

# Many objective robust decision making for complex environmental systems undergoing change

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## ABSTRACT

This paper introduces many objective robust decision making (MORDM). MORDM combines concepts and methods from many objective evolutionary optimization and robust decision making (RDM), along with extensive use of interactive visual analytics, to facilitate the management of complex environmental systems. Many objective evolutionary search is used to generate alternatives for complex planning problems, enabling the discovery of the key tradeoffs among planning objectives. RDM then determines the robustness of planning alternatives to deeply uncertain future conditions and facilitates decision makers' selection of promising candidate solutions. MORDM tests each solution under the ensemble of future extreme states of the world (SOW). Interactive visual analytics are used to explore whether solutions of interest are robust to a wide range of plausible future conditions (i.e., assessment of their Pareto satisficing behavior in alternative SOW). Scenario discovery methods that use statistical data mining algorithms are then used to identify what assumptions and system conditions strongly influence the cost-effectiveness, efficiency, and reliability of the robust alternatives. The framework is demonstrated using a case study that examines a single city's water supply in the Lower Rio Grande Valley (LRGV) in Texas, USA. Results suggest that including robustness as a decision criterion can dramatically change the formulation of complex environmental management problems as well as the negotiated selection of candidate alternatives to implement. MORDM also allows decision makers to characterize the most important vulnerabilities for their systems, which should be the focus of ex post monitoring and identification of triggers for adaptive management.

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## 1. Introduction

This paper contributes the many objective robust decision making (MORDM) framework, which combines many objective evolutionary optimization, robust decision making (RDM), and interactive visual analytics to facilitate the management of complex environmental systems. The MORDM framework seeks to address several key challenges for environmental systems undergoing change. The first is how to evaluate the performance of alternative planning and management strategies. To make these evaluations, planners have traditionally used cost benefit analysis, in which a project's benefits are commensurated to their expected monetary value and then compared to a project's costs to determine whether the project will be funded (Griffin, 1998). Aggregating these multiple performance measures into a single value can yield negative decision biases that result because different aspects of performance

are rewarded and penalized in ways that cannot be predicted *a priori* (Franssen, 2005). The approach has also been shown to inadequately compensate for non-monetary benefits (Bromley and Beattie, 1973) especially when multiple policies are considered (Hoehn and Randall, 1989). Multiobjective approaches instead seek to quantify the large number of conflicting objectives that characterize planning. In addition to cost, it has been recognized that complex planning efforts often reveal additional critical performance objectives (Hitch, 1960), such as maximizing reliable performance, minimizing environmental damages, and improving system efficiency. The Harvard Water Program was one of the earliest efforts to advocate for the multiobjective planning approach by emphasizing the importance of both economic objectives and engineering performance objectives (Maass et al., 1962; Reuss, 2003; Banzhaf, 2009). Considering "many" objectives explicitly and simultaneously can also aid planners in avoiding cognitive myopia (Hogarth, 1981). Cognitive myopia arises when decision makers inadvertently ignore aspects of the problem (such as important decision alternatives or key planning objectives) by

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focusing on a limited number of *a priori* specified alternatives or a narrowly defined, highly aggregated definition of “optimality” (Brill et al., 1990).

As introduced in Cohon and Marks (1975), a problem with multiple objectives can be defined by minimizing a vector,  $F(\mathbf{l})$ , demonstrated in Equation (1),

$$\text{minimize } \mathbf{F}(\mathbf{l}) = (f_1, f_2, \dots, f_p) \quad \forall \mathbf{l} \in \Omega \quad (1)$$

Subject to:

$$c_i(\mathbf{l}) = 0 \quad \forall i \in [1, q] \quad (2)$$

$$c_j(\mathbf{l}) \leq 0 \quad \forall j \in [1, r] \quad (3)$$

where  $\mathbf{l}$  is a vector of design levers<sup>1</sup> (decision variables) in the decision space,  $\Omega$ . The decisions (design levers) can be expressed as real-valued, integer, or binary variables. The problem formulation also can impose  $q$  equality constraints (Equation (2)) and  $r$  inequality constraints (Equation (3)). Feasible solutions are then defined as those that can meet all the imposed constraints. The solution to a multiobjective problem in Equation (1) is the set of non-dominated or non-inferior solutions. Assuming minimization of all objectives, a solution  $F(\mathbf{a}) = [f_1(\mathbf{a}), \dots, f_p(\mathbf{a})]$  dominates  $F(\mathbf{b}) = [f_1(\mathbf{b}), \dots, f_p(\mathbf{b})]$  if and only if  $\forall i, f_i(\mathbf{a}) \leq f_i(\mathbf{b})$  and there exists  $j$  such that  $f_j(\mathbf{a}) < f_j(\mathbf{b})$ . Extending this definition to the entire decision space  $\Omega$ , a solution is termed Pareto optimal if it is non-dominated with respect to all other feasible solutions. Since it is often intractable to enumerate all feasible solutions for a complicated problem, we seek the best-known approximation to the Pareto optimal set, termed the Pareto approximate set.

In practice, there are several factors that make it difficult to find Pareto approximate solutions for complex problems. Severe performance constraints may arise due to regulatory requirements (Sophocleous, 2000), resource limitations (Cooper and Sehlke, 2012), or risk aversion (Characklis et al., 2006; Kasprzyk et al., 2009; Brekke et al., 2009). The presence of constraints severely limits the number of feasible solutions. Also, the simulation models that quantify performance for complex environmental systems may include many interacting subsystems, such as agricultural, municipal, and environmental concerns, that introduce non-linearities and non-separable dependencies (Haimes, 1977; Cai, 2008; Nicklow et al., 2010; Reed et al., 2012). These complexities have motivated a growing number of researchers to exploit multi-objective evolutionary algorithms (MOEAs), which represent a class of solution techniques that can perform well when solving constrained, non-linear problems with decision spaces that can be large in dimension, discrete, non-convex, and stochastic (Coello Coello et al., 2007; Nicklow et al., 2010; Reed et al., 2012). These challenging properties often emerge when MOEA search is coupled with complex simulation models to discover alternative outcomes. Although the benefits of combining simulation models were clearly recognized in seminal water resources planning and management efforts (see for example Maass et al., 1962; Loucks et al., 1981; Simonovic, 1992), very recent breakthroughs in computational power and MOEA-based search are now enabling studies to fully realize the benefits of these tools.

The mathematical challenges that have motivated the growing use of MOEAs are very relevant to the planning and management of complex environmental systems. These systems require a risk-based, stochastic approach in which decision makers can evaluate the resilience of their system (Hashimoto et al., 1982) as well as the adaptability and robustness of their decisions (Rockstrom et al., 2009). A growing number of studies have recently explored using MOEAs under uncertainty (Liu and Frangopol, 2005; Deb and Gupta, 2006; Medaglia et al., 2007; Kourakos and Mantoglou, 2008; Singh and Minsker, 2008; Salazar Aponte et al., 2009; Kasprzyk et al., 2009, 2012; Fu and Kapelan, 2011). When considering uncertainty, previous work using MOEAs has relied on expected value calculations of objectives and constraints that assume well-characterized probability distributions (Morgan and Henrion, 1990). However, such expected value calculations of the objectives and constraints may not sufficiently characterize risk under conditions of deep uncertainty. The term *deep uncertainty* refers to components of a planning or management problem where decision makers cannot agree upon the full set of risks to a system or their associated probabilities (Knight, 1921; Langlois and Cosgel, 1993; Lempert, 2002; Lempert et al., 2003). Land use change, the depletion of resources, and climate change are three examples of human-induced changes that introduce deep uncertainties into planning problems (Milly et al., 2008; Reed and Kasprzyk, 2009; Polasky et al., 2011). Decision makers often use scenario analysis (Mahmoud et al., 2009) to help plan for such challenges. Scenario analysis uses a small number of plausible values for key planning variables (such as economic growth) to create storylines for future conditions in a system. As highlighted by Groves and Lempert (2007), there are several important limitations associated with using scenario analysis for deeply uncertain factors within a planning problem. First, a large number of different factors can shape the future, and a small number of scenarios cannot adequately cover all interactions between the different factors. Additionally, there is little guidance on how scenarios should inform decision making. For example, in climate change planning, a single value for global population growth is typically used in each scenario (IPCC, 2000; Arnell et al., 2004), causing subsequent decision making to be contingent on the assumed value.

Decision support strategies that use the concept of *robustness* can help address these challenges, by identifying strategies that perform well across many different assumptions regarding the deeply uncertain factors (Lempert, 2002; Brown, 2010; Hine and Hall, 2010; Brown et al., 2011). This study uses robust decision making (RDM) (Lempert, 2002; Lempert et al., 2003; Bryant and Lempert, 2010; Hall et al., 2012) to characterize robustness. RDM evaluates the performance of policy strategies over an ensemble of deeply uncertain trajectories of the future. Decision makers select plausible ranges for each deeply uncertain factor, but they do not *a priori* select scalar values for a small number of scenarios. Instead, RDM employs statistical algorithms to “discover” scenarios, which are ranges of the exogenous factors<sup>2</sup> that in combination cause poor performance (Bryant and Lempert, 2010). RDM therefore provides a tool for decision makers to determine how changes in their assumptions about exogenous factors affect the performance of their planning strategies. For complex environmental systems, this is especially useful because it can help planners determine the impacts of their predictions and assumptions on the decision making process. Drought forecasting, for example, requires forecasts of runoff, water storage, and return flow. Mitigation strategies are

<sup>1</sup> In this study, we use terminology that differs slightly from terms typically used with MOEAs and optimization. Decision variables for optimization are the same as the levers in this study, and quantitative measures refer both to objectives and also to constraints.

<sup>2</sup> Exogenous factors are those in the modeled system considered not under the direct control of the decision makers, in contrast to the modeled factors considered part of the decision makers’ choice set.

based on the assumptions behind these forecasts, but there are severe political ramifications when these estimates are wrong (Glantz, 1982). By avoiding *a priori* estimates of values of the deeply uncertain factors, RDM avoids a common decision maker bias of being overly confident about current management strategies and timid in adopting planning innovations that reduce risk (Kahneman and Lovallo, 1993).

The purpose of this paper is to propose and demonstrate the MORDM framework, by combining the strengths of MOEA optimization and RDM. The MORDM framework makes two primary contributions. First, RDM has not previously incorporated global optimization techniques such as MOEAs to discover planning alternatives. MORDM exploits MOEAs to solve many objective problems of four or more objectives, thus providing a rich set of alternatives as inputs to RDM. Second, we address the issue of selecting a preferred solution from MOEA-generated tradeoffs. Solution robustness is a promising way to ensure acceptable performance even if system conditions strongly deviate from those used to evaluate the optimality of alternatives (i.e., Pareto satisficing<sup>3</sup> behavior in extreme states of the world). MORDM represents *a posteriori* decision support, in that it does not require assumptions about decision maker preferences before the analysis begins. This study demonstrates how interactive visual analytics (Thomas and Cook, 2006; Keim et al., 2006; Thomas and Kielman, 2009) can support collaborative decision making and enhance planners' ability to effectively process the large amount of information generated by the MORDM framework. This research builds off the historical work in joint cognitive systems (Woods, 1986; Woods and Hollnagel, 2006) with the intent of maximizing the combined analytical strengths of humans and computers when addressing planning and management problems for complex environmental systems. Furthermore, the MORDM framework is designed to emphasize learning and stakeholder feedbacks as part of the decision making process. Our focus on learning and stakeholder feedbacks reflects the fact that public planning problems are rarely static, and have formulations that must change over time (Rittel and Webber, 1973).

