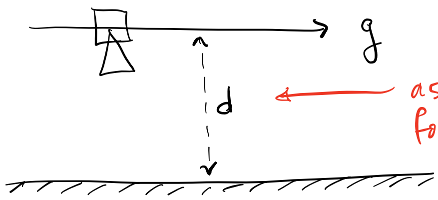
Apply nonlinear observability to mono camera example



assume this is constant for simplicity

Dynamics

$$\dot{x} = \begin{bmatrix} g + u \\ d \end{bmatrix} = \underbrace{\begin{bmatrix} g \\ d \end{bmatrix}}_{f_0(x)} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{f_u(x)} u$$

$$\dot{x} = f_0(x) + f_u(x) \cdot u$$

Measurement

$y =$ optic flow
i.e.
image angular velocity

$$= \begin{bmatrix} g/d \end{bmatrix}$$

We need to calculate:

$$G = \left\{ h,\ L_{f_0} h,\ L_{f_1},\ L_{f_0}^2 h,\ L_{f_0} L_{f_1} h,\ \ldots \right\}$$

First interesting term:

$$L_{f_0} h = \frac{\partial}{\partial x} \begin{bmatrix} g/d \end{bmatrix} \cdot f_0$$

$$= \begin{bmatrix} \frac{\partial}{\partial g} \frac{g}{d} & \frac{\partial}{\partial d} \frac{g}{d} \end{bmatrix} \cdot \begin{bmatrix} g \\ d \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{d} & \frac{-g}{d^2} \end{bmatrix} \begin{bmatrix} g \\ d \end{bmatrix}$$

$$= \frac{g}{d} - \frac{g}{d^{\cancel{2}}} \cdot \cancel{d}$$

$$= [0]$$

$$L_{f_1} h = \frac{\partial}{\partial \underline{x}} \left[g/d \right] f_1$$

$$= \begin{bmatrix} \frac{1}{d} & \frac{-g}{d^2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \left[\frac{1}{d} \right]$$

Any term w/ L_{f_0} will be zero.

Check $L_{f_1}^2 h$:

$$L_{f_1}^2 h = \frac{\partial}{\partial \underline{x}} \left[L_{f_1} h \right] f_1$$

$$= \frac{\partial}{\partial \underline{x}} \left[\frac{1}{d} \right] f_1$$

$$= \begin{bmatrix} 0 & \frac{-1}{d^2} \end{bmatrix} \begin{bmatrix} 1 \\ \end{bmatrix}$$

$$\begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \end{bmatrix}$$

independerdent

$$G = \left\{ \begin{bmatrix} \frac{g}{d} \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{d} \end{bmatrix} u_1, \begin{bmatrix} 0 \end{bmatrix} u_1 \right\}$$

remember these terms exist if $u_1 \neq 0$

Next: how many terms in G?

$$\mathrm{Jac}(G)\Big|_{x_0, u_0} = \left| \begin{bmatrix} 1/d & -g/d^2 \\ 0 & 0 \\ 0 \cdot u_1 & 1/d^2 \cdot u_1 \\ 0 \cdot u_1 & 0 \cdot u_1 \end{bmatrix} \right|_{x_0, u_0} = \mathcal{O}$$

If $u_1 = 0$, $\mathrm{Jac}(G)$ is not full rank for any x_0

If $u_1 \neq 0$, $\mathrm{Jac}(G)$ is full rank if $d \neq \infty$ and $d \neq 0$.

Therefore, system is observable iff $u_1 \neq 0$

$\Rightarrow$ system must be accelerating.

Doing the derivatives is tedious... can be done symbolically.