In[485]:=

```
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
SetOptions[$FrontEndSession, NotebookAutoSave → True];
NotebookSave[];
AppendTo[$Path, FileNameJoin[{$HomeDirectory, "Dropbox", "EpidCRNmodels"}]];
Needs["EpidCRN`"];
(*Latex dictionary*)
Format[mu] := μ; Format[muP] := Subscript[μ, P];
Format[ga] := ga;
Format[ga1] := Subscript[γ, 1];
Format[ga2] := Subscript[γ, 2];
Format[ga12] := Subscript[γ, 12]; Format[ga2] := Subscript[γ, 21];
Format[th1] := Subscript[θ, 1];
Format[th2] := Subscript[θ, 2];
Format[th3] := Subscript[θ, 12];
Format[thv] := Subscript[θ, v];
Format[th] := θ;
Format[La] := Λ; Format[LaP] := Subscript[Λ, p];
Format[be1] := Subscript[β, 1]; Format[be2] := Subscript[β, 2];
Format[be12] := Subscript[β, 12]; Format[be21] := Subscript[β, 21];
Format[de1] := Subscript[δ, 1]; Format[de2] := Subscript[δ, 2];
Format[de12] := Subscript[δ, 12]; Format[de21] := Subscript[δ, 21];
Format[deS] := Subscript[δ, s];
Format[mu1] := Subscript[μ, 1]; Format[mu2] := Subscript[μ, 2];
Format[mu12] := Subscript[μ, 12]; Format[mu21] := Subscript[μ, 21];
Format[al1] := Subscript[α, 1];
Format[al2] := Subscript[α, 2];
Format[alv] := Subscript[α, v];
Format[si] := σ;
Format[rh] := ρ;
Format[si1] := Subscript[σ, 1]; Format[si2] := Subscript[σ, 2];
Format[et1] := Subscript[η, 1]; Format[et2] := Subscript[η, 2];
Format[i1] := Subscript[i, 1]; Format[i2] := Subscript[i, 2];
Format[i12] := Subscript[i, 12]; Format[i21] := Subscript[i, 21];
Format[r1] := Subscript[r, 1];
Format[r2] := Subscript[r, 2];
Format[r12] := Subscript[r, 12];
(*Gavish two-strain model*)(*Test cont with real bdAnalEx outputs*)(*Your setup*)
RN = {(*epidemic reactions:12 total*)"S" + "I1" → 2 * "I1", "S" + "I21" → "I1" + "I21",
   "S" + "I2" → 2 * "I2", "S" + "I12" → "I2" + "I12", "R1" + "I2" → "I12" + "I2",
   "R1" + "I12" → 2 * "I12", "R2" + "I1" → "I21" + "I1", "R2" + "I21" → 2 * "I21", "I1" → "R1",
   "I2" → "R2", "I12" → "R12", "I21" → "R12", (*"I1"→"D","I2"→"D","I12"→"D",
   "I21"→"D",*)(*ecological reactions:16 total*)0 → "S", "S" → 0, "R1" → 0, "R2" → 0,
   "R12" → 0, 0 → "P", "P" → 0, "I1" → 0, "I2" → 0, "I12" → 0, "I21" → 0, "P" + "S" → 2 * "P",
   "P" + "I1" → 2 * "P", "P" + "I2" → 2 * "P", "P" + "I12" → 2 * "P", "P" + "I21" → 2 * "P"};
rts = {(*epidemic transitions*)be1 * s * i1, be1 * s * i21, be2 * s * i2,
```

```
    be2 * s * i12, be12 * r1 * i2, be12 * r1 * i12, be21 * r2 * i1, be21 * r2 * i21, ga1 * i1,
    ga2 * i2, ga12 * i12, ga21 * i21, (*mu1*i1,mu2*i2,mu12*i12,mu21*i21,*)
    (*ecological transitions*)La, (*inflow 0→s*)mu * s, (*natural loss of s*)
    mu * r1, mu * r2, mu * r12, LaP, (*inflow 0→p*)muP * p, (*natural loss of p*)
    mu1 * i1, mu2 * i2, mu12 * i12, mu21 * i21, deS * p * s, (*predation on s*)
    de1 * p * i1, (*predation on i1*)de2 * p * i2, (*predation on i2*)
    de12 * p * i12, (*predation on i12*)de21 * p * i21  (*predation on i21*)};
Print["reactions and transitions: ", Transpose[{RN, rts}] // MatrixForm]
{RHS, var, par, cp, mSi, Jx, Jy, cDFE, E0, K, R0A, infV, alp,
 bet, gam, ngm} = bdAn[RN, rts];
Print["RHS=", RHS // FullSimplify // MatrixForm, "mSi=", mSi, "E0=", E0,
  " K= ", K // MatrixForm, "where the order is ", infV, " after permutation ",
  K[[{1, 4, 2, 3}, {1, 4, 2, 3}]] // MatrixForm, "the repr. funs are", R0A];
```

reactions and transitions:

