Structural Change, Land Use and Urban Expansion

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Abstract

We develop a multi-sector spatial equilibrium model with endogenous land use: land is used either for agriculture or housing. Urban land, densely populated due to commuting frictions, expands out of agricultural land. With rising productivity, the reallocation of workers away from agriculture frees up land for cities to expand, limiting the increase in land values despite higher income and increasing urban population. Due to the reallocation of land use, the area of cities expands at a fast rate and urban density persistently declines, as in the data over a long period. As structural change slows down, cities sprawl less and land values start increasing at a faster rate, as in the last decades. Quantitative predictions of the joint evolution of density and land values across time and space are confronted with historical data assembled for France over 180 years.

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1 Introduction

Since the early years of the industrial revolution, the population massively migrated from rural areas towards cities. This widespread phenomenon of urbanization went together with the reallocation of workers away from the agricultural sector towards manufacturing and service sectors—a phenomenon of structural change. How do cities grow when these well-known phenomena occur? Cities can become denser for a given area—growth at the intensive margin. They can also become larger in surface to accommodate more workers—via growth at the extensive margin. Over a long period, cities have been growing essentially in area, at such a fast speed that their average density has been falling. In other words, over time, cities expanded faster in area than in population. We precisely document this stylized fact for France since 1870 but it is also documented on a global scale in Angel et al. (2010). In France, the population of the main cities has been multiplied by about 4 since 1870, while their area increased by a factor 30: the average urban density has thus been divided by almost 8. This paper shows that this persistent decline in density, despite the process of urbanization, is well explained by the most conventional theories of structural change with non-homothetic (Stone-Geary) preferences and augmented with endogenous land use—whereby land can be used for agriculture or urban housing.

A crucial insight of our theory is to consider that the value of agricultural land at the urban fringe determines the opportunity cost of expanding the area of cities for housing purposes. With low agricultural productivity, agricultural goods and farmland are expensive. High agricultural land values make cities initially small in area and very dense as households cannot afford large homes—a manifestation of the 'food problem' (Schultz (1953)). With structural change driven by rising (agricultural) productivity, workers move away from rural areas towards cities, freeing up agricultural land. As the value of land at the urban fringe falls and households free up resources to buy larger homes, cities expand in area at a fast rate. Together with the reallocation of workers across sectors, reallocation of land use occurs—from agricultural use to urban use. We document that for France, since 1840, about 15% of French land formerly used for agriculture is no longer used for this purpose. As long as the transitory process of reallocation away from agriculture continues, cities grow faster in area than population and average urban density keeps falling with urban expansion. Thus, our theory provides a novel mechanism explaining the sprawl and the suburbanisation of cities. This complements the traditional urban view that cities have sprawled following improvements in the commuting technologies that have allowed households to live further away from their workplace.

Our framework also provides novel predictions regarding the historical evolution of land values. When productivity is low and agricultural goods are in high demand for subsistence needs, the value of farmland is high relative to income. With economic development, structural change frees up farmland for urban expansion and puts downward pressure on its price. The value of agricultural land as share of income falls and, over time, the value of urban land constitutes the largest fraction of aggregate land values. These predictions are in line with the data as shown in Piketty and Zucman

(2014). Moreover, despite rising housing demand, the fast expansion of cities at the extensive margin due to structural change initially limits the increase in urban land rents and housing prices. When the reallocation of workers slows down, so does the reallocation of land use at the fringe of cities. If workers' productivity increases further, the value of land must adjust to prevent further expansion of cities with rising housing demand. Land values start to increase at a faster rate. Our theory thus predicts flat land and housing values for decades before shooting up as the process of structural change ends. This prediction resembles very much the data for France and most advanced economies as best illustrated in Knoll et al. (2017): real housing prices being flat for decades since the nineteenth century before increasing at a fast rate in the recent decades—a *hockey-stick* pattern of housing prices and land values. Our theory thus provides novel insights on the joint evolution of the density of cities and land values along the process of economic development. It also helps understanding how the structure of cities, e.g. their urban extent and density evolves with the process of structural transformation. It sheds new light on the origins of urban sprawl in the process of economic development—a central matter in the artificialization of soils and their environmental impact (IPCC (2018)).

The contribution of our paper is threefold. First, we document new stylized facts on land use and urban expansion for France since the mid-nineteenth century. In particular, using historical maps and satellite data for the more recent period, we document the historical decline of the density of French cities. Between 1870 and 1950, the average density was divided by about 3 and again by about 2.5 until 1975—the thirty years post-World War II being characterized in France by a faster structural change and *rural exodus* (Mendras (1970), Bairoch (1989), Toutain (1993)). Together with the slowdown of structural change in the more recent decades, average urban density did not fall much since. These facts, together with the historical evolution of urban and agricultural land values in France, motivate our theory.

The second contribution is to develop a spatial general equilibrium model of structural change with endogenous land use—agricultural or residential land use. The production side features three sectors: rural, urban and housing. The rural (resp. urban) sector produces agricultural (resp. non-agricultural) tradable goods, the production of the rural good being more land intensive. The housing sector produces location-specific housing units using the urban good and land in the process. Land is in fixed supply and land use rivalrous: land is either used for agriculture or for housing. Following the traditional monocentric model after Alonso et al. (1964), Muth (1969), and Mills (1967), urban land use (cities) emerges endogenously due to commuting costs for workers to produce urban goods: urban land is thus more densely populated than rural land and the urban fringe corresponds to the longest commute of a worker producing urban goods. Importantly, the rental value of land at the fringe of the city must be equalized across potential usages—the marginal productivity of land in the rural sector (agriculture) determining the opportunity cost of expanding further urban land. The last important components of our theory are the drivers of structural change. Structural change is driven by the combination of non-homothetic preferences on the demand side, particularly a subsistence consumption for the rural good, and increasing (agricultural) productivity on the supply side. This generates transitory dynamics with rising productivity in agriculture that are at the heart of our story: in the old times, due to low agricultural productivity, land is scarce with high values of farmland with respect to income. Moreover, households devote a large fraction of their resources to feed themselves and cannot afford large homes. Few urban workers are concentrated on a very small area and urban land is very densely populated. Later on, with agricultural development, farmland is getting less valuable. This frees up rural land for cities to expand, accommodating rising demand for housing of more numerous urban workers. The city sprawls and average urban density falls through two channels: the fall in the rental value of farmland at the urban fringe and the increasing share of spending towards housing. Note that the decline in urban density occurs even without improvements in the commuting technology—the usual source of sprawling in urban economics. At the latest stages of the transition, in more recent times, the reallocation of workers and land use slows down. Urban expansion slows, urban density declines less and land prices increase at faster rate. As a side-product, we also show how commuting frictions together with location-specific land values generate a wedge between the workers marginal productivities in the rural and urban sector, an 'agricultural productivity gap' (Gollin et al. (2014)).

The other natural candidate to account for urban sprawl over time is the development of faster urban commutes, which made urban households live further away from work. Building upon LeRoy and Sonstelie (1983) and DeSalvo and Huq (1996), we incorporate into our theory a commuting mode choice model, which allows for an endogenous decision of individuals of how to commute, based on their opportunity cost of time and location. More specifically, as the opportunity cost of time in the city increases with rising urban productivity, workers optimally choose faster commuting modes and live further away from the center: the city expands at the expense of rural land.[1] Thus, although the mechanisms are entirely different, both urban *and* rural productivity growth lead to sprawling and suburbanization together with a decline in average urban density. However, the implications for density across urban locations are different. Increasing urban productivity and faster commutes lead to a reallocation of urban workers away from the center towards the city fringe. As a consequence, central density falls more than average urban density since suburban density increases. To the contrary, increasing agricultural productivity and structural change lead to the addition of lower and lower density settlements at the fringe of cities: suburban density falls more than the average urban density. While central density did fall since the mid-nineteenth century, historical data for Paris shows that it fell less than the average urban density. This suggests that both channels—the structural change and the commuting speed channels—have been playing a role in driving the density decline.

Regarding land rents, we also show that agricultural productivity growth and structural change are crucial to understand their evolution. If land reallocation away from agriculture towards urban use was only driven by urban productivity growth and faster commutes, rural land would be getting scarcer and more valuable: the value of farmland and agricultural land rents (as a share of income)

[1]In our theory, commuting costs (as a share of income) falls endogenously as individuals choose faster commuting modes when urban income increases. Results are qualitatively similar is one assumes an exogenous fall in commmuting costs.

would increase. Agricultural land rents would also become relatively more important than urban ones – predictions that are widely counterfactual to the evidence in Piketty and Zucman (2014). Quite differently, structural change driven by increasing rural productivity frees up farmland, lowering its value relative to income and reducing the importance of agricultural land rents to the profit of urban ones. These predictions are much more in line with the data.

In a third contribution, we develop a quantitative version of our spatial equilibrium model applied to the French context since 1840. Using data from various historical sources, we measure sectoral factors of production and productivities over long period and calibrate our model to fit the process of structural change in France. We show that the quantitative predictions of the model match relatively well the joint evolution of population density and land values over time and across space. We also disentangle the relative importance of falling commuting costs relative to our novel mechanisms based on structural change in explaining the persistent decline in urban density—emphasizing further the quantitative importance of improvements in agricultural productivity for the expansion of cities.

Related literature. The paper relates to several strands of literature in macroeconomics and spatial economics. From a macro perspective, it relates to the literature linking productivity changes and land values, starting with Ricardo (1817). This traditional view would imply that a fixed factor such as land should continuously rise in value with economic development (see, among others, Nichols (1970) and Grossman and Steger (2017) for a recent contribution). However, such a prediction would not fit well the measurement of housing prices and land values over a long period as in Piketty and Zucman (2014) and Knoll et al. (2017) (see also Davis and Heathcote (2007) for related U.S. evidence). An alternative view developed in Miles and Sefton (2020) argues that the rise in land and housing prices can be mitigated by improvements in commuting technologies, which allow cities to expand outwards. Our approach, in the tradition of the theory of structural change, also argues that land used to be scarce and valuable with low productivity in agriculture but rising productivity alleviates pressure on land—putting downward pressure on its value. In a sense, our theory reconciles these different views in a unified framework. From a theoretical perspective, we contribute to the literature on structural change, surveyed in Herrendorf et al. (2014), by considering a spatial dimension—adding an endogenous use of land and a housing sector—in the most conventional multi-sector model with non-homothetic preferences (Kongsamut et al. (2001), Gollin et al. (2007), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2015)). Structural change and urbanization are known to be tightly linked (Lewis (1954)). Gollin et al. (2016) shows that not only economic development but also natural resources rents lead to urbanization. However, the literature has rarely investigated the spatial dimension of structural change, largely abstracting from spatial frictions. Michaels et al. (2012) and Eckert et al. (2018) are notable exceptions. The crucial difference is the ability of our framework to replicate the evolution of population density within locations—putting emphasis on the internal structure and density of cities—, while their focus is more on the distribution of population and the sectoral specialization across regions. We also emphasize the implications for land values across time and space, largely absent in these studies.

Adding a spatial dimension to a multi-sector model of structural change also generates endogenously an 'agricultural productivity gap' (Gollin et al. (2014)) due to the mere presence of commuting frictions and location-specific housing. This provides a complementary explanation to urban-rural wage gaps, different from migration costs or selection of migrants towards cities (Restuccia et al. (2008), Lagakos and Waugh (2013), Young (2013)).

Our paper also contributes to the literature in spatial economics on urban expansion surveyed in Duranton and Puga (2014, 2015). An important feature of our framework is the existence of preferential residential locations within cities, shaping the population density across space, due to the presence of commuting frictions (Alonso et al. (1964); Muth (1969); Mills (1967)). We expand this literature by bringing the endogenous sectoral allocation of factors and the general equilibrium structure at the heart of the macro literature. Importantly, contrary to the bare bone urban monocentric model, land is in fixed supply and the price of land at the boundary of the city becomes an endogenous object itself affected by the process of structural change. The most related work to our approach developed in Brueckner (1990) shows how location-specific land values pin down rural-urban migrations and the extent of urbanization in a spatial equilibrium (see also Brueckner and Lall (2015) for a survey). However, without the drivers of structural change and endogenous land values at the urban fringe as in our framework, this approach stays relatively silent regarding the long-run dynamics of urbanization and land values. In this latter dimension, our work relates to the literature measuring and explaining land values across space (see Glaeser et al. (2005), Albouy (2016), Albouy et al. (2018) and Combes et al. (2018) for recent contributions). In particular, we show that the dispersion of land values across space and the scarcity of land in some locations depend very much on the extent of economic development and structural change. Our approach also provides an alternative mechanism generating a large sprawling of cities together with economic development. More specifically, it explains, why, over time, most cities expand faster in area than in population as documented on a global scale by Angel et al. (2010). In the French context, we also relate to the historical measurement of urban land use in Combes et al. (2021). Our story is complementary to the usual explanations based on the improvement of commuting technologies and/or the relocation of economic activity within cities (see references in Glaeser and Kahn (2004) and Heblich et al. (2018), Redding (2021) for recent contributions). Lastly, our paper contributes to the literature on quantitative spatial economics surveyed in Redding and Rossi-Hansberg (2017) (see also Ahlfeldt et al. (2015)) by emphasizing the extensive margin of cities.

The paper is organized as follows. Section 2 provides motivating empirical evidence on land use, land values, urban expansion and population density across space over long period in France. Section 3 provides a baseline spatial general equilibrium model of land use and structural change which enlightens the main mechanisms. Section 4 develops a quantitative version calibrated to French historical data. Section 5 concludes.

2 Historical Evidence from France

2.1 Land use and Employment in Agriculture.

Data. Using various sources described in Appendix A, we assemble aggregate data on employment shares in agriculture and agricultural land use since 1840.[2] Historical data on land use in agriculture are available roughly every 30 years (or less) until the 1980s and then at higher frequency. They are largely extracted from secondary sources based on the Agricultural Census (Recensement Agricole), and cross-checked with various alternative historical sources (Toutain (1993) among others). Post-1950, data are from the Ministry of Agriculture.

Employment. As all countries going through structural transformation, France exhibits a large reallocation of labor away from agriculture over the period, from about 60% employed in agriculture in 1840 to about 2.5% today (Figure 1, dotted line).[3] The process of structural change accelerated significantly over the period 1945-1975: in 1945, 36% of the working population are still in agriculture and this number falls below 10% in 1975. In this sense, France is a bit peculiar relative to the other advanced economies: it is still a very agrarian economy right after World War II—much more than the U.K. or the U.S..

Land use. Although measurement is sometimes difficult for the very early periods, one can confidently argue that, in the aggregate, the share of French land used for agriculture significantly fell since 1840 (Figure 1, solid line).[4] Our preferred estimates are that about two thirds of French land was used for agriculture in 1840. In 2015, this number is down to 52%. In other words, about 15% of French land use has been reallocated away from agriculture. While this might not seem like a very large number, this is very large from the perspective of urban expansion. 15% of the French territory is actually more than the total amount of land with artificial use in France nowadays (about 9% of total land today).[5] While this is difficult to assess over such a long period, the novel usage of the land formerly used in agriculture, it is likely that a significant fraction of this land has been artificialized—allowing cities to expand. More precise data on land use over the period 1982-2015 show that the surface of artificialized soils increased by about 2 millions of hectares (3.7% of the French territory), about 70% of the quantity of the land no longer used for agriculture over the same period.[6] The measurement of cities area (presented below) provides further compelling evidence that a significant fraction of agricultural land was reallocated towards urban land use.

[2] Data for employment shares are available since 1806. Regional data (21 regions) are also available since the mid-nineteenth century for all metropolitan French regions but Corsica.

[3] Estimates of rural population are also available for the same time-period (see Appendix A). Rural population follows a similar path with, as expected, higher levels as many people in rural areas do not work directly in agriculture. One needs to be cautious though when using data on rural vs. urban population as the (ad-hoc) definition by official statistics varies over the period.

[4] The main issue is the definition of agricultural land (in particular, the allocation of grazing fields) which is not entirely consistent across years before World War II. See Appendix A for details.

[5] Since 1982, data on land use beyond agricultural land use are available on a regular basis from the Enquetes Teruti and Teruti-Lucas.

[6] The rest of agricultural land is to a large extent converted into forests and woods (Enquetes Teruti and Teruti-Lucas). Their surface, including groves and hedges, increased by almost 1 million of hectares between 1982 and 2015.

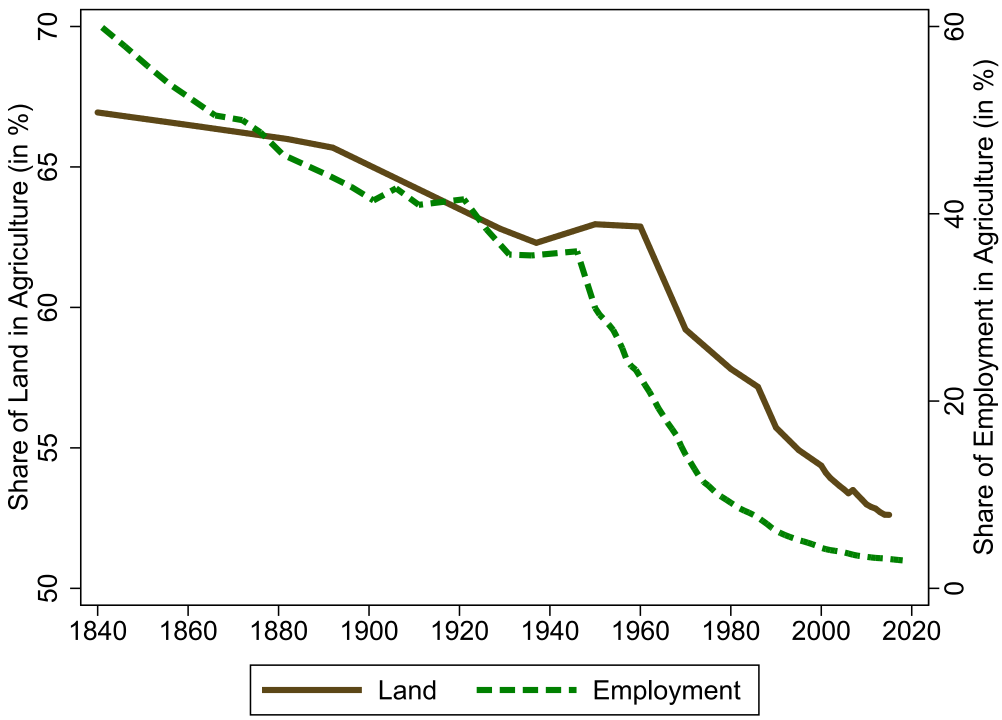


Figure 1: Land use and labor reallocation in France (1840-2015).
Notes: The solid line shows the share of French land used for agriculture (left-axis). The dotted line shows the share of workers in the agricultural sector (right-axis). *Source*: See Appendix A.

2.2 Urban Expansion

Data. We use maps, aerial photographs and satellite data to measure the area of the main French cities at different dates: 1866 (military maps, e.g. carte d'Etat Major), 1950 (maps and/or photographs), and every ten to fifteen years after 1975 using satellite data from GHSL.[7] One caveat of our area measurement is that we cannot have any measurement between 1866 and 1950. Data on the measurement of the urban extent across French cities are detailed in Appendix A. Measurement of the urban extent using maps in 1866 and 1950 is performed for the 100 largest cities in population in the initial period. For a given city, the urban extent ends when the land is not continuously built. For the satellite data, it is delimited by grid cells where the fraction built is below 25%.[8] As an example, Figures 25 and 26 in Appendix A show the area measurement for a medium-size French city, Reims, in 1866 and 1950 using maps. Figure 27 shows the same city of Reims in 2015 viewed from the sky, with an area of about 50 km^2—about 20 times larger than its 1866 counterpart. This last Figure also clearly shows how the city is surrounded by agricultural land—a crucial element

[7]We also double-check the quality of photo/map measurement in the most recent period relative to satellite data measurement. The cross-sectional correlation between measurement using photos and satellite data measurement is very high.

[8]Measurement is not very sensitive to alternative higher thresholds. See Appendix A for sensitivity analysis. Figures 28 in the same Appendix shows how GHSL satellite data are used to delineate the urban boundaries of Paris.

for our story where urban land expands out of farmland. This feature is not specific to Reims. Recent satellite observations (Corine Land Cover) of these 100 main French cities show that they are largely surrounded by agricultural land: apart from their coastal part and water bodies, two thirds of land use in the near surroundings of cities is agricultural.[9]

Using French Census data, we relate the measured land area used by cities to the corresponding population. Data for the first available Census in 1876 are used for the initial period of study. Census data defines population at the municipality level ('commune') and an urban area can incorporate more than one municipality. In 1870, this is not much of an issue as the main 'commune' of the city is the whole city population. In the later periods, one needs to group municipalities ('communes') into an urban area. Post 1975, GHSL data combines satellite images with the Census data on population. This directly provides the population of every grid cells of our measured urban area, circumventing the issue. However, for the 1950 period, the different municipalities that are part of our measured areas must be selected. This is done municipalities by municipalities looking at the map of each of the 100 largest urban areas. This way, we make sure that the overall population of the area incorporates all the corresponding municipalities' population.[10]

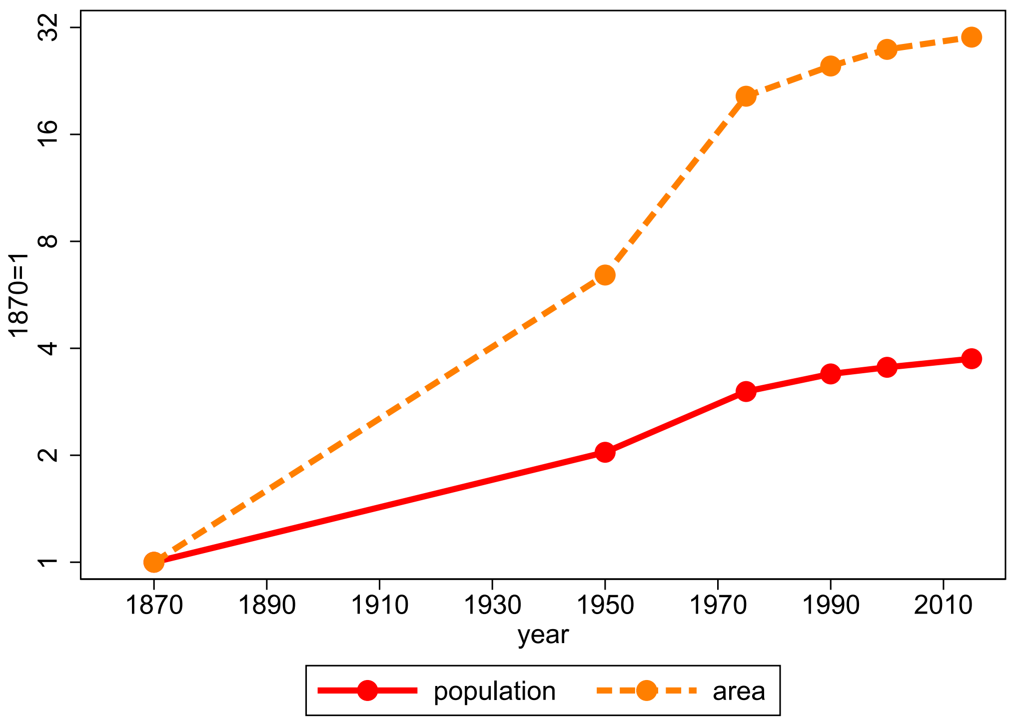


Figure 2: Urban area and population of the 100 largest cities in France (1870-2015).

