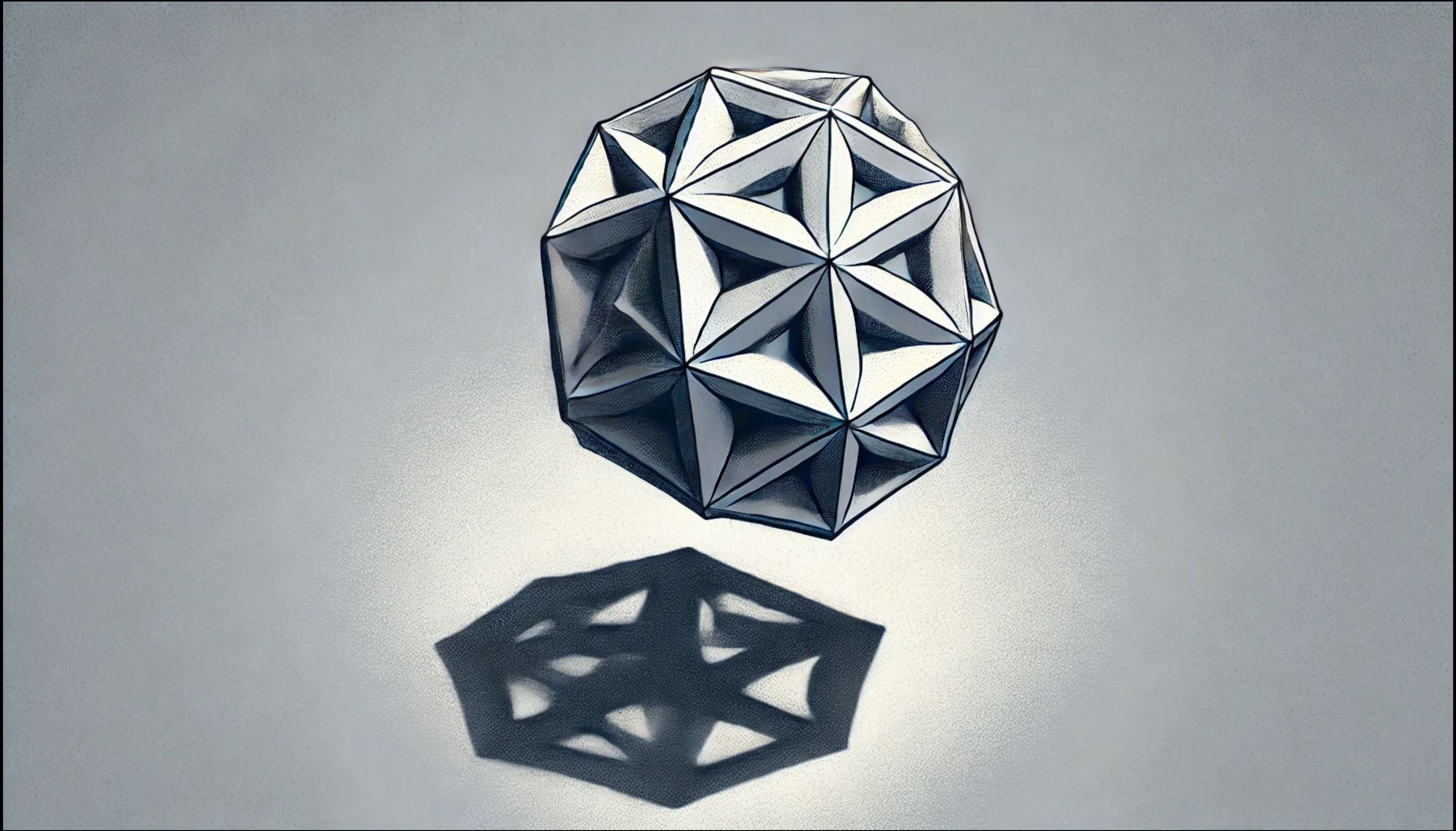
WATERMELON



ALBERS



ROBINSON



Determining the Right Distribution

Graphical methods:

- Histograms
- Q-Q plots

Statistical tests:

- Minimize negative log-likelihood function (scipy.stats.fit)
- Kolmogorov-Smirnov test (scipy.stats.kstest)