Fig. 1 presents the MORDM framework, consistent with the iterative "deliberation with analysis" decision support process recommended by National Research Council (2009). Initially, one or more problem formulations begin the process. Each problem formulation is a formalized hypothesis about what the decision makers feel is most important for their problem, and it is continually updated as we learn more about the system (Liebman, 1976; Zeleny, 1981; Rosenhead, 1996). For example, Kasprzyk et al. (2012) introduce the de Novo planning where a variety of decision strategies, objectives, and constraints are explored simultaneously to carefully evaluate the strengths and weaknesses of alternative problem formulations. The acronym XLRM (Lempert et al., 2003, 2006) is used to describe the four problem formulation components. The formulation identifies uncertainties ("X"), factors beyond the decision maker's control, using two main categories. The first category of uncertainties have well-characterized distributions. The second category, termed deep uncertainties, occur when the model representing a system, key model parameters, or the probability distributions representing classical uncertainties are not known or cannot be agreed upon (Lempert et al., 2006). Decision levers represent actions the decision makers can take to modify their system ("L"). A quantitative relationship ("R") maps decision maker actions to outcomes, typically using a simulation model. Finally, performance measures ("M") are used to gauge success. After defining an initial problem formulation, the second

step of MORDM, termed Generating Alternatives, uses an MOEA to find a Pareto approximate set of solutions. This step incorporates the classical uncertainties by including them in the relationships governing the system. The third step, uncertainty analysis, interrogates solutions' performance under deep uncertainties (e.g., climate change effects, population changes, land-use changes, etc.). To do so, we globally sample deeply uncertain exogenous factors that strongly influence the plausible future states of the world (SOW), and evaluate the performance measures of each solution in every SOW. Highly interactive visual analytics are then exploited to understand Pareto approximate tradeoffs as well as the robustness of the component solutions that compose the tradeoffs. The decision maker can choose candidate solutions for further exploration after step 3. In step 4, we subject the candidate solutions to a scenario discovery process (Bryant and Lempert, 2010), which seeks to identify the specific combinations of deeply uncertain exogenous factors that most strongly influence the ability of potentially robust solutions to meet their multi-measure goals. Concepts and methods from RDM contribute to multiple steps of the MORDM framework found in Fig. 1. RDM informs the problem formulation, in particular identifying deep uncertainties and contributing the XLRM framework. RDM informs the uncertainty analysis, by evaluating the performance of candidate solutions over a wide range of plausible SOW. Finally, RDM's SD identifies the key drivers of those SOW in which the candidate solutions fail to perform well.

The remainder of this paper proceeds as follows. Section 2 explains the rationale of each step in MORDM. The case study used to demonstrate the framework is introduced in Section 3. Results are presented in Section 4. Sections 5 and 6 provide general discussion and concluding remarks.

## 2. Methods

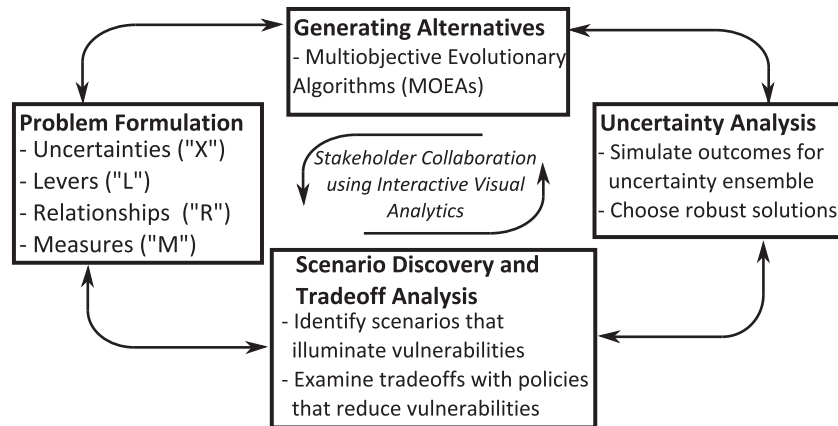
### 2.1. Problem formulation

As illustrated in Fig. 1, the problem formulation component of the MORDM framework represents decision makers' evolving hypotheses about the most important uncertainties, levers, relationships and measures (XLRM) for their system. Decision levers quantify an action or policy that can be taken to influence the system. Creative formulation of decision levers is critical for discovering planning alternatives that can dramatically improve system performance. For example, risk-based contracts based on insurance instruments can dramatically improve how water utilities confront hydrologic extremes and growing planning uncertainties (Palmer and Characklis, 2009). Measures quantify the performance of decision makers' actions with respect to multiple outcomes. A "relationship" represents a mapping from actions to outcomes as quantified using the output measures. The relationship can vary from a simple screening model (Groves and Lempert, 2007) to an agent-based model that considers multiple actors interacting with each other (Lempert, 2002; Yang et al., 2009).

When formulating a planning problem in the modern context of environmental change, it is vital to very carefully consider its associated uncertainties. Literature on complex environmental systems often distinguishes natural variability from epistemic uncertainties that can be reduced through further observation or knowledge (Oberkampf et al., 2004). Epistemic uncertainty can include errors in estimated probabilities for extreme events (IPCC, 2000) and errors associated with model structure (Haimes, 1977). MORDM focuses on epistemic uncertainty from several sources: assumptions for estimated probabilities, model structures, and alternative configurations of decision levers. We build on the concept of "deep" uncertainties that emerge from the suite of risks in a system or their associated probabilities that are not known or cannot be agreed upon (Lempert et al., 2006). The motivating idea is to characterize our uncertainties with currently available data while also acknowledging the nonstationarity of environmental systems (Milly et al., 2008). Nonstationarity can be incorporated in planning by exploring how changing conditions can cause environmental systems to deviate from their expected behavior.

The feedback arrows in Fig. 1 illustrate the importance of learning and feedbacks across all of the steps of the MORDM framework. Treating problem formulation as a learning process supports the idea that decision support should improve stakeholders' conceptual understanding of complex environmental systems (Liebman, 1976; Roy, 1999). For example, after initial trials of optimization, some measures may be removed and others added (Zeleny, 2005; Kasprzyk et al., 2012). The decision makers may also condition their decision on some measures that are not considered during the optimization (Loughlin et al., 2001).

<sup>3</sup> By Pareto satisficing, we mean that a solution's performance remains close to the Pareto optimal surfaces for each of many future states of the world.



**Fig. 1.** The four steps of the many objective robust decision making (MORDM) framework. The process typically begins with problem formulation. Each step facilitates stakeholder involvement using interactive visual analytics.

## 2.2. Generating alternatives using MOEAs

Many objective problem formulations aim to show decision makers critical tradeoffs between their performance measures (Haimes and Hall, 1977; Cohon and Marks, 1975; Kasprzyk et al., 2009). MOEAs provide an effective way to discover such tradeoff solutions for complex environmental systems. Their population-based search yields approximations to the Pareto optimal front in a single algorithm run (see Deb, 2001; Coello Coello et al., 2007; Efstratiadis and Koutsogiannis, 2010; Nicklow et al., 2010; Reed et al., 2012, for reviews).

For MOEAs to successfully attain high quality approximations to the Pareto frontier, they must converge to a diverse group of solutions that cover the full extent of an application's tradeoffs. The concept of convergence is an important measure of MOEA performance for a problem. It measures how close the MOEA's approximation set has come to the theoretical Pareto optimal front or a best-known approximation to the front. Diversity maintenance is the second critical component to successful MOEA search. It emphasizes that a MOEA must find points "well-spread" across the entire Pareto front. Although early MOEAs had difficulties in maintaining convergence and diversity for challenging problems (Reed et al., 2012), several modern MOEAs have theoretic proofs of convergence and diversity (Rudolph, 1998; Rudolph and Agapie, 2000; Laumanns, 2002; Reed et al., 2012). Recent diagnostic assessments of top-performing MOEAs on severely challenging many objective test problems (Hadka and Reed, 2012) and on many objective water resources applications (Reed et al., 2012) emphasize that recently introduced auto-adaptive MOEAs are effective, efficient, and easier-to-use. These studies highlight that top performing MOEAs can maintain both convergent and diverse search for problems with up to 10-objectives that have a broad range of properties (i.e., nonlinearity, non-convexity, discreteness, stochasticity, and severe constraints).

## 2.3. Uncertainty analysis

The previous step results in a set of Pareto approximate solutions contingent on the best-estimate values and probability distributions for input parameters to the simulation model. This step explores how the solutions in this set perform when these best-estimate assumptions are relaxed. The MORDM uncertainty analysis interrogates each solution in the Pareto approximate set with a large number of alternative SOWs to see how the solution performs under a range of assumptions regarding the exogenous factors. Performing the analysis on all points shows the analyst which regions of the space are robust, which may be very different than the performance attained in the baseline SOW. The uncertainty analysis ultimately helps the decision maker choose a formulation and one or more constituent solutions that have acceptable (Pareto satisficing) performance across a wide range of plausible future scenarios.

There are three important design considerations for uncertainty analysis in MORDM. The first is how to sample deeply uncertain exogenous factors (e.g., future climate conditions, population growth, market pricing, etc.) to create an uncertainty ensemble.<sup>4</sup> Our demonstration in the current paper first sets the upper and lower

ranges of the deeply uncertain dimensions and then uses Latin Hypercube Sampling (LHS, McKay et al., 1979)<sup>5</sup> to create an ensemble of possible values for the uncertainties. Each ensemble member, or SOW, represents a set of values for each exogenous factor whose impact on future conditions is being explored.<sup>6</sup> Further extensions of this approach could also use multiple simulation model structures or different governing equations across each SOW. A second consideration is how many measures to use for calculating robustness. Some previous RDM work has considered robustness over multiple attributes (Lempert et al., 2003; Dixon et al., 2008; Popper et al., 2009), but these studies have not systematically identified the full range of strategies that might satisfy different preferences among the objectives. In contrast, MORDM provides such a capability. Finally, the analyst should determine appropriate statistical thresholds for defining robustness. For its example application, this paper selects potentially robust candidate solutions based on each solution's performance in the most extreme SOW samples. This does not mean that we believe the most extreme ensemble members will actually occur. Rather, we consider these extreme cases in order to understand the conditions where a proposed strategy may fail to meet its performance goals.

## 2.4. Scenario discovery

Scenario discovery (SD) uses statistical cluster analysis on databases of simulation model runs to identify simple, easy-to-understand descriptions of the combinations of uncertain model inputs that best predict the future states of the world where the strategies identified as robust in the uncertainty analysis nonetheless perform poorly (Lempert, 2012). We first set thresholds for each of the performance measures, in accordance with plausible stakeholder preferences. Uncertainty ensemble members that violate these thresholds are hereafter termed "vulnerable". Fig. 2 shows a hypothetical example of the SD process. Step 1 illustrates thresholds for a measure,  $m$ , that represent the delineation between acceptable and vulnerable performance. In our example, the threshold is set at the 90th percentile of the measure data. In step 2, the points with vulnerable performance are colored in black, with the other points shown without a fill color. The horizontal and vertical axes in step 2 are values for uncertainties,  $x_1$  and  $x_2$ . As is shown in the figure, it can be difficult to describe the cluster of black points (i.e., the values that cause vulnerabilities) in simple terms. SD therefore employs the Patient Rule Induction Method (PRIM, Friedman and Fisher, 1999)<sup>7</sup> to automatically calculate "scenario boxes" such as the gray shading in the figure. Each scenario represents ranges of exogenous parameter values over which the candidate solution performs poorly. PRIM's boxes are similar to

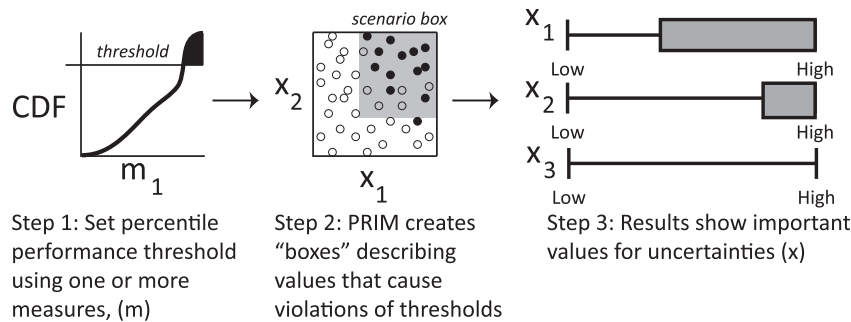
<sup>5</sup> LHS was chosen as a sampling method following prior work (Bryant and Lempert, 2010). LHS breaks the sampling space into evenly-spaced grid boxes and chooses a random value within each grid box. This method of stratification can more evenly sample a space relative to uniform random sampling (Saltelli et al., 2000).

