SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Linear Regression Extensions

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SciencesPo Paris
2021-03-22

Quick "Quiz" on Last Week's Material

1. From your ***computer*** 👉 connect to ***www.wooclap.com/SCPOMLR***

   OR

2. From your ***phone*** 👉 flash QR code below

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Empirical applications:

(i) *class size* and *student performance*, (ii) *college tuition* and *earnings potential*, (iii) *wage*, *education* and *gender*

Standardized Regression

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Let's define what *standardizing* a variable means.

> ***Standardizing*** a variable z means to *demean* the variable and to divide the demeaned value by its own standard deviation:

$$z_i^{stand} = \frac{z_i - \bar{z}}{\sigma(z)}$$

where $\bar{z}$ is the mean of z and $\sigma(z)$ is the standard deviation of z, i.e. $\sigma(z) = \sqrt{\text{Var}(z)}$.

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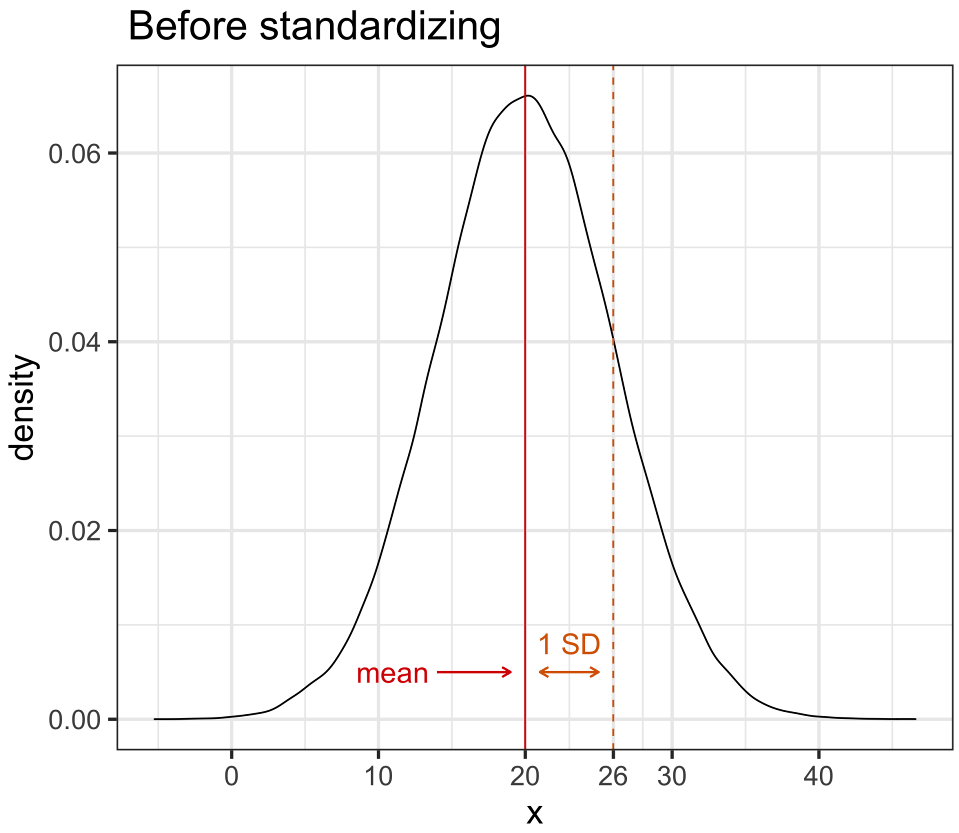
Intuitively, standardizing ***puts variables on the same scale*** so we can compare them.

In our class size and student performance example, it will help to interpret:

- The **magnitude** of the effects,
- The **relative importance of each variable**.

Standardized Regression: Graphically



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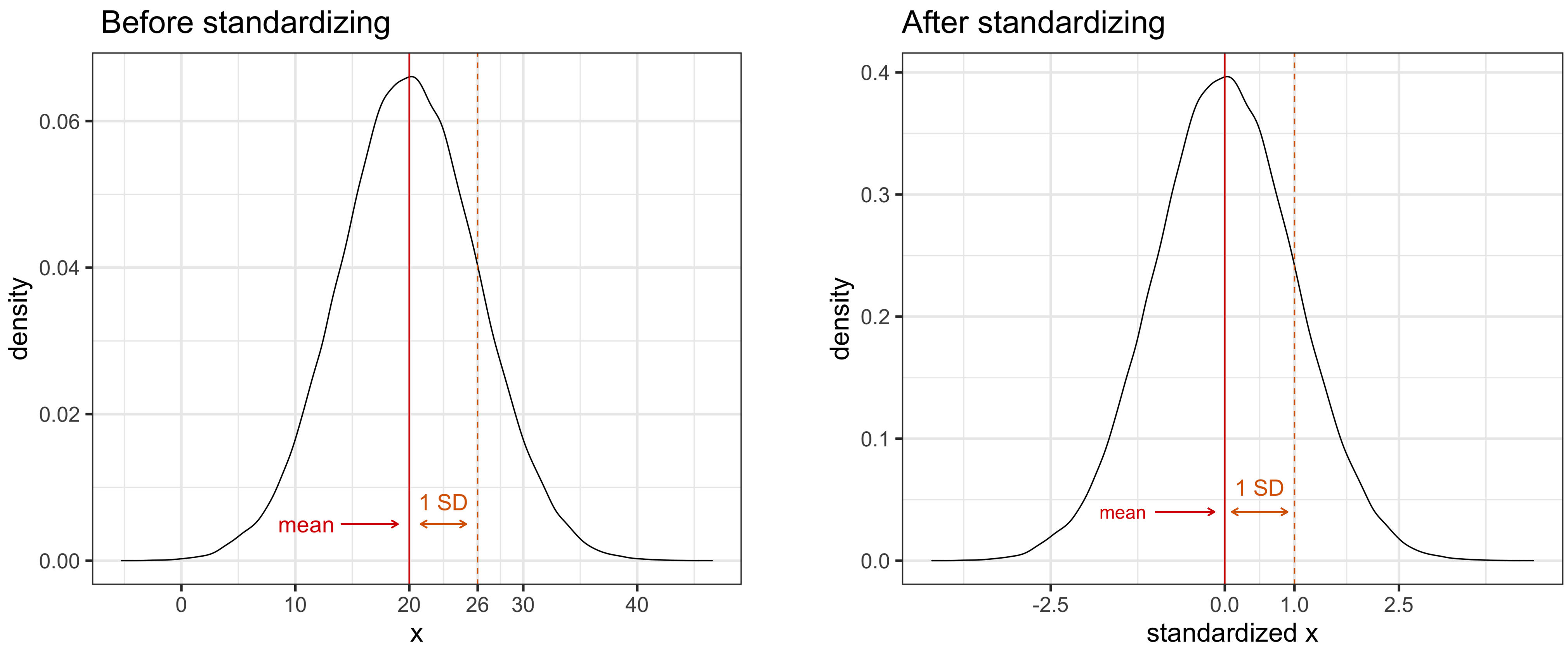

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Task 1: Standardized regression

07:00

Let's go back our grades dataset. Remember that to load the data you need to use the `read_dta()` function from the `haven` package. These are the estimates we got from regressing average math test scores on the full set of regressors.

```
##       (Intercept)          classize      disadvantaged school_enrollment            female         religious
##       78.560725298       0.003320773      -0.389333008       0.000758258       0.923710499       2.876146701
```

1. Create a new variable `avgmath_stand` equal to the standardized math score. You can use the `scale()` function (combined with `mutate`) or do it by hand with base `R`.

2. Run the full regression using the standardized math test score as the dependent variable. Interpret the coefficients and their magnitude.

3. Create the standardized variables for each *continuous* regressor as `<regressor>_stand`.

    - Would it make sense to standardize the `religious` variable?

4. Regress `avgmath_stand` on the full set of standardized regressors and `religious`. Discuss the relative influence of the regressors.

Non-Linear Relationships

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1. *Log* models
2. *Polynomial* models

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  - This *level* can be: euros, years, number of students,... and even percentage.
- Taking the *natural* log of the dependent and/or the independent variable(s) leads us to define 3 other types of regressions:
  - ***Log - level***: $log(y_i) = b_0 + b_1 x_{1,i} + \ldots + e_i$
  - ***Level - log***: $\mathrm{y}_i = b_0 + b_1 log(x_{1,i}) + \ldots + e_i$
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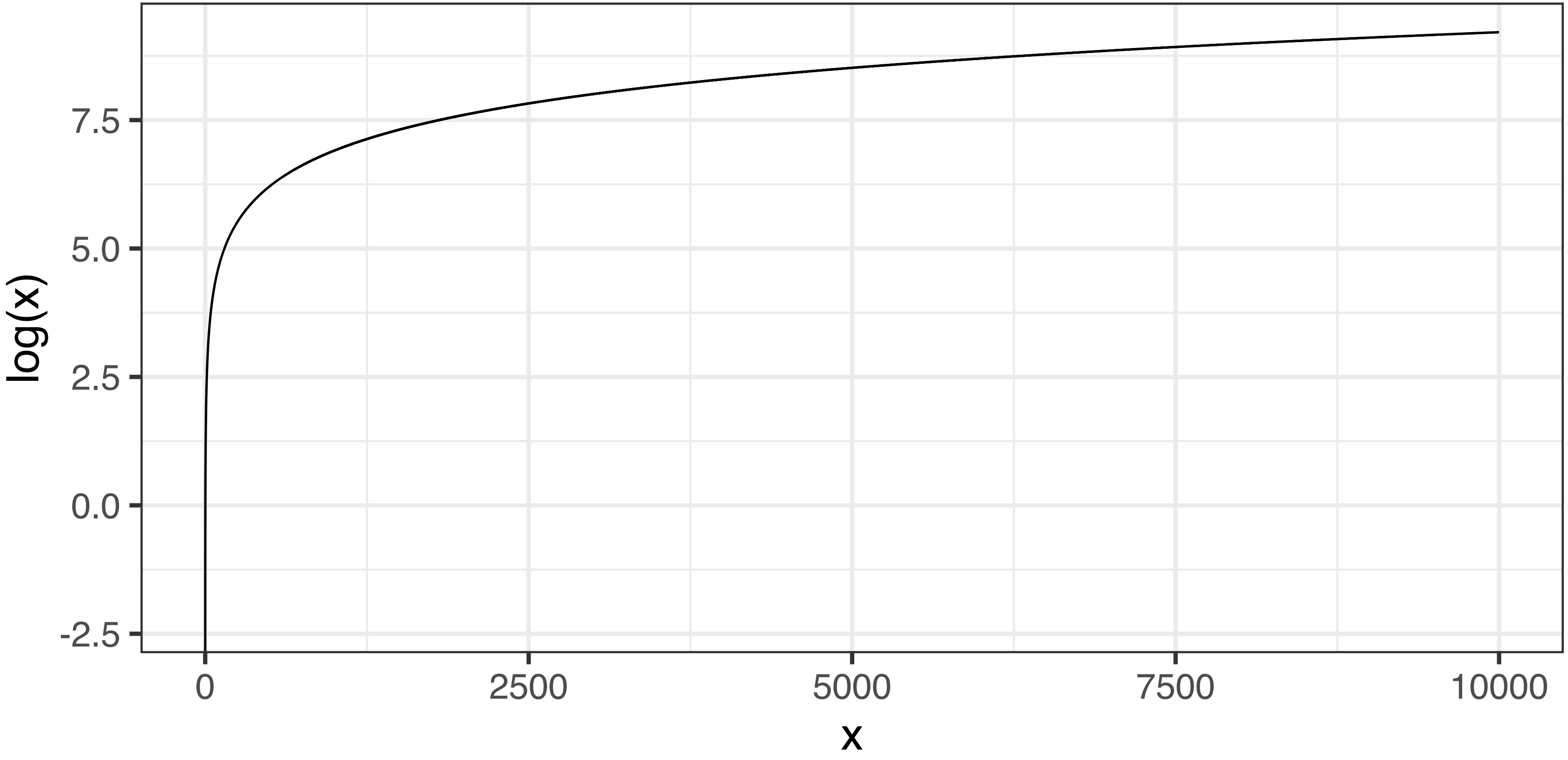
The (natural) log Function: A Primer 😉



