In[1]:=

```
(*0: Latex dictionary*)
Format[g3]:=Subscript[γ,3];
Format[x1]:=Subscript[x,1];Format[x2]:=Subscript[x,2];Format[x3]:=Subscript[x,3];
Format[x4]:=Subscript[x,4];Format[x5]:=Subscript[x,5];
Format[x6]:=Subscript[x,6];Format[x7]:=Subscript[x,7];Format[x8]:=Subscript[x,8];Format[be]:=β;
Format[la]:=λ;Format[mu]:=μ;Format[m1]:=Subscript[μ,1];
Format[m2]:=Subscript[μ,2];Format[m3]:=Subscript[μ,3];Format[m4]:=Subscript[μ,4];
Format[g4]:=Subscript[γ,4];Format[g5]:=Subscript[γ,5];
Format[g6]:=Subscript[γ,6];
Format[g7]:=Subscript[γ,7];Format[g8]:=Subscript[γ,8];
Format[ga1]:=Subscript[γ,1];Format[ga2]:=Subscript[γ,2];Format[et]=η;Format[be1]:=Subscript[β,1
Format[a1]:=Subscript[α,1];Format[a2]:=Subscript[α,2];Format[al]:=α;Format[de]:=δ;
Format[ep1]:=Subscript[ϵ,1];Format[ep2]:=Subscript[ϵ,2];Format[La]:=Λ;
```

In[13]:=
```
(*1: First steps*)
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];SetOptions[$FrontEndSession, NotebookAutoSave → True];
NotebookSave[];
AppendTo[$Path, FileNameJoin[{$HomeDirectory, "Dropbox", "EpidCRNmodels"}]];
asoRea[RN_]:=Module[{parseSide},parseSide[str_]:=If[str===0,{},StringSplit[ToString[str],"+"]//
Map[Function[r,Association["Substrates"→parseSide[r[[1]]],"Products"→parseSide[r[[2]]]]],RN]]

(*Corrected minimal siphon finder*)
minSiph[species_List,reactions_List]:=Module[{ns,sm,specs,constraints,solutions,siphons,minimal},
sm=AssociationThread[species→Range[ns]];
specs=Array[Symbol["s"<>ToString[#]]&,ns];
(*Build constraints*)constraints={Or@@specs};(*At least one species in siphon*)Do[Module[{subIdx
products=reaction["Products"];
(*Convert species names to indices*)subIdx=If[substrates==={}||substrates==={""},{},Select[Look
prodIdx=If[products==={}||products==={""},{},Select[Lookup[sm,products,Nothing],IntegerQ]];
(*Add constraints for each product*)Do[If[Length[subIdx]==0,(*Empty product:product cannot be in
Print["Constraints generated: ",Length[constraints]];
Print["Sample constraints: ",Take[constraints,Min[5,Length[constraints]]]];
(*Find solutions with moderate limit to avoid crashes*)solutions=FindInstance[constraints,specs,I
If[solutions==={},Return[{}]];
siphons=Map[Flatten@Position[specs/. #,True]&,solutions];
siphons=DeleteDuplicates[siphons];
Print["All found siphons: ",siphons];
(*Proper minimality check:remove any siphon that contains another*)minimal={};
Do[If[Not[AnyTrue[siphons,Function[other,other=!=siphon&&SubsetQ[siphon,other]]]],AppendTo[minim
Print["minimal siphons: ",minimal];
minimal]



var=Array[Symbol["x" <> ToString[#]] &, 8];
va1=Symbol["x" <> ToString[#]] & /@ Range[2, 8];
v46={x4,x6};v57={x5,x7};
l4=b1 x2 x6/(1+a1 x2+ep1 x6);
l5=b2 x2 x7/(1+a2 x2+ep2 x7); g45=g4 x4 + g5 x5;
(*{R1,R2}={(b2*mu)/(g7*(et + a2*mu)),(b1*mu)/(g6*(et + a1*mu))};*)
sd=La/mu;mR1=be1/(ga1+mu);mR2=be2/(ga2+mu);RR1=mR1 sd;RR2=mR2 sd;
k1=ga1/(ga1+mu+t1);a2c=1/k1;R12=mR2 (1/mR1+ si2 r11);r11=k1(sd-1/mR1);
R2c=R2/R12;
cRFE={x4→0,x6→0,x5→0,x7→0,x8→0};cRFEp={x3→0,x4→0,x6→0,x5→0,x7→0,x8→0};

cpol={a1→0,a2→0,ep1→0,ep2→0};
c2345={x2→0,x3→0,x4→0,x5→0};c2=x2→0;cde=de→0;c46={x4→0,x6→0};c57={x5→0,x7→0};
cx1=x1→z+1/mR;
cph=ph→La-mu x1;cLa=La→mu^2(al+et)/(al *et);
RN={0 → "x1",  "x1" + "x2"→2 "x2", "x3" + "x2"→2 "x3",0 → "x4" ,0 →  "x5","x2"→ "x3","x4" →"x6"
"x5"→"x7","x6"→"x8","x7"→"x8","x8"→"x3","x2"→0,"x3"→0};
minSiph[ToString/@var,asoRea[RN]]
```

```
Constraints generated: 1

Sample constraints: {s1 || s2 || s3 || s4 || s5 || s6 || s7 || s8}

All found siphons: {{1, 2, 3, 4, 5, 6, 7, 8}, {2, 3, 4, 5, 6, 7, 8}, {3, 4, 5, 6, 7, 8}, {4, 5, 6, 7, 8},
  {5, 6, 7, 8}, {6, 7, 8}, {7, 8}, {8}, {1, 3, 4, 5, 6, 7, 8}, {2, 4, 5, 6, 7, 8}, {3, 5, 6, 7, 8},
  {4, 6, 7, 8}, {5, 7, 8}, {6, 8}, {7}, {1, 2, 4, 5, 6, 7, 8}, {2, 3, 5, 6, 7, 8}, {3, 4, 6, 7, 8},
  {4, 5, 7, 8}, {5, 6, 8}, {6, 7}, {1, 4, 5, 6, 7, 8}, {2, 5, 6, 7, 8}, {3, 6, 7, 8}, {4, 7, 8}}

minimal siphons: {{8}, {7}}
```

Out[33]=

```
{{8}, {7}}
```

```
Needs["ReactionKinetics`"];reacD=ReactionsData[RN];
Γ= reacD["γ"]//Normal;Γ//MatrixForm;
expo= reacD["α"]//Normal//Transpose;
{comp,r,nR,spec,nS,vol,vars,defi}=reacD["complexes","reactionsteps","R","species","M","volpertgra
"variables","deficiency"];
spec;Print["rates are product of",
rv=expM[var,expo]," and ",
tk={ph,al,et,l4,l5,de,g4,g5,g6,g7,g8,mu,mu}," only case δ=0 is studied"];
Rv=tk*rv/.cde/.cph;
RHS=Γ.Rv//Simplify;
Print["RHS ",RHS//MatrixForm]
par=Par[RHS,var];cp=Thread[par>0];cv=Thread[var≥0];
ct=Join[cp,cv];eq=Thread[(RHS/.c2/.cph)==0];
so=Solve[eq,var]//Flatten;
jac=Grad[RHS,var];
jD=jac/.cDFE;
Print[" At cOSNDE",cDFE=Append[so,c2]," chp of jac/.cDFE factorizes linearly "]
chD=CharacteristicPolynomial[jD,u]//Factor
RH1=Drop[RHS,1]/.cde;va1=Drop[var,1];
mod={RHS,var,par};
inf=Range[2,8];
ng=NGM[mod,inf];M=ng[[1]];
K=ng[[6]];
Print["OSNDFE eigs of K are",K//Eigenvalues]
so=FullSimplify[#]&/@Solve[Thread[(RHS/.cRFE)==0],var];
Print["at RFE, fps are ",so]
jRp=jac/.cRFEp;
Print[" At cRFEp, chp of jac factorizes"]
chRp=CharacteristicPolynomial[jRp,u]//FullSimplify//Factor
eig=(jRp//Eigenvalues)/.so[[3]];eigl=Take[eig, UpTo[Length[eig] - 4]];
Print[" At cRFEp, the 4 linear eigs are",eigl]
minSiph[var,asoRea[RN]]
```

ReactionKinetics Version 1.0 [March 25, 2018] using Mathematica Version 13.3.0 for Microsoft Windows (64-bit) (June 3, 2023) (Version 13.3, Release 0) loaded 29 May 2025 at 16:30 TimeZone
GNU General Public License (GPLv3) Terms Apply.

Please report any issues, comments, complaint related to ReactionKinetics at
jtoth@math.bme.hu, nagyal@math.bme.hu or dpapp@iems.northwestern.edu

rates are product of $\{1, x_1 x_2, x_2 x_3, 1, 1, x_2, x_4, x_5, x_6, x_7, x_8, x_2, x_3\}$ and

$\left\{ph, \alpha, \eta, \frac{\beta_1 x_2 x_6}{1+\alpha_1 x_2+\epsilon_1 x_6}, \frac{\beta_2 x_2 x_7}{1+\alpha_2 x_2+\epsilon_2 x_7}, \delta, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8, \mu, \mu\right\}$ only case δ=0 is studied

$$\text{RHS} \begin{pmatrix} \Lambda - x_1 (\mu + \alpha x_2) \\ -x_2 (\mu - \alpha x_1 + \eta x_3) \\ -\mu x_3 + \eta x_2 x_3 + \gamma_8 x_8 \\ -\gamma_4 x_4 + \frac{\beta_1 x_2 x_6}{1+\alpha_1 x_2+\epsilon_1 x_6} \\ -\gamma_5 x_5 + \frac{\beta_2 x_2 x_7}{1+\alpha_2 x_2+\epsilon_2 x_7} \\ \gamma_4 x_4 - \gamma_6 x_6 \\ \gamma_5 x_5 - \gamma_7 x_7 \\ \gamma_6 x_6 + \gamma_7 x_7 - \gamma_8 x_8 \end{pmatrix}$$

Solve: Equations may not give solutions for all "solve" variables.