<sup>6</sup> Other factors not included in the SOW could also affect the policy's performance. Subsequent use of SOW in this paper refer only to changes in the dimensions chosen for the model, similar to a *ceteris paribus* assumption in economics.

<sup>7</sup> Lempert et al. (2008) compared PRIM to scenario discovery using Classification and Regression Tree (CART) analysis, favoring PRIM due to its high level of user interactivity. SD using PRIM is implemented in R, and is freely available (Bryant, 2009).

<sup>4</sup> Exogenous factors are those that can be considered external to the modeled system. Note that whether a factor is exogenous or endogenous depends on the specific model. For example, if a population growth depends on a factor that is explicitly included in a simulation (such as resource availability), the population growth would be endogenous. If no population model is included in a simulation and decision makers assume a rate, the assumed rate is considered exogenous.





**Fig. 2.** The three main steps of scenario discovery using the Patient Rule Induction Method (PRIM). Each step uses interactive visualizations in the R statistical package, with the user setting performance thresholds and levels of coverage and density for PRIM scenario boxes.

traditional scenarios because they provide simple descriptions of future trajectories. An important difference between our scenario boxes and traditional point-valued scenarios, though, is that the boxes are developed using quantitative descriptions of system performance. PRIM functions by “peeling” away thin layers of the uncertainty space to identify scenarios and also iteratively increases or decreases dimensions of the proposed candidate box (the number of uncertainties that are restricted) depending on how well those dimensions capture the cluster of points of interest. The resulting boxes are expressed in the form  $B = \{a \leq x_j \leq b, j \in L\}$ . In other words, a subset of dimensions of uncertainty  $x_j$  are constrained to be between lower and upper ranges  $a$  and  $b$ .

In an interactive process, PRIM suggests alternative candidate boxes from which the users choose. The algorithm identifies boxes that lie on a Pareto optimal surface described by three measures. Coverage quantifies how many of the vulnerable points are captured within a scenario. Density indicates how many of the captured points within a scenario are actually in the vulnerable set. Interpretability, which indicates how easily users can understand the information, is considered to decrease with the number of parameters used to define the box (Bryant and Lempert, 2010).<sup>8</sup> Importantly, scenario discovery also suggests which uncertain parameters are less important in describing the vulnerable cases. Step 3 in Fig. 2 is an example of how the constrained dimensions in the scenario box are visualized. In the example only 2 dimensions out of 3 are constrained, indicating that the first two dimensions are the most important uncertainties for the system where the gray shading shows the specific ranges of variables where vulnerabilities emerge. Some RDM analyses use such scenarios to help identify policies that might ameliorate these vulnerabilities and then present tradeoff curves to help participants decide whether these new policies are worth adopting (Lempert et al., 2006; Lempert and Groves, 2010; Popper et al., 2009; Hall et al., 2012). In these analyses, the potentially ameliorating policies are chosen from an already-existing set of options or handcrafted by analysts with input from stakeholders. A MORDM analysis might significantly improve the choice set by reapplying its evolutionary algorithms specifically to search for policies that ameliorate the vulnerabilities identified by SD with minimal tradeoffs with other goals. However, we leave this potentially promising but challenging step for future work.

## 2.5. Interactive visual analytics

Since the MORDM framework represents a form of constructive *a posteriori* decision support (Roy, 1999; Lempert, 2002; Tsoukias, 2008; Reed and Kasprzyk, 2009), a large amount of information is available to inform decision makers' choices. A successful system therefore requires a method for processing and viewing the data that makes it tractable for decision makers to interpret and analyze. Interactive visual analytics (Keim et al., 2006; Thomas and Cook, 2006; Thomas and Kielman, 2009; Andrienko et al., 2010) uses multiple linked views of decision-relevant information to facilitate the data processing procedure. In our proposed MORDM framework, visual analytics is critical for facilitating insights and the articulation of preferences (i.e., MOEA-generated tradeoffs, PRIM assessment of uncertainties, etc.).

As shown in the center of Fig. 1, visual analytics can enhance all four MORDM steps. The first major use of interactive visual analytics for MORDM is visualizations of decision levers, performance measures, and metrics of robustness for each

solution. These components can be combined with other variables that were not included in optimization formulations (Loughlin et al., 2001) but may ultimately influence a decision maker's analysis.<sup>9</sup> Several plotting types can be used, with the main purpose of comparing solution properties and ultimately choosing a robust solution. Glyph plots use 3-dimensional Cartesian coordinates, as well as points' size, color, rotation, and transparency to show trends in up to 7 dimensions simultaneously (Kollat and Reed, 2007b). Prior work has used these visualizations to compare competing problem formulations (Kasprzyk et al., 2009, 2012) and trade-offs through time (Reed and Kollat, 2012). In addition to glyphs, parallel coordinate plots (Inselberg, 1985; Wegman, 1990; Fleming et al., 2005) can show many dimensions at once, but only allow pairwise comparison between subsequent dimensions. Both plotting types are interactive, with the analyst changing which variables appear on each axis in real-time. The analyst can also “brush” solutions that meet user-defined criteria (Inselberg, 1997), reducing the number of plotted solutions at any one time.

Interactive visual analytics are also used in uncertainty analysis and scenario discovery. The analyst can visualize how each solution responds to the sampled uncertainty ensemble using the multiple performance measures. Within SD, visuals for coverage and density are integral in allowing the user to guide PRIM in creating scenarios. This process can inform modifications to the performance thresholds, or perhaps even demonstrate that additional dimensions of uncertainty should be included in subsequent analyses.

## 3. Lower Rio Grande Valley case study implementation

### 3.1. Motivation

Although climate change and urbanization pose serious threats to water management, the rising cost of building new infrastructure motivates non-structural approaches such as water marketing (Anderson and Hill, 1997) for ensuring a sufficient quantity of supply. In a water market, agents transfer quantities of water, either across different regions (Israel and Lund, 1995) or different user sectors within the same water system (Hadjigeorgalis, 2008). The presence of a water market does not imply that the city can easily ascertain the best combination of traditional and market-based instruments for their supply. Adding a new portfolio instrument (spot leasing) to a strategy that uses traditional supply and an options contract can reduce costs and surplus water (Kasprzyk et al., 2009). The significant performance improvement achieved by adding new planning instruments underscores an important uncertainty in defining the planning process itself. In a water marketing system, alternative system assumptions and formulations can dramatically influence the discovery and exploitation of critical planning tradeoffs (Kasprzyk et al., 2012).

These issues motivate water marketing as a challenging planning context that can serve as an excellent test case for demonstrating the MORDM framework. Our case study develops portfolio-based strategies for a hypothetical single city in the Lower Rio

<sup>8</sup> In Fig. 2, 18 points are captured in the scenario box. Of the points captured in the box, only 12 are vulnerable, but 13 points in total are vulnerable. Therefore, the coverage of our candidate box is  $12/13 = 92\%$ , whereas the density of the box is  $12/18 = 67\%$ . Boxes that have high density may “miss” many vulnerable points because the vulnerable points are spread through the uncertainty space. Conversely, scenarios with high coverage may not constrain the uncertainty dimensions very much, leading to low density and a poor amount of scenario precision. For a more detailed discussion please refer to Bryant and Lempert (2010).

<sup>9</sup> Decision makers often see decision variables, objectives, and other variables as interchangeable (Tsoukias, 2008). In other words, they would like to see how the decision space, performance space, and other factors interact concurrently.

Grande Valley (LRGV) of Texas (see Characklis et al., 2006; Kirsch et al., 2009; Kasprzyk et al., 2009, 2012). The city uses water market transfers from agriculture to municipal use to augment their traditional reservoir-based supply. The LRGV case study assumes that municipal use has the same priority as water for irrigation, with the goal of determining reliable portfolio strategies that can increase the city's reliability while ensuring sufficient water for other regional uses (Characklis et al., 1999).

Our use of the LRGV test case in this study builds on the de Novo planning results and assumptions from Kasprzyk et al. (2012). The de Novo planning framework proposed by Kasprzyk et al. (2012) seeks to formalize the discovery and evaluation of alternative formulations for challenging planning problems. De Novo planning does this by starting with an *a priori* selection of decision variables, objectives, and constraints, and subjecting this formulation to global sensitivity analysis. We then search a suite of alternative problem formulations, using evolutionary multiobjective optimization to develop alternative tradeoffs for the LRGV. The study ultimately sought to identify which of these alternative formulations were “non-dominated”, versus the more traditional focus on non-dominated solutions in a single formulation. In our prior work, our choice of a preferred model case and candidate solutions, though, was strongly dependent on the expected value calculations used to assess performance tradeoffs. These expectations assume that our LRGV Monte Carlo simulation using historical data fully captures the breadth of uncertain futures that occur for the system. Unfortunately, these assumptions could very plausibly be violated in future planning periods. For example, Levine (2007) highlights concerns over reduced inflow from the Mexican tributaries feeding the LRGV's reservoir system. Climate change is projected to cause increasing temperatures, which could increase reservoir evaporation and modify streamflows (e.g. Zhu et al., 2005). Fluctuations in agricultural commodity prices also influence irrigators' willingness to participate in a water market (Ranjan, 2010), possibly increasing market prices. Each of these challenges illustrates that the “expected” performance assessment for the LRGV could be negatively impacted by deeply uncertain risks for unexpected system changes and shifts in estimated likelihoods.

Our goal is to demonstrate how the MORDM framework addresses the following questions: Was our choice of a preferred model case and its component solutions critically biased by our use of historical data to assess the expected behavior for the LRGV system? If so, what are the controlling assumptions or conditions we should consider to improve water planning for the LRGV test case? The following sections discuss our computational experiment for each of the steps shown in Fig. 1: problem formulation, alternatives generation, uncertainty analysis, and scenario discovery.

### 3.2. Problem formulation

Our problem formulation is constructed using the four components of the XLRM framework: uncertainties, levers, relationships, and measures. This section focuses on the decision levers, performance measures, and quantitative relationship for the LRGV, with the uncertainties discussed in Section 3.4.

Decision levers are used to construct the city's portfolio of three water supply instruments: permanent rights, spot leases, and options. Permanent rights are a non-market based instrument in which the city is allocated a percentage of reservoir inflow in each month, using a ratio of the volume of the city's rights (the decision lever,  $N_R$ ) to the total volume of regional rights. Spot leases can be acquired at any month in the year with a variable price. An adaptive options contract reduces lease price volatility by guaranteeing a fixed price for water acquisitions made later in the year. The options contracts are controlled by up to three variables: a high and

low volume ( $N_{O_{low}}$ ,  $N_{O_{high}}$ ), with a threshold ( $\xi$ ) that decides which contract to activate depending on the available water supply (Kirsch et al., 2009). Acquisitions of water from both leases and options are controlled by anticipatory thresholds that relate expected supply to demand. The thresholds control “when” ( $\alpha$ ) and “how much” ( $\beta$ ) water the city must acquire when using the market. Since 85% of the water in the region is used for agricultural use, we assume that there will always be enough water to meet the transfer requests specified by the alpha and beta variables (Characklis et al., 2006).