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⚠️ You can only log your variables if they don't take 0 or negative values! Always think about this when taking the log of your dependent or independent variable(s)

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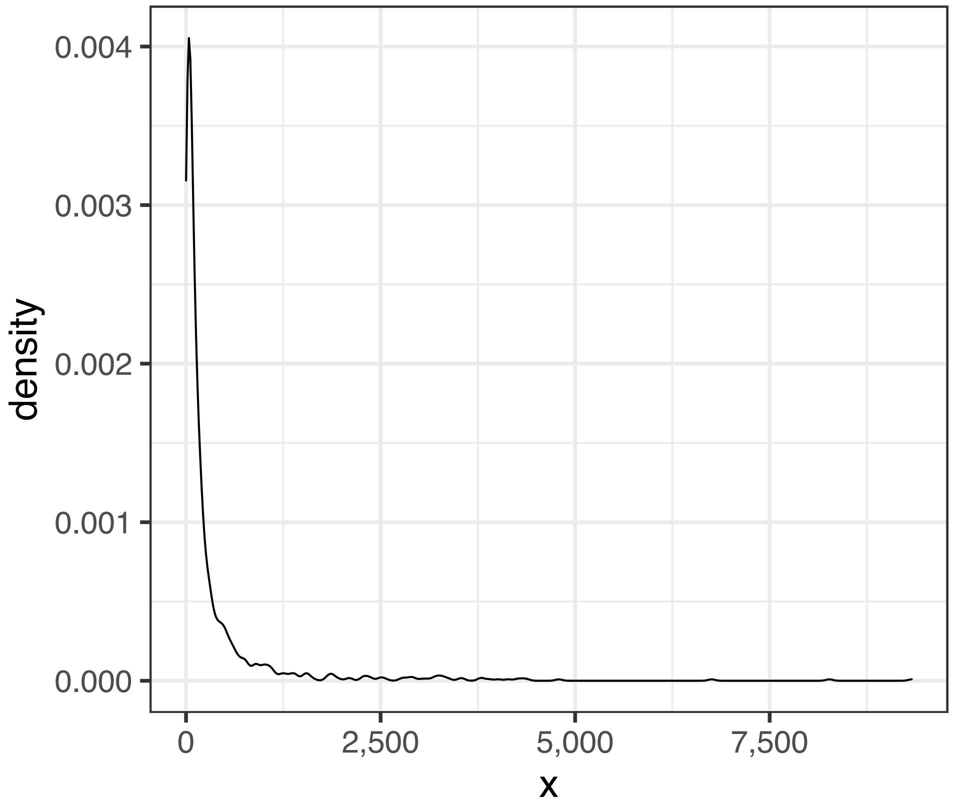

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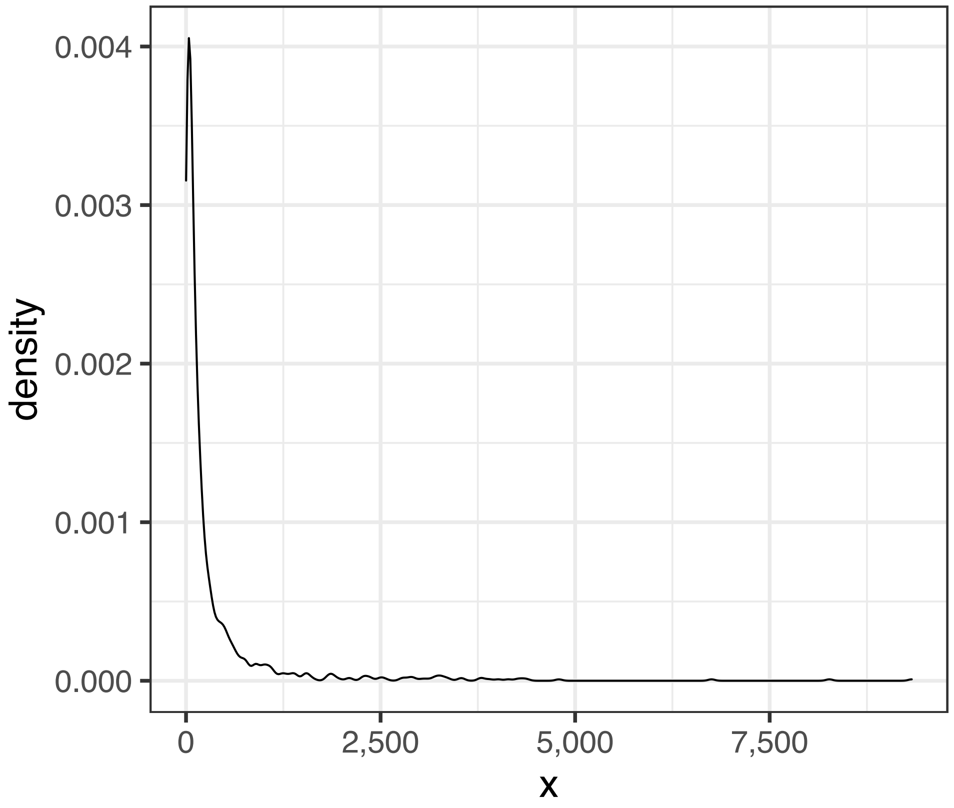

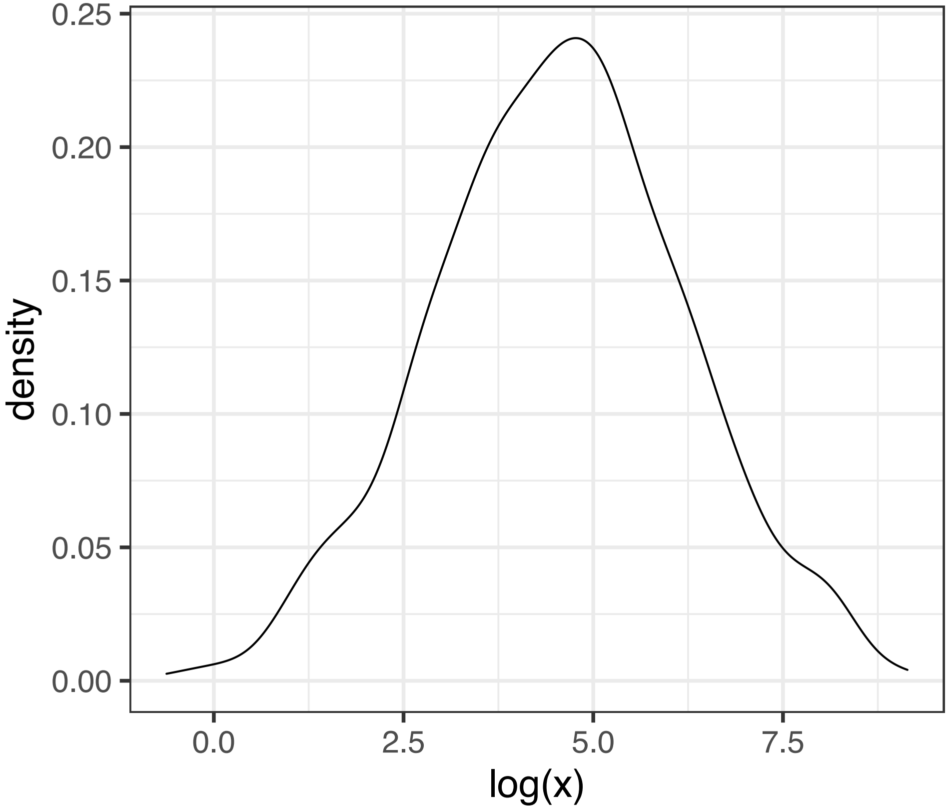

Log Models: Simplified Interpretations

Specification	Model	Interpretation of b_1
Level - Level	$y = b_0 + b_1 x + e$	A **one unit** increase in x is associated, on average, with a b_1 **unit change** in y
Log - Level	$log(y) = b_0 + b_1 x + e$	A **one unit** increase in x is associated, on average, with a $b_1 \times 100$ **percent change** in y
Level - Log	$y = b_0 + b_1 log(x) + e$	A **one percent** increase in x is associated, on average, with a $b_1 / 100$ **unit change** in y
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- This may look like cooking recipes but of course it can be derived with some relatively simple maths.
- ⚠️ these interpretations are only true for ***small*** changes in x and small/or b_1. What happens if we want to know the change in y for big changes in x or when b_1 is large?