$I1 + S \to 2\,I1$	$\beta_1\, i_1\, s$
$I21 + S \to I1 + I21$	$\beta_1\, i_{21}\, s$
$I2 + S \to 2\,I2$	$\beta_2\, i_2\, s$
$I12 + S \to I12 + I2$	$\beta_2\, i_{12}\, s$
$I2 + R1 \to I12 + I2$	$\beta_{12}\, i_2\, r_1$
$I12 + R1 \to 2\,I12$	$\beta_{12}\, i_{12}\, r_1$
$I1 + R2 \to I1 + I21$	$\beta_{21}\, i_1\, r_2$
$I21 + R2 \to 2\,I21$	$\beta_{21}\, i_{21}\, r_2$
$I1 \to R1$	$\gamma_1\, i_1$
$I2 \to R2$	$\gamma_{21}\, i_2$
$I12 \to R12$	$\gamma_{12}\, i_{12}$
$I21 \to R12$	$ga21\, i_{21}$
$0 \to S$	Λ
$S \to 0$	$\mu\, s$
$R1 \to 0$	$\mu\, r_1$
$R2 \to 0$	$\mu\, r_2$
$R12 \to 0$	$\mu\, r_{12}$
$0 \to P$	Λ_p
$P \to 0$	$\mu_P\, p$
$I1 \to 0$	$i_1\, \mu_1$
$I2 \to 0$	$i_2\, \mu_2$
$I12 \to 0$	$i_{12}\, \mu_{12}$
$I21 \to 0$	$i_{21}\, \mu_{21}$
$P + S \to 2\,P$	$\delta_s\, p\, s$
$I1 + P \to 2\,P$	$\delta_1\, i_1\, p$
$I2 + P \to 2\,P$	$\delta_2\, i_2\, p$
$I12 + P \to 2\,P$	$\delta_{12}\, i_{12}\, p$
$I21 + P \to 2\,P$	$\delta_{21}\, i_{21}\, p$

$$\text{RHS}=\begin{pmatrix} -i_1(\gamma_1+\mu_1+\delta_1 p)+\beta_1(i_1+i_{21})s \\ \Lambda-(\beta_2(i_{12}+i_2)+\beta_1(i_1+i_{21})+\mu+\delta_s p)s \\ -i_{21}(ga21+\mu_{21}+\delta_{21} p)+\beta_{21}(i_1+i_{21})r_2 \\ -i_2(\gamma_{21}+\mu_2+\delta_2 p)+\beta_2(i_{12}+i_2)s \\ -i_{12}(\gamma_{12}+\mu_{12}+\delta_{12} p)+\beta_{12}(i_{12}+i_2)r_1 \\ \gamma_1 i_1-(\beta_{12}(i_{12}+i_2)+\mu)r_1 \\ \gamma_{21} i_2-(\beta_{21}(i_1+i_{21})+\mu)r_2 \\ \gamma_{12} i_{12}+ga21\, i_{21}-\mu r_{12} \\ \Lambda_p+p(\delta_1 i_1+\delta_{12} i_{12}+\delta_2 i_2+\delta_{21} i_{21}-\mu_P+\delta_s s) \end{pmatrix}\text{mSi=}$$

$$\{\{i1, i21\}, \{i2, i12\}\}\text{E0=}\Bigg\{\Bigg\{p\to\frac{\delta_s\Lambda+\delta_s\Lambda_p-\mu\mu_P+\sqrt{4\delta_s\Lambda_p\mu\mu_P+(\delta_s\Lambda+\delta_s\Lambda_p-\mu\mu_P)^2}}{2\delta_s\mu_P},$$