To test the de Novo results of prior work, we adapt our treatment of decision levers from that study (Kasprzyk et al., 2012). To choose an appropriate configuration of decision levers, Sobol' variance decomposition (Sobol', 1993) was used to analyze the effect of each decision lever on the LRGV's performance measures in the initial many objective formulation (Kasprzyk et al., 2009). The results suggested that only the volume of permanent rights and the alpha strategy variables significantly influenced performance, and the performance was less sensitive to the adaptive options contract variables and beta. The fact that a small subset of variables had this sensitivity performance motivated Kasprzyk et al. (2012) to explore the question: “what is the minimum level of formulation complexity that is justified and effective for the LRGV test case?” The four candidate formulations of decision levers are summarized in Table 1. Case I uses the volume of permanent rights, a single-volume non-adaptive options contract, and one variable to determine both when to go to the market and how much water to acquire. Case II varies that threshold decision by the time of the year, and case III separates the when and how much decision by explicitly searching for separate alpha and beta values. Case IV adds in the adaptive options contract to meet the full complexity of the *a priori* problem formulation from the initial study (Kasprzyk et al., 2009).

The “relationship” for our problem formulation uses a 10-year expected performance Monte Carlo simulation and an extreme drought, both with a monthly timestep. For further details about the simulation model and its implementation, the reader is encouraged to consult a series of prior studies (Characklis et al., 2006; Kirsch et al., 2009; Kasprzyk et al., 2009, 2012). The simulation model samples historical lease pricing, demand, and reservoir inflows to test how the supply portfolios would perform under a single best estimate of the LRGV's uncertainties. We also use an extreme drought scenario, which combines the driest year in the historical record with the maximum demands from the discrete demand distributions used in the simulation. The city begins with a volume of water controlled by initial rights, and must satisfy its demands using its portfolio planning strategy. The drought scenario provides a challenging test for the portfolios, since the drought conditions maximize the difference between the expected supply and demand used in alpha/beta calculations and the city's actual demand (Kasprzyk et al., 2009).

**Table 1**  
Model cases.

Case	Volumetric decisions	Strategy decisions	Notes
I	$N_R, N_O$	$\alpha$	Single opt. contract, one alpha controls “when” and “how much”
II	$N_R, N_O$	$\alpha_{\text{May–Dec}}, \alpha_{\text{Jan–Apr}}$	Single opt. contract, two alphas control “when” and “how much”
III	$N_R, N_O$	$\alpha_{\text{May–Dec}}, \alpha_{\text{Jan–Apr}}, \beta_{\text{May–Dec}}, \beta_{\text{Jan–Apr}}$	Single opt. contract, alphas control “when”, betas control “how much”
IV	$N_R, N_{O_{low}}, N_{O_{high}}$	$\xi, \alpha_{\text{May–Dec}}, \alpha_{\text{Jan–Apr}}, \beta_{\text{May–Dec}}, \beta_{\text{Jan–Apr}}$	Adaptive opt. contract, Full-complexity formulation

Each configuration of decision levers in Table 1 is tested using a many objective formulation of measures (objectives and constraints) as shown in the following equations.

$$\mathbf{F}(\mathbf{I}_k) = \left( f_{10 \text{ yr. cost}}, f_{10 \text{ yr. surplus}}, f_{10 \text{ yr. crit. rel.}}, f_{10 \text{ yr. dropped}}, \right. \\ \left. f_{10 \text{ yr. num. leases}}, f_{\text{dr. trans. cost}} \right) \\ \forall \mathbf{I}_k \in \Omega$$

(4)

Subject to:

$$c_{10 \text{ yr. rel.}} : f_{10 \text{ yr. rel.}} \geq 0.98 \quad (5)$$

$$c_{10 \text{ yr. costvar}} : f_{10 \text{ yr. costvar}} \leq 1.1 \quad (6)$$

$$c_{10 \text{ yr. crit. rel.}} : f_{10 \text{ yr. crit. rel.}} \geq 0.99 \quad (7)$$

$$c_{\text{dr. rel.}} : f_{\text{dr. rel.}} = 1.00 \quad (8)$$

In the equations, “10 yr.” refers to the 10-year Monte Carlo simulation and “dr.” refers to the drought. The vector  $\mathbf{I}_k$  denotes the levers (decision variables), with the subscript  $k$  indicating that the set of objectives and constraints is used with each model case from Table 1. Recall that the term “measure” indicates a quantification of system performance, which is used as an objective or constraint for optimization. This section briefly reviews each of the measures from Equations (4) through (8), but for the full definitions please consult Kasprzyk et al. (2012). These measures are also used later in this study to define the robustness of selected solutions. They are broken into three groups, depending on the type of attribute they are meant to quantify.

The first group of measures quantifies the cost of the supply portfolio.  $f_{10 \text{ yr. cost}}$  is the sum of the average annual costs for rights, options, and leases calculated using the 10-year simulation. The high-tail of the cost distribution is measured by cost variability,  $c_{10 \text{ yr. costvar}}$ , defined as the ratio of the mean of the costs falling above the 95th percentile divided by the average annual cost. In the drought,  $f_{\text{dr. trans. cost}}$  quantifies the cost of market transactions.

The second group of measures uses various metrics of reliability (Hashimoto et al., 1982) to quantify the performance of each portfolio. Reliability,  $f_{10 \text{ yr. rel}}$  quantifies the likelihood that the city will meet its required demand in the 10-year Monte Carlo simulation. Drought Reliability,  $f_{\text{dr. rel}}$ , is the same calculation, performed within the drought scenario. The Critical Reliability measure  $f_{10 \text{ yr. crit. rel}}$  treats “success” of each month differently than the basic Reliability; the city must meet at least 60% of sampled demand. Critical Reliability therefore measures the likelihood of very large failure events that would be difficult to mitigate using demand management or other techniques.

The third group considers measures associated with market use and the efficiency of each portfolio. Acquisitions of exercised options and leases are fully controlled by the supply/demand threshold. Therefore, some portfolios could specify a large number of leases or options and incur high transactions costs. Number of Leases ( $f_{10 \text{ yr. num. leases}}$ ), minimizes the expected number of leases obtained in each portfolio. Transfers are modeled to expire after 12 months of nonuse, so the Dropped Transfers measure quantifies the volume of water dropped from nonuse ( $f_{10 \text{ yr. dropped}}$ ). Finally, the Surplus Water measure ( $f_{10 \text{ yr. surplus}}$ ) determines the average amount of water carried over from year to year by the portfolio. This measure is minimized as a proxy for other regional users; portfolios that carry too much surplus use water that could be put to other purposes.

In summary, the decision levers are the volumetric and strategy decisions shown in Table 1. The relationship is a Monte Carlo simulation and extreme drought scenario. The cost, reliability, and market use metrics defined in this section are the performance measures. Finally, the uncertainties in this problem formulation include both classical and deep uncertainties. The classical uncertainties are assumed distributions of lease prices, inflows, losses, reservoir variation, and demand, and are incorporated into the model using Monte Carlo simulations. We then examine the extent to which solutions identified by the MOEA step are robust across a variety of deep uncertainties, which we discuss in more detail in Section 3.4.

### 3.3. Multi-objective evolutionary algorithm

The  $\epsilon$ -NSGAII (Kollat and Reed, 2006, 2007a; Reed et al., 2007) was chosen to generate alternatives for this study. This algorithm extends the original NSGAII (Deb et al., 2002) by adding epsilon dominance archiving (Laumanns, 2002) and adaptive population sizing (Harik et al., 1997) to change the size of the population commensurate with problem difficulty. At present,  $\epsilon$ -NSGAII is a top performing search tool, as tested by recent diagnostic comparisons of modern MOEAs (Reed et al., 2012; Hadka and Reed, 2012). Other top-performing MOEAs could be used in subsequent MORDM applications, but the MOEA used should be selected carefully to ensure that it can reliably solve challenging problems (see Section 2.2).

The MOEA was used to generate alternatives for each of the four problem cases in Table 1. The algorithm's parameterization can strongly control its performance (Reed et al., 2012), so choosing proper values for parameters such as crossover and mutation is important. Our study parameters are presented in Table 3 and fully discussed in Kasprzyk et al. (2012). In order to reduce the effects of random number generation within the initial random population and the search operators, all algorithm runs are replicated fifty times with different random seeds. A subsequent nondominated sort combines the best individuals across all runs (as performed in Kollat et al., 2008; Kasprzyk et al., 2009, 2012; Reed et al., 2012). Epsilon precision within the  $\epsilon$ -NSGAII allows users to control significant precision on each measure used within search. This study's epsilon precisions, presented in Table 2, were adjusted to minimize the effect of simulation noise, as described in detail in Kasprzyk et al. (2012).

**Table 2**  
Objectives' epsilon settings.

Objective	Value
10-yr. cost	\$30,000
10-yr. surplus water	1233 cubic meters
10-yr. critical reliability	0.002
10-yr. drops	2467 cubic meters
10-yr. number of leases	0.3
Drought trans. cost	\$10,000

**Table 3**  
Parameters for MOEA search.

Symbol	Value	Description
$M$	5000	Monte Carlo sample size
$T$	10	Planning period [years]
$p_m$	$1/n$	Probability of mutation ( $n$ : num. of decision levers)
$p_c$	1.0	Probability of crossover
$\eta_c$	15	Distribution index (crossover)
$\eta_m$	20	Distribution index (mutation)
$NFE$	500,000	Number of function evaluations

### 3.4. Uncertainty sampling

The previous step identified solutions on the Pareto approximate surface given the base case assumptions. This uncertainty sampling step aims to test how significantly the performance of each of these solutions varies if the base case assumptions turn out to be wrong. This step focuses on two types of deeply uncertain parameters: 1) those that define the base case probability distributions used in the Monte Carlo simulation model and 2) other parameters that the Monte Carlo simulation treats as fixed values. Table 4 lists parameters of the first type and Table 5 lists those of the second type.

We use a LHS sample over both types of uncertainties to generate 10,000 alternative SOW's against which to test the solutions (the base case represents one SOW). For the real-valued, non-probabilistic parameters (e.g. the initial reservoir level) each SOW has a value between the lower and upper bounds listed in Table 5. For the parameters defining the probability distributions (e.g. the distribution of lease prices), we use “scaling factors” to renormalize the tails of the distribution as described in Section 3.4.1 below. Each SOW has a value for each scaling factor between the lower and upper bounds listed in Table 4.

We then run the Monte Carlo simulation model for each of the 10,000 SOW's and record its performance according to each of the measures. It is useful to compare the different purposes of the LHS sample used to generate the 10,000 SOW's and the Monte Carlo samples used by the simulation model to generate results for each individual SOW. The Monte Carlo samples weight points according to their estimated likelihood, so the model can sum over the sample to calculate in each SOW the mean values and variances for the model outputs. The LHS sample is a quasi-random design that weights points equally. It is used here to explore the performance of strategies over a wide range of plausible cases. There is no claim that all SOW in the sample are equally likely. Rather the sample aims to provide data that allows decision makers to understand which solutions are more or less sensitive to deviations from their base case assumptions and, in the scenario discovery step, what particular combinations of uncertainties would cause particular solutions to perform poorly. It should be noted that the specific LHS sample used here is meant only as one pragmatic example of how to explore deeply uncertain factors or probabilistic assumptions that influence the Pareto satisficing behavior of tradeoff solutions. This step offers a rich opportunity for future research to explore alternative schemes for exploring deep uncertainties.