Notes: The upper dotted line shows the total urban area of the 100 cities relative to the intial period (sum of all the urban areas) . The bottom solid line shows the total population relative to the initial period in the same cities. Both area and population are normalized to unity in the initial period. *Source*: See Appendix A.

[9]The rest is made of forest/moors and discontinuous urban land (e.g. leisure/transport infrastructure, industrial/commercial sites, ...)—both categories in roughly equal proportions. See details in Appendix A.

[10]For most cities in 1950, only very few 'communes' are agglomerated into one city. Only the largest cities, and particularly Paris, are the results of the agglomeration of many different 'communes'.

The area and population of French cities. Not surprisingly, more populated cities are larger in area. In the cross-section, at a given date, a 10% increase in the population of a city corresponds to a 8.5% increase in its area and this elasticity varies little across the different time periods.[11] Thus, in the cross-section, the urban area increases less than proportionately with urban population—larger cities being denser on average. This stands in contrast with their evolution in the time-series. Over time, cities have been increasing much faster in area than in population. Let us give some order of magnitude and describe the average evolution over time for the most populated 100 French cities in 1876. Figure 2 shows the evolution of the total area and population of these 100 cities over the period considered—both variables being normalized to 1 to show the increase in size. Since 1870, the area of cities has been multiplied by a factor close to 30 on average. This is very large. Between 1870 and 1950, the area of cities was roughly multiply by a factor 6. Between 1950 and today, the area of cities was multiplied again by a factor 5 on average—the fastest rate of increase being observed over the period 1950-1975. For comparison, the population of these largest cities has been multiplied by a factor close to 4 since 1870.[12] As urban area increased at a much faster rate than urban population, the average urban density significantly declined over the period.

The density of French cities. Using the population and the area of cities at the different dates, one can measure the evolution of urban densities across the different cities over 150 years. While in the cross-section larger cities are denser, the density of French cities declined over time—area expanding at a faster rate than population. This is shown in Figure 3 for the population-weighted average of density across the 100 largest French cities (and in Appendix A for the 3 largest French cities: Paris, Lyon and Marseille). The average urban density fell massively over the period: density has been divided by a factor of the order of magnitude of 8. Urban density fell at the fastest rate over the period 1950-1975 and barely falls thereafter. Thus, urban density fell the most over the period when people massively left rural areas and the employment share in agriculture also fell the most. The later slowdown of the decline in density coincides with the slowdown in the rate of structural transformation.[13]

Ideally, one would like to explore how density evolved in different locations of a city (within-city variations). This would provide information on whether density fell in the central locations or in the outskirts of the city. Unfortunately, for most cities we are not able to differentiate the central density to the suburban one as most cities expand the area of their main historical 'commune', particularly so over the period 1860-1950. Thus, we cannot measure the historical population in different parts of a city. However, this can be done for Paris which is divided into districts. Figure 29 shows the evolution of the density of Central Paris relative to the average urban density of the metropolitan area: the central density of Paris did fall over time but significantly less than the average density of the city. This is suggestive that the decline in average urban density is not only

[11] See Appendix A for scatter plots of log area on log population at various dates.

[12] French population was multiplied by a bit less than 2 over the entire period. Due to the reallocation of people way from rural areas towards cities, we get roughly a factor 4 over the period.

[13] The historical decline in urban density is observed across all cities although the magnitude differs across cities: for instance, in Lille, urban density fell from 67,300 to about 4,200 people/km^2—divided by a factor 15 between 1866 and 2015; over the same period, urban density was divided by les than 4 in Nancy, from 13,400 to 3,500 people/km^2.

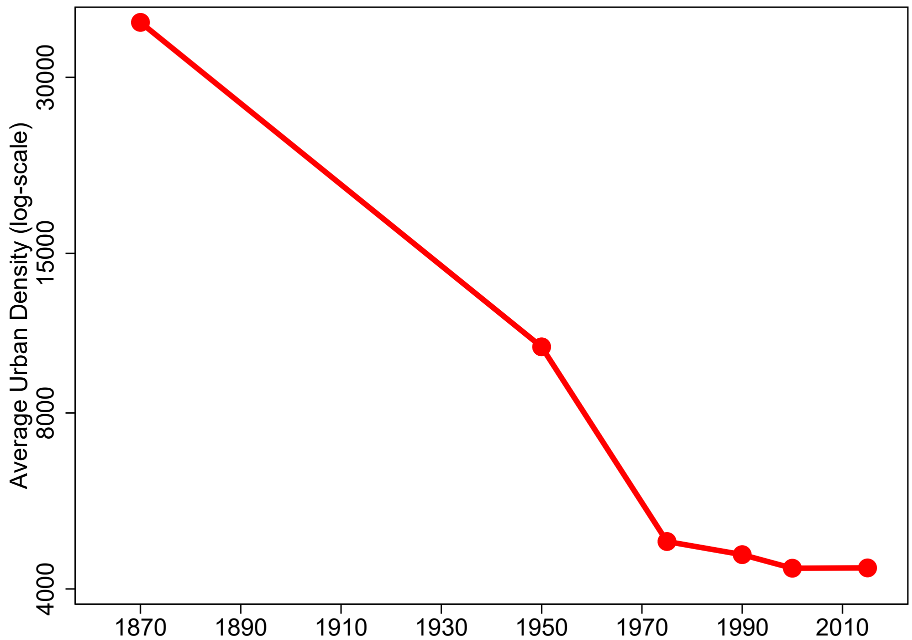


Figure 3: The historical decline in urban density.

Notes: The solid line shows the urban density averaged across the top 100 French cities (weighted average with 1975 population weights). *Source*: Etat major, IGN, GHSL and Census.

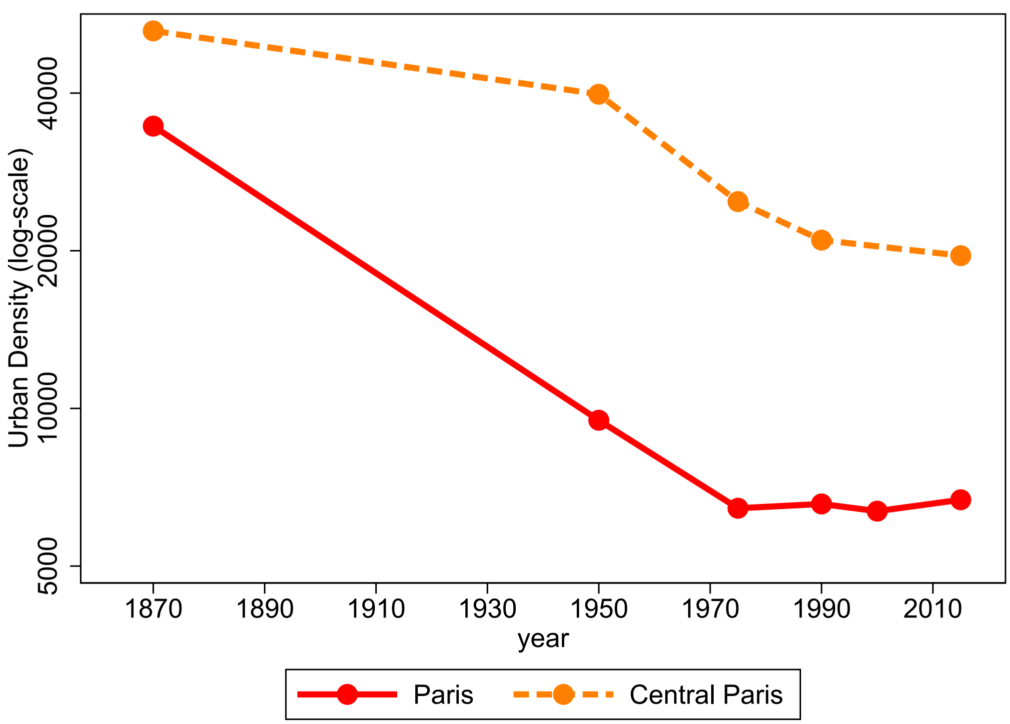


Figure 4: The historical decline in urban and central density in Paris.

Notes: The solid line shows the average urban density in Paris; the dotted line shows the density in Central Paris (districts 1 to 6). *Source*: Etat major, IGN, GHSL and Census.

due to a reallocation of urban residents away from dense centers but also due to the addition of less and less dense suburban areas over time.

2.3 Land values

Data. Data on land and housing values (over income) for France over a long period can be found in Piketty and Zucman (2014). Using various data sources detailed in Appendix A, we also computed a measure of farmland prices per unit of land. Historical data for the real housing price index for France are provided in Knoll et al. (2017).

Historical evolution. Figure 5 shows the evolution of tha aggregate value of French land over income since 1820. The fall in the value of housing and land wealth (as a share of income) in the pre-World War II period is essentially driven by a declining value of farmland. While farmland was expensive relative to income in the nineteenth century, it is today relatively cheap. This is confirmed by data on average farmland prices: since 1850, the average value of an agricultural field (per ha) as a share of per capita income has been divided by a factor 15 in France. This fact is at the heart of our story: structural change puts downward pressure on farmland values—allowing cities to expand at a fast rate. As a consequence, there is an important reallocation of land values across usage, from agricultural land towards housing (or urban) land. While the value of agricultural land accounted for more than 70% of housing and land wealth in 1820, it accounts for only 3% in 2010. Lastly, despite the falling value of farmland as share of income, the value of land wealth (as share of income) grows at an increasing rate after 1950.

This steep increase, arguably driven by the increasing value of urban land where most of the population is concentrated, echoes the findings of Knoll et al. (2017).[14] They show that for developed countries, including France, housing prices have been quite stable until the 1950s before rising at an increasing pace—a *hockey-stick* shape of housing prices as shown in Figure 6.

To sum-up, our historical data shows a set of salient facts over the last 180 years: beyond the well-known reallocation of labor away from agriculture, land has been reallocated away from agricultural use. Migrations away from the rural areas were accompanied with urban expansion both in area and population. However, as the urban area grew at a significantly faster pace than urban population, the average urban density massively declined over the period, particularly so in the decades following World War II in France. Together with this process of structural change, the value of farmland as a share of income shrank a lot to the benefits of non-agricultural (urban) land.

These stylized facts motivate our subsequent theoretical analysis where we introduce a spatial dimension together with endogenous land use to the most standard theory of structural change with non-homothetic preferences.

[14] Bonnet et al. (2019) show that this increase in the price of housing is largely driven by the price of land and not by the capital and structure component.

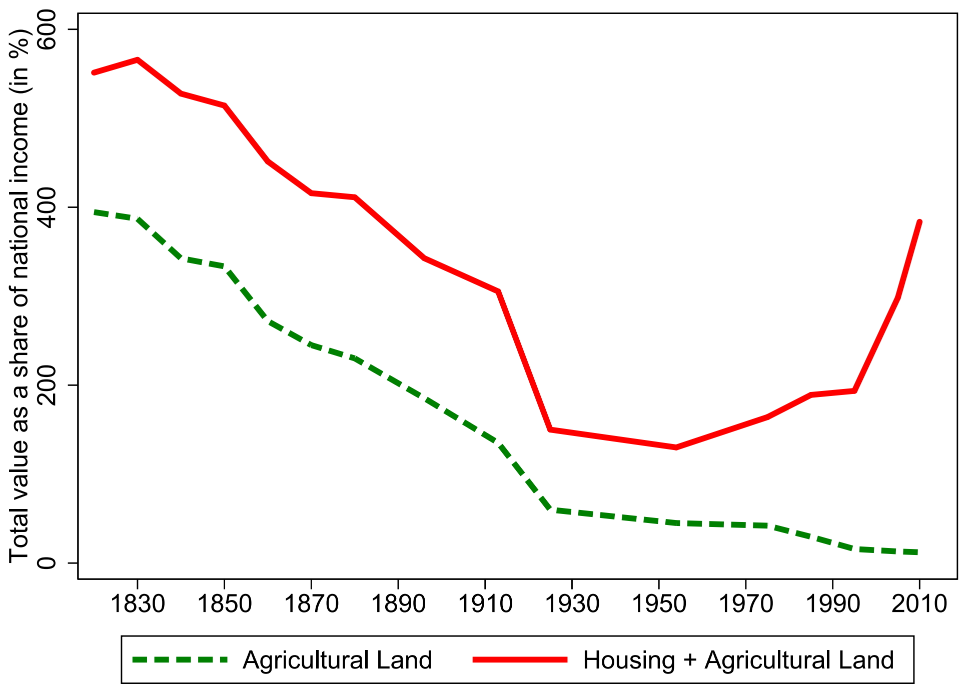


Figure 5: Agricultural Land and Housing Wealth (1820-2010)

Source: Piketty and Zucman (2014).

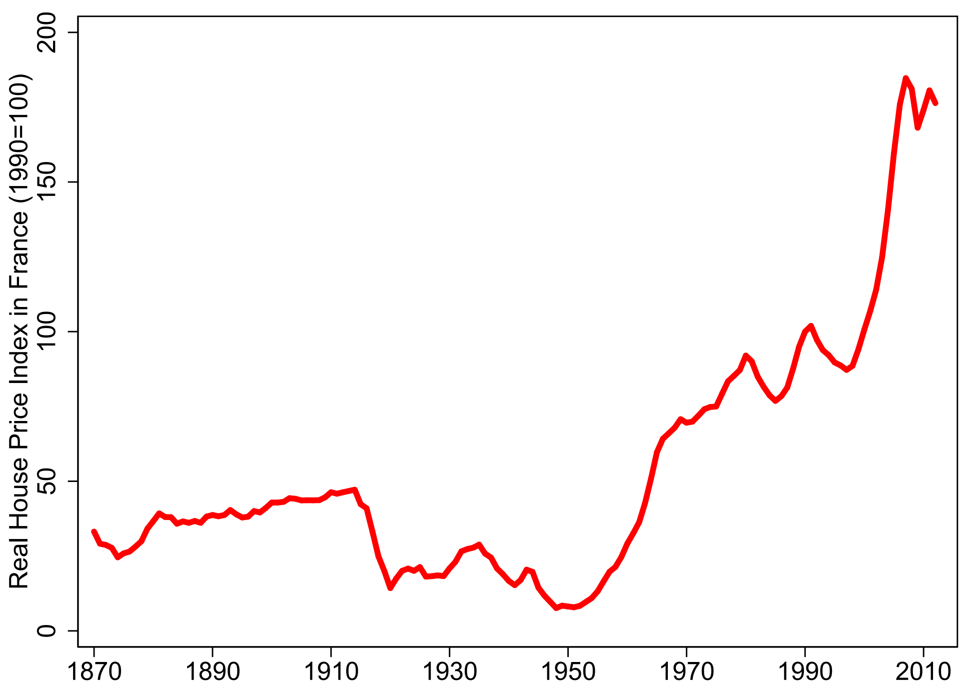


Figure 6: Real Housing Price Index in France (1870-2010).

Source: Knoll et al. (2017).

3 A Baseline Model

3.1 Production

We consider an economy producing a urban good (u) and a rural good (r). The urban good can be thought as a composite of manufacturing good and services, while the rural good can be thought as an agricultural good. Goods and factor markets are perfectly competitive. Both goods are perfectly tradable.

Factor Endowments. The economy is endowed with land and a continuum of workers, both in fixed supply. Land can be used to produce the rural good or for residential purposes. Land area is normalized to unity. Each worker is endowed with one unit of labour and we denote by L the total population of workers.

Production and Factor Payments. The production of the urban good only uses labour as input. One unit of labour produces θ_u units of the urban good. Perfect competition insures that the urban wage is

$$w_u = \theta_u, \tag{1}$$

in terms of units of the urban good, which is used as numeraire. For now, we consider the urban productivity θ_u (and thus the urban wage) as exogenous. We consider agglomeration forces in Section 4. Aggregate production of the urban good is

$$Y_u = \theta_u L_u,$$

where L_u denotes the number of workers working in the urban sector.

The production of the rural good uses labor and land according to the following constant returns to scale technology

$$Y_r = \theta_r (L_r)^{\alpha} (S_r)^{1-\alpha},$$

where L_r denotes the number of workers working in the rural (agricultural) sector, S_r the amount of land used for production and θ_r a Hicks-neutral productivity parameter. $0 < \alpha < 1$ is the intensity of labor use in production, $1 - \alpha > 0$ ensures that land is used more intensively to produce the rural good.[15]

Define p the relative price of the rural good in terms of the numeraire urban good. Rural workers and land are paid their marginal productivities,

$$w_r = \alpha p \theta_r \left(\frac{S_r}{L_r}\right)^{1-\alpha}, \tag{2}$$

$$\rho_r = (1-\alpha) p \theta_r \left(\frac{L_r}{S_r}\right)^{\alpha}, \tag{3}$$

where w_r is the rural wage and ρ_r the rental price of land anywhere in the rural sector.

[15] The quantitative section 4 explores a more general CES production function for the rural good.

Remarks. The important technology assumption is that the rural sector uses a fixed factor, land, for production, which implies (stronger) decreasing returns to scale to labor in this sector compared to the urban sector. The fact that the urban sector does not use land is not crucial as long as this sector is less land intensive than the rural one.

3.2 Spatial Structure and Commuting Costs

Spatial structure. The production of the urban good takes place in the city, while the production of the rural good, being more land intensive, takes place in the rural area. For now, we assume that production of the urban good takes place in only one location $\ell = 0$. Extension with multiple locations (multiple cities) is provided in Section 4.

One can think of $\ell = 0$ as the Central Business District (CBD) in a standard urban model. Workers' locations of residence ℓ are ordered from zero to unity depending on the magnitude of the spatial frictions $\tau(\ell)$, that workers have to pay to work in the urban sector. A worker residing in location ℓ and working in the urban sector earns wage *net of spatial frictions* equal to $w(\ell) = w_u - \tau(\ell)$, with $\tau(0) = 0$ and $\partial\tau(\ell)/\partial\ell \geq 0$. The commuting cost $\tau(\ell)$ incorporates all spatial frictions which lowers disposable income available for consumption when living further away from the location of production. It includes time-costs of commuting and the effective spending on transportation. It could also incorporate an income reduction if it is harder to find a job when living further away from the location of production. The commuting cost is partly endogenous in our framework as urban households adjust their mode of commuting depending on their income and their location, as described in further details below.

Since spatial frictions increase with ℓ, urban workers locate as close as possible to $\ell = 0$. If one denotes $\ell = \phi < 1$ the further away location of a urban worker, ϕ is endogenous in our framework and represents the fringe of the city. Workers residing in locations above ϕ produce the rural good, which does not involve spatial frictions.

Remarks. The spatial structure calls for a number of important remarks. First, if it were possible for all workers to locate at $\ell = 0$, it would save the spatial frictions. Second, one should note that for $\ell \leq \phi$, land will be used for residential purposes to host urban workers. As a consequence, land available for rural production would also be maximized if all workers could locate at $\ell = 0$. This case could correspond to an entirely 'vertical' city, where land use and spatial frictions are irrelevant. We view this extreme case as a standard two-sector model of structural transformation. Last, the spatial frictions $\tau(\ell)$ do not involve traffic congestion—the reason why a more compact city (lower ϕ) always saves on commuting costs in our baseline economy. The case of traffic congestion is explored in Section 4.

Commuting costs. We provide a micro-foundation for the commuting costs, $\tau(\ell)$, where urban workers choose a commuting mode m depending on their location ℓ and opportunity cost of time (wage rate w_u). This modelling approach helps mapping commuting costs into observables from commuting data but results do not depend qualitatively on the micro-foundation as long as commuting costs are increasing in the opportunity cost of time and commuting distance.

Commuting costs in location ℓ, $\tau(\ell)$, are the sum of spending on commuting using transport mode m, $f(m)$, and time-costs proportional to $w_u \cdot t(\ell)$, where $t(\ell)$ denotes the time spent on daily commutes of an individual located in ℓ, such that

$$\tau(\ell) = f(m) + \zeta w_u \cdot t(\ell), \tag{4}$$

whereby $0 < \zeta \leq 1$ represents the valuation of commuting time in terms of foregone wages. Transportation modes m available are optimally chosen. They are continuously ordered by their speed, as in DeSalvo and Huq (1996), such that m denotes both the mode and the speed of commute. Faster commutes are more expensive and $f(m)$ is increasing in m. For tractability, we use the following functional form, $f(m) = \frac{c_\tau}{\eta_m} m^{\eta_m}$, with $\eta_m > 0$ and c_τ a cost parameter measuring the efficiency of the commuting technology. With speed m, the commuting time (both ways) is equal to $\frac{2\ell}{m}$. This yields the following expression for the commuting costs,

$$\tau(\ell) = \frac{c_\tau}{\eta_m} m^{\eta_m} + 2\zeta w_u \left(\frac{\ell}{m}\right). \tag{5}$$

This expression of the commuting costs facilitates parametrization and preserves some tractability, while elucidating the main mechanisms.[16] We turn to the optimal choice of transportation mode.

Optimal mode of transportation. At any given moment in time, prevailing technology offers different transportation modes ordered by their respective speed m. An individual in location ℓ chooses the mode of transportation corresponding to speed m in order to minimize the commuting costs $\tau(\ell)$. By equalizing the marginal cost of a higher speed m to its marginal benefits in terms foregone wage, the optimal chosen mode/speed satisfies,

$$m = \left(\frac{2\zeta w_u}{c_\tau}\right)^{1-\xi} \cdot \ell^{1-\xi}, \tag{6}$$

where $\xi \equiv \frac{\eta_m}{1+\eta_m} \in (0, 1]$. Individuals living further away choose faster commuting modes. The speed of commuting also increases with the wage rate as a higher wage increases the opportunity cost of time. Using Eqs. 5-6, we get that equilibrium commuting costs satisfy,

$$\tau(\ell) = a \cdot (w_u \ell)^\xi, \tag{7}$$

[16]The cost $f(m)$ has several possible interpretations. At a more macro level, it can represent the fixed cost of installing public transportation, where a faster mode is more expensive (a train line versus the horse drawn omnibus). At a more individual level, it represents the cost of buying an individual mean of transportation—a bike being cheaper than an automobile. However, this reduced-form approach sets aside the possibility that the implemented commuting technologies and the effective speed of commuting depends in a more sophisticated way on the equilibrium allocation in the city (e.g. traffic congestion or the construction of transportation infrastructures may depend on the whole spatial allocation of urban residents). The quantitative section 4 uses a more general function for the spending on commuting f, also increasing in the commuting distance ℓ and urban wages w_u: $f = f(m, \ell, w_u)$. Longer commutes are more expensive and higher urban labour costs also increase commuting costs.

where $a \equiv \left(\frac{1+\eta_m}{\eta_m}\right) c_\tau^{\frac{1}{1+\eta_m}} (2\zeta)^{\frac{\eta_m}{1+\eta_m}} > 0$. Commuting costs are falling with improvements in the commuting technology (a lower a).[17] They are increasing with the wage rate (the opportunity cost of time) and the distance of commuting trips with constant elasticities. Since individuals optimally choose the commuting speed, the elasticity ξ of commuting cost to the wage rate is strictly smaller than unity. This is important as it implies that, for a given residential location, the share of resources devoted to commuting falls with rising urban productivity and wages. In equilibrium, this tends to make individuals willing to live further away when productivity increases in order to enjoy larger homes. Lastly, our derivation of commuting costs enlightens the calibration as the elasticity of commuting costs to income (resp. commuting distance) is directly tied to the elasticity of commuting speed to income (resp. commuting distance), which have data counterparts (Eq. 6).