Log Models: General Interpretations

For ***any increase in x, Δx, and any b_1*** $(\Delta x = 5\% = 0.05 \implies 1 + \Delta x = 1.05)$:

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$\rightarrow$ for $\Delta x = 20\% = 0.20$, $log(1 + \Delta x) = log(1.20) = 0.18$

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$\rightarrow$ for $\Delta x = 10\% = 0.10$ and $b_1 = 10$,
$((1+\Delta x)^{b_1}-1)\times 100=(1.1^{10}-1)\times 100=159.37$

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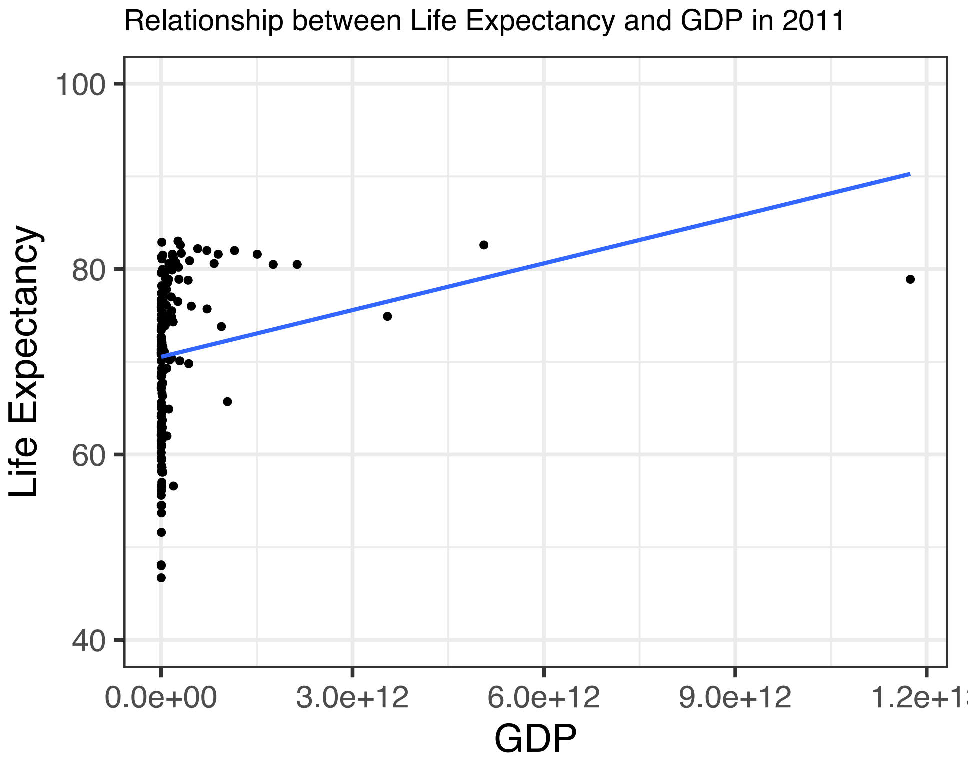

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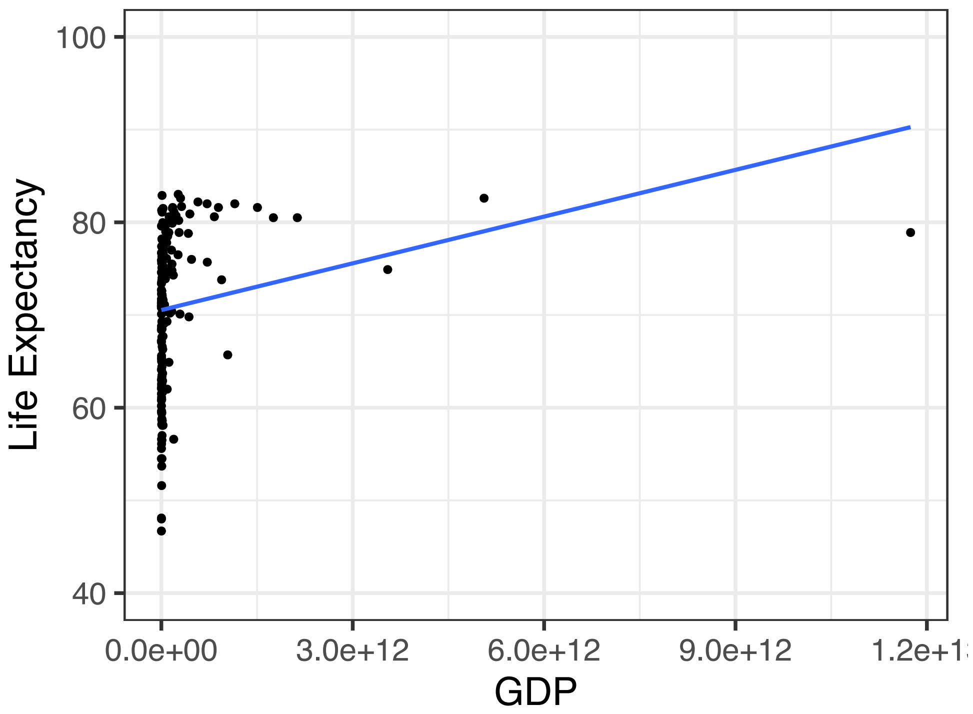

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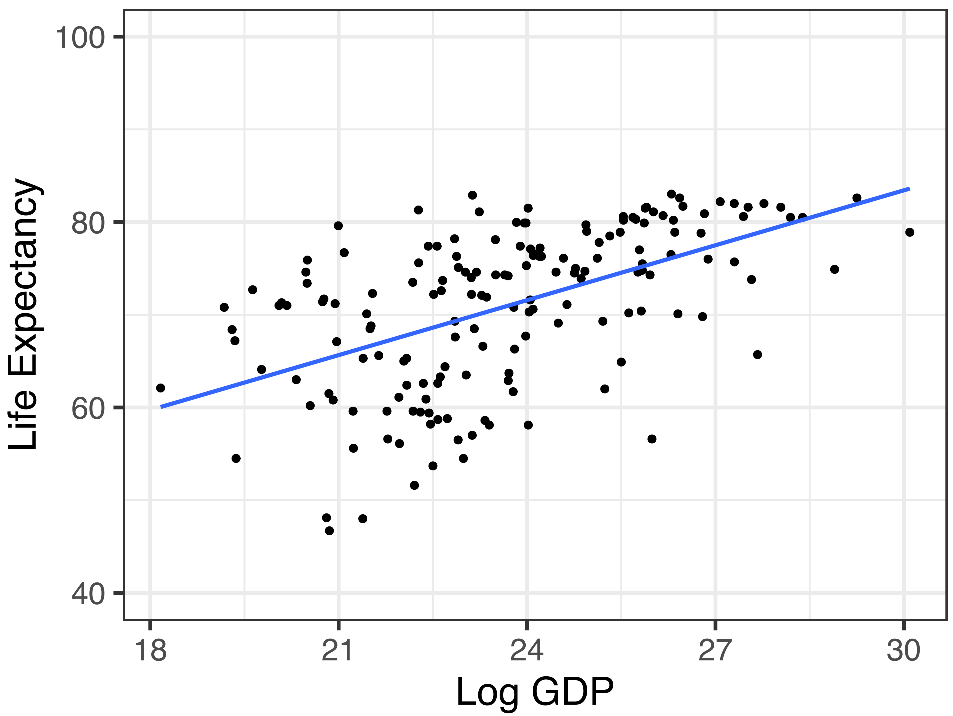

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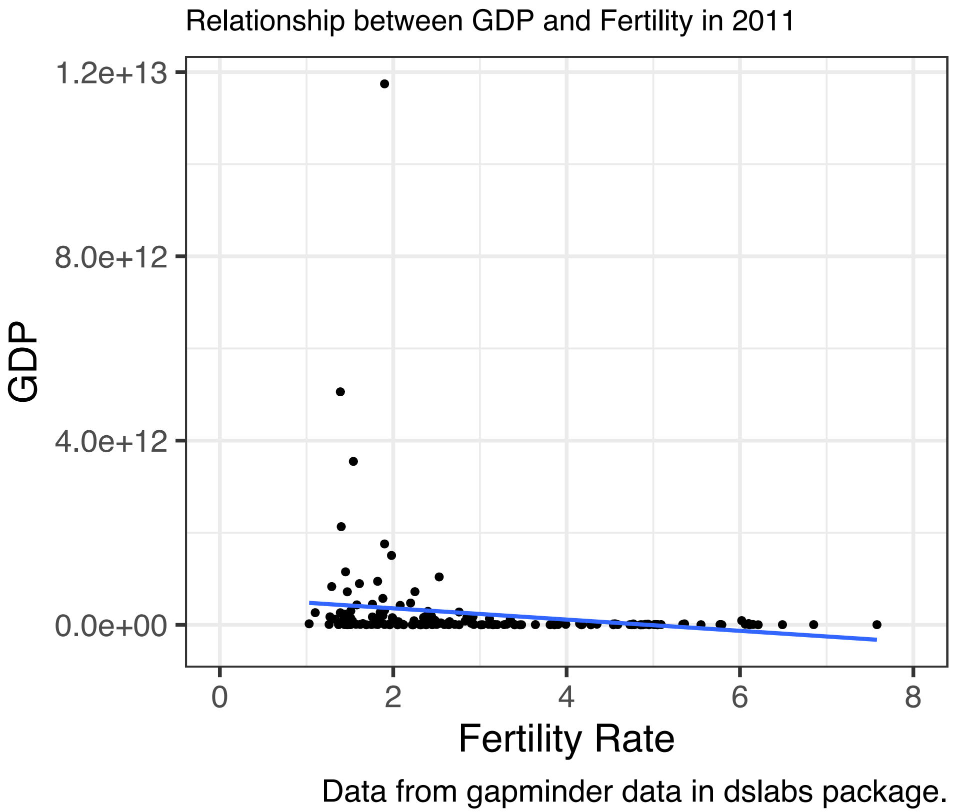