#### 3.4.1. Scaling factors

The scaling methodology for our study is adapted from Dixon et al. (2008), in which probability distributions are renormalized to explore the consequences of potential mis-estimation of the likelihood of extreme events in the assumed baseline distribution. Here we renormalize the weight in the highest or lowest 25% of the distribution and use an integer scaling factor between 1 and 10 to control the reweighting. We re-run the simulation, where a non-uniform sampling procedure is used such that the highest or lowest 25% of the data becomes 1–10 times likelier, depending on the

**Table 4**  
Scaling factors.

Input variable	Lower bound	Upper bound
Low inflows	1	10
High losses	1	10
High demands	1	10
High lease prices	1	10
Losses in Reservoir storage	1	10

**Table 5**  
Sampled model parameters.

Parameter	Lower bound	Upper bound
Initial rights	0.0	0.4
Demand growth rate (%)	1.1	2.3
Initial reservoir level [10 <sup>6</sup> m <sup>3</sup> ]	987	2714

scaling factor. Each scaling factor is treated as an integer in the LHS. Note that subsequent MORDM analyses can use alternative scaling or distributional sampling methodologies. The key issue is to quantitatively explore the impacts of alternative likelihood assumptions on measures of system performance.

Table 4 presents the data scaling factors, and Fig. 3 illustrates how these factors modify the cumulative distribution function (CDF) of the input data. To demonstrate how each SOW has different scaling factors across the data types, the figure shows example results attained using the 2, 4, 6, and 10 scaling factors, with the thick blue line indicating the original baseline data. Additionally, the figure shows how the data changes across two representative months: January (Fig. 3a–c and g–i) and August (Fig. 3d–f and j–l).

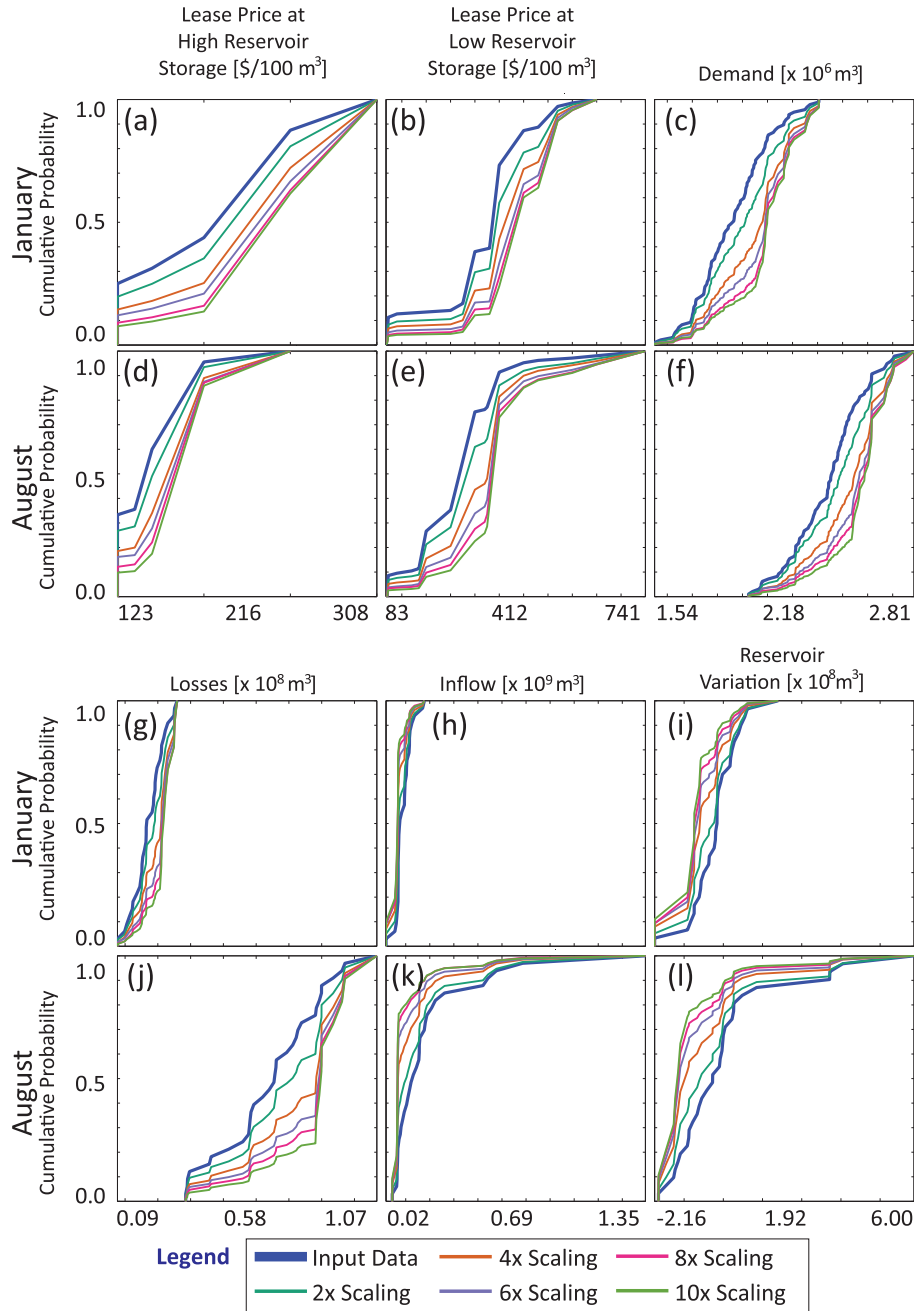
Lease pricing distributions are given in Fig. 3a, b, d and e. Leases are purchased by sending a document to the Watermaster's office. The office arranges a one-time transfer of water from the irrigator to the municipality, at a variable price, from the main reservoir system. Lease pricing is modeled as a random variable sampled from an empirical monthly distribution. A prior analysis (Characklis et al., 2006) showed that there are two distributions of lease prices, depending on whether or not the reservoir volume is below  $1.76 \times 10^{12}$  cubic meters. Changing agricultural commodity prices could impact willingness to participate in the market and therefore modify prices (Ranjan, 2010). Additionally, the simulation assumes that any requested transfer can be fulfilled entirely, so in reality there could be volume limitations that would make prices increase. Finally, empirical price distributions developed for the LRGV test case are based on a limited number of data points, so the distribution could have a slightly different shape than what was observed. Lease prices are scaled to emphasize the highest 25% of the data distributions; hereafter the scaled lease prices are termed “High Lease Prices”.<sup>10</sup>

Demand distributions are presented in Fig. 3c and f. Uncertain demand in the simulation is sampled based on Gaussian distributions with parameters estimated using historical data. The entire demand distribution also grows exponentially subject to a demand growth rate, as discussed below. Characklis et al. (1999) suggested that regional demands could increase by a factor of three from 1990 to 2050. The assumption of normality in modeling demand could also be incorrect, motivating exploration of distributions that have larger “tails” with a larger proportion of likelihood in higher values. Demand distributions are scaled to emphasize the highest 25% of demand, termed “High Demands” hereafter.

Losses and inflows are presented in Fig. 3g, h, j and k. Water allocated to permanent rights is calculated from reservoir inflows *pro rata* based on the volume of permanent rights compared to the total amount of regional rights. For example, if the modeled city has 50 units of rights, and all regional rights holders hold 100 units of rights, our modeled city would obtain half of all reservoir inflows in every month. The water available for allocation is the difference

<sup>10</sup> Two distributions of prices are still maintained, depending on reservoir storage (see Fig. 3). Therefore, under the sampled SOW, the simulation determines which lease price distribution to use in a given month, and applies the same scaling factor for both distributions.





**Fig. 3.** Comparison of cumulative distribution functions under different scaling factors. a–c and g–i show the scaled data for January, whereas d–f and j–l show the data for August. The thick line shows the baseline data, with the colored lines illustrating scaling factors between 2 and 10. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

between a sampled inflow volume and a volume of losses, both estimated from empirical historical distributions. Inflows and losses are deeply uncertain since in future planning periods, climate change could shift the distribution of values toward higher losses and lower inflows (Zhu et al., 2005). Inflows are scaled to emphasize the lowest 25% and termed “Low Inflows”, whereas losses are scaled to emphasize the highest 25% and termed “High Losses”.

The reservoir variation is presented in Fig. 3i and l. Reservoir variation refers to the aggregate change in reservoir storage for the water source; the variable can be positive for gains and negative for losses. The reservoir volume is monitored by the International Boundary and Water Commission (IBWC) that is charged with maintaining a 1944 treaty between the United States and Mexico.

Recent concerns about lower inflows coming from Mexican tributaries (Levine, 2007) could cause the reservoir supply to decrease relative to historical conditions. Increased evaporation from climate change could also play a role. Therefore, the reservoir variation scaling focuses on the lowest 25% of the data. This is termed “Losses in Reservoir Storage” since the scaled reservoir variation tends to cause more losses than in the baseline historical dataset.

### 3.4.2. Scalar model parameters

A second group of deeply uncertain variables represents point values of model parameters. The first sampled parameter is the initial water rights. This is an initial condition that represents the amount of water available to the city’s supply in the first month of

the simulation. The initial condition relates to the water carried over from year to year by the portfolio, and has been shown to be important in determining portfolios' performance (Kasprzyk et al., 2012). The lower bound of 0.0 represents no water in the beginning of the simulation, such as a situation in which a supply failure occurs in the month before the simulation begins. The upper bound, 0.4, approximates a situation in which the city starts their supply with a volume equal to 40% of their permanent rights volume. Under extreme supply conditions of higher demands coupled with lower inflows, the city may not be able to maintain their preferred amount of supply from year to year. Thus this variable is treated as deeply uncertain, and sampled within the range 0.0–0.4.

The second deeply uncertain model parameter is the demand growth percentage. Based on an analysis of USGS water supply data and projections from the Texas Water Development Board presented in Kasprzyk et al. (2012), we posited that the LRGV's demands are likely to grow with a rate between 1.1% and 2.3%. Since future demand growth will depend on a variety of factors that are difficult to predict, including future population growth, housing stock and weather patterns, this variable is sampled between 1.1% and 2.3% in the ensemble.

The initial reservoir volume is the final sampled model parameter. The main effect of the reservoir level in the simulation is on lease pricing, as mentioned earlier. There is also a “dead storage zone” modeled in the mass balance simulation, below which no water is allocated. The range used in sampling, 987 million cubic meters to 2714 million cubic meters, is adapted from prior work (Characklis et al., 2006).

#### 3.4.3. Quantifying robustness

MORDM requires both a method to sample uncertainty as well as a method to quantify robustness for each tradeoff solution. For each tradeoff solution, there exists a distribution of performance for each output measure across all SOWs in the LHS ensemble. Our treatment of robustness focuses on the most extreme SOWs in the ensemble. We assume that if a solution performs well in these worst-case SOWs, it will also have robust performance for many deeply uncertain trajectories of the future. Specifically, we calculate a deviation metric, percent deviation ( $d_i$ ):

$$d_i = \begin{cases} \frac{f_{i\ 90} - f_{i\ base}}{f_{i\ base}} & \text{if } i \text{ is minimized} \\ \frac{f_{i\ base} - f_{i\ 10}}{f_{i\ base}} & \text{if } i \text{ is maximized} \end{cases} \quad (9)$$

where  $i$  is the measure of interest, the 90 or 10 subscript indicates the 90th or 10th percentile in the uncertainty ensemble, and “base” indicates the measure value from the baseline simulation. In other words, the percent deviation calculation shows the magnitude that the most extreme 10% of the ensemble members deviates from the expected measure value in the baseline state of the world. We identify promising candidate solutions as those that have low percentage deviation across many output measures of interest.

