

Amadae Hawk-Dove Binary model of systemic discrimination

INTRODUCTION

The Amadae Hawk-Dove Binary (HDB) model of discrimination uses game theoretic agent-based modeling to study how localized individual choices result in overarching social patterns. This model provides a minimalist account of the sufficient conditions to yield a systemic pattern of discrimination. It uses a Hawk Dove game with the added feature that a binary marker is applied to otherwise homogenous agents throughout a population to create two groups, with members of one group with an arbitrary tag (such as Red), and members of the second group with an arbitrary tag (such as Blue).

In this agent-based HDB model, give the introduction of the binary tag throughout the population, even though proportions of the two group sizes do not change, still the symmetry of the Mixed Strategy Nash Equilibrium (MSNE) play is broken, and one of the two pure Nash equilibria arise. HDB shows that in contexts represented by a Hawk Dove game, in which competition over scarce resources involves costly conflict, that introducing a binary tag is sufficient to create an overarching pattern of hierarchy between members of the two groups. See also Cailin O'Connor for a similar analysis of the Nash Mini Demand game (2017, 2019).

	Dove	Hawk	
Dove	$V/2, V/2$	$0, V$	$V = \text{Reward}$ $C = \text{Costs}$
Hawk	$V, 0$	$(V-C)/2, (V-C)/2$	

MSNE*: $V/C = \text{probability Hawk play}$; $(1-V/C) = \text{probability Dove play}$

*MSNE: Mixed Strategy Nash Equilibrium

Simulation operating instructions

HDB is set up to run on default settings, which can be changed by the user. The payoff matrix is calculated according to the user's input. Either using the default settings, or entering new values, the user can then run the simulation.

The user can enter values for:

- Number of rounds for Stage 1
- Number of rounds for Stage 2

- Agent count
- Red agents (%)
- Reward V (greater than 0)
- Cost C (0 or higher)
- Mode (initialization rule for first encounter with other-colored player: MSNE; expect Dove, play Hawk; expect Hawk, play Dove; play Hawk with probability of 0.5 and Dove otherwise).

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🔒 flowa.github.io/amadae-hawk-dove-binary-model/

Settings

Duration

Stage 1

Mode: Color-blind play

Play Mixed Strategy Nash Equilibrium ▾

Rounds

0

Stage 2

Mode: Color-reactive play w/1st decision rule

Play Hawk with p = 0.5, Dove otherwise ▾

Rounds

100

Agents

Agent count

200

Red agents (%)

5

Count

Red

Blue

10

190

Payoff

Reward (V)

10

Cost (C)

20

	Hawk	Dove
Hawk	-5 / -5	10 / 0
Dove	0 / 10	5 / 5

Authors

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Results

Run simulation

Description of the model

This computer simulation is of a multi-agent Hawk-Dove game played for a given number of rounds in which agents are randomly paired during each round. The model has two stages, Stage

1 and Stage 2, each of a number of rounds R specified by the user. In both stages the population number is input by the user, and a number of 100-150 is recommended for results given the need to have a sufficiently large population, but not to over-burden computational efficiency. The Hawk-Dove payoff matrix is specified above with the variables V for reward and C for cost. In the model V has a default value of 10, and C is variable according to user input with a default of 5. The purpose of the model is to demonstrate the impact of introducing a binary marker, here Red and Blue, tagging every player, on the progression of play. The proportion of Red and Blue players is input by the user. Note that for populations with very small minorities, play takes extensive numbers of rounds (possibly 500-2000) to complete the impact of introducing a binary tag into a population of otherwise homogenous actors.

The variables running this model are:

N : Number of players, each labeled with a unique number ranging from 1 to N , enter an even number so pairings are complete.

I : Every individual is each labeled $I_1, I_2, I_3 \dots I_x \dots$ to I_N . (The variable x denotes a player's number).

R' and R'' : Number of rounds, here designated as R' for Stage 1 and R'' for Stage 2. Each round is numbered from $R'_1, R'_2, R'_3 \dots R'_y \dots R'_{R'}$ in Stage 1, and $R''_1, R''_2, R''_3 \dots R''_y \dots R''_{R''}$ in Stage 2. (The variable y denotes the number of the round). The total number of rounds for Stage 1 and Stage 2 is input by the user, and can be 0. Each round is visible by manipulating the sliding bar on the top of the simulation data outcome panel. Clicking in the left-corner of the sliding bar enables viewing sequential play as an animation.

