Statistics on Wave Heights and Periods for the North Atlantic Ocean

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Abstract—This paper gives the frequency distributions of wave heights and wave periods obtained from weather ships stationed in the North Atlantic, together with an evaluation of the reliability of the visual wave-height estimates comprising the basic data from which the distributions are derived. Visual estimates are compared with values determined from stereo-photographs. An additional check is provided by wave-meter measurements. It is shown that a log-normal distribution is applicable to the frequency distribution of wave heights experienced over a typical year and that this distribution is a useful guide to the determination of the incidence of a particular sea state at a given location.

Data from weather ships—Wave heights and periods for ten ocean stations were obtained from U. S. Weather Bureau records. These data are visual estimates covering a period of six years and were made every three hours by trained weather observers. By international agreement, all ocean-station vessels use the same methods of estimating, as described by the *U. S. Weather Bureau* [1950; pp. 57–58]. Only one quantitative value for wave height and one for wave period were reported each time the sea was observed.

Visual estimates for correlation against stereo data—In order to evaluate the accuracy of the visual wave estimates and their usefulness in the statistical analyses, stereocameras were installed on one of the weather ships, the USCGC *Unimak* (WAVP 379), and on the USS *Valley Forge* (CVS 45).

Visual estimates of the sea surface were made by trained observers at the same times that stereo-photographs were taken. The *Unimak* estimates were made by regularly assigned Weather Bureau observers. The *Valley Forge* estimates were made by trained Navy aerological personnel; two or more of them made independent estimates twice each hour during most observation periods. By special arrangement, five or six of the Navy aerological personnel made independent wave estimates three times each day. Subsequently, the *Valley Forge* estimates were averaged and plotted against time of observation and a smooth curve was fitted to these points. From these curves, characteristic wave heights were obtained and are entered in Column 7 of Table 1.

The times shown in Columns 3 and 4 of Table 1 are local civil time values for the time zone in which each observation was taken. Each of the ship positions shown in Column 4 of Table 1 was obtained from the vessels' regularly developed navigation data; positions shown are those that were closest in time to each of the observations of wave data.

In making their observations, the Weather Bureau personnel on the *Unimak* followed the instructions of the *U. S. Weather Bureau* [1950], which specifies "larger, well-formed waves." The Navy observers were guided by the *U. S. Hydrographic Office* [1955], which calls for "higher, well-defined waves." It is apparent that the estimating procedures specified by these two different organizations are essentially the same.

Throughout this report the term 'characteristic height' will be used to denote *the average height of the higher, well-defined waves.* The statement italicized will serve as a definition of this quantity. The term 'significant wave height' will be used only in its statistical sense *as the average of the $\frac{1}{3}$ highest waves.* The 'characteristic wave period' is *the average period* of the higher well-defined waves.

Data from sea surface profiles—Of the many hundred stereophotograph pairs taken during the sea trials, 60 were selected for analysis to cover as wide a range of sea conditions as practicable. Each of these was converted into sea-surface profiles by photogrammetric specialists at the Naval Photographic Interpretation Center or Navy Hydrographic Office. The Wild A5 Autograph and Zeiss Stereo Planigraph Model C5 were used by the respective agencies, and vertical mapping techniques were adapted to this horizontal application. Next, the sea-surface profiles were analyzed by the Oceanographic Division of the Hydrographic Office. The accuracy of a wave profile varies with the distance from the camera to the profile. The average accuracy is about ± 0.5 ft at a distance of 2000 ft and is better than this at shorter distances.

The sea-surface profiles were used to determine the characteristic wave heights by following the same general procedure as that used by the shipboard observers in making their estimates; thus these two independent determinations of characteristic wave height should be comparable. These values are shown in Column 9 of Table 1.

Statistical background—Wave heights and wave periods from the weather-ship data for ten ocean stations are presented in the form of their distribution functions. For example, all wave heights reported by the shipboard observers are considered to be members of a statistical population of wave heights. The distribution function of wave heights indicates the relative probability of encountering a wave of a given height as a function of that height. Figure 1 illustrates this distribution function. The area under the curve to a value x_i is the integral of the function up to x_i and is equal to the fraction of all members of the population of wave heights which have a height less than x_i. Mathematically

$$P(x) = \int_0^x p\,dx \quad \text{and} \quad P(x \to \infty) = \int_0^\infty p\,dx = 1$$

where p is the probability density and P is a function of x designated as the cumulative distribution function of x. $P(x)$ is numerically equal to the probability that a value chosen at random from the population is less than x.

A detailed discussion of the statistical methods utilized in this report is given by *Jasper* [1956]. Only a few of the major concepts will be described here. The distribution applicable to a given sea condition is here called a 'short-term' distribution, whereas the distribution applicable when the sea conditions are allowed to vary over a wide range, such as over a year's time, is called a 'long-term' distribution. Thus the long-term distribution is the result of a summation of a number of short-term distributions. Oceanographers have held that the short-term distribution of wave heights x is approximately of the Rayleigh type (a narrow power-spectrum is assumed) for which

$$p(x) = \frac{2x}{E_i} e^{-x^2/E_i}$$

where E_i is the mean square of all the individual wave heights x corresponding to sea condition i. Note that numerically the value of E computed for wave height will be four times the value of E computed for wave amplitude because wave height is taken equal to twice the wave amplitude. See *Pierson* and others [1955] and *St. Denis* and *Pierson* [1953] for a discussion of the distribution of wave heights in terms of the power-spectrum concept. The short-term distribution is approximately valid if measurements are taken over a relatively short period of time, of the order of one hour, during which interval the sea conditions do not change appreciably. It can be shown that this distribution is the same as that representing the wave heights, in the area under consideration, at one instant of time.

Jasper [1956] suggested that the long-term distribution of wave heights and wave periods is of the log-normal type, that is, that the logarithms of these heights and periods are approximated by a normal distribution. Thus

$$p(\log x)d(\log x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\log x - u)^2/2\sigma^2} d(\log x)$$

$$P(x) = P(\log x) = \int_{\log x = -\infty}^{\log x} p(\log x)d(\log x)$$

where $p(\log x)$ is the probability density of the variate, $\log x$, u is the mean value of $\log x$, and σ^2 is the variance of $\log x$. Then the parameters u and σ define this distribution completely.

