Two strain dengue

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ABSTRACT (original article):
Keywords:

CITATION (original article):

In[ ]:=
```
(*Latex dictionary*)
Format[mu]:=μ;
Format[ga]:=γ;Format[ga1]:=Subscript[γ,1];Format[ga2]:=Subscript[γ,2];
Format[t1]:=Subscript[θ,1];Format[t2]:=Subscript[θ,2];Format[th]:=θ;
Format[La]:=Λ;Format[be1]:=Subscript[β,1];Format[be2]:=Subscript[β,2];
Format[si1]:=Subscript[σ,1];Format[si2]:=Subscript[σ,2];
Format[et1]:=Subscript[η,1];Format[et2]:=Subscript[η,2];
Format[i1]:=Subscript[i,1];Format[i2]:=Subscript[i,2];
Format[y1]:=Subscript[y,1];Format[y2]:=Subscript[y,2];
Format[r1]:=Subscript[r,1];Format[r2]:=Subscript[r,2];
```

In[9]:=

```
(*entering the closed model, packages*)
ClearAll["Global`*"];
SetDirectory[NotebookDirectory[]];SetOptions[$FrontEndSession, NotebookAutoSave → True];
NotebookSave[];
AppendTo[$Path,"C:\\Users\\flori\\Dropbox\\EpidCRNmodels"];<<EpidCRN`;(*
Needs["RobertNachbar`CompartmentalModeling`"]*)

(*particular cases, key formulas*)
cDFE={i1→0,i2→0,y1→0,y2→0};cE2={i1→0,r1→0,y2→0};cE1={i2→0,r2→0,y1→0};cLa=La→mu;
csd=s→La/mu;
csym={ga1→ga,ga2→ga,t1→th,t2→th,(*La→0,mu→0,*)et1→1,et2→1};
csymG={ga1→ga,ga2→ga,t1→th,t2→th,La→0,mu→0,et1→1,et2→1};
cet={et1→1,et2→1};cChu={t1→0,t2→0,th→0,La→mu,et1→1,et2→1};
sd=La/mu;mR1=be1/(ga1+mu);mR2=be2/(ga2+mu);R1=mR1 sd;R2=mR2 sd;
k1=ga1/(ga1+mu+t1);a2c=1/k1;R12=mR2 (1/mR1+ si2 r11);r11=k1(sd-1/mR1);
R2c=R2/R12;

(*enter closed model, as first step*)
RNc={"S"+"I1"→2  "I1", "S"+"Y1" →  "Y1"+  "I1", "I1"→  "R1",
"S"+"I2" →2  "I2","S"+"Y2" →  "Y2"+ "I2", "I2"→  "R2",
 "R1"+  "I2"→ "I2"+  "Y2","R1"+  "Y2"→2  "Y2","Y2"→"R",
 "R2"+"I1"→"I1"+"Y1", "R2"+ "Y1"→2"Y1","Y1"→"R",
  "R1"→"S","R2"→"S","R"→"S"};
  var={S,I1,Y1,R1,I2,Y2,R2,R};
  (*enter open model, adding in and out 9 reactions *)
RN=Join[{0→"S"},RNc,{"S"→0,"I1" →0,"Y1" →0,"R1" →0,"I2" →0,"Y2" →0,"R2" →0,"R"→0}];
minSiph[ToString/@var,asoRea[RN]]
```

```
Constraints generated: 15

Sample constraints:
 {s1 || s2 || s3 || s4 || s5 || s6 || s7 || s8, ! s1, s2 ⇒ s1 || s3, s3 ⇒ s1 || s3, s5 ⇒ s1 || s6}

All found siphons: {{2, 3, 4, 5, 6, 7, 8}, {2, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 8}, {2, 3, 4, 5, 6},
  {2, 3, 4, 7}, {2, 3, 4}, {2, 3, 5, 6, 7, 8}, {2, 3, 5, 6, 7}, {2, 3, 5, 6, 8}, {2, 3, 5, 6},
  {2, 3, 7}, {2, 3}, {4, 5, 6, 7}, {4, 5, 6}, {4, 7}, {4}, {5, 6, 7}, {5, 6}, {7}}

After minimality filter: {{2, 3}, {4}, {5, 6}, {7}}
```

Out[24]=

```
{{2, 3}, {4}, {5, 6}, {7}}
```

```
Needs["ReactionKinetics`"];
(*RNDc=ReactionsData[RNc];Γc= RNDc["γ"]//Normal;

Γt=perR[Γc,2,1];Γc=Γt;
con=cons[Γc];al=RNDc["α"]//Normal;
expoc= perR[al,2,1]//Transpose;Print["nR=",nRc=RNDc["R"]]
{nS,defic,comc}=
RNDc["M","deficiency","complexes"];
Print["Gavish closed",Γc//MatrixForm]
Print["has rank ",MatrixRank[Γc],
"  deficiency ", defic, " and ",con//Length," cons ",con(*x=con.var*)]

(*,"=",var,var//Length]*)
monoc=expM[var,expoc];
Print["rates"];
tkc={be1,be1 et1,ga1,be2,be2 et2,ga2,si2 be2,si2 be2,ga2,si1 be1,si1 be1,ga1,t1,t2,th}
Rvc=tkc* monoc;
cv=Thread[var≥0];ct=Join[cpc,cv];
RHSc=Γc.Rvc//Simplify;
Print["Gavish RHSc:",RHSc//MatrixForm," has Rv= ",Rvc//Transpose//MatrixForm]*)
```

nR=15

Gavish closed $\begin{pmatrix} -1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$

has rank 7 deficiency δ=N−L−S=22−7−7=8 and 1 cons {{1, 1, 1, 1, 1, 1, 1, 1}}

rates

Out[∘]=

$\{\beta_1, \beta_1 \eta_1, \gamma_1, \beta_2, \beta_2 \eta_2, \gamma_2, \beta_2 \sigma_2, \beta_2 \sigma_2, \gamma_2, \beta_1 \sigma_1, \beta_1 \sigma_1, \gamma_1, \Theta_1, \Theta_2, \Theta\}$

Gavish RHSc: $\begin{pmatrix} r_1 \Theta_1 + r_2 \Theta_2 + r\,\Theta - \beta_1 s\,(i_1 + \eta_1 y_1) - \beta_2 s\,(i_2 + \eta_2 y_2) \\ -\gamma_1 i_1 + \beta_1 s\,(i_1 + \eta_1 y_1) \\ -\gamma_1 y_1 + \beta_1 r_2 \sigma_1\,(i_1 + y_1) \\ \gamma_1 i_1 - r_1\,(\Theta_1 + \beta_2 \sigma_2\,(i_2 + y_2)) \\ -\gamma_2 i_2 + \beta_2 s\,(i_2 + \eta_2 y_2) \\ -\gamma_2 y_2 + \beta_2 r_1 \sigma_2\,(i_2 + y_2) \\ \gamma_2 i_2 - r_2\,(\Theta_2 + \beta_1 \sigma_1\,(i_1 + y_1)) \\ -r\,\Theta + \gamma_1 y_1 + \gamma_2 y_2 \end{pmatrix}$ has Rv= $\begin{pmatrix} \beta_1 i_1 s \\ \beta_1 \eta_1 s\,y_1 \\ \gamma_1 i_1 \\ \beta_2 i_2 s \\ \beta_2 \eta_2 s\,y_2 \\ \gamma_2 i_2 \\ \beta_2 i_2 r_1 \sigma_2 \\ \beta_2 r_1 \sigma_2 y_2 \\ \gamma_2 y_2 \\ \beta_1 i_1 r_2 \sigma_1 \\ \beta_1 r_2 \sigma_1 y_1 \\ \gamma_1 y_1 \\ r_1 \Theta_1 \\ r_2 \Theta_2 \\ r\,\Theta \end{pmatrix}$

```
RND=ReactionsData[RN];
Γ= RND["γ"]//Normal;
(*cycle={6,8,3,4,5,7};
Γt=perC[Γ,cycle];
Γ=Γt;*)
con=cons[Γ];
expo= RND["α"]//Normal//Transpose;Print["nR=",nR=RND["R"]]
{nS,defi,com}=
RND["M","deficiency","complexes"];
Print["Gavish SM",Γ//MatrixForm]
Print["has rank ",MatrixRank[Γ],
"  deficiency ", RND["deficiency"], " and ",con//Length," cons ",con(*x=con.var*)]
Print["check of order of variables:"]
Print[RND["variables"]//Length," variables=",RND["variables"],"=",var]
inf={2,3,5,6};
 mono=expM[var,expo];(*{ xA xB, xB xC, xC,  xD,xE};*)
tk=Join[{La}, tkc,{mu,mu,mu,mu,mu,mu,mu,mu}];
Rv=tk* mono;
RHS=Γ.Rv//Simplify;
Print["Gav RHS:",RHS//MatrixForm," has Rv= ",Rv//Transpose//MatrixForm]
Print["Check sum ",Total[RHS]//FullSimplify," reveals N=La/mu is conserved"]
par=Par[RHS,var]
cp=Join[Thread[par>0],{ga>0,s>0}];
cp1=Join[cp,{ga>0,s>0,r1>0,i1>0}];
cv=Thread[var>0];ct=Join[cp,cv];
```

nR=24

$$\text{Gavish SM}\begin{pmatrix}1 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1\end{pmatrix}$$

has rank 8 deficiency δ=N−L−S=23−7−8=8 and 0 cons {}

check of order of variables:

