# Distributies

Raoul Grouls, 12 maart 2025

# Distributions

## Definition

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- A mathematical description

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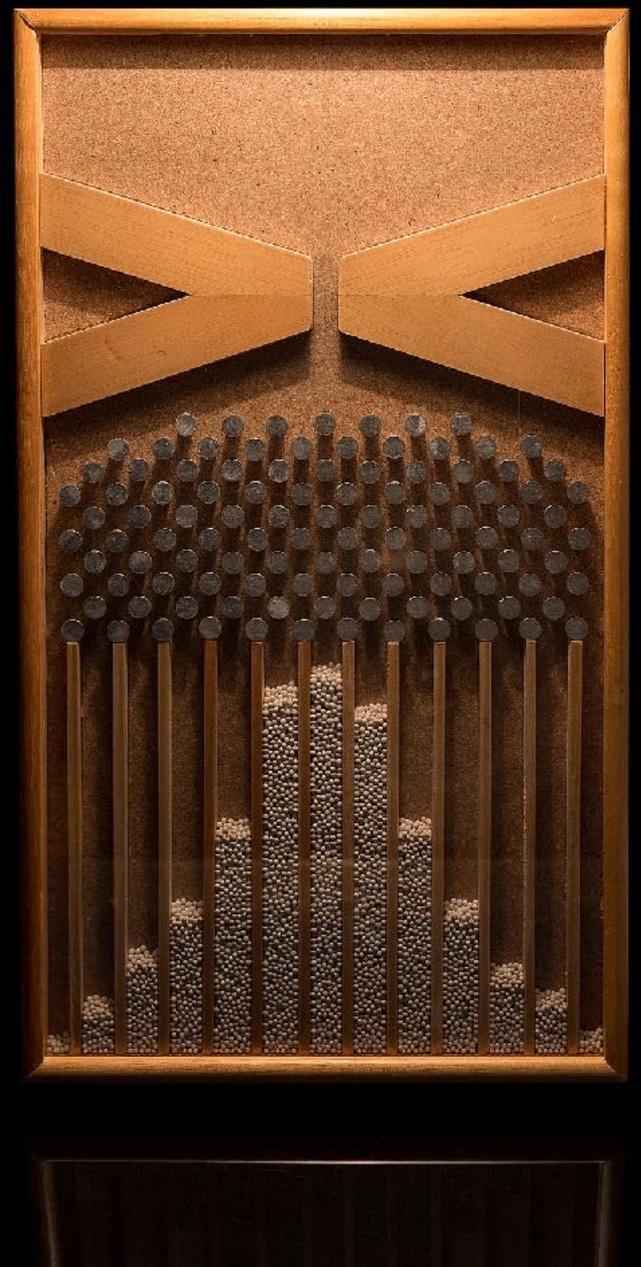
- A mathematical description
- Of a random phenomenon