In this report log-normal distributions are fitted to the characteristic wave heights and periods reported by the weather ships. The resultant graphs give the probability with which a given value of the variate x will or will not be exceeded in an average year. Although the distributions given here are for a six-year period, study of distributions for individual years making up the six-year period indicates that a single year gives a

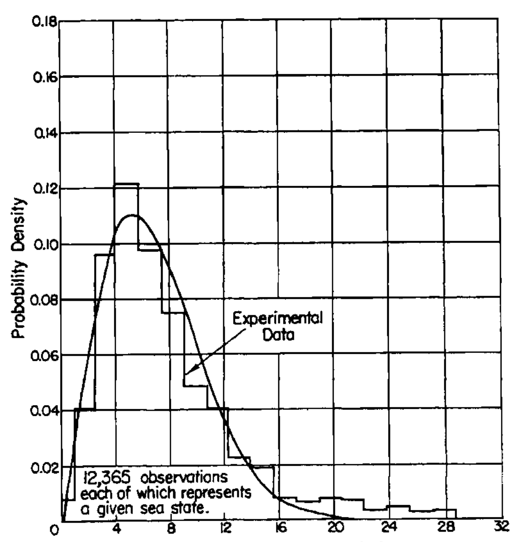

FIG. 1 – Distribution function of wave heights

TABLE 1 – *Correlation of wave data with ship positions*

Ship	Stereophotographs: Date	Stereophotographs: Time of photo	Stereophotographs: Position: Time	Position: Lat. N	Position: Long. W	Stereophoto and sea surface profile number	Visual observations made aboard ship: Number of estimators	Visual: Characteristic wave height (Average of larger waves): Sea	Visual: Characteristic wave height: Swell	DTMB stereoanalysis: Number of largest waves N: Sea	DTMB: Number of largest waves N: Swell	DTMB: Characteristic wave height (Average of larger waves): Sea	DTMB: Characteristic wave height: Swell	USN Hydrographic Office stereoanalysis: Number of waves in sample	USN: Cutoff wave length	USN: Significant wave height	USN: E^a (estimated from wave height)	USN: E^a (estimated from wind data)
1955		h m	h m	°	°			ft	ft			ft	ft		ft	ft	ft²	ft²
CVS 45	Sep. 11	14 35	12 00	40	58	A-0119	3	5.0	...	12	12	5.4	6.5	200	23	3.6	6.6	13.6
	Sep. 12	17 01	20 00	40	51	A-0200	4	4.8	...	11	7	4.4	6.9	197	18	4.2	8.3	9.2
	Sep. 20	11 13	12 00	45	17	B-0163	4	...	9.3	11	15	6.1	11.6	201	42	7.2	25.6	20.0
	Sep. 20	12 02	12 00	45	17	B-0165	3	...	8.0	12	6	9.6	10.0	199	33	6.8	23.6	32.0
	Sep. 15	09 25	08 00	43	37	B-0218	2	7.2	...	25	12	6.3	5.9	200	32	4.1	14.3	24.0
	Sep. 22	14 34	12 00	48	8	C-0233	3	...	10.0	11	7	5.9	11.2	198	42	6.1	17.0	28.0
	Sep. 28	17 15	20 00	42	13	C-0418	4	...	9.0	11	14	5.5	9.0	200	..	5.3	12.5	24.0
	Sep. 30	11 00	12 00	40	15	C-0452	3	...	8.0	12	21	6.6	8.6	182	30	5.9	17.2	32.0
	Sep. 30	14 17	12 00	40	15	C-0466	3	...	7.7	12	12	8.1	8.2	161	34	6.0	17.7	24.0
	Sep. 30	15 15	12 00	40	15	C-0470	3	...	7.8	12	15	9.1	7.5	209	..	4.4	8.9	24.0
	Oct. 1	12 45	12 00	43	13	C-0495	6	...	9.0	12	16	5.8	7.0	196	31	4.2	10.7	28.0
	Oct. 1	14 15	12 00	43	13	C-0500	3	...	8.8	10	11	6.3	8.0	110	34	6.0	14.3	32.0
	Oct. 2	13 00	12 00	39	13	D-0017	3	...	4.4	12	8	2.8	4.5	177	20	3.4	5.1	9.2
	Oct. 11	09 48	08 00	37	15	D-0085	5	...	6.2	15	11	4.2	5.4	208	21	3.7	6.1	10.4
	Oct. 11	16 16	20 00	37	18	D-0122	7	...	8.0	16	8	6.2	6.5	241	20	4.8	9.9	12.0
	Oct. 12	08 12	08 00	36	21	D-0148	6	...	5.5	14	5	8.5	5.4	223	..	6.7	19.6	18.0
	Oct. 12	09 46	08 00	36	21	D-0159	5	...	5.0	13	10	7.1	6.7	184	20	5.6	13.6	9.2
	Oct. 12	16 00	20 00	35	22	D-0199	3	...	6.9	10	14	6.5	6.3	190	..	4.3	8.9	16.0
	Oct. 13	10 46	08 00	35	26	E-0017	5	...	4.0	10	8	2.2	3.5	184	..	2.1	2.1	5.2
	Oct. 13	12 15	12 00	35	27	E-0021	7	...	3.7	12	7	5.6	4.5	200	..	3.3	5.3	5.2
	Oct. 13	16 46	20 00	35	29	E-0034	7	...	2.3	13	..	5.0	...	148	30	4.5	8.9	5.2
	Oct. 14	13 15	12 00	34	34	E-0062	3	...	6.1	10	5	7.2	7.1	200	26	5.2	11.8	16.0
	Oct. 14	13 45	12 00	34	34	E-0064	3	...	6.1	15	12	6.9	6.9	199	26	5.9	15.8	16.0
	Oct. 14	14 15	12 00	34	34	E-0068	4	4.6	6.0	..	..	...	...	180	33	6.0	17.0	16.0
	Oct. 14	14 45	12 00	34	34	E-0071	4	...	6.0	10	14	6.9	5.9	178	28	3.4	9.3	18.0