ReplaceAll: {cDFE} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

At cOSNDE$\left\{x_1 \to \frac{\Lambda}{\mu}, x_3 \to 0, x_4 \to 0, x_5 \to 0, x_6 \to 0, x_7 \to 0, x_8 \to 0, x_2 \to 0\right\}$

chp of jac/.cDFE factorizes linearly

Out[∘]=

$$\frac{(\gamma_4 + u)(\gamma_5 + u)(\gamma_6 + u)(\gamma_7 + u)(\gamma_8 + u)(\mu + u)^2 \left(-\alpha \Lambda + \mu^2 + \mu u\right)}{\mu}$$

OSNDFE eigs of K are$\left\{\frac{\alpha x_1}{\mu}, 0, 0, 0, 0, 0, 0\right\}$

Solve: Equations may not give solutions for all "solve" variables.

at RFE, fps are

$$\left\{\left\{x_1 \to \frac{\Lambda}{\mu}, x_2 \to 0, x_3 \to 0\right\}, \left\{x_1 \to \frac{\eta \Lambda}{(\alpha+\eta)\mu}, x_2 \to \frac{\mu}{\eta}, x_3 \to \frac{\alpha \Lambda}{(\alpha+\eta)\mu} - \frac{\mu}{\eta}\right\}, \left\{x_1 \to \frac{\mu}{\alpha}, x_2 \to \frac{\Lambda}{\mu} - \frac{\mu}{\alpha}, x_3 \to 0\right\}\right\}$$

At cRFEp, chp of jac factorizes

Out[∘]=

$$\frac{1}{(1+\alpha_1 x_2)(1+\alpha_2 x_2)} (\gamma_8 + u)(\mu + u)(\mu + u - \alpha x_1 + \alpha x_2)(\mu + u - \eta x_2)$$
$$\left(\gamma_4 \gamma_6 + \gamma_4 u + \gamma_6 u + u^2 - \beta_1 \gamma_4 x_2 + \alpha_1 \gamma_4 \gamma_6 x_2 + \alpha_1 \gamma_4 u x_2 + \alpha_1 \gamma_6 u x_2 + \alpha_1 u^2 x_2\right)$$
$$\left(\gamma_5 \gamma_7 + \gamma_5 u + \gamma_7 u + u^2 - \beta_2 \gamma_5 x_2 + \alpha_2 \gamma_5 \gamma_7 x_2 + \alpha_2 \gamma_5 u x_2 + \alpha_2 \gamma_7 u x_2 + \alpha_2 u^2 x_2\right)$$

At cRFEp, the 4 linear eigs are$\left\{-\gamma_8, -\mu, -\alpha\left(\frac{\Lambda}{\mu} - \frac{\mu}{\alpha}\right), -\mu + \eta\left(\frac{\Lambda}{\mu} - \frac{\mu}{\alpha}\right)\right\}$

Part: The expression {{}, {}} cannot be used as a part specification.

Part: The expression {{}, {}} cannot be used as a part specification.

Part: The expression {{}} cannot be used as a part specification.

General: Further output of Part::pkspec1 will be suppressed during this calculation.

FindInstance: $\frac{\text{ga1}+\mu}{\text{be1}}$ is not a valid variable.

ReplaceAll:

$\left\{\frac{\text{ga1}+\mu}{\text{be1}} \,||\, \frac{\text{ga2}+\mu}{\text{be2}} \,||\, \text{s3} \,||\, \text{s4} \,||\, \text{s5} \,||\, \text{s6} \,||\, \text{s7} \,||\, \text{s8}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, !\,\{\}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \ll 2 \gg\right\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindInstance: $\frac{\text{ga1}+\mu}{\text{be1}}$ is not a valid variable.

ReplaceAll:

$\left\{\frac{\text{ga1}+\mu}{\text{be1}} \,||\, \frac{\text{ga2}+\mu}{\text{be2}} \,||\, \text{s3} \,||\, \text{s4} \,||\, \text{s5} \,||\, \text{s6} \,||\, \text{s7} \,||\, \text{s8}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, !\,\{\}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \ll 3 \gg\right\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

FindInstance: $\frac{\text{ga1}+\mu}{\text{be1}}$ is not a valid variable.

General: Further output of FindInstance::ivar will be suppressed during this calculation.

ReplaceAll:

$\left\{\frac{\text{ga1}+\mu}{\text{be1}} \,||\, \frac{\text{ga2}+\mu}{\text{be2}} \,||\, \text{s3} \,||\, \text{s4} \,||\, \text{s5} \,||\, \text{s6} \,||\, \text{s7} \,||\, \text{s8}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}, \{\}\}, !\,\{\}, !\,\{\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \{\} \Rightarrow \left\{\frac{\text{ga1}+\mu}{\text{be1}}, \frac{\text{ga2}+\mu}{\text{be2}}, \text{s3}, \text{s4}, \text{s5}, \text{s6}, \text{s7}, \text{s8}\right\} \,||\, \{\{\}\}, \ll 4 \gg\right\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

Out[∘]=

```
$Aborted
```

In[∘]:=

In[∘]:=
```
ch2=CoefficientList[chRp[[8]],u]/.so[[3]]//FullSimplify
ch1=CoefficientList[chRp[[7]],u]/.so[[3]]//FullSimplify
in1=Hur2[ch1];in2=Hur2[ch2];ine=Append[Join[cp,in1,in2],La>al mu^2];
re=red[Reduce[ine],ine]//FullSimplify
re//Length
rec=reCL[Reduce[ine]]//FullSimplify
rec==re

re[[1]]//Length
re[[1]]
re[[1]][[1]]

re[[2]]//Length
re[[2]]
re[[2]][[1]]
```

Out[∘]=

$$\left\{\gamma_5\left(\gamma_7-\frac{(\beta_2-\alpha_2\gamma_7)\Lambda}{\mu}+\frac{(\beta_2-\alpha_2\gamma_7)\mu}{\alpha}\right),\ \gamma_5+\gamma_7+\frac{\alpha_2(\gamma_5+\gamma_7)\Lambda}{\mu}-\frac{\alpha_2(\gamma_5+\gamma_7)\mu}{\alpha},\ 1+\frac{\alpha_2\Lambda}{\mu}-\frac{\alpha_2\mu}{\alpha}\right\}$$

Out[∘]=

$$\left\{\gamma_4\left(\gamma_6-\frac{(\beta_1-\alpha_1\gamma_6)\Lambda}{\mu}+\frac{(\beta_1-\alpha_1\gamma_6)\mu}{\alpha}\right),\ \gamma_4+\gamma_6+\frac{\alpha_1(\gamma_4+\gamma_6)\Lambda}{\mu}-\frac{\alpha_1(\gamma_4+\gamma_6)\mu}{\alpha},\ 1+\frac{\alpha_1\Lambda}{\mu}-\frac{\alpha_1\mu}{\alpha}\right\}$$

Out[∘]=

$$\Bigg(\alpha>0\ \&\&\ \alpha<\frac{\Lambda}{\mu^2}\ \&\&\ \Lambda\le\mu^2\ \&\&\ \Bigg(\left(\alpha_2>0\ \&\&\ \alpha_2+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}<0\right)\ ||$$
$$\left(\alpha_2+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}>0\ \&\&\ (\alpha_2\alpha\Lambda+\alpha\mu-\alpha_2\mu^2)\left(-\alpha\beta_2\Lambda+(\beta_2-\alpha_2\gamma_7)\mu^2+\alpha\gamma_7(\alpha_2\Lambda+\mu)\right)>0\right)\Bigg)\ \&\&$$
$$\Bigg(\left(\alpha_1>0\ \&\&\ \alpha_1+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}<0\right)\ ||$$
$$\left(\alpha_1+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}>0\ \&\&\ (\alpha_1\alpha\Lambda+\alpha\mu-\alpha_1\mu^2)\left(-\alpha\beta_1\Lambda+(\beta_1-\alpha_1\gamma_6)\mu^2+\alpha\gamma_6(\alpha_1\Lambda+\mu)\right)>0\right)\Bigg)\Bigg)\ ||$$
$$\Bigg(\Lambda>\mu^2\ \&\&\ \Bigg(\Bigg(\alpha>0\ \&\&\ \alpha<\frac{\mu^2}{\Lambda}\ \&\&\ \Bigg(\left(\alpha_2>0\ \&\&\ \alpha_2+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}<0\right)\ ||\ \Bigg(\alpha_2+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}>0\ \&\&$$
$$(\alpha_2\alpha\Lambda+\alpha\mu-\alpha_2\mu^2)\left(-\alpha\beta_2\Lambda+(\beta_2-\alpha_2\gamma_7)\mu^2+\alpha\gamma_7(\alpha_2\Lambda+\mu)\right)>0\Bigg)\Bigg)\ \&\&$$
$$\Bigg(\left(\alpha_1>0\ \&\&\ \alpha_1+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}<0\right)\ ||\ \Bigg(\alpha_1+\frac{\alpha\mu}{\alpha\Lambda-\mu^2}>0\ \&\&$$
$$(\alpha_1\alpha\Lambda+\alpha\mu-\alpha_1\mu^2)\left(-\alpha\beta_1\Lambda+(\beta_1-\alpha_1\gamma_6)\mu^2+\alpha\gamma_6(\alpha_1\Lambda+\mu)\right)>0\Bigg)\Bigg)\Bigg)\ ||$$
$$\alpha==\frac{\mu^2}{\Lambda}\ ||\ \left(\frac{\mu^2}{\Lambda}<\alpha<\frac{\Lambda}{\mu^2}\ \&\&\ \gamma_7>\frac{\beta_2(\alpha\Lambda-\mu^2)}{\alpha_2\alpha\Lambda+\alpha\mu-\alpha_2\mu^2}\ \&\&\ \gamma_6>\frac{\beta_1(\alpha\Lambda-\mu^2)}{\alpha_1\alpha\Lambda+\alpha\mu-\alpha_1\mu^2}\right)\Bigg)\Bigg)$$