8 variables=$\{c_S, c_{I1}, c_{Y1}, c_{R1}, c_{I2}, c_{Y2}, c_{R2}, c_R\}=\{s, i_1, y_1, r_1, i_2, y_2, r_2, r\}$

$$\text{Gav RHS:}\begin{pmatrix}\Lambda - \beta_2\, i_2\, s - \mu\, s + r_1\,\Theta_1 + r_2\,\Theta_2 + r\,\Theta - \beta_1\, s\,(i_1 + \eta_1\, y_1) - \beta_2\,\eta_2\, s\, y_2\\ -\gamma_1\, i_1 - i_1\,\mu + \beta_1\, i_1\, s + \beta_1\,\eta_1\, s\, y_1\\ -((\gamma_1 + \mu)\, y_1) + \beta_1\, r_2\,\sigma_1\,(i_1 + y_1)\\ \gamma_1\, i_1 - r_1\,(\mu + \Theta_1 + \beta_2\,\sigma_2\,(i_2 + y_2))\\ -\gamma_2\, i_2 - i_2\,\mu + \beta_2\, i_2\, s + \beta_2\,\eta_2\, s\, y_2\\ -((\gamma_2 + \mu)\, y_2) + \beta_2\, r_1\,\sigma_2\,(i_2 + y_2)\\ \gamma_2\, i_2 - r_2\,(\mu + \Theta_2 + \beta_1\,\sigma_1\,(i_1 + y_1))\\ -\mu\, r - r\,\Theta + \gamma_1\, y_1 + \gamma_2\, y_2\end{pmatrix}\quad\text{has Rv=}\begin{pmatrix}\Lambda\\ \beta_1\, i_1\, s\\ \beta_1\,\eta_1\, s\, y_1\\ \gamma_1\, i_1\\ \beta_2\, i_2\, s\\ \beta_2\,\eta_2\, s\, y_2\\ \gamma_2\, i_2\\ \beta_2\, i_2\, r_1\,\sigma_2\\ \beta_2\, r_1\,\sigma_2\, y_2\\ \gamma_2\, y_2\\ \beta_1\, i_1\, r_2\,\sigma_1\\ \beta_1\, r_2\,\sigma_1\, y_1\\ \gamma_1\, y_1\\ r_1\,\Theta_1\\ r_2\,\Theta_2\\ r\,\Theta\\ \mu\, s\\ i_1\,\mu\\ \mu\, y_1\\ \mu\, r_1\\ i_2\,\mu\\ \mu\, y_2\\ \mu\, r_2\\ \mu\, r\end{pmatrix}$$

Check sum $\Lambda - \mu\,(i_1 + i_2 + r + r_1 + r_2 + s + y_1 + y_2)$ reveals N=La/mu is conserved

Out[•]=

$\{\beta_1, \beta_2, \eta_1, \eta_2, \gamma_1, \gamma_2, \Lambda, \mu, \sigma_1, \sigma_2, \Theta_1, \Theta_2, \Theta\}$

In[•]:=

```
minSiph[ToString/@var,asoRea[RN]]
```

```
Constraints generated: 1

Sample constraints: {s1 || s2 || s3 || s4 || s5 || s6 || s7 || s8}

All found siphons: {{1, 2, 3, 4, 5, 6, 7, 8}, {2, 3, 4, 5, 6, 7, 8}, {3, 4, 5, 6, 7, 8}, {4, 5, 6, 7, 8},
  {5, 6, 7, 8}, {6, 7, 8}, {7, 8}, {8}, {1, 3, 4, 5, 6, 7, 8}, {2, 4, 5, 6, 7, 8}, {3, 5, 6, 7, 8},
  {4, 6, 7, 8}, {5, 7, 8}, {6, 8}, {7}, {1, 2, 4, 5, 6, 7, 8}, {2, 3, 5, 6, 7, 8}, {3, 4, 6, 7, 8},
  {4, 5, 7, 8}, {5, 6, 8}, {6, 7}, {1, 4, 5, 6, 7, 8}, {2, 5, 6, 7, 8}, {3, 6, 7, 8}, {4, 7, 8}}

After minimality filter: {{8}, {7}}
```

Out[•]=

```
{{8}, {7}}
```

```
(*Solve  E1 boundary and DFE, jac*)
so1=FullSimplify[#]&/@Solve[(RHS/.cE1)==0,var]
so1=red[Reduce[Append[cp,(RHS/.cE1)==0],var]//FullSimplify,cp];
Print["E1 eq has ",so1//Length," sols ",so1[[1]]]
so1[[2]]
so1=reCL[Reduce[Append[cp,(RHS/.cE1)==0],var]//FullSimplify];
Print["E1 eq has ",so1//Length," sols, first is ",so1[[1]], " sec is"]
so1[[2]]
sos=Reduce[Append[cp,(RHS/.cE1/.csd)==0],DeleteElements[var,{1}]]//FullSimplify
sos//Length
red[sos,cp]
(*Symmetric case*)
so=SolveValues[Thread[(RHS/.csymG)==0],var];
Print["Gav  symm case  without demog and etas has ",so//Length," sols"]
sef=seZF[so]//FullSimplify;
Print["with ",sef//Length," endemic ",sef]
(*Chung case
so=SolveValues[Thread[(RHS/.cChu)==0],var];
Print["Chu case   has ",so//Length," sols"]
sef=seF[so]//FullSimplify;
Print["with ",sef//Length," endemic "]
sef*)

jac=Grad[(RHS),var];
ch=CharacteristicPolynomial[jac,u];
chD=ch/.cDFE//Factor;
Print["CharacteristicPolynomial at DFE has ",chD//Length," factors, two are quadratic;when eta=1"
chDe=chD/.cet

ch1=ch/.cet/.cE1//Factor;
Print["CharacteristicPolynomial at E1 has ",ch1//Length," factors, last has degree ",Exponent[ch1
Print["Conjectured inequality R2c<R1 when"]
red[Reduce[Append[cp,R2c<R1&&R1>1],var]//FullSimplify,cp]
Print["Conjectured inequality R2c >1 when si2 k1 <1"]
red[Reduce[Append[cp,R2c>1&&R1>1],var]//FullSimplify,cp]
```

••• Solve: Equations may not give solutions for all "solve" variables.

Out[∘]=

$$\left\{\left\{s \to \frac{\gamma_1+\mu}{\beta_1},\ i_1 \to \frac{(\beta_1 \Lambda - \mu (\gamma_1+\mu))\ (\mu+\Theta_1)}{\beta_1 \mu (\gamma_1+\mu+\Theta_1)},\ r_1 \to \frac{\beta_1 \gamma_1 \Lambda - \gamma_1 \mu (\gamma_1+\mu)}{\beta_1 \mu (\gamma_1+\mu+\Theta_1)},\ y_2 \to 0,\ r \to 0\right\},\ \left\{s \to \frac{\Lambda}{\mu},\ i_1 \to 0,\ r_1 \to 0,\ y_2 \to 0,\ r \to 0\right\}\right\}$$

E1 eq has 2 sols $i_1 == 0 \&\& r_1 == \frac{-\Lambda+\mu s}{\Theta_1} \&\& \left(\left(\frac{\Lambda}{\mu} == s \&\& r == \frac{\gamma_2 y_2}{\mu+\Theta} \&\& \frac{(\Lambda - \mu s + r_1 \Theta_1)\ (\mu+\Theta)}{-\gamma_2 \Theta + \beta_2 \eta_2 s (\mu+\Theta)} == y_2 \&\& \left(\mu \le \frac{\beta_2 \eta_2 \Lambda}{\gamma_2} \mid\mid \left(\mu > \frac{\beta_2 \eta_2 \Lambda}{\gamma_2} \&\& \left(\frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta < 0 \mid\mid \frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta > 0\right)\right)\right)\right) \mid\mid \left(r == 0 \&\& s == \frac{\gamma_2 \Theta}{\beta_2 \eta_2 \mu + \beta_2 \eta_2 \Theta} \&\& \frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta == 0 \&\& y_2 == 0 \&\& \mu > \frac{\beta_2 \eta_2 \Lambda}{\gamma_2}\right)\right)$

Out[∘]=

$$i_1 == \frac{(\Lambda - \mu s)\ (\mu+\Theta_1)}{-\gamma_1 \Theta_1 + \beta_1 s (\mu+\Theta_1)} \&\& r == 0 \&\& r_1 == \frac{-\Lambda + (\beta_1 i_1 + \mu) s}{\Theta_1} \&\& s == \frac{\gamma_1+\mu}{\beta_1} \&\& y_2 == 0$$