$$r_1\to 0,\ r_{12}\to 0,\ r_2\to 0,\ s\to\frac{\Lambda+\Lambda_p+\frac{\mu\mu_P}{\delta_s}-\frac{\sqrt{4\delta_s\Lambda_p\mu\mu_P+(\delta_s\Lambda+\delta_s\Lambda_p-\mu\mu_P)^2}}{\delta_s}}{2\mu}\Bigg\},$$

$$\Bigg\{p\to-\frac{-\delta_s\Lambda-\delta_s\Lambda_p+\mu\mu_P+\sqrt{4\delta_s\Lambda_p\mu\mu_P+(\delta_s\Lambda+\delta_s\Lambda_p-\mu\mu_P)^2}}{2\delta_s\mu_P},\ r_1\to 0,$$

$$r_{12}\to 0,\ r_2\to 0,\ s\to\frac{\Lambda+\Lambda_p+\frac{\mu\mu_P}{\delta_s}+\frac{\sqrt{4\delta_s\Lambda_p\mu\mu_P+(\delta_s\Lambda+\delta_s\Lambda_p-\mu\mu_P)^2}}{\delta_s}}{2\mu}\Bigg\}\Bigg\}\ \text{K=}$$

$$\begin{pmatrix} \frac{\beta_1 s}{\gamma_1+\mu_1+\delta_1 p} & 0 & 0 & \frac{\beta_1 s}{ga21+\mu_{21}+\delta_{21} p} \\ 0 & \frac{\beta_{12} r_1}{\gamma_{12}+\mu_{12}+\delta_{12} p} & \frac{\beta_{12} r_1}{\gamma_{21}+\mu_2+\delta_2 p} & 0 \\ 0 & \frac{\beta_2 s}{\gamma_{12}+\mu_{12}+\delta_{12} p} & \frac{\beta_2 s}{\gamma_{21}+\mu_2+\delta_2 p} & 0 \\ \frac{\beta_{21} r_2}{\gamma_1+\mu_1+\delta_1 p} & 0 & 0 & \frac{\beta_{21} r_2}{ga21+\mu_{21}+\delta_{21} p} \end{pmatrix}\text{ where the order is }\{i_1, i_{12}, i_2, i_{21}\}$$

$$\text{after permutation }\begin{pmatrix} \frac{\beta_1 s}{\gamma_1+\mu_1+\delta_1 p} & \frac{\beta_1 s}{ga21+\mu_{21}+\delta_{21} p} & 0 & 0 \\ \frac{\beta_{21} r_2}{\gamma_1+\mu_1+\delta_1 p} & \frac{\beta_{21} r_2}{ga21+\mu_{21}+\delta_{21} p} & 0 & 0 \\ 0 & 0 & \frac{\beta_{12} r_1}{\gamma_{12}+\mu_{12}+\delta_{12} p} & \frac{\beta_{12} r_1}{\gamma_{21}+\mu_2+\delta_2 p} \\ 0 & 0 & \frac{\beta_2 s}{\gamma_{12}+\mu_{12}+\delta_{12} p} & \frac{\beta_2 s}{\gamma_{21}+\mu_2+\delta_2 p} \end{pmatrix}$$

$$\text{the repr. funs are}\Bigg\{\frac{\beta_{12}\gamma_{21} r_1+\beta_{12}\mu_2 r_1+\beta_{12}\delta_2 p r_1+\beta_2\gamma_{12} s+\beta_2\mu_{12} s+\beta_2\delta_{12} p s}{(\gamma_{12}+\mu_{12}+\delta_{12} p)(\gamma_{21}+\mu_2+\delta_2 p)},$$

$$\frac{\beta_{21}\gamma_1 r_2+\beta_{21}\mu_1 r_2+\beta_{21}\delta_1 p r_2+\beta_1\, ga21\, s+\beta_1\mu_{21} s+\beta_1\delta_{21} p s}{(\gamma_1+\mu_1+\delta_1 p)(ga21+\mu_{21}+\delta_{21} p)}\Bigg\}$$

In[•]:=
```
bdfp = bdFp[RHS, var, mSi];
Print["rat sols on first siphon facet are"]
bd1 = bdfp[[1, 1]] // FullSimplify
bdfp[[1, 2]]
```

```
fps on siphon facet {i1, y1}: 3 boundary points
fps on siphon facet {i2, y2}: 3 boundary points
rat sols on first siphon facet are
```