3.3 Preferences and Consumption

Preferences. Preferences over urban urban and rural goods are non-homothetic as in Kongsamut et al. (2001) and Herrendorf et al. (2013) among others. Consider a worker living in a location ℓ. Denote $c_r(\ell)$ the consumption of rural (agricultural) goods, $c_u(\ell)$ the consumption of urban goods (used a a numeraire) and $h(\ell)$ the consumption of housing. The composite consumption good is

$$C(\ell) = (c_r(\ell) - \underline{c})^{\nu(1-\gamma)} \left(c_u(\ell) + \underline{s}\right)^{(1-\nu)(1-\gamma)} h(\ell)^\gamma \tag{8}$$

where $\underline{c}$ denotes the minimum consumption level for the agricultural good, and where $\underline{s}$ stands for the initial endowment of the urban good. Preference parameters ν and γ belong to $(0, 1)$. Workers derive utility only from consumption. The utility of a household in location ℓ is thus equivalent to $C(\ell)$.

Budget contraint. The household earns a wage income net of spatial frictions $w(\ell)$ in location ℓ. Given the spatial structure, $w(\ell) = w_u - \tau(\ell)$ for $\ell \le \phi$ and $w(\ell) = w_r$ for $\ell > \phi$. The households also earns land rents, r. Land rents are redistributed lump-sum equally across workers and are thus assumed to independent on the location. The budget constraint of the worker in location ℓ satisfies

$$pc_r(\ell) + c_u(\ell) + q(\ell)h(\ell) = w(\ell) + r, \tag{9}$$

with $q(\ell)$ the rental price per unit of housing in location ℓ.

Expenditures. Maximizing Eq. (8) subject to the budget constraint Eq. (9), expenditures on each good satisfy

$$pc_r(\ell) = (1-\gamma)\nu(w(\ell) + r + \underline{s} - p\underline{c}) + p\underline{c} \tag{10}$$

$$c_u(\ell) = (1-\gamma)(1-\nu)(w(\ell) + r + \underline{s} - p\underline{c}) - \underline{s} \tag{11}$$

$$q(\ell)h(\ell) = \gamma(w(\ell) + r + \underline{s} - p\underline{c}). \tag{12}$$

[17] a is alike a relative price of commuting: if technology improves relatively faster in the commuting sector, the relative price a of commuting (in terms of urban goods) falls.

Due to the presence of subsistence needs ($\underline{c} > 0$), individuals reallocate consumption away from the rural good with rising income, increasing the consumption share of the urban good and housing. The reallocation of demand towards the urban good is stronger when $\underline{s} > 0$.

3.4 Equilibrium Sorting

Mobility equations. We consider an equilibrium, where ex-ante identical workers sort across locations. Since the rural and the urban good are perfectly tradable, urban workers, which would all prefer locations closer to $\ell = 0$, compete for these locations. Adjustment of housing prices through the price of land, make sure that households remain indifferent across different locations. Using Eqs. (10)-(12), this implies the following mobility equation, where consumption is equalized to $\overline{C}$ across locations ℓ,

$$\overline{C} = C(\ell) = \kappa \frac{w(\ell) + r + \underline{s} - p\underline{c}}{q(\ell)^{\gamma}}, \tag{13}$$

with κ constant across locations, equal to $((1-\gamma)\nu)^{(1-\gamma)\nu} ((1-\gamma)(1-\nu))^{(1-\gamma)(1-\nu)} \gamma^{\gamma}/p^{\nu(1-\gamma)}$. The mobility Eq. (13) implies that $\left(\frac{w(\ell)+r+\underline{s}-p\underline{c}}{q(\ell)^{\gamma}}\right)$ is constant across locations. This holds within urban locations ($\ell \leq \phi$), within (identical) rural locations as well as when comparing a urban and rural worker. Since workers in the rural sector do not face spatial frictions and live in ex-post identical locations, $\ell \geq \phi$, the price of housing must be the same across these locations. We denote by q_r the price of housing in the rural sector, where $q_r = q(\ell \geq \phi)$. A worker in the rural sector is paid his marginal productivity w_r, receives land rents r and faces the same housing rental price $q_r = q(\phi)$ than a urban worker at the fringe. Therefore we have

$$w(\phi) = w_r = w_u - \tau(\phi). \tag{14}$$

In other words, the urban worker at the fringe of the city must have the same wage net of commuting frictions than a rural worker. Eq. (14) shows how the spatial structure matters to understand the urban-rural wage gap. Higher spatial frictions at the fringe ϕ reduce incentives of rural households to move to the urban sector.

Housing Rental Price Gradient. Within city locations ($\ell \leq \phi$), the rental price of one unit of housing adjusts such that workers are indifferent across locations. Using Eqs. (13) and (14), we get

$$q(\ell) = q_r \left(\frac{w(\ell) + r + \underline{s} - p\underline{c}}{w(\phi) + r + \underline{s} - p\underline{c}} \right)^{1/\gamma} = q_r \left(\frac{w(\ell) + r + \underline{s} - p\underline{c}}{w_r + r + \underline{s} - p\underline{c}} \right)^{1/\gamma}. \tag{15}$$

Within the city, $q(\ell)$ is falling with ℓ to compensate workers who live in worse locations. For ℓ above ϕ, the housing price is constant across locations and equal to q_r. A crucial difference compared to the standard urban model is that the price at the fringe q_r is endogenously determined in our general equilibrium model.

3.5 Housing Market Equilibrium

Housing Demand. Using Eq. (15), the demand for housing space per worker in each location $h(\ell)$ is increasing with ℓ for $\ell \leq \phi$,

$$h(\ell) = \gamma \left(\frac{w(\ell) + r + \underline{s} - p\underline{c}}{q(\ell)} \right) = \left(\frac{\gamma}{q_r} \right) (w(\phi) + r + \underline{s} - p\underline{c})^{1/\gamma} (w(\ell) + r + \underline{s} - p\underline{c})^{1-1/\gamma}. \quad (16)$$

Facing higher housing prices, household closer to the CBD demand less housing space. For locations in the rural area, housing demand per rural worker is constant equal to $h(\ell \geq \phi) = \gamma \left(\frac{w_r + r + \underline{s} - p\underline{c}}{q_r} \right)$.

Housing Supply. The supply of housing (floorspace) is provided by land developers, which can use more or less intensively the land for residential purposes. In each location ℓ, developers supply housing space $H(\ell)$ per unit of land with a convex cost, $\frac{H(\ell)^{1+1/\epsilon}}{1+1/\epsilon}$ with $\epsilon > 0$, paid in units of the numeraire.[18] Profits per unit of land of the developers are

$$\pi(\ell) = q(\ell) H(\ell) - \frac{H(\ell)^{1+1/\epsilon}}{1 + 1/\epsilon} - \rho(\ell),$$

where $\rho(\ell)$ is the rental price of a unit of land in location ℓ. Similarly to the housing price $q(\ell)$ above, for locations beyond the fringe ϕ, the land rent is constant, hence $\rho_r = \rho(\ell \geq \phi)$.

Maximizing profits gives the following supply of housing $H(\ell)$ in a given location ℓ,

$$H(l) = q(\ell)^{\epsilon}, \quad (17)$$

where the parameter ϵ is the price elasticity of housing supply. More convex costs to build intensively on a given plot of land reduces the supply response of housing to prices.[19]

Lastly, free entry imply zero profits of land developers. This pins down land prices in a given location,

$$\rho(\ell) = \frac{q(\ell) H(\ell)}{1 + \epsilon} = \frac{q(\ell)^{1+\epsilon}}{1 + \epsilon}, \quad (18)$$

Eq. (18), together with Eq. (15), implies that land prices are higher in locations closer to the city center, more so if land developers can build more intensively (higher ϵ).

Arbitrage across land usage imply that the latter land price must be in equilibrium above the marginal productivity of land for production of the rural good (Eq. (3)), where the condition holds with equality in the rural part of the economy, for $\ell \geq \phi$,

$$\rho_r = \frac{q_r^{1+\epsilon}}{1 + \epsilon} = (1 - \alpha) p \theta_r \left(\frac{L_r}{S_r} \right)^{\alpha}. \quad (19)$$

[18] The urban good is used as an intermediary input for the production of housing space. $1/\epsilon > 0$ is a cost parameter measuring the convexity of the cost function. In the quantitative section 4, we use a more general cost function. The parameter $\epsilon = \epsilon(\ell)$ can depend on the location.

[19] Some equivalent formulation holds for a Cobb-Douglas production function of housing used for example in Combes et al. (2018).

This last equation shows that a fall in the relative price of rural goods and/or a reallocation of workers away from the rural sector lowers the price of urban land at the fringe of cities.

Housing Market Clearing. Consider first locations within the city, $\ell \leq \phi$. Market clearing for housing in each location implies $H(\ell) = D(\ell)h(\ell)$, where $D(\ell)$ denotes the density (number of urban workers) in location ℓ. Within the city, the density $D(\ell)$ follows immediately from Eqs. (16) and (17),

$$D(\ell) = \frac{H(\ell)}{h(\ell)} = \frac{q(\ell)^{1+\epsilon}}{\gamma(w(\ell) + r + \underline{s} - p\underline{c})}. \tag{20}$$

Density for $\ell \leq \phi$ can be rewritten using Eq. (15) and Eq. (18) as,

$$D(\ell) = \rho_r \frac{1+\epsilon}{\gamma}(w(\phi) + r + \underline{s} - p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell) + r + \underline{s} - p\underline{c})^{\frac{1+\epsilon}{\gamma}-1}. \tag{21}$$

Importantly, lower rural land rents ρ_r at the urban fringe lowers density across all urban locations. Integrating density defined in Eq. 21 across urban locations gives the total urban population,

$$L_u = \int_0^\phi D(\ell)dl = \rho_r \int_0^\phi \frac{1+\epsilon}{\gamma}(w(\phi) + r + \underline{s} - p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell) + r + \underline{s} - p\underline{c})^{\frac{1+\epsilon}{\gamma}-1}d\ell. \tag{22}$$

Eq. (22) pins down the city size ϕ. It says that if more workers are willing to move in the urban sector, the city will have to be bigger in area to host them—ϕ is increasing with L_u. One should also notice that the city's area increases if the price of land ρ_r at the fringe is lower, if housing supply conditions are tighter (low ϵ), and if commuting frictions $\tau(\ell)$ are lower.

In the rural area, $\ell \geq \phi$, market clearing for residential housing imposes

$$L_r\gamma\left(w_r + r + \underline{s} - p\underline{c}\right) = S_{hr}\left(q_r\right)^{1+\epsilon} = S_{hr}(1+\epsilon)\rho_r,$$

where S_{hr} is the amount of land demanded in the rural area for residential purposes. This leads to the following demand of land for residential purposes in the rural area,

$$S_{hr} = \frac{L_r\gamma\left(w_r + r + \underline{s} - p\underline{c}\right)}{(1+\epsilon)\rho_r}. \tag{23}$$

Land and labor market clearing. Land is used for residential or productive purposes. With total land available in fixed supply, the land market clearing condition is

$$S_r + S_{hr} + \phi = 1.$$

Using Eq. (23), this is equivalent to

$$S_r = 1 - \phi - \frac{L_r\gamma\left(w_r + r + \underline{s} - p\underline{c}\right)}{(1+\epsilon)\rho_r}. \tag{24}$$

The labor market clearing is such that the total population L is located either in the city or in the rural area,

$$L_u + L_r = L. \tag{25}$$

Land rents. Aggregate land rents, rL, include the land rents generated both in the city and in the rural area,

$$rL = \int_0^{\phi} \rho(\ell)d\ell + \rho_r \times (1-\phi), \tag{26}$$

where it is useful to notice that the rental value of land in the city exceed the rental value a farmland for the same area due to spatial frictions.

3.6 Goods markets equilibrium

A last step consists in clearing the goods market for rural and urban goods to pin down the allocation of labor across sectors for a given equilibrium city size ϕ.

Aggregate per capita income. Let us introduce y as the aggregate per capita income in the economy net of spatial frictions that is spent on both goods,

$$y = r + \frac{L_r}{L}w_r + \frac{1}{L}\int_0^{\phi} w(\ell)D(\ell)dl.$$

Goods market clearing conditions. Aggregating Eqs. (10)-(11) across locations, we get that aggregate per capita consumption of rural good and urban good satisfy

$$\begin{aligned} pc_r &= \nu(1-\gamma)(y + \underline{s} - p\underline{c}) + p\underline{c} \\ c_u &= (1-\nu)(1-\gamma)(y + \underline{s} - p\underline{c}) - \underline{s} \end{aligned}$$

The rural good is only used for consumption. This gives the following market clearing condition for the rural good,

$$\nu(1-\gamma)y + \nu(1-\gamma)(\underline{s} - p\underline{c}) + p\underline{c} = py_r, \tag{27}$$

where $y_r = \frac{Y_r}{L}$ denotes the production per worker of the rural good.

The urban good market clearing is more involved as urban goods are either consumed, used as intermediary inputs to build residential housing (in all locations) or used to pay for commuting costs. The sum of these three uses equals the supply of the urban good, expressed per capita,

$$c_u + \frac{1}{L}\int_0^{\phi} \tau(\ell)D(\ell)d\ell + \frac{1}{L}\frac{\epsilon}{1+\epsilon}\int_0^1 q(\ell)H(\ell)d\ell = y_u, \tag{28}$$

where $y_u = \frac{Y_u}{L}$ denotes the production per worker of the urban good.

3.7 Equilibrium allocation

For a given set of exogenous parameters, technological parameters $(\theta_u, \theta_r, \alpha)$, commuting cost parameters (a, ξ) and resulting spatial frictions $\tau(\ell)$ at each location $\ell \in \mathcal{L}$, housing supply conditions ϵ, and preference parameters, $(\nu, \gamma, \underline{c}, \underline{s})$, the equilibrium is defined as follows.

Definition 1. *An equilibrium is a sectoral labor allocation (L_u, L_r), a city fringe (ϕ) and rural land used for production (S_r), sectoral wages (w_u, w_r), a rental price of farmland (ρ_r), a relative price of rural goods (p) and land rents (r), such that:*

- *Factors are paid the marginal productivity, Eqs. (1)-(3).*
- *Workers are indifferent in their location decisions, Eq. (14).*
- *The demand for urban residential land (or the city fringe ϕ) satisfies Eq. (22).*
- *Land and labor markets clear, Eqs. (24) and (25).*
- *Land rents satisfy Eq. (49).*
- *Rural and urban goods markets clear, Eqs. (27) and (28).*

The main intuition for the equilibrium allocation goes as follows: if the urban sector hosts more workers, the area of the city has to be larger (ϕ tends to increase with L_u). However, if the city is larger in area, the worker in the further away urban location commutes more, making the urban sector less attractive for workers: a higher ϕ reduces the incentives of workers to move from the rural to the urban sector (L_u tends to decrease with an increasing ϕ). Given technology, the combination of these two forces pins down the allocation of workers across sectors together with the land used for urban residential housing.

However, the equilibrium cannot be described analytically. Thus, we use numerical illustrations to explain the main mechanisms through which increasing productivity, in the rural and urban sectors, change the population, area and density of cities in our framework. The numerical simulations are not aiming at being quantitative but at elucidating the main channels at play to understand urban expansion when economies go through the process of structural change. A quantitative evaluation in the context of France is provided in Section 4.

3.8 Numerical illustrations

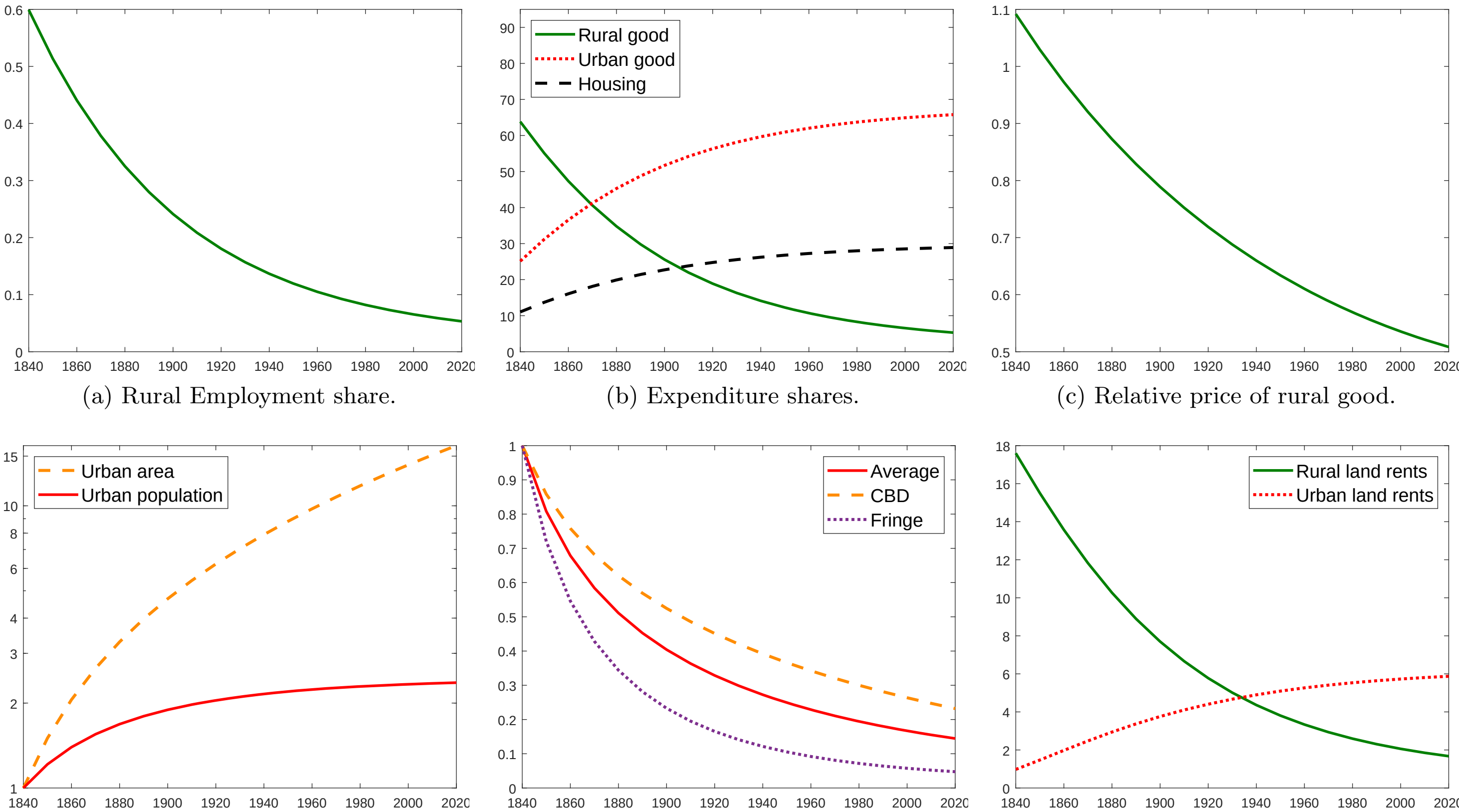
Parameters values. We consider an economy as described above endowed with land and labor, both normalized to 1. While the exercise is not quantitative, we nevertheless set parameters values in a reasonable range with respect to the data. The share of land in production is set to 25% ($\alpha = 0.75$). We set the constant elasticity of housing supply ϵ to 4, corresponding to a share of land in housing value of 20%. Preferences towards the different goods are set to roughly match the employment share in agriculture and the housing spending share in the recent period in France—$\nu = 2.5\%$ and $\gamma = 30\%$. At each date t, the productivity is assumed to be the same in both sectors, $\theta_{u,t} = \theta_{r,t}$, and the initial productivity is normalized to unity. Both sectors are growing at the same constant rate of productivity growth of 1.25% per annum. Most importantly, together with rising productivity, structural change emerges due to the presence of subsistence needs for rural goods, $\underline{c} = 2/3$. As we focus on subsistence needs, we set $\underline{s}$ to zero. With such preferences, the share of employment is the rural sector is about 60% at start. For comparison, we explore at a later stage the model dynamics when structural change is driven by increasing demand for urban goods rather than subsistence needs ($\underline{s} >> \underline{c}$). The values for the commuting costs parameters are set such that the urban area remains small relative to land used in agriculture, $a = 2$.[20] The parameter determining the elasticity of commuting costs to urban income and commuting distance, ξ, is set to 2/3 to generate an increase in the average urban commuting speed comparable to the data (see Miles and Sefton (2020)).[21]

Baseline. Figure 7 summarizes the model dynamics following rising productivity in both sectors—starting at an initial period labeled 1840 for illustration purposes. The top panel of Figure 7 shows the evolution of employment, spending shares and relative prices. As is well known in the literature, due to low initial (rural) productivity, the share of workers needed to produce rural goods is high at start to satisfy subsistence needs. The demand for rural goods for subsistence make them initially relatively expensive and households spend a disproportionate share of their income on rural goods. With rising (rural) productivity solving the 'food problem', workers move away from the rural to the urban sector, the relative price of rural goods falls as well as the share of spending towards rural goods.

The bottom panel of Figure 7 shows outcomes that are more specific to our theory with endogenous land use: urban area (compared to urban population), urban densities (average, central and fringe) and land rents (as a share of income). Along the process of structural change, urban area grows faster than urban population, leading to a fall in the average urban density (plots (d) and (e) of Figure 7). This is the outcome of two different forces. On one hand, this is the natural consequence of *rural* productivity growth solving the 'food problem': higher rural productivity frees up farmland for cities to expand, lowering farmland rents relative to income. Moreover, as workers spend less on rural goods, they can afford larger homes and spend relatively more on housing. The city expands outwards at a fast rate. As land at the city fringe is getting cheaper (relative to income), the city expands by adding a less and less dense suburban fringe over time, contributing to the fall in average urban density (plot (e) of Figure 7).

[20] In our formulation of the commuting costs, a is a transformation of the different commuting costs parameters but one can always set the commuting efficiency c_τ to target a given a (see Eq. 7).

[21] In Miles and Sefton (2020), the average speed of commuting in England has been multiplied by almost 5 since 1840—in line with our baseline experiment described below. French data detailed in Appendix A shows a similar increase.



(a) Rural Employment share. (b) Expenditure shares. (c) Relative price of rural good.

(d) City size (Index, 1815=1). (e) Urban densities (Index, 1815=1). (f) Land rents over income (%).

Figure 7: Baseline numerical illustration.

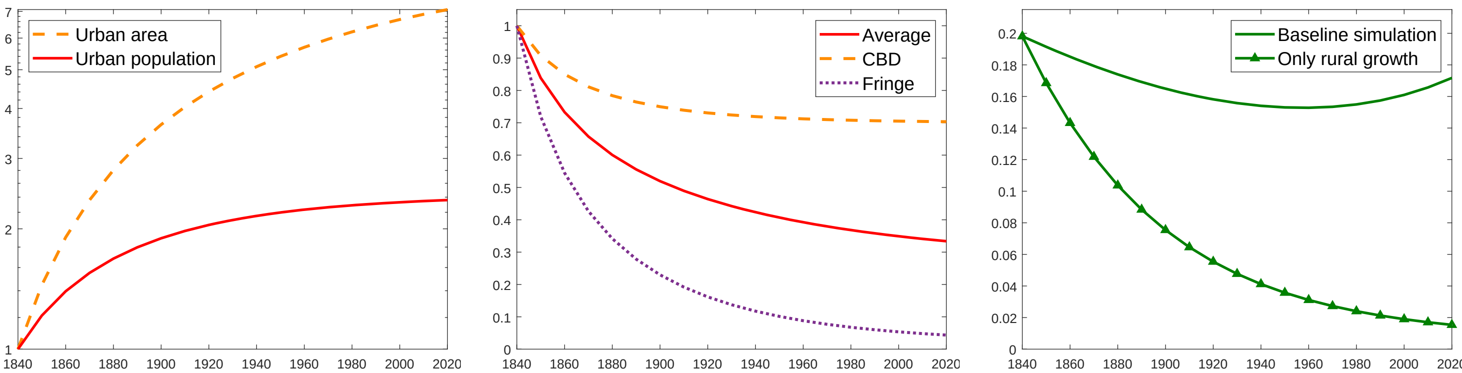
Notes: Simulation with 1.25% constant productivity growth in both sectors, $\underline{c} > 0$, and $\underline{s} = 0$.