E1 eq has 2 sols, first is

$i_1 == 0 \&\& r_1 == \frac{-\Lambda+\mu s}{\Theta_1} \&\& \left(\left(\frac{\Lambda}{\mu} == s \&\& r == \frac{\gamma_2 y_2}{\mu+\Theta} \&\& \frac{(\Lambda - \mu s + r_1 \Theta_1)\ (\mu+\Theta)}{-\gamma_2 \Theta + \beta_2 \eta_2 s (\mu+\Theta)} == y_2 \&\& \left(\mu \le \frac{\beta_2 \eta_2 \Lambda}{\gamma_2} \mid\mid \left(\mu > \frac{\beta_2 \eta_2 \Lambda}{\gamma_2} \&\& \left(\frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta < 0 \mid\mid \frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta > 0\right)\right)\right)\right) \mid\mid \left(r == 0 \&\& s == \frac{\gamma_2 \Theta}{\beta_2 \eta_2 \mu + \beta_2 \eta_2 \Theta} \&\& \frac{\beta_2 \eta_2 \Lambda \mu}{\beta_2 \eta_2 \Lambda - \gamma_2 \mu} + \Theta == 0 \&\& y_2 == 0 \&\& \mu > \frac{\beta_2 \eta_2 \Lambda}{\gamma_2}\right)\right)$ sec is

Out[∘]=

$$i_1 == \frac{(\Lambda - \mu s)\ (\mu+\Theta_1)}{-\gamma_1 \Theta_1 + \beta_1 s (\mu+\Theta_1)} \&\& r == 0 \&\& r_1 == \frac{-\Lambda + (\beta_1 i_1 + \mu) s}{\Theta_1} \&\& s == \frac{\gamma_1+\mu}{\beta_1} \&\& y_2 == 0$$

Out[∘]=

$$i_1 == 0 \&\& r_1 == 0 \&\& y_2 == 0 \&\& \beta_1 > 0 \&\& \eta_1 > 0 \&\& \eta_2 > 0 \&\& \gamma > 0 \&\& \gamma_1 > 0 \&\& \gamma_2 > 0 \&\& \Lambda > 0 \&\& \mu > 0 \&\& s > 0 \&\& \sigma_1 > 0 \&\& \sigma_2 > 0 \&\& \Theta_1 > 0 \&\& \Theta_2 > 0 \&\& \Theta > 0 \&\& r == 0 \&\& \left(\beta_2 \ge \frac{\gamma_2 \mu \Theta}{\eta_2 \Lambda \mu + \eta_2 \Lambda \Theta} \mid\mid \beta_2 > 0\right)$$

Out[∘]=

19

Out[∘]=

$$i_1 == 0 \&\& r_1 == 0 \&\& y_2 == 0 \&\& r == 0$$

Solve: Equations may not give solutions for all "solve" variables.

Gav symm case without demog and etas has 15 sols

with 1 endemic $\left\{\left\{s, \frac{s (-\gamma + \beta_2 s)\ (\beta_1 s (\sigma_1 - \sigma_2) + \gamma \sigma_2)\ \Theta}{\gamma^2 (\gamma - (\beta_1+\beta_2) s)\ \sigma_1 \sigma_2},\ \frac{(\gamma - \beta_1 s)\ (\gamma - \beta_2 s)\ (\beta_1 s (\sigma_1 - \sigma_2) + \gamma \sigma_2)\ \Theta}{\beta_1 \gamma^2 (-\gamma + (\beta_1+\beta_2) s)\ \sigma_1 \sigma_2},\ \frac{\gamma - \beta_2 s}{\beta_2 \sigma_2},\ -\frac{s (\gamma - \beta_1 s)\ (\gamma \sigma_1 + \beta_2 s (-\sigma_1 + \sigma_2))\ \Theta}{\gamma^2 (\gamma - (\beta_1+\beta_2) s)\ \sigma_1 \sigma_2},\ \frac{(\gamma - \beta_1 s)\ (\gamma - \beta_2 s)\ (\gamma \sigma_1 + \beta_2 s (-\sigma_1 + \sigma_2))\ \Theta}{\beta_2 \gamma^2 (-\gamma + (\beta_1+\beta_2) s)\ \sigma_1 \sigma_2},\ \frac{\gamma - \beta_1 s}{\beta_1 \sigma_1},\ \frac{(\gamma - \beta_1 s)\ (\gamma - \beta_2 s)\ (\beta_1 \sigma_1 + \beta_2 \sigma_2)}{\beta_1 \beta_2 (-\gamma + (\beta_1+\beta_2) s)\ \sigma_1 \sigma_2}\right\}\right\}$

CharacteristicPolynomial at DFE has 6 factors, two are quadratic;when eta=1

Out[ ]=

$$(\mu + u)\ (\mu + \Theta_1 + u)\ (\mu + \Theta_2 + u)\ (\mu + \Theta + u)$$
$$\left(\gamma_1^2 + 2\,\gamma_1\,\mu + \mu^2 - \beta_1\,\gamma_1\,s - \beta_1\,\mu\,s - \beta_1\,\gamma_1\,r_2\,\sigma_1 - \beta_1\,\mu\,r_2\,\sigma_1 + 2\,\gamma_1\,u + 2\,\mu\,u - \beta_1\,s\,u - \beta_1\,r_2\,\sigma_1\,u + u^2\right)$$
$$\left(\gamma_2^2 + 2\,\gamma_2\,\mu + \mu^2 - \beta_2\,\gamma_2\,s - \beta_2\,\mu\,s - \beta_2\,\gamma_2\,r_1\,\sigma_2 - \beta_2\,\mu\,r_1\,\sigma_2 + 2\,\gamma_2\,u + 2\,\mu\,u - \beta_2\,s\,u - \beta_2\,r_1\,\sigma_2\,u + u^2\right)$$

```
CharacteristicPolynomial at E1 has 2 factors, last has degree 7
```

Out[ ]=

$$\mu\ (\gamma_1 + \mu)\ < \beta_1\,\Lambda\ \&\&\ \gamma_1\,\sigma_2 < \gamma_1 + \mu + \Theta_1$$

Out[ ]=

$$\beta_1 > \frac{\mu\ (\gamma_1 + \mu)}{\Lambda}$$

In[•]:=
```
(*NGM cell for DFE*)
mod={RHS,var,par};
ng=NGM[mod,inf];
K=ng[[6]];
Print["Eigs of K as functions of s, r, are"]
eig=K//Eigenvalues
Print[" cDFE is"]
cDFE=DFE[mod,inf][[1]]//FullSimplify
Print["Eigs of K at DFE are"]
eigD=FullSimplify[eig/.cDFE,cp]
(*NGM cell for DFE; same answer
inf={2,3,4,5,6,7};
mod={RHS,var,tk};
ng=NGM[mod,inf];
K=ng[[6]];
Print["Eigs of K as functions of s, r, are"]
eig=K//Eigenvalues
Print[" cDFE is"]
cDFE=DFE[mod,inf][[1]]//FullSimplify
Print["Eigs of K at DFE are"]
eigD=FullSimplify[eig/.cDFE,cp]*)
```

Eigs of K as functions of s, r, are

Out[•]=

$$\left\{\frac{\beta_1\gamma_2 s+\beta_1\mu s+\beta_1\gamma_2 r_2\sigma_1+\beta_1\mu r_2\sigma_1-\beta_1(\gamma_2+\mu)\sqrt{s^2-2r_2 s\sigma_1+4\eta_1 r_2 s\sigma_1+r_2^2\sigma_1^2}}{2(\gamma_1+\mu)(\gamma_2+\mu)},\right.$$
$$\frac{\beta_1\gamma_2 s+\beta_1\mu s+\beta_1\gamma_2 r_2\sigma_1+\beta_1\mu r_2\sigma_1+\beta_1(\gamma_2+\mu)\sqrt{s^2-2r_2 s\sigma_1+4\eta_1 r_2 s\sigma_1+r_2^2\sigma_1^2}}{2(\gamma_1+\mu)(\gamma_2+\mu)},$$
$$\frac{\beta_2\gamma_1 s+\beta_2\mu s+\beta_2\gamma_1 r_1\sigma_2+\beta_2\mu r_1\sigma_2-\beta_2(\gamma_1+\mu)\sqrt{s^2-2r_1 s\sigma_2+4\eta_2 r_1 s\sigma_2+r_1^2\sigma_2^2}}{2(\gamma_1+\mu)(\gamma_2+\mu)},$$
$$\left.\frac{\beta_2\gamma_1 s+\beta_2\mu s+\beta_2\gamma_1 r_1\sigma_2+\beta_2\mu r_1\sigma_2+\beta_2(\gamma_1+\mu)\sqrt{s^2-2r_1 s\sigma_2+4\eta_2 r_1 s\sigma_2+r_1^2\sigma_2^2}}{2(\gamma_1+\mu)(\gamma_2+\mu)}\right\}$$

cDFE is

Out[•]=

$$\left\{s\to\frac{\Lambda}{\mu},\ r_1\to 0,\ r_2\to 0,\ r\to 0\right\}$$

Eigs of K at DFE are

Out[•]=

$$\left\{0,\ \frac{\beta_1\Lambda}{\gamma_1\mu+\mu^2},\ 0,\ \frac{\beta_2\Lambda}{\gamma_2\mu+\mu^2}\right\}$$