Out[∘]=

$$\left\{\left\{i_2 \to 0,\ R \to 0,\ r_1 \to 0,\ r_2 \to 0,\ s \to \frac{\Lambda}{\mu},\ y_2 \to 0\right\},\right.$$

$$\left\{i_2 \to \frac{(\beta_2 \Lambda - \mu\ (\gamma_2 + \mu))\ (\mu + \Theta_2)}{\beta_2\ \mu\ (\gamma_2 + \mu + \Theta_2)},\ R \to 0,\ r_1 \to 0,\ r_2 \to \frac{\beta_2\ \gamma_2\ \Lambda - \gamma_2\ \mu\ (\gamma_2 + \mu)}{\beta_2\ \mu\ (\gamma_2 + \mu + \Theta_2)},\ s \to \frac{\gamma_2 + \mu}{\beta_2},\ y_2 \to 0\right\},$$

$$\{i_2 \to -(((\mu + \Theta_1)\ (\mu + \Theta_2)\ (\gamma_2\ \mu\ (\Theta_1 - \Theta_3) + \beta_2\ \eta_2\ \Lambda\ \sigma_2\ (\mu + \Theta_3) + \mu\ \Theta_1\ (\mu + \Theta_3)))\ /$$
$$(\beta_2\ \mu\ \sigma_2\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3))))),$$

$$R \to \left(\gamma_2\ (\mu + \Theta_1)\ \left(-\left(\left(\mu^2\ (-1 + \sigma_2) - \beta_2\ \Lambda\ \sigma_2 - \mu\ \Theta_1\right)\ (\mu + \Theta_2)\right) + \gamma_2\ \mu\ (\mu - \mu\ \sigma_2 + \Theta_1 - \sigma_2\ \Theta_2)\right)\right) \Big/$$
$$(\beta_2\ \mu\ \sigma_2\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3)))),\ r_1 \to$$

$$-\left(\left((\gamma_2 + \mu)\ \left(-\left(\left(\mu^2\ (-1 + \sigma_2) - \beta_2\ \Lambda\ \sigma_2 - \mu\ \Theta_1\right)\ (\mu + \Theta_2)\right) + \gamma_2\ \mu\ (\mu - \mu\ \sigma_2 + \Theta_1 - \sigma_2\ \Theta_2)\right)\ (\mu + \Theta_3)\right) \Big/\right.$$
$$(\beta_2\ \mu\ \sigma_2\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$\left.(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3))))\right),$$

$$r_2 \to -((\gamma_2\ (\mu + \Theta_1)\ (\gamma_2\ \mu\ (\Theta_1 - \Theta_3) + \beta_2\ \eta_2\ \Lambda\ \sigma_2\ (\mu + \Theta_3) + \mu\ \Theta_1\ (\mu + \Theta_3)))\ /$$
$$(\beta_2\ \mu\ \sigma_2\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3))))),$$

$$s \to ((\gamma_2 + \mu)\ (\mu + \Theta_2)\ (\gamma_2\ \mu\ (\Theta_1 - \Theta_3) + \beta_2\ \eta_2\ \Lambda\ \sigma_2\ (\mu + \Theta_3) + \mu\ \Theta_1\ (\mu + \Theta_3)))\ /$$
$$(\beta_2\ \mu\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3)))),$$

$$y_2 \to \left((\mu + \Theta_1)\ \left(-\left(\left(\mu^2\ (-1 + \sigma_2) - \beta_2\ \Lambda\ \sigma_2 - \mu\ \Theta_1\right)\ (\mu + \Theta_2)\right) + \gamma_2\ \mu\ (\mu - \mu\ \sigma_2 + \Theta_1 - \sigma_2\ \Theta_2)\right)\ (\mu + \Theta_3)\right) \Big/$$
$$(\beta_2\ \mu\ \sigma_2\ (\eta_2\ ((\mu\ (-1 + \sigma_2) - \Theta_1)\ (\mu + \Theta_2) + \gamma_2\ (\mu\ (-1 + \sigma_2) - \Theta_1 + \sigma_2\ \Theta_2))\ (\mu + \Theta_3) +$$
$$\left.(\mu + \Theta_2)\ (\gamma_2\ (\Theta_1 - \Theta_3) + \Theta_1\ (\mu + \Theta_3))))\}\right\}$$

Out[∘]=

```
AllSolsRational
```

In[∘]:=

```
{E1, E2, R12, R21, coP} = invN2[bdfp[[1, 1]], bdfp[[2, 1]], R0A, E0, par, cp, 2, 2];
Print["invasion numbers R12, R21 are ", R12 // Apart, R21 // Apart]
```