On the other hand, rising *urban* productivity leads to a reallocation of workers away from the dense center towards the fringe—contributing further to the fall in average urban density. With a rising urban income, workers move towards the suburbs to enjoy larger homes despite a rising opportunity cost of commuting time. This is so because they optimally choose faster commuting modes when moving towards the suburbs. Thus, although the mechanisms are entirely different, both rural and urban productivity growth contribute to urban sprawl and falling urban density in this experiment.

Regarding land rents, the reallocation of workers away from agriculture and the fall in the relative price of rural goods exerts downward pressure of the price of farmland. Thus, land rents are reallocated away from the rural part towards the urban part (plot (f) of Figure 7)).

To sum up, beyond the well-known predictions regarding employment shares across sectors, our theory is able to qualitatively reproduce the salient facts described in Section 2 for France regarding the expansion of the urban area, the evolutions of urban density and land values.

Rural versus urban productivity growth. To disentangle further the mechanisms at play, it is useful to investigate the model's implications when only rural or urban productivity growth occurs. Figure 8 shows selected model's outcomes with only rural productivity growth—θ_r growing at 1.25% per year, while θ_u is set to unity throughout. The qualitative implications are similar to the baseline illustration. Workers move away from the rural sector, the rural good and farmland are getting less expensive and urban density falls despite rising urban population. However, the city sprawls significantly less: without urban productivity growth, there is less reallocation away from central locations towards the fringe. Average urban density mostly falls due to the addition of lower density habitat at the urban fringe where land gets cheaper—central density falling significantly less.



(a) City size (Index, 1815=1). (b) Urban densities (Index, 1815=1). (c) Farmland rental price.

Figure 8: Numerical illustration with only rural growth.

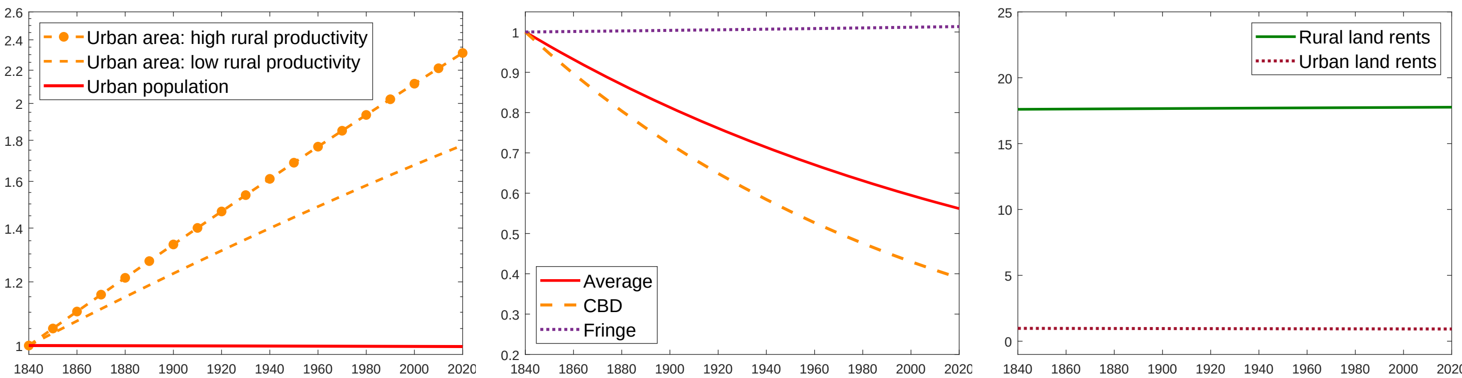
Notes: Simulation with 1.25% constant rural productivity growth and constant urban productivity; $\underline{c} > 0$ and $\underline{s} = 0$.

Figure 9 shows model's outcomes with only urban productivity growth—θ_u growing at 1.25% per year, while θ_r is set to unity throughout (resp. a high value for comparison). Here, the qualitative implications are more widely different from the baseline illustration. Urban productivity growth leads to urban expansion in area but not in population: many rural workers are required to satisfy subsistence needs and feed the population (plot (a) of Figure 9).[22] The city expands in area as higher urban productivity reallocates urban workers away from the center towards the urban fringe. As the demand for land at the fringe rises, so does the price: farmland is getting more expensive. This, in turn, increases suburban density, mitigating the overall fall in urban density. As a consequence, central density is falling more than the average one (plot (b) of Figure 9). With only urban productivity growth, rural land rents (as a share of income) do not fall and there is no reallocation of land values towards the urban areas (plot (c) of Figure 9). Thus, rising urban productivity and faster urban commutes are not sufficient to account for the evolution of urban densities and land rents across space.

Lastly, it is important to note that the reallocation of urban residents away from the center towards the suburbs is significantly stronger at a higher level of rural productivity. In other words, the interaction between rural and urban productivity matters for the area expansion of cities (plot (a) of Figure 9). If rural productivity is low, people spend most of their resources on rural necessity goods, limiting their ability to expand their housing space when urban productivity increases. As

[22]Urban population might even very slightly fall as more workers are required to produced subsistence needs with less land available for agriculture.

a consequence, rising urban productivity reallocate significantly less people towards the suburbs and cities stay dense despite higher urban wages. To the opposite, when rural productivity is high enough, rising urban productivity expands the urban area much more as urban residents expand more their housing space. In this sense, beyond the direct effect of rural productivity on urban expansion, rural productivity is also crucial as it provides the necessary incentive for people to relocate towards the city fringe and use faster commutes when urban productivity increases.



(a) City size (Index, 1815=1). (b) Urban densities (Index, 1815=1). (c) Land rents over income (%).

Figure 9: Numerical illustration with only urban growth.

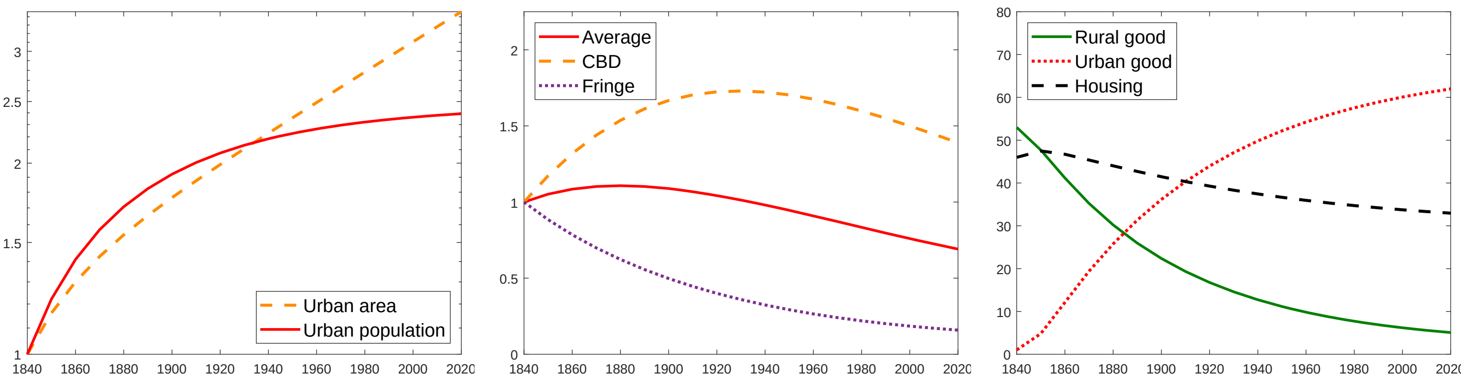
Notes: Simulation with 1.25% constant urban productivity growth and constant rural productivity; $\underline{c} > 0$ and $\underline{s} = 0$. The line with circles corresponds to the simulation with rural productivity being equal to the last period value, while the others correspond to the simulation with rural productivity being equal to the initial period value.

To sum up, our numerical illustrations show how, in the presence of subsistence needs, agricultural productivity growth not only matters for urbanization and the reallocation of workers away from the rural sector, but it is also essential to replicate the large historical decline in urban density, the fall in farmland prices (relative to income) and the reallocation of land rents towards urban areas.

Labor push versus labor pull. In the baseline illustration, the driver of structural change is rural productivity growth combined with subsistence needs for rural goods—a model where rising productivity frees up resources for the urban sector to expand ('rural labor push'). An alternative view on structural change would emphasize a rising demand for (luxury) urban goods as income rises ('urban labor pull'). In our set-up, this would correspond to a high $\underline{s}$ relative to $\underline{c}$. For comparison, we simulate the economy with a value for $\underline{s}$ twice as big as $\underline{c}$ ($\underline{s} = 2\underline{c} = 1.2$), such that, keeping all other parameters to their baseline values, the initial share of employment in the rural sector remains close to 60%.[23] Under such preferences, Figure 10 shows the model dynamics following rising productivity in both sectors. While such a calibration can generate employment shares broadly in line with the data, it cannot generate the fall in urban density observed. Indeed, as income increases, the spending share on housing falls and workers shrink their housing size to consume more of the urban good. Thus, the city does not expand much in area to host more numerous urban workers and urban density does not fall. Urban density tends to increase due to

[23]Preference parameters are also such that the share of spending towards urban goods remains positive throughout.

the reallocation of workers towards the urban center (plot (b) of Figure 10): as they shrink their housing size, urban workers relocate away from the suburbs towards central locations, increasing central density—the opposite of the data.[24] A high enough subsistence need is important for urban density to decline as it leads to an increase in the housing spending share following structural change. Note also that the evolution of the spending share on housing is informative regarding the relative magnitude of $\underline{c}$ and $\underline{s}$ (plot (b) of Figure 7 and plot (c) of Figure 10). An increasing share of housing spending, as in the data (see the calibration description in Section 4), points towards a calibration where $\underline{c}$ is significantly larger than $\underline{s}$.



(a) City size (Index, 1815=1). (b) Urban densities. (c) Expenditure shares.

Figure 10: Numerical illustration with $\underline{s} > \underline{c} > 0$.

Notes: Simulation with 1.25% constant productivity growth in both sectors. The preference parameters are such that $\underline{s} = 2\underline{c} > 0$, while keeping the initial rural employment share close to 60%.

4 Quantitative Model [in progress]

We develop a quantitative version of the model to account for the process of structural change and urban expansion in France. The quantitative model is calibrated using French historical data since 1840.[25] Data are described in details in Appendix A. At this stage, we implement a single city economy without agglomeration economies or traffic congestion. One can interpret the following quantitative simulations as aggregate outcomes for the 'average' French city.

We extend our baseline theory in several dimensions to compare model outcomes to data. First, we consider a surface instead of a segment as the given initial land endowment. The city is circular with endogenous radius ϕ and area $\pi\phi^2$. Second, we allow for a more flexible parametrization of the commuting costs and of the construction costs faced by land developers. The latter are such that the housing supply elasticities depend on the location within the city (as in Baum-Snow and Han (2019)). Lastly, we consider a dynamic version of the model where households maximize an

[24] Suburban (fringe) density does fall in this experiment (plot (b) of Figure 10). The same mechanisms as in our baseline illustration also play a role: farmland is getting cheaper at the city fringe due to structural change.

[25] 1840 is the first date of observation for agricultural land use, which is necessary to compute the path of productivity in the rural sector.

intertemporal log-utility with borrowing and lending in a riskfree asset in net-zero supply. Given a discount factor β, this pins down the path of the equilibrium real interest rate in our simulations and allows the computation of land values beyond rents. Extensions of the baseline theory for quantitative purposes are detailed in Appendix B.

4.1 Calibration

The production technology and demographics are parametrized externally. The remaining parameters are set to match some outcomes observed in the data. While the parameters are jointly determined to minimize the distance between the model's outcomes and a set of specified moments in the data, we provide, for sake of space, the main intuitions behind the identification of the model's parameters. Details of the joint estimation and the minimization procedure are provided in Appendix B.

Technology. In the rural sector, the share of land used in agriculture is set to 25%, $\alpha = 0.75$ as in Boppart et al. (2019). Boppart et al. (2019) provide an estimate very close to unity for the elasticity of substitution between land and labor in agriculture. Thus, as in our baseline model, rural production is Cobb-Douglas but we perform sensitivity with respect to the elasticity of substitution between land and labor (see Appendix B for details).

The path for productivity in both sectors, θ_r and θ_u, is calibrated to match its data counterpart using French sectoral data on production, employment and agricultural land use.[26] The estimated path for θ_r and θ_u is displayed in Figure 23 in Appendix A. The evolution of productivity is in line with the evolution of the standards of living in France over the same period. Such a path for productivity is also consistent with the conventional view that the nineteenth century is characterized by faster productivity growth in non-agricultural sectors, manufacturing in particular, while agricultural productivity grew at a significantly faster rate post-World War II. More specifically, starting the agricultural crisis in late nineteenth century, technological progress in the French agriculture was particularly slow and delayed relative to other countries, before catching up at a fast rate post World War II (Bairoch (1989)). Going forward (post 2015), we assume that productivity in both sectors grow at a constant rate of 1% annually.

Demographics. Population, L_t, is normalized to unity in the first period and set a each date to match the increase of the French population since 1840 according to Census data. Over the period considered, the French population roughly doubled and the increase in the labor force is of the same magnitude. Going forward, we use the projections for the French population by INSEE until 2040.

Preferences. The parameter of preferences towards the rural good, ν, is set to 2.5% leading to a targeted long-run employment of share in agriculture in line with recent observations. Given technology, demographics and the other preference parameters, the subsistence needs in agriculture, $\underline{c}$, is set to match at best the evolution of the employment share in agriculture since 1840. This gives a value of $\underline{c} = 0.78$.

[26] Due to the normalization of price indices, θ_r and θ_u are set equal to unity in the initial period.

The preference parameter towards housing services, γ is set to match the housing spending share in the recent years (31.4% in 2015). Given other parameters, the endowment of urban good, $\underline{s}$, is set such that the housing spending share matches the data for the year 1900 (23.7% if we take a 5-year average around 1900)—our initial period of observation regarding consumption expenditures. More precisely, the preference parameters, γ and $\underline{s}$ are jointly set such that the housing spending share is in line with the data for its initial and final observations. This yield a value for $\underline{s}$ of 0.21. Note that the values of $\underline{c}$ and $\underline{s}$ are such that the high initial share of employment in agriculture is largely due to the subsistence need for rural goods.

The last preference parameter, the discount factor β, is irrelevant for the equilibrium allocation but pins down the rate of interest and thus matters for the value of land at each date. It is set externally to 0.94 such that the value of agricultural land over income matches the data in 1840.

Housing supply conditions. Existing estimates of the housing supply elasticities, ϵ, typically varies between 2 and 5, depending on the location as well as on the estimation technique (see, among others, Albouy et al. (2018), Combes et al. (2017) and Baum-Snow and Han (2019)).[27] Baum-Snow and Han (2019) provides evidence of the *within-city* variation of the housing supply elasticities, ranging from about 2.5 at the CBD to about 5 at the fringe of cities. For the purpose of the quantitative analysis, we extend the baseline theory by allowing location-specific housing supply elasticities, $\epsilon(\ell)$ with $\partial\epsilon(\ell)/\partial\ell \geq 0$ (see Appendix B for details). This is meant to capture that it might be more costly for developers to build closer to the center than in the suburbs or the rural part of the economy. Following Baum-Snow and Han (2019), we set an elasticity of 2 at the CBD and 5 at the fringe and the rural area.[28] For comparison purposes, we perform sensitivity analysis with a constant elasticity of housing supply, $\epsilon = 3$.

Commuting costs. For the purpose of our quantitative analysis, we expand the commuting choice model by introducing a more general spending cost on commuting f, which still depends on the mode choice m but also on the commuting distance ℓ and the labour costs w_u (see Appendix B for details). Intuitively, beyond its speed, the pecuniary cost of a commuting mode depends on the distance travelled (e.g. cost of gazoline/energy) as well as the overall level of wages (e.g. wage of the bus driver). Under some parametric assumptions, commuting costs under an optimal mode choice are of the following form (comparable to Eq. 7),

$$\tau(\ell) = a \cdot w_u^{\xi_w} \cdot \ell^{\xi_\ell},$$

where the elasticities of commuting costs to income, ξ_w, and to distance, ξ_ℓ, are both positive and below unity. The parameter a is inversely related to the efficiency of the commuting technology. We use commuting data detailed in Appendix A to set the elasticities ξ_w and ξ_ℓ.

[27] With Cobb-Douglas production of housing using land and structure, there is a mapping between the elasticity ϵ and the land share in housing production. Typical estimates of the land share varies between 0.2 and 0.3, corresponding to elasticities between 2 and 4.

[28] We assume that the elasticities $\epsilon(\ell)$ evolve linearly from the central value to the fringe value. We perform sensitivity analysis with respect to the functional form of $\epsilon(\ell)$, allowing the elasticity to fall at a faster rate towards 2 as one gets closer to the CBD. Results are barely affected.

In the model, the elasticity of speed to commuting distance is equal to $1 - \xi_\ell$. Using French individual commuting data, this elasticity is very precisely estimated within a narrow range around 0.45—depending on the sample used and on the controls.[29] Thus, we set externally ξ_ℓ equal to 0.55.

The parameter ξ_w is tied to the evolution of urban speed when average income increases. We collected historical data on the use of different commuting modes since 1840 for Paris to provide an estimate of the evolution of the average commuting speed in Paris (see Appendix A for details). We find that the average commuting speed was multiplied by about 5 for Paris since 1840.[30] Such historical data are not available for the rest of France but recent data indicates that this is likely to be a lower bound of the overall increase since car use is more limited in the Parisian urban area than elsewhere. Thus, we target a five-fold increase of the average urban commuting speed over the period 1840-2010 in our baseline but perform some sensitivity analysis with a slightly higher increase. Contrary to ξ_ℓ, the parameter ξ_w is set jointly with the other parameters to match the target since commuting speed is an endogenous model's outcome (see Appendix B for details). In our baseline, ξ_w is set to 0.75.[31]

The remaining parameter a is set such that the urban area, $\pi\phi^2$, represents 18% of agricultural land in the recent period—in the data (INSEE), the measured artificial land is 18% of the land used for agriculture in 2015. Results are not very sensitive to a as long as $\pi\phi^2$ is a relatively small fraction of the available land.

The baseline parameter values are summarized in Table 1.

4.2 Results [preliminary]

We present model's predictions over the period 1840-2020 under the baseline calibration with only one representative city. Data counterparts, when available, are described in Appendix A. Sensitivity analysis and an extension of the baseline model to account for multiple cities are relegated to later sections.

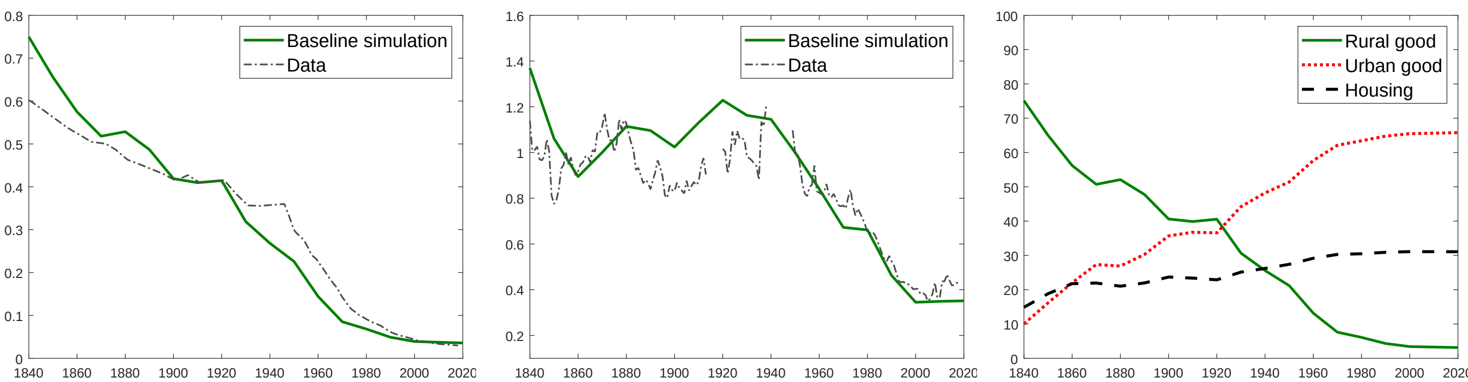
[29] Commuting data also shows that the relationship between speed and commuting distance is very close to log-linear as in the model. See Appendix A.

[30] Miles and Sefton (2020) find a very similar estimate for the U.K..

[31] Using individual commuting data since the eighties, one can estimate the percentage change in speed over 30 years *for a given commuting distance*. Over the period 1983-2006/2013, this increase is about 10% for an increase in measured urban productivity of about 40%. This yields an alternative estimate very close to the calibrated value, $\xi_w \approx 1 - \frac{10}{40} = 0.75$. See Appendix A for details.

Parameter	Description	Value
S	Total Space	1.0
L_0	Total Population in 1840	1.0
θ_0	Initial productivity in 1840	1.0
α	Labor Weight in Rural Production	0.75
σ	Land-Labor Elasticity of Substitution	1.0
ν	Utility Weight for Rural Consumption	0.025
γ	Utility Weight for Housing	0.31
$\underline{c}$	Rural Consumption Good Subsistence Level	0.779
$\underline{s}$	Initial Urban Good Endowment	0.207
β	Discount Factor	0.935
ξ_ℓ	Elasticity of commuting cost wrt location	0.55
ξ_w	Elasticity of commuting cost wrt urban wages	0.75
a	Commuting Cost Shifter	2.14
ϵ_r	Housing supply elasticity in rural area	5.0
$\epsilon(0)$	Housing supply elasticity at the center	2.0

Table 1: Parameter values

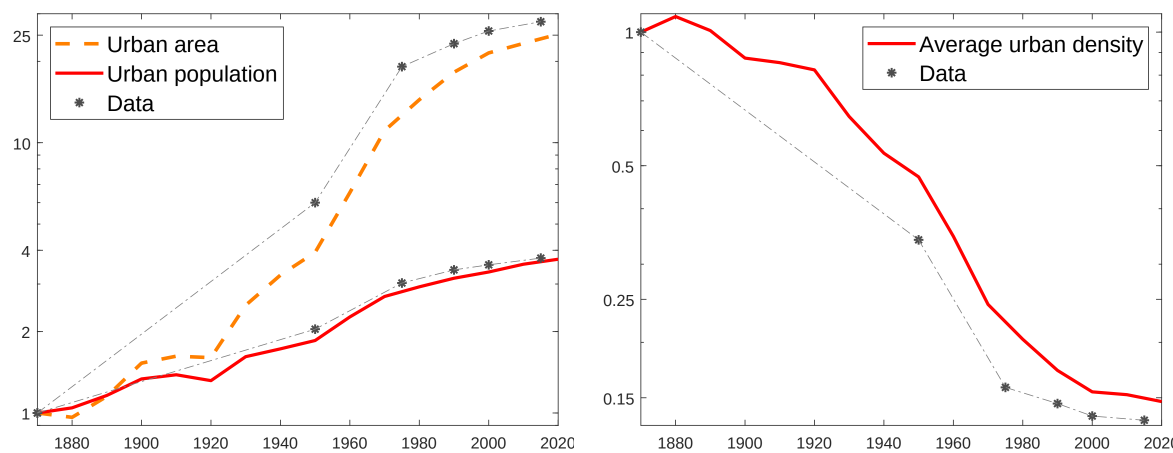


(a) Rural employment share. (b) Relative price of rural goods. (c) Spending Shares.