	Oct. 14	15 15	20 00	34	35	E-0074	3	...	5.9	17	12	7.3	6.5	200	..	4.9	11.3	18.0
	Oct. 16	06 45	08 00	34	45	E-0145	1	...	10.3	13	9	8.0	10.4	148	42	5.4	18.4	24.0
	Oct. 17	09 45	08 00	35	57	F-0222	4	...	7.3	16	10	4.1	5.7	200	..	5.2	7.1	20.0
	Oct. 18	12 45	12 00	35	59	F-0289	4	...	6.1	15	..	5.6	...	200	26	3.3	6.8	16.0
	Oct. 19	10 15	12 00	35	67	F-0344	4	4.8	...	12	9	5.8	5.1	176	25	5.0	11.6	13.6
	Oct. 19	14 45	12 00	35	67	F-0362	4	...	6.6	11	3	9.2	6.3	166	30	5.9	15.9	20.0
	Oct. 19	15 51	20 00	35	68	F-0365	1	7.7	...	13	4	8.2	11.0	150	26	6.0	15.3	20.0
	Oct. 20	17 15	20 00	35	71	G-0049	5	...	3.7	15	7	5.0	5.0	163	23	4.4	9.1	8.0
	Oct. 22	07 15	08 00	37	71	G-0113	4	...	5.9	14	11	5.7	5.1	200	26	4.8	10.4	20.0
	Oct. 22	12 15	12 00	36	72	G-0127	6	...	6.6	15	10	5.5	6.7	63	40	6.1	19.8	16.8
	Oct. 22	16 17	20 00	37	73	G-0150	2	...	6.5	14	8	4.2	7.1	200	..	4.1	8.1	18.4
	Nov. 15	07 50	08 00	37	73	H-0047	2	...	3.7	12	9	3.6	3.9	200	..	3.2	5.0	5.2
	Nov. 15	09 50	08 00	37	73	H-0060	2	...	2.7	14	10	3.6	3.7	181	20	3.3	4.6	9.2
	Nov. 15	10 55	12 00	38	73	H-0078	4	...	2.7	14	..	3.8	...	162	20	2.5	3.0	9.2
	Nov. 15	11 15	12 00	38	73	H-0087	1	...	2.7	14	10	2.9	3.7	200	..	2.8	3.5	9.2
	Nov. 15	11 55	12 00	38	73	H-0100	3	...	2.8	14	12	3.5	4.5	200	20	3.4	5.2	9.2
	Nov. 16	07 15	08 00	37	72	H-0155	6	...	3.6	14	15	6.0	5.3	188	20	5.1	12.0	9.2
	Nov. 16	12 55	12 00	37	72	H-0201	2	...	7.0	16	11	5.1	5.3	169	30	5.2	11.9	32.0
	Nov. 16	15 45	20 00	37	72	I-0014	3	...	7.5	16	13	6.4	7.9	148	32	6.9	22.0	30.0
	Nov. 17	08 15	08 00	37	74	I-0052	5	...	7.1	20	13	3.8	6.0	200	..	4.6	9.7	18.4
	Nov. 17	13 45	12 00	37	74	I-0073	2	...	4.3	17	12	4.3	6.0	200	..	4.6	10.3	10.4
	Dec. 9	15 30	12 00	36	70	J-0003	4	10.5	9.9	16	11	7.3	9.3	139	34	6.8	21.5	24.0
	Dec. 10	09 45	08 00	36	72	J-0035	3	12.5	12.7	18	9	15.3	12.2	144	38	8.1	29.4	40.0
	Dec. 10	10 45	12 00	36	72	J-0040	3	12.0	12.1	13	..	16.8	...	150	40	10.9	58.8	48.0
WAVP 379	Oct. 16[b]	16 29	15 00	44	41	U-359	1	13.0	...	14	..	11.6	...	146	..	9.4	...	...
	Oct. 22[b]	13 18	12 00	44	41	U-415	1	5.0	...	11	..	7.1	...	31	..	5.5	...	...
	Jan. 16	12 40	12 00	57	51	U-505	1	14.0	...	10	..	14.7	...	91	..	8.1	...	...
	Jan. 16	16 28	15 00	57	51	U-517	2	19.5	...	10	..	21.6	...	87	..	9.0	...	...
	Jan. 28	15 16	15 00	57	51	U-525	2	5.5	...	20	..	6.5	...	90, 66 } 156	..	5.1, 6.9 } 6.2	...	...
	Jan. 28	15 36	15 00	57	51	U-526	2	5.5	...	8	..	8.4	...		..		...	...
	Jan. 31	12 26	12 00	57	51	U-537	1	18.0	...	7	..	15.8	...	37	..	8.2	...	...
	Feb. 1	12 20	12 00	57	51	U-540	1	16.0	...	12	..	17.3	...	111	..	7.4	...	...
	Feb. 1	12 21	12 00	57	51	U-541	1	16.0	...	8	..	20.7	...	33 } 258	..	12.0 } 7.9	...	...
	Feb. 1	12 22	12 00	57	51	U-542	1	16.0	...	12	..	17.4	...	114	..	7.2	...	...
	Feb. 2	12 18	12 00	57	51	U-548	1	18.0	...	9	..	19.1	...	34	..	8.4	...	...

[a] E is the mean square value of wave heights.

[b] 1954, not 1955.

