Introduction to the monocentric Urban Model

Graduate Labor 2017

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Sciences Po

ToC

Intro

- The von Thünen Model
- Spatial Equilibrium Assumption
- Tradeoffs in Urban Economic environment:
  - Accessibility vs
  - Congestion costs
- Related to Firm location decisions:
  - Agglomeration forces
    - silicon valley
  - Agglomeration costs
    - traffic, house prices, crime, waste, etc

The Von Thünen Model (1828), a simple version

- There is an isolated city in a featureless plain (i.e. $\mathbb{R}^2$)
  - no streets, woods, rivers or mountains.
  - land is equally productive everywhere.
- Individuals can earn a wage $w$in the city as laborer or a price p if they work the land sell the crops.
- Crop production is Leontief, i.e the farmer needs for 1 unit of crop the farmer needs:
  1. one unit of labor (he supplies that)
  2. one unit of land (he rents that from a landlord).
- Transport cost to the city (the market) is linear in distance x.
- The rent of land at distance x is $P(x)$.
- Therefore, net income of a farmer at x is

$$y(x) = p - \tau x - P(x)$$

Von Thünen Land Rent Function

- We need a spatial equilibrium condition s.t. a stable number of people choose to become wage workers and farmers:

$$y(x) = w, \text{ for } x \leq \bar{x}$$

- Then the von Thünen rent is the maximum rent a farmer could pay at x before making a loss:

$$P(x) = p - w - \tau x, \text{ for } x \leq \bar{x}$$

  - Rent decreases with distance to the city.
- If we assume that beyond $\bar{x}$ the rent is zero, i.e. $P(\bar{x}) = 0$, we get the radius of arrable land as

$$\bar{x} = \frac{p - w}{\tau}$$

  - Higher price p or lower transport τpushes the maximal distance $\bar{x}$further out.

Which Crops are planted where?

- Suppose we have multiple crops i with $p_i > p_{i+1}$ and $\tau_i > \tau_{i+1}$
- Farmers will put land to it's most productive use.
- Higher yield crops that are more expensive to transport are produced closer to the market.
    - Dairy Farming
- This produces a rent function that is convex over distance.
- Could have setup with different labor intensity for producing different products.
    - most labor intensive product is closest to city

Von Thünen Rings

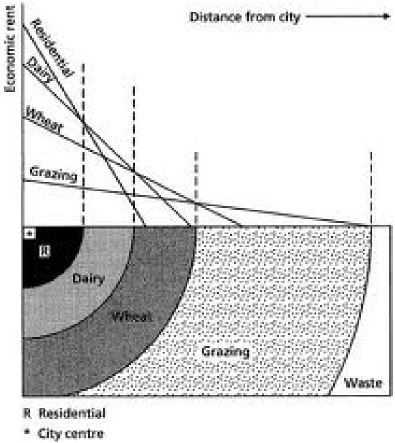


Figure: http://postoilgeography.blogspot.fr/2012/10/remembering-von-thunen.html

The Monocentric Model of the City

1. We assume a city has one unique center, the *central business district*, CBD, where all firms are.
2. The shape of the city could be circular, or a line. We will work with a line. (It's a line segment on $\mathbb{R}$)
3. The CDB is represent by a point $x = 0$.
4. All workers have to commute to the CDB to work, and they face commuting costs.
5. They have to acquire housing services.
6. This model allows us to study how house prices vary with distance from the CDB, along with housing consumption, land prices, construction density and population density.
7. It is a good model to illustrate the costs associated with agglomeration effects.

Preferences

- Consumers consume a numeraire composite good z and housing h, and

$$u(h, z)$$

  is a utility function that's increasing in both arguments.
- Housing is allocated competitively to the highest bidder at each location.
- Commuting costs are linear in distance
- If $P(x)$ is price of housing, and w is the wage, the budget constraint is

$$w - \tau x = P(x)h + z$$

Population

- There are N individuals living as workers in the city.
- They all have identical preferences (in particular, nobody intrinsically values a certain location over another, given h, z)

First Simple Consumer Problem: von Thünen Consumers

- To start, assume there is no choice about housing $h = \bar{h}$.
- Then, given the price function, the consumer chooses where to locate

$$\max_{x>0} u(w - \tau x - P(x)\bar{h}, \bar{h}) \tag{1}$$

- Given **perfect mobility** (zero moving costs), utility is the same everywhere:

$$u(w - \tau x - P(x)\bar{h}, \bar{h}) = \bar{u}, \forall x \leq \bar{x}$$

- The FOC of (1) yields

$$P(x)' = -\frac{\tau}{\bar{h}}$$

- Alternative use of land beyond $\bar{x}$ at rent $\bar{p} \geq 0$ is the boundary condition to get equilibrium rent:

$$P(x) = \bar{p} + \frac{1}{\bar{h}} \int_x^{\bar{x}} \tau d\tau$$

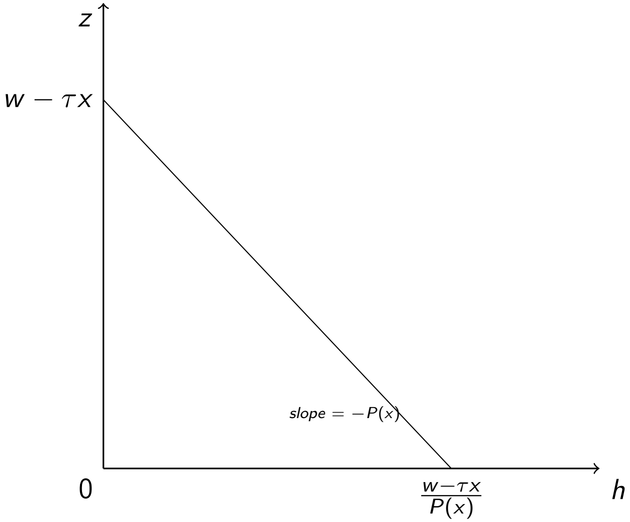
[Aside] Why do we need a special theory for that?