In this study, our initial exploration of how each solution performed across different SOW suggested that we should focus on a subset of the 9 performance measures that were initially identified. We found that many of the solutions actually improved their performance across 3 performance measures – surplus, dropped transfers, and drought transfers costs – in many SOW. Since the goal of the MORDM exercise is to find SOW in which the candidate solutions are vulnerable to plausible futures, and to suggest ways to mitigate those vulnerabilities, we focused on the remaining 6 measures (cost, cost variability, reliability, critical reliability, drought reliability, and number of leases).

### 3.5. Scenario discovery

Our exploration of the “percent deviation” outcomes suggested a potential candidate solution that performed relatively well across many of the measures of interest. We then applied the SD process to this candidate solution in order to identify future SOW (values of uncertainties) under which it would perform poorly. To identify poor performance, we defined a set of thresholds for each group of performance measures: cost, reliability, and market use. Table 6 defines the threshold sets used to delineate SOWs that cause poor solution performance. In general, the candidate solution was considered to perform poorly in SOW in which any of the measures in the threshold set fell into the most extreme 10% of values.

## 4. Results

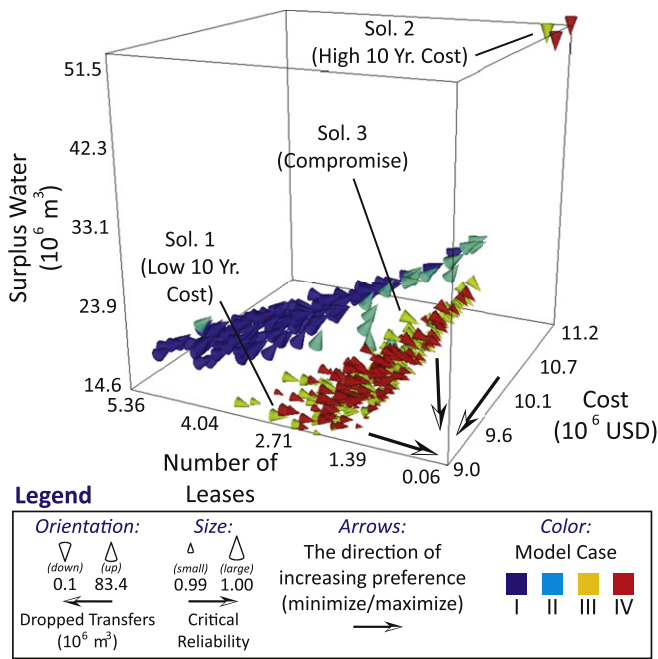
### 4.1. Generating alternatives

Fig. 4 presents multi-objective tradeoffs generated using a MOEA for each of the four model cases in Table 1. These results, adapted from Kasprzyk et al. (2012), compare the model cases' performance with respect to multiple output measures. In this study, our purpose is two-fold: (1) evaluate which formulation yields solutions that are robust across a broad suite of SOW and (2) assess if our prior choice of preferred model case III was appropriate given the LRGV's deep uncertainties. Fig. 4 is a glyph plot in which each portfolio solution is represented by a cone. The cone's coordinates represent the cost ( $f_{10\ yr.\ cost}$ ), number of leases ( $f_{10\ yr.\ num.\ leases}$ ), and surplus water ( $f_{10\ yr.\ surplus}$ ) measures. Additionally, the orientation of the cones shows the dropped transfers ( $f_{10\ yr.\ dropped}$ ) and the cones' size shows the critical reliability ( $f_{10\ yr.\ crit.\ rel.}$ ). On each axis, arrows indicate the direction of increasing preference (i.e., whether a measure is minimized or maximized). Overall, Fig. 4 shows that model cases III and IV have better expected performance with respect to the three measures plotted on the spatial axes, providing good performance with fewer leases and lower surplus water at every level of cost. Cases III and IV could be argued to be the “non-dominated” problem formulations based on their expected performance attained using the Monte Carlo simulation.

This result motivated choice of model case III in the prior study (Kasprzyk et al., 2012) and subsequent exploration of three selected solutions shown in Fig. 4. Case III uses the distinct alpha variable to determine “when” to go to the market and beta to determine “how much” to buy. Additionally, the case III problem formulation separates these variables between January–April and May–December, but unlike case IV it does not use the adaptive options contract. In general, use of distinct alpha and beta values allows the city to “tune” market acquisitions to the input data and to buy only as many transfers as needed to meet reliability under the modeled conditions (Characklis et al., 2006). If the city typically has enough surplus water to avoid market transactions from January through April, for example, water managers can choose a portfolio that has lower values of alpha and beta in those months. These trends led to excellent performance with respect to their expected value measures (especially lowering the dropped transfers and surplus water due to efficient market use). However, our choice of the selected

**Table 6**  
Threshold sets.

Name	Measures	Thresholds
Market use	Number of leases	>90th percentile
Reliability	Reliability, critical reliability Drought reliability	<10th percentile in any
Cost	Cost, cost variability	>90th percentile in any

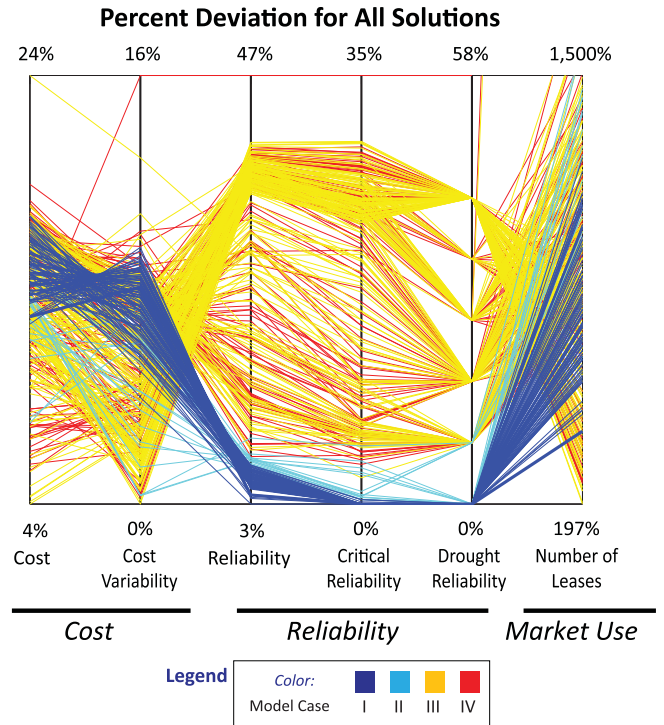


**Fig. 4.** Non-dominated tradeoffs generated in prior work (Kasprzyk et al., 2012). Each cone is an individual water portfolio solution, with spatial axes plotting solutions' cost, number of leases, and surplus water. The annotated solutions from model case III were chosen because of their performance in the plotted performance measures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

solutions is based on the single baseline SOW used to assess the tradeoffs. In the remainder of our results, the MORDM framework is employed to rigorously explore our choice of a formulation and solutions of interest for the LRGV test case. MORDM explores how changing assumptions about this historical baseline SOW can affect the performance of these solutions and possibly motivate selection of one of the alternative problem formulation's strategies for managing the city's water supply.

#### 4.2. Percent deviation of performance measures

MORDM uncertainty analysis globally samples many different combinations of exogenous assumptions (or alternate future conditions). Each set of exogenous parameters yields a new Monte Carlo simulation for the LRGV system that deviates from its original Monte Carlo simulation based on the historical SOW. Specifically, each tradeoff solution is tested with an ensemble of 10,000 SOWs as described in Tables 4 and 5. The percent deviation metric is used to interpret each solution's performance in the uncertainty ensemble, which calculates the difference between the most extreme 10% of the values in the uncertainty ensemble and the value in the baseline SOW. Fig. 5 is an initial visualization of the percent deviation results for each solution across several measures. The figure is in the form of a parallel coordinate plot, with color distinguishing the 4 model cases in the same manner as was used in Fig. 4. Each line in the figure shows a single solution, and the line's vertical position on each axis shows relative values of deviation for each measure, with values closer to the bottom axis representing more robust performance. Fig. 5 allows us to observe the magnitude of percent deviation for each solution across the three groups of performance measures, and to determine trends across the model cases. Ideal performance in Fig. 5 would be a horizontal line that intersects all of the vertical axes at zero deviation.



**Fig. 5.** Parallel axis plot showing the percent deviation for all solutions, defined as the difference between the most extreme 10% measure value from the uncertainty ensemble compared to the baseline state of the world. Color shows the model case. Measures are grouped into three categories: cost, reliability, and market use.

Cases I and II exhibit moderate to high deviation with respect to the cost and number of leases measures but improved performance with respect to reliability when compared to cases III and IV. Cases I and II use a single variable to exercise their options and leases and do not separate the “when” and “how much” decisions (see Table 1). Portfolios in these cases would need to maintain a high value for their alpha strategy variable in order to ensure a sufficient supply. The high threshold may lead to reduced performance with respect to the number of leases and surplus water measures in the baseline SOW (see Fig. 4). However, these large acquisitions of water from the market helped portfolios in cases I and II maintain high reliability in the uncertainty ensemble. A decision maker would likely be willing to accept higher costs, though, if it means that supply reliability is maintained under extreme conditions. In summary, the robust reliability performance of cases I and II exhibited by Fig. 5 is a surprising result that conflicts with our prior choice of case III (Kasprzyk et al., 2012). This shows that choosing solutions with respect to robustness can lead to markedly different decisions compared to observing performance measure results in a single SOW based on historical likelihoods.

#### 4.3. Negotiation of a robust solution

Visual analytics within MORDM provides a rich opportunity for decision makers to interact with decision-relevant data for each of the suggested tradeoff solutions. This section demonstrates two specific techniques for interacting directly with the data visualizations. First, the decision makers can change the ordering of variables on visualization axes to better illustrate conflicts.<sup>11</sup> These

<sup>11</sup> When all axes are arranged such that the preferred direction is toward one half of the figure (i.e., at the bottom), crossing lines indicate that one cannot achieve good performance in a given measure without suffering degradation in another.

plots show how it would be very difficult to ascertain *a priori* what conflicts exist between measures (Franssen, 2005) and which solutions would be robust to a wide array of plausible futures (Lempert, 2002). Moreover, decision makers can interact directly with solutions shown in the plot, using “brushing” to impose limits on the plotted data to reflect decision maker preferences (Inselberg, 1997). Brushing yields a reduced group of diverse alternatives, allowing the decision maker to choose between a small number of maximally different alternatives (Brill et al., 1990). Fig. 6 expands our treatment of the percent deviation results to demonstrate these techniques. The measure axes have been rearranged to better illustrate the conflicts between percent deviation in reliability measures and the other performance measures. The figure also reflects brushing to focus on solutions that have lower than 5% deviation in any of the three reliability measures. Our focus in this demonstration is to choose solutions that have excellent performance in meeting reliability; decision makers who have other preferences can express them by using different arrangements of the axes and different brushing criteria. As a result of the brushing, solutions that do not meet the reliability criteria are shown in gray, with the remaining desired solutions shown in a color gradient. Choosing an appropriate color mapping is an interactive process, and the colors should be chosen to clearly illustrate trends in the data. In Fig. 6 the color scale demonstrates the percent deviation in cost of the remaining solutions (from 12% to 15%).