Selected sol when i1=0 is (solution 2):

$$\left\{i_2 \to \frac{(\beta_2 \Lambda - \gamma_2 \mu - \mu^2)\ (\mu + \Theta_2)}{\beta_2 \mu\ (\gamma_2 + \mu + \Theta_2)},\ R \to 0,\ r_1 \to 0,\ r_2 \to \frac{\gamma_2\ (\beta_2 \Lambda - \gamma_2 \mu - \mu^2)}{\beta_2 \mu\ (\gamma_2 + \mu + \Theta_2)},\ s \to \frac{\gamma_2 + \mu}{\beta_2},\ y_2 \to 0\right\}$$

Selected sol when i2=0 is (solution 2):

$$\left\{i_1 \to \frac{(\beta_1 \Lambda - \gamma_1 \mu - \mu^2)\ (\mu + \Theta_1)}{\beta_1 \mu\ (\gamma_1 + \mu + \Theta_1)},\ R \to 0,\ r_1 \to \frac{\gamma_1\ (\beta_1 \Lambda - \gamma_1 \mu - \mu^2)}{\beta_1 \mu\ (\gamma_1 + \mu + \Theta_1)},\ r_2 \to 0,\ s \to \frac{\gamma_1 + \mu}{\beta_1},\ y_1 \to 0\right\}$$

under coP: $\left\{\beta_1 \to 4,\ \beta_2 \to 3,\ \eta_1 \to 1,\ \eta_2 \to 1,\ \gamma_1 \to 1,\ \gamma_2 \to 1,\ \Lambda \to 1,\ \mu \to 1,\ \sigma_1 \to 1,\ \sigma_2 \to 1,\ \Theta_1 \to \frac{1}{2},\ \Theta_2 \to 1,\ \Theta_3 \to 1\right\}$

invasion nrs are{1.05, 1.55556} repr nrs are{2., 1.5}

END invNr OUTPUT

invasion numbers R12, R21 are $\frac{\beta_2\ (\gamma_1 + \mu)}{\beta_1\ (\gamma_2 + \mu)} + \frac{\beta_1 \beta_2 \eta_2 \gamma_1 \Lambda \sigma_2 - \beta_2 \eta_2 \gamma_1^2 \mu \sigma_2 - \beta_2 \eta_2 \gamma_1 \mu^2 \sigma_2}{\beta_1 \mu\ (\gamma_2 + \mu)\ (\gamma_1 + \mu + \Theta_1)}$

$$\frac{\beta_1\ (\gamma_2 + \mu)}{\beta_2\ (\gamma_1 + \mu)} + \frac{\beta_1 \beta_2 \eta_1 \gamma_2 \Lambda \sigma_1 - \beta_1 \eta_1 \gamma_2^2 \mu \sigma_1 - \beta_1 \eta_1 \gamma_2 \mu^2 \sigma_1}{\beta_2 \mu\ (\gamma_1 + \mu)\ (\gamma_2 + \mu + \Theta_2)}$$

```
(*E1*)
cE2 = bd1[[2]]
cE1 = bdfp[[2, 1]][[2]]
jac = Grad[RHS, var];
j1 = jac /. cE1 /. {i2 → 0, y2 → 1} // FullSimplify;
cChu = {et1 → 1, et2 → 1, th3 → 0, th1 → 0, th2 → 0, La → mu};
Print[" At cE1, chp of jac, ch1, factorizes as"]
ch1 = Numerator[Together[CharacteristicPolynomial[j1, u] /. cChu]] // Factor
(*{lSta,qSta,hDeg,ll,ql}=sta[ch1,par];
Print["Stability of E1 holds iff"]
  re1=Reduce[Join[cp,lSta,qSta]]//FullSimplify*)
```

Out[∘]=

$$\left\{i_2 \to \frac{(\beta_2 \Lambda - \mu (\gamma_2 + \mu)) (\mu + \Theta_2)}{\beta_2 \mu (\gamma_2 + \mu + \Theta_2)}, R \to 0, r_1 \to 0, r_2 \to \frac{\beta_2 \gamma_2 \Lambda - \gamma_2 \mu (\gamma_2 + \mu)}{\beta_2 \mu (\gamma_2 + \mu + \Theta_2)}, s \to \frac{\gamma_2 + \mu}{\beta_2}, y_2 \to 0\right\}$$