- Why can't we use the standard consumer model for this? Arrow-Debreu?
- Standard model is based on convex production sets for firms, i.e. no *increasing returns to scale (IRS).*
- We think that IRS, i.e. agglomeration forces, are an important feature of cities. Why else are they so productive?
- Endowed with *space,* the standard model predicts a form of *backyard capitalism*: we all work at home.
- Spatial Impossibility Theorem, see [Fujita and Thisse(2013)] chapter 2.
    - The standard model is unable to produce differential land rent if space is homogeneous (i.e. a featureless plain)

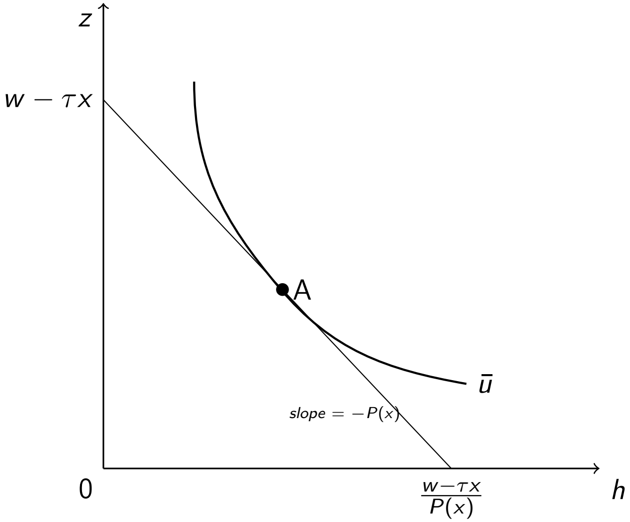
More complete Consumer's Problem

- Now we allow for the choice of h as well.
- Where to locate (x)?
- How much z?
- How are these choice going to influence the price function $P(x)$?

Back to Intuition



Back to Intuition



Differences to standard model

- 2 differences to standard model:
    - choose location x
    - choose between z and h, where $P(x)$ varies endogenously.
- Consumer computes optimal z, h at each location, and then picks location with highest utility.

Recap of Main Assumptions

1. The City is a line.
2. Only reason for travel is commute to work.
3. Proportionally increasing commuting cost, paid for in numeraire good.
4. Static model.
5. Exogenous geography of jobs.
6. Homogeneous residents.
7. Perfect mobility, i.e. there is spatial equilibrium.

The Standard Approach

- This is a standard constrained utility maximization problem.
- How to bundle z, h in order to achieve maximal u under the budget constraint?

$$\max_{z(x),h(x)} u(h, z) \text{ subject to } w - \tau x = P(x)h + z$$

- We can substitute for z in the utility function, and obtain

$$\frac{\partial u}{\partial h} - \frac{\partial u}{\partial z} P(x) = 0 \Rightarrow P(x) = \frac{\frac{\partial u}{\partial h}}{\frac{\partial u}{\partial z}} \tag{2}$$

- Your standard first order condition: the ratio of relative prices is equal to the ratio of marginal utilities.
- We get the marshallian demand for $z$by using Marshallian demand for housing and the budget constraint:

$$z(x) = w - \tau x - P(x)h(x)$$

The Standard Approach

- Given equal utility for all individuals, we get

$$u(h(x), w - \tau x - P(x)h(x)) = \bar{u} \tag{3}$$

- Totally differentiate that wrt x:

$$\frac{\partial u}{\partial h}\frac{\partial h(x)}{\partial x} - \frac{\partial u}{\partial z}P(x)\frac{\partial h(x)}{\partial x} - \frac{\partial u}{\partial z}\left(\tau + h(x)\frac{dP(x)}{dx}\right) = 0$$

- By the envelope theorem the first 2 terms cancel out, (just plug in (2) for P) and we get

$$\frac{dP(x)}{dx} = -\frac{\tau}{h(x)} < 0 \tag{4}$$

which is the *Alonso-Muth* condition.