V : Reward, input by user, value must be greater than 0, default value is 10

C : Costs, specified by user, value must be 0 or greater, default is 20

P_R and P_B : Proportion of agents; P_R is the proportion of Red agents, input by user as a defined percentage of actors; $(1-P_R)$ is the proportion of Blue agents, P_B . Each individual in the population I_1 through I_N is randomly assigned a Red or Blue designation to uphold the designated proportion.

H : Accumulated history of Hawk and Dove play by other-type actors; H_H is the % of Hawk play experienced by an agent, and $H_D = (1-H_H)$ is the % of Dove play experienced by an agent, in all previous rounds when encountering other actors. H_D and H_H are unique for every player I_x in every round R''_1 to $R''_{R''}$.

Stage 1 of the model anticipates that each player is tagged as either Red or Blue, but no agent acts on this information; all actors are colorblind. Stage 1 simulates the multi-agent N -player Hawk Dove game in which in every round of play, players encounter each other in randomized pairs. The program tabulates the cumulative and average score for each player. It anticipates their Red and Blue labels, and provides an average and cumulative score for Red-type and Blue-type actors. Each player is programmed to play the MSNE equilibrium of the game in

accordance with user's input of V and C : each plays Hawk (V/C) % of the time and Dove ($1 - V/C$)% of the time. The frequency of play is governed by shuffling a deck preassigned the appropriate ratio of Hawk versus Dove play, and correlating the values randomly to each player.

Stage 2 of the model displays the color labels in the proportion of Red and Blue input by the user. Stage 2 plays for the number of rounds (R'') input by the user. Stage 1 or Stage 2 can be played in isolation by using a value of 0 for the other stage. As in Stage 1, agents play successive rounds in randomized pairs. In the default setting “Play Mixed Strategy Nash Equilibrium,” players play the MSNE in their first encounter with outgroup players, regardless of which round this occurs in. Throughout Stage 2, players play MSNE with like-players, and cumulative scores are kept for all interactions.

Users can change the default initialization mode for Stage 2 and can select “Expect Dove” which entails Hawk play for first outgroup encounters with other colored agents; “Expect Hawk” which entails Dove play for first outgroup encounters; or “Play Hawk with $p = 0.5$, and Dove Otherwise.” Throughout the progression of play, in every round agents play MSNE when they encounter like-colored players.

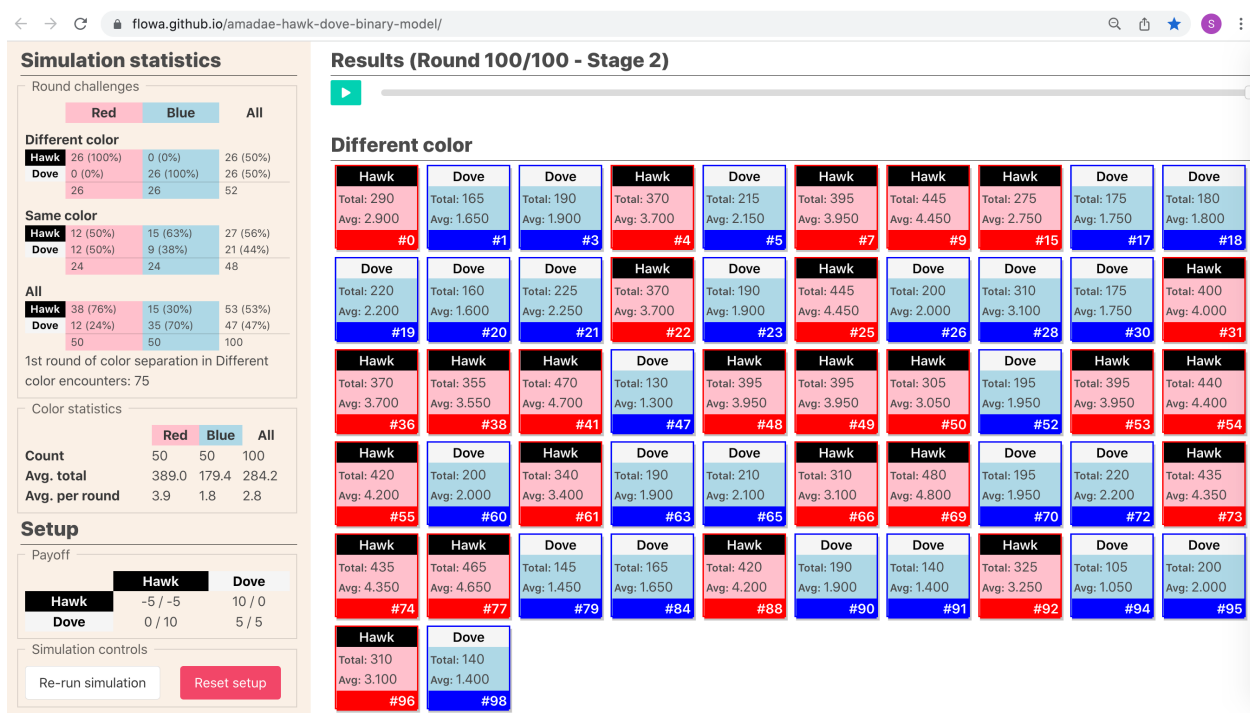
When agents encounter unlike players, after their first such encounter, they use an expected utility (EU) calculation based on their history of encounters with the other type actor, H_H and H_D . The EU calculation is completed for each round R''_1 to $R''_{R''}$ in Stage 2, with a unique calculation in each round for each player.