Out[∘]=

$$\left\{i_1 \to \frac{\left(\beta_1 \Lambda - \gamma_1 \mu - \mu^2\right) (\mu + \Theta_1)}{\beta_1 \mu (\gamma_1 + \mu + \Theta_1)}, R \to 0, r_1 \to \frac{\gamma_1 \left(\beta_1 \Lambda - \gamma_1 \mu - \mu^2\right)}{\beta_1 \mu (\gamma_1 + \mu + \Theta_1)}, r_2 \to 0, s \to \frac{\gamma_1 + \mu}{\beta_1}, y_1 \to 0\right\}$$

At cE1, chp of jac, ch1, factorizes as

Out[∘]=

$$(\mu + u) \left(-\beta_1 \beta_2 \gamma_1^5 \gamma_2 \mu^3 + \beta_2 \gamma_1^6 \gamma_2 \mu^3 + \beta_1^2 \gamma_1^4 \gamma_2^2 \mu^3 - \beta_1 \gamma_1^5 \gamma_2^2 \mu^3 - \beta_1 \beta_2 \gamma_1^5 \mu^4 + \beta_2 \gamma_1^6 \mu^4 + 2 \beta_1^2 \gamma_1^4 \gamma_2 \mu^4 - 5 \beta_1 \beta_2 \gamma_1^4 \gamma_2 \mu^4 - 2 \beta_1 \gamma_1^5 \gamma_2 \mu^4 + 6 \beta_2 \gamma_1^5 \gamma_2 \mu^4 + 4 \beta_1^2 \gamma_1^3 \gamma_2^2 \mu^4 - 5 \beta_1 \gamma_1^4 \gamma_2^2 \mu^4 + \beta_1^2 \gamma_1^4 \mu^5 - 5 \beta_1 \beta_2 \gamma_1^4 \mu^5 - \beta_1 \gamma_1^5 \mu^5 + 6 \beta_2 \gamma_1^5 \mu^5 + \cdots 2049 \cdots + 2 \beta_1^2 \gamma_1 \mu^2 \sigma_1 u^6 - 3 \beta_1 \gamma_1^2 \mu^2 \sigma_1 u^6 + \beta_1^2 \mu^3 \sigma_1 u^6 - 3 \beta_1 \gamma_1 \mu^3 \sigma_1 u^6 - \beta_1 \mu^4 \sigma_1 u^6 + \beta_2 \gamma_1^4 \sigma_2 u^6 + \beta_1 \beta_2 \gamma_1^2 \mu \sigma_2 u^6 + 3 \beta_2 \gamma_1^3 \mu \sigma_2 u^6 + 2 \beta_1 \beta_2 \gamma_1 \mu^2 \sigma_2 u^6 + 3 \beta_2 \gamma_1^2 \mu^2 \sigma_2 u^6 + \beta_1 \beta_2 \mu^3 \sigma_2 u^6 + \beta_2 \gamma_1 \mu^3 \sigma_2 u^6 + \beta_1 \gamma_1^3 u^7 + 3 \beta_1 \gamma_1^2 \mu u^7 + 3 \beta_1 \gamma_1 \mu^2 u^7 + \beta_1 \mu^3 u^7\right)$$

Full expression not available (original memory size: 459.3 kB)

Stability of E1 holds iff

Out[∘]=

$$\beta_1 > 0 \,\&\&\, \beta_2 > 0 \,\&\&\, \eta_1 > 0 \,\&\&\, \eta_2 > 0 \,\&\&\, \gamma_1 > 0 \,\&\&\, \gamma_2 > 0 \,\&\&\, \Lambda > 0 \,\&\&\, \mu > 0 \,\&\&\, \sigma_1 > 0 \,\&\&\, \sigma_2 > 0 \,\&\&\, \Theta_1 > 0 \,\&\&\, \Theta_2 > 0 \,\&\&\, \Theta_3 > 0$$