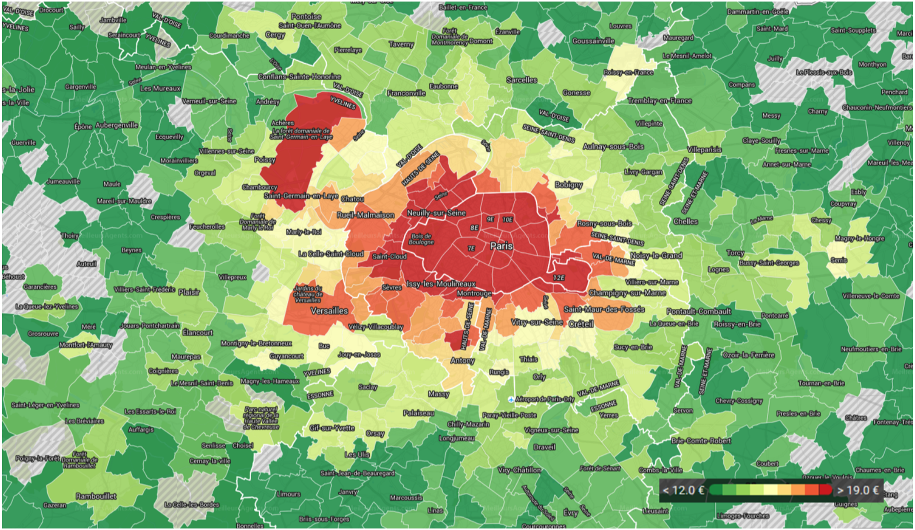
Alonso-Muth Condition

- [Alonso et al.(1964), Mills(1967), Muth(1969)] were the main developers of the urban land use model.
- The condition in (4) is the first of 5 gradients predicted by the monocentric model.

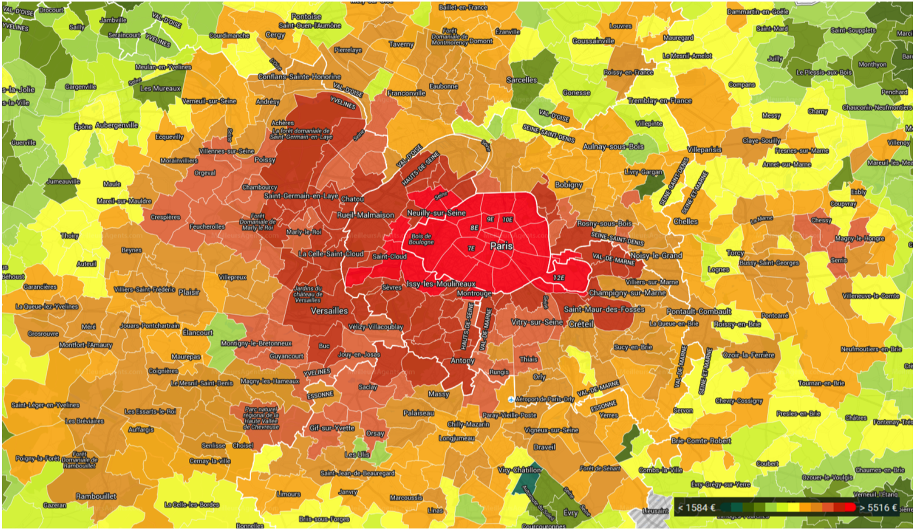
Gradient Number 1:

As consumers move further away from the CDB, the house price $P(x)$ declines. Furthermore, transport costs rise in proportion.

Gradient number 1: Parisian Rents per m^2



Gradient number 1: Parisian House price per m^2



The Bid Rent Approach

- Can get the Alonso-Muth condition more directly.
- Let $\Psi(x,\bar{u})$ be the maximum rent a resident would pay at x, achieving common $\bar{u}$

$$\Psi(x,\bar{u}) = \max_{h(x),z(x)} \left[P(x) | u(h,z) = \bar{u}, w - \tau x = P(x)h(x) + z(x)\right]$$

- Substitute budget constraint for P:

$$\Psi(x,\bar{u}) = \max_{h(x),z(x)} \left[\frac{w - \tau x - z(x)}{h(x)} | u(h,z) = \bar{u}\right]$$

- Recall the definition of the hicksian demand function in this case:

$$z(h(x),\bar{u}) \equiv \arg\min_{z} \frac{w - \tau x - z}{h(x)}, \text{ s.t. } u(h,z) = \bar{u}$$

The Alonso-Muth Condition, again

- Sub hicksian demand for z:

$$\Psi(x, \bar{u}) = \max_{h(x)} \left[\frac{w - \tau x - z(h(x), \bar{u})}{h(x)} \right] \tag{5}$$

- In Equilibrium, how do housing costs change as one moves a bit away from the CBD?

$$\left. \frac{d\Psi(x, \bar{u})}{dx} \right|_{h(x)=h\left(\underbrace{\Psi(x,\bar{u})}_{\text{maximal } P}, \bar{u}\right)} = -\frac{\tau}{h(x)} < 0 \tag{6}$$

- Again Alonso-Muth.
- In equilibrium (i.e. if $h(x) = h\left(\Psi(x, \bar{u}), \bar{u}\right)$), moving slightly further from CBD, housing costs (the highest bid) decrease proportionally to transport costs τ.

Housing Consumption

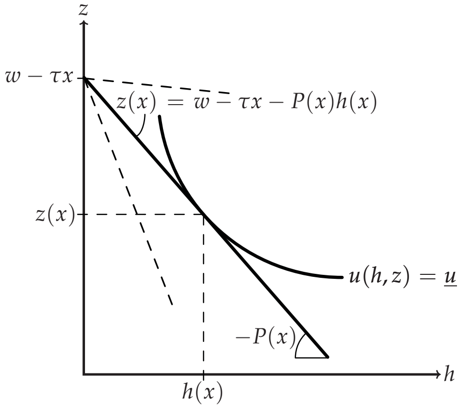
- Get the amount of housing consumption from the FOC of (5):

$$\frac{\partial z(h(x), \bar{u})}{\partial h(x)} h(x) + w - \tau x - z(h(x), \bar{u}) = 0$$

or

$$\underbrace{\frac{\partial z(h(x), \bar{u})}{\partial h(x)}}_{\text{slope of indiff curve}} = \underbrace{\frac{w - \tau x - z(h(x), \bar{u})}{h(x)}}_{\text{slope of BC}}$$

Finding Housing Demand



Panel (a)
Deriving housing prices in x

- differs from standard expenditure min problem
- there, shift budget parallel
- here, pivot.