Out[∘]=

2

Out[ ]=

$$\left(\alpha > 0 \,\&\&\, \alpha < \frac{\Lambda}{\mu^2} \,\&\&\, \Lambda \le \mu^2 \,\&\&\, \left(\left(\alpha_2 > 0 \,\&\&\, \alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2\right)\,\left(-\alpha\,\beta_2\,\Lambda + \left(\beta_2 - \alpha_2\,\gamma_7\right)\,\mu^2 + \alpha\,\gamma_7\,\left(\alpha_2\,\Lambda + \mu\right)\right) > 0\right)\right) \,\&\&\, \left(\left(\alpha_1 > 0 \,\&\&\, \alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2\right)\,\left(-\alpha\,\beta_1\,\Lambda + \left(\beta_1 - \alpha_1\,\gamma_6\right)\,\mu^2 + \alpha\,\gamma_6\,\left(\alpha_1\,\Lambda + \mu\right)\right) > 0\right)\right)\right) \,||\, \left(\Lambda > \mu^2 \,\&\&\, \left(\left(\alpha > 0 \,\&\&\, \alpha < \frac{\mu^2}{\Lambda} \,\&\&\, \left(\left(\alpha_2 > 0 \,\&\&\, \alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2\right)\,\left(-\alpha\,\beta_2\,\Lambda + \left(\beta_2 - \alpha_2\,\gamma_7\right)\,\mu^2 + \alpha\,\gamma_7\,\left(\alpha_2\,\Lambda + \mu\right)\right) > 0\right)\right) \,\&\&\, \left(\left(\alpha_1 > 0 \,\&\&\, \alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2\right)\,\left(-\alpha\,\beta_1\,\Lambda + \left(\beta_1 - \alpha_1\,\gamma_6\right)\,\mu^2 + \alpha\,\gamma_6\,\left(\alpha_1\,\Lambda + \mu\right)\right) > 0\right)\right)\right) \,||\, \alpha == \frac{\mu^2}{\Lambda} \,||\, \left(\frac{\mu^2}{\Lambda} < \alpha < \frac{\Lambda}{\mu^2} \,\&\&\, \gamma_7 > \frac{\beta_2\,\left(\alpha\,\Lambda - \mu^2\right)}{\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2} \,\&\&\, \gamma_6 > \frac{\beta_1\,\left(\alpha\,\Lambda - \mu^2\right)}{\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2}\right)\right)\right)$$

Out[ ]=

True

Out[ ]=

5

Out[ ]=

$$\alpha > 0 \,\&\&\, \alpha < \frac{\Lambda}{\mu^2} \,\&\&\, \Lambda \le \mu^2 \,\&\&\, \left(\left(\alpha_2 > 0 \,\&\&\, \alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2\right)\,\left(-\alpha\,\beta_2\,\Lambda + \left(\beta_2 - \alpha_2\,\gamma_7\right)\,\mu^2 + \alpha\,\gamma_7\,\left(\alpha_2\,\Lambda + \mu\right)\right) > 0\right)\right) \,\&\&\, \left(\left(\alpha_1 > 0 \,\&\&\, \alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \,\&\&\, \left(\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2\right)\,\left(-\alpha\,\beta_1\,\Lambda + \left(\beta_1 - \alpha_1\,\gamma_6\right)\,\mu^2 + \alpha\,\gamma_6\,\left(\alpha_1\,\Lambda + \mu\right)\right) > 0\right)\right)$$

Out[ ]=

$\alpha > 0$

Out[ ]=

2

Out[#]=

$$\Lambda > \mu^2 \&\& \left(\left(\alpha > 0 \&\& \alpha < \frac{\mu^2}{\Lambda} \&\& \left(\left(\alpha_2 > 0 \&\& \alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_2 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \&\& \left(\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2\right)\left(-\alpha\,\beta_2\,\Lambda + (\beta_2 - \alpha_2\,\gamma_7)\,\mu^2 + \alpha\,\gamma_7\,(\alpha_2\,\Lambda + \mu)\right) > 0\right)\right) \&\& \left(\left(\alpha_1 > 0 \&\& \alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} < 0\right) \,||\, \left(\alpha_1 + \frac{\alpha\,\mu}{\alpha\,\Lambda - \mu^2} > 0 \&\& \left(\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2\right)\left(-\alpha\,\beta_1\,\Lambda + (\beta_1 - \alpha_1\,\gamma_6)\,\mu^2 + \alpha\,\gamma_6\,(\alpha_1\,\Lambda + \mu)\right) > 0\right)\right)\right) \,||\, \alpha == \frac{\mu^2}{\Lambda} \,||\, \left(\frac{\mu^2}{\Lambda} < \alpha < \frac{\Lambda}{\mu^2} \&\& \gamma_7 > \frac{\beta_2\left(\alpha\,\Lambda - \mu^2\right)}{\alpha_2\,\alpha\,\Lambda + \alpha\,\mu - \alpha_2\,\mu^2} \&\& \gamma_6 > \frac{\beta_1\left(\alpha\,\Lambda - \mu^2\right)}{\alpha_1\,\alpha\,\Lambda + \alpha\,\mu - \alpha_1\,\mu^2}\right)\right)$$

Out[#]=

$$\Lambda > \mu^2$$

```
(*2: simplified RHs*)
RHs={ph-x1 (x2 al), (de+mu-x1 al+x3 et),de x2-mu x3+g8 x8+x2 x3 et,-g4 x4+(b1 x2 x6)/(1+a1 x2+ep1 x6),
-g5 x5+(b2 x2 x7)/(1+a2 x2+ep2 x7),
g4 x4-g6 x6,g5 x5-g7 x7,g6 x6+g7 x7-g8 x8}/.cde;
(*Solve first linear equations*)
Print["For simplified RHs, The  variables ",el={2,3,6,7,8}, "are rationally eliminable by cel:"];
cel=Solve[Thread[RHs[[el]]==0],var[[el]]][[1]]
(*Solve  equations 4,5*)
Print["The fourth and fifth of remaining equations Rc factor:"]
Rc=RHs//.cel//Flatten//Factor
fq={4,5};
so=Solve[Thread[Rc[[{4,5}]]==0],var[[fq]]]//FullSimplify;

(*obtain RUR equation Rc[[1]] in all cases*)
rur=FullSimplify[#]&/@Collect[#,ph]&/@(Rc[[1]]//.so);
Print["Solving the factored equations yields ",so//Length," cases so 2,E,RFE,1",so," in which
the system is reducible to rational  equations rur in x1:"]
rur
rurp=Numerator[Together[rur/.cph]];
rurcofs=CoefficientList[rurp,x1];
(*eq=(Rc/.c57)[[4]]==0;so4=SolveValues[eq,x4]//FullSimplify*)
g45s=FullSimplify[#]&/@g45//.so;
Print["The ",rurcofs//Length," polynomial  equations rurcofs of degree",Length/@rurcofs-1," have
FullSimplify/@(rurcofs)
Print["and have exactly one root when "]
ine=onePR[rurcofs,cp] (*Append[cp,(rurcofs[[#]]//First) (rurcofs[[#]]//Last)<0]&/@Range[4]*)
cOPR=seZF/@%
```

For simplified RHs, The variables {2, 3, 6, 7, 8}are rationally eliminable by cel:

Out[∘]=

$$\left\{x_2 \to \frac{\mu^2 - \alpha\,\mu\,x_1 + \eta\,\gamma_4\,x_4 + \eta\,\gamma_5\,x_5}{\eta\,(\mu - \alpha\,x_1)},\ x_3 \to \frac{-\mu + \alpha\,x_1}{\eta},\ x_6 \to \frac{\gamma_4\,x_4}{\gamma_6},\ x_7 \to \frac{\gamma_5\,x_5}{\gamma_7},\ x_8 \to \frac{\gamma_4\,x_4 + \gamma_5\,x_5}{\gamma_8}\right\}$$

The fourth and fifth of remaining equations Rc factor:

Out[∘]=

$$\Big\{\frac{-\eta\,\mu\,\mathrm{ph} + \alpha\,\mu^2\,x_1 + \alpha\,\eta\,\mathrm{ph}\,x_1 - \alpha^2\,\mu\,x_1^2 + \alpha\,\eta\,\gamma_4\,x_1\,x_4 + \alpha\,\eta\,\gamma_5\,x_1\,x_5}{\eta\,(-\mu + \alpha\,x_1)},$$
$$0,\ 0,\ \Big(\gamma_4\,x_4\,\big(-\eta\,\gamma_6\,\mu + \beta_1\,\mu^2 - \alpha_1\,\gamma_6\,\mu^2 + \alpha\,\eta\,\gamma_6\,x_1 - \alpha\,\beta_1\,\mu\,x_1 + \alpha_1\,\alpha\,\gamma_6\,\mu\,x_1 +$$
$$\beta_1\,\eta\,\gamma_4\,x_4 - \alpha_1\,\eta\,\gamma_4\,\gamma_6\,x_4 - \epsilon_1\,\eta\,\gamma_4\,\mu\,x_4 + \alpha\,\epsilon_1\,\eta\,\gamma_4\,x_1\,x_4 + \beta_1\,\eta\,\gamma_5\,x_5 - \alpha_1\,\eta\,\gamma_5\,\gamma_6\,x_5\big)\Big)\Big/$$
$$\big(\eta\,\gamma_6\,\mu + \alpha_1\,\gamma_6\,\mu^2 - \alpha\,\eta\,\gamma_6\,x_1 - \alpha_1\,\alpha\,\gamma_6\,\mu\,x_1 + \alpha_1\,\eta\,\gamma_4\,\gamma_6\,x_4 + \epsilon_1\,\eta\,\gamma_4\,\mu\,x_4 - \alpha\,\epsilon_1\,\eta\,\gamma_4\,x_1\,x_4 + \alpha_1\,\eta\,\gamma_5\,\gamma_6\,x_5\big),$$
$$\Big(\gamma_5\,x_5\,\big(-\eta\,\gamma_7\,\mu + \beta_2\,\mu^2 - \alpha_2\,\gamma_7\,\mu^2 + \alpha\,\eta\,\gamma_7\,x_1 - \alpha\,\beta_2\,\mu\,x_1 + \alpha_2\,\alpha\,\gamma_7\,\mu\,x_1 + \beta_2\,\eta\,\gamma_4\,x_4 -$$
$$\alpha_2\,\eta\,\gamma_4\,\gamma_7\,x_4 + \beta_2\,\eta\,\gamma_5\,x_5 - \alpha_2\,\eta\,\gamma_5\,\gamma_7\,x_5 - \epsilon_2\,\eta\,\gamma_5\,\mu\,x_5 + \alpha\,\epsilon_2\,\eta\,\gamma_5\,x_1\,x_5\big)\Big)\Big/$$
$$\big(\eta\,\gamma_7\,\mu + \alpha_2\,\gamma_7\,\mu^2 - \alpha\,\eta\,\gamma_7\,x_1 - \alpha_2\,\alpha\,\gamma_7\,\mu\,x_1 + \alpha_2\,\eta\,\gamma_4\,\gamma_7\,x_4 + \alpha_2\,\eta\,\gamma_5\,\gamma_7\,x_5 + \epsilon_2\,\eta\,\gamma_5\,\mu\,x_5 - \alpha\,\epsilon_2\,\eta\,\gamma_5\,x_1\,x_5\big),$$
$$0,\ 0,\ 0\Big\}$$

