Nonlinear Observability Matrix (Discrete)

Linear case: $\dfrac{\partial^2}{\partial x_0 \partial x_0^T} \dfrac{1}{2} Y_w^T Y_w = \left[\dfrac{\partial Y_w}{\partial x_0}\right]^T \left[\dfrac{\partial Y_w}{\partial x_0}\right] = O_w^T O_w$

length of discrete window

$$O_w = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^w \end{bmatrix}$$

Nonlinear using linearizations:

$$O_w = \begin{bmatrix} C_0 \\ C_1 A_1 \\ C_2 A_2 A_1 \\ \vdots \\ C_w A_w \cdots A_2 A_1 \end{bmatrix}$$

linearizations along specified trajectory

expensive to compute

alternative

Empirical observability matrix:

$$O_w = \frac{1}{2\varepsilon}\left[Y_w^{+j} - Y_{w,k}^{-j} \right]$$

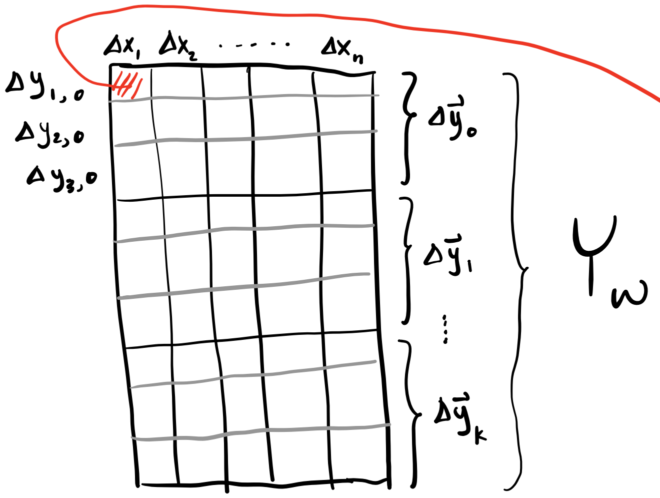
initial state vector

perturbation in of ε for each individual state

$$Y_w^{+j} = Y_w(x_0 + \varepsilon e_j, u)$$

$$Y_w^{-j} = Y_w(x_0 - \varepsilon e_j, u)$$

keep controls constant

$O_w =$ 

how much does measurement y_1 change at timestep 0 when state x_1 is perturbed?

i.e. $\dfrac{1}{2\varepsilon}\left(y_{1,0}^{+j} - y_{1,0}^{-j}\right)$ for $j=1$

Perform $2n$ simulations: 2 simulations for each state variable corresponding to $+\varepsilon$ and $-\varepsilon$ perturbations.
Each sim. yields Y_w^{+j} and Y_w^{-j}, from which we can calculate the n^{th} column of O_w