Bid Rent: Example with Cobb-Douglas Utility

- Assume $u(h, z) = h^{\alpha} z^{1-\alpha}, 0 < \alpha < 1$
- what is $z(h(x), \bar{u})$?

Bid Rent: Example with Cobb-Douglas Utility

- Assume $u(h,z) = h^{\alpha} z^{1-\alpha}, 0 < \alpha < 1$
- what is $z(h(x), \bar{u})$?

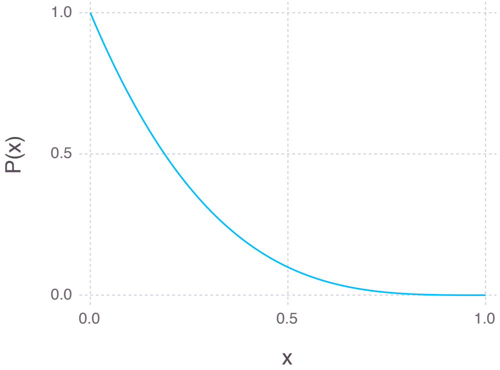
- just plug in $h^{\alpha} z^{1-\alpha} = \bar{u}$ to find $z(h(x), \bar{u}) = h(x)^{\frac{-\alpha}{1-\alpha}} \bar{u}^{\frac{1}{1-\alpha}}$
- in equation (5):

$$\Psi(x, \bar{u}) = \max_{h(x)} \left[\frac{w - \tau x - h(x)^{\frac{-\alpha}{1-\alpha}} \bar{u}^{\frac{1}{1-\alpha}}}{h(x)} \right]$$

Cobb-Douglas Price function

taking FOC and solving for h gives:

- $h(x) = \left(\frac{\bar{u}}{(1-\alpha)^{1-\alpha}(w-\tau x)^{1-\alpha}} \right)^{\frac{1}{\alpha}}$
- $\Psi(x, \bar{u}) = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\frac{w-\tau x}{\bar{u}} \right)^{\frac{1}{\alpha}}$



Lower House Price $\Rightarrow$ more housing

- Lower price $P(x)$ leads consumers to consume more housing.
- Differentiate the hicksian demand for housing wrt x

$$\frac{\partial h\left(P(x), \bar{u}\right)}{\partial u} = \underbrace{\frac{\partial h\left(P(x), \bar{u}\right)}{\partial P(x)}}_{(-)} \underbrace{\frac{dP(x)}{dx}}_{(-)} \geq 0 \tag{7}$$

- This is the second gradient:

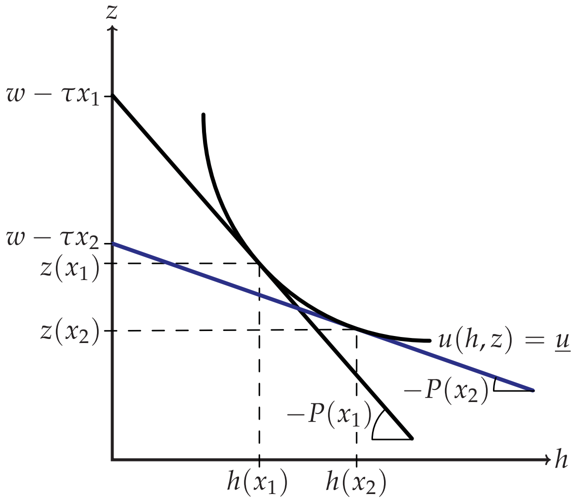
Gradient Number 2

Consumption of Housing increases with distance to the CDB. Note: this is a pure *substitution effect* (away from z and towards more h) since $\bar{u}$ is fixed.

Convex Price Function

- We have seen above that $P(x)$ is a convex, decreasing function.
- This is not an artefact of functional form assumptions.
- Taking the second derivative of $P(x)$ in the Alonso-Muth condition (4) gives $\frac{d^2 P(x)}{dx^2} > 0$

Compare locations $x_1 < x_2$: Shape of $P(x)$



- more remote x_2 has lower P
- adding more x_i's traces convex envelope P

Location Choice

- Remember the Alonso-Muth condition in (4)

$$\frac{dP(x)}{dx} = -\frac{\tau}{h(x)} < 0$$

and it's counterpart in (6)

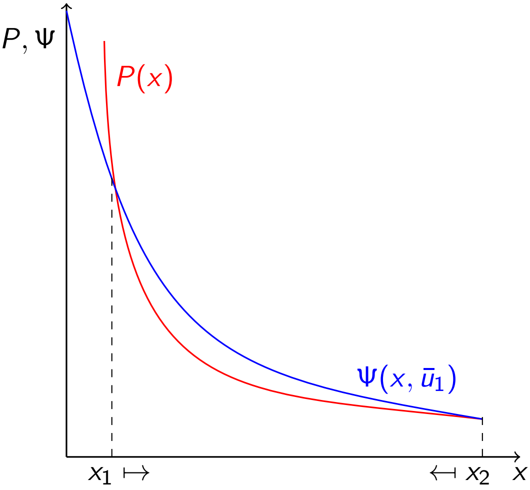
$$\left.\frac{d\Psi(x,\bar{u})}{dx}\right|_{h(x)=h\left(\underbrace{\Psi(x,\bar{u})}_{\text{maximal } P},\bar{u}\right)} = -\frac{\tau}{h(x)} < 0$$

