How do we incorporate the information that is available from a nonlinear system into our KF?

Extended Kalman Filter:

Assume $\hat{x}_k \approx x_k$, then linearize the nonlinear eqns at each step and use those linearizations in the filter.

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Table 3.1: Discrete-Time Linear Kalman Filter

Model	$x_{k+1} = \Phi_k x_k + \Gamma_k u_k + \Upsilon_k w_k, \quad w_k \sim N(0, Q_k)$ $\tilde{y}_k = H_k x_k + v_k, \quad v_k \sim N(0, R_k)$
Initialize	$\hat{x}(t_0) = \hat{x}_0$ $P_0 = E\{\tilde{x}(t_0)\tilde{x}^T(t_0)\}$
Gain	$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1}$
Update	$\hat{x}_k^+ = \hat{x}_k^- + K_k[\tilde{y}_k - H_k \hat{x}_k^-]$ $P_k^+ = [I - K_k H_k] P_k^-$
Propagation	$\hat{x}_{k+1}^- = \Phi_k \hat{x}_k^+ + \Gamma_k u_k$ $P_{k+1}^- = \Phi_k P_k^+ \Phi_k^T + \Upsilon_k Q_k \Upsilon_k^T$

Switching Notation:

$A = \Phi$

$B = \Gamma$

$C = H$

Model (Linear)

$$x_{k+1} = A_k x_k + B_k u_k + \gamma_k w_k, \quad w_k \sim \mathcal{N}(0, Q_k)$$

$$\tilde{y}_k = C_k x_k + v_k, \quad v_k \sim \mathcal{N}(0, R_k)$$

Model (nonlinear)

$$x_{k+1} = f(x_k, u_k) + \gamma_k w_k$$

$$\tilde{y}_k = h(x_k, u_k) + v_k$$

Gain (linear)

$$K_k = P_k^- C_k^T \left[C_k P_k^- C_k^T + R_k \right]^{-1}$$

Gain (ekf)

$$C_k = \mathrm{Jac}(h) \Big|_{\hat{x}_k^-, u_k}$$

Update $\hat{x}$ (linear)

$$\hat{x}_k^+ = \hat{x}_k^- + K_k \Big(\tilde{y}_k - \underbrace{\left[C_k \hat{x}_k^- + D_k u_k \right]}_{\text{use } h(x_k^-, u_k)} \Big)$$

Update $\hat{x}$ (ekf)

Update P (linear)

$$P_k^+ = [I - K_k C_k] P_k^-$$

Update P (ekf)

$$C_k = \mathrm{Jac}(h)\Big|_{\hat{x}_k^-,\, u_k}$$

Propagate $\hat{x}$ (linear)

$$\hat{x}_{k+1}^- = A_k \hat{x}_k^+ + B_k u_k$$

Propagate $\hat{x}$ (ekf)

$$\hat{x}_{k+1}^- = f(\hat{x}_k^+, u_k)$$

Propagate P (linear)

$$P_{k+1}^- = A_k P_k^+ A_k^T + \gamma_k Q_k \gamma_k^T$$

Propagate P (ekf)

$$A_k = \mathrm{Jac}(f)\Big|_{\hat{x}_k^+,\, u_k}$$

realistically, $\hat{x}_k^-$ is fine too.