The solutions highlighted by brushing in Fig. 6 mostly come from cases I or II, and have a range of performance with respect to deviations in cost, number of leases, and cost variability. The blue colored solutions, which have low deviations in cost, also have low deviations in terms of number of leases but exhibit higher deviations in cost variability. These solutions typically perform well across all of the cost and reliability measures, so we selected solution 4 since it has the lowest deviation with respect to number of leases in this set. Table 7 compares the decision levers of solution 4, termed the “robust” solution, with the previously selected solutions 1–3 from case III. Solution 4’s strategy variables are each 1.69, which means that its expected supply must be 1.69 times its expected demand in all months.<sup>12</sup> Though this may lead to the portfolio acquiring a higher number of leases and wasting more dropped transfers, this higher amount of market use leads to more robust performance in extreme SOWs. The solution’s volumes of rights and options are also comparable to those of low cost solution 1. Furthermore, solution 4 comes from a simpler decision lever formulation (case I) that may be easier to implement in practice, with fewer decision levers and more straightforward rules for acquiring market transfers.

Fig. 7 examines solutions’ robustness performance in the context of the original expected-value performance measures in the optimization. The figure retains the same spatial axes, cone size, and orientation from Fig. 4 and uses color gradients to superimpose the percent deviation results for cost (Fig. 7a) and critical reliability (Fig. 7b). This is a unique way to show the robustness of each region of the original tradeoff set. In prior work, solutions 1–3 were selected because of their good performance in cost, number of leases, and surplus water under the baseline SOW. Fig. 7a illustrates that some of the solutions in this region also have low percent deviation in cost. The same region, though, exhibits poor performance with respect to percent

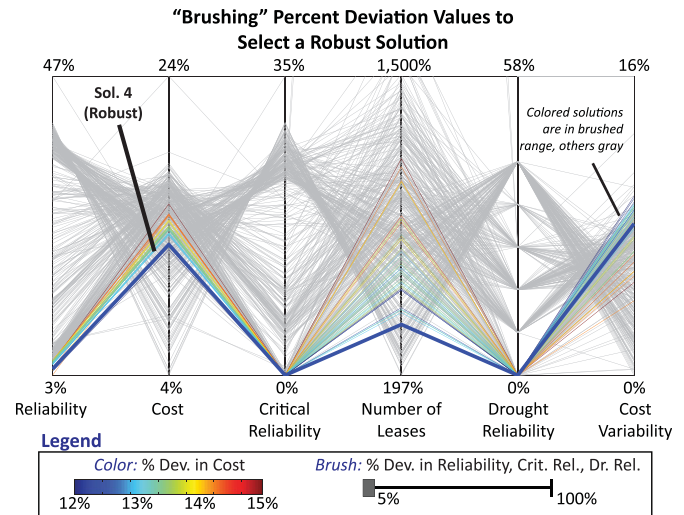


Fig. 6. Parallel coordinate plot of percent deviation for all solutions. Here, the axes were rearranged to show maximal conflict between measures. Brushing was used to plot in gray all solutions that had greater than 5% deviation in the three reliability measures. Color was then rescaled to show the percent deviation in cost of the remaining solutions. Solution 4 was selected for further analysis.

deviation in critical reliability (see Fig. 7b). This could lead to severe failures if assumptions about the LRGV’s supply and demand conditions are wrong. In our initial tradeoff exploration (Kasprzyk et al., 2012), we did not select solution 4 because it has a high number of leases compared to the selected solutions in case III. Solutions in the region containing solution 4, however, are much more robust with respect to critical reliability, avoiding critical failures in SOW with low inflows, high demands, and other plausible future conditions.

#### 4.4. Scenario discovery

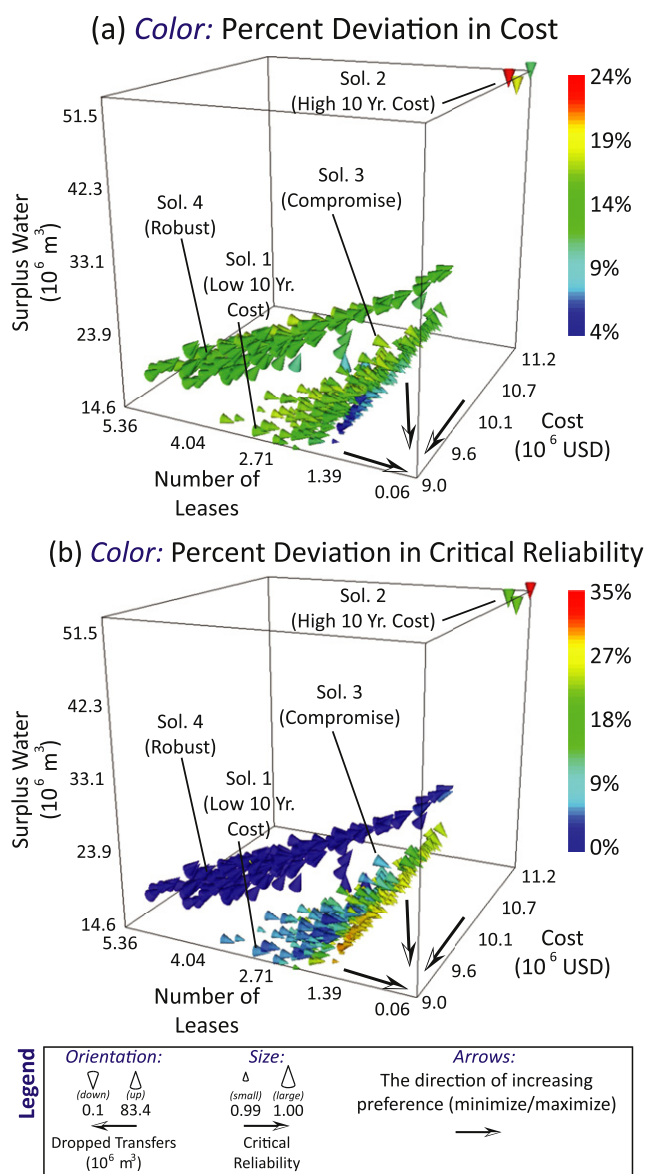
The prior sections focused solely on solutions’ worst case performance in the uncertainty analysis compared to their Pareto approximate performance from optimization using the baseline SOW. Solution 4 was selected because of its acceptable performance with respect to a percent deviation metric across several measures. However, the analysis did not indicate what values of the uncertain exogenous factors caused Solution 4 to have poor performance in alternative SOW. This section presents the results of computer-aided scenario discovery (SD) on Solution 4, with the goal of providing straightforward scenario descriptions of what factors cause poor performance for the solutions. Recall that SD uses PRIM to construct “scenario boxes” for three sets of performance thresholds defined in Table 6. The use of SD within MORDM follows the methodology of Bryant and Lempert (2010), using

Table 7  
Selected solutions’ properties.

Solution	1	2	3	4
Name	Low 10 yr. cost	High 10 yr. cost	Compromise	Robust
Model case	3	3	3	1
$N_R$ ( $10^6$ m <sup>3</sup> )	37	61	46	38
$N_O$ ( $10^6$ m <sup>3</sup> )	17	0	21	20
$\alpha_{\text{May-Dec}}$	1.29	1.29	1.24	1.69
$\beta_{\text{May-Dec}}$	1.53	1.34	1.60	1.69
$\alpha_{\text{Jan-Apr}}$	1.20	0.06	1.39	1.69
$\beta_{\text{Jan-Apr}}$	1.28	0.09	1.39	1.69

<sup>12</sup> According to Table 1, case I specified one alpha/beta threshold whereas case III specified four distinct values. Table 7 presents all solutions using four distinct variables, although in the MOEA search there was only one threshold value searched in case I.





**Fig. 7.** The selected solution in the context of the prior objectives. The cone coordinates, orientation, and size are retained from Fig. 4, but the color shows the percent deviation in cost (a) and critical reliability (b). Note that the selected solution 4 is in a different region of the space than what was previously selected.

metrics of coverage and density to select appropriate scenarios.<sup>13</sup> Rows in Fig. 8 show the factors sampled in the MODRM uncertainty ensemble (scaling factors in the first five rows and model parameters in the second three rows). The columns show the three threshold sets, or groups of thresholds on performance measures that were used to delineate poor performance from acceptable performance. The gray bars show the values that triggered vulnerabilities in the discovered scenarios. Each threshold set developed a unique set of vulnerable points. For example, points that caused high cost may not cause poor reliability, and so forth. The

sets of box constraints are therefore independent across the three different threshold sets.

The cost threshold set was violated when low inflows were scaled to be 3.5 times likelier, high losses scaled 1.9 times likelier, high demands 3.4 times likelier, the demand growth rate higher than 1.4%, and the initial reservoir volume below 1.8 billion cubic meters. Each of the dimensions included in this scenario would cause the city to buy more water on the market and thus incur higher costs. Dimensions included in the scenario lower the amount of water supply (low inflows, high losses, and low reservoir volume) and increase the amount of demand (high demand growth rate). Moving to the reliability threshold set, we find that inflows, losses, and demands also appear in the scenario box. Poor performance in reliability also occurs when the initial rights variable is lower than 0.24. The initial rights variable is included in the simulation as a way to model the amount of water carried over from year to year by a portfolio (i.e., the amount of water available to the city before the simulation time period begins). Low initial rights is representative of the effects of a long term (potentially multi-year) drought (Kasprzyk et al., 2009). The appearance of low initial rights in the reliability threshold set's results indicates that the portfolio's market use strategy may sometimes fail if there is not sufficient water in the city's supply account, especially in these drought conditions. The third threshold set is market use. This set focuses only on SOW when the number of leases are in the highest 10%. The gray bars here show that inflows, losses, and demands are the most important factors for causing vulnerabilities.

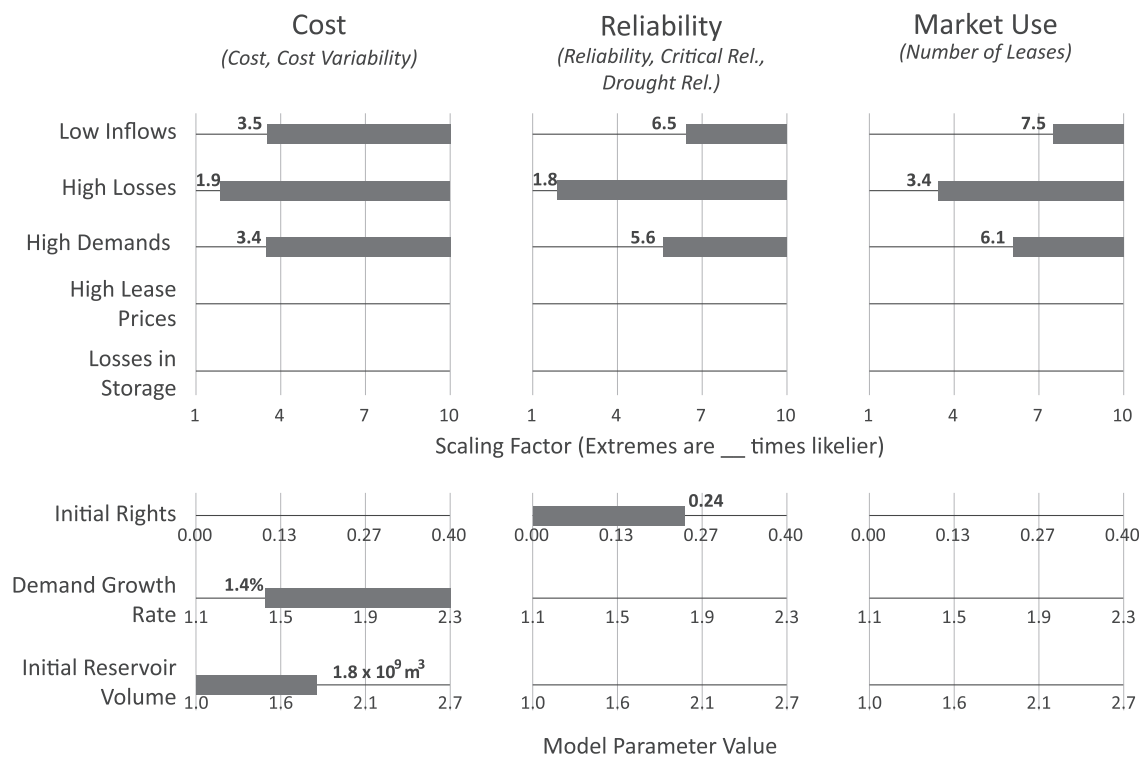
It is not our goal to claim that each of the SOW in the discovered scenarios will actually occur in future planning. Ultimately, the decision makers would use these scenarios to inform discussions of what factors are most important for future planning. As an example, losses were critically important in all three threshold sets. Scaling factors that caused vulnerabilities for the losses were relatively low, triggered at 1.9 for cost, 1.8 for reliability, and 3.4 for market use. Low to moderate values for scaling factors could very plausibly occur in future planning periods, since they do not represent drastic changes compared to the distributions attained using LRGV test case's historical data (see Fig. 3). Therefore, the system should likely be monitored to ensure that high losses are not being observed in the system, since low losses are important for ensuring robust portfolio performance. Triggered scaling values for other dimensions indicate a higher level of robustness; for example, vulnerabilities in cost, reliability, and market use were only triggered when low inflows were scaled to be 3.5, 6.5, and 7.5 times likelier, respectively.