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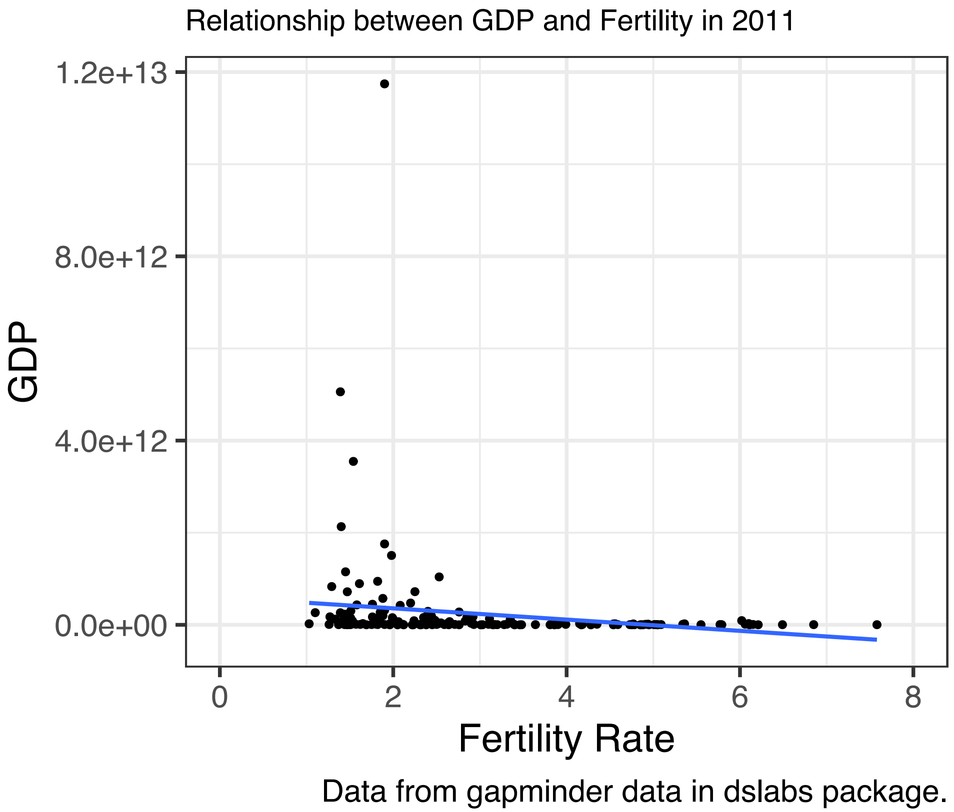
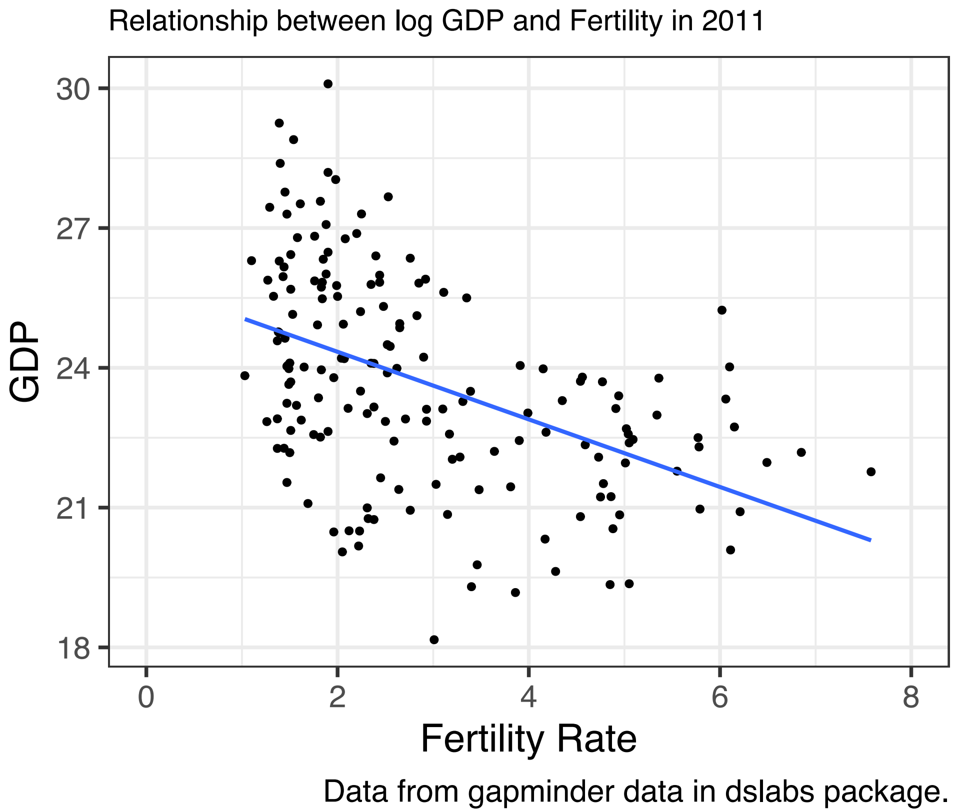




When Should You Use log Models?

1. If the relationship betwen x and y looks like a log or exponential function.

2. To easily interpret coefficients as ***elasticities*** which play a central role in economic theory.

Elasticity of y with respect to x: percent change in y following a 1% increase in x.

Accounting for Other Types Non-Linear Relationships

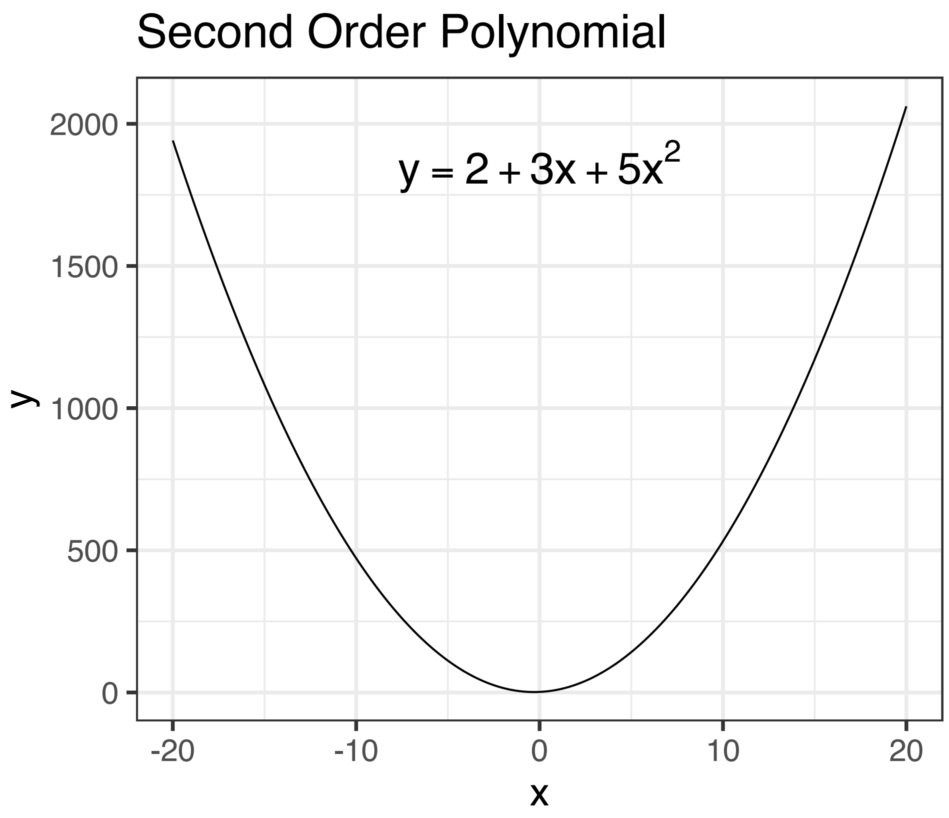
What if the relationship between x and y is not exponential/log?

Accounting for Other Types Non-Linear Relationships

What if the relationship between x and y is not exponential/log?

$\rightarrow$ ***polynomial*** regressions: just take a polynomial function of the regressor!

Polynomial Wut? 😟



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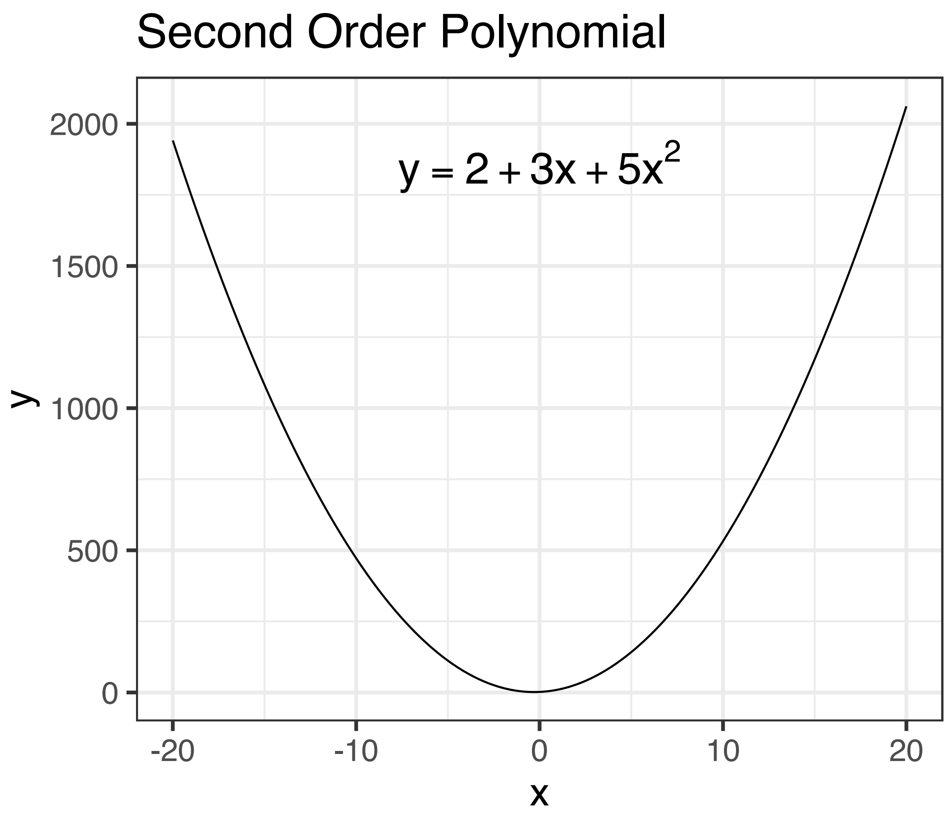