- This implies

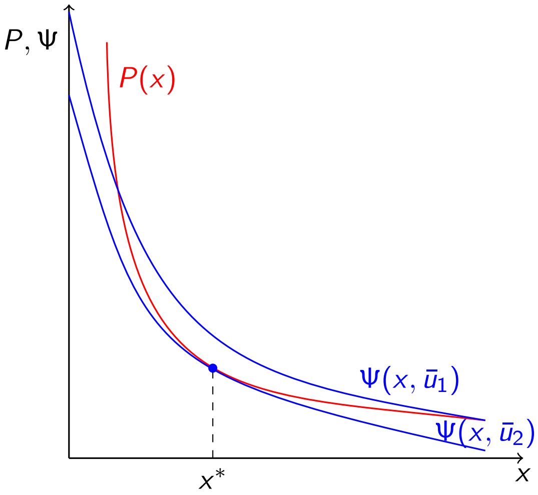
$$\frac{dP(x)}{dx} = \frac{d\Psi(x,\bar{u})}{dx} \tag{8}$$

i.e. optimal location choice occurs when the bid rent curve Ψ and the rental price curve P are tangent.

Location Choice



Location Choice



Land Use Equilibrium

- What happens at the city egde $\bar{x}$?
- Assume there is *other* use for land, here: agriculture.
    - Agricultural activity does *not* require commuting to CBD (we are not in 1828 anymore!).
    - Therefore famers' willingness to pay for land should be independent of x.
- The land market needs to be in equilibrium at any distance x.

Land Use Equilibrium within City

- Landlords let land to the highest bidder at each location.
- We know from equation (8) and the previous graph that optimality of consumers required that

$$\frac{dP(x)}{dx} = \frac{d\Psi(x, \bar{u})}{dx}, x < \bar{x}$$

- Landlords let land to the highest bidder at each location, i.e.

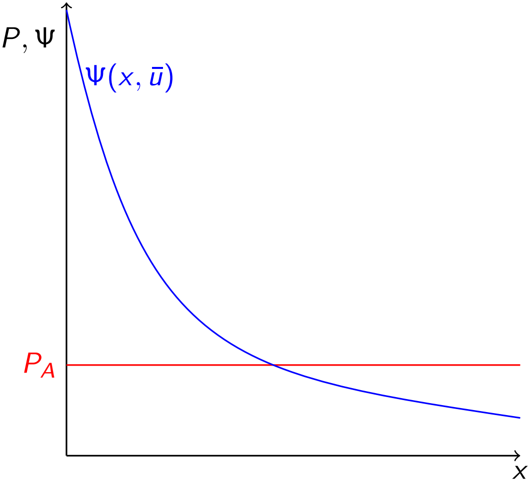
$$P(x) = \max\left(\Psi(x, \bar{u}), \text{farmer's bid}\right)$$

- How much is the farmer going to bid for land?

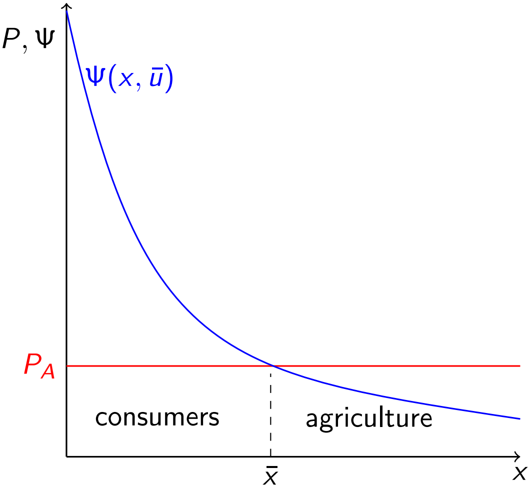
Farmer's Land Bid

- No commute $\Rightarrow$ no importance of being close to CBD.
- Assume that produces $Q = aL$, where $a > 0$ and L is land.
- Profit: $\pi_A = p_q Q - P(x)L = (ap_q - P(x))L$,
  - p_q is the price of agricultural good Q
  - $P(x)$ is still the rental price of land
- Free entry: $\pi_A = 0 \Rightarrow P(x) = ap_q$, i.e. $P(x) = P_A$, independent of x!

Equilibrium Land Price



Equilibrium Land Price



Equilibrium Land Price

- We can rewrite the price function as the *upper envelope* of those bids:

$$P(x) = \max\left(\Psi(x, \bar{u}), P_A\right) \tag{9}$$

- Given the Result on Ψ (ie. the Alonso-Muth condition), and the flatness of P_A we get

Gradient Number 3:

The Land Price function as the upper envelope of consumers' bid rent and the agricultural land price is non-increasing in x.

Comparative Statics for $\bar{x}$

- increasing in N: higher demand for housing
- decreasing in $\tau'(x)$: it becomes costlier to be further away.
- increasing in weight of h in utility function: given prices are lower further away, consumers are willing to move out further to enjoy h
- increasing with wage at CBD
- decreasing in farmer's income.