In[∘]:=
```
(*NGM cell for EE1*)
inf={5,6,7};
mod={RHS,var,tk};
ng=NGM[mod,inf];M=ng⟦1⟧;
K=ng⟦6⟧;
Print["Eigs of K when 1 resident=",K//MatrixForm,"  in asym Gav case are",eig=K//Eigenvalues," po
po=Collect[Numerator[Together[CharacteristicPolynomial[K,u]/u]]//FullSimplify,u]
R12G=eig⟦3⟧;
Kc=K/.cet;
Print["Eigs without asym eta are",eig=Kc//Eigenvalues]
Print[" at E1, R21="]
R12n=eig⟦3⟧
R12==R12n

fus=FullSimplify[Reduce[And @@ cp1&&R1>1&&R12<1, {be1,be2}]/.cE1, Assumptions → cp1];
Print[" in Chu case, the E1 stab cond is "]
fus
fuG=FullSimplify[Reduce[And @@ cp1&&R1>1&&R12G<1, {be1,be2}]/.cE1, Assumptions → cp1];
Print[" in Gav case, the E1 stab cond is "]
fuG
```

Eigs of K when 1 resident= $\begin{pmatrix} \frac{\beta_2 s}{\gamma_2+\mu} & \frac{\beta_2 \eta_2 s}{\gamma_2+\mu} & 0 \\ \frac{\beta_2 r_1 \sigma_2}{\gamma_2+\mu} & \frac{\beta_2 r_1 \sigma_2}{\gamma_2+\mu} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

in asym Gav case are $\left\{0, \frac{\beta_2 \left(s + r_1 \sigma_2 - \sqrt{s^2 - 2 r_1 s \sigma_2 + 4 \eta_2 r_1 s \sigma_2 + r_1^2 \sigma_2^2}\right)}{2 (\gamma_2 + \mu)},\right.$
$\left.\frac{\beta_2 \left(s + r_1 \sigma_2 + \sqrt{s^2 - 2 r_1 s \sigma_2 + 4 \eta_2 r_1 s \sigma_2 + r_1^2 \sigma_2^2}\right)}{2 (\gamma_2 + \mu)}\right\}$ pol is

Out[∘]=
$$\beta_2^2 (-1 + \eta_2) r_1 s \sigma_2 + \beta_2 (\gamma_2 + \mu) (s + r_1 \sigma_2) u - (\gamma_2 + \mu)^2 u^2$$

Eigs without asym eta are $\left\{0, 0, \frac{\beta_2 (s + r_1 \sigma_2)}{\gamma_2 + \mu}\right\}$

at E1, R21=

Out[∘]=
$$\frac{\beta_2 (s + r_1 \sigma_2)}{\gamma_2 + \mu}$$

Out[∘]=
True

in Chu case, the E1 stab cond is

Out[∘]=
$$\beta_1 \Lambda > \mu (\gamma_1 + \mu) \&\& \beta_2 (s + r_1 \sigma_2) < \gamma_2 + \mu$$

in Gav case, the E1 stab cond is

Out[•]=

$$\beta_1 \Lambda > \mu\ (\gamma_1 + \mu)\ \&\&\ \Big((\eta_2 == 1\ \&\&\ \beta_2\ (s + r_1 \sigma_2) < \gamma_2 + \mu)\ ||\ (-1 + \eta_2) \left((\gamma_2 + \mu)\ s + r_1\ (\gamma_2 + \mu + 2\,\beta_2\ (-1 + \eta_2)\ s)\ \sigma_2 - (\gamma_2 + \mu)\ \sqrt{s^2 + 2\ (-1 + 2\,\eta_2)\ r_1\ s\ \sigma_2 + r_1^2\,\sigma_2^2}\right) < 0\Big)$$

In[•]:=

```
(*A rational substitution*)
RHSc=RHS/.cChu;RH1=Drop[RHSc,1];va1=Drop[var,1];so=Solve[RH1==0,va1]//FullSimplify;
Print["in Chung case, there are ", so//Length," rat fps, suggesting some Kol fact; end pt EE (s)
sef=seZF[so];sef//Length
rat=sef//Flatten
cf=FullSimplify[#]&/@CoefficientList[Numerator[Together[RHSc⟦1⟧/mu/.rat]],s]
```

in Chung case, there are 4 rat fps, suggesting some Kol fact; end pt EE (s) is

Out[•]=

1

Out[•]=

$$\Big\{i_1 \to \frac{\mu\ s\ (\gamma_2 + \mu - \beta_2\ s)\ (-\beta_1\,\gamma_2\ s\ \sigma_1 - (\gamma_2 + \mu)\ (\gamma_1 + \mu - \beta_1\ s)\ \sigma_2)}{\left((\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2 - (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (\beta_2\ (\gamma_1 + \mu) + \beta_1\ (\gamma_2 + \mu))\ s + \beta_1\,\beta_2\,\mu\ (\gamma_1 + \gamma_2 + \mu)\ s^2\right)\ \sigma_1\,\sigma_2},$$

$$y_1 \to (\mu\ (\gamma_1 + \mu - \beta_1\ s)\ (\gamma_2 + \mu - \beta_2\ s)\ (\beta_1\,\gamma_2\ s\ \sigma_1 + (\gamma_2 + \mu)\ (\gamma_1 + \mu - \beta_1\ s)\ \sigma_2)) \Big/ \left(\beta_1\ \left(-(\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2 + (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (\beta_2\ (\gamma_1 + \mu) + \beta_1\ (\gamma_2 + \mu))\ s - \beta_1\,\beta_2\,\mu\ (\gamma_1 + \gamma_2 + \mu)\ s^2\right)\ \sigma_1\,\sigma_2\right),\ r_1 \to \frac{\gamma_2 + \mu - \beta_2\ s}{\beta_2\,\sigma_2},\ i_2 \to$$

$$\frac{\mu\ s\ (\gamma_1 + \mu - \beta_1\ s)\ (-((\gamma_1 + \mu)\ (\gamma_2 + \mu - \beta_2\ s)\ \sigma_1) - \beta_2\,\gamma_1\ s\ \sigma_2)}{\left((\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2 - (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (\beta_2\ (\gamma_1 + \mu) + \beta_1\ (\gamma_2 + \mu))\ s + \beta_1\,\beta_2\,\mu\ (\gamma_1 + \gamma_2 + \mu)\ s^2\right)\ \sigma_1\,\sigma_2},$$

$$y_2 \to (\mu\ (\gamma_1 + \mu - \beta_1\ s)\ (\gamma_2 + \mu - \beta_2\ s)\ ((\gamma_1 + \mu)\ (\gamma_2 + \mu - \beta_2\ s)\ \sigma_1 + \beta_2\,\gamma_1\ s\ \sigma_2)) \Big/ \left(\beta_2\ \left(-(\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2 + (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (\beta_2\ (\gamma_1 + \mu) + \beta_1\ (\gamma_2 + \mu))\ s - \beta_1\,\beta_2\,\mu\ (\gamma_1 + \gamma_2 + \mu)\ s^2\right)\ \sigma_1\,\sigma_2\right),\ r_2 \to \frac{\gamma_1 + \mu - \beta_1\ s}{\beta_1\,\sigma_1},\ r \to ((\gamma_1 + \mu - \beta_1\ s)\ (\gamma_2 + \mu - \beta_2\ s)$$

$$(-\beta_2\,\gamma_1\ (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ \sigma_2 + \beta_1\ (-\gamma_2\ (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ \sigma_1 + \beta_2\,\gamma_2\,\mu\ s\ \sigma_1 + \beta_2\,\gamma_1\,\mu\ s\ \sigma_2))) \Big/ \left(\beta_1\,\beta_2\ \left((\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2 - (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (\beta_2\ (\gamma_1 + \mu) + \beta_1\ (\gamma_2 + \mu))\ s + \beta_1\,\beta_2\,\mu\ (\gamma_1 + \gamma_2 + \mu)\ s^2\right)\ \sigma_1\,\sigma_2\right)\Big\}$$

Out[•]=

$$\Big\{(\gamma_1 + \mu)^2\ (\gamma_2 + \mu)^2\ \sigma_1\,\sigma_2,\ (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ ((\gamma_1 + \mu)\ (\gamma_2 + \mu)\ \sigma_1 - (-((\gamma_1 + \mu)\ (\gamma_2 + \mu)) + \beta_2\ (\gamma_1 + \mu)\ \sigma_1 + (\beta_1 + \gamma_1 + \mu)\ (\gamma_2 + \mu)\ \sigma_1)\ \sigma_2),$$

$$-\beta_2\ (\gamma_1 + \mu)^2\ (\gamma_2 + \mu)\ \sigma_1 + \beta_2\ \left(-\mu\ (\gamma_1 + \mu)\ (\gamma_2 + \mu) + \left(\gamma_1^2\ (\gamma_2 + \mu) + \mu\ (\beta_1 + \mu)\ (\gamma_2 + \mu) + \gamma_1\,\mu\ (\beta_1 + 2\ (\gamma_2 + \mu))\right)\ \sigma_1\right)\ \sigma_2 + \beta_1\ (\gamma_1 + \mu)\ (\gamma_2 + \mu)\ (-\mu\,\sigma_1 + (\gamma_2 + \mu)\ (-1 + \sigma_1)\ \sigma_2),$$

$$\beta_1\,\beta_2\,\mu\ ((\gamma_1 + \mu)\ \sigma_1 + (\gamma_2 + \mu - (\gamma_1 + \gamma_2 + \mu)\ \sigma_1)\ \sigma_2)\Big\}$$