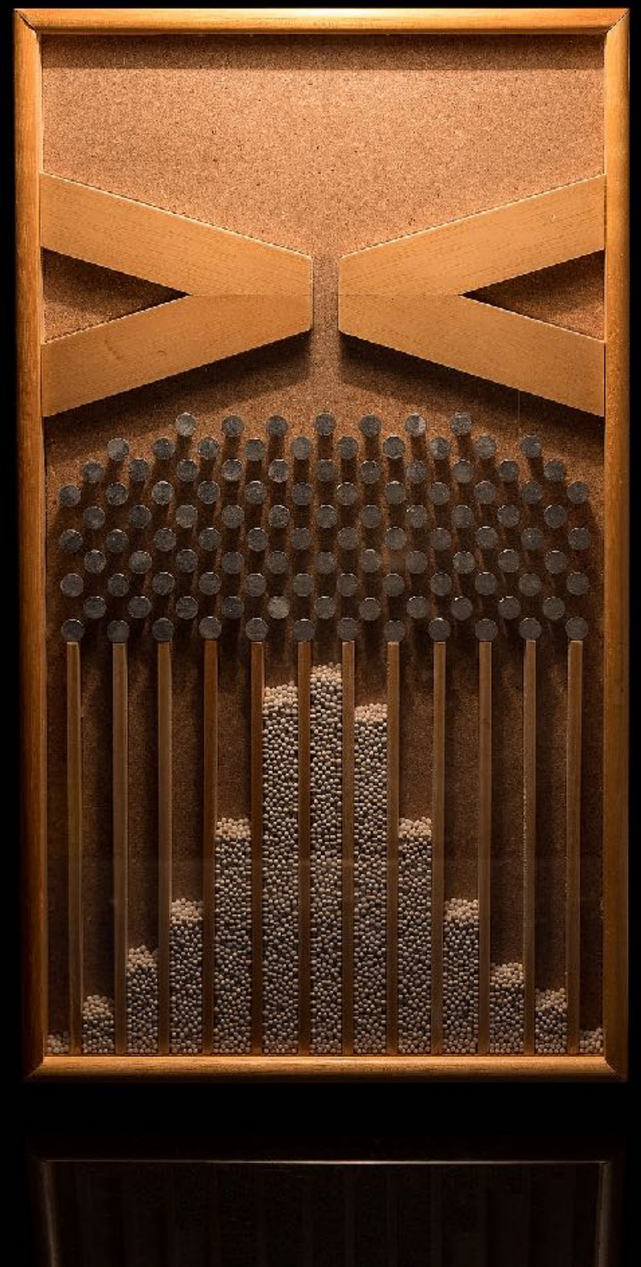


# Distributions

## Definition

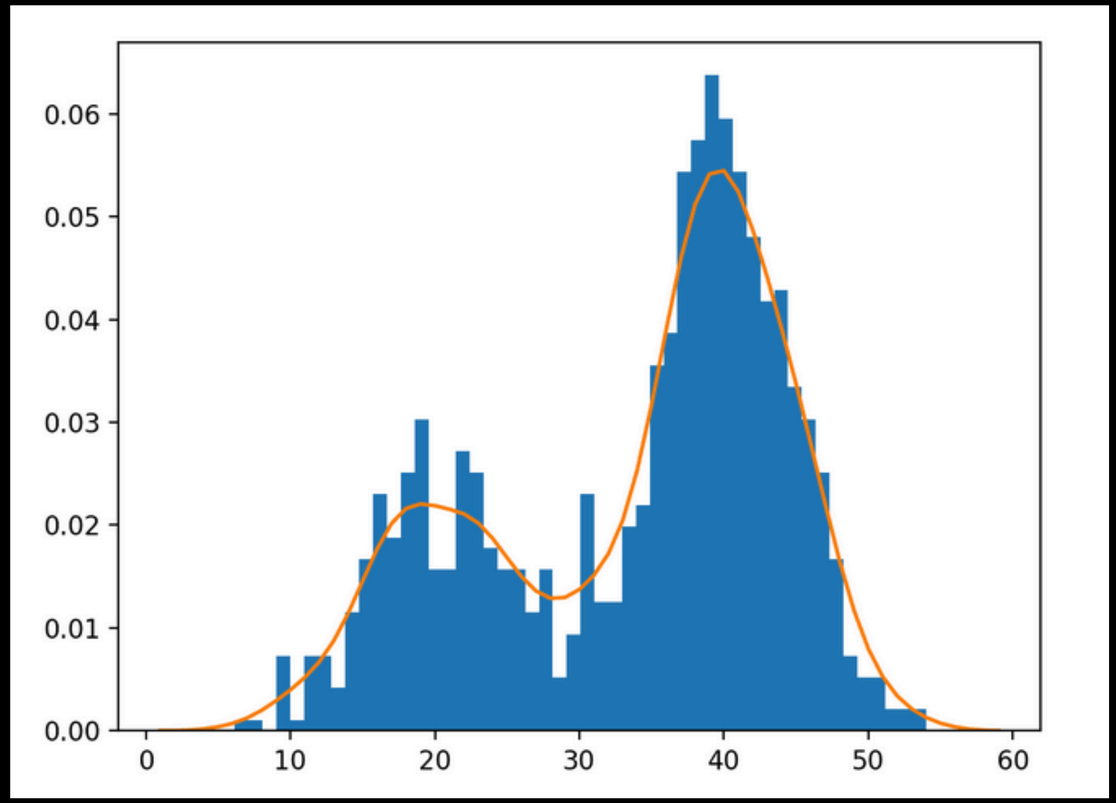
A probability distribution is:

- A mathematical description
- Of a random phenomenon
- In terms of all its possible outcomes and their associated probabilities



# Distributions

- In this image, what are:
  - All the possible outcomes?
  - The associated probabilities?



# Distributions

## Types

The main types of distributions are:

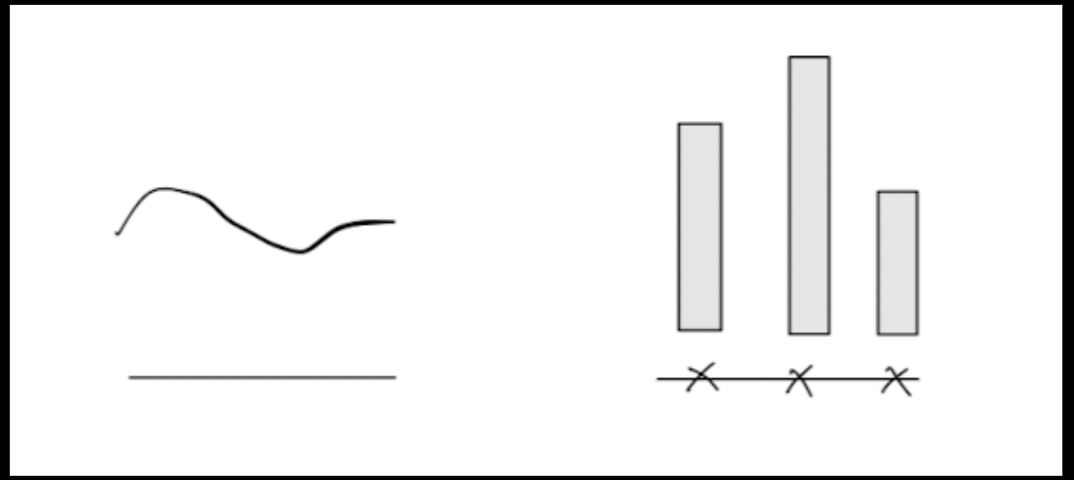
- **Discrete** : when an outcome can only take discrete values (e.g. number of birds)
- **Continuous** : when outcomes take continuous values (e.g. blood pressure)

# Distributions

## Basic visualisation type

**Every** horizontal line you draw can be interpreted as a continuous distribution.
**Every** barplot as a discrete distribution.

All the distributions we are going to discuss are variations of these two basic types!



# Distributions

## Basic visualisation type

For *parametric distributions*, we have a formula that describes the line / bars. You just put in the parameters, and the output is the line / bars.

# Discrete distributions

## PMF

A **probability mass function** (pmf) describes the probability distribution of discrete variables.

Consider a toin coss:

$$f(x) = \begin{cases} 0.5 & x \text{ is head} \\ 0.5 & x \text{ is tails} \end{cases}$$

This is the pmf of the *Bernoulli distribution*

# Conditions for a PMF

## Plain English

1. An event cannot have a negative probability

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1. An event cannot have a negative probability
2. The sum of probabilities of all events must be 1
3. The probability of a subset $X$ of outcomes $T$ is the same as adding the probabilities of the individual elements.