Solving the factored equations yields 4

cases so 2,E,RFE,1 $\Big\{\Big\{x_4 \to 0,\ x_5 \to \frac{(\eta\,\gamma_7 - \beta_2\,\mu + \alpha_2\,\gamma_7\,\mu)\,(\mu - \alpha\,x_1)}{\eta\,\gamma_5\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu + \alpha\,\epsilon_2\,x_1)}\Big\},$

$$\Big\{x_4 \to \frac{-\beta_2\,\eta\,\gamma_6 + \gamma_6\,\big(-\alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2 - \alpha\,\epsilon_2\,(\eta + \alpha_1\,\mu)\,x_1\big) + \beta_1\,(\eta\,\gamma_7 + \epsilon_2\,\mu\,(-\mu + \alpha\,x_1))}{\eta\,\gamma_4\,(\beta_2\,\epsilon_1 - \alpha_2\,\epsilon_1\,\gamma_7 + \epsilon_2\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu + \alpha\,\epsilon_1\,x_1))},$$
$$x_5 \to \frac{-\gamma_7\,\big(\beta_1\,\eta - \alpha_1\,\eta\,\gamma_6 + \alpha_2\,\eta\,\gamma_6 - \epsilon_1\,\eta\,\mu - \alpha_2\,\epsilon_1\,\mu^2 + \alpha\,\epsilon_1\,(\eta + \alpha_2\,\mu)\,x_1\big) + \beta_2\,(\eta\,\gamma_6 + \epsilon_1\,\mu\,(-\mu + \alpha\,x_1))}{\eta\,\gamma_5\,(\beta_2\,\epsilon_1 - \alpha_2\,\epsilon_1\,\gamma_7 + \epsilon_2\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu + \alpha\,\epsilon_1\,x_1))}\Big\},$$

$\{x_4 \to 0,\ x_5 \to 0\},\ \Big\{x_4 \to \frac{(\eta\,\gamma_6 - \beta_1\,\mu + \alpha_1\,\gamma_6\,\mu)\,(\mu - \alpha\,x_1)}{\eta\,\gamma_4\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu + \alpha\,\epsilon_1\,x_1)},\ x_5 \to 0\Big\}\Big\}$ in which

the system is reducible to rational equations rur in x1:

Out[∘]=

$$\Big\{\mathrm{ph} - \frac{\alpha\,x_1\,(\eta\,\gamma_7 + \epsilon_2\,\mu\,(-\mu + \alpha\,x_1))}{\eta\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu + \alpha\,\epsilon_2\,x_1)},\ \mathrm{ph} - \frac{\alpha\,x_1\,(\epsilon_2\,\eta\,\gamma_6 + \epsilon_1\,\eta\,\gamma_7 + \epsilon_1\,\epsilon_2\,\mu\,(-\mu + \alpha\,x_1))}{\eta\,(\beta_2\,\epsilon_1 - \alpha_2\,\epsilon_1\,\gamma_7 + \epsilon_2\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu + \alpha\,\epsilon_1\,x_1))},$$
$$\mathrm{ph} - \frac{\alpha\,\mu\,x_1}{\eta},\ \mathrm{ph} - \frac{\alpha\,x_1\,(\eta\,\gamma_6 + \epsilon_1\,\mu\,(-\mu + \alpha\,x_1))}{\eta\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu + \alpha\,\epsilon_1\,x_1)}\Big\}$$

The 4 polynomial equations rurcofs of degree $\{2, 2, 1, 2\}$ have coefs

Out[∘]=

$$\Big\{\Big\{\eta\,\Lambda\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu),\ \eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu) + \alpha\,\big(-\eta\,\gamma_7 + \epsilon_2\,\eta\,\Lambda + \epsilon_2\,\mu^2\big),\ -\alpha\,\epsilon_2\,(\alpha + \eta)\,\mu\Big\},$$
$$\Big\{\eta\,\Lambda\,(-\beta_2\,\epsilon_1 - \beta_1\,\epsilon_2 + \alpha_1\,\epsilon_2\,\gamma_6 + \alpha_2\,\epsilon_1\,\gamma_7 + \epsilon_1\,\epsilon_2\,\mu),\ \alpha\,\eta\,(\epsilon_1\,\gamma_7 + \epsilon_2\,(\gamma_6 - \epsilon_1\,\Lambda)) +$$
$$\eta\,(\beta_2\,\epsilon_1 + \beta_1\,\epsilon_2 - \alpha_1\,\epsilon_2\,\gamma_6 - \alpha_2\,\epsilon_1\,\gamma_7)\,\mu - \epsilon_1\,\epsilon_2\,(\alpha + \eta)\,\mu^2,\ \alpha\,\epsilon_1\,\epsilon_2\,(\alpha + \eta)\,\mu\Big\},\ \{\eta\,\Lambda,\ -((\alpha + \eta)\,\mu)\},$$
$$\Big\{\eta\,\Lambda\,(\beta_1 - \alpha_1\,\gamma_6 - \epsilon_1\,\mu),\ \eta\,\mu\,(-\beta_1 + \alpha_1\,\gamma_6 + \epsilon_1\,\mu) + \alpha\,\big(-\eta\,\gamma_6 + \epsilon_1\,\eta\,\Lambda + \epsilon_1\,\mu^2\big),\ -\alpha\,\epsilon_1\,(\alpha + \eta)\,\mu\Big\}\Big\}$$

and have exactly one root when

Out[•]=

$$\{\{\alpha_1>0,\alpha_2>0,\alpha>0,\beta_1>0,\beta_2>0,\epsilon_1>0,\epsilon_2>0,\eta>0,\gamma_4>0,\gamma_5>0,\gamma_6>0,\gamma_7>0,\gamma_8>0,\Lambda>0,\mu>0,\left(-\alpha^2\epsilon_2\mu-\alpha\epsilon_2\eta\mu\right)(\beta_2\eta\Lambda-\alpha_2\eta\gamma_7\Lambda-\epsilon_2\eta\Lambda\mu)<0\},\{\alpha_1>0,\alpha_2>0,\alpha>0,\beta_1>0,\beta_2>0,\epsilon_1>0,\epsilon_2>0,\eta>0,\gamma_4>0,\gamma_5>0,\gamma_6>0,\gamma_7>0,\gamma_8>0,\Lambda>0,\mu>0,\left(\alpha^2\epsilon_1\epsilon_2\mu+\alpha\epsilon_1\epsilon_2\eta\mu\right)(-\beta_2\epsilon_1\eta\Lambda-\beta_1\epsilon_2\eta\Lambda+\alpha_1\epsilon_2\eta\gamma_6\Lambda+\alpha_2\epsilon_1\eta\gamma_7\Lambda+\epsilon_1\epsilon_2\eta\Lambda\mu)<0\},\{\alpha_1>0,\alpha_2>0,\alpha>0,\beta_1>0,\beta_2>0,\epsilon_1>0,\epsilon_2>0,\eta>0,\gamma_4>0,\gamma_5>0,\gamma_6>0,\gamma_7>0,\gamma_8>0,\Lambda>0,\mu>0,\eta\Lambda(-\alpha\mu-\eta\mu)<0\},\{\alpha_1>0,\alpha_2>0,\alpha>0,\beta_1>0,\beta_2>0,\epsilon_1>0,\epsilon_2>0,\eta>0,\gamma_4>0,\gamma_5>0,\gamma_6>0,\gamma_7>0,\gamma_8>0,\Lambda>0,\mu>0,\left(-\alpha^2\epsilon_1\mu-\alpha\epsilon_1\eta\mu\right)(\beta_1\eta\Lambda-\alpha_1\eta\gamma_6\Lambda-\epsilon_1\eta\Lambda\mu)<0\}\}$$

Out[•]=

$$\{\{\left(-\alpha^2\epsilon_2\mu-\alpha\epsilon_2\eta\mu\right)(\beta_2\eta\Lambda-\alpha_2\eta\gamma_7\Lambda-\epsilon_2\eta\Lambda\mu)<0\},\{\left(\alpha^2\epsilon_1\epsilon_2\mu+\alpha\epsilon_1\epsilon_2\eta\mu\right)(-\beta_2\epsilon_1\eta\Lambda-\beta_1\epsilon_2\eta\Lambda+\alpha_1\epsilon_2\eta\gamma_6\Lambda+\alpha_2\epsilon_1\eta\gamma_7\Lambda+\epsilon_1\epsilon_2\eta\Lambda\mu)<0\},\{\eta\Lambda(-\alpha\mu-\eta\mu)<0\},\{\left(-\alpha^2\epsilon_1\mu-\alpha\epsilon_1\eta\mu\right)(\beta_1\eta\Lambda-\alpha_1\eta\gamma_6\Lambda-\epsilon_1\eta\Lambda\mu)<0\}\}$$

```
(*3: RFE*)
Print["The positivity conds in case j=",j=3," for ",ine⟦j⟧," are"]
re=seZF[Reduce[ine⟦j⟧]]
inRFE={4,5,6,7};
ng=NGM[mod,inRFE];M=ng⟦1⟧;
K=ng⟦6⟧;
eig47=K/.cRFE/.cel/.so⟦j⟧//Eigenvalues;
Print["RFE eigs",K/.cRFE//Eigenvalues," =",eig47]
(*{R1, R2} =  {eig47⟦3⟧, eig47⟦4⟧}*)
R2===eig47⟦4⟧
```

The positivity conds in case j=3 for $\{\alpha_1>0,\alpha_2>0,\alpha>0,\beta_1>0,\beta_2>0,\epsilon_1>0,\epsilon_2>0,\eta>0,\gamma_4>0,\gamma_5>0,\gamma_6>0,\gamma_7>0,\gamma_8>0,\Lambda>0,\mu>0,\eta\Lambda(-\alpha\mu-\eta\mu)<0\}$ are

Out[•]=

True

RFE eigs$\left\{0,0,\frac{\beta_2 x_2}{\gamma_7+\alpha_2\gamma_7 x_2},\frac{\beta_1 x_2}{\gamma_6+\alpha_1\gamma_6 x_2}\right\}$ =$\left\{0,0,\frac{\beta_2\mu}{\gamma_7(\eta+\alpha_2\mu)},\frac{\beta_1\mu}{\gamma_6(\eta+\alpha_1\mu)}\right\}$