In[∘]:=
```
(*A rational substitution*)
RHSc=RHS/.cet;RH1=Drop[RHSc,1];va1=Drop[var,1];so=Solve[RH1==0,va1]//FullSimplify;
Print["in eta=1 case, there are ", so//Length," rat fps, suggesting some Kol fact; end pt EE (s)
sef=seZF[so];sef//Length
rat=sef//Flatten
cf=FullSimplify[#]&/@CoefficientList[Numerator[Together[RHSc[[1]]/.rat]],s]
```

in eta=1 case, there are 4 rat fps, suggesting some Kol fact; end pt EE (s) is

Out[∘]=

1

Out[∘]=

$$\Big\{i_1 \to \big(s\,(-\gamma_2-\mu+\beta_2\,s)\,((\gamma_1+\mu)\,(\gamma_2+\mu)\,\sigma_2\,(\mu+\Theta_2)+\beta_1\,s\,(-\mu\,\sigma_2\,(\mu+\Theta_2)+\gamma_2\,(\mu\,\sigma_1-\mu\,\sigma_2+\sigma_1\,\Theta_1-\sigma_2\,\Theta_2)))\big)\Big/\Big(\big((\gamma_1+\mu)^2\,(\gamma_2+\mu)^2-(\gamma_1+\mu)\,(\gamma_2+\mu)\,(\beta_2\,(\gamma_1+\mu)+\beta_1\,(\gamma_2+\mu))\,s+\beta_1\,\beta_2\,\mu\,(\gamma_1+\gamma_2+\mu)\,s^2\big)\,\sigma_1\,\sigma_2\Big),\ y_1 \to \big((\gamma_1+\mu-\beta_1\,s)\,(\gamma_2+\mu-\beta_2\,s)\,((\gamma_1+\mu)\,(\gamma_2+\mu)\,\sigma_2\,(\mu+\Theta_2)+\beta_1\,s\,(-\mu\,\sigma_2\,(\mu+\Theta_2)+\gamma_2\,(\mu\,\sigma_1-\mu\,\sigma_2+\sigma_1\,\Theta_1-\sigma_2\,\Theta_2)))\big)\Big/\Big(\beta_1\,\big(-(\gamma_1+\mu)^2\,(\gamma_2+\mu)^2+(\gamma_1+\mu)\,(\gamma_2+\mu)\,(\beta_2\,(\gamma_1+\mu)+\beta_1\,(\gamma_2+\mu))\,s-\beta_1\,\beta_2\,\mu\,(\gamma_1+\gamma_2+\mu)\,s^2\big)\,\sigma_1\,\sigma_2\Big),\ r_1 \to \frac{\gamma_2+\mu-\beta_2\,s}{\beta_2\,\sigma_2},$$

$$i_2 \to -\Big(\Big(s\,(\gamma_1+\mu-\beta_1\,s)\,\big(\mu\,(\gamma_2+\mu-\beta_2\,s)\,\sigma_1\,(\mu+\Theta_1)+\gamma_1\,(\mu^2\,\sigma_1+\beta_2\,\mu\,s\,(-\sigma_1+\sigma_2)+\mu\,\sigma_1\,\Theta_1+\gamma_2\,\sigma_1\,(\mu+\Theta_1)+\beta_2\,s\,(-\sigma_1\,\Theta_1+\sigma_2\,\Theta_2))\big)\Big)\Big/\Big(\big((\gamma_1+\mu)^2\,(\gamma_2+\mu)^2-(\gamma_1+\mu)\,(\gamma_2+\mu)\,(\beta_2\,(\gamma_1+\mu)+\beta_1\,(\gamma_2+\mu))\,s+\beta_1\,\beta_2\,\mu\,(\gamma_1+\gamma_2+\mu)\,s^2\big)\,\sigma_1\,\sigma_2\Big)\Big),$$

$$y_2 \to \Big((\gamma_1+\mu-\beta_1\,s)\,(\gamma_2+\mu-\beta_2\,s)\,\big(\mu\,(\gamma_2+\mu-\beta_2\,s)\,\sigma_1\,(\mu+\Theta_1)+\gamma_1\,(\mu^2\,\sigma_1+\beta_2\,\mu\,s\,(-\sigma_1+\sigma_2)+\mu\,\sigma_1\,\Theta_1+\gamma_2\,\sigma_1\,(\mu+\Theta_1)+\beta_2\,s\,(-\sigma_1\,\Theta_1+\sigma_2\,\Theta_2))\big)\Big)\Big/\Big(\beta_2\,\big(-(\gamma_1+\mu)^2\,(\gamma_2+\mu)^2+(\gamma_1+\mu)\,(\gamma_2+\mu)\,(\beta_2\,(\gamma_1+\mu)+\beta_1\,(\gamma_2+\mu))\,s-\beta_1\,\beta_2\,\mu\,(\gamma_1+\gamma_2+\mu)\,s^2\big)\,\sigma_1\,\sigma_2\Big),\ r_2 \to \frac{\gamma_1+\mu-\beta_1\,s}{\beta_1\,\sigma_1},$$

$$r \to -\Big(\Big((\gamma_1+\mu-\beta_1\,s)\,(\gamma_2+\mu-\beta_2\,s)\,\big(\beta_1\,\gamma_2\,\mu\,(\gamma_2+\mu-\beta_2\,s)\,\sigma_1\,(\mu+\Theta_1)+\beta_2\,\gamma_1\,(\gamma_1+\mu)\,(\gamma_2+\mu)\,\sigma_2\,(\mu+\Theta_2)+\beta_1\,\gamma_1\,(\gamma_2^2\,\sigma_1\,(\mu+\Theta_1)+\gamma_2\,\mu\,\sigma_1\,(\mu+\Theta_1)-\beta_2\,\mu\,s\,\sigma_2\,(\mu+\Theta_2))\big)\Big)\Big/\Big(\beta_1\,\beta_2\,\big((\gamma_1+\mu)^2\,(\gamma_2+\mu)^2-(\gamma_1+\mu)\,(\gamma_2+\mu)\,(\beta_2\,(\gamma_1+\mu)+\beta_1\,(\gamma_2+\mu))\,s+\beta_1\,\beta_2\,\mu\,(\gamma_1+\gamma_2+\mu)\,s^2\big)\,\sigma_1\,\sigma_2\,(\mu+\Theta)\Big)\Big)\Big\}$$

Out[•]=

$$\{(\gamma_1+\mu)^2\,(\gamma_2+\mu)^2\,(\beta_1\,\sigma_1\,(\gamma_2\,\mu\,(\Theta_1-\Theta)+\beta_2\,\Lambda\,\sigma_2\,(\mu+\Theta)+\mu\,\Theta_1\,(\mu+\Theta))+\beta_2\,\mu\,\sigma_2\,(\gamma_1\,(\Theta_2-\Theta)+\Theta_2\,(\mu+\Theta))),$$

$$-\Big((\gamma_1+\mu)\,(\gamma_2+\mu)\,\Big(\beta_1\,\beta_2\,\Big(\mu\,(\gamma_1+\mu)\,\Big(\beta_2\,\Lambda\,\sigma_1\,\sigma_2+\mu^2\,(-\sigma_1+(-1+\sigma_1)\,\sigma_2)+\mu\,(\sigma_1\,\Theta_1+\sigma_2\,\Theta_2)+\gamma_2\,(-\mu\,\sigma_1+\mu\,(-1+\sigma_1)\,\sigma_2+\sigma_1\,\Theta_1+\sigma_2\,\Theta_2)\Big)+\Big(\gamma_1\,\Big(\gamma_2\,\mu\,(\sigma_1\,(-2+\sigma_2)-2\,\sigma_2)+\beta_2\,\Lambda\,\sigma_1\,\sigma_2+\mu^2\,(-\sigma_1+(-3+\sigma_1)\,\sigma_2)+\mu\,(\sigma_1\,\Theta_1-\sigma_2\,\Theta_2)\Big)+\mu\,\Big(\beta_2\,\Lambda\,\sigma_1\,\sigma_2+\mu^2\,(-\sigma_1+(-1+\sigma_1)\,\sigma_2)+\gamma_2\,(-3\,\mu\,\sigma_1+\mu\,(-1+\sigma_1)\,\sigma_2-\sigma_1\,\Theta_1+\sigma_2\,\Theta_2)+\mu\,(\sigma_1\,\Theta_1+\sigma_2\,\Theta_2)\Big)\Big)\,\Theta\Big)+\beta_1^2\,(\gamma_2+\mu)\,\sigma_1\,(\gamma_2\,\mu\,(\Theta_1-\Theta)+\beta_2\,\Lambda\,\sigma_2\,(\mu+\Theta)+\mu\,\Theta_1\,(\mu+\Theta))+\beta_2^2\,\mu\,(\gamma_1+\mu)\,\sigma_2\,(\gamma_1\,(\Theta_2-\Theta)+\Theta_2\,(\mu+\Theta))\Big)\Big),$$