# Conditions for a PMF

## Mathematical

The probability is a function $f$ over the sample space $\mathcal{S}$ of a discrete random variable $X$, which gives the probability that $X$ is equal to a certain value.

$$f(x) = P(X = x)$$

Each pmf satisfies these conditions:

1. $f(x) \geq 0, \forall x \in X$
2. $\Sigma_{x \in \mathcal{S}} f(x) = 1$
3. For a collection $\mathcal{A}$, $P(\mathcal{A} \in \mathcal{S}) = \Sigma_{x_i \in \mathcal{A}} f(x_i)$

# Continuous Distributions

## PDF

For continuous distributions, we use a probability density function (pdf)

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## PDF

For continuous distributions, we use a probability density function (pdf)

1. $f(x) > 0, \forall x \in X$
2. The integral of the probabilities of all possible events must be 1 (area under the curve)
3. The probability $X$ of values in the interval $[a, b]$ is the integral from $a$ to $b$

# Continuous Distributions

## PDF

This might look like unnecessary mathematical details. But it is actually important to understand the difference.

Example: can you answer the question “What is the probability your body temperature is 37.0 C?”

# Continuous Distributions

## PDF

The answer might be unexpected: 0!

Let’s say your answer is 25%. But what if your temperature is 37.1? does that count? Or 37.01?

# Continuous Distributions
## PDF

- Because the distribution is *continuous* you can only say something about the *range*
- “What is the probability your temperature is between 36.5 and 37.2 C?”

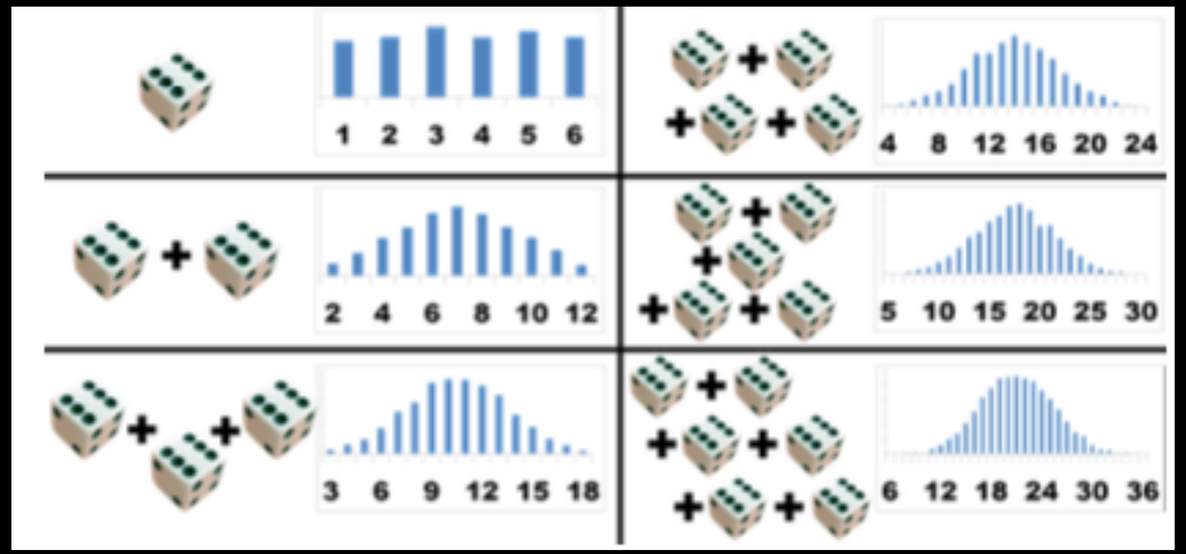
# Quiz time

# Normal distributions

## Central limit theorem

The central limit theorem states that:

- the distribution of a normalized version of the sample mean converges to a standard normal distribution.
- This holds even if the original variables themselves are not normally distributed.

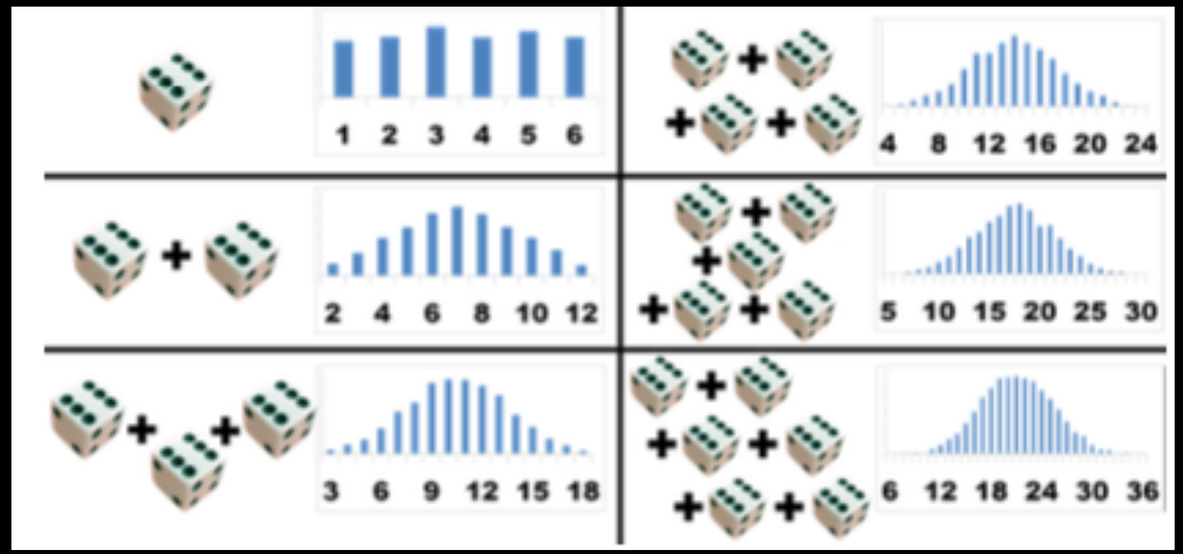


# Normal distributions

## Central limit theorem

The Normal distribution is one of the distributions that is used most often.