Figure 11: Structural change.

Structural change. Figure 11 shows that our model is able to account for the patterns of structural change observed in the data. Rising rural productivity reallocates labor away from the rural sector and makes rural necessity goods less valuable. The relative price of rural goods falls as productivity increases. Our model fits the data on the historical evolution of the relative price remarkably well despite not being targeted (Figure 11b). Moreover, rising income leads to a reallocation of spending away from rural goods towards the urban good and housing services: the spending share on the rural good gradually falls, the share spent on the urban good continuously increases, and so does the spending share on housing services, although at a slower speed (Figure 11c). Overall, the spending share patterns are broadly in line with the data if one abstracts from fluctuations in the interwar period (see Figure 24 in Appendix A).

Urban expansion. Figure 12 shows the model's outcomes regarding the evolution of city size (area versus population) and the average urban density. For comparison with our data on urban expansion, the plots start in 1870—normalizing the 1870-value to unity. In line with the data, cities expand much faster in area than in population (Figure 12a). While our model does not account for overall expansion of the urban area, it explains a very large fraction. As a consequence, the model predicts a large fall in average urban density—density is divided by more than 6.5 since 1870, slightly less than in the data (Figure 12b). As structural change slowdowns, so does the fall in urban density.



(a) City size (1870=1).

(b) Average urban density (1870=1).

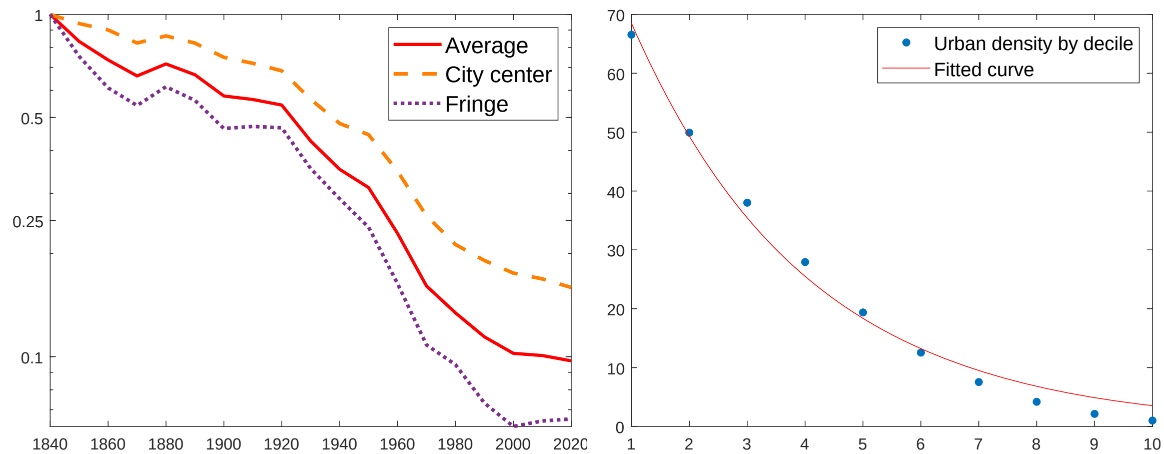
Figure 12: Urban expansion.

Density across space. Figure 13 shows the model's predictions for density in different locations. Figure 13a depicts the evolution of the central density and the density at the fringe of the city (relative to the average), where densities are normalized to 1 in 1840 for readibility.[32] It shows that the fall in average density is driven both by a fall in central density and a fall in density at the urban fringe. Central density falls because households find it worth to use faster commuting modes and to move towards the suburbs as their income rises. The fall in density at the suburban fringe is the natural consequence of structural change: the reallocation of workers away from agriculture combined with less valuable agricultural goods puts downward pressure on the price of farmland. Households can afford larger homes in the suburban parts of the city. The latter mechanism, more specific to our theory, is crucial to generate a fall in average density that is larger than the fall in the central one—in line with the Parisian data discussed in Section 2. Our model predicts that the overall fall in the central density is about 60% of the fall in the average density—in the ballpark of the estimates for Paris. Lastly, one can measure the density gradient by distance deciles within the urban area, both in the data and in the model in the recent period.[33] The model's predictions

[32]The fringe of the city center is at 15% of the initial radius of the city in 1840.

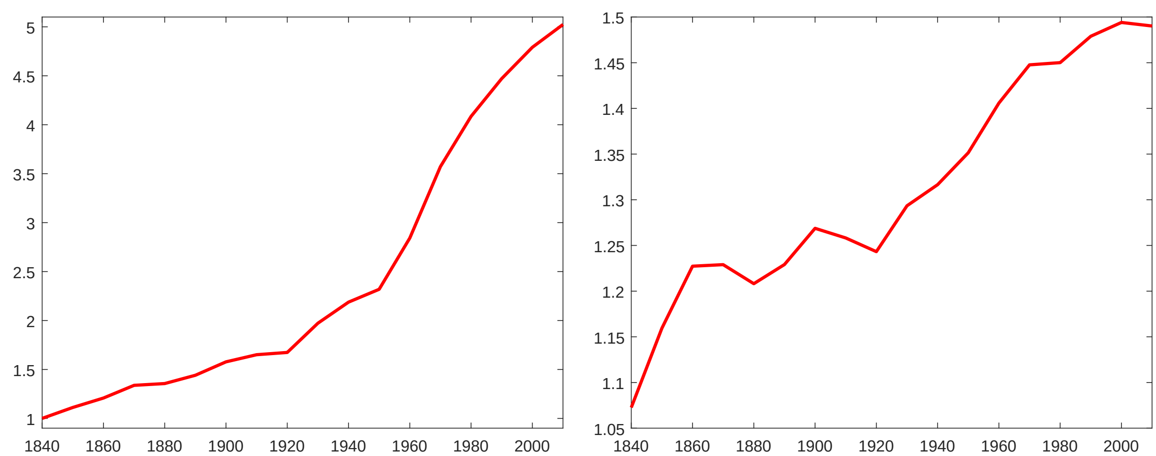
[33]In the data, this can be done using the satellite data merged with census data (see Appendix A for details). We compute the average gradient in the data as a population-weighted average the density gradients across cities.

are shown in Figure 13b. The shape of the curve is very close to an exponential (fitted curve) as in the data. The exponential coefficient of the fitting curve is also close to its data counterpart—0.15 against 0.16 in the data. Thus, our quantitative model provides a reasonable fit of the data regarding the density of urban settlements across time and space.



(a) Urban density (1840=1). (b) Density gradient (2015).

Figure 13: Density across space.



(a) Average urban commuting speed (1840=1). (b) Agricultural productivity gap.

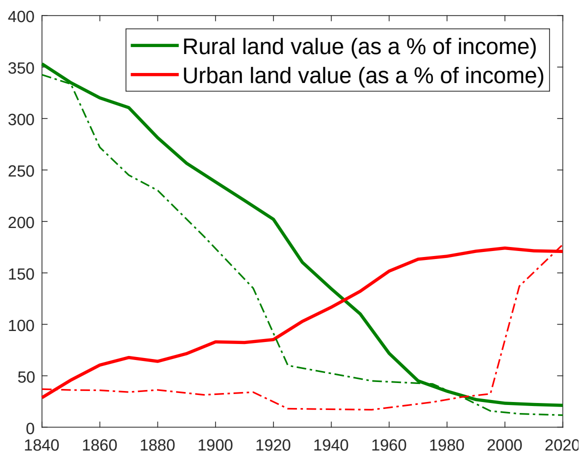
Figure 14: Commuting speed and the 'agricultural productivity gap'.

Commuting speed and the 'agricultural productivity gap'. Our model with endogenous commuting costs generates predictions regarding the evolution of commuting speed across locations and across time. Moreover, the marginal urban worker, which has the longest commute, needs to be compensated relative to the rural worker. Our model thus predicts an endogenous urban-rural wage gap, which depends on the city fringe (ϕ) and the endogenous commuting costs in this further away location. These predictions are shown in Figure 14. Over time, our model generates a five-fold rise in the average commuting speed (Figure 14a). Beyond this targeted increase, the predictions over the whole period line up relatively well with the evolution of commuting speed estimated using data for the Parisian area. The increase by a factor of about 2 until 1930 reflects the more intensive usage of public transport in Paris and their increase in speed over this period (from the initial omnibus to the metro as detailed Appendix A). The later increase, more specifically post-World War II, reflects the increasing car usage.

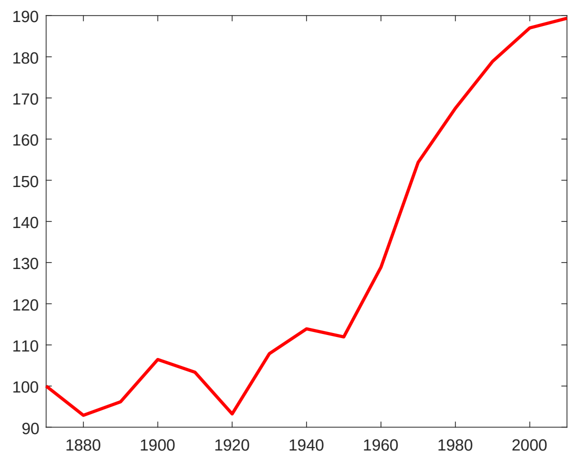
Following Gollin et al. (2014), Figure 14b shows the 'agricultural productivity gap'—a monotonic transformation of commuting costs at the fringe of the city proportional to the urban-rural wage gap, w_u/w_r. We compute the raw 'agricultural productivity gap' as,

$$\text{Raw-APG} = \frac{L_r/L_u}{VA_r/VA_u},$$

where VA_i denotes the value added in sector i. The value predicted by the model for the recent period, around 1.5, is in line with the values computed by Gollin et al. (2014) for France—lying in between their Raw-AGP and Adjusted-APG. Computing the Raw-APG for the entire sample period directly from historical national accounts data, we find that our model falls short of the entire gap, especially for the initial years, but explains a large fraction since 1960.[34] Our quantitative model suggests that spatial frictions combined with location-specific housing can generate urban-rural wage gaps of a significant economic magnitude. It also provides insights on the persistence of fairly large gaps even in developed countries, where labor misallocation is arguably less relevant.



(a) Urban/Rural Wealth/income.



(b) Real Housing Price Index (1870=100).

Figure 15: Land values and housing price.

Land values and housing price. The model's predictions about land values and housing prices are shown in Figure 15. Due to structural change, the value of rural land as a share of income fell dramatically whereas the value of urban land increased significantly (Figure 15a). This is broadly in line with data from Piketty and Zucman (2014) even though the model misses the timing of

[34]Using wage data, Sicsic (1992) provides estimates of the urban-rural wage gap in France over the period 1852-1911. Like in the U.K., he finds a significant increase of the gap over the period, in line with our predictions.

the increase in the value of land used for housing. Importantly, the value of urban land increased faster than income—particularly so in the recent decades. This mirrors the evolution of the housing price index since 1870 (Figure 15b), whose shape reminds of the hockey-stick shown in Figure 6. The model generates about half of the increase in housing prices described in Knoll et al. (2017) post-World War II.

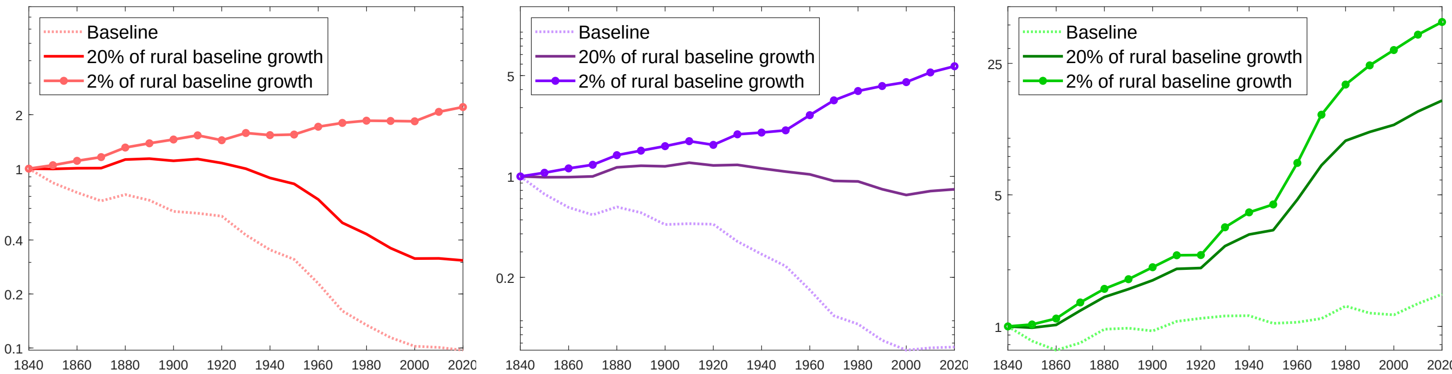
4.3 Sensitivity analysis

In order to shed further lights on the mechanisms at play and discuss the sensitivity of our results to the different elements of the model, we perform alternative experiments. More specifically, these experiments aim at showing how structural change and the use of faster commutes interact in driving the urban expansion. We also discuss the robustness of our findings to the production side in the rural and housing sector.

The role of rural productivity growth. To emphasize further the crucial role of technological progress in agriculture and structural change for our results, it is useful perform sensitivity analysis with a lower rural productivity growth. We perform simulations with a stagnating (resp. slowly growing) rural productivity, where the growth rate of θ_r is 2% (resp. 20%) of the baseline at each date. Results of these simulations are shown in Figure 16 for some variables of interest together with the baseline simulation for comparison. With low improvements of the rural technology, the urban density falls significantly less and might even increase if rural productivity stays sufficiently low (Figure 16a). The growth of population and urban productivity puts pressure on land in the rural area to feed an increasing and richer population. This increases the relative price of rural goods and the price of farmland at the urban fringe (Figure 16c)—preventing the city to expand.[35] Furthermore, facing higher price of rural goods, households tend to reduce their housing spending share to feed themselves, reducing the demand for urban land. These forces tend to make the city much denser than our baseline—more so at the urban fringe due to rising farmland values (Figure 16b). Thus, urban density might increase despite the reallocation of urban workers towards the less dense part of the city as they commute faster due to rising urban productivity. It is worth emphasizing that population growth, by putting pressure on land, makes improvements in agricultural productivity even more crucial to generate a sizeable expansion in urban area.

This simulation does not say that improvements in commuting technologies do not matter for the expansion in area of cities. However, it makes clear that they matter only when combined with rural productivity growth and structural change. The next experiment provides further insights on the quantitative role of commuting costs for our results.

[35]In the simulation with stagnating rural productivity, the city even shrinks in size. Workers move away from cities despite urban productivity growth as more rural workers are needed to feed the increasing population.



(a) Average urban density (1840=1). (b) Density at the urban fringe (1840=1). (c) Rental value of farmland (1840=1).

Figure 16: Sensitivity to rural productivity growth.

Notes: Productivity growth in the rural sector is set to 2% of the baseline rural productivity growth (solid line), resp. 20% of the baseline (solid line with circles). All other parameters are kept to their baseline value of Table 1. Simulation for the baseline rural productivity growth is shown in dotted for comparison.

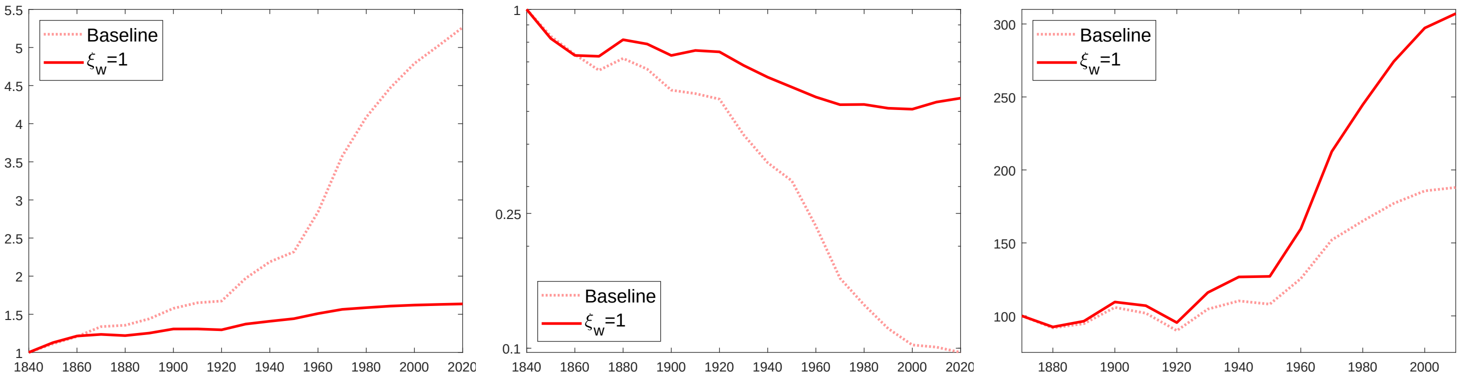
The elasticity of commuting costs to income. To shed further light on the quantitative importance of falling commuting costs and rising commuting speed, we set the elasticity of commuting costs to income, ξ_w, to unity, $\tau(\ell) = a.w_u.\ell^{\xi_\ell}$. With such a calibration, the fraction of wages devoted to commuting in given location does not fall with rising urban productivity and wage, contrary to our baseline. This is so because the speed of commuting does not increase with a rising opportunity cost of time (urban wage).[36] When compared to our baseline, this illustrates the quantitative role of the use of faster commutes when urban productivity increases. Figure 17 shows the results in this alternative calibration together with our baseline for comparison purposes. Figure 17a makes clear that increasing the elasticity of commuting costs to income limits the increase in the average commuting speed over the period.[37] As the cost of faster commutes increases more than in our baseline, urban workers do not relocate away from central locations towards the suburban part of the city. This severely limits the expansion in area of the city relative to the baseline and the average urban density falls significantly less (Figure 17b).

Thus, when combined with rural productivity growth, the use of faster commutes and the corresponding decline in commuting costs (as a share of the urban wage) is quantitatively important to account for the overall decline in urban density—particularly so in central locations. In this alternative simulation, as the urban area expands much less but urban population grows essentially as much due to structural change, urban land values and housing prices increase much more than in our baseline (Figure 17c). This mirrors the role of improvements in commuting modes to limit the increase in urban land values emphasized in Heblich et al. (2018) and Miles and Sefton (2020).[38]

[36] In this knife-edge calibration, workers do not switch to faster modes when facing an increase in their wage: the increase in the operating cost of faster commutes offsets the benefits due to a rising opportunity cost of time.

[37] The average speed still increases slightly as, due to structural change, workers locate in suburban locations where they are willing to use faster commuting modes.

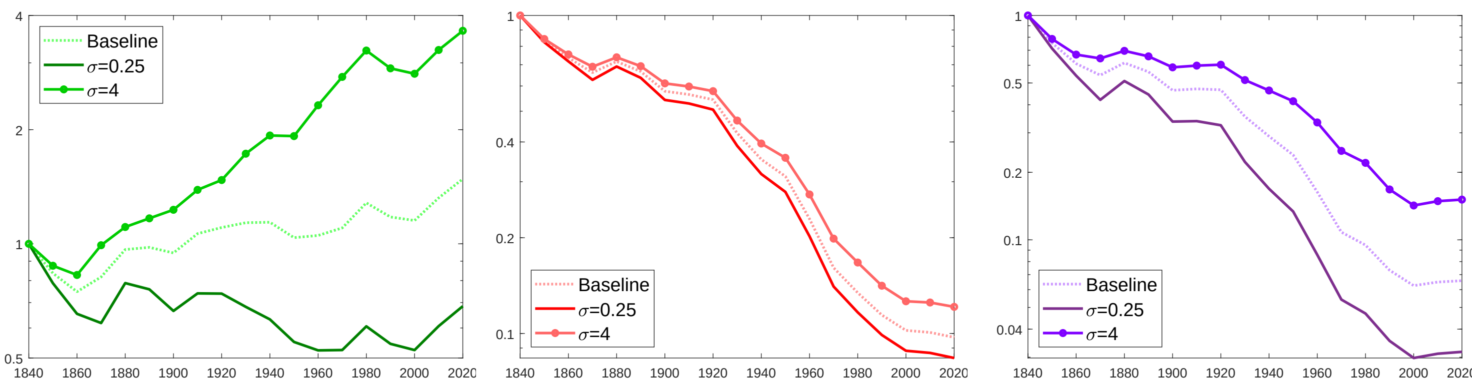
[38] Higher urban housing prices generate an agricultural productivity gap about twice as large as in our baseline simulation in the recent period. Equivalently, the urban resident at the fringe faces much higher commuting costs.



(a) Average commuting speed (1840=1). (b) Average urban density (1840=1). (c) Real housing price index (1870=100).

Figure 17: Sensitivity to the elasticity of commuting costs to income.

Notes: The elasticity of commuting cost to income, ξ_w, is set to 1. All other parameters are kept to their baseline value of Table 1. Simulation for the baseline calibration shown in dotted for comparison.



(a) Rental value of farmland (1840=1). (b) Average urban density (1840=1). (c) Density at the urban fringe (1840=1).

Figure 18: Sensitivity to the elasticity of substitution between land and labor σ.

Notes: The elasticity of substitution between land and labor σ is set to a low value of 0.25 (resp. a high value of 4). All other parameters are kept to their baseline value of Table 1. Simulation for the baseline calibration shown in dotted for comparison.

The elasticity of substitution between land and labor in the rural sector. Our baseline assumes a unitary elasticity of substitution between land and labor, $\sigma = 1$. Values for this parameter used in the literature typically range between 0 and 1 (see Bustos et al. (2016) and Leukhina and Turnovsky (2016)). We perform sensitivity analysis with a lower value of 0.25. We also show results for a high value of 4 to enlighten further the quantitative importance of the adjustment of land values at the fringe of the city for our results.[39] Results for this sensitivity analysis are shown in Figure 18 for variables of interest. With a lower elasticity of substitution, the rental price of farmland falls more (increases less) following structural change as land and labor are more complement in the rural sector (Figure 18a). As the opportunity cost of expanding the city is lower, the urban area

[39] A higher σ limits the fall of farmland values at the fringe of cities when workers move towards the urban sector.

increases more and the average urban density falls more (Figure 18b). This is driven by a larger fall of density in the cheaper suburban parts (Figure 18c). With $\sigma = 0.25$, the model matches the expansion in area and the corresponding decline in average density observed in French cities since 1870. To the opposite, if land and labor are more substitutes in the rural sector ($\sigma = 4$), the reallocation of workers away from agriculture puts less downward pressure on the value of farmland, limiting the expansion of the urban area and the decline in density, which falls short of the data. These experiments further illustrate the importance of the farmland price adjustment at the fringe of cities to understand the reallocation of land use.

The housing supply elasticity. Our baseline features location-specific housing supply elasticities with a lower elasticity at the city center relative to the fringe. As sensitivity analysis, we perform a simulation where the elasticity is set to 3 in all locations, in the mid-range of empirical estimates.[40] Results regarding the time evolution of the aggregate variables of interests—employment, relative price of rural goods, urban area, average urban density and land values—are barely affected. The most noticeable difference is that the model with a constant housing supply elasticity generates a city center much denser relative to the suburban part. Compared to our baseline, a more elastic housing supply at the center leads to a larger provision of housing in these locations. As a consequence, the gradient of density is significantly steeper than in the data (Figure 32 in Appendix B).