$$\beta_1\,\beta_2\,\mu\,\Big(\beta_2\,\Big(\mu\,\Big(\mu^2\,(\sigma_1\,(-1+\sigma_2)-\sigma_2)+\beta_1\,\Lambda\,\sigma_1\,\sigma_2+\mu\,\sigma_2\,\Theta_2\Big)\,(\mu+\Theta)+\gamma_1\,\Big(\mu^3\,(2\,\sigma_1\,(-1+\sigma_2)-\sigma_2)+\gamma_2\,\mu\,(2\,\mu\,\sigma_1\,(-1+\sigma_2)-\mu\,\sigma_2+2\,\sigma_2\,\Theta_2)+\beta_1\,\Lambda\,\sigma_1\,\sigma_2\,\Theta+\gamma_2\,(-3\,\mu\,\sigma_1+2\,\mu\,(-1+\sigma_1)\,\sigma_2-\sigma_1\,\Theta_1+\sigma_2\,\Theta_2)\,\Theta+\mu^2\,(3\,\sigma_2\,\Theta_2-2\,\sigma_1\,\Theta+(-3+2\,\sigma_1)\,\sigma_2\,\Theta)+\mu\,\sigma_2\,(\beta_1\,\Lambda\,\sigma_1+\Theta_2\,\Theta)\Big)+\gamma_2\,\Big(\mu^3\,(\sigma_1\,(-1+\sigma_2)-\sigma_2)+\beta_1\,\Lambda\,\sigma_1\,\sigma_2\,\Theta+\mu\,(\beta_1\,\Lambda\,\sigma_1\,\sigma_2-\sigma_1\,\Theta_1\,\Theta+\sigma_2\,\Theta_2\,\Theta)+\mu^2\,(-2\,\sigma_1\,\Theta+\sigma_2\,(\Theta_2+(-1+\sigma_1)\,\Theta))\Big)+\gamma_1^2\,(\mu\,(\mu\,\sigma_1\,(-1+\sigma_2)+2\,\sigma_2\,\Theta_2-\sigma_1\,\Theta+(-2+\sigma_1)\,\sigma_2\,\Theta)+\gamma_2\,(\mu\,\sigma_1\,(-1+\sigma_2)-\sigma_1\,\Theta+\sigma_2\,(\Theta_2+(-1+\sigma_1)\,\Theta)))\Big)+\beta_1\,(\gamma_2+\mu)\,\Big(\gamma_1\,\Big(\mu^2\,(\sigma_1\,(-1+\sigma_2)-\sigma_2)+\gamma_2\,\sigma_1\,\Theta_1+(-\gamma_2\,\sigma_1+\gamma_2\,(-1+\sigma_1)\,\sigma_2+\sigma_1\,\Theta_1-\sigma_2\,\Theta_2)\,\Theta+\mu\,(\gamma_2\,(-1+\sigma_1)\,\sigma_2-2\,\sigma_2\,\Theta+\sigma_1\,(\Theta_1+(-1+\sigma_2)\,\Theta))\Big)+\mu\,\Big(\mu^2\,(\sigma_1\,(-1+\sigma_2)-\sigma_2)+2\,\gamma_2\,\sigma_1\,\Theta_1+(-2\,\gamma_2\,\sigma_1+\gamma_2\,(-1+\sigma_1)\,\sigma_2+\sigma_1\,\Theta_1)\,\Theta+\mu\,(\gamma_2\,(-1+\sigma_1)\,\sigma_2-\sigma_2\,\Theta+\sigma_1\,(\Theta_1+(-1+\sigma_2)\,\Theta))\Big)\Big)\Big),$$

$$\beta_1^2\,\beta_2^2\,\mu^2\,\Big(\gamma_1\,\mu\,\sigma_1+\mu^2\,\sigma_1+\gamma_2\,\mu\,\sigma_2+\mu^2\,\sigma_2-\gamma_1\,\mu\,\sigma_1\,\sigma_2-\gamma_2\,\mu\,\sigma_1\,\sigma_2-\mu^2\,\sigma_1\,\sigma_2-\gamma_2\,\sigma_1\,\Theta_1-\gamma_1\,\sigma_2\,\Theta_2-(\gamma_1+\gamma_2+\mu)\,(\sigma_1\,(-1+\sigma_2)-\sigma_2)\,\Theta\Big)\}$$

In[∘]:=

```
(*test RUR wrong ? *)par=Variables[tk];
modc={RHSc,var,par};ru=RUR[modc,ind];
Print["in Chung case, RUR finds the pol ", pol=ru⟦2⟧," of deg ", Exponent[pol,var⟦ind⟧]];
```