Population Density

- Let $n(x)$ be the densitiy of consumers at $x$and let's define total city population as

$$N = \int_0^{\bar{x}} n(x) dx$$

- Normalizing the amount of housing at each x to $\bar{H} = 1$, we get

$$n(x) h^*(x, \bar{u}) = 1 = \bar{H}$$

- Then, since we know that h^* is increasing in distance, we get:

Gradient Number 4:

Population Density is decreasing in distance from the CBD.

Different City Configurations

- We can have open and closed cities, and resident or absentee landlords.
- Closed: population is given.
- Open: There are several cities, utility is assumed the same everywhere, and population sizes are endogenous.
- absentee landlords: Land revenue disappears.
- Resident landlords: doesn't disappear.

Supply of Housing

- Assume a neoclassical housing production function $H(K, L)$: capital and land.
- We assume that the parcel of land is given to the developer.
  - in intensive form: $S \equiv \frac{K}{L}, h(S) = H(L, K)/L$
  - S is capital per unit of land, i.e. density of structure, or how much floorspace per m^2 of lot size.
- Consumers:
  - Now bid for price *per unit of housing services* h.
  - Has the same properties as $P(x)$, but we will call it $P^h(x)$
- Developers
  - buy land at the land price $P(x)$ per unit of L
  - buy capital K at price r
  - build the house.
  - sell $h(s)$ units of housing space at price $P^h(x)$ to consumers.

Optimal Supply of housing

- Developers maximize profit at location x

$$\begin{aligned} \Pi &= \left(P^h(x)h(S) - rS - P(x)\right) L \\ \pi = \frac{\Pi}{L} &= P^h(x)h(S) - rS - P(x) \end{aligned}$$

- First order condition for S:

$$P^h(x)h'(S) = r$$

- Zero profit condition per unit of land:

$$P(x)^h h(S) = rS + P(x)$$

Supply of Housing

- Total differential of FOC wrt x is

$$\begin{aligned}
\frac{\partial h'(S(x))}{\partial S(x)}\frac{dS(x)}{dx} + \frac{\partial P^h(x)}{\partial x} &= 0 \\
S'(x) &= -\frac{\partial P^h(x)}{\partial x}\frac{1}{h''(S)} < 0
\end{aligned}$$

Gradient number 5:

capital intensity (building height) decreases with distance from the CDB.

Different Income Groups

- Suppose there are high and low income groups $w_2 > w_1$ in the city, with $\bar{u}_2 > \bar{u}_1$
- (5) and (6): clear that higher w means higher bid (if housing is a normal good.)
- So, we have that $h_2(x) > h_1(x), \forall x$
- But, by Alonso-Muth (6), this implies

$$-\frac{\tau}{h_2(x)} > -\frac{\tau}{h_1(x)} \Leftrightarrow \frac{d\Psi(x,\bar{u}_2)}{dx} > \frac{d\Psi(x,\bar{u}_1)}{dx}$$

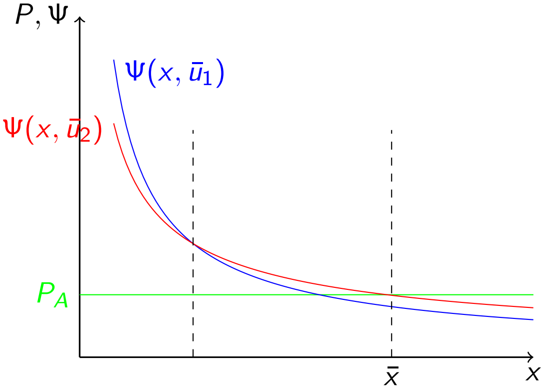
- (Note that $\frac{d\Psi(x,\bar{u})}{dx} < 0$ in general, so this means that $\Psi(x,\bar{u}_1)$ has a steeper descent)

Different Income Groups

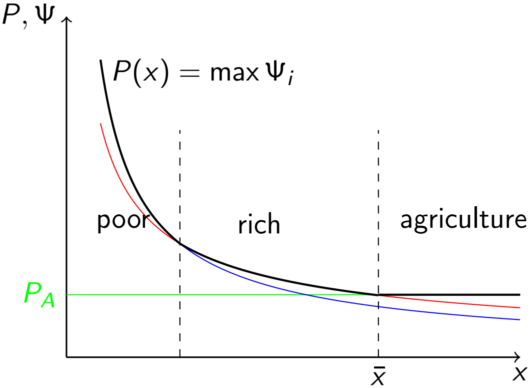
Given Single Crossing of Bid rents:

If housing is a normal good (it's budget share increases with income), poorer residents will locate closer to the CBD, richer ones further away. There is perfect separation between both groups.