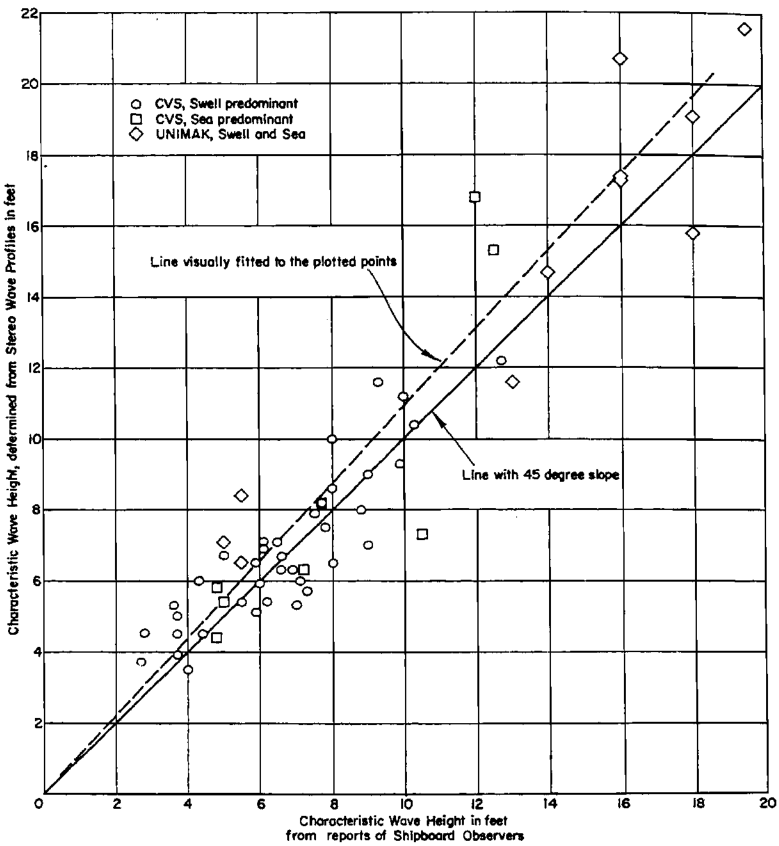

FIG. 2 – Scatter diagram for comparison of characteristic wave heights determined independently from the wave-profile analysis and by shipboard observers; data from Table 1, Columns 7 and 9

typical sample of the distribution obtained for many years. Therefore, the six-year distribution may be considered valid for an average year.

Evaluation of reliability of visual estimates of wave heights—The characteristic wave heights obtained by the shipboard observers are compared with those derived from the stereograms for the same sea condition in Figure 2, where values from Columns 7 and 9 of Table 1 are plotted as abscissas and ordinates, respectively. If exact agreement existed between visual estimates and the results of photogrammetric analysis, then all points would lie on a straight line with a 45° slope. The points plotted in Figure 2 scatter fairly well about a straight line which has a slope somewhat greater than 45°. The average deviation of the points from the line is expected to decrease as the number of points is increased. It should be noted that each stereophotograph covers a limited field of view compared with the field of view of the shipboard observer (Fig. 3).

It is considered that Figure 2 shows good correlation between the visual estimates and quantitative height determinations made from the stereophotographs. Individual estimates may not be accurate, but when the number of estimates is large the correlation is good.

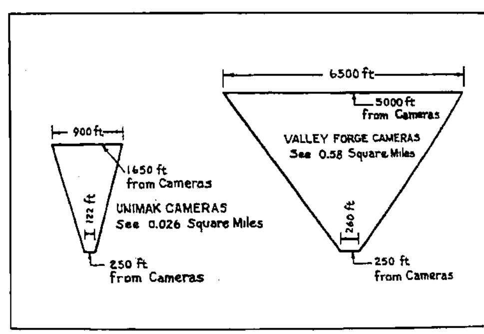

FIG. 3 – Ocean perspectives seen by stereocameras on *Unimak* and *Valley Forge* (areas indicated are fixed by properties of cameras, film, stereoplanigraphs, and camera separation).

The *Unimak* data and the *Valley Forge* data do not indicate divergent trends, that is, although the observers on the *Unimak* and the *Valley Forge* used slightly different criteria for observing waves, their results are substantially identical.

The visual observations on which the long-term distributions given later in this report are based, comprise between 11,000 and 18,000 separate observations for each ocean station. It is concluded

