

Definitions

We want to understand what the measurements of a system tell us about the state.

Specifically, we will focus on how measurements from the future can help inform the initial state.

"observability" = "to what degree is the initial state uniquely determined by the subsequent measurements?"

Similar approaches exist to relate past measurements to the current state

"constructability" → "to what extent is the final state uniquely determined by the prior measurements?"

tricky for nonlinear sys b/c requires either inverting dynamics, or linearizing at each step.

We will take 4 different approaches that yield similar results:

1. observability matrix - derived by solving for x_0 from the measurements

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^n \end{bmatrix}$$

2. observability Gramian - derived by solving for the sensitivity of the output energy w/ respect to the initial state

$$W_o = O^T O$$

3. Fisher Information Matrix - derived like Gramian, but when considering uncertainty.

$$FIM = O^T \tilde{R}^{-1} O$$

↑
 \tilde{R} can be
noisy.

4. Empirical observability - derived through numerical simulations yields O computationally.