Calculation of EU for player I_x , round $R'' = y$ when playing an unlike player for each strategy choice of Dove and Hawk is a function of **each player's history of other-colored encounters specified for each round of play**:

$$EU(I_x, R''_y) (\text{Dove}) = (H_D * V/2) + (H_H * 0)$$

$$EU(I_x, R''_y) (\text{Hawk}) = (H_D * V) + (H_H * \{V-C\}/2)$$

In subsequent rounds of encounter paired with an unlike agent (after the first such round), in each round $R''_2 \dots R''_{R''}$, every agent I_x plays Hawk or Dove according to whether $EU I_x (\text{Hawk})$ or $EU I_x (\text{Dove})$ is higher, and randomizes between 50% Hawk and 50% Dove if they are equal. Note that in experimental trials (Amadae and Watts 2022), it made no appreciable difference if agents played MSNE instead of Hawk play with $p = 0.5$ and Dove play with $p = 0.5$ in the case of an expected utility tie.



Analysis

The Hawk-Dove Binary (HDB) model builds on a research using evolutionary games to provide the basis of this agent-based approach (Maynard Smith 1974). Evolutionary game theory demonstrates that a multi-agent Hawk Dove game played indefinitely among randomized pairs in a population of actors is in equilibrium when all actors play the Mixed Strategy Nash Equilibrium (MSNE). When an arbitrary binary marker is introduced throughout the population, one of the pure strategy Nash equilibriums results, with members of one group always playing Hawk and members of the other group always playing Dove, or vice versa. This model has been used by the evolutionary game theorist John Maynard Smith to provide a potential mechanism for the evolution of property rights (Smith 1982; Gintis 2007). New findings from HDB simulations show that majorities gain the advantage in high-cost games under neutral initialization conditions (a decision rule of playing the Mixed Strategy Nash Equilibrium in first encounters with outgroup members), and minorities gain the advantage in low-cost games under neutral initialization conditions (Amadae and Watts 2022).

The HDB model of systemic discrimination is simplified and abstract. Here the Hawk Dove game is applied to agent-based modeling rather than to evolutionary replicator dynamics. Its target is to explain a possible mechanism for the emergence and persistence of discriminatory conventions. This explanatory tactic resembles the Checkerboard model of segregation developed by James Sakoda and Thomas Schelling (Sakoda 1978; Rainer Hegselmann 2017; Schelling 1969; Schelling 1971; Aydinonat 2008) It also is similar to Robert Axelrod's analysis of the evolution of cooperation using the Prisoner's Dilemma (Robert Axelrod 1984). Social scientists use models to represent the empirical world in a simplified form (Sugden 2000; Ylikoski and Aydinonat 2014). Models can provide "how possibly" representations of how

outcomes arise from specified initial conditions and variables determining a system's evolution. Models are also useful for analyzing possible causal mechanisms determining outcomes.

HDB does not assume discriminatory attitudes, as for example racist or sexist inclinations. Hawk-Dove shows that in a population wherein all members are indistinguishable, relatively egalitarian distributional outcomes will result. However, in these repeating bargaining scenarios in which agents resort to costly conflict when they spar over scarce resources, if a binary tag is introduced an asymmetric equilibrium evolves. Resources can range from money and status to tangible goods and territory. Varying forms of HDB with differing population ratios and costs of conflict can be tested in empirical studies to see under what conditions the abstract model makes accurate predictions of agents' actions (see e.g. Hargreaves Heap and Varoufakis 2002).

HOW TO INSTALL THE MODEL

The online version is available in <https://flowa.github.io/amadae-hawk-dove-binary-model/>

If you wish to run the simulation with varying parameters, there is also command line version of the simulation. See installation instruction here: <https://github.com/flowa/amadae-hawk-dove-binary-model>

License

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References

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