2-legged dynamics of a walking gait

M-SLIP 2L
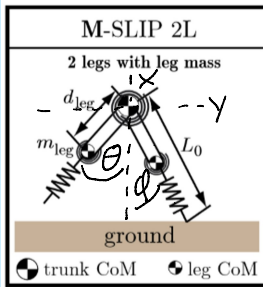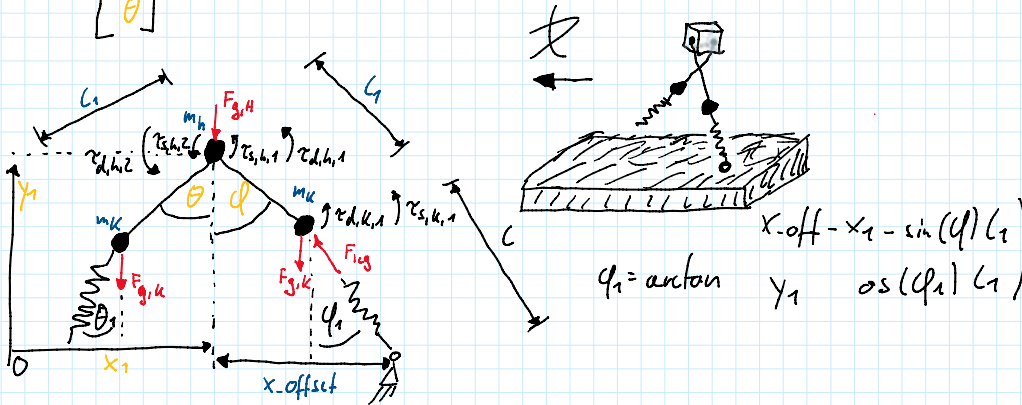2 legs with leg mass
d_{leg} --- Y
m_{leg} θ L_0
ground
🌑 trunk CoM   🌑 leg CoM

1) No flight-phase
2) Two alternating modes:
 · Stance of leg 1 and retraction of leg 2
 · Stance of leg 2 and retraction of leg 1
3) Symmetric motion in both modes
 → same dynamics with switched coordinates $(\varphi, \theta, \dot\varphi, \dot\theta, \ddot\varphi, \ddot\theta)$
4) As the retracting leg strikes the ground the standing leg lifts off
5) Actuation: leg retraction of non-stance leg
6) No movement in knee of retracting leg → no damping/stiffness
7) Spring-Damping of retracting leg does not affect stance-leg

$$q = \begin{bmatrix} x \\ y \\ \varphi \\ \theta \end{bmatrix} \rightarrow 4 \text{ unknown} \rightarrow 4 \text{ equ. needed}$$

$q_1 = arctan \left(\dfrac{x\_off - x_1 - sin(\varphi)\,L_1}{y_1 \quad os(\varphi_1)\,L_1} \right)$

$\theta_1 = const \rightarrow 6)$

$$\text{I)} \quad \ddot{x} = \Sigma F_x / M = \left(-c_1 (l_0 - l) \cdot sin(\varphi - q_1)\right)/M$$

$$\text{II)} \quad \ddot{y} = \Sigma F_y / M = \left(-g M + c_1 (l_0 - l)\cdot sin(\varphi - q_1)\right)/M$$

$$\text{III)} \quad \ddot{\varphi} = \Sigma M_z'' / I_\varphi = \left(- F_{g,k} \cdot L_1 sin(\varphi) + F_{g,k}\cdot L_1 sin(\theta) - \dot\varphi\, d_\varphi - \varphi c_\varphi + \right.$$
$$\left. + c_1 (L_0 - L) cos(\varphi - q_1) \cdot L_1 sin(\varphi) - c_1 (L_0 - L) sin(\varphi - q_1) \cdot L_1 cos(\varphi) - c_{q_1} q_1 - d_{q_1} \dot q_1 \right) I_\varphi$$

$$\text{IV)} \quad \ddot{\theta} = \Sigma M_z'' / I_\theta = \left(- F_{g,k}\cdot L_1 sin(\varphi) + F_{g,k} \cdot L_1 sin(\theta) - \dot\theta d_\theta - \theta c_\theta + 0 \rightarrow 7)\right) I_\theta$$

$$I_\varphi = m_L \cdot L_1^2$$

$$I_\theta = m_L \cdot L_1^2$$