Out[∘]=

$Aborted

in Chung case, RUR finds the pol

$$
\begin{aligned}
&\{-\Lambda\,\sigma_2 + s\,(-\mu + \mu\,\sigma_2),\ s^2\left(-\beta_1\beta_2\gamma_2\Lambda\mu - \beta_1\beta_2\Lambda\mu^2 + \beta_1\beta_2\gamma_2\Lambda\mu\sigma_1\right) + 2\beta_1\gamma_1\gamma_2\Lambda^2\sigma_2 + \beta_2\gamma_1\gamma_2\Lambda^2\sigma_2 +\\
&\quad \beta_1\gamma_2^2\Lambda^2\sigma_2 - 2\beta_1\beta_2\Lambda^3\sigma_2 + \beta_1\gamma_1\Lambda^2\mu\sigma_2 + \beta_2\gamma_1\Lambda^2\mu\sigma_2 + 2\beta_1\gamma_2\Lambda^2\mu\sigma_2 + \beta_2\gamma_2\Lambda^2\mu\sigma_2 + \beta_1\Lambda^2\mu^2\sigma_2 +\\
&\quad \beta_2\Lambda^2\mu^2\sigma_2 - \beta_1\gamma_1\gamma_2\Lambda^2\sigma_1\sigma_2 - \beta_1\gamma_2^2\Lambda^2\sigma_1\sigma_2 - \beta_1\gamma_2\Lambda^2\mu\sigma_1\sigma_2 - \gamma_1^2\gamma_2^2\Lambda\sigma_2^2 + 2\beta_1\gamma_1\gamma_2\Lambda^2\sigma_2^2 +\\
&\quad \beta_2\gamma_1\gamma_2\Lambda^2\sigma_2^2 + \beta_1\gamma_2^2\Lambda^2\sigma_2^2 - \beta_1\beta_2\Lambda^3\sigma_2^2 - 2\gamma_1^2\gamma_2\Lambda\mu\sigma_2^2 - 2\gamma_1\gamma_2^2\Lambda\mu\sigma_2^2 + \beta_1\gamma_1\Lambda^2\mu\sigma_2^2 + \beta_2\gamma_1\Lambda^2\mu\sigma_2^2 +\\
&\quad 2\beta_1\gamma_2\Lambda^2\mu\sigma_2^2 + \beta_2\gamma_2\Lambda^2\mu\sigma_2^2 - \gamma_1^2\Lambda\mu^2\sigma_2^2 - 4\gamma_1\gamma_2\Lambda\mu^2\sigma_2^2 - \gamma_2^2\Lambda\mu^2\sigma_2^2 + \beta_1\Lambda^2\mu^2\sigma_2^2 + \beta_2\Lambda^2\mu^2\sigma_2^2 -\\
&\quad 2\gamma_1\Lambda\mu^3\sigma_2^2 - 2\gamma_2\Lambda\mu^3\sigma_2^2 - \Lambda\mu^4\sigma_2^2 + s\,\big(2\beta_1\gamma_1\gamma_2\Lambda\mu + \beta_2\gamma_1\gamma_2\Lambda\mu + \beta_1\gamma_2^2\Lambda\mu - 2\beta_1\beta_2\Lambda^2\mu +\\
&\qquad \beta_1\gamma_1\Lambda\mu^2 + \beta_2\gamma_1\Lambda\mu^2 + 2\beta_1\gamma_2\Lambda\mu^2 + \beta_2\gamma_2\Lambda\mu^2 + \beta_1\Lambda\mu^3 + \beta_2\Lambda\mu^3 - \beta_1\gamma_1\gamma_2\Lambda\mu\sigma_1 - \beta_1\gamma_2^2\Lambda\mu\sigma_1 -\\
&\qquad \beta_1\gamma_2\Lambda\mu^2\sigma_1 - \beta_1\beta_2\gamma_2\Lambda^2\sigma_2 - \beta_1\gamma_1\gamma_2^2\Lambda\sigma_1\sigma_2 + \beta_1\beta_2\gamma_2\Lambda^2\sigma_1\sigma_2 + \beta_1\gamma_1\gamma_2^2\Lambda\sigma_2^2 - \beta_1\beta_2\gamma_2\Lambda^2\sigma_2^2\big),\\
&\ s\left(\beta_1\beta_2\gamma_2\Lambda^2\mu + \beta_1\beta_2\Lambda^2\mu^2 - \beta_1\beta_2\gamma_2\Lambda^2\mu\sigma_1\right) + \beta_1\beta_2\gamma_2\Lambda^3\sigma_2 + \beta_1\beta_2\Lambda^3\mu\sigma_2 - \beta_1\beta_2\gamma_2\Lambda^3\sigma_1\sigma_2 +\\
&\quad \beta_1\beta_2\gamma_2\Lambda^3\sigma_2^2 - \gamma_1^2\gamma_2^2\Lambda\mu\sigma_2^2 + \beta_1\beta_2\Lambda^3\mu\sigma_2^2 - 2\gamma_1^2\gamma_2\Lambda\mu^2\sigma_2^2 - 2\gamma_1\gamma_2^2\Lambda\mu^2\sigma_2^2 - \gamma_1^2\Lambda\mu^3\sigma_2^2 -\\
&\quad 4\gamma_1\gamma_2\Lambda\mu^3\sigma_2^2 - \gamma_2^2\Lambda\mu^3\sigma_2^2 - 2\gamma_1\Lambda\mu^4\sigma_2^2 - 2\gamma_2\Lambda\mu^4\sigma_2^2 - \Lambda\mu^5\sigma_2^2 + \beta_1\gamma_1\gamma_2^2\Lambda^2\sigma_1\sigma_2^2 - \beta_1\beta_2\gamma_2\Lambda^3\sigma_1\sigma_2^2 +\\
&\quad \beta_1\gamma_1\gamma_2\Lambda^2\mu\sigma_1\sigma_2^2 + \beta_1\gamma_2^2\Lambda^2\mu\sigma_1\sigma_2^2 + \beta_1\gamma_2\Lambda^2\mu^2\sigma_1\sigma_2^2 - \beta_1\gamma_1\gamma_2^2\Lambda^2\sigma_2^3 + \beta_1\beta_2\gamma_2\Lambda^3\sigma_2^3 +\\
&\quad \gamma_1^2\gamma_2^2\Lambda\mu\sigma_2^3 - 2\beta_1\gamma_1\gamma_2\Lambda^2\mu\sigma_2^3 - \beta_2\gamma_1\gamma_2\Lambda^2\mu\sigma_2^3 - \beta_1\gamma_2^2\Lambda^2\mu\sigma_2^3 + \beta_1\beta_2\Lambda^3\mu\sigma_2^3 + 2\gamma_1^2\gamma_2\Lambda\mu^2\sigma_2^3 +\\
&\quad 2\gamma_1\gamma_2^2\Lambda\mu^2\sigma_2^3 - \beta_1\gamma_1\Lambda^2\mu^2\sigma_2^3 - \beta_2\gamma_1\Lambda^2\mu^2\sigma_2^3 - 2\beta_1\gamma_2\Lambda^2\mu^2\sigma_2^3 - \beta_2\gamma_2\Lambda^2\mu^2\sigma_2^3 + \gamma_1^2\Lambda\mu^3\sigma_2^3 +\\
&\quad 4\gamma_1\gamma_2\Lambda\mu^3\sigma_2^3 + \gamma_2^2\Lambda\mu^3\sigma_2^3 - \beta_1\Lambda^2\mu^3\sigma_2^3 - \beta_2\Lambda^2\mu^3\sigma_2^3 + 2\gamma_1\Lambda\mu^4\sigma_2^3 + 2\gamma_2\Lambda\mu^4\sigma_2^3 + \Lambda\mu^5\sigma_2^3,\\
&\ \beta_1\beta_2\Lambda^3\sigma_2 - \gamma_1^2\gamma_2^2\Lambda\sigma_2^2 + \beta_1\beta_2\Lambda^3\sigma_2^2 - 2\gamma_1^2\gamma_2\Lambda\mu\sigma_2^2 - 2\gamma_1\gamma_2^2\Lambda\mu\sigma_2^2 - \gamma_1^2\Lambda\mu^2\sigma_2^2 - 4\gamma_1\gamma_2\Lambda\mu^2\sigma_2^2 -\\
&\quad \gamma_2^2\Lambda\mu^2\sigma_2^2 - 2\gamma_1\Lambda\mu^3\sigma_2^2 - 2\gamma_2\Lambda\mu^3\sigma_2^2 - \Lambda\mu^4\sigma_2^2 + \beta_1\gamma_1\gamma_2\Lambda^2\sigma_1\sigma_2^2 + \beta_1\gamma_2^2\Lambda^2\sigma_1\sigma_2^2 +\\
&\quad \beta_1\gamma_2\Lambda^2\mu\sigma_1\sigma_2^2 + \gamma_1^2\gamma_2^2\Lambda\sigma_2^3 - 2\beta_1\gamma_1\gamma_2\Lambda^2\sigma_2^3 - \beta_2\gamma_1\gamma_2\Lambda^2\sigma_2^3 - \beta_1\gamma_2^2\Lambda^2\sigma_2^3 + \beta_1\beta_2\Lambda^3\sigma_2^3 +\\
&\quad 2\gamma_1^2\gamma_2\Lambda\mu\sigma_2^3 + 2\gamma_1\gamma_2^2\Lambda\mu\sigma_2^3 - \beta_1\gamma_1\Lambda^2\mu\sigma_2^3 - \beta_2\gamma_1\Lambda^2\mu\sigma_2^3 - 2\beta_1\gamma_2\Lambda^2\mu\sigma_2^3 - \beta_2\gamma_2\Lambda^2\mu\sigma_2^3 +\\
&\quad \gamma_1^2\Lambda\mu^2\sigma_2^3 + 4\gamma_1\gamma_2\Lambda\mu^2\sigma_2^3 + \gamma_2^2\Lambda\mu^2\sigma_2^3 - \beta_1\Lambda^2\mu^2\sigma_2^3 - \beta_2\Lambda^2\mu^2\sigma_2^3 + 2\gamma_1\Lambda\mu^3\sigma_2^3 + 2\gamma_2\Lambda\mu^3\sigma_2^3 +\\
&\quad \Lambda\mu^4\sigma_2^3 + s\,\big(\beta_1\beta_2\Lambda^2\mu - \beta_1\gamma_1\gamma_2^2\Lambda\sigma_1\sigma_2 + \beta_1\gamma_1\gamma_2^2\Lambda\sigma_2^2 + \beta_1\gamma_1\gamma_2^2\Lambda\sigma_1\sigma_2^2 - \beta_1\beta_2\gamma_2\Lambda^2\sigma_1\sigma_2^2 -\\
&\qquad \beta_1\gamma_1\gamma_2^2\Lambda\sigma_2^3 + \beta_1\beta_2\gamma_2\Lambda^2\sigma_2^3\big),\ s^3\left(-\beta_1\beta_2\gamma_2\mu^2 - \beta_1\beta_2\mu^3 + \beta_1\beta_2\gamma_2\mu^2\sigma_1\right) +\\
&\quad s^2\,\big(\beta_1\gamma_1\gamma_2^2\mu - \beta_1\beta_2\gamma_2\Lambda\mu + 2\beta_1\gamma_1\gamma_2\mu^2 + \beta_2\gamma_1\gamma_2\mu^2 + \beta_1\gamma_2^2\mu^2 - \beta_1\beta_2\Lambda\mu^2 + \beta_1\gamma_1\mu^3 + \beta_2\gamma_1\mu^3 +\\
&\qquad 2\beta_1\gamma_2\mu^3 + \beta_2\gamma_2\mu^3 + \beta_1\mu^4 + \beta_2\mu^4 - \beta_1\gamma_1\gamma_2^2\mu\sigma_1 - \beta_1\gamma_1\gamma_2\mu^2\sigma_1 - \beta_1\gamma_2^2\mu^2\sigma_1 - \beta_1\gamma_2\mu^3\sigma_1\big) -\\
&\quad \gamma_1^2\gamma_2^2\Lambda\sigma_2 + 2\beta_1\gamma_1\gamma_2\Lambda^2\sigma_2 + \beta_2\gamma_1\gamma_2\Lambda^2\sigma_2 + \beta_1\gamma_2^2\Lambda^2\sigma_2 - \beta_1\beta_2\Lambda^3\sigma_2 - 2\gamma_1^2\gamma_2\Lambda\mu\sigma_2 -\\
&\quad 2\gamma_1\gamma_2^2\Lambda\mu\sigma_2 + \beta_1\gamma_1\Lambda^2\mu\sigma_2 + \beta_2\gamma_1\Lambda^2\mu\sigma_2 + 2\beta_1\gamma_2\Lambda^2\mu\sigma_2 + \beta_2\gamma_2\Lambda^2\mu\sigma_2 - \gamma_1^2\Lambda\mu^2\sigma_2 -\\
&\quad 4\gamma_1\gamma_2\Lambda\mu^2\sigma_2 - \gamma_2^2\Lambda\mu^2\sigma_2 + \beta_1\Lambda^2\mu^2\sigma_2 + \beta_2\Lambda^2\mu^2\sigma_2 - 2\gamma_1\Lambda\mu^3\sigma_2 - 2\gamma_2\Lambda\mu^3\sigma_2 - \Lambda\mu^4\sigma_2 +\\
&\quad s\,\big(-\gamma_1^2\gamma_2^2\mu + 2\beta_1\gamma_1\gamma_2\Lambda\mu + \beta_2\gamma_1\gamma_2\Lambda\mu + \beta_1\gamma_2^2\Lambda\mu - \beta_1\beta_2\Lambda^2\mu - 2\gamma_1^2\gamma_2\mu^2 - 2\gamma_1\gamma_2^2\mu^2 +\\
&\qquad \beta_1\gamma_1\Lambda\mu^2 + \beta_2\gamma_1\Lambda\mu^2 + 2\beta_1\gamma_2\Lambda\mu^2 + \beta_2\gamma_2\Lambda\mu^2 - \gamma_1^2\mu^3 - 4\gamma_1\gamma_2\mu^3 - \gamma_2^2\mu^3 + \beta_1\Lambda\mu^3 +\\
&\qquad \beta_2\Lambda\mu^3 - 2\gamma_1\mu^4 - 2\gamma_2\mu^4 - \mu^5 + \beta_1\gamma_1\gamma_2^2\Lambda\sigma_2 - \beta_1\beta_2\gamma_2\Lambda^2\sigma_2\big)\}\ \text{of deg}\ \{1, 2, 1, 1, 3\}
\end{aligned}
$$

```
(*Direct stab anal for EE1*)
jac=Grad[RHSc,var];
jac1=jac/.so1[[2]]//Factor;ch1=CharacteristicPolynomial[jac1,u]//Factor
ch1//Length
Exponent[ch1[[6]],u]
```

