$$\Rightarrow \cos x + j\sin x = \frac{\cancel{T_s j\Omega} + T_s^2 + \Omega^2 - \cancel{j\Omega T_s}}{T_s^2 + \Omega^2}$$

$$\cos x + j\sin x = 1 = \frac{T_s^2+\Omega^2}{T_s^2+\Omega^2}$$ se verifică.

• $$\omega = 2\,\mathrm{arctg}\left(\frac{\Omega T_s}{2}\right) \Leftarrow \omega = \arccos\frac{1-\Omega^2T_s^2}{1+\Omega^2T_s^2}$$

$$\omega = \arctan\left(\frac{\sqrt{1-\frac{1-\Omega^4T_s^4-2\Omega^2T_s^2}{1+\Omega^4T_s^4+2\Omega^2T_s^2}}}{\frac{1-\Omega^2T_s^2}{1+\Omega^2T_s^2}}\right) \Rightarrow \omega = \cancel{\arctan\left(\frac{\frac{\sqrt{2}\,\Omega^2T_s^2}{\sqrt{1+\Omega^2T_s^2}}}{}\right)}$$

$$\omega = \arctan\left(\frac{\frac{2\Omega^3T_s^3+4\Omega^2T_s^2}{1+\Omega^2T_s^2}}{\frac{1-\Omega^2T_s^2}{1+\Omega^2T_s^2}}\right) = \arctan\left(\frac{\Omega^2T_s^2(2+\Omega^2T_s^2)}{1-\Omega^2T_s^2}\right)$$

$$= \arctan\left(\frac{2\Omega^3T_s^3+4\Omega^2T_s^2}{1-\Omega^2T_s^2}\right).$$

• $$\cos\omega = \frac{1-\Omega^2T_s^2}{1+\Omega^2T_s^2} \Rightarrow \frac{1}{\sqrt{1+\mathrm{tg}^2\omega}} = \frac{1-\Omega^2T_s^2}{1+\Omega^2T_s^2}$$

$$\Rightarrow 1+\mathrm{tg}^2\omega = \frac{(1+\Omega^2T_s^2)^2}{(1-\Omega^2T_s^2)^2} \Rightarrow \mathrm{tg}\,\omega = \sqrt{\frac{(1+\Omega^2T_s^2)^2-(1-\Omega^2T_s^2)^2}{(1-\Omega^2T_s^2)^2}}$$

$$\mathrm{tg}\,\omega = \sqrt{\frac{\cancel{1}+\cancel{\Omega^4T_s^4}+2\Omega^2T_s^2\cancel{-1}-\cancel{\Omega^4T_s^4}+2\Omega^2T_s^2}{1+\Omega^4T_s^4-2\Omega^2T_s^2}} = \frac{2\Omega T_s}{1-\Omega^2T_s^2}$$

$$\mathrm{tg}\,\omega = \frac{2\Omega T_s}{(1+\Omega T_s)(1-\Omega T_s)} \Rightarrow \boxed{\omega = 2\,\mathrm{arctg}(\Omega T_s)}.$$

$$\Rightarrow \Omega T_s = 2\,\mathrm{tg}\frac{\omega}{2} = \cancel{2}\,\mathrm{tg}\,\mathrm{arctg}\,\Omega T_s \Rightarrow \boxed{\text{invers: } \Omega T_s = \mathrm{tg}\frac{\omega}{2}}$$

Pr_3. Marin Flaviu - Daniel

Tema 1 - B.

$s = \frac{1}{T_s}\frac{z-1}{z+1}$ ← Transformarea inversă Biliniară pseudo- (Tustin) ($\frac{1}{T_s}$ în loc de $\frac{2}{T_s}$)

=> Filtrul Bw discret G:

1. F.T. a filtrului analogic este:

$$H(s) = \frac{(-1)^M \prod_{m=1}^{M} s_m}{\prod_{m=1}^{M} (s - s_m)}$$

2. Discretizare cu tr. inversă Tustin de mai sus:
(Analog → Discret)

$$G(z) = \frac{(-1)^M \prod_{m=1}^{M} s_m}{\prod_{m=1}^{M}\left(\frac{1}{T_s}\frac{z-1}{z+1} - s_m\right)} = \frac{(-1)^M\left(\prod_{m=1}^{M}\frac{s_m T_s}{1 - s_m T_s}\right)(z+1)^M}{\prod^{M}\left(z - \frac{1 + s_m T_s}{1 - s_m T_s}\right)} \begin{matrix} \to B \\ \to A \end{matrix}$$

=>

• ω și Ω ω → puls. în discret (digital)
Ω → puls. în continuu (analog)

$$j\Omega = \frac{1}{T_s}\frac{e^{j\omega}-1}{e^{j\omega}+1} \Rightarrow \frac{e^{j\omega}-1}{e^{j\omega}+1} = \frac{j\Omega}{T_s} \Rightarrow \exp$$

$r = \sqrt{T_s^2 +}$

$$\Rightarrow e^{j\omega} = + \frac{\frac{j\Omega}{T_s} + 1}{1 - \frac{j\Omega}{T_s}} = + \frac{j\Omega + T_s}{T_s - j\Omega}$$

exp

$$\cancel{\omega = \arccos\left(\frac{2j\Omega + T_s}{T_s - 2j\Omega}\right)} \quad \cancel{\omega = -j \ln \frac{j\Omega + T_s}{T_s - j\Omega}}$$

$\omega =$

$$e^{j\omega} = \cos x + j\sin x \Rightarrow \frac{\cos x + j\sin x - 1}{\cos x + j\sin x + 1} \Rightarrow \cos x + j\sin x = \frac{(j\Omega + T_s)(T_s - j\Omega)}{T_s^2 + \Omega^2}$$