A major reason for this is, that if you keep sampling and **adding** from a population you *always* end up with a normal distribution.



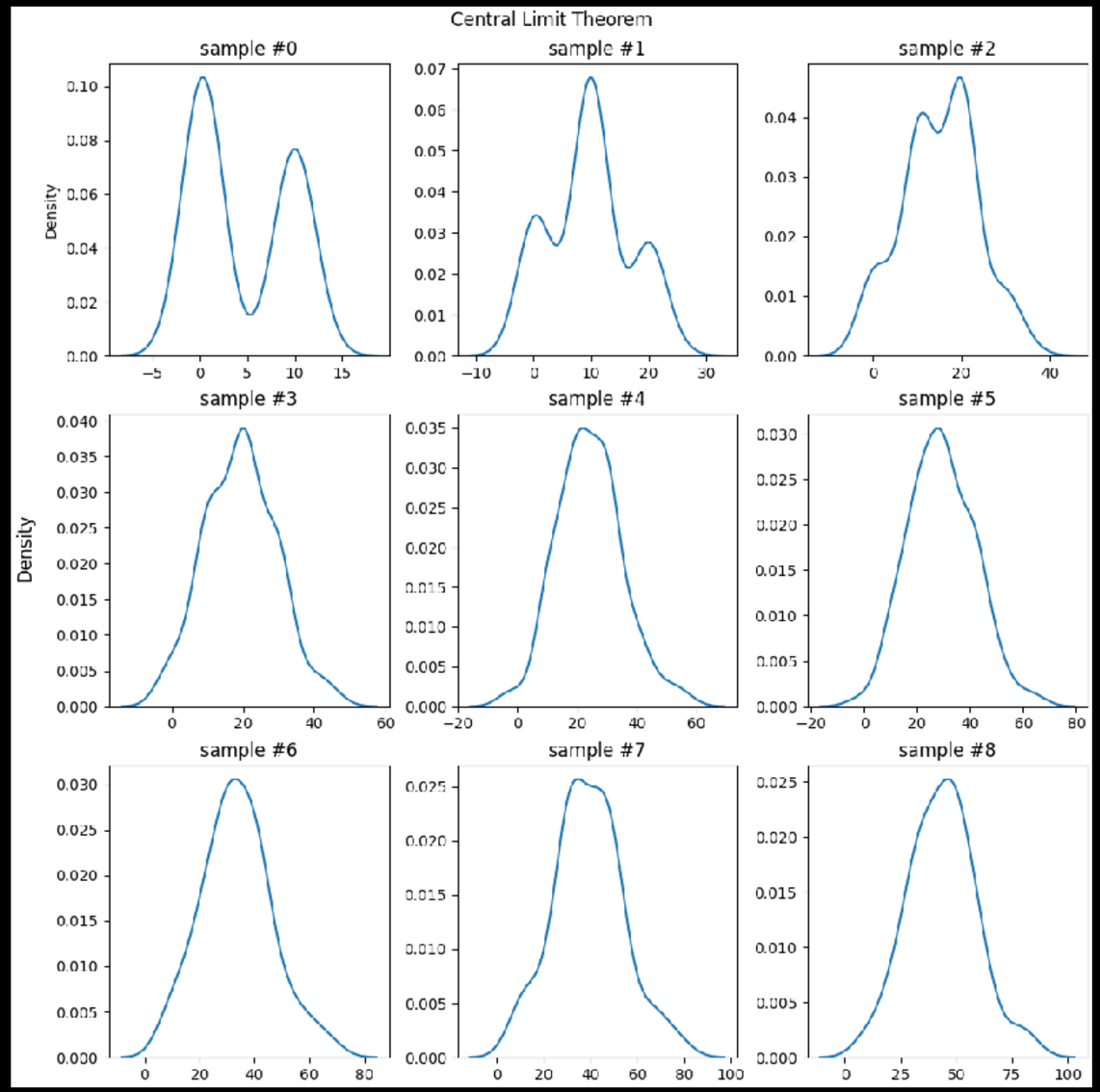
# Normal distributions

## Central limit theorem

Take a persons height.

- This is determined by a combination of 180 genes.
- One gene will contribute to a longer neck, the other to longer legs
- If we assume the genes contribute independently, height equals the sum of 180 genes.

Thus, height will be normally distributed. So will the weight of wolves or the length of a penguins wing.