Out[•]=

True

In[•]:=

```
(*x4=0*)
Print["The positivity conds in case j=",j=1," are"]
ca[j]=Delete[Thread[(var/.cel/.so⟦j⟧)≥0],{{4},{6}}];
re5=Reduce[Join[cp,{(x5)>0},ca[j]]//.so⟦j⟧]//FullSimplify

in46={4,6,8};
ng=NGM[mod,in46];M=ng⟦1⟧;
K=ng⟦6⟧;
eig46=K/.c46/.cel/.so⟦j⟧//Eigenvalues;
Print["c46 eigs",K/.c46//Eigenvalues," =",eig46]
Print["stability cond is"]
Timing[re5=seZF[Reduce[Join[cp,{(x5)>0&&eig46<1&&rurp⟦j⟧==0},ca[j]]//.so⟦j⟧,x1]//FullSimplify]]
```

The positivity conds in case j=1 are

Out[•]=

$$\alpha_1 > 0 \,\&\&\, \beta_1 > 0 \,\&\&\, \epsilon_1 > 0 \,\&\&\, \gamma_4 > 0 \,\&\&\, \gamma_6 > 0 \,\&\&\, \Lambda > 0 \,\&\&\, \gamma_7 > 0 \,\&\&\, \alpha_2 > 0 \,\&\&\, \eta > 0 \,\&\&\, x_1 > 0 \,\&\&\, \epsilon_2 > 0 \,\&\&\, \gamma_8 > 0 \,\&\&\, \gamma_5 > 0 \,\&\&\, \alpha_2\,\gamma_7 < \beta_2 \,\&\&\, \gamma_7\,(\eta + \alpha_2\,\mu) < \beta_2\,\mu \,\&\&\, \mu < \alpha\,x_1$$

c46 eigs$\left\{\frac{\beta_1\,x_2}{\gamma_6 + \alpha_1\,\gamma_6\,x_2}, 0, 0\right\}$ =

$$\left\{-\frac{\beta_1\left(\eta\,\gamma_7 - \epsilon_2\,\mu^2 + \alpha\,\epsilon_2\,\mu\,x_1\right)}{\gamma_6\left(-\beta_2\,\eta - \alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2 - \alpha\,\epsilon_2\,\eta\,x_1 - \alpha_1\,\alpha\,\epsilon_2\,\mu\,x_1\right)}, 0, 0\right\}$$

stability cond is

Out[•]=

$$\Bigg\{376.531, \frac{\alpha\,\eta\,(-\gamma_7 + \epsilon_2\,\Lambda) + \eta\,(-\beta_2 + \alpha_2\,\gamma_7)\,\mu + \epsilon_2\,(\alpha + \eta)\,\mu^2}{\alpha\,\epsilon_2\,(\alpha + \eta)\,\mu} + \sqrt{\left(\frac{1}{\alpha^2\,\epsilon_2^2\,(\alpha + \eta)^2\,\mu^2}\left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \alpha^2\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\right)} = 2\,x_1 \,\&\&\, \alpha > \frac{\mu^2}{\Lambda} \,\&\&$$

$$\Bigg(\Bigg(\epsilon_2 > 0 \,\&\&\, \epsilon_2 < \frac{\beta_2 - \alpha_2\,\gamma_7}{\mu} \,\&\&\, \alpha_2 > 0 \,\&\&\, \Bigg(\left(\beta_2 > \frac{\alpha\,\gamma_7\,\mu}{\alpha\,\Lambda - \mu^2} \,\&\&\, \eta > 0 \,\&\&\, \alpha_2 < \frac{\beta_2}{\gamma_7} + \frac{\alpha\,\mu}{-\alpha\,\Lambda + \mu^2} \,\&\&\, \eta \le \frac{\alpha\,\mu^2}{\alpha\,\Lambda - \mu^2}\right) \,||\, \left(\beta_2 > \frac{\eta\,\gamma_7}{\mu} \,\&\&\, \eta > \frac{\alpha\,\mu^2}{\alpha\,\Lambda - \mu^2} \,\&\&\, \alpha_2 + \frac{\eta}{\mu} < \frac{\beta_2}{\gamma_7}\right)\Bigg) \,\&\&$$

$$\Bigg(\Bigg(\frac{\eta\,\mu\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu) + \alpha\left(-\eta\,\gamma_7 + \epsilon_2\,\eta\,\Lambda + \epsilon_2\,\mu^2\right)}{-((\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda)\,\mu) + \epsilon_2\,\mu^3} + \sqrt{\left(\frac{1}{\mu^2\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)^2}\left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \alpha^2\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\right)} = 2\,\alpha_1 \,\&\&\, \gamma_6 >$$

$$\frac{\beta_1\,\mu\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)}{\alpha\,\eta\,(-\gamma_7 + \epsilon_2\,\Lambda) + \alpha\,\epsilon_2\,\mu\,(2\,\alpha_1\,\Lambda + \mu) - \mu\left(-\beta_2\,\eta - 2\,\alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + 2\,\alpha_1\,\epsilon_2\,\mu^2\right)}\Bigg) \,||$$

$$\Bigg(\gamma_6 > \frac{1}{2}\Bigg(\Big(\beta_1\left(\alpha\,\eta\,(\gamma_7 - \epsilon_2\,\Lambda) - \alpha\,\epsilon_2\,\mu\,(2\,\alpha_1\,\Lambda + \mu) + \mu\left(-\beta_2\,\eta - 2\,\alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + 2\,\alpha_1\,\epsilon_2\,\mu^2\right)\right)\Big) \Big/ \left(\alpha\left(\eta\,(\beta_2 + \alpha_1\,\gamma_7 - \alpha_2\,\gamma_7 - \alpha_1\,\epsilon_2\,\Lambda) - \epsilon_2\left(\eta + \alpha_1^2\,\Lambda\right)\mu - \alpha_1\,\epsilon_2\,\mu^2\right) + \alpha_1\,\mu\left(-\eta\,(\beta_2 + (\alpha_1 - \alpha_2)\,\gamma_7) + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2\right)\right) + \sqrt{\Big(\Big(\beta_1^2\left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \alpha^2\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\Big) \Big/ \left(\alpha\left(\eta\,(\beta_2 + \alpha_1\,\gamma_7 - \alpha_2\,\gamma_7 - \alpha_1\,\epsilon_2\,\Lambda) - \epsilon_2\left(\eta + \alpha_1^2\,\Lambda\right)\mu - \alpha_1\,\epsilon_2\,\mu^2\right) + \alpha_1\,\mu\left(-\eta\,(\beta_2 + (\alpha_1 - \alpha_2)\,\gamma_7) + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2\right)\right)^2\Big)}\Bigg) \,\&\&\, \alpha_1 > 0 \,\&\&$$

$$\alpha_1 < \frac{1}{2} \left(\frac{\eta\, \mu\, (\beta_2 - \alpha_2\, \gamma_7 - \epsilon_2\, \mu) + \alpha \left(-\eta\, \gamma_7 + \epsilon_2\, \eta\, \Lambda + \epsilon_2\, \mu^2\right)}{-((\eta\, \gamma_7 + \alpha\, \epsilon_2\, \Lambda)\, \mu) + \epsilon_2\, \mu^3} + \sqrt{\left(\frac{1}{\mu^2 \left(\eta\, \gamma_7 + \alpha\, \epsilon_2\, \Lambda - \epsilon_2\, \mu^2\right)^2} \left(\eta^2\, \mu^2\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu)^2 - 2\, \alpha\, \eta\, \mu\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu) \left(\eta\, (\gamma_7 + \epsilon_2\, \Lambda) - \epsilon_2\, \mu^2\right) + \alpha^2 \left(\eta^2\, (\gamma_7 - \epsilon_2\, \Lambda)^2 + \epsilon_2^2\, \mu^4 - 2\, \epsilon_2\, \eta\, \mu\, (-2\, \beta_2\, \Lambda + 2\, \alpha_2\, \gamma_7\, \Lambda + \gamma_7\, \mu + \epsilon_2\, \Lambda\, \mu)\right)\right)\right)} \right) \;||$$

$$\left(\alpha_1 > \frac{1}{2} \left(\frac{\eta\, \mu\, (\beta_2 - \alpha_2\, \gamma_7 - \epsilon_2\, \mu) + \alpha \left(-\eta\, \gamma_7 + \epsilon_2\, \eta\, \Lambda + \epsilon_2\, \mu^2\right)}{-((\eta\, \gamma_7 + \alpha\, \epsilon_2\, \Lambda)\, \mu) + \epsilon_2\, \mu^3} + \sqrt{\left(\frac{1}{\mu^2 \left(\eta\, \gamma_7 + \alpha\, \epsilon_2\, \Lambda - \epsilon_2\, \mu^2\right)^2} \left(\eta^2\, \mu^2\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu)^2 - 2\, \alpha\, \eta\, \mu\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu) \left(\eta\, (\gamma_7 + \epsilon_2\, \Lambda) - \epsilon_2\, \mu^2\right) + \alpha^2 \left(\eta^2\, (\gamma_7 - \epsilon_2\, \Lambda)^2 + \epsilon_2^2\, \mu^4 - 2\, \epsilon_2\, \eta\, \mu\, (-2\, \beta_2\, \Lambda + 2\, \alpha_2\, \gamma_7\, \Lambda + \gamma_7\, \mu + \epsilon_2\, \Lambda\, \mu)\right)\right)\right)} \right) \;\&\&\right.$$