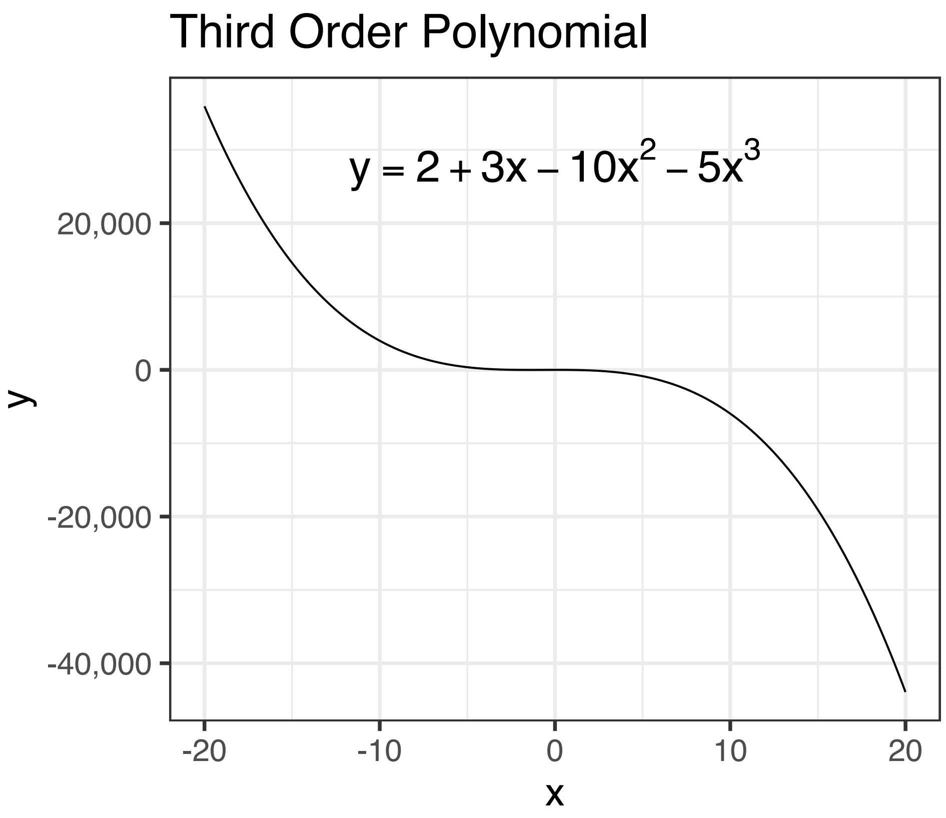

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Several ways of doing this in `R`:

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lm(y ~ x + I(x^2) + I(x^3), data)
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Several ways of doing this in `R`:

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lm(y ~ x + I(x^2) + I(x^3), data)
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```
lm(y ~ poly(x, 3, raw = TRUE), data)
```

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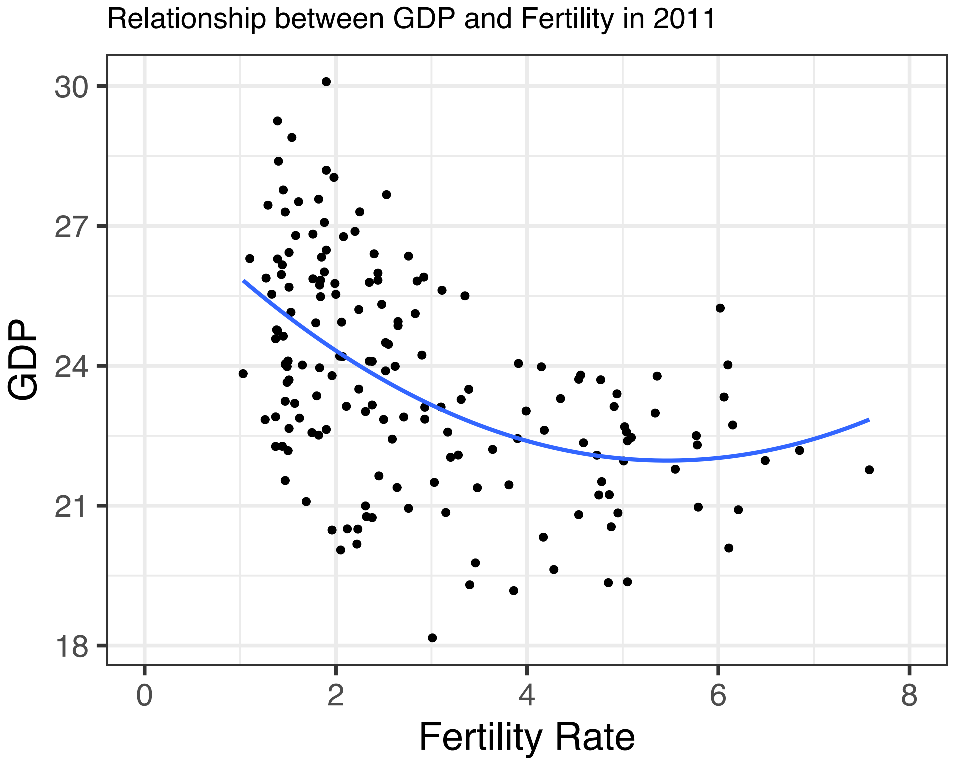
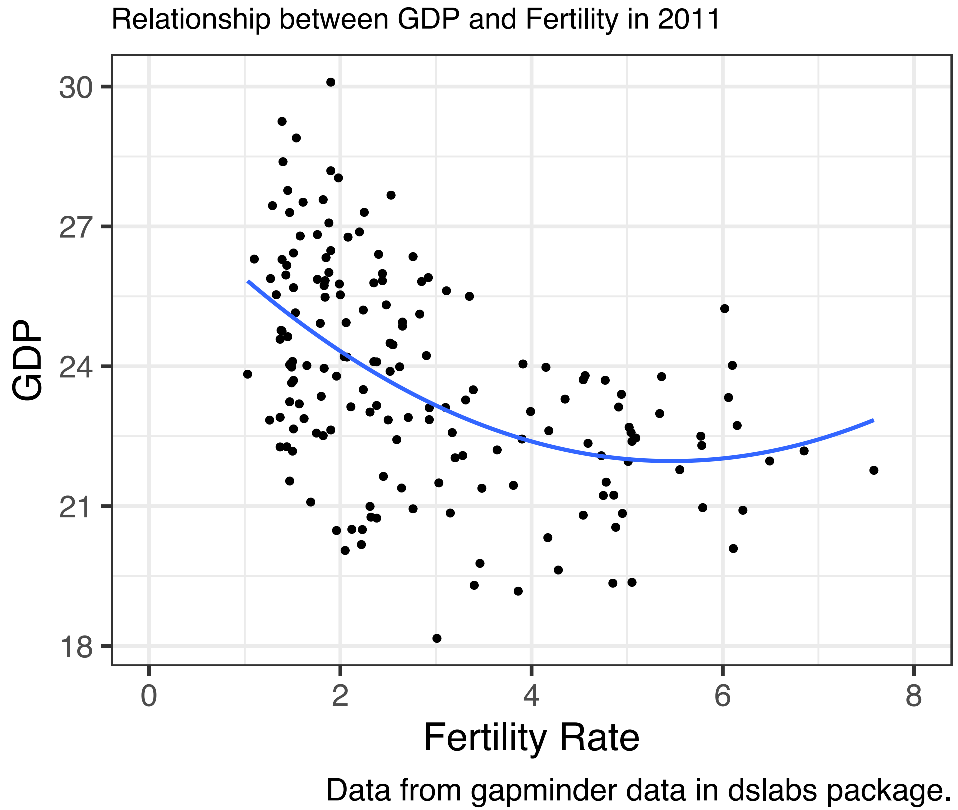
2nd order:



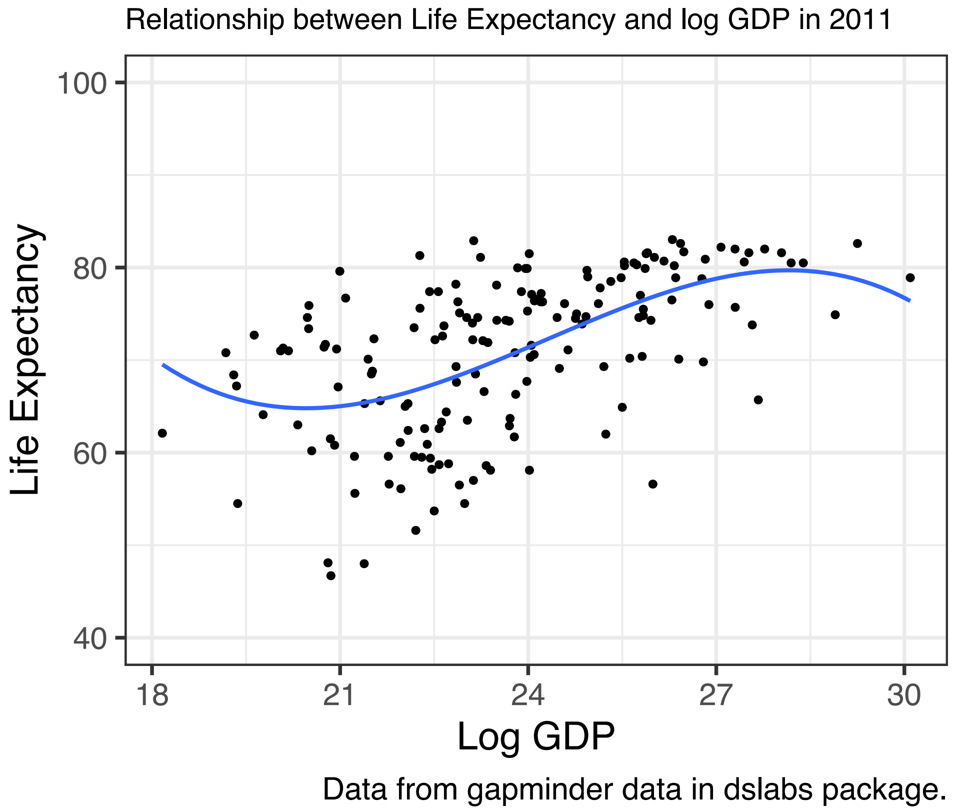
Data from gapminder data in dslabs package.

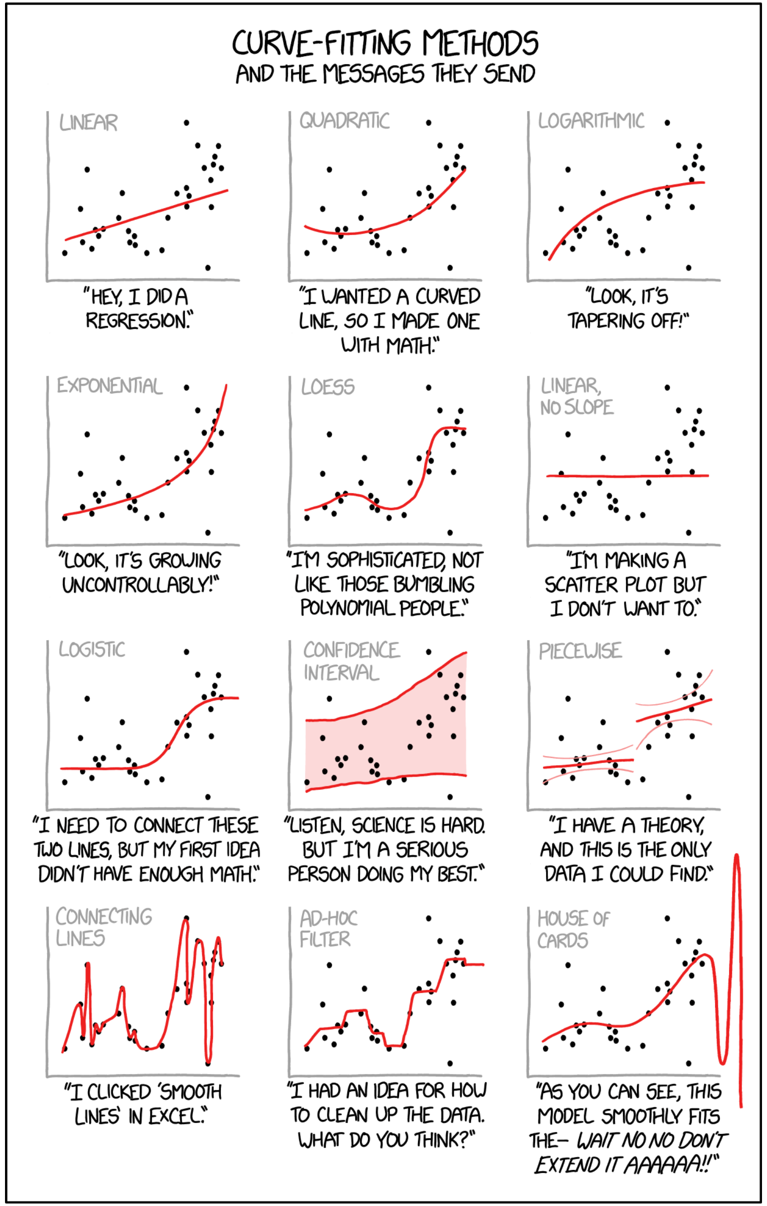
Polynomial Regressions

2nd order:



3rd order:




CURVE-FITTING METHODS
AND THE MESSAGES THEY SEND
LINEAR
"HEY, I DID A REGRESSION."
QUADRATIC
"I WANTED A CURVED LINE, SO I MADE ONE WITH MATH."
LOGARITHMIC
"LOOK, IT'S TAPERING OFF!"
EXPONENTIAL
"LOOK, IT'S GROWING UNCONTROLLABLY!"
LOESS
"I'M SOPHISTICATED, NOT LIKE THOSE BUMBLING POLYNOMIAL PEOPLE."
LINEAR, NO SLOPE
"I'M MAKING A SCATTER PLOT BUT I DON'T WANT TO."
LOGISTIC
"I NEED TO CONNECT THESE TWO LINES, BUT MY FIRST IDEA DIDN'T HAVE ENOUGH MATH."
CONFIDENCE INTERVAL
"LISTEN, SCIENCE IS HARD. BUT I'M A SERIOUS PERSON DOING MY BEST."
PIECEWISE
"I HAVE A THEORY, AND THIS IS THE ONLY DATA I COULD FIND."
CONNECTING LINES
"I CLICKED 'SMOOTH LINES' IN EXCEL."
AD-HOC FILTER
"I HAD AN IDEA FOR HOW TO CLEAN UP THE DATA. WHAT DO YOU THINK?"
HOUSE OF CARDS
"AS YOU CAN SEE, THIS MODEL SMOOTHLY FITS THE– WAIT NO NO DON'T EXTEND IT AAAAAA!!"

Task 2: Non-linear relationships

10:00

1. Load the data here. This dataset contains information about tuition and estimated incomes of graduates for universities in the US. More details can be found here.

2. Create a scatter plot of estimated mid career pay (`mid_career_pay`) $(y - axis)$ as a function of out of state tuition (`out_of_state_tuition`) $(x - axis)$. Would you say the relationship is broadly linear or rather non-linear? Use `geom_smooth(method = "lm", se = F) + geom_smooth(method = "lm", se = F, formula = y ~ poly(x, 2, raw= T))` to fit both a linear and 2nd order regression line. This time which seems most appropriate?

3. Create a variable equal to out of state tuition divided by 1000. Regress mid career pay on out of state tuition divided by 1000. Interpret the coefficient.

4. Regress mid career pay on out of state tuition divided by 1000 and its square. *Hint:* you can use either `poly(x, 2, raw = T)` or `x + I(x^2)`, where x is your regressor. What does the positive sign on the squared term imply?

Interaction Terms

Interacting Regressors

- We interact two regressors when we believe ***the effect of one depends on the value of the other***.
    - *Example:* The returns to education on wage vary by gender.

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- In practice, if we interact x_1 and x_2, we would write our model like this :

$$y_i = b_0 + b_1 x_{1,i} + b_2 x_{2,i} + b_3 x_{1,i} * x_{2,i} + \ldots + e_i$$

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- The interpretation of b_1, b_2, and b_3 will depend on the type of x_1 and x_2.
- Let's focus on the cases where one regressor is a ***dummy/categorical*** variable and the other is ***continuous***.
- It will give you the intuition for the other cases:
  - Both regresors are dummies/categorical variables,
  - Both regresors are continuous variables.

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$$\begin{aligned} &\mathbb{E}[\text{score}_i | \text{small}_i = 1 \ \& \ \text{experience}_i = 10] - \mathbb{E}[\text{score}_i | \text{small}_i = 0 \ \& \ \text{experience}_i = 10] \\ &= b_0 + b_1 + b_2 * 10 + b_3 * 10 - (b_0 + b_2 * 10) \\ &= b_1 + b_3 * 10 \end{aligned}$$

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Running the regression for the `math` score (for all grades), we obtain:

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lm(math ~ small+ experience + small*experience, star_df)
```