# Log Normal distribution

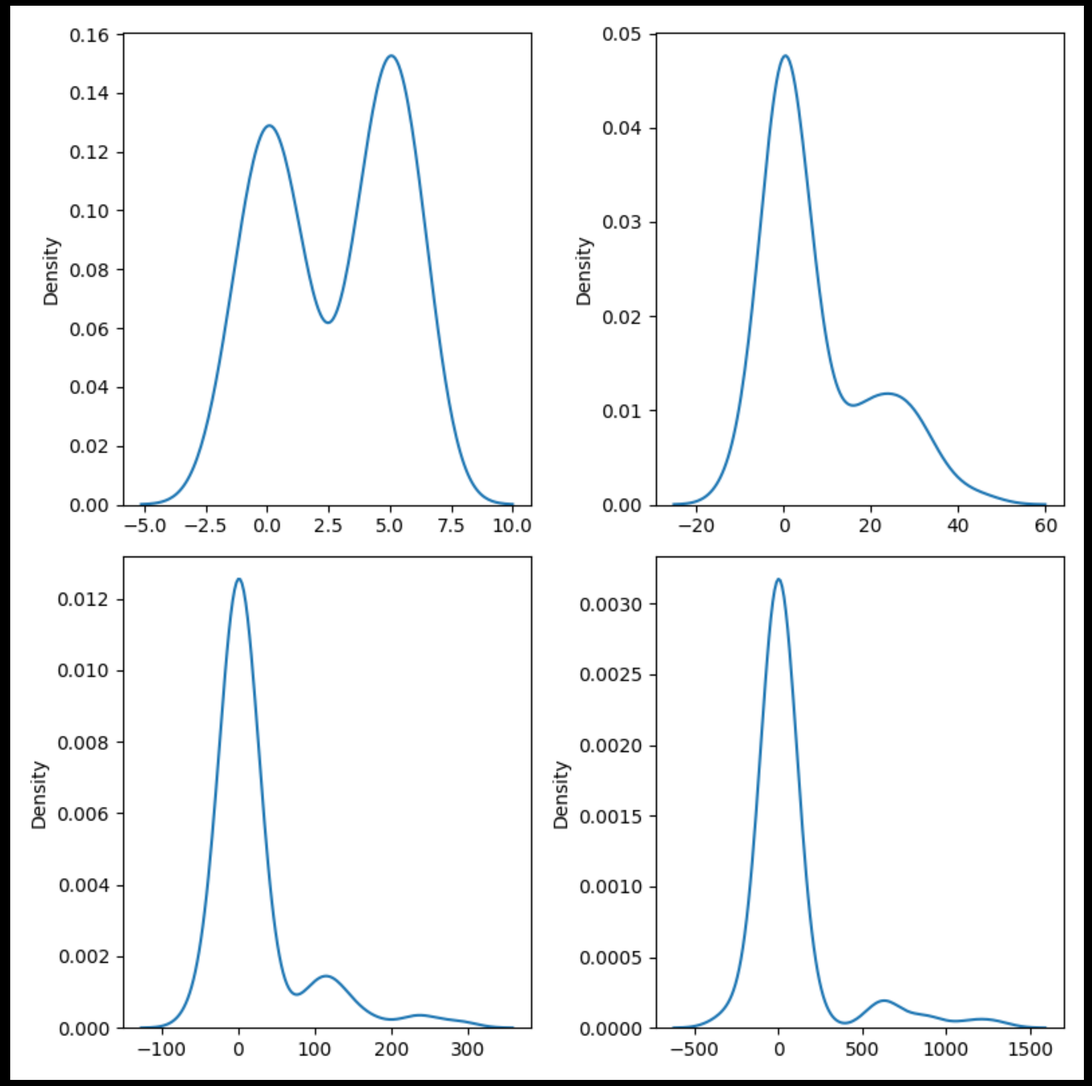
However, **multiplying** values will give you a long tail!

This is the case when variables interact in some way, and are not independent.

$$4 + 4 + 4 + 4 = 16$$

but

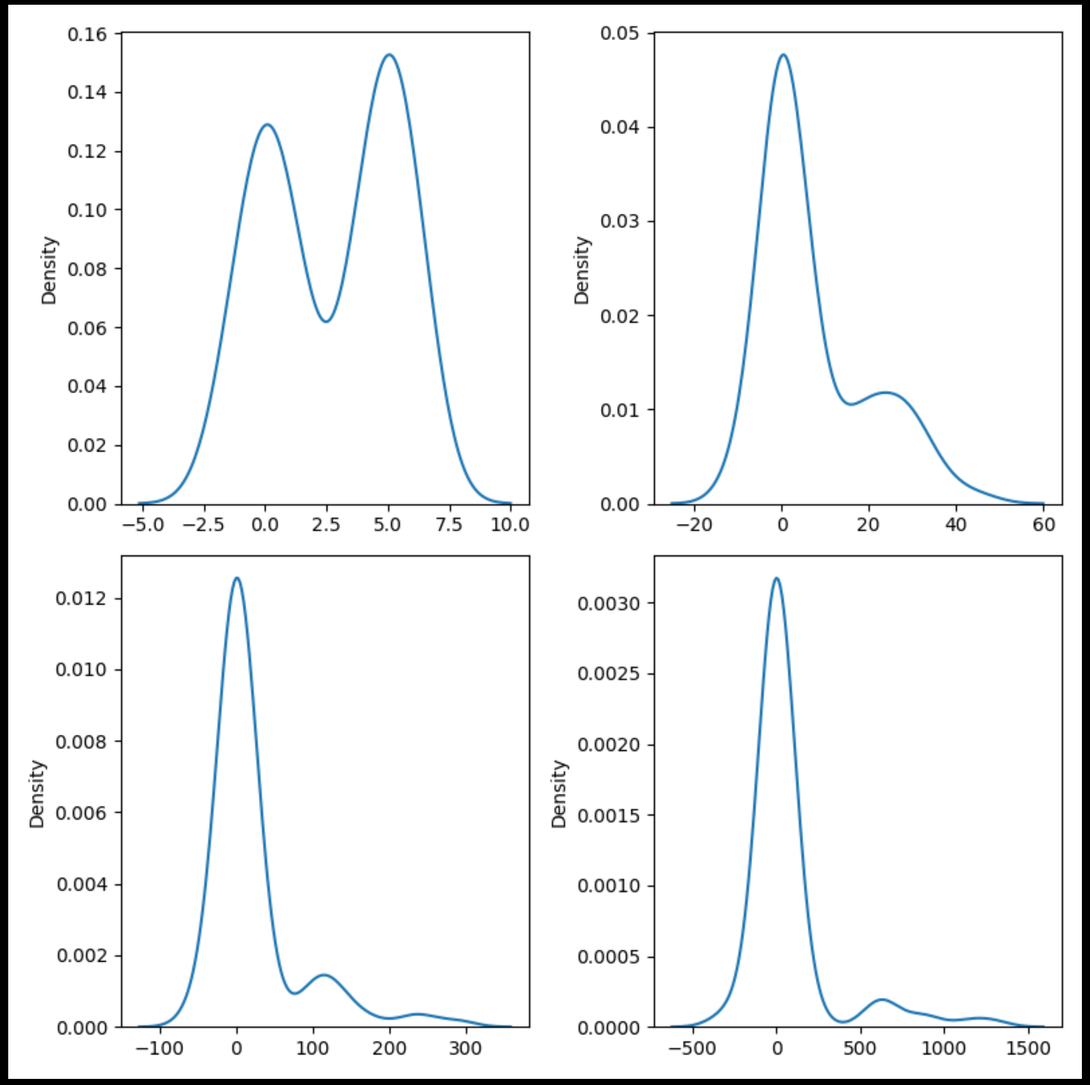
$$4 \times 4 \times 4 \times 4 = 256$$



# Log Normal distribution

Multiplying should be expected if variables interact with each other.