2 Income Groups



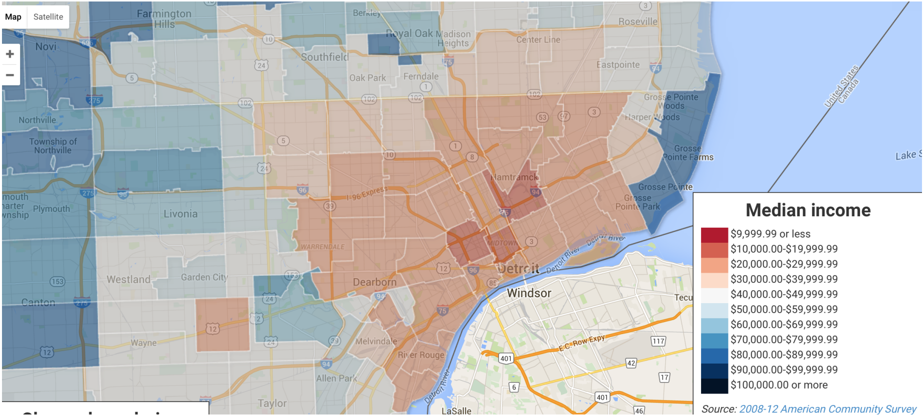
2 Income Groups



Social Stratification

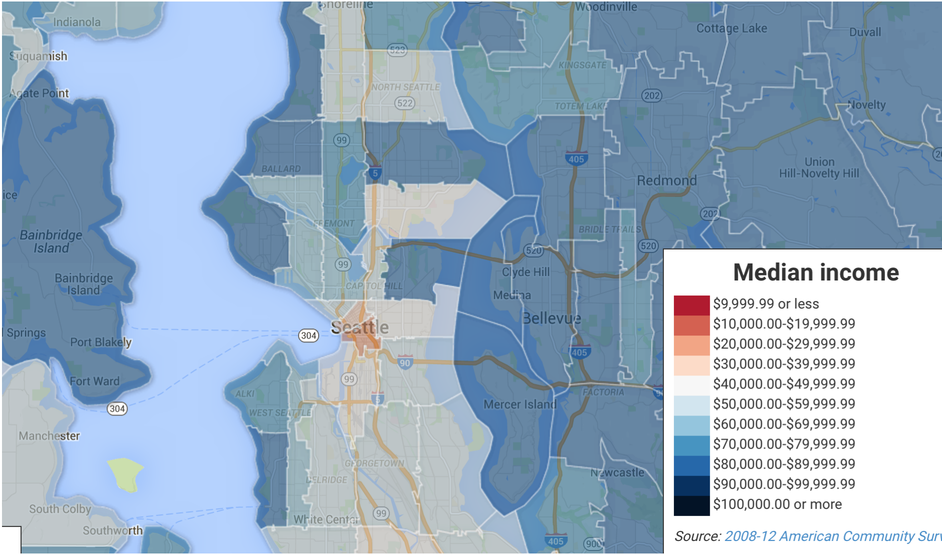
- The previous result gives strong predictions about which type of consumer lives where in the city.
- We found people getting richer as distance increaes.
- For many US cities, this works well
  - pictures from `http://www.richblockspoorblocks.com`
- Not so well for many European cities
  - Paris: `https://upload.wikimedia.org/wikipedia/commons/7/70/Jms_pc_median_income_2010.png`
  - London, interactive: `http://data.london.gov.uk/apps/ons-small-area-income-estimates/`

Detroit



Source: 2008-12 American Community Survey

Seattle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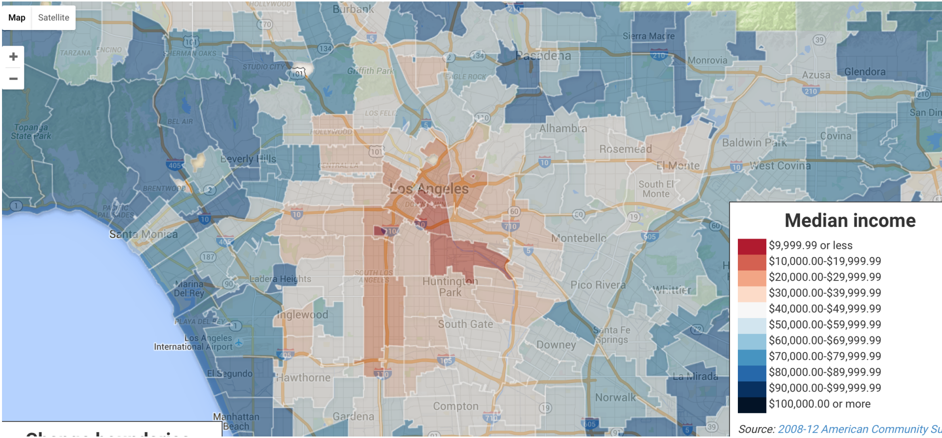
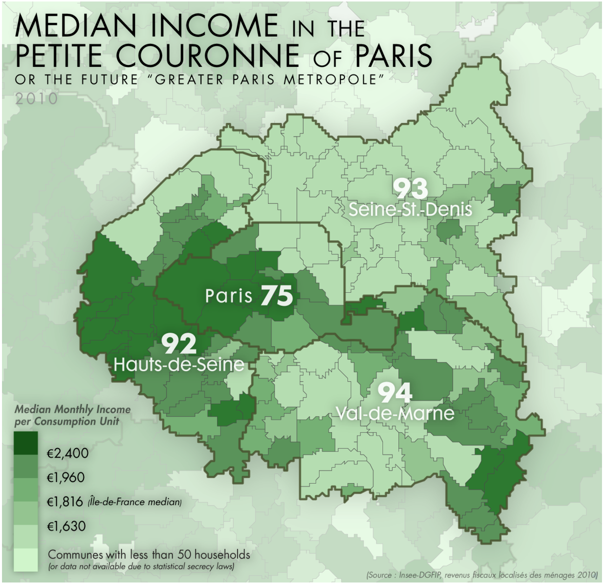
Suquamish
Agate Point
Bainbridge Island
Bainbridge Island
Springs
Port Blakely
Fort Ward
Manchester
South Colby
Southworth
Shoreline
NORTH SEATTLE
BALLARD
FREMONT
CAPITOL HILL
Seattle
ALKI
WEST SEATTLE
GEORGETOWN
DELRIDGE
White Center
Woodinville
Cottage Lake
Duvall
KINGSGATE
TOTEM LAKE
Novelty
Union Hill-Novelty Hill
Redmond
BRIDLE TRAILS
Clyde Hill
Medina
Bellevue
FACTORIA
Mercer Island
Newcastle
Median income
$9,999.99 or less
$10,000.00-$19,999.99
$20,000.00-$29,999.99
$30,000.00-$39,999.99
$40,000.00-$49,999.99
$50,000.00-$59,999.99
$60,000.00-$69,999.99
$70,000.00-$79,999.99
$80,000.00-$89,999.99
$90,000.00-$99,999.99
$100,000.00 or more
Source: 2008-12 American Community Surv