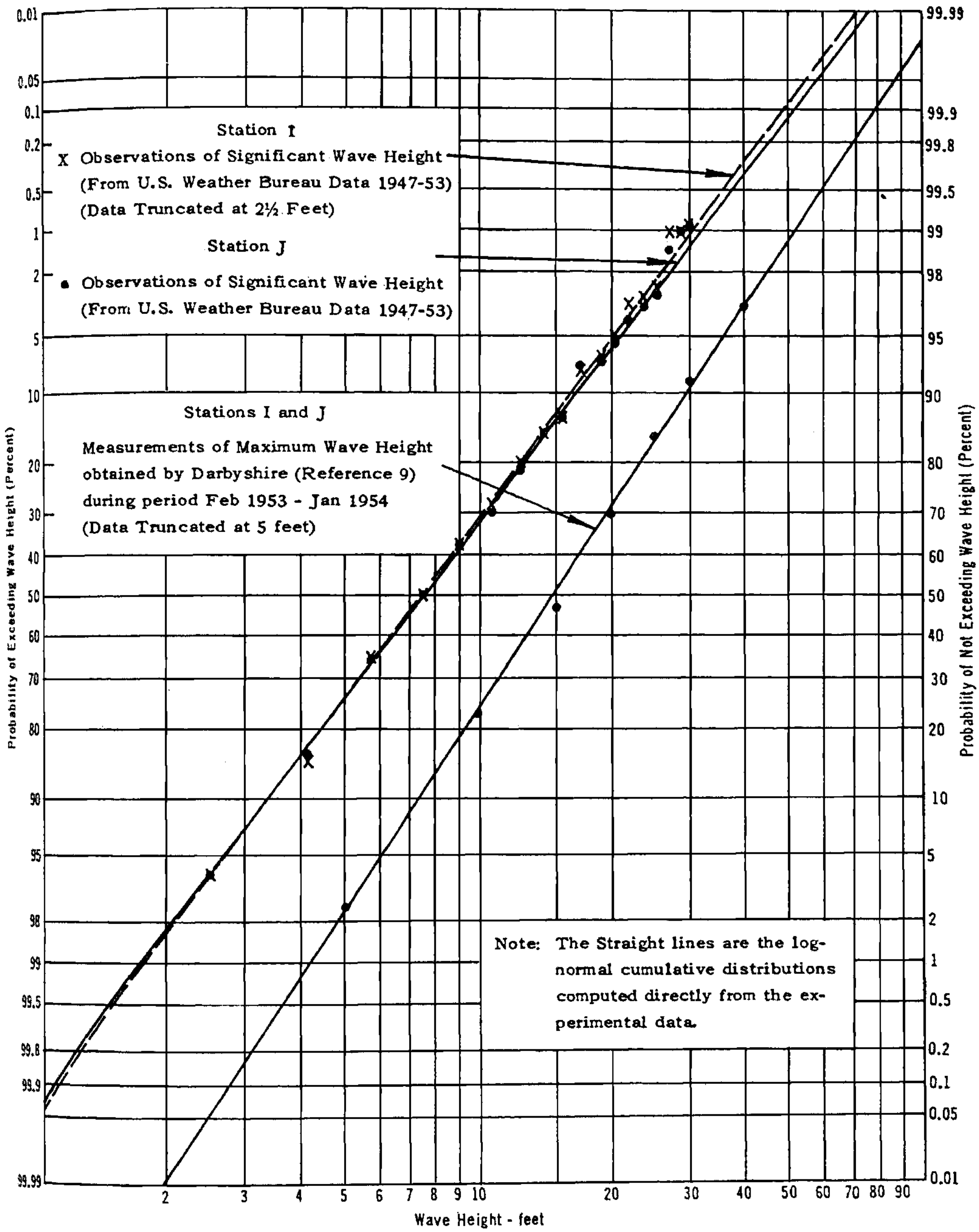

FIG. 4 – Comparison of wave-height distributions derived from visual observations and from measurements of wave heights at Atlantic Ocean Stations I and J; the distribution fitted to the Darbyshire data corresponds to a standard deviation of 0.57 for $\log_e$ (maximum wave height) and a median value of the maximum wave height equal to 15 ft.

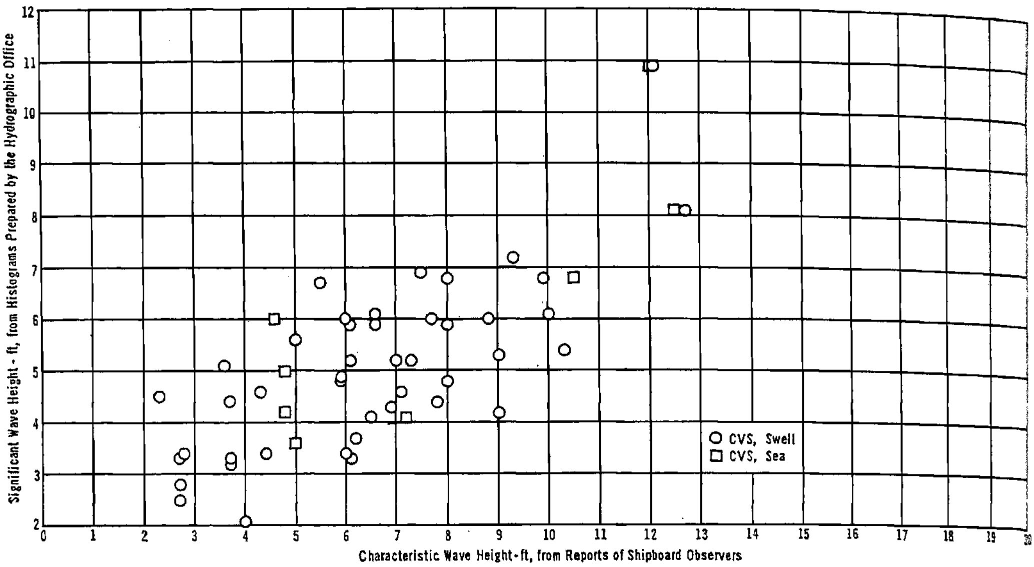

FIG. 5 – Scatter diagram showing a plot of significant wave height against characteristic wave height; data from Table 1, Columns 7 and 12

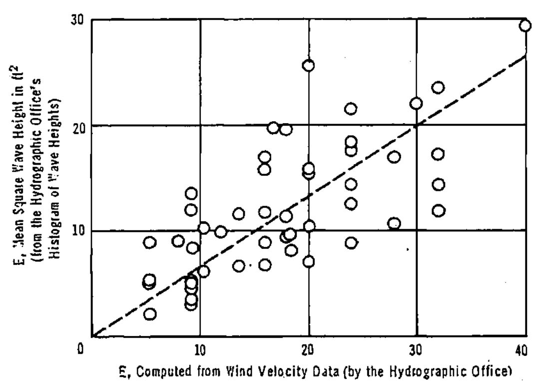

FIG. 6 – Scatter diagram for comparison of values of E derived from wind and wave data; each plotted point corresponds to the analysis of one stereophotograph; the computation of E from the wind data neglects the presence of decaying swell; the values plotted are taken from the last two columns of Table 1.

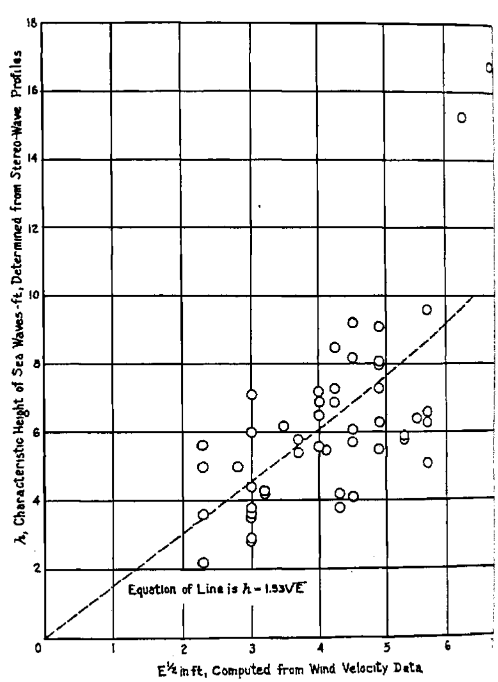

FIG. 7 – Scatter diagram showing a plot of characteristic wave height against the statistic $\sqrt{E}$; data from Table 1, Columns 9 and 14.

that the errors associated with the visual observations are fairly well averaged out when such a large number of observations is utilized to define the distribution and that the reported characteristic wave heights are therefore proportional to the severity of the sea.