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##
## Call:
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## Coefficients:
##          (Intercept)              smallTRUE             experience   smallTRUE:experience
##             534.1919                15.8906                 1.3305                -0.3034
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```

Interpretation:

- The interaction term allows the impact of being in a small class to vary with the experience of the teacher.
- In particular, we still observe a *positive impact of being in a small class* on math score,
- but this *effect is decreasing in the experience of the teacher*.

Interacting Regressors: Visually

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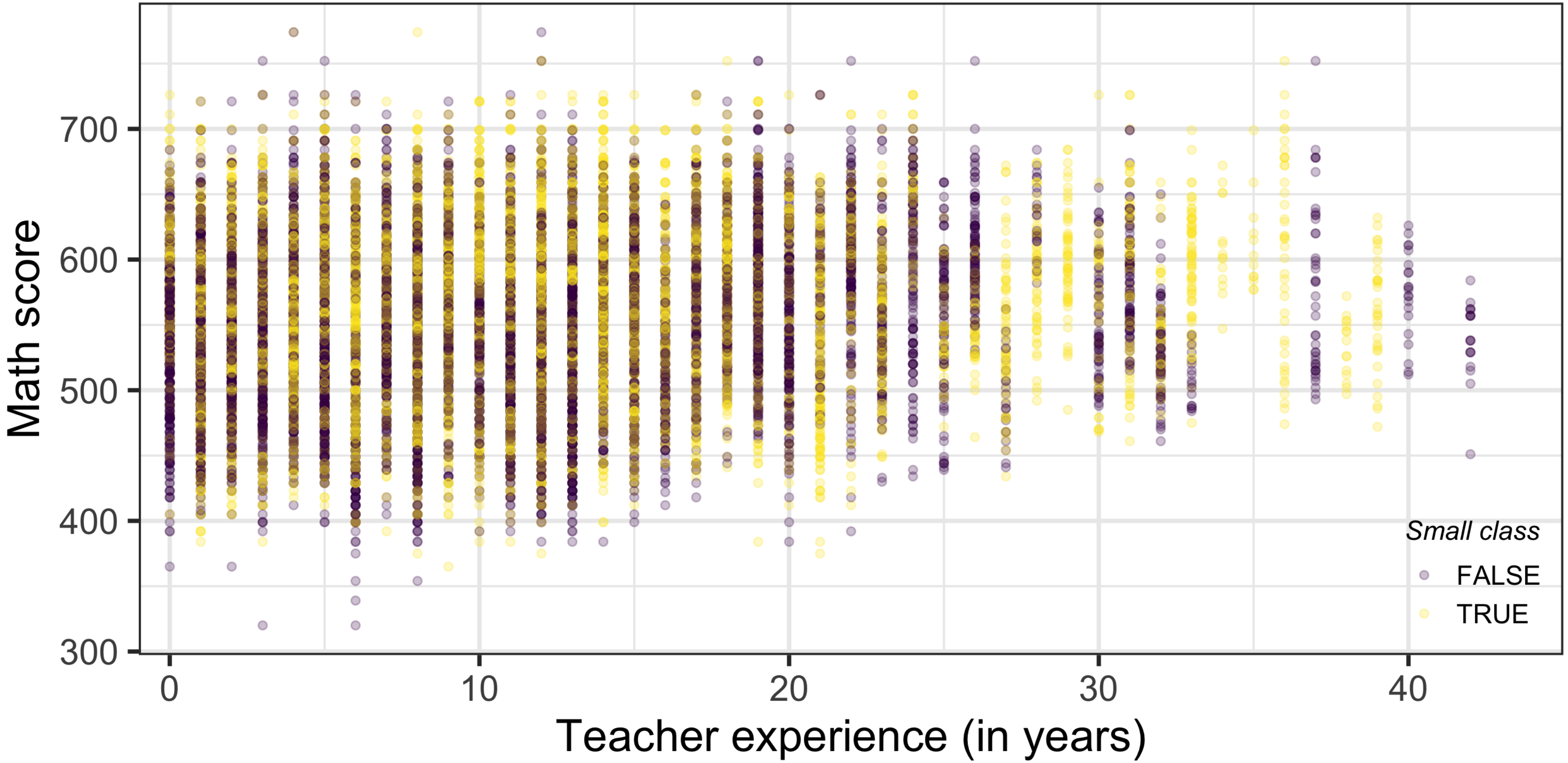