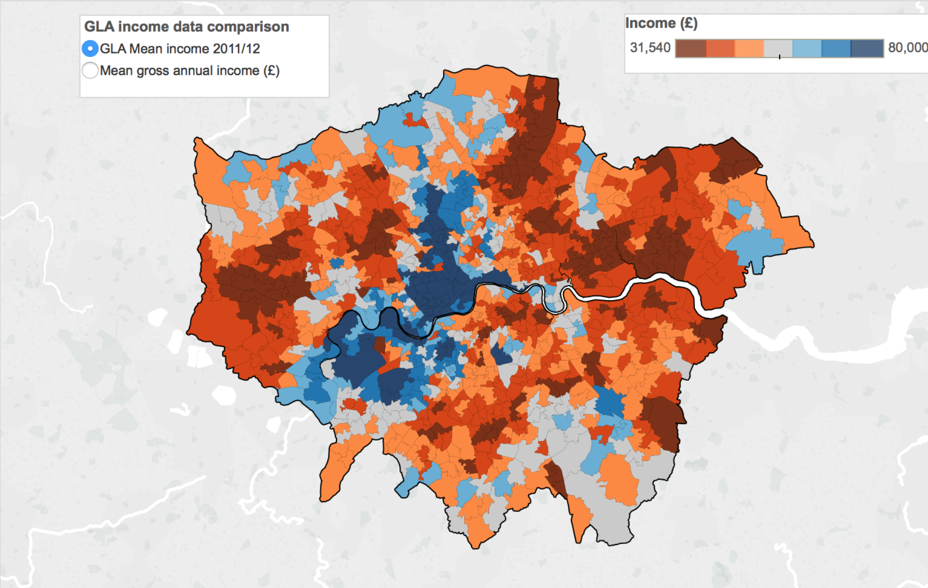
Los Angeles


Map
Satellite
Burbank
Sierra Madre
Pasadena
Monrovia
Azusa
Glendora
Tarzana
Encino
Sherman Oaks
Studio City
Griffith Park
Eagle Rock
Los Feliz
Hollywood
Alhambra
Rosemead
El Monte
Baldwin Park
Covina
West Covina
Topanga State Park
Bel Air
Beverly Hills
Central LA
South El Monte
Brentwood
Los Angeles
Downtown
Pacific Palisades
Santa Monica
Montebello
Marina Del Rey
Ladera Heights
South Los Angeles
Huntington Park
Pico Rivera
Whittier
Inglewood
South Gate
Playa Del Rey
Los Angeles International Airport
Downey
Santa Fe Springs
El Segundo
Hawthorne
La Mirada
Norwalk
Compton
Manhattan Beach
Gardena
Median income
$9,999.99 or less
$10,000.00-$19,999.99
$20,000.00-$29,999.99
$30,000.00-$39,999.99
$40,000.00-$49,999.99
$50,000.00-$59,999.99
$60,000.00-$69,999.99
$70,000.00-$79,999.99
$80,000.00-$89,999.99
$90,000.00-$99,999.99
$100,000.00 or more
Source: 2008-12 American Community Su

Paris


MEDIAN INCOME IN THE
PETITE COURONNE OF PARIS
OR THE FUTURE "GREATER PARIS METROPOLE"
2010
93
Seine-St.-Denis
Paris 75
92
Hauts-de-Seine
94
Val-de-Marne
Median Monthly Income
per Consumption Unit
€2,400
€1,960
€1,816 (Île-de-France median)
€1,630
Communes with less than 50 households
(or data not available due to statistical secrecy laws)
(Source : Insee-DGFIP, revenus fiscaux localisés des ménages 2010)

London


GLA income data comparison
GLA Mean income 2011/12
Mean gross annual income (£)
Income (£)
31,540
80,000

Amenity Based Theory

- [Brueckner et al.(1999)Brueckner, Thisse, and Zenou] propose an amenity based theory
- Assume there is an *amenity index* $a(x)$ that everyone agrees on.
- $a(x)$ is how *cool* the area around x is.
- If the weight of amenity in the utility is sufficiently high, rich consumers will outbid poor consumers where $a(x)$ is high.
- In many European cities with historical centres, $a(x)$ is high in the centre.

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