4.4 Extensions

Agglomeration and congestion. We introduce urban agglomeration forces by assuming that the urban productivity increases (externally) with urban employment, $\theta_u(L_u) = \theta_u \cdot L_u^\lambda$. Urban wages, $w_u = \theta_u . L_u^\lambda$, thus increase faster relative to our baseline when the city expands. Further details of this extension are provided in Appendix B. We set $\lambda = 0.05$, in the range of empirical estimates for France (Combes et al. (2010)). Other parameters are left identical to the baseline for comparison. For variables of interests, results in presence of agglomeration forces are displayed in Figure 33 in Appendix B together with the baseline. While the city expands slightly more in area, there is barely no effect of agglomeration forces on urban population. The faster increase in the urban wage due to agglomeration forces increases urban housing demand and reduces urban commuting costs (as a share of inccome). This relocates urban households towards the suburbs where they can enjoy larger homes and the city sprawls more. However, a higher urban income makes also rural goods more valuable increasing (almost equally) rural workers' wage. General equilibrium forces thus prevent workers' reallocation towards cities. They also make rural land more valuable—mitigating the area expansion of the city. As a consequence, despite higher incomes driven by urban expansion, the economy with agglomeration forces behave quantitatively similarly to our baseline.

We also consider additional urban congestion costs by assuming that commuting costs are increasing with urban population, $a(L_u) = a \cdot L_u^\mu$. This summarizes the potential channels through which

[40] This corresponds to a land share in housing of 25%. This is broadly in line with the average in the data over the period 1980-2010, even though the land share in housing sharply increased since the early 2000s (Bonnet et al. (2019)).

larger cities might involve longer and slower commutes for a given commuting distance. Further details of this extension are provided in Appendix B. We set $\mu = 0.05$, leaving other parameters to their baseline values. The evolution of the variables of interest is shown on the same Figure 33 in Appendix B for comparison. Congestion forces move the equilibrium in the opposite direction of agglomeration forces. They reduce the expansion in area and the extent of suburbanisation. By increasing commuting costs, they also increase urban housing prices. However, in general equilibrium, they also make rural goods and rural land less valuable—severely mitigating the direct effect of congestion costs on urban expansion.

Multiple cities. Make the point that larger cities are denser but the average fall in urban density remains unchanged. Ideally see how we match Paris. And investigate how density falls in large versus small cities.

4.5 Counterfactuals

t.b.d.

5 Conclusion

t.b.d.

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A Data Appendix

A.1 Agricultural land use

Data sources and definitions. Data for the land used in agriculture are available in various secondary sources based on the French Agricultural Statistics (Statistique Agricole). We checked the consistency of the measures across the different sources.

The variable of interest is the area of land used for agriculture (SAU, for 'Surface Agricole Utilisée'). It is important to note that it includes land that is cultivated but excludes all land that is not (woods and forests, rocky land unfit for agriculture, mountains, swamps. . .).

Post World War 2 (WW2), data for the SAU are provided by the Ministry of Agriculture (data available in Desriers (2007) until 2000 by decade and available on annual basis since 2000 on the website of the Ministry (Agreste)).

Before WW2, agricultural statistics on land use are also available but on a very irregular basis.[41] Through a search across various sources, we compute a measure for the SAU from the first Agricultural Census in 1840 until today. It is worth noting that one must be cautious with such a measure before WW2 in the earlier periods. While it is quite clear that the share of land used in agriculture fell over the whole period, the variations throughout the 19th century (before the 1882 Census) must be taken with caution.

The main difficulty is to make the data presented in various sources comparable across years. First, woods and forest, accounting for 15-20% of French land in the 19th century (and more than 25% today) were initially included in the cultivated agricultural land. We made sure to exclude them from the SAU consistently over the whole time period considered. A second difficulty arises because the French territory varied since 1790: some variations being due to measurement, some due to the loss (or addition) of some parts of France — loss of Alsace and Moselle after the war of 1870 until 1918 and addition of Savoye and Comté de Nice in 1860 (see discussion in Augé-Laribé (1945)). This makes comparison difficult across time, even though we show our measure of the SAU as a share of the French territory at the time. A third difficulty for the early periods (before 1882), detailed below, regards the treatment of pasture and grazing fields in a consistent way across years.

Period 1945-2015. Let us start with the most recent period where the data are arguably of better quality and coherent across time and then present our measures going further back in time. Since 1945, the land used in agriculture has clearly been clearly falling over the period 1950-2015: while land used for agriculture accounted for 62% of total French land post-WW2, this numbers falls to 52% in 2015.

Interwar Period. In between the world wars, we could find measures for the years 1929 and 1937. Two slightly different measures are available for 1929: one in Toutain (1993) and one in Mauco

[41] In the 19th century, starting 1840, France aimed at organizing every decade a detailed data collection of agricultural statistics (Agricultural Census, 'Statistique Agricole'). See for instance description in Flechey (1898) and Augé-Laribé (1945). A comparison across years during the 19th century is available in the report of the 1892 Census. Before 1840, Lavoisier provides the first measure of land use in France, in 1790, as described in Mauguin (1890).

(1937). We take the average between the two, a SAU of 34 483 thousands of ha in 1929. A measure, very similar to 1929, is available in Augé-Laribé (1945) for 1937: 34 207 thousands of ha and 33 285 if one excludes Alsace-Moselle for comparison with earlier periods. This corresponds to about 62% of the French territory.[42]

Nineteenth century. Before World War 1, we have measures in 1882 and 1892 (see Mauguin (1890), Flechey (1898), Hitier (1899) and Toutain (1993) for further details). Both measures are consistent across sources, including the main results of the 1892 Agricultural Census as a more primary source.[43] This gives an SAU of 34 882 thousands of ha in 1882 and 34720 in 1892—slightly higher than the values in between the wars despite a smaller French territory. Figure 19 provides the details of the measurement for the 1892 Agricultural Census.[44]

The first measurement in 1840 constitutes our first observation. However, in the 1840 data, an important difficulty is the treatment of meadows, pasture and grazing fields (prés, herbages, pâturages,...). These should be included in the SAU to the extent that the land is used for agricultural purposes (feeding cattle). As grazing fields and meadows account for a large share of French agricultural land (up to 11% in 1892), their inclusion (or not) in the cultivated part of agricultural land (SAU) matters. However, in 1840, a significant share of grazing fields ('pâturages', 'pâtis communaux/vaines pâtures') is excluded from the SAU. The non-cultivated part of agricultural land thus appears to be a much larger measured area than in all subsequent years.[45] As discussed in the results of the 1892 Agricultural Census, comparison across years is difficult due to the reallocation of grazing fields into the cultivated part of French land over the period 1840-1880. This reallocation is quite artificial—mostly a statistical artefact coming from the earlier exclusion of common pasture. Excluding entirely the measured non-cultivated part from the SAU in 1840 gives thus a lower bound, while including it entirely to account for all grazing fields gives an upper bound. To solve this issue, Toutain (1993) provides an estimate of agricultural land in 1840, in between these two values, of 35 500 thousands of ha. While this is just a matter of definition and any solution is somehow arbitrary, we proceed in a similar fashion as Toutain (1993) and assume that the grazing fields later reallocated in the cultivated part are part of the SAU in 1840. This gives a land use in agriculture of 35 497 thousands of ha in 1840—a very similar number to Toutain (1993). Proceeding exactly in the same way for the year 1862 gives an SAU of 36 088 ha—a higher value but for a larger territory. Both values correspond to about two thirds of French land used in agriculture.

We find that about 1.5% of French land was reallocated away from agriculture between 1840 and

[42] Mauco (1937) compares to the 1892 value and find very similar numbers than ours once woods are excluded from his measurement. Augé-Laribé (1945) compares to the 1882 value and the measure given for 1882 is also consistent with our data.

[43] Statistique Agricole de la France: Résultats généraux de l'Enquête Décennale de 1892. The online archives are available at: https://gallica.bnf.fr/ark:/12148/bpt6k855121k/f1.item

[44] Comparison of land use as a share of total French land across the 19th century is also available in the report of the 1892 Census.

[45] As shown in Figure 19, in 1892, the non-cultivated part includes moor and rocky land arguably unfit for agriculture, accounting for about 11% of French land. The corresponding non-cultivated part in 1840 accounts for 17% of French land as it includes a significant share of grazing fields.

RÉSUMÉ DES CULTURES.

A. — SITUATION EN 1892.

1. TERRITOIRE.

Nous donnons ci-après, par grandes catégories, la répartition du territoire de la France, telle qu'elle résulte des relevés opérés en 1892 :

CATÉGORIES DU TERRITOIRE.			SUPERFICIES.	RÉPARTITION et PROPORTION.
			hectares.	p. 100.
1° TERRITOIRE AGRICOLE.				
Superficie cultivée.	Terres labourables.	Céréales	14,827,085	28.06
		Grains autres que les céréales	319,705	0.60
		Pommes de terre	1,474,144	2.68
		Autres tubercules et racines pour l'alimentation humaine	128,238	0.24
		Cultures industrielles	531,508	1.00
		Cultures fourragères [1]	4,736,394	9.08
		Jardins potagers et maraîchers	386,827	0.73
		Jachères	3,367,518	6.37
		Terres labourables	25,771,419	48.76
	Vignes		1,800,489	3.40
	Prés naturels		4,402,836	8.33
	Herbages pâturés [2]		1,810,608	3.42
	Bois et forêts		9,521,568	18.03
	Cultures arborescentes, etc.		934,800	1.76
	Cultures permanentes non assolées.		18,470,301	34.94
	Totaux de la superficie cultivée		44,241,720	83.70
Superficie non cultivée.	Landes, pâtis, bruyères		3,898,530	7.37
	Terrains rocheux et montagneux, incultes		1,972,994	3.73
	Terrains marécageux		316,373	0.60
	Tourbières		38,292	0.07
	Totaux de la superficie non cultivée.		6,226,189	11.77
Totaux du territoire agricole.			50,467,909	95.47
2° TERRITOIRE NON AGRICOLE			2,389,290	4.53
Totaux généraux du Territoire			52,857,199	100.00

(1) Non compris les cultures dérobées.
(2) Y compris les herbages alpestres.

Figure 19: Land Use in the 1892 Recensement Agricole.

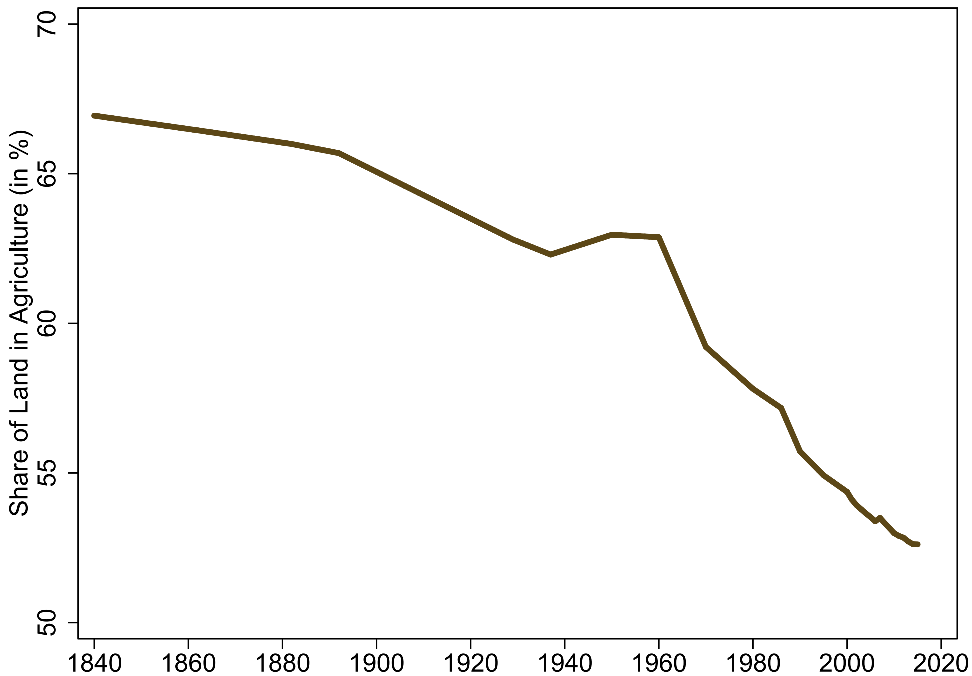


Figure 20: Shares of Land used in Agriculture (1840-2015).

1892 and one can use the data provided in the report of the 1892 Census to investigate how this land was reallocated. The majority was used for larger forests but the land built (non agricultural land) also increased by about 0.5% of the French territory (corresponding to 233 thousands of ha).

The measured cultivated agricultural land (as a share of French territory) over the period 1840-2015 is summarized in Figure 20.

Pre-1800. Lastly, Lavoisier provided in 1790 the very first measure of French agricultural land before the creation of the Agricultural Census. Comparison of Lavoisier's measurement with the later 'Statistiques Agricoles' is however difficult. Like for the later measurements, a large fraction of land ('vaines patûres') includes grazing fields as well as rocky land and moor unfit for agriculture (see Mauguin (1890) for an attempt to compare with the 1882 Census). Excluding woods but including the 'vaines patûres' (common pasture) in 1790 gives a surface of almost 40 000 thousands of ha. Excluding all the 'vaines patûres' provides a lower bound of about 31 000 thousands of ha. This gives a reasonable but fairly wide bracket for the total land used for agriculture. Assuming that the non-cultivated part due to rocky land is comparable to the later measures gives a SAU in 1790 around 34 000 thousands of ha—comparable to the later years (on a smaller territory)—about 65% of French land measured at the time. While this measure should be taken with great caution, it nevertheless comforts us in using a value close to the 1840 measurement to start our time series in the quantitative analysis. We will thus assume a constant area used for agriculture from 1815 to 1840 to avoid missing data (national accounts and employment data described below being available over this early period).

A.2 Sectoral employment

Sources. Data on employment are available in three different sources covering different time periods: Marchand and Thélot (1991) ('Deux siècles de travail en France') for the period 1806-1990; Herrendorf et al. (2014) for the period 1856-2006; OECD for the period 1950-2018. When overlapping, the different sources are largely consistent with each other.[46] We use the three sources allowing to span the entire 1806-2018 period. For the pre-WW2 period, data available in Marchand and Thélot (1991) and Herrendorf et al. (2014) are on an irregular basis—typically one or two observations per decade (corresponding to Census years). Annual data are available from 1950 onwards.

Over the nineteenth century (until 1901), we use the data from Marchand and Thélot (1991) as the serie goes further back in time. Over the period 1901-1950, we use the data from Herrendorf et al. (2014). Over the period 1950-2018, we use data provided by the OECD on an annual basis, where the measure of employment is expressed in full-time equivalent.

Share of employment in agriculture. This gives the share of employment in agriculture over the entire period (1806-2018) in Figure 21. Data are linearly interpolated in between two values when data are not available on an annual basis (pre-1950). It starts with about 2/3 of the employment in agriculture in 1806 and falls progressively to 3% in 2018. One can notice the acceleration in the process of reallocation post WW2. In the matter of three decades, the employment share in agriculture went from 36% in 1946 to 10% in 1976.

A.3 Sectoral national accounts

Sources. Data on value added at the sectoral level together with aggregate value added (GDP) at current prices are available in two different sources covering different time periods. Historical national accounts from Toutain (1987) are used to cover the period 1815-1938. They are directly available at the Groningen Growth and Development Centre (Historical National Accounts Database, http://www.ggdc.net/).

Post WW2, INSEE provides sectoral value added at current prices for the period 1949-2019. For both series, we use agricultural value added and aggregate GDP at current prices. Using both sources gives covers the entire period 1815-2019. The series are interrupted at war times: observations are missing for the periods 1914-1919 and 1939-1948.

Toutain (1987) also provides volume indices for GDP in agriculture and for aggregate GDP over the period 1815-1982 (also available Groningen Growth and Development Centre). The serie for agricultural volumes is extended in Toutain (1993) until 1990. Together with the value added at current prices, these series will be used to compute an agricultural price deflator and a GDP deflator.

[46] Marchand and Thélot (1991) gives a slightly lower share of employment in agriculture in the first half of the 20th century relative to Herrendorf et al. (2014). Our results do not depend on the use of one serie or the other.

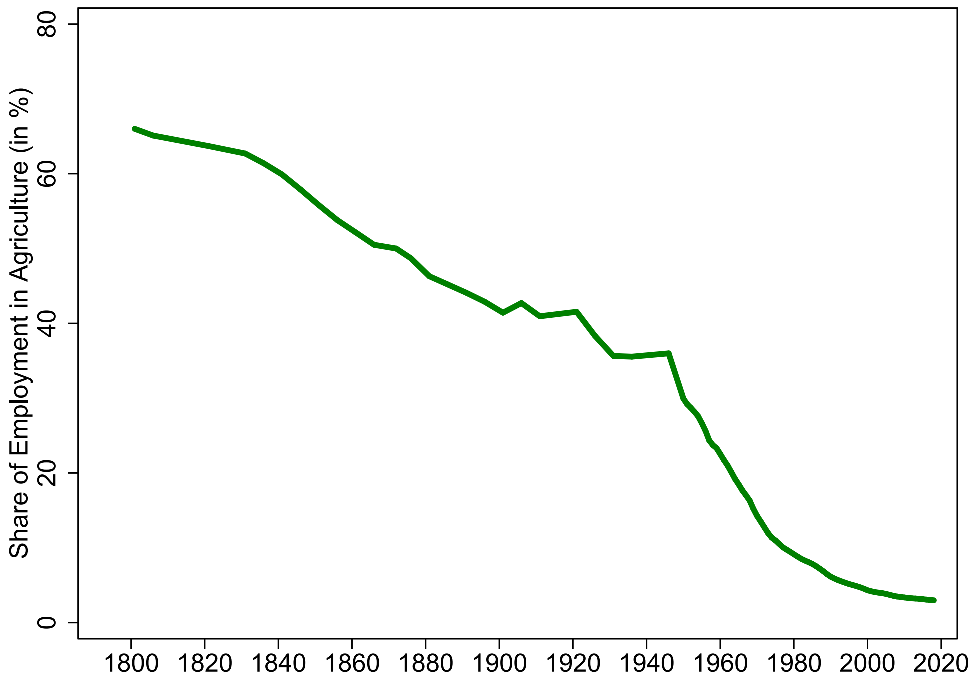


Figure 21: Shares of Employment in Agriculture (1806-2018).

A.4 Prices

Sources for sectoral prices. Data on agricultural producer prices are available over the period 1815-2019 using two different data sources: one derived from the national accounts in value added and volume from Toutain (1987, 1993) and one from INSEE post-1949.

Using Toutain (1993), we compute a price index of agricultural goods using the value added in agriculture divided by the production volume index in agriculture (period 1815-1990). Post WW2, INSEE directly provides a producer price index for agricultural goods (Indice des prix agricoles à la production, IPPAP)—the serie can be retropolated back to 1949.[47] These two series will be used to construct a price index for agriculture goods over the period 1815-2019 (with interruptions at war times). Similarly, a GDP deflator over the period 1815-1960 can be computed using GDP at current prices and a GDP volume index from Toutain (1987). Post-1960, we use the GDP deflator from the World Bank.[48] The price index for agricultural products and the GDP-deflator are both normalized to 100 in 1949.

Relative price for agricultural goods. Using the computed historical time-series for the agricultural producer price index and the GDP-deflator, one can take the ratio of the two series to shed some lights on the evolution of the relative prices of agricultural goods. The serie for the

[47] The IPPAP serie is the 'Base 2000 rétropolée' available in Insee Méthodes 114 (INSEE (2006)). Until 1970, the retropolated serie from INSEE excludes fruits and vegetables. The serie including fruits and vegetables and the one excluding them are almost identical when both are available.

[48] We checked consistency with the consumer price index available over the period 1820-2015 (INSEE). Inflation is very similar in both series.

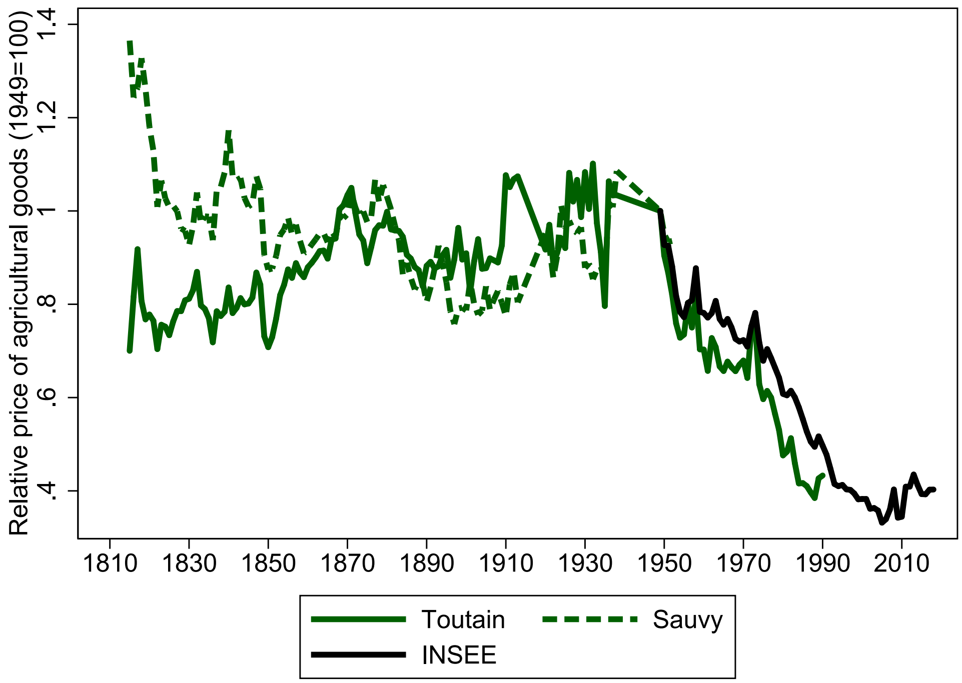


Figure 22: Relative prices of agricultural goods, 1949=100 (1815-2019).

relative price based on Toutain production data (solid green) over the period 1815-1990 and the INSEE producer price (solid black) starting 1949 are shown in Figure 22. While the relative price of agricultural appears fairly stable until 1910, it exhibits later a clear downward trend over the twentieth century. Both series show a similar trend post WW2.

Our baseline price index of agricultural goods (denoted P_{agri}) uses the series computed using the national accounts of Toutain prior to WW2 (1815-1938) and the agricultural producer prices by INSEE post WW2 (1949-2019). The two series are linked by the same normalization to 100 in 1949. The final serie for P_{agri} is only interrupted during the wars.

The model counterpart of our data is the relative price of rural/agricultural goods over the price of urban/non-agricultural goods. The latter is not observed but can be backed out using the GDP-deflator under some assumptions. Let us denote $P_{agri,t}$ the price index for agricultural goods at date t, $P_{non-agri,t}$ the price index for non-agricultural goods and $P_{GDP,t}$ the GDP-deflator. We assume the following relationship at each date t,

$$P_{GDP,t} = P_{agri,t}^{\omega_t} P_{non-agri,t}^{1-\omega_t}, \tag{29}$$

where ω_t is the share in value-added of agricultural goods computed using historical national accounts. Since we observe in the data all the variables but $P_{non-agri,t}$, we can invert Eq. 29 to back out a price index for non-agricultural goods (urban goods including manufacturing and services),

$$P_{non-agri,t} = P_{agri,t}^{-\frac{\omega_t}{1-\omega_t}} P_{GDP,t}^{\frac{1}{1-\omega_t}}.$$

We are now equipped with a price index for agricultural goods, non-agricultural goods and a GDP deflator over the period 1815-2019.