Examples are stock prices, failures of machines, ping times on a network, income distribution.



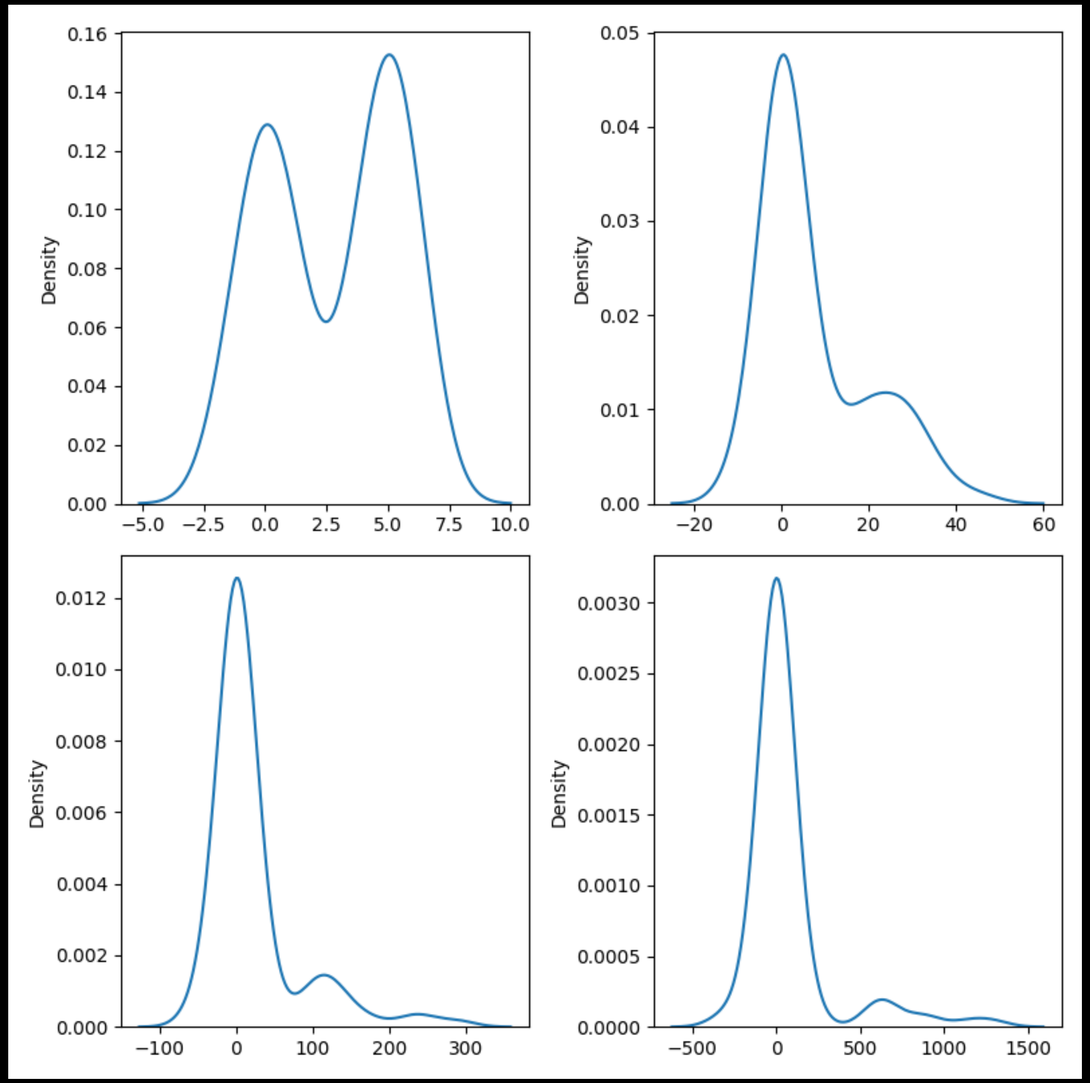
# Log Normal distribution

**multiplying** values will give you a fat-tail distribution! This will typically be a log-normal distribution:

If $X$ is log-normal distibuted, then

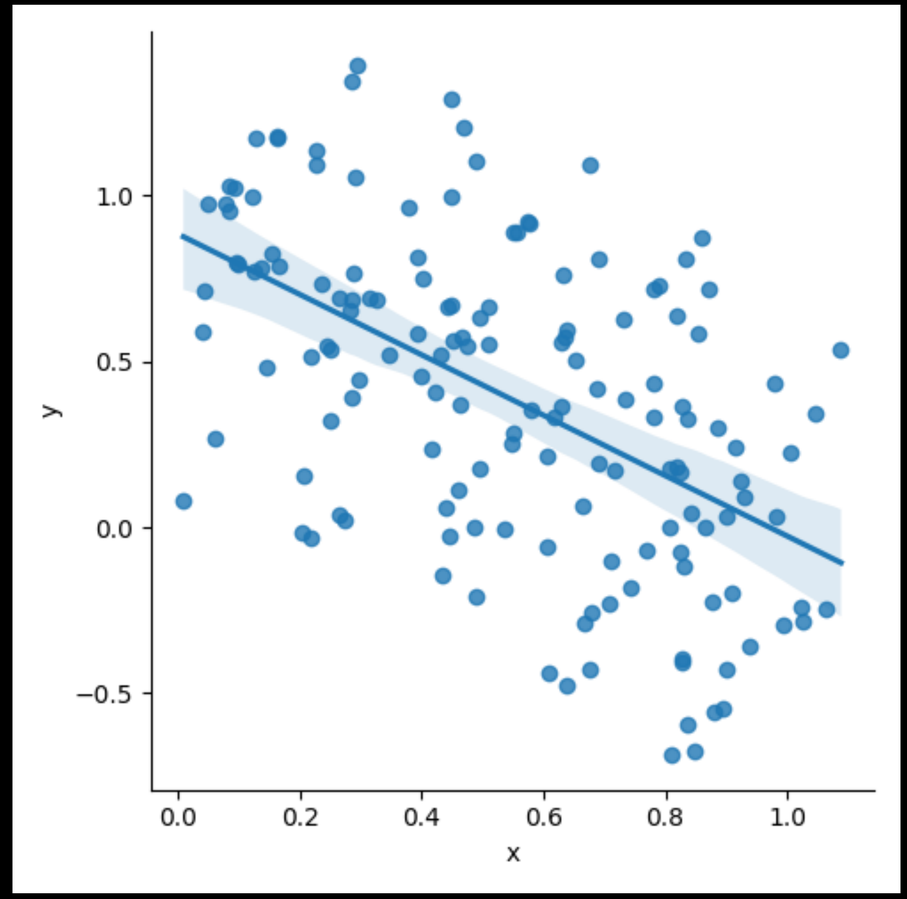
$$y = log(X)$$

will be a normal distribution.