Further evidence to support the validity of the visual observations can be drawn from an analysis of measurements of wave height made by *Darbyshire* [1955] by means of a wave meter installed on a weather ship. These measurements were made over a period of about one year, February 1953 to January 1954, at North Atlantic Weather Stations I and J (Fig. 4). Darbyshire reported the maximum wave height for each three-hour period for which visual wave observations were made while the ship was at sea. The visual observations made by weather observers are reported as the 'characteristic' wave height. In the discussion of Figure 5 in the following section, the characteristic height is indicated as being proportional to the significant height. It is of interest to compare the visual observations with the measurements obtained with the wave meter. If the hypothesis is accepted that the short-term distribution of wave height follows the Rayleigh distribution, then the maximum, significant, and characteristic wave heights for a given sea condition are related by constant factors. Thus, the long-term distributions of maximum and characteristic wave heights should be of the same type, log-normal in this case, and should differ only in their mean values. The standard deviation of $\log_e$ (characteristic wave height in feet) is 0.622 at Station J and 0.612 at Station I from visual observations, as compared with a value of 0.57 for $\log_e$ (maximum wave height) for the measurements at Station I and J reported by Darbyshire (Fig. 4). A log-normal distribution has been fitted to the wave-meter data on the assumption that the distribution of maximum wave heights is log normal. The experimental data indicate excellent agreement with the fitted distribution, well within the accuracy of the measurements. The latter fact, together with the good agreement between the standard deviations of characteristic (visual estimates) and maximum (measurements) wave heights, supports the hypothesis that the distribution of wave heights may be approximated by Rayleigh and log-normal distributions for the short term and for the long term, respectively. *Darbyshire* [1956] tests the applicability of the long-term log-normal distribution to extensive data on maximum wave heights obtained by use of the

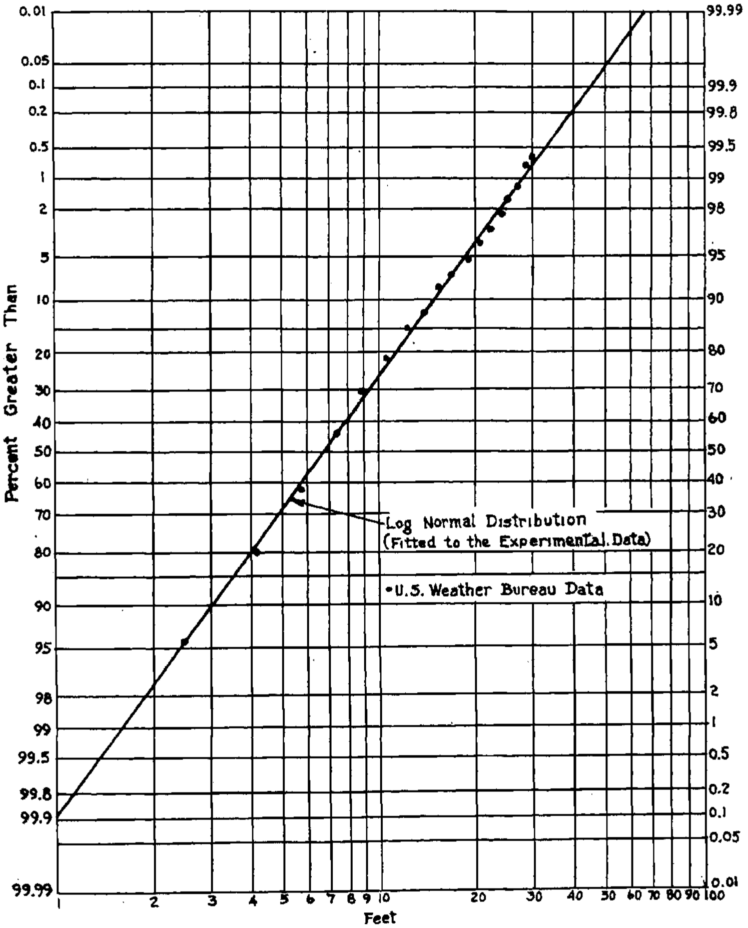

FIG. 8 - Distribution of characteristic wave heights at Station C

British wave meter. He concludes that the logarithmic law appears to be a useful guide to determine the incidence of a particular wave state at a given location.

It is concluded on the basis of the foregoing discussion that the visual estimates by weather observers of sea state, reported as a 'characteristic' wave height, may be used with confidence in establishing distribution patterns such as are given in a following section.

Comparison of wave statistics—It is of interest to utilize the wave data obtained during the sea tests of the *Valley Forge* and the *Unimak* to gain some insight into the validity of a few of the assumptions often made in the forecasting and analysis of ocean waves. For this purpose, the following items were computed:

(a) The characteristic wave height was determined from the wave profiles.

(b) The wave profiles were analyzed statistically to obtain the frequency distribution of individual wave heights and of the corresponding wave lengths above a certain cutoff length, which is that wave length below which lies three per cent of the area under the power spectrum.

(c) The mean value of the squares of all individual wave heights corresponding to waves longer than the cutoff length was determined from the data obtained under (b). These values are denoted by the symbol E, and are listed in Table 1.

(d) The average value of the upper third of the waves having the largest magnitudes (significant wave height) was determined from the data obtained under (b).