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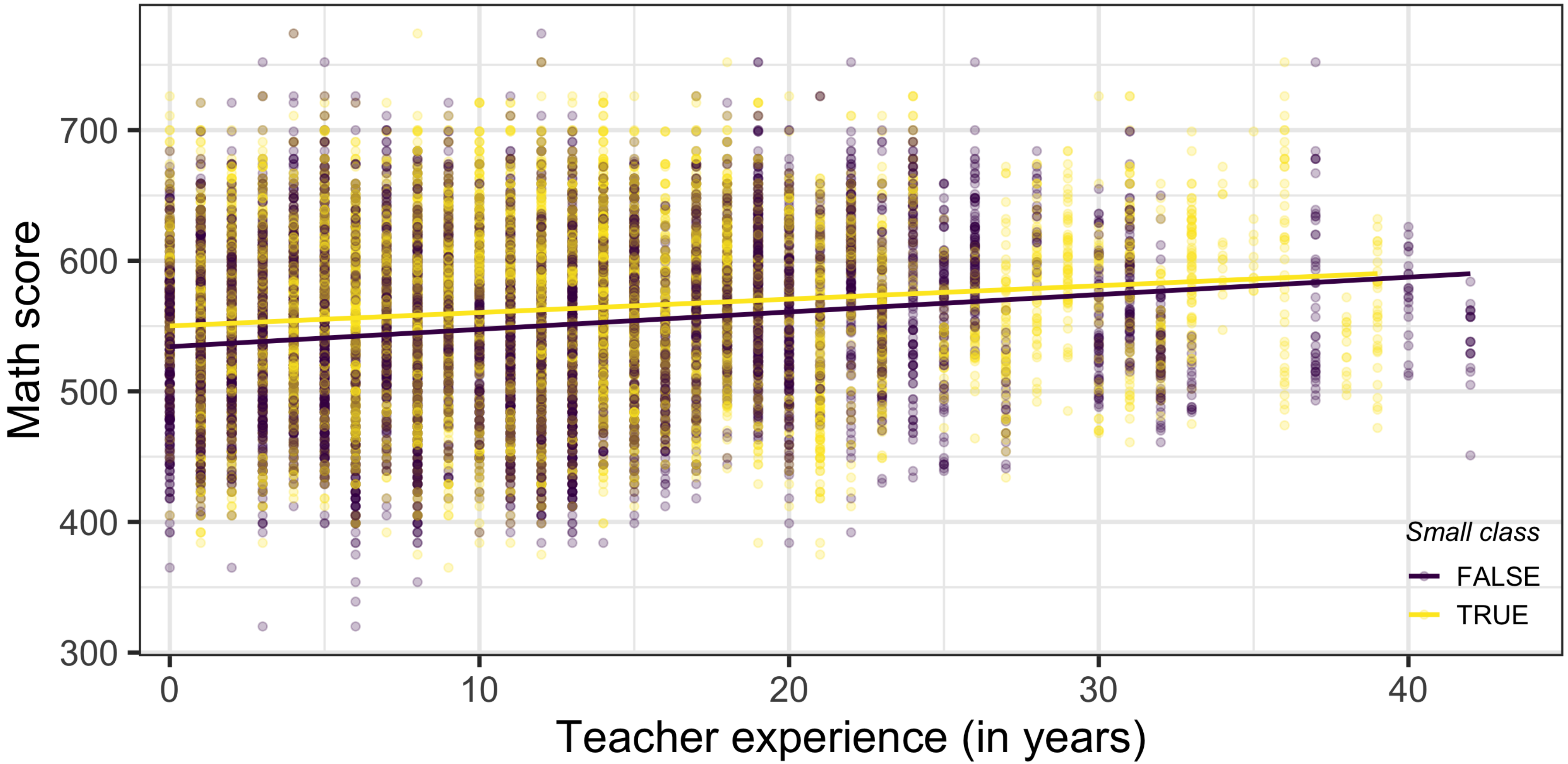

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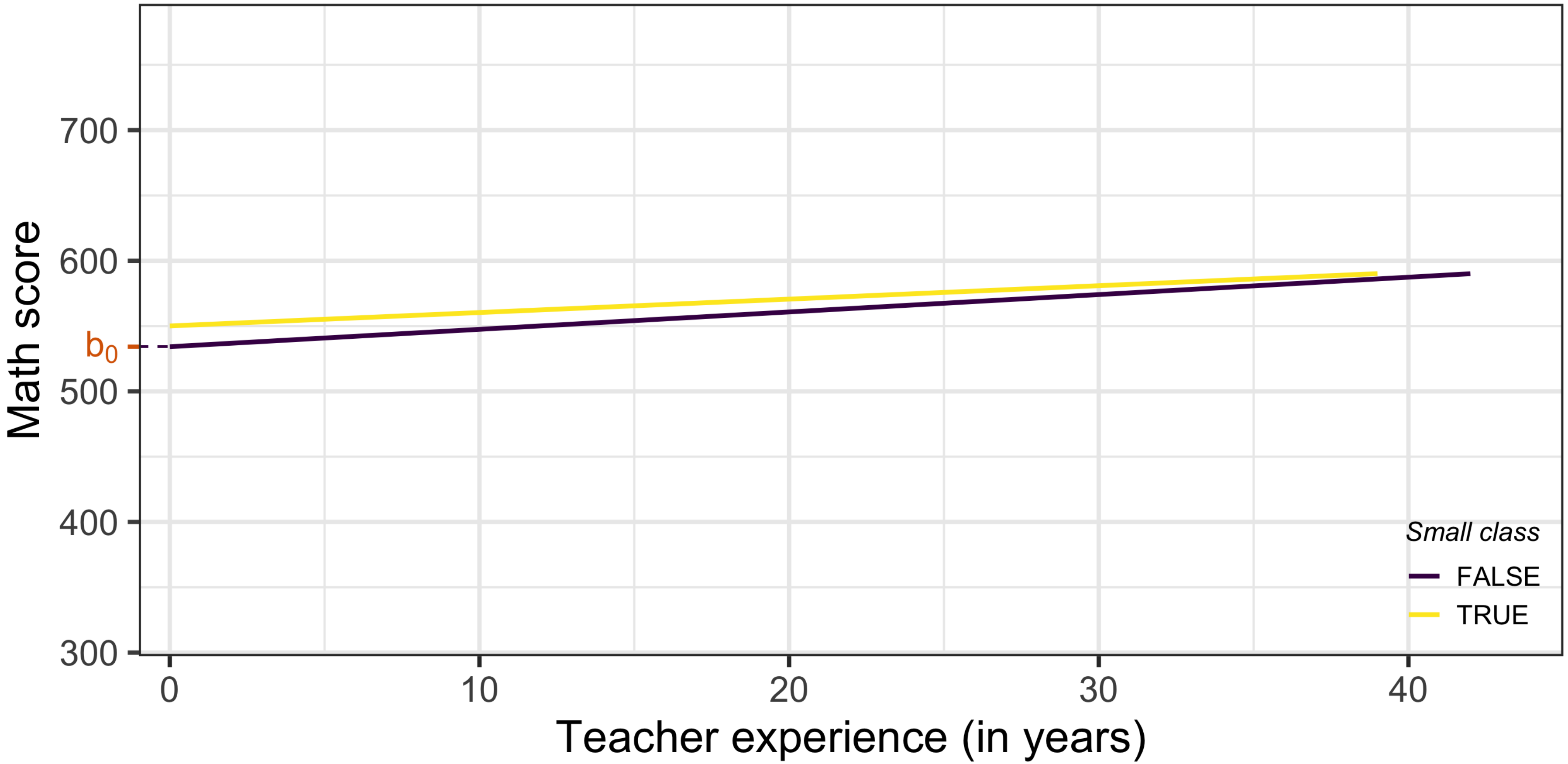

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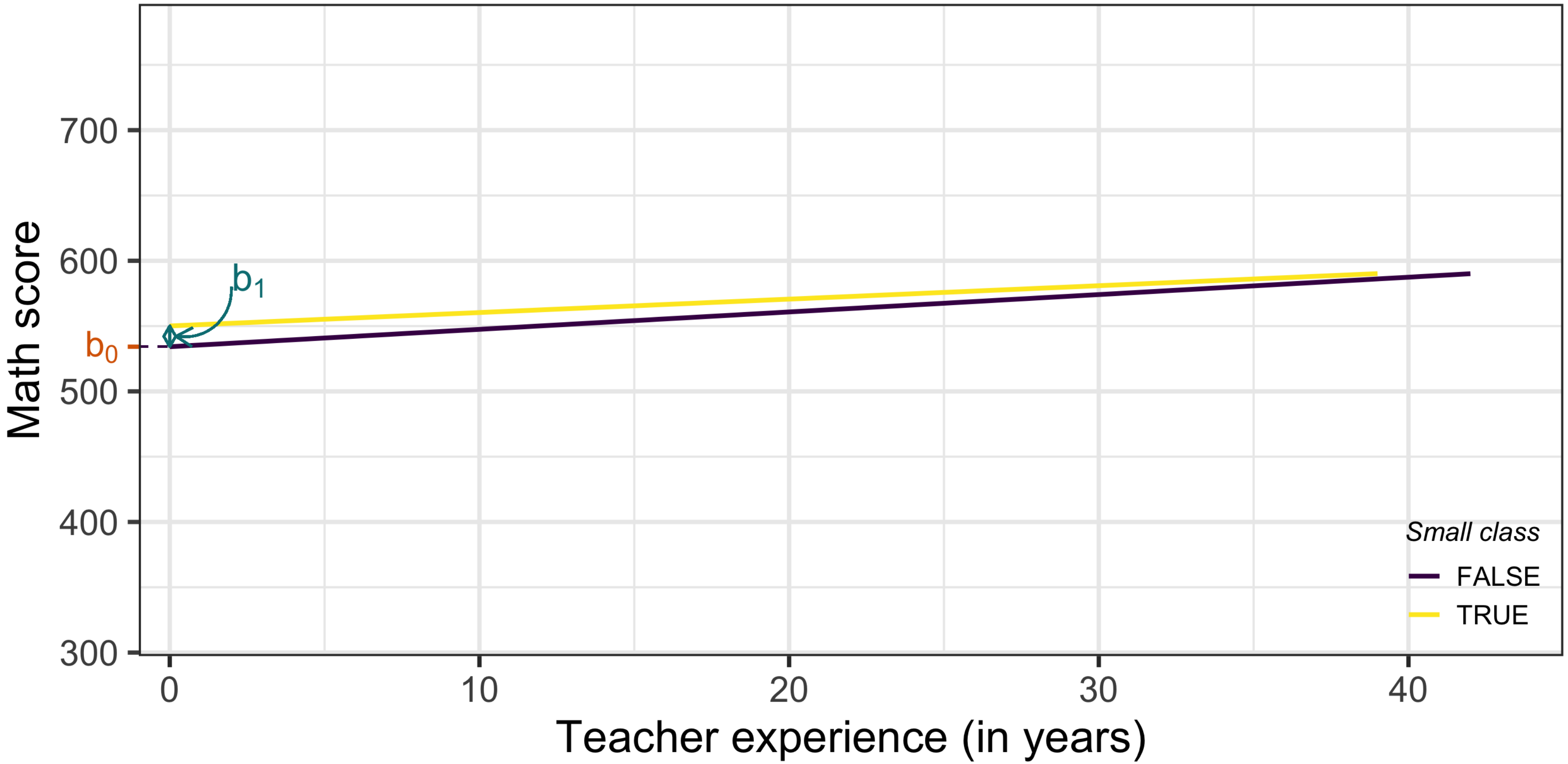