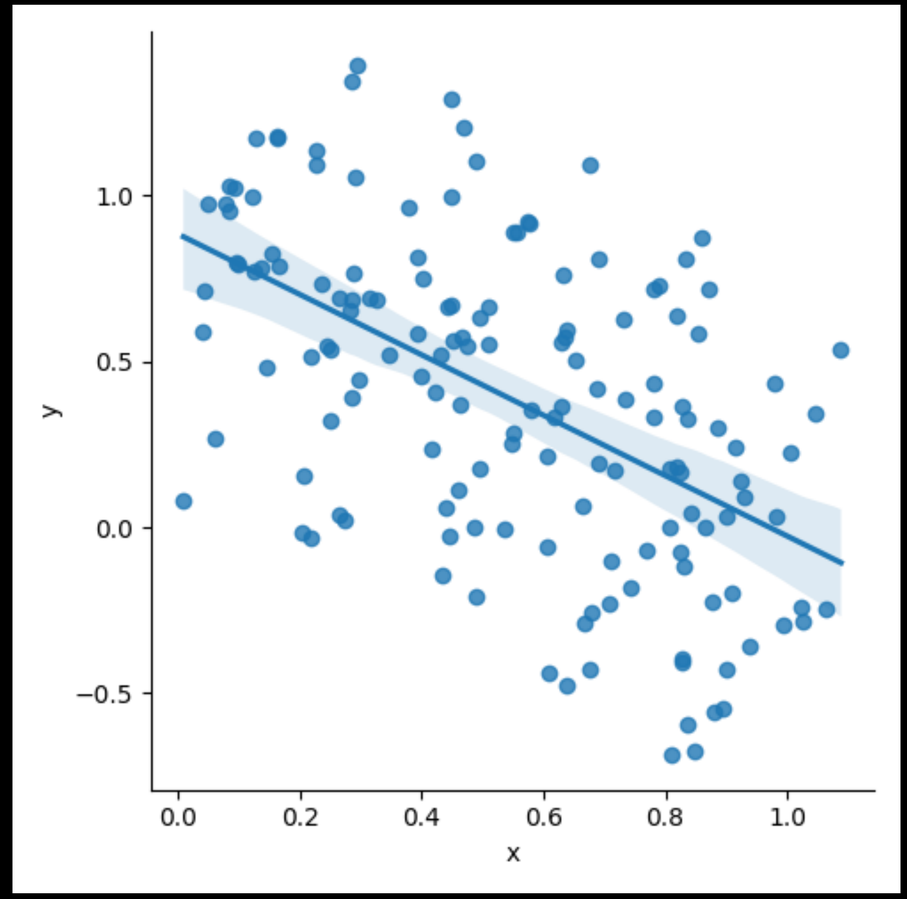
# The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- What is your conclusion about the data?



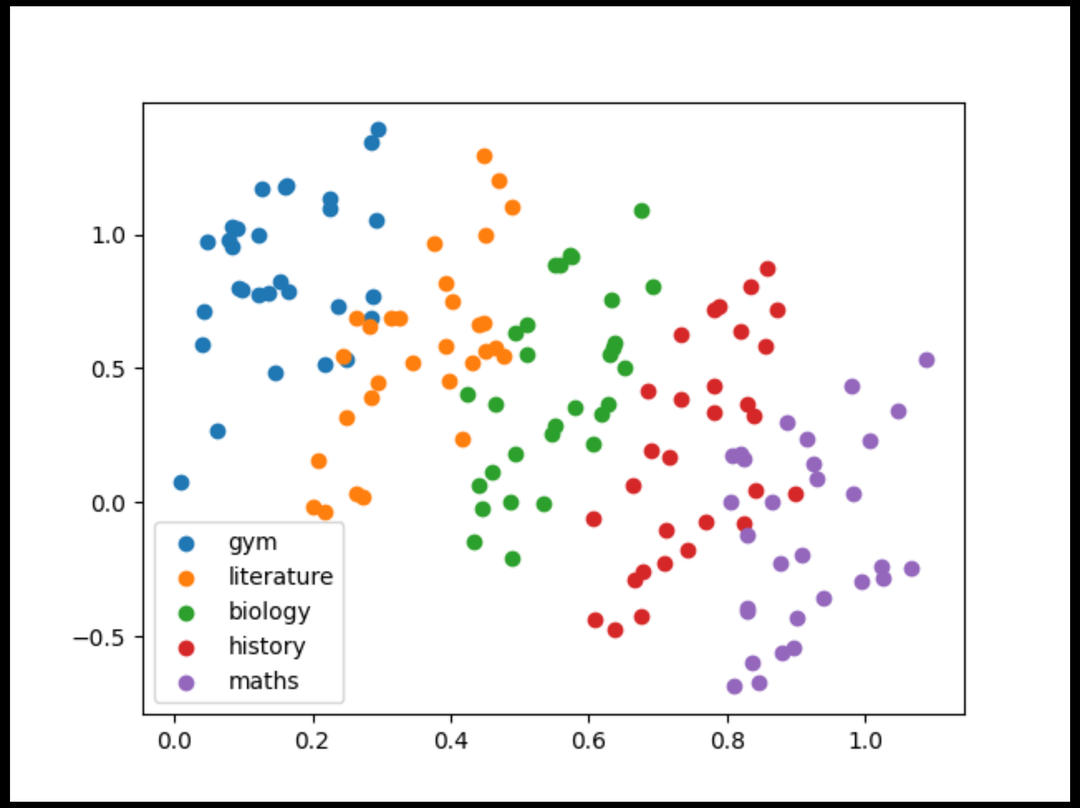
# The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- Does your conclusion change if I tell you that the $x$ axis is the amount of hours invested in study, and the $y$ axis is the average grade of a student? Why?