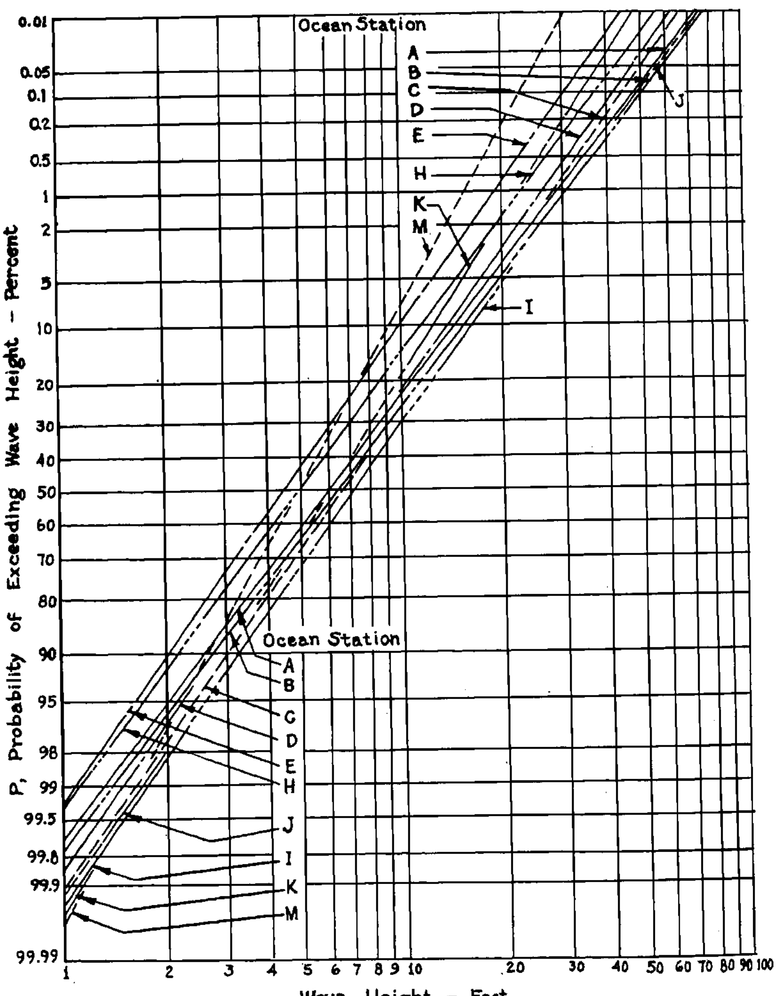

FIG. 9 – Distribution of characteristic wave heights for ten ocean stations

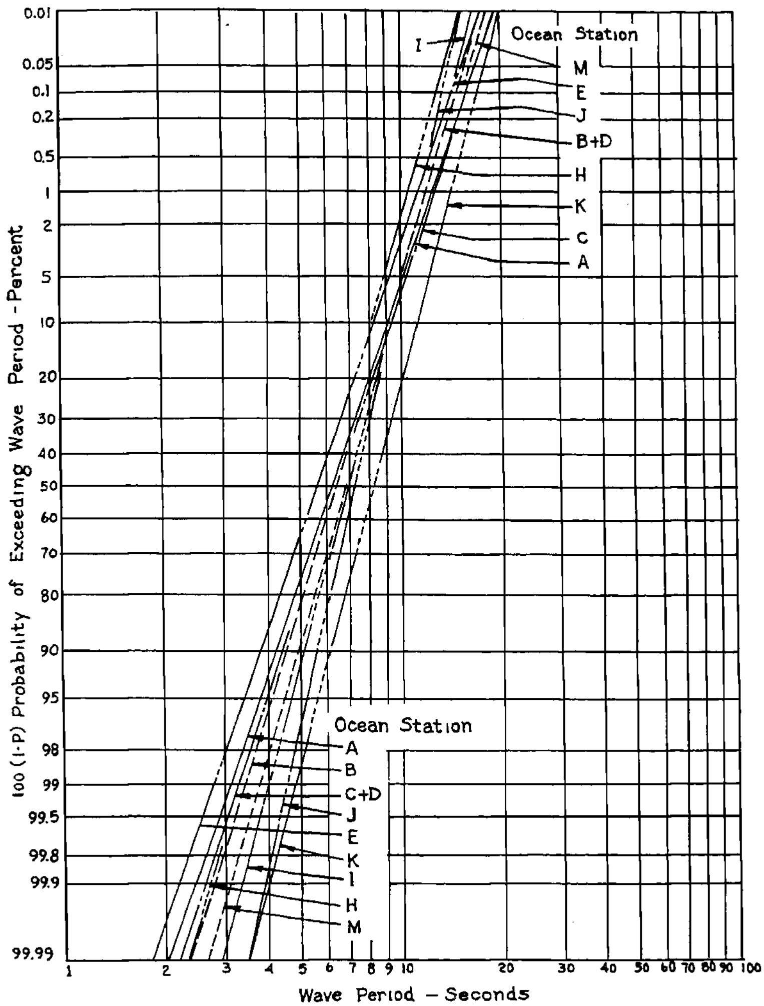

FIG. 10 – Distribution of characteristic wave periods for ten ocean stations

(e) The average value of the characteristic wave heights determined by the shipboard observers was tabulated.

(f) The Hydrographic Office computed a theoretical value of E on the basis of the distribution of wind velocities that generated the sea. This computation does not take account of swell that may have been present in the wave system. The method of *Pierson* and others [1955] was used, according to which E is proportional to the area under the power spectrum of the sea. These values are also given in Table 1.

In Figure 2, (a) is plotted against (e). In Figure 6, (c) is plotted against (f). In Figure 7, (a) is plotted against the square root of (f). In Figure 5, (d) is plotted against (e).

Figure 2 indicates that trained shipboard observers can, on the average, estimate the heights of the predominant waves reasonably well.

The value of E determined from the wind data should agree with the E obtained from the wave profiles provided the power-spectrum theory is valid, a narrow sea spectrum exists, swell is negligible, and the stereophotograph covered a representative area of the ocean. One may expect considerable deviations from these assumptions; for example, the sea surface profiles sometimes indicate considerable deviation from a narrow spectrum as well as the presence of swell. Nevertheless, Figure 6 suggests a linear relationship between the E's determined by two independent methods. It is concluded that the wind data may be used to determine the sea state, at least qualitatively.

Figure 7 suggests a linear relation between the characteristic wave heights h, as determined by the methods of *Darbyshire* [1956] and *U. S. Hydrographic Office* [1955] and the square root of E, except for very severe sea conditions.

Figure 5 would be expected to indicate a linear relationship between the significant wave height and the visual shipboard estimates of the characteristic wave height since the latter is presumably proportional to E in accordance with the indications of Figures 6 and 7. Figure 5 does not contradict such a linear relationship. The scatter of values is most likely due to errors in determination of the significant wave height, inasmuch as Figure 2 shows that the visual shipboard estimates are reasonably correct. The computed value of the significant wave height (d) is very much a function of the cutoff length. The *Unimak* stereophotographs did not furnish sufficient data, in the opinion of the Hydrographic Office oceanographers, to permit an evaluation of the significant wave height; and therefore these data were not available for the plots.

From an overall point of view, consideration of Figures 2, 5, 6, and 7 suggests that

(1) The methods of *Pierson* and others [1955] may be applied to make a rough estimate of wind waves.

(2) Trained observers can, on the average, make reasonably accurate observations of the heights of the larger, well-formed waves that are present in a given sea.