$$\gamma_6 + \frac{1}{2} \left(\left(\beta_1 \left(\alpha\, \eta\, (-\gamma_7 + \epsilon_2\, \Lambda) + \eta\, (\beta_2 + 2\, \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7)\, \mu + 2\, \alpha_1\, \alpha\, \epsilon_2\, \Lambda\, \mu + \epsilon_2\, (\alpha - \eta)\, \mu^2 - 2\, \alpha_1\, \epsilon_2\, \mu^3\right)\right) \Big/ \left(\alpha \left(\eta\, (\beta_2 + \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7 - \alpha_1\, \epsilon_2\, \Lambda) - \epsilon_2 \left(\eta + \alpha_1^2\, \Lambda\right) \mu - \alpha_1\, \epsilon_2\, \mu^2\right) + \alpha_1\, \mu \left(-\eta\, (\beta_2 + (\alpha_1 - \alpha_2)\, \gamma_7) + \epsilon_2\, \eta\, \mu + \alpha_1\, \epsilon_2\, \mu^2\right)\right) + \sqrt{\left(\left(\beta_1^2 \left(\eta^2\, \mu^2\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu)^2 - 2\, \alpha\, \eta\, \mu\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu) \left(\eta\, (\gamma_7 + \epsilon_2\, \Lambda) - \epsilon_2\, \mu^2\right) + \alpha^2 \left(\eta^2\, (\gamma_7 - \epsilon_2\, \Lambda)^2 + \epsilon_2^2\, \mu^4 - 2\, \epsilon_2\, \eta\, \mu\, (-2\, \beta_2\, \Lambda + 2\, \alpha_2\, \gamma_7\, \Lambda + \gamma_7\, \mu + \epsilon_2\, \Lambda\, \mu)\right)\right)\right) \Big/ \left(\alpha \left(\eta\, (\beta_2 + \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7 - \alpha_1\, \epsilon_2\, \Lambda) - \epsilon_2 \left(\eta + \alpha_1^2\, \Lambda\right) \mu - \alpha_1\, \epsilon_2\, \mu^2\right) + \alpha_1\, \mu \left(-\eta\, (\beta_2 + (\alpha_1 - \alpha_2)\, \gamma_7) + \epsilon_2\, \eta\, \mu + \alpha_1\, \epsilon_2\, \mu^2\right)\right)^2 \right)} \right) > 0 \left.\right) \;||$$

$$\left(\alpha_1 > 0 \;\&\&\; \gamma_6 + \frac{1}{2} \left(\left(\beta_1 \left(\alpha\, \eta\, (-\gamma_7 + \epsilon_2\, \Lambda) + \eta\, (\beta_2 + 2\, \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7)\, \mu + 2\, \alpha_1\, \alpha\, \epsilon_2\, \Lambda\, \mu + \epsilon_2\, (\alpha - \eta)\, \mu^2 - 2\, \alpha_1\, \epsilon_2\, \mu^3\right)\right) \Big/ \left(\alpha \left(\eta\, (\beta_2 + \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7 - \alpha_1\, \epsilon_2\, \Lambda) - \epsilon_2 \left(\eta + \alpha_1^2\, \Lambda\right) \mu - \alpha_1\, \epsilon_2\, \mu^2\right) + \alpha_1\, \mu \left(-\eta\, (\beta_2 + (\alpha_1 - \alpha_2)\, \gamma_7) + \epsilon_2\, \eta\, \mu + \alpha_1\, \epsilon_2\, \mu^2\right)\right) + \sqrt{\left(\left(\beta_1^2 \left(\eta^2\, \mu^2\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu)^2 - 2\, \alpha\, \eta\, \mu\, (-\beta_2 + \alpha_2\, \gamma_7 + \epsilon_2\, \mu) \left(\eta\, (\gamma_7 + \epsilon_2\, \Lambda) - \epsilon_2\, \mu^2\right) + \alpha^2 \left(\eta^2\, (\gamma_7 - \epsilon_2\, \Lambda)^2 + \epsilon_2^2\, \mu^4 - 2\, \epsilon_2\, \eta\, \mu\, (-2\, \beta_2\, \Lambda + 2\, \alpha_2\, \gamma_7\, \Lambda + \gamma_7\, \mu + \epsilon_2\, \Lambda\, \mu)\right)\right)\right) \Big/ \left(\alpha \left(\eta\, (\beta_2 + \alpha_1\, \gamma_7 - \alpha_2\, \gamma_7 - \alpha_1\, \epsilon_2\, \Lambda) - \epsilon_2 \left(\eta + \alpha_1^2\, \Lambda\right) \mu - \alpha_1\, \epsilon_2\, \mu^2\right) + \alpha_1\, \mu \left(-\eta\, (\beta_2 + (\alpha_1 - \alpha_2)\, \gamma_7) + \epsilon_2\, \eta\, \mu + \alpha_1\, \epsilon_2\, \mu^2\right)\right)^2 \right)} \right) > 0 \;\&\&\right.$$

$$\epsilon_2 \geq \frac{\beta_2 - \alpha_2\, \gamma_7}{\mu} \;\&\&\; \alpha_2 > 0 \;\&\&\; \left(\left(\beta_2 > \frac{\alpha\, \gamma_7\, \mu}{\alpha\, \Lambda - \mu^2} \;\&\&\; \eta > 0 \;\&\&\; \alpha_2 < \frac{\beta_2}{\gamma_7} + \frac{\alpha\, \mu}{-\alpha\, \Lambda + \mu^2} \;\&\&\; \eta \leq \frac{\alpha\, \mu^2}{\alpha\, \Lambda - \mu^2}\right) \;||\; \left(\beta_2 > \frac{\eta\, \gamma_7}{\mu} \;\&\&\; \eta > \frac{\alpha\, \mu^2}{\alpha\, \Lambda - \mu^2} \;\&\&\; \alpha_2 + \frac{\eta}{\mu} < \frac{\beta_2}{\gamma_7}\right)\right)\left.\right)\left.\right)\}$$

In[∘]:=

```
re5//Length
re5⟦2⟧
re5⟦3⟧
```

Out[∘]=

3

Out[∘]=

$$\alpha > \frac{\mu^2}{\Lambda}$$

Out[∘]=

$$\Bigg(\epsilon_2 > 0 \&\& \epsilon_2 < \frac{\beta_2 - \alpha_2\,\gamma_7}{\mu} \&\& \alpha_2 > 0 \&\& \left(\left(\beta_2 > \frac{\alpha\,\gamma_7\,\mu}{\alpha\,\Lambda - \mu^2} \&\& \eta > 0 \&\& \alpha_2 < \frac{\beta_2}{\gamma_7} + \frac{\alpha\,\mu}{-\alpha\,\Lambda + \mu^2} \&\& \eta \le \frac{\alpha\,\mu^2}{\alpha\,\Lambda - \mu^2}\right) \,||\, \left(\beta_2 > \frac{\eta\,\gamma_7}{\mu} \&\& \eta > \frac{\alpha\,\mu^2}{\alpha\,\Lambda - \mu^2} \&\& \alpha_2 + \frac{\eta}{\mu} < \frac{\beta_2}{\gamma_7}\right)\right) \&\&$$

$$\Bigg(\Bigg(\frac{\eta\,\mu\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu) + \alpha\left(-\eta\,\gamma_7 + \epsilon_2\,\eta\,\Lambda + \epsilon_2\,\mu^2\right)}{-((\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda)\,\mu) + \epsilon_2\,\mu^3} + \sqrt{\left(\frac{1}{\mu^2\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)^2}\right.} \\ \left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \alpha^2\right. \\ \left.\left.\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\right) == 2\,\alpha_1 \&\& \gamma_6 >$$

$$\frac{\beta_1\,\mu\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)}{\alpha\,\eta\,(-\gamma_7 + \epsilon_2\,\Lambda) + \alpha\,\epsilon_2\,\mu\,(2\,\alpha_1\,\Lambda + \mu) - \mu\left(-\beta_2\,\eta - 2\,\alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + 2\,\alpha_1\,\epsilon_2\,\mu^2\right)}\Bigg) \,||$$

$$\Bigg(\gamma_6 > \frac{1}{2}\Bigg(\Big(\beta_1\left(\alpha\,\eta\,(\gamma_7 - \epsilon_2\,\Lambda) - \alpha\,\epsilon_2\,\mu\,(2\,\alpha_1\,\Lambda + \mu) + \mu\left(-\beta_2\,\eta - 2\,\alpha_1\,\eta\,\gamma_7 + \alpha_2\,\eta\,\gamma_7 + \epsilon_2\,\eta\,\mu + \right.\right. \\ \left.\left.2\,\alpha_1\,\epsilon_2\,\mu^2\right)\right)\Big) \Big/ \Big(\alpha\left(\eta\,(\beta_2 + \alpha_1\,\gamma_7 - \alpha_2\,\gamma_7 - \alpha_1\,\epsilon_2\,\Lambda) - \epsilon_2\left(\eta + \alpha_1^2\,\Lambda\right)\mu - \right. \\ \left.\alpha_1\,\epsilon_2\,\mu^2\right) + \alpha_1\,\mu\left(-\eta\,(\beta_2 + (\alpha_1 - \alpha_2)\,\gamma_7) + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2\right)\Big) + \\ \sqrt{\Big(\Big(\beta_1^2\left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \right.} \\ \left.\alpha^2\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\Big) \Big/ \\ \Big(\alpha\left(\eta\,(\beta_2 + \alpha_1\,\gamma_7 - \alpha_2\,\gamma_7 - \alpha_1\,\epsilon_2\,\Lambda) - \epsilon_2\left(\eta + \alpha_1^2\,\Lambda\right)\mu - \alpha_1\,\epsilon_2\,\mu^2\right) + \\ \alpha_1\,\mu\left(-\eta\,(\beta_2 + (\alpha_1 - \alpha_2)\,\gamma_7) + \epsilon_2\,\eta\,\mu + \alpha_1\,\epsilon_2\,\mu^2\right)\Big)^2\Big)\Bigg) \&\& \alpha_1 > 0 \&\&$$

$$\alpha_1 < \frac{1}{2}\left(\frac{\eta\,\mu\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu) + \alpha\left(-\eta\,\gamma_7 + \epsilon_2\,\eta\,\Lambda + \epsilon_2\,\mu^2\right)}{-((\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda)\,\mu) + \epsilon_2\,\mu^3} + \sqrt{\left(\frac{1}{\mu^2\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)^2}\right.}\right. \\ \left(\eta^2\,\mu^2\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)^2 - 2\,\alpha\,\eta\,\mu\,(-\beta_2 + \alpha_2\,\gamma_7 + \epsilon_2\,\mu)\left(\eta\,(\gamma_7 + \epsilon_2\,\Lambda) - \epsilon_2\,\mu^2\right) + \right. \\ \left.\left.\left.\alpha^2\left(\eta^2\,(\gamma_7 - \epsilon_2\,\Lambda)^2 + \epsilon_2^2\,\mu^4 - 2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda + 2\,\alpha_2\,\gamma_7\,\Lambda + \gamma_7\,\mu + \epsilon_2\,\Lambda\,\mu)\right)\right)\right)\right)\Bigg) \,||$$

$$\Bigg(\alpha_1 > \frac{1}{2}\Bigg(\frac{\eta\,\mu\,(\beta_2 - \alpha_2\,\gamma_7 - \epsilon_2\,\mu) + \alpha\left(-\eta\,\gamma_7 + \epsilon_2\,\eta\,\Lambda + \epsilon_2\,\mu^2\right)}{-((\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda)\,\mu) + \epsilon_2\,\mu^3} + \sqrt{\Bigg(\frac{1}{\mu^2\left(\eta\,\gamma_7 + \alpha\,\epsilon_2\,\Lambda - \epsilon_2\,\mu^2\right)^2}}$$

