Dynamic Modelling

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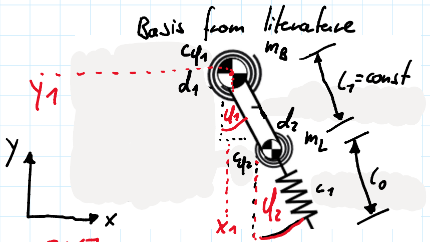
Monopod - Robot

1) Graphical model
2) Generalized coordinates
3) Position vectors of point masses
4) Dynamics via Euler-Lagrange // Free body diagramm
    a. Air-Phase
    b. Ground-Phase (Additional contact/spring forces)

1)



2) $q = \begin{bmatrix} x \\ y \\ \varphi_1 \\ \theta \end{bmatrix}$; φ_2 describes the relative angle between leg-segment and spring-segment

3) $r_{m_B} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$; $r_{m_L} = \begin{bmatrix} x_1 + \sin(\varphi_1) L_1 \\ y_1 - \cos(\varphi_1) L_1 \end{bmatrix}$

4) Using Euler-Lagrange:

$$\boxed{\frac{d}{dt}\frac{\partial T}{\partial \dot q} - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} - Q_{ncons} = 0}$$

$$T = \frac{1}{2}\sum_{i=1}^{2} m_i \dot r_i^T \dot r_i =$$

$$= \frac{1}{2}\left(m_L \cdot \left[\dot x_1 + \dot\varphi_1 \cos(\varphi_1) L_1 ,\ \dot y_1 + \dot\varphi_1 \sin(\varphi_1) L_1\right] \begin{bmatrix} \dot x_1 + \dot\varphi_1 \cos(\varphi_1) L_1 \\ \dot y_1 + \dot\varphi_1 \sin(\varphi_1) L_1 \end{bmatrix} + \right.$$

$$\left. + m_B \cdot \left[\dot x_1 ,\ \dot y_1\right] \begin{bmatrix} \dot x_1 \\ \dot y_1 \end{bmatrix} \right) =$$

$$= \frac{1}{2}\left[m_L \left(\dot x_1^2 + 2\dot x_1 \dot\varphi_1 \cos(\varphi_1) L_1 + \dot\varphi_1^2 \cos^2(\varphi_1) L_1^2 + \dot y_1^2 + 2 \dot y_1 \dot\varphi_1 \sin(\varphi_1) L_1 + \dot\varphi_1^2 \sin^2(\varphi_1) L_1^2\right) + m_B\left(\dot x_1^2 + \dot y_1^2\right)\right]$$

$$\boxed{= \frac{1}{2}\left[m_L \left(\dot x_1^2 + \dot y_1^2 + \dot\varphi_1^2 L_1^2 + 2\dot\varphi_1 L_1 \left(\dot x_1 \cos(\varphi_1) + \dot y_1 \sin(\varphi_1)\right)\right) + m_B\left(\dot x_1^2 + \dot y_1^2\right)\right]}$$

$$\boxed{V = V_G + V_{\varphi_c} + V_c}$$

$$V_G = + m_B y_1 g - m_L \left[x_1 + \sin(\varphi_1) L_1 ,\ y_1 - \cos(\varphi_1) L_1\right] \begin{bmatrix} 0 \\ -g \end{bmatrix} =$$

$$= + m_B y_1 g + m_L \left(y_1 - \cos(\varphi_1) L_1\right) g$$

$$\underline{V_{\varphi_c} = V_{\varphi_{c1}} + V_{\varphi_{c2}} = c_{\varphi_1}\varphi_1 + c_{\varphi_2}\varphi_2}$$

$$\underline{V_c = c_1\left(l - l_0\right) =}$$

$$\underline{l} = \sqrt{r_2^T r_2} = \sqrt{x_1^2 + 2x_1\sin(\varphi_1)l_1 + \sin^2(\varphi_1)l_1^2 + y_1^2 - 2y_1\cos(\varphi_1)l_1 + \cos^2(\varphi_1)l_1^2} =$$

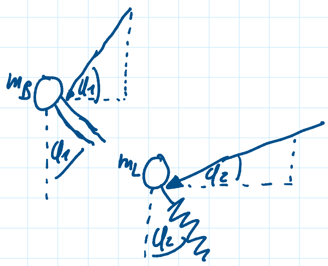
$$= \sqrt{x_1^2 + 2l_1\left(x_1\sin(\varphi_1) - y_1\cos(\varphi_1)\right) + y_1^2 + l_1^2}$$

$$\underline{l_0} = \sqrt{x_{1,0}^2 + 2l_1\left(x_{1,0}\sin(\varphi_{1,0}) - y_{1,0}\cos(\varphi_{1,0})\right)y_{1,0}^2 + l_1^2}$$

$$Q_{ncons} = \begin{bmatrix} Q^{nc}_{x1} \\ Q^{nc}_{x2} \\ Q^{nc}_{\varphi_1} \\ Q^{nc}_{\varphi_2} \end{bmatrix} = \begin{bmatrix} d_1\dot{\varphi}_1\cos(\varphi_1) + d_2\dot{\varphi}_2\cos(\varphi_2) \\ d_1\dot{\varphi}_1\sin(\varphi_1) + d_1\dot{\varphi}_2\sin(\varphi_2) \\ d_2\dot{\varphi}_2 l_1 \\ 0 \end{bmatrix}$$

$$X_1 = -d_1\dot{\varphi}_1\begin{bmatrix} -\cos\varphi_1 \\ -\sin\varphi_1 \end{bmatrix}$$

$$X_2 = -d_2\dot{\varphi}_2\begin{bmatrix} -\cos\varphi_2 \\ -\sin\varphi_2 \end{bmatrix}$$



$$Q^{nc}_{s_1} = \left(\frac{\partial r_{mB}}{\partial s_1}\right)^T X_1 + \left(\frac{\partial r_{mL}}{\partial s_1}\right)^T X_2 =$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} X_1 + \begin{bmatrix} 1 & 0 \end{bmatrix} X_2 =$$

$$= d_1\dot{\varphi}_1\cos(\varphi_1) + d_2\dot{\varphi}_2\cos(\varphi_2)$$

$$Q^{nc}_{s_2} = \left(\frac{\partial r_{mB}}{\partial s_2}\right)^T X_1 + \left(\frac{\partial r_{mL}}{\partial s_2}\right)^T X_2 =$$

$$= d_1\dot{\varphi}_1\sin(\varphi_1) + d_2\dot{\varphi}_2\sin(\varphi_2)$$

$$Q^{nc}_{\varphi_1} = \left(\frac{\partial r_{mB}}{\partial \varphi_1}\right)^T X_1 + \left(\frac{\partial r_{mL}}{\partial \varphi_1}\right)^T X_2 =$$

$$= \begin{bmatrix} 0 & 0 \end{bmatrix} X_1 + \begin{bmatrix} \cos(\varphi_1)l_1 \, , & \sin(\varphi_1)l_1 \end{bmatrix} X_2 =$$

$$= d_2 \dot{\varphi}_2 \left(\cos^2(\varphi_1) l_1 + \sin^2(\varphi_1) l_1 \right) = d_2 \dot{\varphi}_2 l_1$$

$$Q_{\varphi_2}^{nc} = \left(\frac{\partial r_{mB}}{\partial \varphi_2} \right)^T X_1 + \left(\frac{\partial r_{mC}}{\partial \varphi_2} \right)^T X_2 = 0$$