(3) The characteristic wave height reported by trained observers is proportional to the square root of the statistic E, corresponding to the sea state considered, except for severe sea states. The empirical relationship between the characteristic wave height h reported by the observers and the statistic $\sqrt{E}$, for the data plotted in Figures 6 and 7, is approximately as follows $h \approx 1.53\sqrt{E}$ when E is derived from wind data, and $h \approx 1.88\sqrt{E}$ when E is derived from the wave data.

(4) The so-called 'significant' wave height is not particularly significant since it is difficult to compute, although it is statistically well defined. The average height of the predominant wave heights, here designated by the term 'characteristic,' as reported by observers, is physically more meaningful and is more easily reproduced on repetitive estimates than is an estimate of the significant wave height.

Distribution patterns of wave heights and wave periods from analysis of visual observations—Cumulative long-term distribution patterns of the characteristic wave heights and periods were plotted as illustrated in Figure 8 for the ten ocean stations. Methods of fitting a log-normal distribution are given by *Hald* [1952, pp. 29, 34–35, 62–63]. The log-normal distributions from the data furnished by the Weather Bureau are shown in Figures 9 and 10.

TABLE 2 – *Statistics for log-normal distributions computed from wave observations made in North Atlantic Ocean*

Ocean station	Latitude North	Longitude West	Characteristic wave heights: Dates of observations From	To	Number of observations	Median value of characteristic wave height	Mean value of logarithm of characteristic wave height[a]	Variance (σ^2) of logarithm of characteristic wave height[a]	Characteristic wave periods: Periods of record From	To	Number of observations	Median value of characteristic wave period	Mean value of logarithm of characteristic wave period[a]	Variance (σ^2) of logarithm of characteristic wave period[a]
						ft						sec		
A	62°00′	33°00′	Jan. 49	June 54	12,891	6.34	1.847	0.4524	Jan. 49	June 54	12,342	6.30	1.840	0.0935
B	56 39	51 00	Jan. 49	Dec. 54	15,547	6.59	1.886	0.4434	Jan. 49	June 55	16,060	6.40	1.857	0.0748
C	52 45	35 30	Jan. 49	Dec. 54	16,857	6.75	1.910	0.3763	Jan. 49	June 55	17,471	6.48	1.869	0.0873
D	44 00	41 00	Jan. 49	Dec. 54	16,804	6.26	1.834	0.3843	Jan. 49	June 55	17,310	6.24	1.831	0.0800
E	35 00	48 00	Jan. 49	Dec. 54	16,777	4.56	1.516	0.3765	Jan. 49	June 55	16,896	5.86	1.768	0.0939
H	36 00	70 00	Jan. 49	June 54	14,607	5.08	1.625	0.4237	May 49	June 54	13,647	5.92	1.779	0.0643
I	61 00	15 20	Jan. 47	June 53	11,274	7.36	1.996	0.3747	Jan. 49	Dec. 51	12,142	7.02	1.948	0.0546
									Jan. 53	Dec. 54				
J	52 30	20 00	Jan. 47	June 53	12,016	7.40	2.002	0.3863	Jan. 49	Dec. 51	11,593	7.29	1.986	0.0423
									Jan. 53	Dec. 54				
K	45 00	16 00	Jan. 49	Dec. 53	11,182	6.20	1.824	0.3033	June 49	Dec. 53	11,906	8.30	2.117	0.0548
M	66 00	02 00	Jan. 49	Dec. 53	14,324	4.99	1.608	0.2344	Jan. 49	Dec. 53	14,188	6.99	1.946	0.0676

Note: The statistical computations are based on truncated data. The truncation point is 2.5 ft for wave heights and 5 sec for wave periods.

[a] Logarithms are to the base e.

The rather good fit of all computed lines to the plotted data suggests that a log-normal distribution is a good approximation to the distribution pattern of characteristic wave heights and periods for values above the truncation point.

Distribution patterns for wave length can be derived from the data for wave periods by applying an approximate conversion: Wave length = constant × (wave period)^2. For fully developed waves the constant is 3.4.

It is apparent that the distribution of wave lengths will be log-normal if that for the periods is log-normal, since the conversion involves only a change in mean value and slope from the distribution of the periods.

In Table 2 mean values and variances are given for the wave height and period data reported from each ocean station. Also the latitudes, longtitudes, and observation periods over which the data were collected are shown.

Summary—Frequency distribution patterns of wave heights and wave periods may be approximated by a one-parameter type of distribution function when the environmental conditions are steady, whereas they will tend to follow the two-parameter logarithmically normal distribution when the environmental conditions are allowed to vary over a wide range. It should be emphasized that the log-normal distribution illustrated in Figure 8 is influenced much more by the usual sea conditions than by the rare occurrences of very high or very low seas. Thus one should expect greater deviations from the fitted line for very small and very large wave heights and wave periods than for those heights and periods which occur more frequently. It is concluded that the long-term distributions of wave height and wave period may be approximated by the log-normal distribution.

Reasonably accurate visual estimates of wave height can be obtained from trained observers, provided a number of independent estimates are averaged. A single estimate may be considerably in error.

Acknowledgments—The research given in this paper was carried out by the David Taylor Model Basin. It would not have been possible without the cooperation of many persons on board the *Valley Forge* and *Unimak*, on the Coast Guard and Navy staffs, in the Weather Bureau, at the Hydrographic Office, at the Naval Photographic Interpretation Center, and at the Taylor Model Basin.

Wilbur Marks of the TMB staff assisted in the oceanographic planning and analysis. The camera installations were made by C. E. Lemich of the Taylor Model Basin. B. M. Wigle and R. J. Dominic, also of the Taylor Model Basin staff, sorted and matched the many photographs and assisted in analyzing the stereograms and in developing the plotted figures. The stereograms were developed by John Davis and John Boyle of the Hydrographic Office and Photographic Interpretation Center, respectively. Much encouragement has been given to the overall program, of which this study is a part, by John Vasta of the Bureau of Ships.

The paper was reviewed by E. H. Kennard, R. T. McGoldrick, and Mary C. Crook of the Taylor Model Basin staff. The authors wish to express their sincere appreciation to the persons mentioned here and to the many others whose assistance made this work possible.

The opinions or assertions in this paper are the private ones of the authors and are not to be construed as official or reflecting the views of the Navy Department.

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(Communicated manuscript received November 8, 1957, and, as revised, June 17, 1958.)