$$\left(\eta^2\,\mu^2\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)^2-2\,\alpha\,\eta\,\mu\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)\,\left(\eta\,(\gamma_7+\epsilon_2\,\Lambda)-\epsilon_2\,\mu^2\right)+\alpha^2\,\left(\eta^2\,(\gamma_7-\epsilon_2\,\Lambda)^2+\epsilon_2^2\,\mu^4-2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda+2\,\alpha_2\,\gamma_7\,\Lambda+\gamma_7\,\mu+\epsilon_2\,\Lambda\,\mu)\right)\right)\Bigg)\Bigg)\ \&\&$$

$$\gamma_6+\frac{1}{2}\left(\left(\beta_1\,\left(\alpha\,\eta\,(-\gamma_7+\epsilon_2\,\Lambda)+\eta\,(\beta_2+2\,\alpha_1\,\gamma_7-\alpha_2\,\gamma_7)\,\mu+2\,\alpha_1\,\alpha\,\epsilon_2\,\Lambda\,\mu+\epsilon_2\,(\alpha-\eta)\,\mu^2-2\,\alpha_1\,\epsilon_2\,\mu^3\right)\right)\Big/\left(\alpha\,\left(\eta\,(\beta_2+\alpha_1\,\gamma_7-\alpha_2\,\gamma_7-\alpha_1\,\epsilon_2\,\Lambda)-\epsilon_2\,\left(\eta+\alpha_1^2\,\Lambda\right)\,\mu-\alpha_1\,\epsilon_2\,\mu^2\right)+\alpha_1\,\mu\,\left(-\eta\,(\beta_2+(\alpha_1-\alpha_2)\,\gamma_7)+\epsilon_2\,\eta\,\mu+\alpha_1\,\epsilon_2\,\mu^2\right)\right)+\sqrt{\left(\left(\beta_1^2\,\left(\eta^2\,\mu^2\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)^2-2\,\alpha\,\eta\,\mu\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)\,\left(\eta\,(\gamma_7+\epsilon_2\,\Lambda)-\epsilon_2\,\mu^2\right)+\alpha^2\,\left(\eta^2\,(\gamma_7-\epsilon_2\,\Lambda)^2+\epsilon_2^2\,\mu^4-2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda+2\,\alpha_2\,\gamma_7\,\Lambda+\gamma_7\,\mu+\epsilon_2\,\Lambda\,\mu)\right)\right)\right)\Big/\left(\alpha\,\left(\eta\,(\beta_2+\alpha_1\,\gamma_7-\alpha_2\,\gamma_7-\alpha_1\,\epsilon_2\,\Lambda)-\epsilon_2\,\left(\eta+\alpha_1^2\,\Lambda\right)\,\mu-\alpha_1\,\epsilon_2\,\mu^2\right)+\alpha_1\,\mu\,\left(-\eta\,(\beta_2+(\alpha_1-\alpha_2)\,\gamma_7)+\epsilon_2\,\eta\,\mu+\alpha_1\,\epsilon_2\,\mu^2\right)\right)^2\right)}\right)>0\Bigg)\Bigg)\Bigg)\ ||\ \Bigg(\alpha_1>0\ \&\&$$

$$\gamma_6+\frac{1}{2}\left(\left(\beta_1\,\left(\alpha\,\eta\,(-\gamma_7+\epsilon_2\,\Lambda)+\eta\,(\beta_2+2\,\alpha_1\,\gamma_7-\alpha_2\,\gamma_7)\,\mu+2\,\alpha_1\,\alpha\,\epsilon_2\,\Lambda\,\mu+\epsilon_2\,(\alpha-\eta)\,\mu^2-2\,\alpha_1\,\epsilon_2\,\mu^3\right)\right)\Big/\left(\alpha\,\left(\eta\,(\beta_2+\alpha_1\,\gamma_7-\alpha_2\,\gamma_7-\alpha_1\,\epsilon_2\,\Lambda)-\epsilon_2\,\left(\eta+\alpha_1^2\,\Lambda\right)\,\mu-\alpha_1\,\epsilon_2\,\mu^2\right)+\alpha_1\,\mu\,\left(-\eta\,(\beta_2+(\alpha_1-\alpha_2)\,\gamma_7)+\epsilon_2\,\eta\,\mu+\alpha_1\,\epsilon_2\,\mu^2\right)\right)+\sqrt{\left(\left(\beta_1^2\,\left(\eta^2\,\mu^2\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)^2-2\,\alpha\,\eta\,\mu\,(-\beta_2+\alpha_2\,\gamma_7+\epsilon_2\,\mu)\,\left(\eta\,(\gamma_7+\epsilon_2\,\Lambda)-\epsilon_2\,\mu^2\right)+\alpha^2\,\left(\eta^2\,(\gamma_7-\epsilon_2\,\Lambda)^2+\epsilon_2^2\,\mu^4-2\,\epsilon_2\,\eta\,\mu\,(-2\,\beta_2\,\Lambda+2\,\alpha_2\,\gamma_7\,\Lambda+\gamma_7\,\mu+\epsilon_2\,\Lambda\,\mu)\right)\right)\right)\Big/\left(\alpha\,\left(\eta\,(\beta_2+\alpha_1\,\gamma_7-\alpha_2\,\gamma_7-\alpha_1\,\epsilon_2\,\Lambda)-\epsilon_2\,\left(\eta+\alpha_1^2\,\Lambda\right)\,\mu-\alpha_1\,\epsilon_2\,\mu^2\right)+\alpha_1\,\mu\,\left(-\eta\,(\beta_2+(\alpha_1-\alpha_2)\,\gamma_7)+\epsilon_2\,\eta\,\mu+\alpha_1\,\epsilon_2\,\mu^2\right)\right)^2\right)}\right)>0\ \&\&$$

$$\epsilon_2\geq\frac{\beta_2-\alpha_2\,\gamma_7}{\mu}\ \&\&\ \alpha_2>0\ \&\&\ \left(\left(\beta_2>\frac{\alpha\,\gamma_7\,\mu}{\alpha\,\Lambda-\mu^2}\ \&\&\ \eta>0\ \&\&\ \alpha_2<\frac{\beta_2}{\gamma_7}+\frac{\alpha\,\mu}{-\alpha\,\Lambda+\mu^2}\ \&\&\ \eta\leq\frac{\alpha\,\mu^2}{\alpha\,\Lambda-\mu^2}\right)\ ||\ \left(\beta_2>\frac{\eta\,\gamma_7}{\mu}\ \&\&\ \eta>\frac{\alpha\,\mu^2}{\alpha\,\Lambda-\mu^2}\ \&\&\ \alpha_2+\frac{\eta}{\mu}<\frac{\beta_2}{\gamma_7}\right)\right)\Bigg)\Bigg)$$

In[∘]:=
```
iEb={4,6};
ng=NGM[mod,iEb];M=ng⟦1⟧;
K=ng⟦6⟧;K//MatrixForm
K//Eigenvalues
Det[M]//FullSimplify
chEb=(-1)^((K//Length)-1) Det[ K-IdentityMatrix[2]]//FullSimplify
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}\frac{\beta_1\,x_2}{\gamma_6+\alpha_1\,\gamma_6\,x_2} & \frac{\beta_1\,x_2}{\gamma_6+\alpha_1\,\gamma_6\,x_2}\\ 0 & 0\end{pmatrix}$$

Out[∘]=

$$\left\{\frac{\beta_1\,x_2}{\gamma_6+\alpha_1\,\gamma_6\,x_2},\,0\right\}$$

Out[∘]=

$$\gamma_4 \left(\gamma_6 - \frac{\beta_1 x_2 (1 + \alpha_1 x_2)}{(1 + \alpha_1 x_2 + \epsilon_1 x_6)^2}\right)$$

Out[∘]=

$$-1 + \frac{\beta_1 x_2}{\gamma_6 + \alpha_1 \gamma_6 x_2}$$

In[∘]:=

```
iEb={5,7};
ng=NGM[mod,iEb];M=ng[[1]];
K=ng[[6]];K//MatrixForm
K//Eigenvalues
Det[M]//FullSimplify
chEb=(-1)^((K//Length)-1) Det[ K-IdentityMatrix[2]]//FullSimplify
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{\beta_2 x_2}{\gamma_7+\alpha_2 \gamma_7 x_2} & \frac{\beta_2 x_2}{\gamma_7+\alpha_2 \gamma_7 x_2} \\ 0 & 0 \end{pmatrix}$$

Out[∘]=

$$\left\{\frac{\beta_2 x_2}{\gamma_7 + \alpha_2 \gamma_7 x_2}, 0\right\}$$

Out[∘]=

$$\gamma_5 \left(\gamma_7 - \frac{\beta_2 x_2 (1 + \alpha_2 x_2)}{(1 + \alpha_2 x_2 + \epsilon_2 x_7)^2}\right)$$

Out[∘]=

$$-1 + \frac{\beta_2 x_2}{\gamma_7 + \alpha_2 \gamma_7 x_2}$$

```
(*Solve linear equations in pol case*)
RHp=RHs/.cpol;
Print["The  variables ",el={2,3,6,7,8}, "are rationally eliminable:"];
cel=Solve[Thread[RHp[[el]]==0],var[[el]]][[1]]

Print["The fourth and fifth of remaining equations factor:"]
Rc=RHp//.cel//Flatten//Factor
fq={4,5};
so=Solve[Thread[Rc[[{4,5}]]==0],var[[fq]]]//Factor;
Print["Solving the factored equations yields ",so//Length," cases:"]
so
Print["The system is reducible to ",so//Length," equations in x1:"]
rur=Collect[#,x1]&/@(Rc[[1]]//.so);
rur//Length
FullSimplify[#]&/@rur
so1=Solve[#==0,x1]&/@rur
```

```
The  variables {2, 3, 6, 7, 8}are rationally eliminable:
```

Out[∘]=

$$\left\{x_2 \to \frac{\mu^2 - \mu x_1 \alpha + \gamma_4 x_4 \eta + \gamma_5 x_5 \eta}{(\mu - x_1 \alpha)\, \eta},\ x_3 \to \frac{-\mu + x_1 \alpha}{\eta},\ x_6 \to \frac{\gamma_4 x_4}{\gamma_6},\ x_7 \to \frac{\gamma_5 x_5}{\gamma_7},\ x_8 \to \frac{\gamma_4 x_4 + \gamma_5 x_5}{\gamma_8}\right\}$$