In[∘]:=

```
{R01, R02} = R0A /. E0;
gridRes = 50;
plotInd = {1, 2};
steTol = 10^(-8);
staTol = 10^(-10);
choTol = 10^(-13);(*tMax=300;
nIc=8;*)
Timing[(*Step 3:Scanning-now with mSi from step 1*)
 {plot, noSol, results} = scan[RHS, var, par, coP, plotInd, mSi,
    (*mSi auto-provided*)gridRes, steTol, staTol, choTol, R01, R02, R12, R21];]
plot
```

```
Auto-determined varInd: {1, 2, 5}
  Susceptible: s
  Strain 1 (first compartment): i1
  Strain 2 (first compartment): i2
Computing R01=1, R02=1 intersection point...
R01-R02 intersection at: β1 = 2., β2 = 2.
Adjusted ranges: β1 ∈ [0.001, 8.], β2 ∈ [0.001, 6.]
Scanning 2500 parameter combinations...
```

```
Checked 730 potential coexistence points
```

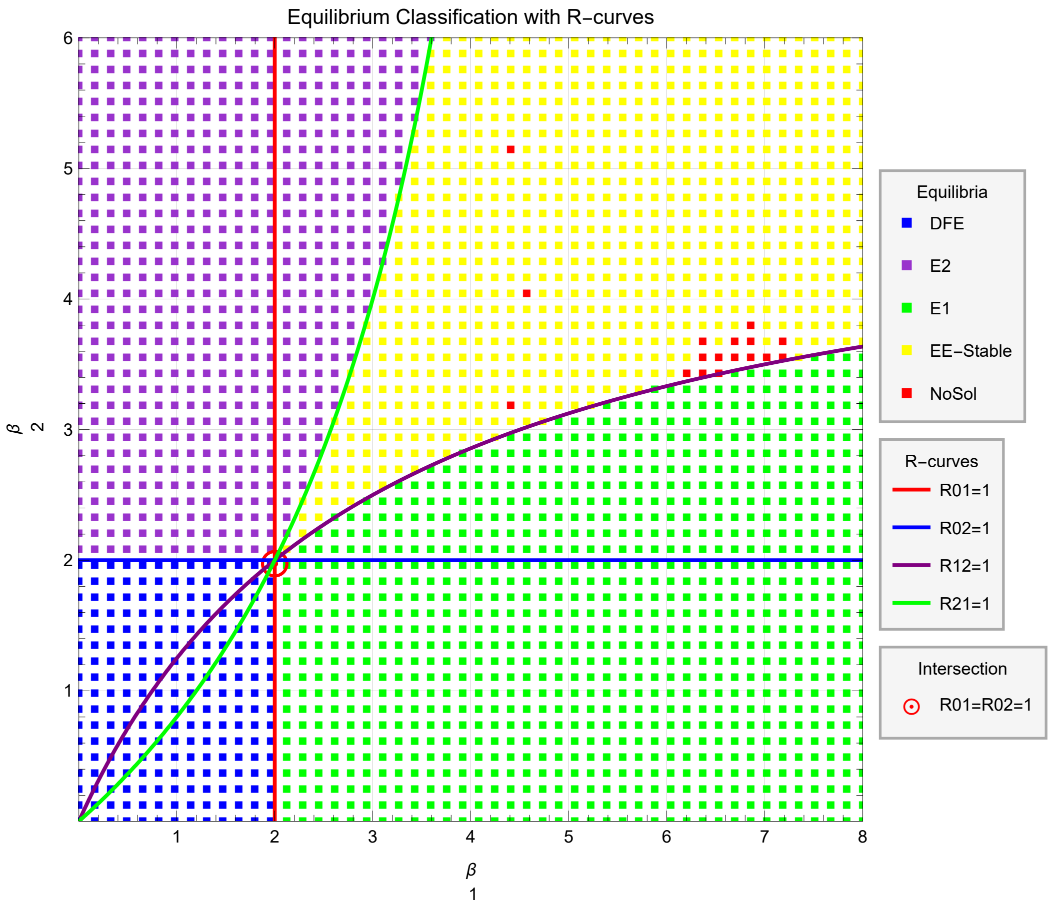
All Rij equations: $\left\{\frac{\beta_1}{2} == 1, \frac{\beta_2}{2} == 1, \frac{(3+\beta_1)\ \beta_2}{5\ \beta_1} == 1, \frac{\beta_1\ (4+\beta_2)}{6\ \beta_2} == 1\right\}$

```
DFE: 221 (9%)
E2: 612 (24%)
E1: 937 (37%)
EE-Stable: 713 (29%)
NoSol: 17 (1%)
```

Out[ ]=

```
{7.26563, Null}
```

Out[ ]=



In[ ]:= **Export["GavScan.pdf", plot]**

Out[ ]=

GavScan.pdf