Out[∘]=

$$-\frac{1}{\beta_1^2\mu^3(\Theta_1+\gamma_1+\mu)^3}(u+\mu)\left(\Lambda^3\Theta_1^3\Theta u^2\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1+\Lambda^3\Theta_1^3 u^3\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1+\Lambda^3\Theta_1^2\Theta u^3\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1+\Lambda^3\Theta_1^2 u^4\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1+\Lambda^3\Theta_1^3\Theta u\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1^2+\Lambda^3\Theta_1^3 u^2\alpha_1\alpha_2\beta_1^4\beta_2\gamma_1^2+\ldots 75116\ldots+\alpha_1\beta_1\beta_2\gamma_2\mu^{12}+\beta_1^2\mu^{13}-\alpha_1\beta_1^2\mu^{13}-\beta_1\beta_2\mu^{13}+\alpha_1\beta_1\beta_2\mu^{13}\right)$$

Full expression not available (original memory size: 21.7 MB)

Out[∘]=

6

In[∘]:=

```
(*Direct stab anal for EE1, symmetric case*)
jac1=jac/.so1[[2]]/.Append[csym,cLa]//Factor;ch1=CharacteristicPolynomial[jac1,u]//Factor
ch1//Length
Exponent[ch1[[5]],u]
```

Out[∘]=

$$-\frac{1}{\beta_1(\Theta+\gamma+\mu)^3}(u+\mu)\left(-\Theta^6 u^4\beta_1-3\Theta^5 u^5\beta_1-3\Theta^4 u^6\beta_1-\Theta^3 u^7\beta_1-\Theta^6 u^3\beta_1^2-3\Theta^5 u^4\beta_1^2-3\Theta^4 u^5\beta_1^2-\Theta^3 u^6\beta_1^2+r_2\Theta^6 u^3\alpha_1\beta_1^2-\Theta^5 u^4\alpha_1\beta_1^2+\ldots 13898\ldots+7\beta_1\gamma\mu^9-7\alpha_1\beta_1\gamma\mu^9-7\beta_2\gamma\mu^9+7\alpha_1\beta_2\gamma\mu^9+\alpha_2\beta_2\gamma\mu^9-\alpha_1\alpha_2\beta_2\gamma\mu^9+\beta_1\mu^{10}-\alpha_1\beta_1\mu^{10}-\beta_2\mu^{10}+\alpha_1\beta_2\mu^{10}\right)$$

Full expression not available (original memory size: 3.3 MB)

Out[∘]=

5

Out[∘]=

7

```
Timing[fu=FullSimplify[Reduce[Rc>1, {be1,be2}], Assumptions → Append[cp,R1s>1]]]
fu//Length
fu[[1]]
```

$$\begin{pmatrix} s\,be_2-ga_2-mu & \eta_2\,s\,be_2 & 0 \\ r_1\,si_2\,be_2 & r_1\,si_2\,be_2-ga_2-mu & 0 \\ ga_2 & 0 & -\Theta_2-i_1\,si_1\,be_1-y_1\,si_1\,be_1-mu \end{pmatrix}$$

Eigs of K are

$$\left\{-\frac{(i_1 + y_1)\, si_1\, be_1}{\theta_2 + mu}, \frac{s\,\theta_2\, be_2 + r_1\,\theta_2\, si_2\, be_2 + s\, be_2\, mu + r_1\, si_2\, be_2\, mu - \sqrt{s^2 - 2\, r_1\, s\, si_2 + 4\,\eta_2\, r_1\, s\, si_2 + r_1^2\, si_2^2}\; be_2\, (\theta_2 + mu)}{2\,(\theta_2 + mu)\,(ga_2 + mu)}, \frac{s\,\theta_2\, be_2 + r_1\,\theta_2\, si_2\, be_2 + s\, be_2\, mu + r_1\, si_2\, be_2\, mu + \sqrt{s^2 - 2\, r_1\, s\, si_2 + 4\,\eta_2\, r_1\, s\, si_2 + r_1^2\, si_2^2}\; be_2\, (\theta_2 + mu)}{2\,(\theta_2 + mu)\,(ga_2 + mu)}\right\}$$

at E1

$$\Big\{0.`, \frac{1}{2\, be_1\, mu\, (\theta_1 + ga_1 + mu)\,(ga_2 + mu)}\, be_2 \Big(\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu) + \sqrt{}\Big(mu^2\,(ga_1 + mu)^2\,(\theta_1 + ga_1 + mu)^2 + 2\, si_2\, ga_1\, mu\,(ga_1 + mu)\,(\theta_1 + ga_1 + mu)\,(-\Lambda\, be_1 + mu\,(ga_1 + mu)) + 4\,\eta_2\, si_2\, mu\,(ga_1 + mu)\,(\theta_1 + ga_1 + mu)\,(\Lambda\, be_1\, ga_1 - ga_1\, mu\,(ga_1 + mu)) + si_2^2\,(\Lambda\, be_1\, ga_1 - ga_1\, mu\,(ga_1 + mu))^2\Big)\Big)\Big\}$$

$$\frac{be_2\,(2\,\Lambda\, si_2\, be_1\, ga_1 + 2\, mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu))}{2\, be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}$$

$$\Big\{21.25`, \Big(be_2 > \frac{be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}{\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu)}\ \&\&\ (\theta_1 + ga_1 + mu == si_2\, ga_1\ ||\ si_2\, ga_1 < \theta_1 + ga_1 + mu\ ||\ (si_2\, ga_1 > \theta_1 + ga_1 + mu\ \&\&\ \Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu) > 0))\Big)\ ||\ \Big(\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu) < 0\ \&\&\ be_2 < \frac{be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}{\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu)}\ \&\&\ si_2\, ga_1 > \theta_1 + ga_1 + mu\Big)\Big\}$$

Out[*]=

2

$$be_2 > \frac{be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}{\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu)}\ \&\&\ (\theta_1 + ga_1 + mu == si_2\, ga_1\ ||\ si_2\, ga_1 < \theta_1 + ga_1 + mu\ ||\ (si_2\, ga_1 > \theta_1 + ga_1 + mu\ \&\&\ \Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu) > 0))$$

In[*]:=

```
fu[[2]]
fu[[2]][[2]][[2]]
```

$$\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu) < 0\ \&\&\ be_2 < \frac{be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}{\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu)}\ \&\&\ si_2\, ga_1 > \theta_1 + ga_1 + mu$$

$$\frac{be_1\, mu\,(\theta_1 + ga_1 + mu)\,(ga_2 + mu)}{\Lambda\, si_2\, be_1\, ga_1 + mu\,(ga_1 + mu)\,(\theta_1 + ga_1 - si_2\, ga_1 + mu)}$$

In[*]:=

```
Timing[fu = FullSimplify[Reduce[Rc > 1], Assumptions → Append[cp, R1s > 1]]]
fu // Length
fu⟦1⟧
```

$$\left\{18.796875`,\ \left(be_2 < \frac{be_1\ mu\ (\theta_1 + ga_1 + mu)\ (ga_2 + mu)}{\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu)}\ \&\&\ \Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu) < 0\right)\ ||\ \left(be_2 > \frac{be_1\ mu\ (\theta_1 + ga_1 + mu)\ (ga_2 + mu)}{\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu)}\ \&\&\ \Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu) > 0\right)\right\}$$

Out[∘]=

2

$$be_2 < \frac{be_1\ mu\ (\theta_1 + ga_1 + mu)\ (ga_2 + mu)}{\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu)}\ \&\&\ \Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu) < 0$$

In[∘]:=

```
fu⟦2⟧
fu⟦2⟧⟦2⟧⟦2⟧
```

$$\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu) < 0\ \&\&\ be_2 < \frac{be_1\ mu\ (\theta_1 + ga_1 + mu)\ (ga_2 + mu)}{\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu)}\ \&\&\ si_2\ ga_1 > \theta_1 + ga_1 + mu$$

$$\frac{be_1\ mu\ (\theta_1 + ga_1 + mu)\ (ga_2 + mu)}{\Lambda\ si_2\ be_1\ ga_1 + mu\ (ga_1 + mu)\ (\theta_1 + ga_1 - si_2\ ga_1 + mu)}$$

```
Needs["RobertNachbar`CompartmentalModeling`"]
FileNameJoin[{$UserBaseDirectory, "Paclets",
  "Repository", "RobertNachbar__CompartmentalModeling-1.11.2"}]
SystemOpen[%]
```

Out[∘]=

```
C:\Users\flori\AppData\Roaming\Mathematica\Paclets\Repository\
  RobertNachbar__CompartmentalModeling-1.11.2
```

In[∘]:=

```
CompartmentalModelGraph[{2 A + B →(k1) X, X →(k2) 2 Y}, GraphTheme → "PetriNet", options]
```