## 5. Discussion

Generating high-quality planning alternatives for complex environmental systems poses several significant challenges. First, the systems are often characterized by multiple, conflicting performance measures. Traditional design approaches often try to aggregate the multiple measures into a single metric of performance (e.g. Cross, 1989). This aggregation, though, favors certain aspects of performance over others in unpredictable ways (Maass et al., 1962, pp. 17–20; Maass, 1966; Maass and Major, 1970; Bromley and Beattie, 1973; Franssen, 2005; Banzhaf, 2009). In contrast, MODRM's problem formulation considers multiple conflicting performance measures explicitly and simultaneously. The solution to a many objective problem formulation is a set of tradeoff solutions, each of which is non-dominated with respect to multiple performance measures. Considering the tradeoff as a whole allows decision makers to learn about trends and properties of their modeled system, such as how sensitive it is to change, how it responds to extreme events, and what its performance is with respect to

<sup>13</sup> In using PRIM, we chose scenarios that had acceptable coverage and density and constrained the smallest number of dimensions possible. For example, the reliability threshold set only uses 4 out of a possible 9 dimensions. While higher-dimensional scenarios could also plausibly explain the dataset, using a small number of dimensions allows the scenario to be easy to interpret and also maximizes the density (i.e., most of the points in the scenario are indeed vulnerable).

## Scenarios Where the “Robust” Solution Performs Poorly



**Fig. 8.** Results of scenario discovery. The bars indicate SOW where the “Robust” solution (see solution 4 on Figs. 6 and 7 and Table 7) performs poorly on three groups of performance measures.

multiple measures (Haimes, 1977). Tradeoffs for the LRGV, for example, showed that a water market can lower surplus water and cost while providing high reliability (Kasprzyk et al., 2009). A second challenge comes from planning problems that impose severe constraints on performance that limit our ability to find feasible solutions (i.e., solutions that meet or exceed all planning constraints). MORDM uses an advanced solution generation technique (MOEAs) to find non-dominated planning alternatives for the formulation. The efficacy of MOEAs on problems with severe constraints was recently demonstrated using a severely constrained engineering systems design problem (Shah et al., 2011). The authors used random Monte Carlo sampling to generate possible alternatives and found that only 4 out of a possible 50 million randomly generated solutions met the feasibility constraints. A MOEA, however, was able to find feasible solutions in a few thousand design evaluations.

MORDM's framework subjects the set of solutions identified by the MOEA to a rigorous, quantitative evaluation under a wide range of plausible future conditions, and uses visual interactive techniques to assist policymakers in selecting robust solutions. Interactive visual analytics is an important part of MORDM because it enables exploration across many performance tradeoffs, robustness measures, and critical exogenous factors simultaneously. We used the percent deviation metric to examine the performance of solutions across various plausible futures, and we started with showing the metric across six different performance measures at once (Fig. 5). Fig. 6 represents a “brushing” exercise that eliminated solutions with high deviation in the three reliability measures and utilized a color gradient to show deviation in cost. This reduced the number of solutions from which to choose and enabled the decision maker to choose from maximally different alternatives (Brill et al., 1990) that were also robust to deep uncertainties. MORDM's use of percent deviation for solution selection is a unique contribution of

this work, compared to prior MOEA studies that focused solely on expected value performance measures in solution selection. The approach is a promising way to include decision maker participation in the planning process and to meet decision makers' desire to maintain a high level of performance even under deeply uncertain future conditions such as climate change (Brekke et al., 2009).

Including climate change in system planning underscores a key motivating question for managing complex environmental systems: how do we generate plausible projections of future system conditions and use them to make decisions? Although planners could previously assume that the statistical properties of their systems would not change under long planning horizons, anthropogenic changes limit our ability to properly characterize expected conditions in the future (Milly et al., 2008; Brown, 2010; Brown et al., 2011; Polasky et al., 2011). To plan for deeply uncertain future risks, decision makers often use scenario analysis, which seeks to provide coherent storylines of plausible future events (Mahmoud et al., 2009). The Special Report on Emissions Scenarios (SRES) is a well-known scenario analysis for climate planning, with a small number of axes designed to cover assumptions about the type of economic development and governance (IPCC, 2000; Arnell et al., 2004). An integrated assessment model quantifies these properties and uses point values of variables such as global carbon emissions as inputs to other models. While useful for creating a formalized discussion for climate change, the SRES example highlights key issues with the classical scenario approach (Groves and Lempert, 2007; Brown et al., 2011). The choice of socioeconomic data for the SRES scenarios is made independently of possible system vulnerabilities such as climate warming. Since traditionally there is no feedback between this choice of scenario data and ultimate system performance measures, there is no way to characterize how changes in those assumptions can affect

likelihood of system vulnerability. In fact, decision makers will often hold optimistic views of future forecasts (and scenarios) while limiting their choice of mitigating actions due to their aversion to risk (Kahneman and Lovallo, 1993). MORDM addresses these issues by simulating a wide array of plausible futures, requiring only an assumption of plausible ranges of exogenous factors. Subsequently, visualizations of solution performance can identify the important values of uncertainties without *a priori* estimates of values that cause performance failures. Problem formulations in the framework promote innovative decision levers (Kahneman and Lovallo, 1993) and facilitate selection of solutions that have acceptable performance across a wide array of simulated futures.

Our MORDM demonstration culminated with interactive scenario discovery (SD) for the LRGV test case. The goal of SD is to find simple explanations of which factors cause vulnerabilities for selected robust solutions (Lempert, 2012). We tested a large number of different scaling factors to perturb the forcing data for the LRGV, and SD indicated that performance vulnerabilities were not strongly controlled by some deeply uncertain variables (such as lease pricing). However, the analysis suggested that even for small perturbations in losses, there could be large deviations in cost, reliability, and market use for our selected solution. Prior studies with the LRGV simulation did not strongly focus on the losses variable (Characklis et al., 2006; Kirsch et al., 2009), and this demonstrates a decision bias termed cognitive hysteresis (Gettys and Fisher, 1979). Cognitive hysteresis refers to a situation in which the initial formulation of a problem strongly motivates the selected solution for the problem. SD can help decision makers formulate new hypotheses about their system and identify the most important factors for future planning. In the LRGV, this means closely monitoring the amount of water coming into the reservoir and properly adjusting design decisions and water balances for losses.

Planning for complex environmental systems should acknowledge that the planning problem itself is constantly evolving. This phenomenon is partly due to new system conditions (such as the impacts of climate change or rapid population growth) that emerge over time. However, the planning formulation also changes due to learning that occurs as decision makers actually solve different iterations of their problem. In the LRGV, solving for tradeoffs showed us the dramatic effect of the market, and SD results demonstrated the importance of certain deeply exogenous factors. MORDM can therefore be considered an iterative process, with each step feeding back to the problem formulation to generate new hypotheses (Hogarth, 1981) about a system's decision levers, performance measures, uncertainties, and the governing relationships between actions and outcomes. MORDM's optimization and systems analysis focus is on enhancing learning and collaborative problem construction when supporting decision makers (Liebman, 1976; Roy, 1999; Tsoukias, 2008; Reed and Kasprzyk, 2009). In this manner our approach echoes the concerns of Rittel and Webber (1973), in that the formulation of a planning problem is uniquely coupled to the actual solution to the problem. Fig. 8 gave an example of three such discovered scenarios for the LRGV test case that could cause concern for water managers. The scenario showed that low inflows and high losses could cause managers' costs to increase, their reliability to suffer, and their market use to be higher than anticipated in a baseline SOW. After examining these results, decision makers may want to pose a new problem that focuses on low inflow simulations and tries to optimize their water acquisitions under these conditions (Harou et al., 2010).

## 6. Conclusion

In their discussion of “wicked” public domain planning problems, Rittel and Webber (1973) argued that these problems have no

definitive solution, since the problem formulation is constantly modified as planners learn more about the system and conditions evolve. The authors also argue that, for such “wicked” problems, the lack of any public consensus undercuts the usefulness of a single objective philosophy (e.g., increasing national welfare), in favor of a multiobjective perspective that can engage diverse stakeholders with differing worldviews. These challenges are compounded by a recent focus on human-induced changes, such as climate and land use change (Milly et al., 2008; Rockstrom et al., 2009; Polasky et al., 2011). The MORDM framework presented in this paper seeks to address these challenges with multiobjective planning that considers deep uncertainties while evaluating alternative problem formulations.

MORDM employs many objective search using multiobjective evolutionary algorithms to address conflicting performance measures by developing Pareto approximate tradeoff sets. MOEA search allows analysts to generate tradeoff sets even under severe performance constraints and uncertainties that arise as systems undergo change. We also address an important issue in analyzing tradeoffs: How do we inform the final negotiated selection of a solution from the tradeoff set? MORDM helps decision makers select solutions that perform well under a wide array of deeply uncertain future trajectories (i.e., robust alternatives). The framework samples solutions under a wide array of plausible futures and calculates robustness metrics across all performance objectives using RDM. Furthermore, MORDM uses statistical data mining algorithms to clarify which exogenous factors control performance failures for the system. The framework represents the first time MOEA optimization has been used in combination with RDM techniques.

The MORDM framework is demonstrated using a risk-based water portfolio planning problem in the Lower Rio Grande Valley of Texas, USA. The tradeoff solutions exhibit a wide range of performance under the MORDM uncertainty ensemble. Our results indicated that the simplest of four problem formulations had the most robust performance. The chosen formulation was able to achieve high performance with respect to critical reliability (i.e., avoiding catastrophic failures). A subsequent demonstration using scenario discovery characterized which exogenous factors strongly controlled LRGV performance failures. Losses in reservoir inflows influenced failures across a broad suite of measures, even when they were scaled only slightly differently than the baseline historical data. This finding frames how the MORDM framework can be used to inform adaptive management of complex environmental systems undergoing change. For the LRGV, this could include monitoring evaporation rates and triggering new planning in the event of extreme droughts. Such iterative, interactive methods aid in helping find more sustainable solutions to “wicked” environmental planning and management problems and can aid in building consensus across a broad range of decision maker preferences.

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