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# Functional Regression

## Using the `fda` Package in R

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Ramsay, Hooker and Graves (2009) *Functional Data Analysis with R and Matlab* (Springer)

# This Presentation

- What Is Functional Regression?
- Different types of Functional Regression
- `fRegress.numeric`: Scalar Response
- `fRegress.fdPar`: Functional response,  $x = \text{scalar}$
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# What Is Functional Regression?

Functional Data Analysis extends spline smoothing to:

- an arbitrary finite basis approximation to a function space
- smoothing with an arbitrary linear differential operator

Functional regression = fitting a model where

- the response or
- an explanatory variable

is a function.

## Different types of Functional Regression

Functional regression = fitting a model where

- the response or
- an explanatory variable

is a function.

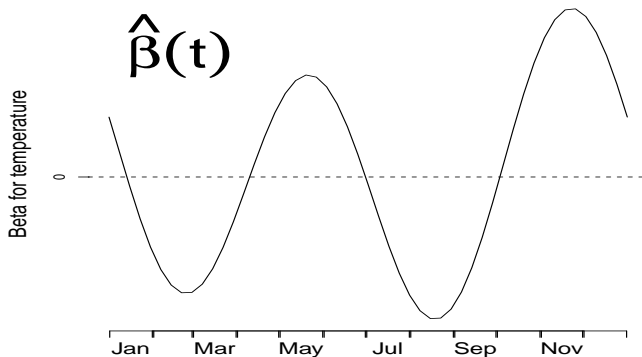
|                 | Explanatory Variable |                                  |
|-----------------|----------------------|----------------------------------|
| response        | <i>scalar</i>        | <i>function</i>                  |
| <i>scalar</i>   | lm                   | fRegress.numeric                 |
| <i>function</i> | fRegress.fdPar       | fRegress.fdPar / linmod / pda.df |

R code for all of these appears in script files in the `fda` package

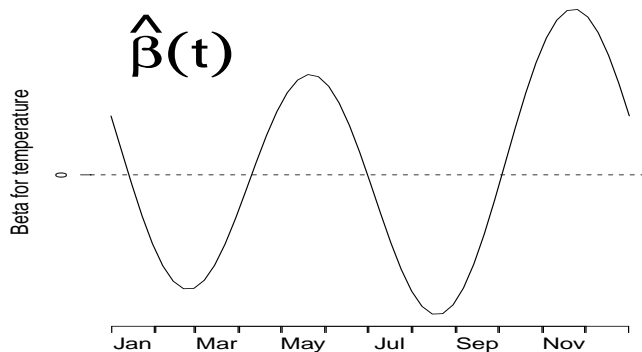
## fRegress.numeric: Scalar Response

$$y_i = \alpha_0 + \int x_i(t)\beta(t)dt + \epsilon_i.$$

log(annual precipitation) ~ (temperature profile)



$\log(\text{annual precipitation}) \sim \text{temperature}(t)$



Conclusion: Wetter locations tend to be

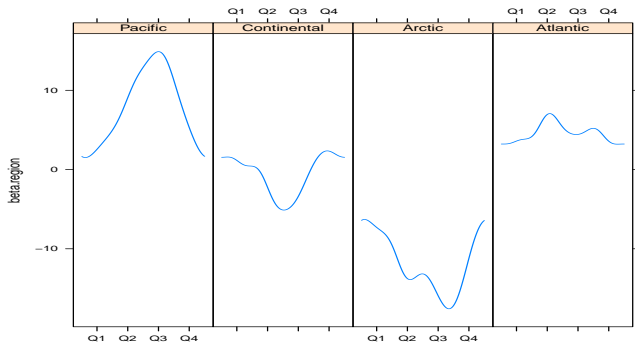
- cooler in February and August and
- warmer in May and November

Ramsay, Hooker, Graves (2009, Fig. 9.1)

fRegress.numeric: functional response,  $x = \text{scalar}$

$$y_i(t) = \beta_0(t) + \sum x_{ij}\beta_j(t) + \epsilon_i(t)$$

temperature  $\sim$  region; Region Deviation:

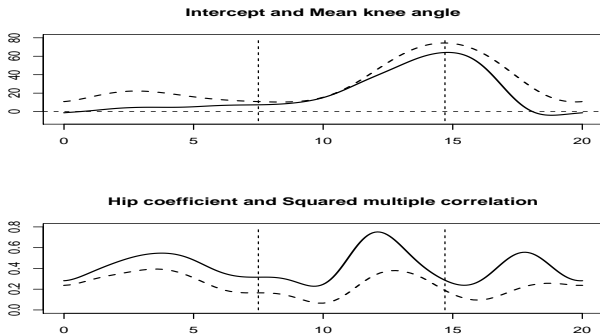


Ramsay, Hooker, Graves (2009, Fig. 10.1)

## fRegress.fdPar: Concurrent Functional Model

$$y_i(t) = \beta_0(t) + \sum x_{ij}(t)\beta_j(t) + \epsilon_i(t)$$

(knee angle)  $\sim$  (hip angle)



Ramsay, Hooker and Graves (2009, Fig. 10.7)



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## fRegress.formula: Simple fRegress Setup

## linmod: Full Integration Regression

Ramsay, Hooker and Graves (2009) *Functional Data Analysis with R and Matlab* (Springer, ch. 10)

## pda.fd: Estimating a Differential Equation

Ramsay, Hooker and Graves (2009) *Functional Data Analysis with R and Matlab* (Springer, ch. 11)

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## Closing Remarks

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## References