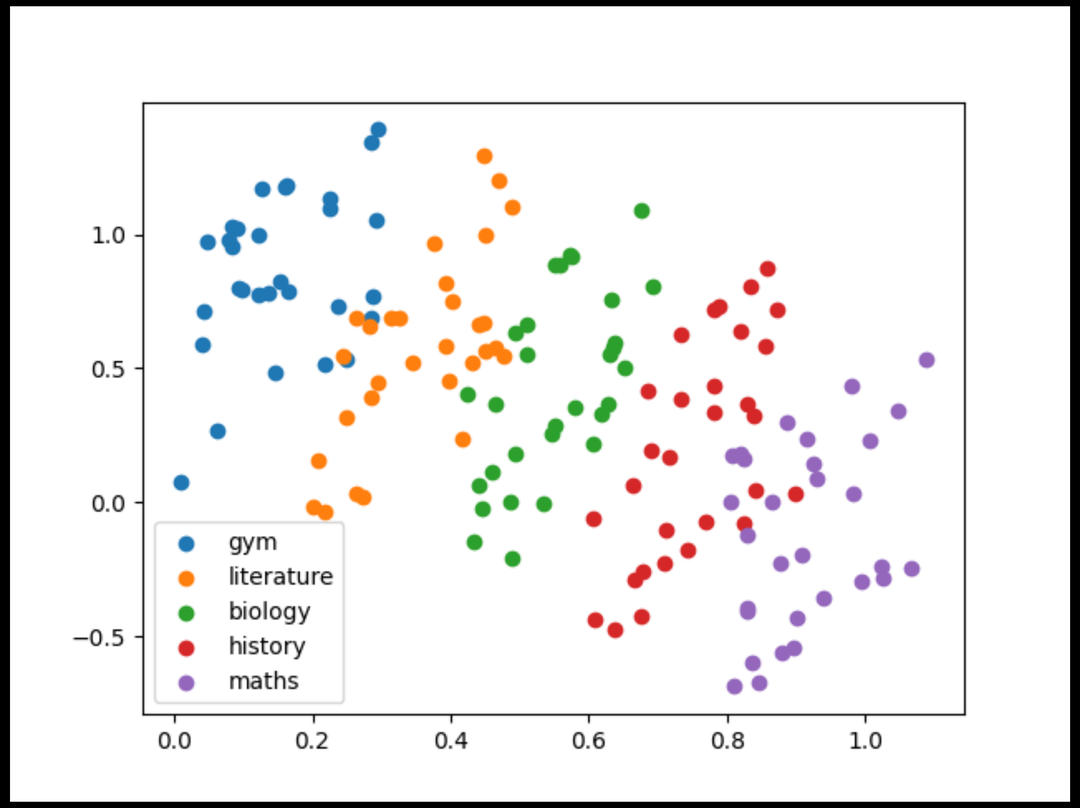
# The Simpsons paradox

- The shaded area is the 99% confidence interval of the linear regression.
- the $x$ axis is the amount of hours invested in study, and the $y$ axis is the average grade of a student
- Does changing the colors change your conclusion?
- If so, should you have changed your inital conclusion, even without this extra information?



# The Simpsons paradox

- Simpson's paradox is a phenomenon in which a trend appears in several groups of data but **disappears** or **reverses** with different groups.
- This result is often encountered in social-science and medical-science statistics
- It is particularly problematic when frequency data are undeservedly given **causal interpretations**



# The Simpsons paradox

## UC Berkely Gender Bias (admission fall 1973)

| | All | | Men | | Women | |
|---|---|---|---|---|---|---|
| | Applicants | Admitted | Applicants | Admitted | Applicants | Admitted |
| **Total** | 12,763 | 41% | 8,442 | 44% | 4,321 | 35% |

# The Simpsons paradox

## UC Berkely Gender Bias (admission fall 1973)

| Department | All | | Men | | Women | |
|---|---|---|---|---|---|---|
| | Applicants | Admitted | Applicants | Admitted | Applicants | Admitted |
| A | 933 | 64% | **825** | 62% | 108 | 82% |
| B | 585 | 63% | **560** | 63% | 25 | 68% |
| C | 918 | 35% | 325 | 37% | **593** | 34% |
| D | 792 | 34% | 417 | 33% | 375 | 35% |
| E | 584 | 25% | 191 | 28% | **393** | 24% |
| F | 714 | 6% | 373 | 6% | 341 | 7% |
| **Total** | **4526** | **39%** | **2691** | **45%** | **1835** | **30%** |