The fourth and fifth of remaining equations factor:

Out[ ]=

$$\left\{\frac{\mu^2\,x_1\,\alpha-\mu\,x_1^2\,\alpha^2-\mu\,\mathrm{ph}\,\eta+\mathrm{ph}\,x_1\,\alpha\,\eta+\gamma_4\,x_1\,x_4\,\alpha\,\eta+\gamma_5\,x_1\,x_5\,\alpha\,\eta}{(-\mu+x_1\,\alpha)\,\eta},\,0,\,0,\right.$$
$$\frac{\gamma_4\,x_4\,\left(\beta_1\,\mu^2-\beta_1\,\mu\,x_1\,\alpha-\gamma_6\,\mu\,\eta+\beta_1\,\gamma_4\,x_4\,\eta-\mathrm{e1}\,\gamma_4\,\mu\,x_4\,\eta+\beta_1\,\gamma_5\,x_5\,\eta+\gamma_6\,x_1\,\alpha\,\eta+\mathrm{e1}\,\gamma_4\,x_1\,x_4\,\alpha\,\eta\right)}{(\gamma_6+\mathrm{e1}\,\gamma_4\,x_4)\,(\mu-x_1\,\alpha)\,\eta},$$
$$\frac{\gamma_5\,x_5\,\left(\beta_2\,\mu^2-\beta_2\,\mu\,x_1\,\alpha-\gamma_7\,\mu\,\eta+\beta_2\,\gamma_4\,x_4\,\eta+\beta_2\,\gamma_5\,x_5\,\eta-\mathrm{e2}\,\gamma_5\,\mu\,x_5\,\eta+\gamma_7\,x_1\,\alpha\,\eta+\mathrm{e2}\,\gamma_5\,x_1\,x_5\,\alpha\,\eta\right)}{(\gamma_7+\mathrm{e2}\,\gamma_5\,x_5)\,(\mu-x_1\,\alpha)\,\eta},$$
$$\left.0,\,0,\,0\right\}$$

Solving the factored equations yields 4 cases:

Out[ ]=

$$\left\{\left\{x_4\to 0,\,x_5\to-\frac{(\mu-x_1\,\alpha)\,(\beta_2\,\mu-\gamma_7\,\eta)}{\gamma_5\,(\beta_2-\mathrm{e2}\,\mu+\mathrm{e2}\,x_1\,\alpha)\,\eta}\right\},\right.$$
$$\left\{x_4\to-\frac{\beta_1\,\mathrm{e2}\,\mu^2-\beta_1\,\mathrm{e2}\,\mu\,x_1\,\alpha+\beta_2\,\gamma_6\,\eta-\beta_1\,\gamma_7\,\eta-\mathrm{e2}\,\gamma_6\,\mu\,\eta+\mathrm{e2}\,\gamma_6\,x_1\,\alpha\,\eta}{\gamma_4\,(\beta_2\,\mathrm{e1}+\beta_1\,\mathrm{e2}-\mathrm{e1}\,\mathrm{e2}\,\mu+\mathrm{e1}\,\mathrm{e2}\,x_1\,\alpha)\,\eta},\right.$$
$$\left.x_5\to-\frac{\beta_2\,\mathrm{e1}\,\mu^2-\beta_2\,\mathrm{e1}\,\mu\,x_1\,\alpha-\beta_2\,\gamma_6\,\eta+\beta_1\,\gamma_7\,\eta-\mathrm{e1}\,\gamma_7\,\mu\,\eta+\mathrm{e1}\,\gamma_7\,x_1\,\alpha\,\eta}{\gamma_5\,(\beta_2\,\mathrm{e1}+\beta_1\,\mathrm{e2}-\mathrm{e1}\,\mathrm{e2}\,\mu+\mathrm{e1}\,\mathrm{e2}\,x_1\,\alpha)\,\eta}\right\},$$
$$\left.\{x_4\to 0,\,x_5\to 0\},\,\left\{x_4\to-\frac{(\mu-x_1\,\alpha)\,(\beta_1\,\mu-\gamma_6\,\eta)}{\gamma_4\,(\beta_1-\mathrm{e1}\,\mu+\mathrm{e1}\,x_1\,\alpha)\,\eta},\,x_5\to 0\right\}\right\}$$

The system is reducible to 4 equations in x1:

Out[ ]=

4

Out[ ]=

$$\left\{\frac{\beta_2\,\mathrm{ph}\,\eta-\gamma_7\,x_1\,\alpha\,\eta+\mathrm{e2}\,(\mu-x_1\,\alpha)\,(\mu\,x_1\,\alpha-\mathrm{ph}\,\eta)}{(\beta_2-\mathrm{e2}\,\mu+\mathrm{e2}\,x_1\,\alpha)\,\eta},\right.$$
$$\frac{\mathrm{e2}\,(\beta_1\,\mathrm{ph}-\gamma_6\,x_1\,\alpha)\,\eta+\mathrm{e1}\,(\beta_2\,\mathrm{ph}-\gamma_7\,x_1\,\alpha)\,\eta+\mathrm{e1}\,\mathrm{e2}\,(\mu-x_1\,\alpha)\,(\mu\,x_1\,\alpha-\mathrm{ph}\,\eta)}{(\beta_2\,\mathrm{e1}+\mathrm{e2}\,(\beta_1-\mathrm{e1}\,\mu+\mathrm{e1}\,x_1\,\alpha))\,\eta},$$
$$\left.\mathrm{ph}-\frac{\mu\,x_1\,\alpha}{\eta},\,\frac{\beta_1\,\mathrm{ph}\,\eta-\gamma_6\,x_1\,\alpha\,\eta+\mathrm{e1}\,(\mu-x_1\,\alpha)\,(\mu\,x_1\,\alpha-\mathrm{ph}\,\eta)}{(\beta_1-\mathrm{e1}\,\mu+\mathrm{e1}\,x_1\,\alpha)\,\eta}\right\}$$

Out[*]=

$$\left\{\left\{\left\{x_1 \to -\frac{-e2\,\mu^2\,\alpha + \gamma_7\,\alpha\,\eta - e2\,ph\,\alpha\,\eta + \sqrt{4\,e2\,\mu\,\alpha^2\,(\beta_2\,ph\,\eta - e2\,\mu\,ph\,\eta) + (e2\,\mu^2\,\alpha - \gamma_7\,\alpha\,\eta + e2\,ph\,\alpha\,\eta)^2}}{2\,e2\,\mu\,\alpha^2}\right\},\right.\right.$$

$$\left.\left\{x_1 \to \frac{e2\,\mu^2\,\alpha - \gamma_7\,\alpha\,\eta + e2\,ph\,\alpha\,\eta + \sqrt{4\,e2\,\mu\,\alpha^2\,(\beta_2\,ph\,\eta - e2\,\mu\,ph\,\eta) + (e2\,\mu^2\,\alpha - \gamma_7\,\alpha\,\eta + e2\,ph\,\alpha\,\eta)^2}}{2\,e2\,\mu\,\alpha^2}\right\}\right\},$$

$$\left\{\left\{x_1 \to -\frac{1}{2\,e1\,e2\,\mu\,\alpha^2}\left(-e1\,e2\,\mu^2\,\alpha + e2\,\gamma_6\,\alpha\,\eta + e1\,\gamma_7\,\alpha\,\eta - e1\,e2\,ph\,\alpha\,\eta + \sqrt{\left(4\,e1\,e2\,\mu\,\alpha^2\,(\beta_2\,e1\,ph\,\eta + \beta_1\,e2\,ph\,\eta - e1\,e2\,\mu\,ph\,\eta) + (e1\,e2\,\mu^2\,\alpha - e2\,\gamma_6\,\alpha\,\eta - e1\,\gamma_7\,\alpha\,\eta + e1\,e2\,ph\,\alpha\,\eta)^2\right)}\right)\right\},$$

$$\left.\left\{x_1 \to \frac{1}{2\,e1\,e2\,\mu\,\alpha^2}\left(e1\,e2\,\mu^2\,\alpha - e2\,\gamma_6\,\alpha\,\eta - e1\,\gamma_7\,\alpha\,\eta + e1\,e2\,ph\,\alpha\,\eta + \sqrt{\left(4\,e1\,e2\,\mu\,\alpha^2\,(\beta_2\,e1\,ph\,\eta + \beta_1\,e2\,ph\,\eta - e1\,e2\,\mu\,ph\,\eta) + (e1\,e2\,\mu^2\,\alpha - e2\,\gamma_6\,\alpha\,\eta - e1\,\gamma_7\,\alpha\,\eta + e1\,e2\,ph\,\alpha\,\eta)^2\right)}\right)\right\}\right\},\ \left\{\left\{x_1 \to \frac{ph\,\eta}{\mu\,\alpha}\right\}\right\},$$

$$\left\{\left\{x_1 \to -\frac{-e1\,\mu^2\,\alpha + \gamma_6\,\alpha\,\eta - e1\,ph\,\alpha\,\eta + \sqrt{4\,e1\,\mu\,\alpha^2\,(\beta_1\,ph\,\eta - e1\,\mu\,ph\,\eta) + (e1\,\mu^2\,\alpha - \gamma_6\,\alpha\,\eta + e1\,ph\,\alpha\,\eta)^2}}{2\,e1\,\mu\,\alpha^2}\right\},\right.$$

$$\left.\left.\left\{x_1 \to \frac{e1\,\mu^2\,\alpha - \gamma_6\,\alpha\,\eta + e1\,ph\,\alpha\,\eta + \sqrt{4\,e1\,\mu\,\alpha^2\,(\beta_1\,ph\,\eta - e1\,\mu\,ph\,\eta) + (e1\,\mu^2\,\alpha - \gamma_6\,\alpha\,\eta + e1\,ph\,\alpha\,\eta)^2}}{2\,e1\,\mu\,\alpha^2}\right\}\right\}\right\}$$

```
so = Solve[Thread[(RH1 /. c46) == 0], va1]
so // Length

e1 = FullSimplify /@ CoefficientList[Numerator[Together[RHS[[1]] //. Join[so[[3]], c46]]], x1]
(*Timing[so=Solve[Thread[(RH1)==0],va1]]
so//Length*)


Print["RUR in x1 "]
```