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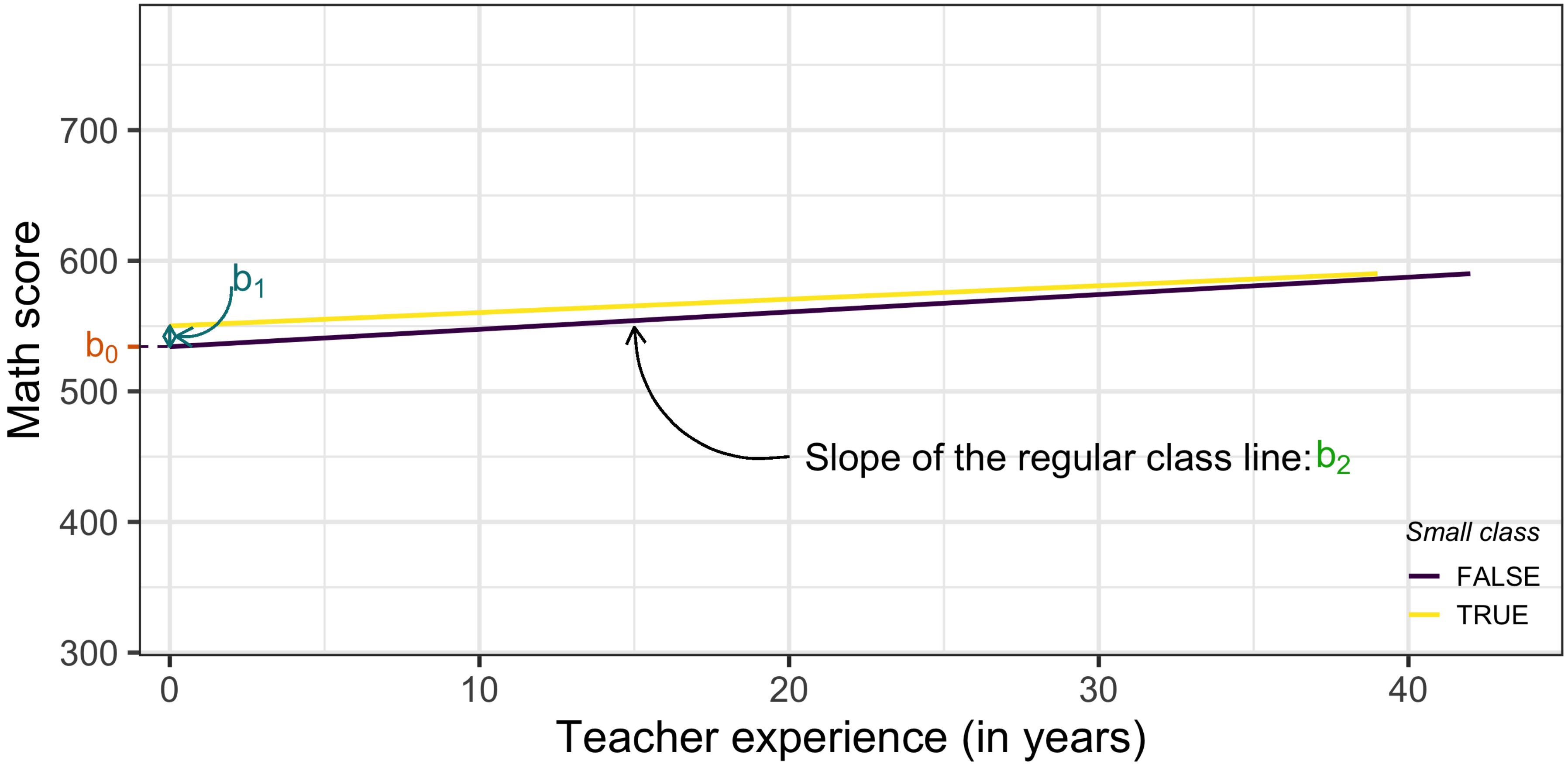

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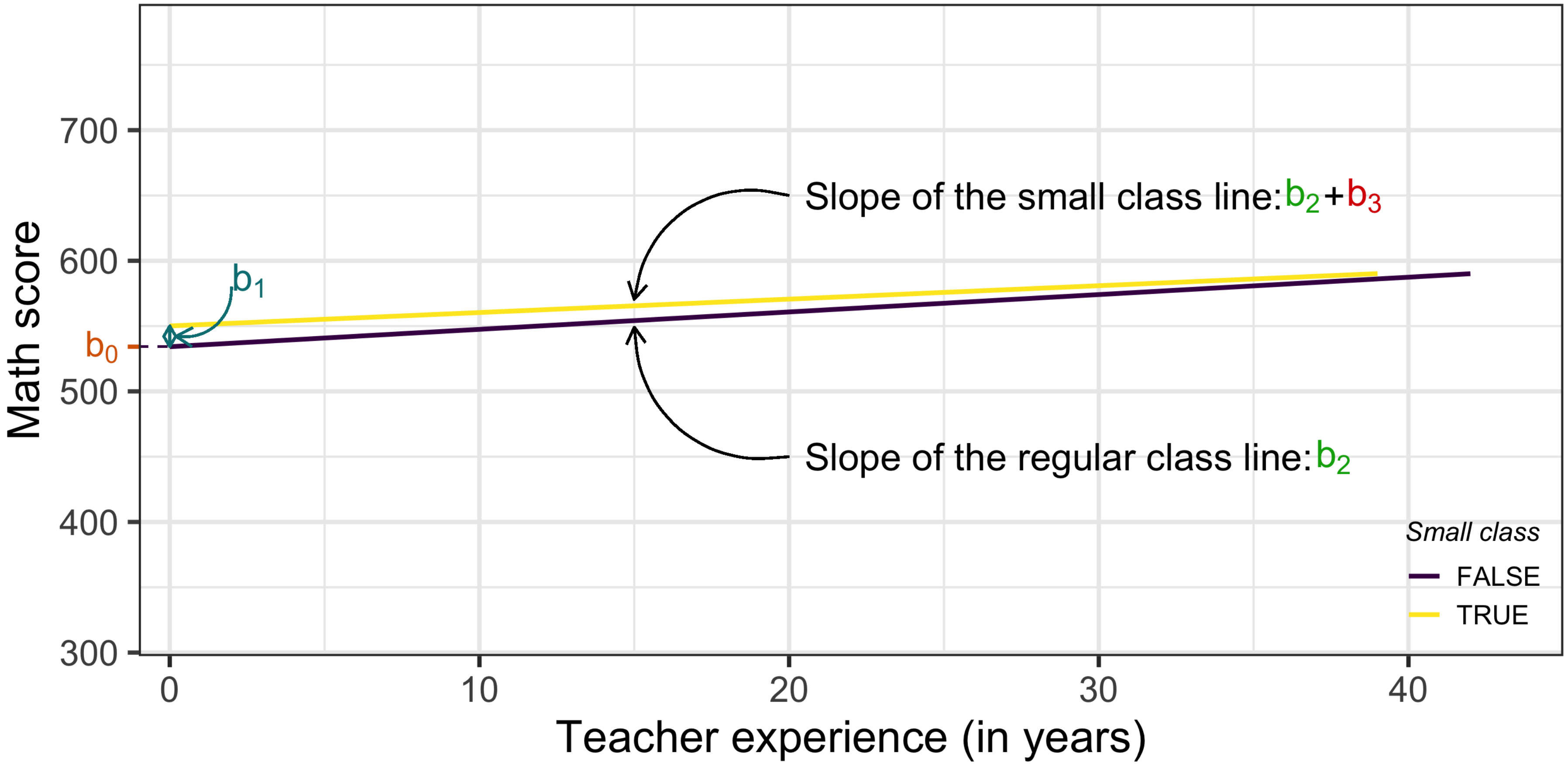

Task 3: Wages, education and gender in 1985

10:00

1. Load the data `CPS1985` from the `AER` package.
2. Look at the `help` to get the definition of each variable: `?CPS1985`
3. We don't know if people are working part-time or full-time, does it matter here?
4. Create the `log_wage` variable equal to the log of `wage`.
5. Regress `log_wage` on `gender` and `education`, and save it as `reg1`. Interpret each coefficient.
6. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg2`. Interpret each coefficient. Does the gender wage gap decrease with education?
7. Create a plot showing this interaction. (*Hint:* use the `color = gender` argument in `aes` and `geom_smooth(method = "lm", se = F)` to obtain a regression line per gender.)

Teaser for the Next 3 Lectures

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  - In other words, how confident can we be that our estimates (sign, magnitude) are not just driven by randomness?

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  - In other words, how confident can we be that our estimates (sign, magnitude) are not just driven by randomness?
- We will answer those kind of questions:
  - We'll present the notion of **sampling**, and
  - Understand what **statistical inference** is and how to do it.

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DEPARTMENT OF ECONOMICS

SEE YOU NEXT WEEK!

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