Sensitivity analysis for the price of agricultural goods. Before WW2, the Statistique Generale de France (former INSEE), in particular thanks to the work of Alfred Sauvy, provides an alternative serie for the price of agricultural goods: 'indice des prix de gros agricoles' which is constituted by a basket of 19 raw agricultural commodities (food related).[49] The serie is retropolated back to 1810 by A. Sauvy (see Sauvy (1952)). This data includes some foreign commodities (e.g. English and US corn prices) and is in part computed using customs price data. For this reason, we use the price of agricultural goods computed using production data of Toutain pre WW2 as baseline. This said, the 'indice des prix de gros agricoles' still contains useful information regarding the price of agricultural goods in France before WW2. Comparison with the price computed using production data from Toutain indicates that the two series exhibit very similar patterns starting 1850. Prior to this date, the 'indice des prix de gros agricoles' from Sauvy (1952) exhibits a significant downward trend, while our baseline from Toutain stays roughly stable (see Figure 22).[50] We use the serie from Sauvy (1952) as sensitivity analysis—our baseline price serie for agricultural goods uses the serie based on Toutain for the period pre WW2.

A.5 Sectoral Productivities

Equipped with sectoral value added at current prices, sectoral price indices, sectoral employment and land use data, one can back out the sectoral productivities (in the agricultural and non-agricultural sector) that are the counterpart of the model (the θs) up to a constant of normalization.

Urban Productivity. Let us start with the urban/non-agricultural sector. According to the model production function, $\theta_u = \frac{Y_u}{L_u}$. We observe the value added in the non-agricultural sector at current prices. Deflating this serie by the constructed price index for non-agricultural goods gives Y_u. Dividing the latter variable by employment in the non-agricultural sector, $L_{non-agri,t}$, allows us to back out the empirical counterpart of $\theta_{u,t}$,

$$\theta_{u,t} = \frac{VA_{non-agri,t}}{P_{non-agri,t} L_{non-agri,t}}.$$

Due to the mere presence of a price index, this serie is defined up to a multiplicative constant. We normalize $\theta_{u,t}$ to unity in the first period (1815). This gives the time-series for $\theta_{u,t}$ plotted in Figure

[49] Details about the index can be found in the 'Etudes spéciales' of the 'Bulletin de la Statistique générale de la France' in 1911. Available online at: https://gallica.bnf.fr/ark:/12148/bpt6k96205098/f73.image

[50] We also compare those series with the relative price of corn. While significantly more volatile, the latter is also fairly consistent with the other series. A period of volatile relative corn price but fairly constant on average until the early 20th century, followed by a downward trend. The downward trend is however more pronounced.

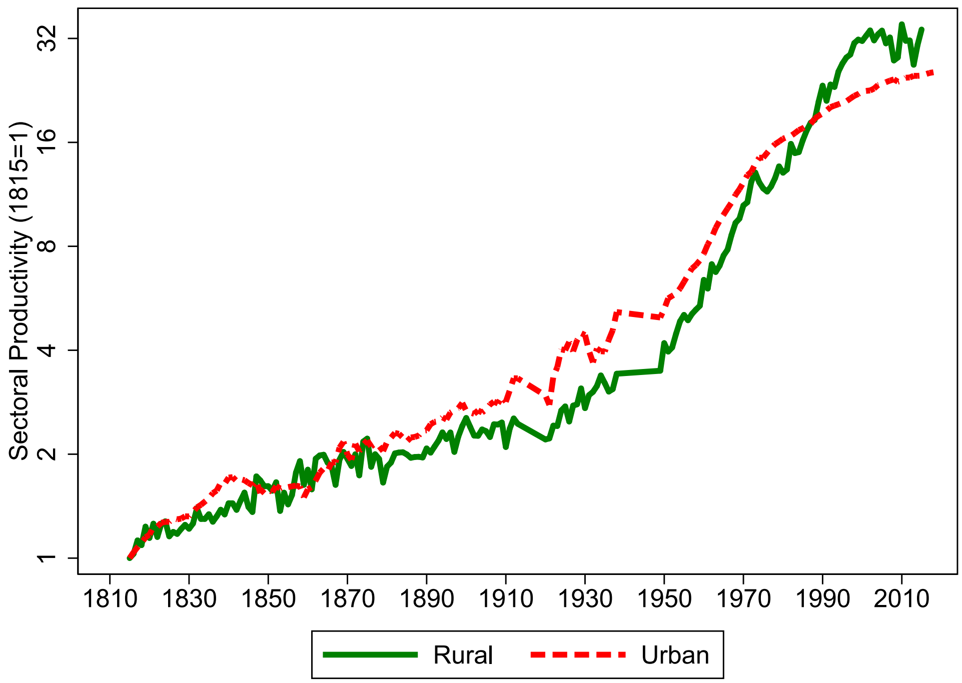


Figure 23: Rural and Urban Productivity, 1815=1 (1815-2019).

23 (dotted red line). This will be our baseline exogenous urban/non-agricultural productivity. It is important to note that the measured urban labour productivity includes technological advances in the non-agricultural sector but also factor accumulation rising labour productivity (physical and human capital accumulation).

Rural Productivity. We proceed in a similar fashion to compute the model's counterpart of the rural productivity, $\theta_{r,t}$, with one important difference: the agricultural output per worker in the rural sector depends also on the land per worker available for agriculture,

$$\frac{Y_r}{L_r} = \theta_r \left(\alpha + (1-\alpha)(\frac{S_r}{L_r})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \theta_r F(\frac{S_r}{L_r}). \tag{30}$$

Thanks to the data on land use in agriculture, one can back out from the data the land per worker in agriculture at each date: it is simply the cultivated area (SAU) divided by employment in agriculture, $\frac{S_r}{L_r} = \frac{SAU}{L_{agri}}$. Using Eq. 30, one can compute the rural productivity parameter, $\theta_{r,t}$, at each date,

$$\theta_{r,t} = \frac{VA_{non-agri,t}}{P_{non-agri,t} L_{non-agri,t}} \frac{1}{F(\frac{SAU_t}{L_{agri,t}})}.$$

With a unitary elasticity of substitution betweeen land and labor ($\sigma = 1$), this gives,

$$\theta_{r,t} = \frac{VA_{agri,t}}{P_{agri,t} L_{agri,t}} (\frac{SAU_t}{L_{agri,t}})^{\alpha-1}.$$

Due to the mere presence of a price index, this serie is defined up to a multiplicative constant. Like $\theta_{u,t}$, we normalize $\theta_{r,t}$ to unity in the first period (1815). This gives the time-series for $\theta_{r,t}$ plotted in Figure 23 (solid green line). This will be our baseline exogenous rural/agricultural productivity shifters.

Comments. Comparing urban and rural productivity, one notices the important common component: this can be due to technological advances benefiting both sectors but also to physical and human capital accumulation, which increase labour productivity across the board. Focusing on the more sectoral specific component, it is visible that non-agricultural productivity grew faster from the late 19th century until WW2. Post WW2, agricultural productivity starts growing at a faster speed, catching-up with the non-agricultural one and eventually outpacing it. This is consistent with Bairoch's view that starting the agricultural crisis in late ninetheeth century, technological progress in the French agriculture is slow and delayed relative to other countries, before a catching up post WW2. The period 1945-1985 period is more broadly characterized by a very fast technological progress in agriculture across developed countries (see Bairoch (1989)). A productivity slowdown is later observed in both sectors.

A.6 Consumption expenditures

Sources. Data on consumption expenditures are available using two different data sources. Pierre Villa provided data on consumption expenditures across 24 different categories of goods for the period 1896-1939.[51] INSEE provides data over the period 1959-2017 on personal consumption expenditures ('Consommation effective des ménages par fonction aux prix courants') across 12 broad categories (food, drinks, clothing, housing, transportation,...) and about 100 narrower categories. INSEE Data are from the Comptes nationaux (Base 2014).[52]

Expenditure shares. We compute expenditure shares on three broad categories: food/drinks, housing and the remaining goods. The expenditure share on food/drinks proxy for the rural goods. The expenditure share outside food, drinks and housing includes manufacturing goods and services. It proxies for the spending share on urban goods. The model's counterparts of the three category are thus respectively expenditure shares on rural goods, housing and urban goods. With some abuse of language, we now refer to these three categories as rural, housing and urban.

The expenditure share on food/drinks is computed by adding all the good categories corresponding to food and drinks consumption divided by aggregate household expenditures (for the pre and post WW2 data). However, it excludes consumption in restaurants that will enter the remaining category (urban goods). The housing expenditure shares include housing related expenses: rents (effective

[51] Data are publicly available thanks to the CEPII. For details and documentation, see http://gesd.free.fr/villadoc.pdf. See also Villa (1993).

[52] Over the period 1950-1958, the CREDOC was providing data on consumption expenditures across broad categories for French households. These data have not been made compatible with the INSEE data post-1959, when INSEE revised the methodology. Investigating data in reports by CREDOC provides some additional insights on consumption expenditure shares in the 1950s across broad categories. As expected, these shares are in between the ones computed using the data from Villa right before WW2 and the later national accounts data of INSEE.

and imputed), energy expenditures, some housing services (garbage, cleaning, repair, ...) but also housing equipments (furniture, tableware, household appliances...).[53]

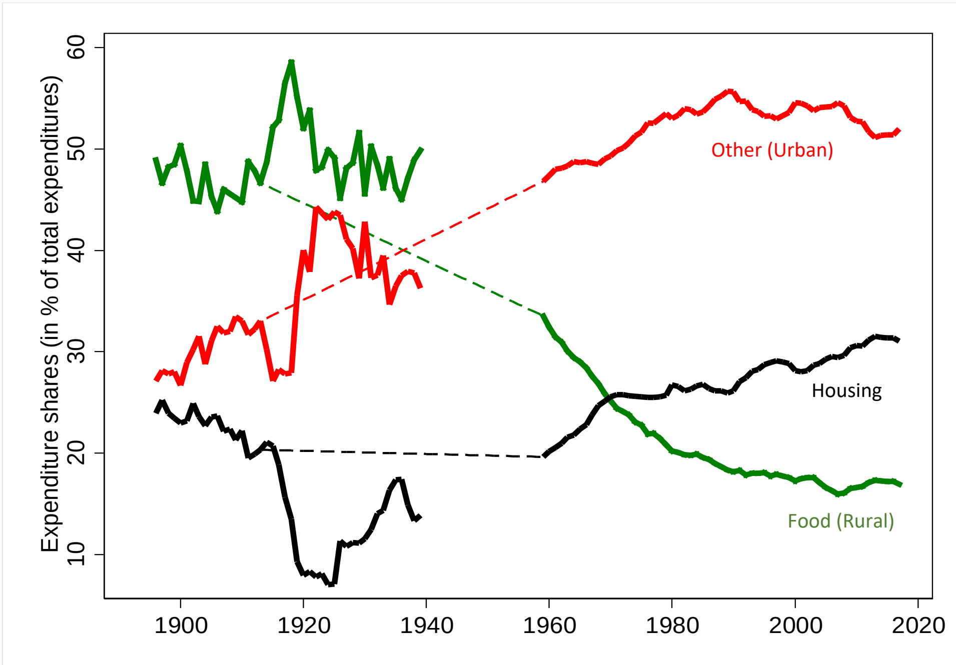


Figure 24: Spending Shares for Rural, Urban and Housing goods.

Notes: The observations around WW2 missing due to difficulties in data collection.

Data on expenditure share across these three broad categories are shown in Figure 24. Comparing the initial periods in the late nineteenth century to today gives the following broad facts: the food (rural) share went down from almost 50% of expenditures to 17%; the housing share increased slightly to 23% to 31%; the share of expenditure on other goods increased as a consequence from 27% to more than 50%. This reallocation of expenditures way from rural goods towards housing and urban goods fits well with the process of structural transformation.

Rent control and the housing expenditure share. An important issue is the significant and persistent dip of the housing expenditure share starting at WW1. This evolution is largely due to the presence of rent controls that were put in place at the beginning of WW1 in France. As the French government wanted families to be able to afford their home during the war, it decreeed that rents would be blocked (in nominal terms). As inflation picked up, this generated a large fall in real housing rents. As rents were very cheap, it freed up resources for households that could be spent on other goods (rural and urban). This is immediately visible on Figure 24, where the share of expenditures on housing went down from 21% in 1914 to less than 10% at the end of the war in 1919—other expenditure shares increasing simultaneously. While the measure want meant

[53]We include housing equiements as (partly) furnished/equipped houses/flats are quite common—even in the early 20th century. Small furnished flats/bedrooms were very common in large cities in the interwar period ('garnis'). However, excluding the latter categorie from housing expenses does not affect our results.

to be temporary, rent control lasted effectively during the whole interwar period despite various modification in the laws. It was eventually profoundly reformed post WW2 in 1948.[54] The reform of 1948 led to a sluggish adjustment of rents and it took some further years before one can reasonably argue that the rent control put in place in 1914 starts playing a more minor role.[55] Given this, our aim is to match the long-run evolution of spending shares while abstracting from the fluctuations in between 1914 and 1959 (first year of observation in the serie provided by INSEE), as illustrated by the dotted lines on Figure 24.

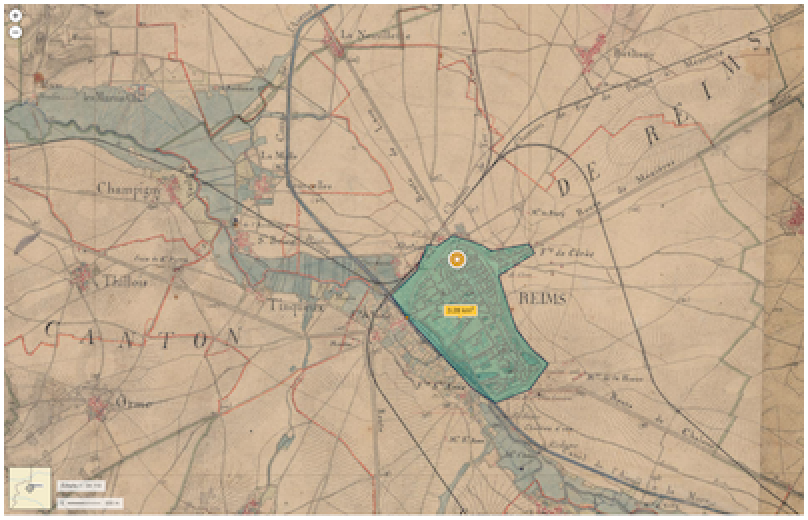
A.7 Measurement of cities

Area Measurement of cities. to be written.

[54] Rents did increase in real terms during the interwar period. However, regulations still significantly limited the rent increases. The reform of 1948 still kept some housing with regulated cheap rents. Rents could be changed for new renters. Few housing with very cheap rents under the special regime of 1948 still subsists.

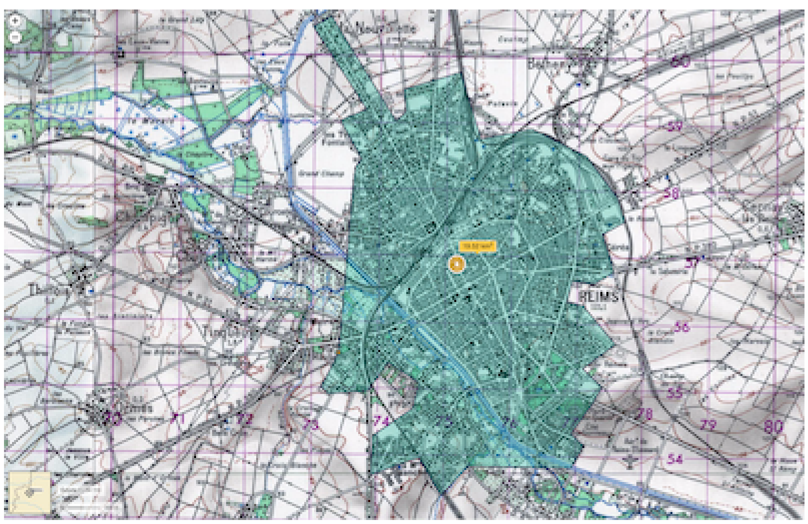
[55] Data from CREDOC in the early 1950s suggests a fairly housing spending share at that time–around 15%.

Figure 25: Reims from above in 1866.



Source: Carte d'Etat Major and IGN.

Figure 26: Reims from above in 1950.

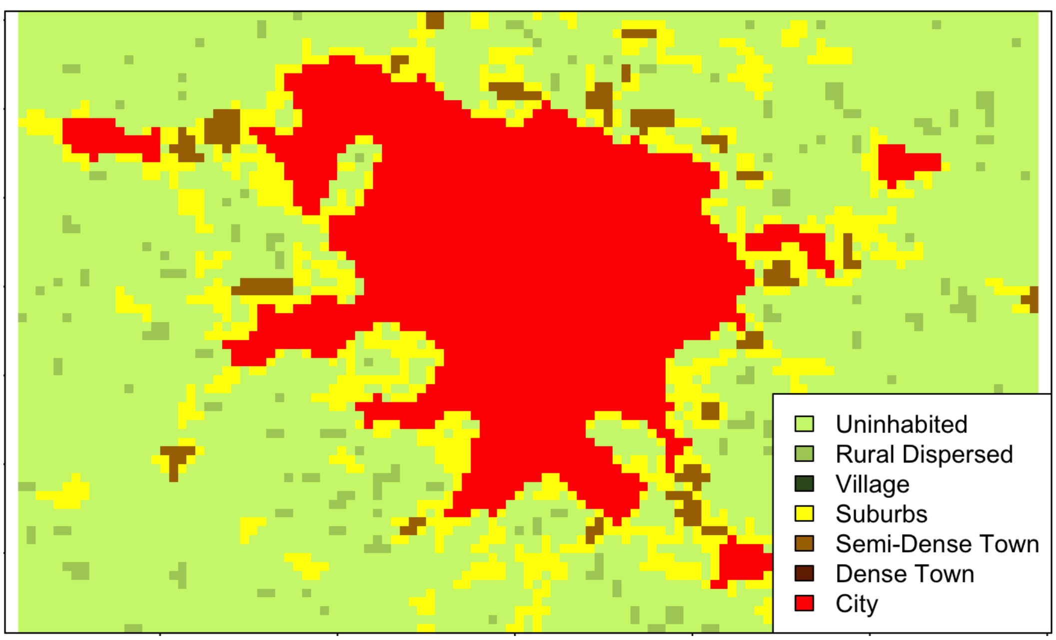


Source: IGN.

Figure 27: Reims from above in 2015.

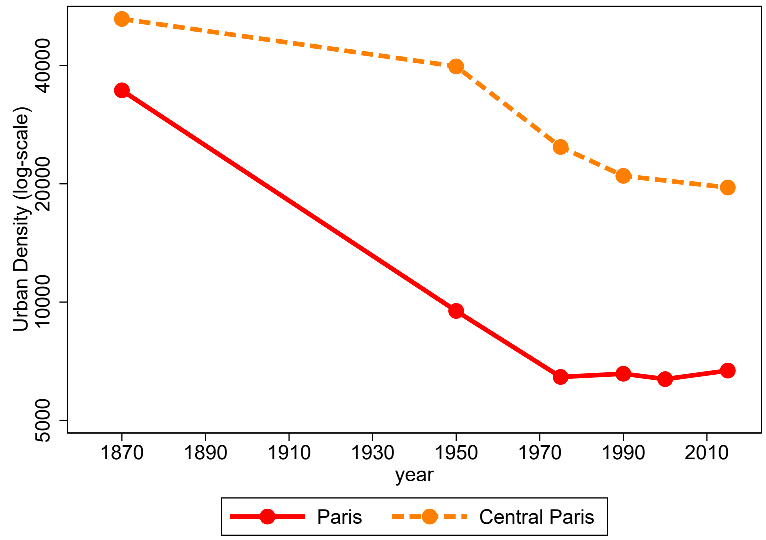
Source: IGN.

Figure 28: Satellite data: Paris from above in 2015.



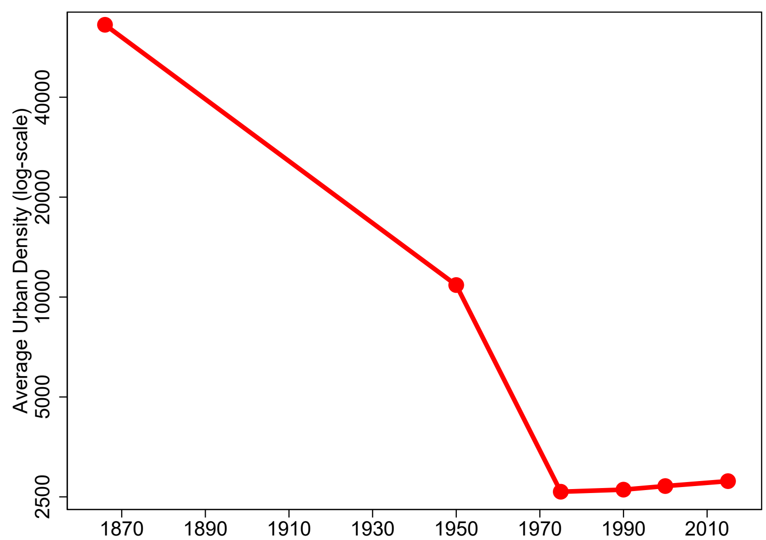
Source: GHSL.

Figure 29: The historical decline in urban density, Paris.



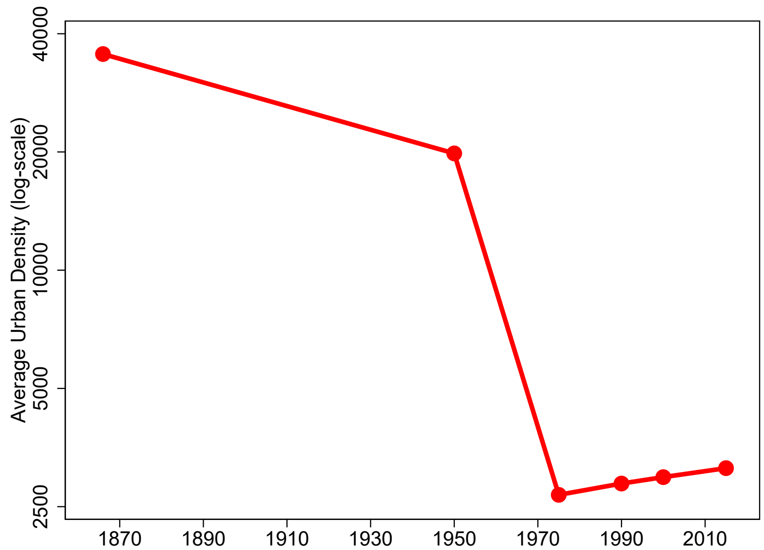
Notes: The solid line shows the urban density in Paris; the dotted line shows the density in Central Paris (districts 1 to 6). *Source*: Etat major, IGN, GHSL and Census.

Figure 30: The historical decline in urban density, Lyon.



Notes: The solid line shows the urban density in Lyon. *Source*: Etat major, IGN, GHSL and Census.

Figure 31: The historical decline in urban density, Marseille.



Notes: The solid line shows the urban density in Marseille. *Source*: Etat major, IGN, GHSL and Census.

B Quantitative Model Appendix

B.1 Extensions to the baseline theory

The quantitative model expands the baseline theory of Section 3 by considering a more general production for the rural good, a more general specification for the commuting costs, by allowing for location-specific housing supply conditions and by considering a circular city on a surface. We also introduce an intertemporal utility to pin down the path for the real interest rate. We describe how these extensions modify the equations of Section 3 necessary to compute the equilibrium allocation.

CES production in the rural sector. We perform sensitivity analysis with respect to the elasticity of substitution between land and labor in the rural sector. In the extended version, the production of the rural good uses labor and land according to the following constant returns to scale technology

$$Y_r = \theta_r \left(\alpha (L_r)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(S_r)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where L_r denotes the number of workers working in the rural (agricultural) sector, S_r the amount of land used for production and θ_r a Hicks-neutral productivity parameter. $0 < \alpha < 1$ is the intensity of labor use in production. $\sigma \geq 0$ is the elasticity of substitution between labor and land, $\sigma = 1$ corresponding to the baseline version. Rural workers and land are paid their marginal productivities such that Eqs. 2 and 3 become,

$$w_r = \alpha p \theta_r \left(\alpha + (1-\alpha) \left(\frac{S_r}{L_r} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}}, \tag{31}$$

$$\rho_r = (1-\alpha) p \theta_r \left(\alpha \left(\frac{L_r}{S_r} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \right)^{\frac{1}{\sigma-1}}, \tag{32}$$

where w_r is the rural wage and ρ_r the rental price of land anywhere in the rural sector and p the relative price of the rural good in terms of the numeraire urban good. Note that it is useful to express the price of land relative to wages,

$$\rho_r = \left(\frac{1-\alpha}{\alpha} \right) w_r \left(\frac{L_r}{S_r} \right)^{\frac{1}{\sigma}}. \tag{33}$$

Note that due to the CES technology, the rental price of land increase with (rural) wages with a unitary elasticity and with population working in the rural sector L_r with an elasticity $1/\sigma$—stronger complementarities between land and labor implying a larger fall of land prices if workers are reallocated to urban production.

Commuting costs. We adopt a more general specification for the commuting costs f. The cost $f = f(\ell, m, w_u)$ depends on the tranportation mode/speed, the location ℓ and labor costs w_u. Faster and longer commutes are more expensive and $f(\ell, m, w_u)$ is increasing in m and ℓ, with $\frac{\partial^2 f}{\partial^2 \ell} \leq 0$. The latter technical assumption makes sure that the importance of the cost f (relative to the opportunity cost of time) decreases as the commuting distance increases. The cost f also increases with the labor costs, w_u, with $\frac{\partial^2 f}{\partial^2 w_u} \leq 0$. This gives the following expression for the commuting costs, similar to Eq. 5,

$$\tau(l) = f(\ell, m, w_u) + 2\zeta w_u. \left(\frac{\ell}{m} \right). \tag{34}$$

For tractability, we will use the following functional form for f,

$$f(\ell, m, w_u) = \frac{c_\tau}{\eta_m}.m^{\eta_m}.l^{\eta_\ell}.w_u^{\eta_w}, \tag{35}$$

with $\eta_m > 0$, $0 \leq \eta_\ell < 1$, $0 \leq \eta_w < 1$ and c_τ a cost parameter measuring the efficiency of the commuting technology.

An individual in location ℓ chooses the mode of transportation corresponding to speed m in order to minimize the commuting costs $\tau(\ell)$. This equalizes the marginal cost of a higher speed m to its marginal benefits in terms foregone wage,

$$\frac{\partial f}{\partial m} = 2\zeta.w_u \left(\frac{\ell}{m^2} \right).$$

Using Eq. 35, the optimal chosen mode/speed satisfies

$$m = \left(\frac{2\zeta}{c_\tau}\right)^{\frac{1}{1+\eta_m}} \cdot w_u^{1-\xi_w} \cdot \ell^{1-\xi_\ell}, \tag{36}$$

where $\xi_w = \frac{\eta_m+\eta_w}{1+\eta_m} \in (0,1]$ and $\xi_\ell = \frac{\eta_m+\eta_l}{1+\eta_m} \in (0,1]$. Using Eqs. 34-36, we get that equilibrium commuting costs satisfy,

$$\tau(\ell) = a \cdot w_u^{\xi_w} \cdot \ell^{\xi_\ell}, \tag{37}$$

where $a = \left(\frac{1+\eta_m}{\eta_m}\right) c_\tau^{\frac{1}{1+\eta_m}} (2\zeta)^{\frac{\eta_m}{1+\eta_m}} > 0$. Commuting costs are falling with improvements in the commuting technology (a lower a). They are increasing with the wage rate (the opportunity cost of time) and the distance of commuting trips with constant elasticities. This corresponds to the more general equation for commuting costs expressed in the quantitative section 4. It is also important to note that the parameters ξ_w (resp. ξ_ℓ) directly maps into elasticities of commuting speed to income (resp. commuting distance) through Eq. 36.

Location-specific housing supply conditions. As shown in Baum-Snow and Han (2019), the elasticity of housing supply to prices is lower closer to the CBD than at the urban fringe. We allow in the quantitative model for location-specific housing supply conditions. To do so, we assume that in each location ℓ, land developers supply housing space $H(\ell)$ per unit of land with a convex cost

$$c(\ell)\frac{H(\ell)^{1+1/\epsilon(\ell)}}{1+1/\epsilon(\ell)}$$

paid in units of the numeraire, where the costs paramters $c(\ell)$ and $1/\epsilon(\ell)$ can depend on the location. This is meant to capture that it might be more costly for developers to build closer to the city center than in the suburbs or the rural part of the economy. Profits per unit of land of the developers are

$$\pi(\ell) = q(\ell)H(\ell) - c(\ell)\frac{H(\ell)^{1+1/\epsilon(\ell)}}{1+1/\epsilon(\ell)} - \rho(\ell),$$

where $\rho(\ell)$ is the rental price of a unit of land in location ℓ. Similarly to the housing price $q(\ell)$ above, for locations beyond the fringe ϕ, the land rent is constant, hence $\rho_r = \rho(\ell \geq \phi)$.

Maximizing profits gives the following supply of housing $H(\ell)$ in a given location ℓ,

$$H(l) = \chi(\ell)q(\ell)^{\epsilon(\ell)}, \tag{38}$$

where the two parameters $\chi(\ell) = (1/c(\ell))^{\epsilon(\ell)} \geq 0$ and $\epsilon(\ell)$ summarize the housing supply conditions. The parameter $\chi(\ell)$ is a supply shifter—higher construction costs reduce the supply of housing in a given location.[56] The parameter $\epsilon(\ell)$ is the price elasticity of housing supply in location ℓ. More

[56] Note that $\chi(\ell)$ could have other interpretations linked to the regulatory environment of housing policy (imposed floor-to-area ratios, developable land constraints, building heights limits...). If each unit of land provides only a fraction that can be developed due to regulations, the supply of housing per unit of land is homogenous to our supply fonction through a lower supply shifter χ—χ incorporating the fraction of developable land per unit of land. χ could

convex costs to build intensively on a given plot of land reduces the supply response of housing to prices. In the rural area, housing supply shifters are assumed to be the same across locations, equal to χ_r— $\chi(\ell) = \chi_r$ for $\ell \geq \phi$. We also assume that supply conditions are more favourable in the rural area such that, $\chi_r \geq \chi(\ell)$ for $\ell \leq \phi$. Similarly, the housing supply elasticity is assumed constant in the rural sector, $\epsilon_r = \epsilon(\ell \geq \phi)$.

Lastly, free entry imply zero profits of land developers. This pins down land prices in a given location,

$$\rho(\ell) = \frac{q(\ell)H(\ell)}{1+\epsilon(\ell)} = \chi(\ell)\frac{q(\ell)^{1+\epsilon(\ell)}}{1+\epsilon(\ell)}, \tag{39}$$

Arbitrage across land usage imply that the latter land price must be in equilibrium above the marginal productivity of land for production of the rural good (Eq. (32)), where the condition holds with equality in the rural part of the economy, for $\ell \geq \phi$,

$$\rho_r = \chi_r \frac{(q_r)^{1+\epsilon_r}}{1+\epsilon_r} = (1-\alpha)p\theta_r \left(\alpha \left(\frac{L_r}{S_r}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right)^{\frac{1}{\sigma-1}}. \tag{40}$$

This last equation shows that a fall in the relative price of rural goods and/or a reallocation of workers away from the rural sector lowers the price of urban land at the fringe of cities.

Consider first locations within the city, $\ell \leq \phi$. Market clearing for housing in each location implies $H(\ell) = D(\ell)h(\ell)$, where $D(\ell)$ denotes the density (number of urban workers) in location ℓ. Within the city, the density $D(\ell)$ follows,

$$D(\ell) = \left(\chi(\ell)\frac{q_r^{1+\epsilon(\ell)}}{1+\epsilon(\ell)}\right)\frac{1}{\gamma_\ell}(w(\phi)+r+\underline{s}-p\underline{c})^{-1/\gamma_l}(w(\ell)+r+\underline{s}-p\underline{c})^{1/\gamma_l-1}, \tag{41}$$

where $\gamma_\ell = \frac{\gamma}{1+\epsilon(\ell)}$ represents the spending share on housing adjusted for the supply elasticity in location ℓ and the fringe housing price q_r satisfies $\rho_r = \chi_r\frac{(q_r)^{1+\epsilon_r}}{1+\epsilon_r}$. Integrating density defined in Eq. 46 across urban locations gives the total urban population,

$$L_u = \int_0^\phi D(\ell)d\ell = \int_0^\phi \left(\chi(\ell)\frac{q_r^{1+\epsilon(\ell)}}{1+\epsilon(\ell)}\right)\frac{1}{\gamma_\ell}(w(\phi)+r+\underline{s}-p\underline{c})^{-1/\gamma_\ell}(w(\ell)+r+\underline{s}-p\underline{c})^{1/\gamma_\ell-1}d\ell \tag{42}$$

Note that with homogenous supply conditions across locations, $\chi(l) = \chi_r$ and $\epsilon(\ell) = \epsilon_r = \epsilon$, Eq. 47 simplifies into the Eq. 22 of the baseline model.

$$L_u = \int_0^\phi D(\ell)dl = \rho_r \int_0^\phi \frac{1+\epsilon}{\gamma}(w(\phi)+r+\underline{s}-p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell)+r+\underline{s}-p\underline{c})^{\frac{1+\epsilon}{\gamma}-1}d\ell.$$

be also interpreted as regulatory limits to expand $H(\ell)$ per unit of land using intermediary input (e.g. limits to build higher). With a 'wedge' $\xi(\ell) < 1$ in the FOC of the maximization for $H(\ell)$, we get: $H(\ell) = \xi(\ell)(q(\ell)/c(\ell))^{\epsilon(\ell)}$, implying $\chi(\ell) = \xi(\ell)\,(1/c(\ell))^{\epsilon(\ell)}$.

In the rural area, $\ell \geq \phi$, market clearing for residential housing imposes

$$L_r\gamma\left(w_r + r + \underline{s} - p\underline{c}\right) = S_{hr}\chi_r\left(q_r\right)^{1+\epsilon_r} = S_{hr}(1+\epsilon_r)\rho_r,$$

where S_{hr} is the amount of land demanded in the rural area for residential purposes. This leads to the following demand of land for residential purposes in the rural area,

$$S_{hr} = \frac{L_r\gamma_r\left(w_r + r + \underline{s} - p\underline{c}\right)}{\rho_r}, \tag{43}$$

where $\gamma_r = \frac{\gamma}{1+\epsilon_r}$. The market clearing condition for land, Eq. 24 needs to be adjusted accordingly,

$$S_r = 1 - \phi - \frac{L_r\gamma_r\left(w_r + r + \underline{s} - p\underline{c}\right)}{\rho_r}. \tag{44}$$

The last modification regards the market clearing for urban goods. The amount of urban good used to produce housing becomes location specific. Eq. 28 becomes,

$$c_u + \frac{1}{L}\int_0^{\phi}\tau(\ell)D(\ell)d\ell + \frac{1}{L}\int_0^1\frac{\epsilon(\ell)}{1+\epsilon(\ell)}q(\ell)H(\ell)d\ell = y_u, \tag{45}$$

where $y_u = \frac{Y_u}{L}$ denotes the production per worker of the urban good.

City on a surface. The baseline model is extended on a surface (instead of a line), where the city is circular around its center—ϕ denotes the radius of the city. All equations are unchanged until the equation determining the city size (Eq. 47 below).

Consider first locations within the city, $\ell \leq \phi$. Market clearing for housing in each location implies $H(\ell) = D(\ell)h(\ell)$, where $D(\ell)$ denotes the density (number of urban workers) in location ℓ. Within the city, the density $D(\ell)$ follows,

$$D(\ell) = \left(\chi(\ell)\frac{q_r^{1+\epsilon(\ell)}}{1+\epsilon(\ell)}\right)\frac{1}{\gamma_\ell}(w(\phi) + r - p\underline{c})^{-1/\gamma_\ell}(w(\ell) + r - p\underline{c})^{1/\gamma_\ell - 1}, \tag{46}$$

where $\gamma_\ell = \frac{\gamma}{1+\epsilon(\ell)}$ represents the spending share on housing adjusted for the supply elasticity in location ℓ and the fringe housing price q_r satisfies $\rho_r = \chi_r\frac{(q_r)^{1+\epsilon_r}}{1+\epsilon_r}$. Integrating density defined in Eq. 46 across urban locations gives the total urban population,

$$L_u = \int_0^{\phi} D(\ell)2\pi\ell d\ell = \int_0^{\phi}\left(\chi(\ell)\frac{q_r^{1+\epsilon(\ell)}}{1+\epsilon(\ell)}\right)\frac{1}{\gamma_\ell}(w(\phi) + r - p\underline{c})^{-1/\gamma_\ell}(w(\ell) + r - p\underline{c})^{1/\gamma_\ell - 1}2\pi\ell d\ell \tag{47}$$

Note that with homogenous supply conditions across locations, $\chi(l) = \chi_r$ and $\epsilon(\ell) = \epsilon_r = \epsilon$, Eq. 47 simplifies into the equivalent of Eq. 22 of the baseline model.

$$L_u = \int_0^{\phi} D(\ell)2\pi\ell d\ell = \rho_r\int_0^{\phi}\frac{1+\epsilon}{\gamma}(w(\phi) + r + \underline{s} - p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell) + r + \underline{s} - p\underline{c})^{\frac{1+\epsilon}{\gamma} - 1}2\pi\ell d\ell.$$

In the rural area, $\ell \geq \phi$, market clearing for residential housing imposes

$$L_r \gamma \left(w_r + r - p\underline{c}\right) = S_{hr} \chi_r \left(q_r\right)^{1+\epsilon_r} = S_{hr}(1+\epsilon_r)\rho_r,$$

where S_{hr} is the amount of land demanded in the rural area for residential purposes. This leads to the following demand of land for residential purposes in the rural area,

$$S_{hr} = \frac{L_r \gamma_r \left(w_r + r + \underline{s} - p\underline{c}\right)}{\rho_r},$$

where $\gamma_r = \frac{\gamma}{1+\epsilon_r}$. The market clearing condition for land, Eq. 24 needs to be adjusted accordingly,

$$S_r = 1 - \pi\phi^2 - \frac{L_r \gamma_r \left(w_r + r + \underline{s} - p\underline{c}\right)}{\rho_r}. \tag{48}$$

Aggregate land rents, rL, include the land rents generated both in the city and in the rural area,

$$rL = \int_0^{\phi} \rho(\ell) 2\pi\ell d\ell + \rho_r \times (1 - \pi\phi^2). \tag{49}$$

The aggregate per capita income y net of spatial frictions in the economy that is spent on both goods becomes,

$$y = r + \frac{L_r}{L} w_r + \frac{1}{L}\int_0^{\phi} w(\ell) D(\ell) 2\pi\ell dl. \tag{50}$$

The market clearing condition for rural goods is unchanged . The last modification regards the market clearing for urban goods,

$$c_u + \frac{1}{L}\int_0^{\phi} \tau(\ell) D(\ell) 2\pi\ell dl + \frac{1}{L}\int_0^{\phi} \frac{\epsilon(\ell)}{1+\epsilon(\ell)} q(\ell) H(\ell) 2\pi\ell dl + \frac{1}{L}\frac{\epsilon_r}{1+\epsilon_r} q_r H_r = y_u, \tag{51}$$

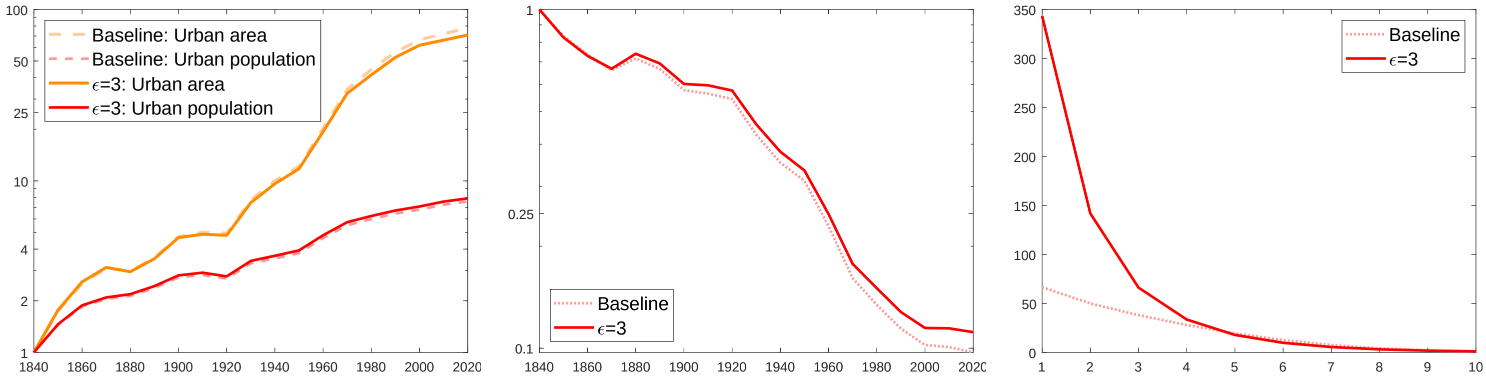
where $y_u = \frac{Y_u}{L}$ denotes the production per worker of the urban good.

Intertemporal Utility. to be written.

B.2 Sensitivity analysis and further extensions

This section shows results of the sensitivity analysis not displyed in the main text and provides further extensions to the baseline quantitative model as robustness. In particular, we set-up the model with multiple cities.

Housing supply elasticity. We perform sensitivity analysis assuming an housing supply elasticity ϵ equal to 3 in all locations. Results are displayed in Figure 32 for the some variables pertaining to urban expansion and density. Outcomes in the baseline simulation are shown for comparison purposes.

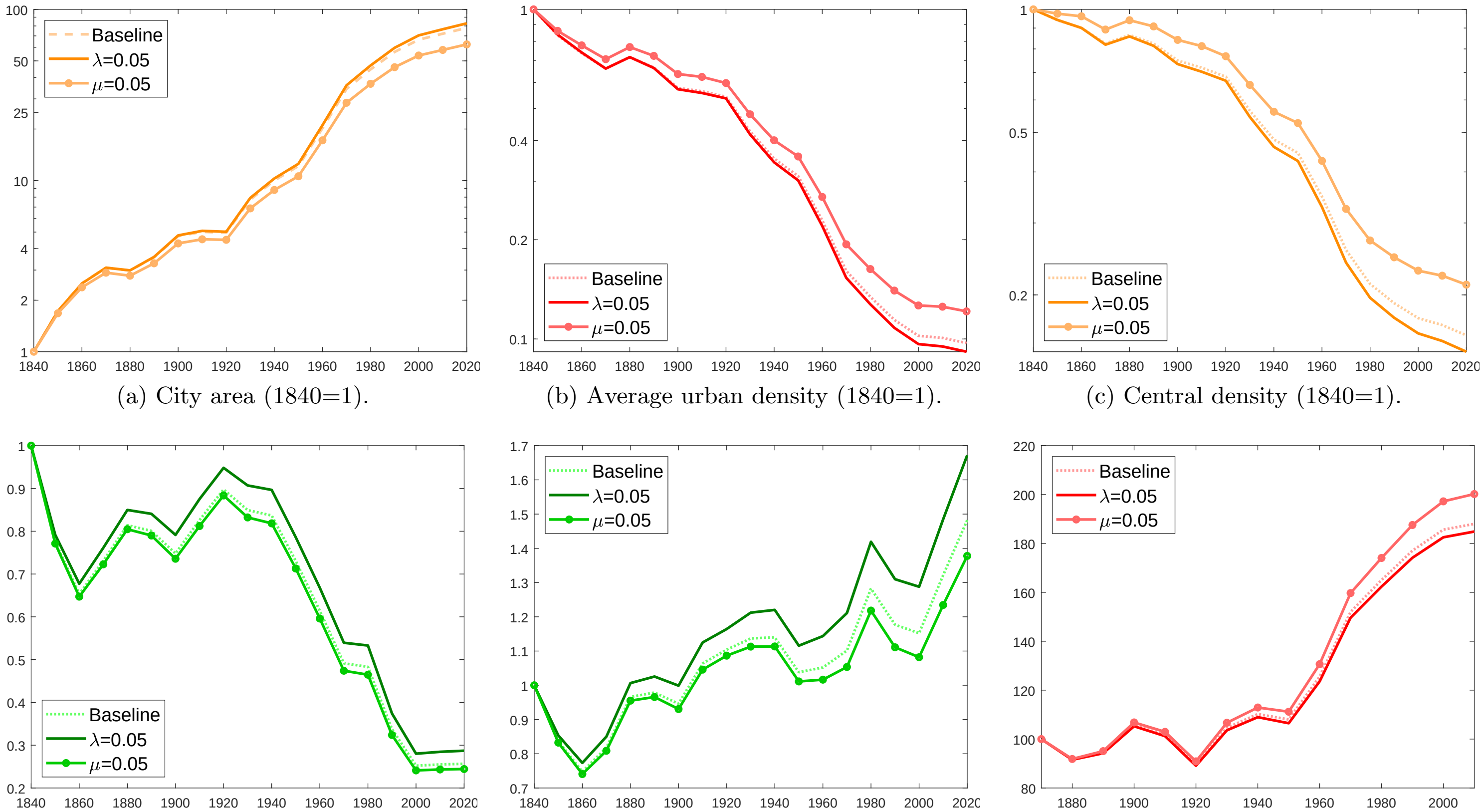


(a) City size (Index, 1840=1). (b) Average urban density (Index, 1840=1). (c) Density by decile (Year 2010).

Figure 32: Constant housing supply elasticity, $\epsilon = 3$.

Notes: The housing supply elasticity ϵ is set to 3 in all locations. Other parameters set to their baseline value of Table 1. Outcomes of interest with constant elasticity, $\epsilon = 3$, are displayed with a solid line. The baseline simulation is shown with a dotted line for comparison.

Agglomeration and Congestion. Develop here the set-up.



(a) City area (1840=1). (b) Average urban density (1840=1). (c) Central density (1840=1).

(d) Relative price of rural good (1840=1). (e) Rental value of farmland (1840=1). (f) Real Housing price index (1870=100).

Figure 33: Agglomeration and congestion forces.

Notes: The solid line represents outcomes in presence of agglomeration forces, with parameter $\lambda = 0.05$. The solid line with dots represents outcomes in presence of congestion forces, with parameter $\mu = 0.05$. Other parameters set to their baseline value of Table 1 up to a normalization of the initial urban productivity. For comparison, outcomes of the baseline simulation are shown with a dotted line.

Multiple cities.

B.3 Solution method

to be written